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Stability Analysis for Some Nonlinear Fifth-Order Equations

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Abstract. The main purpose of this work is to obtain many travelling wave solutions for general Kaup-Kuperschmidt (KK), general Lax, general Sawada-Kotera (SK) and general Ito equations with the aid of symbolic computation by employing the extended direct algebraic method. The stability of these solutions and wave motion have been analyzed by illustrative plots.

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1. Introduction

Life is a constant change in our world. The mathematical model of such changes are best expressed by differential equations. Therefore, even if we do not realize it, differential equations are appearing at every stage of life. So the solution of differential equations gives life better sense. Numerous methods have been developed to solve differential equations especially in the last thirty years. For example, Hirota's bilinear method , inverse scattering method, the tanh method, Backlund transformation, homogeneous balance method, the sine-cosine function method, the exp-function method, Jacobi elliptic function method, algebraic method, the (G'/G)-expansion method, the first integral method, the modified simple equation method, the auxiliary equation method, differential transform method, exponential rational function method, extended direct algebraic method, extended simple equation method, Khater method, so on [1–4, 7, 10–15, 17–21, 23, 25–38, 41, 42, 44, 46, 47].

Consider the family of fifth-order Korteweg-de Vries (fKdV) equation in its standard form as

$$u_t + \alpha u^2 u_x + \delta u_x u_{xx} + \gamma u u_{3x} + u_{5x} = 0, \tag{1.1}$$

where α, β and γ are arbitrary nonzero and real parameters, and $u = u(x,t)$ is a differentiable function. The fKdV equation (1.1) describes motions of long waves in shallow water under gravity and in one-dimensional nonlinear lattice and has wide applications in quantum mechanics and nonlinear optics.

The fKdV equation (1.1) involves two dispersive terms u_{3x} and u_{5x} . Since the parameters α, β and γ are arbitrary constants then, the small changes on these parameters drastically change the characteristics of the fKdV equation. Therefore, different forms of the equation can be obtained by changing these.

However, four well known forms of the fKdV that are of particular interest given by

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(i) The Kaup-Kuperschmidt (KK) equation reads

$$u_t + \frac{1}{5}\gamma^2 u^2 u_x + \frac{5}{2}\gamma u_x u_{xx} + \gamma u u_{3x} + u_{5x} = 0, \tag{1.2}$$

(ii) The Lax equation reads

$$u_t + \frac{3}{10}\gamma^2 u^2 u_x + 2\gamma u_x u_{xx} + \gamma u u_{3x} + u_{5x} = 0, \tag{1.3}$$

(iii) The Sawada-Kotera (SK) equation is given by

$$u_t + \frac{1}{5}\gamma^2 u^2 u_x + \gamma u_x u_{xx} + \gamma u u_{3x} + u_{5x} = 0, \tag{1.4}$$

(iv) The Ito equation is given as

$$u_t + \frac{2}{9}\gamma^2 u^2 u_x + 2\gamma u_x u_{xx} + \gamma u u_{3x} + u_{5x} = 0. \tag{1.5}$$

The first three equations KK, Lax and SK equations are completely integrable equations that have infinite sets of conserved quantities and give multiple soliton solutions. However, the Ito equation is not completely integrable but has a limited number of conserved quantities.

Exact solutions for several forms of Eq.(1.1) have been obtained by many researchers with various methods [5, 6, 9, 16, 22, 43, 45].

2. Analysis of The Extended Direct Algebraic Method

The following is a given nonlinear partial differential equations with two variables x and t,

$$P\left(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, ...\right) = 0 \tag{2.1}$$

P is a polynomial function with respect to the indicated variables or some functions which can be reduced to a polynomial function by using some transformations.

Step 1: Assume that Eq. (2.1) has the following formal solution as:

$$u\left(x, t\right) = u\left(\xi\right) = \sum_{i=0}^{m} a_i \vartheta^i\left(\xi\right) \tag{2.2}$$

where

$$\vartheta' = \sqrt{\alpha\vartheta^2 + \beta\vartheta^4} \quad \text{and} \qquad \xi = kx + \omega t, \tag{2.3}$$

where α, β are arbitrary constants and k, ω, are the wave length and frequency, respectively.

Step 2: Balancing the highest order derivative term and the highest order nonlinear term of Eq. (2.1), then the coefficients of series, α, β, a_0, $a_{1,}\ldots$, a_m, k, ω, can be determined.

Step 3: Substituting Eqs. (2.2) and (2.3) into Eq. (2.1) and collecting coefficients of $\vartheta^i\vartheta^{(i)}$ (which $\vartheta^{(i)}$ shows i^{th} derivative w.r.t. ξ), then setting coefficients equal zero, we will obtain a set of algebraic equations. By solving this system, the parameters α, β, a_0, $a_{1,}\ldots$, a_m, k, ω, can be determined.

Step 4: By substituting the parameters, α, β, a_0, $a_{1,}\ldots$, a_m, k, ω, and $\vartheta\left(\xi\right)$, into Eq. (2.2), the solutions of Eq. (2.1) can be easily obtained.

3. Stability Analysis

Hamiltonian system is a mathematical formalism to describe the evolution equations of a physical system. By using the form of a Hamiltonian system for which the momentum is given as

$$M = \frac{1}{2}\int_{-\infty}^{\infty} u^2 d\xi, \tag{3.1}$$

where M is the momentum, u is the travelling wave solutions in Eq. (2.1). The sufficient condition for soliton stability is

$$\frac{\partial M}{\partial \omega} > 0, \tag{3.2}$$

where ω is the frequency [8, 24, 39, 40].

4. Applications of The Extended Direct Algebraic Method for The Family of Fifth-Order Korteweg-de Vries Equation

In this section, we apply the extended direct algebraic method to find the travelling wave solutions for general KK, general Lax, general SK and general Ito equations.

4.1. The Extended Direct Algebraic Method for General KK equation. Consider the travelling wave solutions Eq. (2.2) and Eq. (2.3), then the general Kaup-Kuperschmidt (KK) equation, Eq. (1.2), becomes

$$\omega u' + \frac{1}{5}\gamma^2 k u^2 u' + \frac{5}{2}\gamma k^3 u' u'' + \gamma k^3 u u''' + k^5 u^{(5)} = 0. \tag{4.1}$$

Balancing the nonlinear uu''' and highest order derivative $u^{(5)}$ in Eq. (4.1) that gives $m = 2$. Since general KK, general Lax, general SK and general Ito equations, which are from the same category, then their solutions can be written as,

$$u(\xi) = a_0 + a_1\vartheta + a_2\vartheta^2. \tag{4.2}$$

By substituting Eq. (4.2) into Eq. (4.1) yields a set of algebraic equations for a_0, $a_{1,}q, s, \alpha, \beta, k, \omega$. The system of equations are found as

$$\begin{aligned}
k^5\alpha^2 a_1 + \omega a_1 + k^3\alpha\gamma a_0 a_1 + \frac{1}{5}k\gamma^2 a_0^2 a_1 &= 0,\\
2\omega a_2 + 8k^3\alpha\gamma a_0 a_2 + \frac{2}{5}k\gamma^2 a_0^2 a_2 + \frac{7}{2}k^3\alpha\gamma a_1^2 + \frac{2}{5}k\gamma^2 a_0 a_1^2 + 32k^5\alpha^2 a_2 &= 0,\\
60k^5\alpha\beta a_1 + \frac{6}{5}k\gamma^2 a_0 a_1 a_2 + 6k^3\beta\gamma a_0 a_1 + 24k^3\alpha\gamma a_1 a_2 + \frac{1}{5}k\gamma^2 a_1^3 &= 0,\\
\frac{4}{5}k\gamma^2 a_1^2 a_2 + 28k^3\alpha\gamma a_2^2 + \frac{4}{5}k\gamma^2 a_0 a_2^2 + 24k^3\beta\gamma a_0 a_2 + 11k^3\beta\gamma a_1^2 + 480k^5\alpha\beta a_2 &= 0,\\
55k^3\beta\gamma a_1 a_2 + k\gamma^2 a_1 a_2^2 + 120k^5\beta^2 a_1 &= 0,\\
\frac{2}{5}k\gamma^2 a_2^3 + 54k^3\beta\gamma a_2^2 + 720k^5\beta^2 a_2 &= 0.
\end{aligned}$$

By solving this algebraic equation system gives,

$$\begin{aligned}
a_0 &= -\frac{40k^2\alpha}{\gamma}, \quad a_1 = 0, \quad a_2 = -\frac{120k^2\beta}{\gamma}, \quad \omega = -176k^5\alpha^2,\\
a_0 &= -\frac{5k^2\alpha}{\gamma}, \quad a_1 = 0, \quad a_2 = -\frac{15k^2\beta}{\gamma}, \quad \omega = -k^5\alpha^2.
\end{aligned}$$

Substituting these parameters into Eq. (4.2), then the following solutions of Eq. (1.2) can be obtained as

$$u_1(x,t) = -\frac{40k^2\alpha}{\gamma} + \frac{120k^2\alpha Sech^2[\sqrt{\alpha}(kx - 176k^5\alpha^2 t)]}{\gamma}, \tag{4.3}$$

$$u_2(x,t) = -\frac{40k^2\alpha}{\gamma} - \frac{1920e^{2\sqrt{\alpha}(kx-176k^5\alpha^2 t)}k^2\alpha\beta}{(e^{(2\sqrt{\alpha}(kx-176k^5\alpha^2 t)} - 4\beta)^2\gamma}, \tag{4.4}$$

$$u_3(x,t) = -\frac{40k^2\alpha}{\gamma} - \frac{1920e^{2\sqrt{\alpha}(kx-176k^5\alpha^2 t)}k^2\alpha\beta}{(1 - 4\beta e^{(2\sqrt{\alpha}(kx-176k^5\alpha^2 t))})^2\gamma}, \tag{4.5}$$

$$u_4(x,t) = -\frac{5k^2\alpha}{\gamma} + \frac{15k^2\alpha Sech^2[\sqrt{\alpha}(kx - k^5\alpha^2 t)]}{\gamma},$$

$$u_5(x,t) = -\frac{5k^2\alpha}{\gamma} - \frac{240e^{2\sqrt{\alpha}(kx-k^5\alpha^2 t)}k^2\alpha\beta}{\left(e^{2\sqrt{\alpha}(kx-k^5\alpha^2 t)} - 4\beta\right)^2\gamma},$$

$$u_6(x,t) = -\frac{5k^2\alpha}{\gamma} - \frac{240e^{2\sqrt{\alpha}(kx-k^5\alpha^2 t)}k^2\alpha\beta}{\left(1 - 4\beta e^{2\sqrt{\alpha}(kx-k^5\alpha^2 t)}\right)^2\gamma}.$$

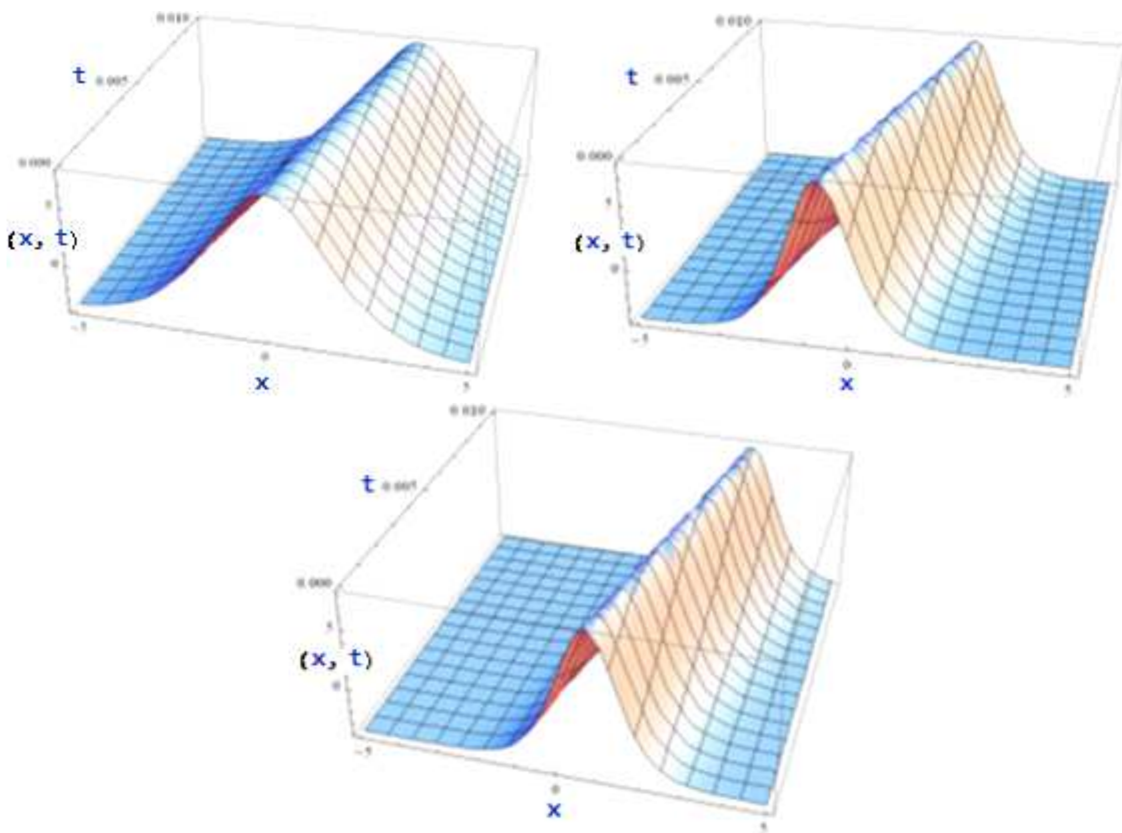

Figure 1. Travelling waves solutions of Eqs. (4.3)-(4.5) in different forms.

The travelling wave solutions of Eqs. (4.3)-(4.5) are shown in Fig.1 $\alpha = k = 1, \beta = -1$ and $\gamma = 10$ with in the interval [-5, 5] and [0,0.01]. According to the conditions of stability Eq. (3.1) and Eq. (3.2), the travelling wave solutions of Eqs. (4.3)-(4.5) are stable in the interval [-5, 5] and [0, 0.01].

4.2. **The Extended Direct Algebraic Method for General Lax equation.** Applying the travelling wave solutions Eqs. (2.2) and (2.3), then general Lax equation, Eq. (1.3), is in the form

$$\omega u' + \frac{3}{10}\gamma^2 k u^2 u' + 2\gamma k^3 u' u'' + \gamma k^3 u u''' + k^5 u^{(5)} = 0. \tag{4.6}$$

Since the solution of Eq. (4.6) in the form of Eq. (4.2), by substituting Eq. (4.2) into Eq. (4.6) yields a set of algebraic equations is for a_0, a_1, a_2, α, β, p, q, r, k, ω. The solution of the system of algebraic equations, can be found as

$$a_0 = -\frac{20k^2\alpha}{\gamma}, \quad a_1 = 0, \quad a_2 = -\frac{60k^2\beta}{\gamma}, \quad \omega = -56k^5\alpha^2,$$
$$a_0 = -\frac{20k^2\alpha}{3\gamma}, \quad a_1 = 0, \quad a_2 = -\frac{20k^2\beta}{\gamma}, \omega = -\frac{8}{3}k^5\alpha^2.$$

Substituting these parameters into Eq. (4.2), then the following solutions of the general Lax equation, Eq. (1.3), can be obtained as

$$u_1(x,t) = -\frac{20k^2\alpha}{\gamma} + \frac{60k^2\alpha Sech^2[\sqrt{\alpha}(kx - 56k^5\alpha^2 t)])}{\gamma}, \tag{4.7}$$

$$u_2(x,t) = -\frac{20k^2\alpha}{\gamma} - \frac{960k^2\alpha\beta e^{2\sqrt{\alpha}(kx-56k^5\alpha^2 t)}}{\left(e^{2\sqrt{\alpha}(kx-56k^5\alpha^2 t)} - 4\beta\right)^2\gamma}, \tag{4.8}$$

$$u_3(x,t) = -\frac{20k^2\alpha}{\gamma} - \frac{960k^2\alpha\beta e^{2\sqrt{\alpha}(kx-56k^5\alpha^2 t)}}{\left(1 - 4\beta e^{2\sqrt{\alpha}(kx-56k^5\alpha^2 t)}\right)^2\gamma}, \tag{4.9}$$

$$u_4(x,t) = -\frac{20k^2\alpha}{3\gamma} + \frac{20k^2\alpha Sech^2[\sqrt{\alpha}(kx - \frac{8}{3}k^5\alpha^2 t)])}{\gamma},$$

$$u_5(x,t) = -\frac{20k^2\alpha}{3\gamma} - \frac{320k^2\alpha\beta e^{2\sqrt{\alpha}(kx-\frac{8}{3}k^5\alpha^2 t)}}{\left(e^{2\sqrt{\alpha}(kx-\frac{8}{3}k^5\alpha^2 t)} - 4\beta\right)^2\gamma},$$

$$u_6(x,t) = -\frac{20k^2\alpha}{3\gamma} - \frac{320k^2\alpha\beta e^{2\sqrt{\alpha}(kx-\frac{8}{3}k^5\alpha^2 t)}}{\left(1 - 4\beta e^{2\sqrt{\alpha}(kx-\frac{8}{3}k^5\alpha^2 t)}\right)^2\gamma}.$$

The travelling wave solutions of Eqs. (4.7)-(4.9) are shown in Fig.2 with $\alpha = k = 1, \beta = -1$ and $\gamma = 10$ in the interval [-5, 5] and [0,0.01]. According to the conditions of stability Eq. (3.1) and Eq. (3.2), the travelling wave solutions of Eqs. (4.7)-(4.9) are stable in the interval [-5, 5] and [0, 0.01].

4.3. **The Extended Direct Algebraic Method for General SK Equation.** Using the Eqs. (2.2) and (2.3), then the general SK equation, Eq. (1.4), becomes

$$\omega u' + \frac{1}{5}\gamma^2 k u^2 u' + 2\gamma k^3 u' u'' + \gamma k^3 u u''' + k^5 u^{(5)} = 0. \tag{4.10}$$

Since the solution of Eq. (4.10) in the in the form Eq. (4.2), by substituting Eq. (4.2) into Eq. (4.10) yields a set of algebraic equations is for $a_0, a_1, \alpha, \beta, k, \omega, p, q$. The solution of the system of algebraic equations, can be found as

$$a_0 = -\frac{20k^2\alpha}{\gamma}, \quad a_1 = 0, \quad a_2 = -\frac{60k^2\beta}{\gamma}, \quad \omega = -16k^5\alpha^2,$$
$$a_0 = -\frac{10k^2\alpha}{\gamma}, \quad a_1 = 0, \quad a_2 = -\frac{30k^2\beta}{\gamma}, \quad \omega = 4k^5\alpha^2.$$

Substituting these parameters into Eq. (4.2), then the following solutions of then the general SK equation, Eq. (1.4), can be obtained as

$$u_1(x,t) = -\frac{20k^2\alpha}{\gamma} + \frac{60k^2\alpha Sech^2[\sqrt{\alpha}(kx - 16k^5\alpha^2 t)])}{\gamma}, \tag{4.11}$$

$$u_2(x,t) = -\frac{20k^2\alpha}{\gamma} - \frac{960k^2\alpha\beta e^{2\sqrt{\alpha}(kx-16k^5\alpha^2 t)}}{\left(e^{2\sqrt{\alpha}(kx-16k^5\alpha^2 t)} - 4\beta\right)^2\gamma}, \tag{4.12}$$

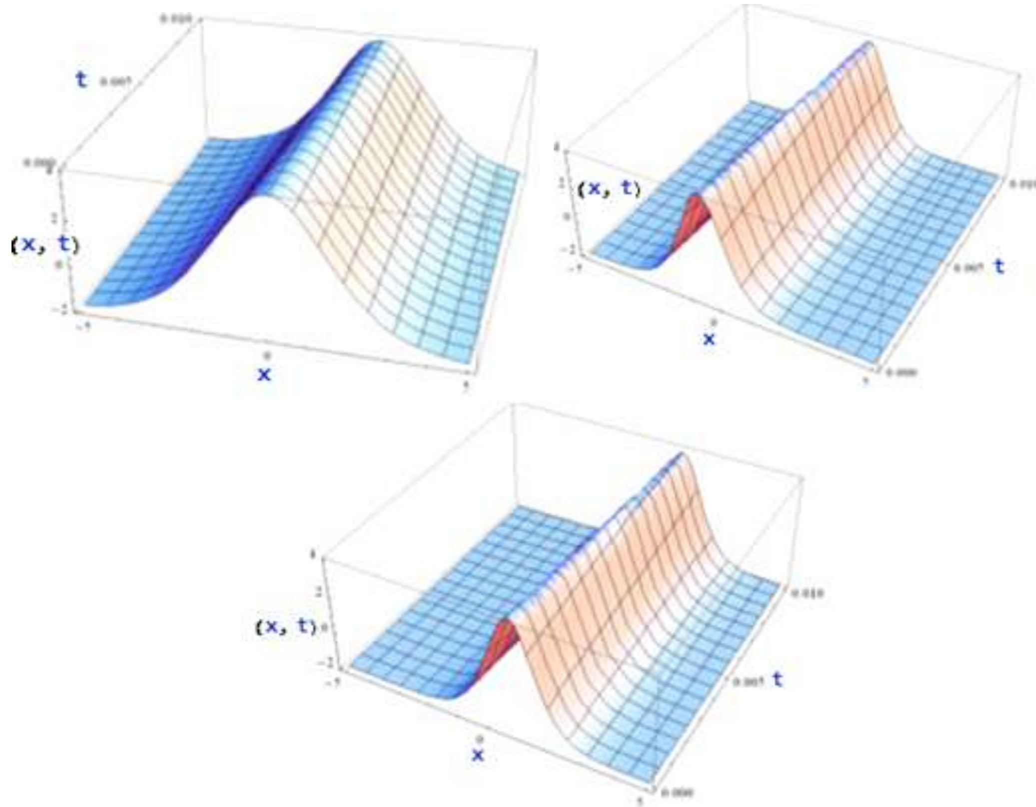

Figure 2. Travelling waves solutions of Eqs. (4.7)-(4.9) in different forms.

$$u_3(x,t) = -\frac{20k^2\alpha}{\gamma} - \frac{960k^2\alpha\beta e^{2\sqrt{\alpha}(kx-16k^5\alpha^2 t)}}{\left(1-4\beta e^{2\sqrt{\alpha}(kx-16k^5\alpha^2 t)}\right)^2\gamma}, \tag{4.13}$$

$$u_4(x,t) = -\frac{10k^2\alpha}{\gamma} + \frac{30k^2\alpha Sech^2[\sqrt{\alpha}(kx+4k^5\alpha^2 t)])}{\gamma},$$

$$u_5(x,t) = -\frac{10k^2\alpha}{\gamma} - \frac{480k^2\alpha\beta e^{2\sqrt{\alpha}(kx+4k^5\alpha^2 t)}}{\left(e^{2\sqrt{\alpha}(kx+4k^5\alpha^2 t)}-4\beta\right)^2\gamma},$$

$$u_6(x,t) = -\frac{10k^2\alpha}{\gamma} - \frac{480k^2\alpha\beta e^{2\sqrt{\alpha}(kx+4k^5\alpha^2 t)}}{\left(1-4\beta e^{2\sqrt{\alpha}(kx+4k^5\alpha^2 t)}\right)^2\gamma}.$$

The travelling wave solutions Eqs. (4.11)-(4.13) are shown in Fig.3, $\alpha = k = 1, \beta = -1$ and $\gamma = 10$ in the interval [-5, 5] and [0, 0.01]. According to the conditions of stability Eq. (3.1) and Eq. (3.2), the travelling wave solutions of Eqs. (4.11)-(4.13) are stable in the interval [-5, 5] and [0, 0.01].

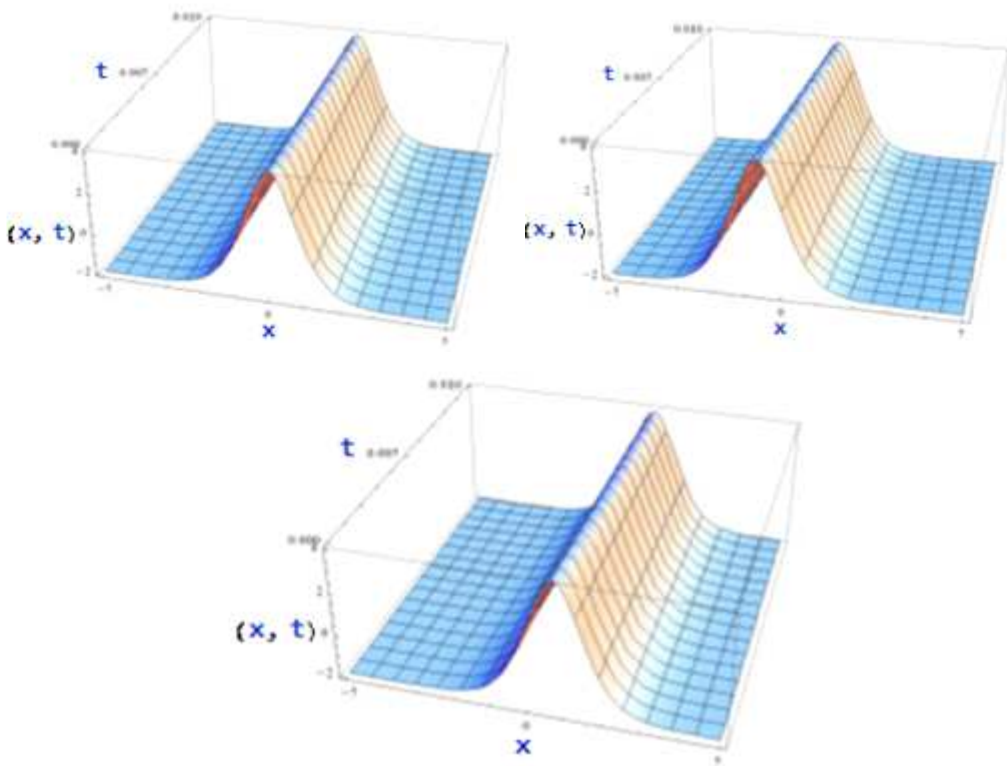

Figure 3. Travelling waves solutions of Eqs. (4.11)-(4.13) in different forms.

4.4. **The Extended Direct Algebraic Method for General Ito Equation.** Consider the travelling wave solutions Eqs. (2.2) and (2.3), then the general Ito equation, Eq. (1.5), becomes

$$\omega u' + \frac{2}{9}\gamma^2 k u^2 u' + 2\gamma k^3 u' u'' + \gamma k^3 u u''' + k^5 u^{(5)} = 0. \tag{4.14}$$

Since the solution of Eq.(4.14) in the form Eq. (4.2), by substituting Eq. (4.2) into Eq.(4.14) yields a set of algebraic equations is for a_0, a_1, α, β, k, ω, p, q, r. The solution of this system of algebraic equations, can be found as

$$a_0 = -\frac{30k^2\alpha}{\gamma}, \quad a_1 = 0, \quad a_2 = -\frac{90k^2\beta}{\gamma}, \quad \omega = -96k^5\alpha^2.$$

Substituting these parameters into Eq. (4.2), then the following solutions of the general Ito equation, Eq. (1.5), can be obtained as

$$u_1(x,t) = -\frac{30k^2\alpha}{\gamma} + \frac{90k^2\alpha Sech^2[\sqrt{\alpha}(kx - 96k^5\alpha^2 t)])}{\gamma}, \tag{4.15}$$

$$u_2(x,t) = -\frac{30k^2\alpha}{\gamma} - \frac{1440k^2\alpha\beta e^{2\sqrt{\alpha}(kx-96k^5\alpha^2 t)}}{\left(e^{2\sqrt{\alpha}(kx-96k^5\alpha^2 t)} - 4\beta\right)^2\gamma}, \tag{4.16}$$

$$u_3(x,t) = -\frac{30k^2\alpha}{\gamma} - \frac{1440k^2\alpha\beta e^{2\sqrt{\alpha}(kx-96k^5\alpha^2 t)}}{\left(1 - 4\beta e^{2\sqrt{\alpha}(kx-96k^5\alpha^2 t)}\right)^2\gamma}. \tag{4.17}$$

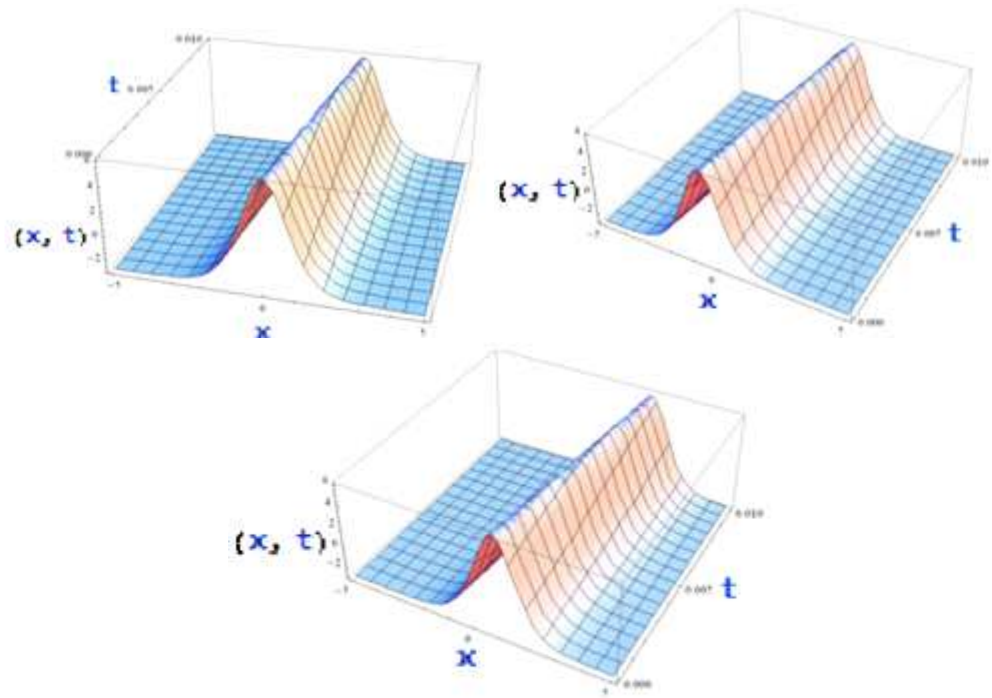

Figure 4. Travelling waves solutions of Eqs. (4.15)-(4.17) in different forms.

The travelling wave solutions of Eqs. (4.15)-(4.17) are shown in Fig.4, $\alpha = k = 1, \beta = -1$ and $\gamma = 10$ with in the interval [-5, 5] and [0, 0.01]. According to the conditions of stability Eq. (3.1) and Eq. (3.2), the travelling wave solutions of Eqs. (4.15)-(4.17) are stable in the interval [-5, 5] and [0, 0.01].

5. Conclusion

In this study, implementing the extended direct algebraic method travelling wave solutions of general KK, general Lax, general SK and general Ito equations obtained with aid of Mathematica program. Graphs of the equations were plotted at specified intervals to demonstrate the stability of the new solutions. It has also been verified that all the solutions found provide the equations. Moreover, many new nonlinear equations that arising in mathematical physics can also be solved by this efficient method.

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