Modelling Travelling Waves on Elastic Blood Vessels

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link to animations: tinyurl.com/modellingbloodflow

traditional methods for modelling blood flow through a vessel are often prohibitively computationally expensive

study literature for cylindrical wave models that use a free boundary formuation

2018 — THE NONLOCAL ABLOWITZ–FOKAS–MUSSLIMANI WATER-WAVE METHOD FOR CYLINDRICAL GEOMETRY* — M. G. BLYTH† AND E. I. PĂRĂU†

2006 — On a new non-local formulation of water waves — By M. J. ABLOWITZ[1], A. S. FOKAS[2] AND Z. H. MUSSLIMANI[3]

2014 — Solitary waves on a ferrofluid jet — M. G. BLYTH AND E. I. PĂRĂU

1970 — Inverse formulation and finite difference solution for flow from a circular orifice — By ROLAND W. JEPPSON

1998 — Axisymmetric capillary waves — J.-M. Vanden-Broeck a,*, T. Miloh b, B. Spivack b

formulation [1]

the free surface Bernoulli condition becomes

$$-K = B^* \quad \text{where} \quad B^* = \frac{B}{\alpha}$$

$$\text{and} \quad K = \frac{F_{zz}}{(1+F_z^2)^{3/2}} - \frac{1}{(1+F_z^2)^{1/2}}\frac{1}{F}$$

define a series solution as

$$F(z) = \sum_{n=0}^{N-1} u_n \cdot cos(2\pi n z)$$

total N+1 unknowns

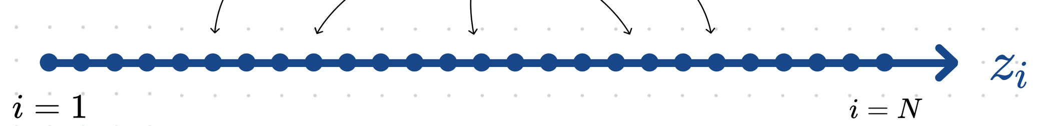
re-create results of axisymmetric capillary waves [1] results using PYTHON

discretize domain

$$F(z=0) - F(z=1/2) = s \longrightarrow \text{1 equation}$$

$$-\left(\frac{F_{zz}}{(1+F_z^2)^{3/2}} - \frac{1}{(1+F_z^2)^{1/2}}\frac{1}{F}\right) = B^* \longrightarrow \text{N equations}$$

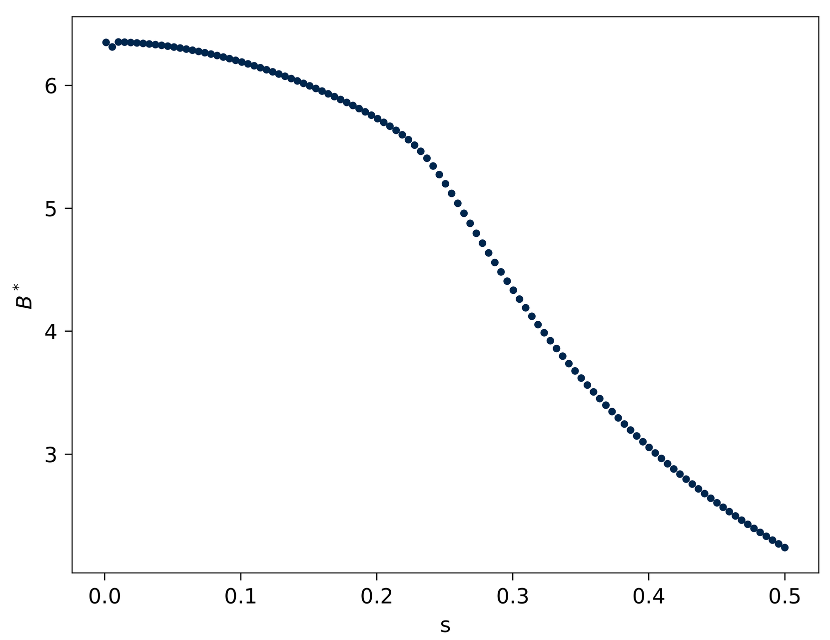
discretize eq'n with finite differences at N domain mesh points



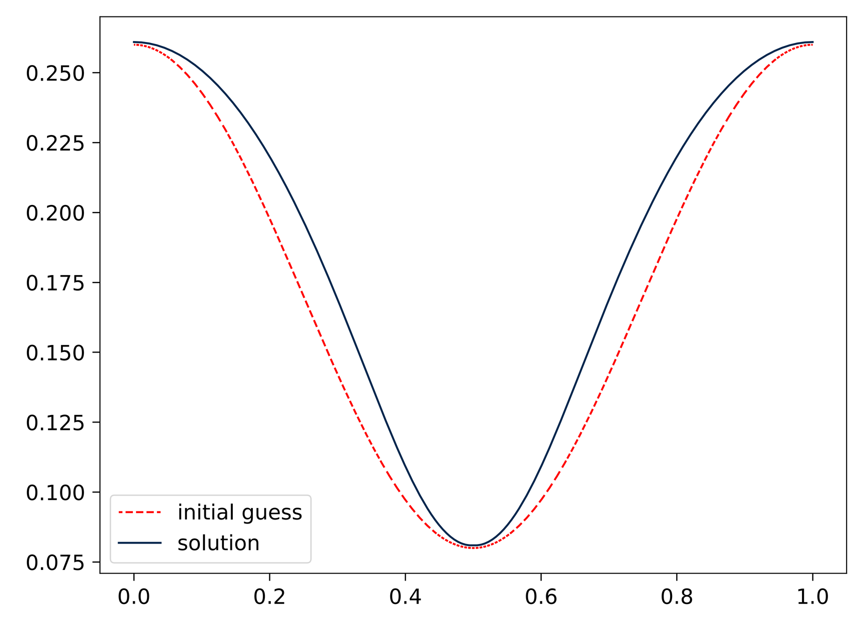
total N+1 equations

solve using Newton's method

* each point on the bifurcation branch corresponds to one wave profile solution



(a)



(b)

Fig 1. (a) Bifurcation branch showing solutions for B* for N = 111 from steepness value s = 0 to s = 0.5. (b) initial guess and final wave profile solution for s = 0.18. (see header for animation link)

study solvers and fluid dynamics software currently used for modelling blood flow

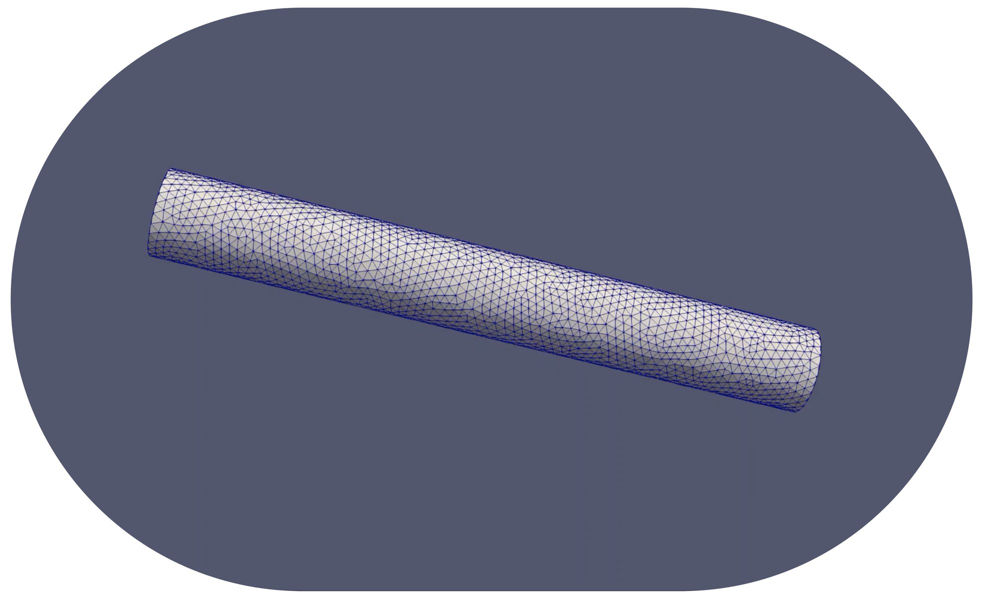
TurtleFSI

OpenFOAM®

SimVascular

simulate a propagating axisymmetric wave on a cylinder using SimVascular [2]

finite element mesh



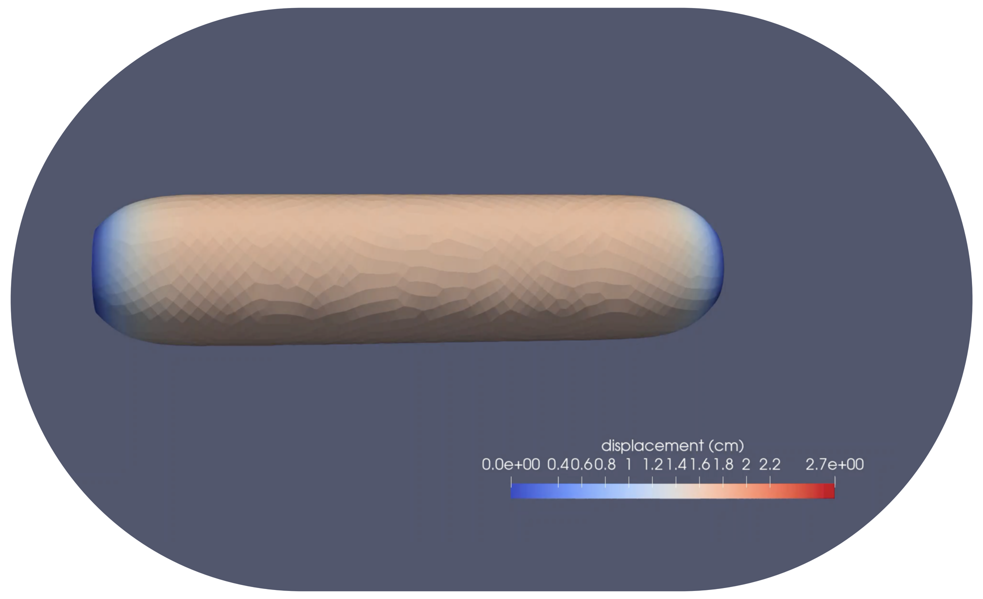
triangular surface elements & tetrehedral volume elements

Nodes: 8595
Elements: 45754
Edges: 8130
Faces: 5420

Fig 2. Discretized finite element mesh of cylinder

set boundary and initial conditions & compute and visualize simulation

wave simulation



total simulated time → 1.51 seconds
computation time → 45.57 minutes

Elastic modulus (wall) = 1.5×10^6 dyn/cm²

Outflow BC resistances

distal = 25
proximal = 10

Fig 3. First animation frame of a propagating wave on the cylinder (see header for animation link)

Future work

- continue to study cylindrical wave models and free boundary solution schemes in literature
- build upon models by applying elastic boundary conditions used for waves under ice and adapting to blood vessels
- assess capabilities of SimVascular and other sofware/solvers, particularly in case of increasingly deformable walls

References

[1] Vanden-Broeck, J.-M., Miloh, T. & Spivack, B. (1998). Axisymmetric capillary waves. *Wave Motion* **27**(3), 245-256.

[2] Updegrove, A., Wilson, N. M., Merkow, J., Lan, H., Marsden, A. L., & Shadden, S. C. (2017). SimVascular: An Open Source Pipeline for Cardiovascular Simulation. *Annals of biomedical engineering*, ***45***(3), 525–541.