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A Sense of Proportion: How humans process relative magnitudes in space and time

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Abstract

Humans perceive ratios for different spatial magnitudes such as length, area, and numerosity, and temporal magnitudes such as duration. Previous studies have shown that spatial ratios may be processed by a common ratio processing system. The aim of the current study was to determine whether ratios across spatial and temporal domains may also be processed by a common system. Two hundred and seventy-five participants completed a series of spatial and temporal ratio estimation and magnitude discrimination tasks. Structural equation modeling (SEM) was used to analyze the relationship between ratio processing across domains when controlling for absolute magnitude processing ability. Results showed a significant relationship between spatial and temporal ratio processing. Absolute magnitude processing was also shown to explain a large part of the variance in both spatial and temporal ratio processing factors. These results have implications for theories of general magnitude processing for both absolute and relative magnitudes.

Keywords

Non-symbolic proportions, proportional reasoning, divided interval durations, number line estimation, ATOM, ratio processing system (RPS)

Summary for lay audience

Imagine a basket containing two green apples and two red apples. Now imagine another basket containing five green apples and five red apples. Although the total number of apples was different between the two baskets, you probably noticed that the proportion of red and green apples was the same. In both baskets, half of the apples are green while the other half are red. This ability to perceive relationships between quantities is called ratio processing. Interestingly, ratio processing can be done for different types of magnitudes like number, length, area, and duration. The aim of the current study was to examine whether spatial and temporal ratios are processing by a general ratio processing mechanism. Two types of tasks were used: ratio estimation tasks, which measured ratio processing abilities, and magnitude discrimination, which measured absolute magnitude processing abilities. In ratio estimation tasks, participants were presented a ratio and asked to represent that ratio on a line. In magnitude discrimination tasks, participants were presented two magnitudes (e.g., two lengths) and asked which of the two magnitudes was the largest. Both types of tasks were done with length, area, numerosity (i.e., number of dots) and duration. Using structural equation modeling, performance on spatial ratio estimation tasks were correlated with performance on temporal ratio estimation tasks while controlling for participant's performance on the magnitude discrimination tasks. Our results showed a significant relation between people's performance on spatial and temporal ratio estimation tasks. This indicates that spatial and temporal ratios may be processed by a common ratio processing system (RPS; Lewis, Matthews, & Hubbard, 2015). Additionally, participants' ability to discriminate absolute magnitudes explained a large part of their performance on ratio estimation tasks. This suggests that, even though participants' performance on ratio estimation tasks can in part be explained by a shared ratio processing mechanism across domains related, another part is also largely explained by absolute magnitude mechanisms associated with either the spatial and/or temporal domain.

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Table of contents

List of Tables

List of Figures

List of Appendices

Chapter 1

Introduction

Relative magnitudes, otherwise known as ratios or proportions, are an integral part of our everyday lives. Although we mostly associate them with mathematics, proportions are necessary to process information in many other domains. For example, artists use proportions as guidelines to draw realistic faces (e.g., the bottom of the nose is halfway between the eyes and the chin). Another example is the progress bar we often see on our screens when watching a video or completing a survey: the progress bar tells us what proportion of the task is done and what proportion is left until completion. These examples describe relationships between lengths, which are proportions in space. However, unbeknown to many, we also use proportions in time. Music is an example of widespread use of temporal proportions. In Western music, rhythms are commonly composed of notes with proportional durations, which is why we can recognize tunes despite tempo changes. When a tune is slowed down, all durations are lengthened such that the relative, or proportional, relationships are maintained. Similar to artists who learn to draw realistic art using proportions, musicians decode symbolic notation of rhythms that indicate how long a note should be played in relation to others.

How do we process proportional relationships for such a wide range of domains? Previous research indicates that proportions may be processed by a general ratio processing system (RPS; Lewis, Matthews, & Hubbard, 2015). However, this field of research has mostly focused on proportions that are symbolic (e.g., fractions) and visuospatial non-symbolic (e.g., ratios in length). Therefore, little is known about ratio processing mechanisms in other domains such as time. This leaves unanswered the question of whether proportions are processed by the same mechanism across magnitudes in space (e.g., numerosity, length, area) and time (e.g., duration)?

The aim of the current research project is to investigate the relation between ratio processing in space and time, and to test whether proportions in these two different domains are processed by a common underlying ratio processing mechanism. More specifically, we aim to examine whether ratio processing is a domain general (i.e., proportions are processed the same way across different types of magnitudes) or a domain specific mechanism (i.e., proportions are processed differently depending on magnitude type).

Ratios in space

The concept of magnitude is ubiquitous: we are constantly confronted with magnitude related decisions. Some of these decisions are based on absolute magnitude, such as the number of objects (e.g., 5 apples) or an amount of something (e.g., 2L of milk). Other decisions are based on more abstract concepts such as relative magnitude (i.e., the relationship between two absolute magnitudes). For example, we can easily tell from the battery icon on electronic devices how much charge is left on our device by comparing the length of the filled bar to the length of the full battery icon, regardless of the overall size of the icon. Relative magnitudes, hereinafter ratios, can take two forms: symbolic and non-symbolic. While symbolic ratios are mostly represented using numbers (e.g., fractions such as 3/6), non-symbolic ratios can be depicted by different types of magnitudes. A set of dots in which half of the dots are black and the other half are gray (e.g., ∴ ∴) is an example of a non-symbolic ratio. The ratio between the lengths of two lines is another example of a non-symbolic ratio. Although some researchers make the distinction between discrete magnitudes such as a number of objects (i.e., numerical magnitudes) and continuous magnitudes such as length or area (i.e., non-numerical magnitudes), both discrete and continuous magnitudes will be grouped under the label of spatial non-symbolic magnitudes in the context of the current study.

The study of non-symbolic ratios is a recent subject of interest in the field of numerical cognition. While the field has a large emerging literature on how absolute magnitudes are processed in the brain, the question of how relative magnitudes are represented in the brain is fairly recent. The first studies on non-symbolic ratio processing (otherwise known as proportional reasoning) were aimed at understanding how humans perceive graphical elements such as bar graphs (Spence, 1990; Spence & Krizel, 1995). More recent studies have focused on questions such as whether our ability to process ratios is innate, just as absolute magnitude processing is posited to be an innate and evolutionary ancient ability (Nieder, Freedman, & Miller, 2002; Tudusciuc & Nieder, 2007a; Vallentin & Nieder, 2010). Similar to absolute magnitude processing, results from both behavioral and neuroimaging studies on ratio processing in infants and animals provide preliminary evidence for an innate ratio processing system (Denison, Reed, & Xu, 2013; McCrink & Wynn, 2007; Vallentin & Nieder, 2008, 2010).

For instance, Vallentin & Nieder (2008) showed that rhesus monkeys can discriminate non-symbolic proportions in a spatial proportion-discrimination task. In this task, monkeys were shown a pair of lines representing a specific ratio followed by a second pair of lines representing either the same or a different ratio. The task was to indicate whether the ratio of the second stimulus matched the ratio of the first stimulus. The monkeys performed well above chance and showed performance similar to human on all trained ratios as well as novel, untrained ratios, indicating that they had generalized the concept of proportionality (Vallentin & Nieder, 2008). In addition to this behavioral evidence, single-cell recordings suggested the presence of ratio selective neurons in the prefrontal cortex (PFC; Vallentin & Nieder, 2008). The authors later replicated these findings and found that similar ratio-tuned neurons were also present in the posterior parietal cortex, a brain region often associated with numerical processing (Vallentin & Nieder, 2010). Altogether, this indicates that ratio processing is an innate ability that humans share with other primates. However, these results are hard to generalize to other types of magnitude given that ratios were depicted using only line length.

Other neuroimaging studies in humans support the findings previously described and extend this literature by investigating ratio processing in various other formats. Using an fMRI adaptation paradigm, Jacob & Nieder (2009b) found that humans encode relative magnitudes in the same areas known to encode absolute magnitudes (i.e., the intraparietal sulcus (IPS) and PFC). In this study, the same ratio with varying overall sizes was repeatedly presented to participants causing the signal in brain areas involved in ratio processing to decrease (a phenomenon often referred to as neural adaptation). Then, after multiple presentations of the same ratio, a deviant ratio was presented causing the signal in these areas to recover (i.e., increase). Participants showed this adaptation response for non-symbolic ratios depicted using both length and numerosity (i.e., sets of dots and triangles). More importantly, the brain activity showed the same adaptation pattern in the same brain areas for both formats (length and numerosity; Jacob & Nieder, 2009b). Another study using the same fMRI adaptation paradigm with number and word fractions (e.g., 3/6 and "a half") uncovered the same pattern of activity, even when the number and word fractions were mixed across trials (Jacob & Nieder, 2009a). These results converge with evidence from previous studies indicating that relative magnitudes are processed by a higher order mechanism that is invariant to format. In other words, once magnitudes are encoded, whether they are symbolic ratios (e.g., number and word fractions) or non-symbolic ratios (e.g., numerosity or length), quantifying the relationship between magnitudes might be done by a single higher order mechanism. Given this convergent body of neuronal and behavioral evidence, Lewis et al., 2015 have proposed the existence of a ratio processing system (RPS) defined as "a set of neurocognitive architectures that support the representation and processing of non-symbolic ratios" (Lewis et al., 2015, p.144). However, this body of literature has mainly focused on ratio processing in the visuo-spatial domain using magnitudes such length and numerosity, and little is known about how ratios are processed in other domains such as time.

Ratios in Time

Many parallels can be drawn between the spatial magnitudes described above and the temporal magnitudes (i.e., durations) that will be described in the following section. Similar to the distinction between absolute and relative spatial magnitudes, the timing literature describes two distinct types of timing: absolute timing and relative timing (Teki, Grube, & Griffiths, 2012). Absolute timing refers to the perception or production of one or multiple intervals based on their absolute duration. This type of timing allows us to determine how much time has passed, whether it is a few seconds, minutes or hours.

Relative timing refers to the perception or production of intervals relative to another interval, most often the beat (i.e., a regular pulse underlying a rhythmic sequence). Relative timing plays an important role in the perception of rhythmic sequences. For one, it is what allows us to rescale rhythmic sequences. For example, whether one sings "Happy Birthday" rapidly or slowly, the listener will likely recognize the same rhythmic pattern, regardless of the rate at which the song is sung. From a standpoint of production, this is also what allows a musician to learn how to play a new piece of music slowly and then gradually increase the playing speed to the true tempo (i.e., the rate at which a musical piece is played). Furthermore, music notation heavily relies on the concept of proportion (e.g., an eighth note is generally half the duration of a quarter note).

Given the strong relationship between relative timing and rhythm perception, the perception and production of time ratios (i.e., "the relative duration between two intervals"; Lutz, 2003) have mostly been studied in the context of multi-interval rhythmic sequences such as sequences of 5 to 6 intervals or short melodies (Lutz, 2003). One recurring finding is humans' proclivity towards integer ratios in rhythmic sequences. For example, sequences composed of simple integer ratios (e.g., 1:2:1) are often better remembered than sequences composed of non-integer ratios (e.g., 1:2.3:1.4; Collier & Logan, 2000). In terms of rhythmic production, humans can easily reproduce sequences composed of integer ratios but struggle to accurately reproduce sequences composed of non-integer ratios (Collier & Wright, 1995). Furthermore, multiple studies have shown how production errors in sequences composed of non-integer ratios gravitate towards integer ratios (Jacoby & McDermott, 2017; Povel, 1981).

However, the advantage for integer ratios does not entirely inform us on how both integer and, more interestingly, non-integer time ratios are perceived or mentally represented. Some researchers have suggested that the bias towards integer ratios is an indicator that time ratios are perceived categorically (Clark, 1987; Schluze, 1989). On the other hand, evidence for categorical perception of time ratios is inconsistent and maybe even accentuated by the experimental design (Schluze, 1989). Furthermore, the bias towards integer ratios could be related in part to the presence of context (e.g., multi-interval rhythmic sequences or an underlying metronome). Few studies have investigated the perception of time ratios in isolation using what are called divided intervals (i.e., a pair of serial intervals delimited by three tones). In one study, Lutz (2003) tested how well musicians could discriminate rescaled divided intervals (i.e., divided intervals with the same ratio but different overall durations) without the presence of context. Participants were presented with a divided interval followed by another divided interval with a different overall duration. The task was to indicate whether the third tone of the second divided interval was early or late compared to the third tone of the first divided interval. Given that the two divided intervals had different overall durations, participants had to rely on the ratio of each divided interval to complete the task. Results showed that participants were poorer for simple integer ratios (i.e., 1:1, and less consistently 1:2 and 2:1) than non-simple integer ratios (e.g., 5:12), suggesting that certain integer ratios like 1:1 may serve as perceptual prototypes (Lutz, 2003). In contrast, another study by Nakajima (1987) examined how adults represent time ratios, this time using an estimation task. Participants were presented divided intervals and their task was to represent on a bounded line when the second tone occurred in relation to the first and third tones. Participants' estimations were mostly linear, indicating that they were fairly accurate in representing these ratios and that time ratios are not solely perceived in a categorical way. Though the aim of the study was to examine how absolute duration is perceived in the context of divided intervals, this study gives important insight into how individuals perceive and represent time ratios without a rhythmic context. These results were later replicated in another study in which participants estimated the ratio presented in the divided interval using symbolic notation (e.g., 1:2; Nakajima, Nishimura, & Teranishi, 1988).

In summary, perception of ratios in time have been studied mostly in the context of rhythmic sequences and beat perception. These studies show that humans are sensitive to ratios in time, but they do not explain how humans perceive relationships between durations. Furthermore, although there is some support for categorical or prototypical perception of time ratios, this evidence is inconsistent and does not converge with evidence showing that humans can estimate both integer and non-integer interval ratios accurately in an isolated context. Therefore, the question remains: what mechanism allows us to quantify relationships between durations and whether these mechanisms are shared with other domains?

Magnitudes in space and time

The literature reviewed thus far has focused on spatial ratios and timing ratios independently. Ratio processing in space, mostly studied in the field of numerical cognition, appears to be notation invariant and similar across non-symbolic and symbolic proportions (Jacob & Nieder, 2009a, 2009b). In the timing literature, humans have been shown to be sensitive to certain types of ratios, though specific literature on how temporal ratios are mentally represented is limited. The following section will review what is known about the relation between spatial and temporal magnitudes.

The idea that all magnitudes, whether they are numerical, spatial or temporal, are processed by the same “generalized magnitude system” is not new. One of the most popularized general magnitude processing theories is the ATOM theory, which stipulates that there are common neural correlates for magnitude processing in the fronto-parietal network (Bueti & Walsh, 2009; Walsh, 2003). This idea stems from linguistic associations as well as numerous behavioral, neuroimaging and lesion studies on the relation between spatial and temporal magnitudes (Marcos & Genovesio, 2017). For example, similar language is often used to describe magnitudes in time (e.g., an event that took a long time) and in space (e.g., a long road; Bottini, Crepaldi, Casasanto, Crollen, & Collignon, 2015; Marcos & Genovesio, 2017). Behaviorally, many studies show interference effects between the spatial and temporal domain (Cai & Connell, 2016; Fabbri, Cancellieri, & Natale, 2012; Ishihara et al., 2008; Srinivasan & Carey, 2010). For example, a study by Srinivasan & Carey (2010) examined how the perception of length was affected when tones of varying durations were presented at the same time. Other studies have directly measured the association between perception of spatial and temporal magnitudes (Mendez, Prado, Mendoza, & Merchant, 2011). Lastly, evidence from neuroimaging and lesion studies also suggests a possible relation between how magnitudes in space and time are processed in the brain (Marcos & Genovesio, 2017). For example, individuals with hemi-spatial neglect have been reported to also have timing deficits (Calabria et al., 2011).

Though the relation between space and time has been studied in numerous ways, the theory of a generalized magnitude system is highly debated (Hamamouche & Cordes, 2019). On a behavioral level, studies on interference between spatial and temporal magnitudes are often asymmetric and inconsistent (Cai & Connell, 2015; Casasanto & Boroditsky, 2008; Marcos & Genovesio, 2017). For example, some studies show a greater influence of duration on spatial judgements (Cai & Connell, 2015) while other studies show the opposite effect in which the spatial magnitude affects the duration judgements (Casasanto & Boroditsky, 2008). Other studies that have directly investigated the relationship between the perception of absolute temporal and spatial magnitudes show similar inconsistencies (Anobile et al., 2018). Mendez et al. (2011) examined this relationship by comparing performance on length and duration categorization tasks in humans and monkeys. If length and duration were indeed processed by a single mechanism, then human and monkey performance on the length categorization task would be expected to correlate with performance on the duration categorization task. However, results showed that length categorization was correlated to duration categorization only for specific lengths and durations (Mendez et al., 2011), a finding which does not provide strong support for a generalized magnitude system. Moreover, neurophysiological studies on the relation between magnitudes in space and time suggest that overlapping neural correlates may correspond to decision-making processes rather than actual magnitude encoding (Genovesio, Tsujimoto, & Wise, 2012; Marcos, Tsujimoto, & Genovesio, 2016). This suggests that neuronal populations encoding duration and length are independent, but the neuronal populations related to decision making (e.g., choosing which of two stimuli is larger) are the same in both the length and duration categorization tasks, making decision making a domain general process. Finally, recent timing neural networks studies (Bi & Zhou, 2020; Merchant & Pérez, 2020) show the role of non-timing, domain general components such as decision making in duration discrimination and categorization tasks.

Though the literature on the relation between time and space has mostly focused on absolute magnitudes, the findings described above leave open the question of how relative magnitudes are processed for different domains? As stated earlier, neuroimaging and single cell studies on ratio processing indicate that spatial ratios are likely processed in the fronto-parietal network. While some of those processes may be specific to processing of spatial magnitudes, ratio processing may also be a higher order process which occurs independently of absolute magnitude encoding. In other words, whether different types of magnitudes, such as length and duration, are encoded independently or by the same neural correlates, ratio processing could be responsible for approximately quantifying the relationship between magnitudes of any type. This would make ratio processing a domain general ability, much like decision making.

Current study

Most research on magnitude processing across domains has been conducted on absolute magnitudes and little is known regarding how relative magnitudes (i.e., ratios) are processed across domains. The aim of the current study was to bridge this gap in the literature by investigating the relation between spatial and temporal ratio processing. More specifically, is ratio processing a domain-specific (i.e., processed separately for each type of magnitude) or domain-general mechanism (i.e., processed by a unique mechanism independent of magnitude type)?

To examine this question, we compared individuals' performance on a battery of ratio estimation and magnitude discrimination tasks both in the visuospatial domain and temporal domain. Twelve tasks were used: three spatial ratio estimation tasks (e.g., estimating the ratio between two lengths), three temporal ratio estimation task (e.g., estimating the ratio between two durations), three spatial magnitude discrimination tasks (e.g., discriminating the longest of two lines) and three temporal magnitude discrimination tasks (e.g., discriminating the longest of two durations). If spatial and temporal ratios are processed by the same underlying mechanism, then individuals' temporal and spatial ratio estimation ability should be related even after controlling for absolute magnitude perception (i.e., how accurately individuals perceive absolute magnitude, such as length and duration). In other words, an individual who is more accurate at estimating spatial ratios, such as the relative length between two lines, would also be more accurate at estimating temporal ratios, such as the relative duration between two intervals, when controlling for their ability to perceive and process absolute magnitudes. We chose to control for absolute magnitude processing to eliminate the possibility that the relationship between ratio processing in space and time is explained by the precision with which people perceive absolute spatial and temporal magnitudes.

To test this hypothesis, we used structural equation modeling (SEM), a useful multivariate technique which allows for the estimation of relationships between multiple latent factors. In contrast to other statistical approaches which assume error-free measures, SEM allows the researcher to separate the variance explain by a latent variable or common factor (e.g., spatial ratio processing) from error. This subsequently allows us to analyze of relationships between error-free variables (e.g., the relationship between spatial and temporal ratio processing). Analysis of SEM models yields two types of information: model fit (i.e., how well does the model fit the data) and parameter estimates (i.e., the magnitude of the relationships between variables). Although model fits were examined in order to evaluate the measurement model for each proposed model, the hypothesis was confirmed based on the magnitude of the parameter estimates.

Four models were tested: a single factor model, a two-factor model, a four-factor model and a bifactor model. Since all tasks involve making judgements about quantity, the first model (i.e., single factor model) tested whether performance on all ratio and absolute magnitude tasks can be explained by a general magnitude processing factor (Figure1a). The residuals of analogous estimation and discrimination tasks were freely estimated to account for common variance due to similar methods (i.e., common shapes and types of auditory intervals). In other words, there may be common variance between some tasks simply because the same type of stimulus is being used. For example, the residuals of the line length ratio estimation task may be correlated with the residuals of the line length magnitude discrimination task because the same shape was used. This single-factor model was not expected to fit the data well but was used to evaluate fit improvement of subsequent models.

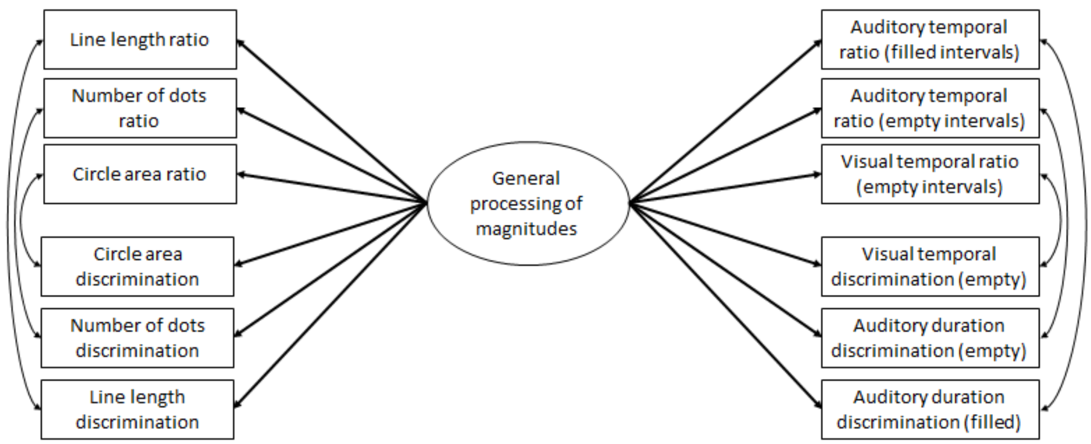

Figure 1a. 1-factor CFA model. The model assumes that performance on all tasks can be explain a single general factor (unidirectional arrows pointing from the latent factor to the observed variables). Curved double headed arrows represent residual correlations between analogous ratio estimation and magnitude discrimination tasks.

The second model tested whether performance on the tasks could be explained by two factors: a general ratio processing factor and an absolute magnitude processing factor (Figure 1b). This model was included in our analyses to test the possibility that both ratio processing and absolute magnitude processing are single, separable constructs. This directly tested the theory of a generalized magnitude system (e.g., ATOM; Walsh, 2001) and a generalized ratio processing system (e.g., RPS; Lewis, Matthews, & Hubbard, 2015). As in the previous model, residual correlations were included between analogous ratio estimation and magnitude discrimination tasks. This model is expected to improve fit significantly compared to the previous single factor model. However, it is not expected to fit the data well and should show poor convergent validity for both factors. This is because previous literature has shown that although space and time have some common neural correlates, there are also many other distinct brain areas involved in processing temporal and spatial stimuli.

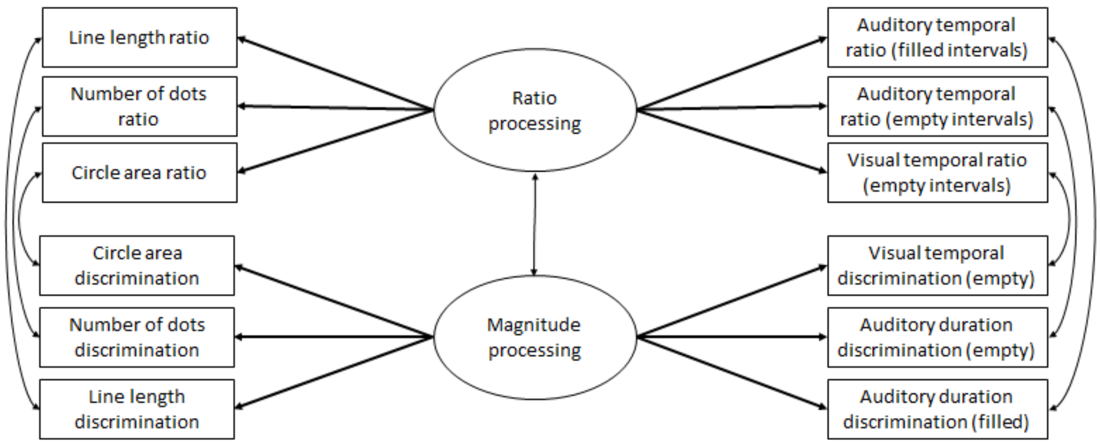

Figure 1b. Two-factor CFA model. The model assumes that performance on all tasks can be explain by a general ratio processing latent factor and a general magnitude processing latent factor. The double headed arrow between ratio and magnitude processing factors represents the correlation between those two latent factors. Curved double headed arrows represent residual correlations between analogous ratio estimation and magnitude discrimination tasks.

The third model was tested using a two-step procedure. The first step consisted of testing the measurement model by estimating a four-factor model using confirmatory factor analysis (CFA) with spatial ratio processing, temporal ratio processing, spatial magnitude processing and temporal magnitude processing as latent factors. This step allowed us to verify that the observed variables could be explained by a four-factor structure (i.e., formed appropriate grouping for each latent factor). The second step tested the structure model of the previous CFA model (i.e., the relationships between the latent factors), and evaluated the strength of the relationship between spatial and temporal ratio processing when controlling for spatial and temporal magnitude processing (Figure 1c). In this model, the single headed arrows between spatial absolute magnitude processing and spatial ratio processing control for absolute magnitude processing abilities. The same rationale is used for the relationship between temporal magnitude and ratio processing.

Single headed arrows were chosen since we assume that absolute (first order) magnitudes are processed before ratios (second order) magnitudes. In addition to controlling for within domain absolute magnitude processing, we also added paths controlling for between domain absolute magnitude processing (dotted lines). Given that the literature is divided on the nature of the relationship between how different types of magnitudes are processed (generalized vs specific processes), we decided to include this path as it might control for additional absolute magnitude processing ability and general cognitive ability. Since we expect the coefficients for the dotted paths to be near zero, we estimated and compared two models, one with the dotted paths and one without the dotted paths, and retained the model with the best fit. Finally, a second-order factor (i.e., general ratio processing) was specified to provide an account for why spatial and temporal ratio processing may covary. If ratio processing is a domain general mechanism, a large coefficient is expected between the general ratio processing factor and the two domain-specific ratio processing factors (spatial and temporal ratio processing).

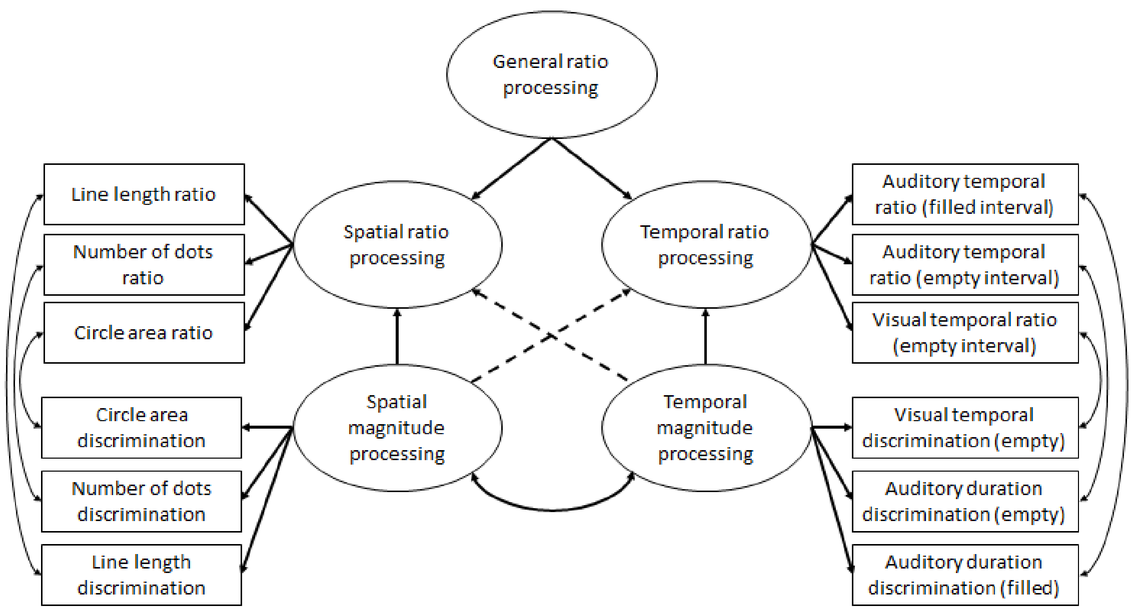

Figure 1c. Four-factor higher order SEM. The model assumes that performance on all tasks can be explain by four latent factors: spatial ratio processing, temporal ratio processing, spatial magnitude processing, temporal magnitude processing. Single headed arrows between magnitude and ratio processing factors

control for variance explained by absolute magnitude perception ability. Curved double headed arrows represent residual correlations between analogous ratio estimation and magnitude discrimination tasks.

Finally, the fourth model tested whether the data could be described using a bifactor model. There are a few notable differences between the hierarchical and bifactor model. However, the main difference is in what each model specification implies theoretically. In the hierarchical model, the general ratio processing factor is what ‘explains’ the common variance between the spatial and temporal ratio processing factors. In contrast, the bifactor model estimates the variance in the ratio tasks that is due to a general factor (i.e., general ratio processing) separate from the variance that is due to specific factors (i.e., ratio processing specific to spatial or temporal magnitudes). In the other words, general and specific ratio processing factors are orthogonal in the bifactor model. The benefit of this model over the hierarchical model previously described is that it will provide us with a more nuanced view of how spatial and temporal ratio processing might be related. One could think of three possible outcomes. The first is that most of the common variance between spatial and temporal ratio processing tasks is explained by the general ratio processing factor, the second is that most of the common variance is explained by specific ratio processing factors, and the third is that the variance is explained by both general and specific ratio processing factors. In summary, the bifactor model allows us to quantify the variance explained by both a general ratio processing variable and two domain specific ratio processing variables (i.e., spatial and temporal ratio processing) when controlling for domain specific magnitude processing (Figure 1d). If ratio processing is a domain general mechanism, large factor loadings are expected between the domain general ratio processing factor and the different ratio estimation tasks, and small factor loadings are expected between the specific factors and the different ratio estimation tasks.

Similar to the hierarchical model, the bifactor model was tested in two steps. The first step consisted of fitting and evaluating the measurement model. In the second step, paths controlling for absolute magnitude processing were added to the specific ratio processing factors as well as for the general ratio processing factor. Although the paths between the general ratio processing factor and specific magnitude processing factor were not present in the hierarchical model (magnitude processing was indirectly controlled through the spatial and temporal ratio processing factor), they are necessary in this bifactor model since the general and specific ratio processing factors are orthogonal.

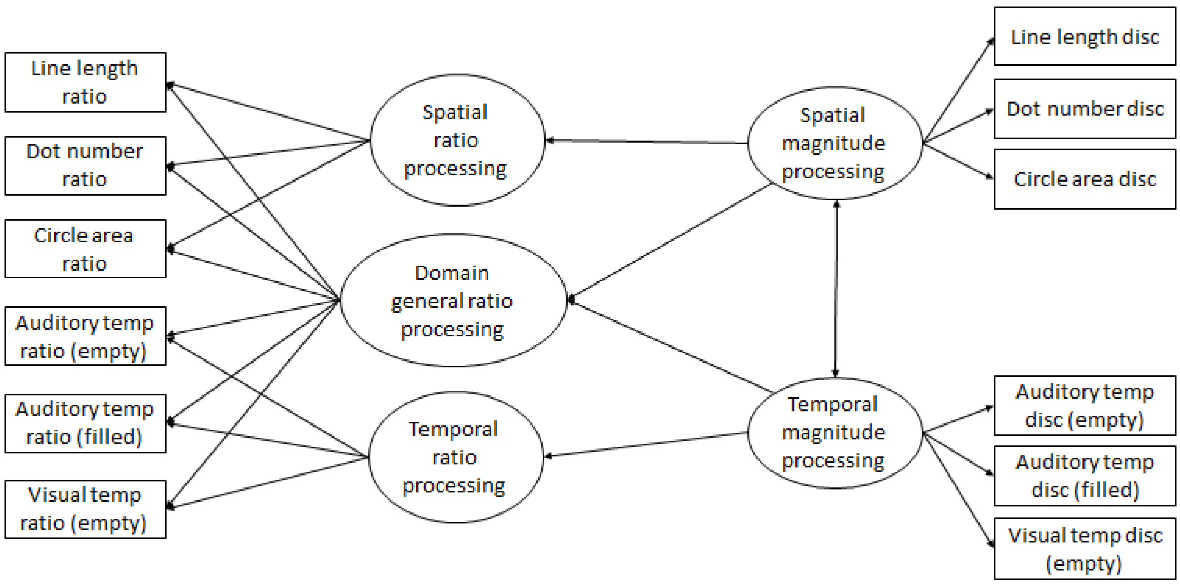

Figure 1d. Bifactor model. The model assumes that performance on ratio estimation tasks can be explain by three orthogonal factors: two specific factors (spatial and temporal ratio processing) and a general ratio processing factor. The model also assumes that performance on magnitude discriminations task can be explained by two latent factors: spatial magnitude processing and temporal magnitude processing. Single headed arrows between magnitude and ratio processing factors control for variance explained by absolute magnitude perception ability. Residual correlations between analogous ratio estimation and magnitude discrimination tasks were also included in the model, though they are not depicted in this figure to avoid cluttering.

Chapter 2

Materials and Methods

Participants

Three hundred twenty-seven participants were recruited from the online survey panel Prolific. Thirty-nine participants withdrew before the start of the study due to technical difficulties and 13 participants withdrew part-way through the study either due to technical difficulties or by choice. The final sample consisted of 275 participants (27.68 ± 8.33 years old; 106 females, 166 males, 3 non-binary; 15.1 ± 3.5 years of education). Participants were residents from the United Kingdom (35.7%), Portugal (32.5%), United States (14.8%), Spain (5.8%), South Africa (4.0%) as well as Ireland, Belgium, Canada, France, Germany, and Sweden (7.2%). To be eligible, participants had to be minimum 18 years old and self-reported normal hearing and normal or corrected to normal vision. Participants also required access to a laptop or desktop computer with a keyboard and sound. Sampling on Prolific was also restricted to adults who were fluent in English to limit cases in which the participants did not understand the instructions well enough to execute the tasks. Sampling was also restricted to adults between the ages of 18 and 50 to limit the potential developmental confounds associated to an older population. Data was collected from April 24th to May 13th, 2021. Participants were paid £7.50 for their participation. The study was approved by the Nonmedical REB at the University of Western Ontario.

Study Design and Materials

Participants completed six ratio estimation tasks and six magnitude discrimination tasks. Tasks were grouped by task type (e.g., they completed all ratio estimations tasks and then all magnitude discrimination tasks), and the task type order was counterbalanced across participants. Participants were permitted to take a 5-minute break between the two sections. The order of tasks within each task type (e.g., line length ratio estimation) was randomized for each participant. The study design is depicted in Figure 2. Once participants had completed all 12 tasks, they completed a short demographics questionnaire. The entire study took approximately one hour to complete. The study was programmed using the free software PsychoPy (version 2020.2.10) and hosted on the platform Pavlovia. The auditory stimuli for the various auditory tasks were generated using MATLAB (version 2019a).

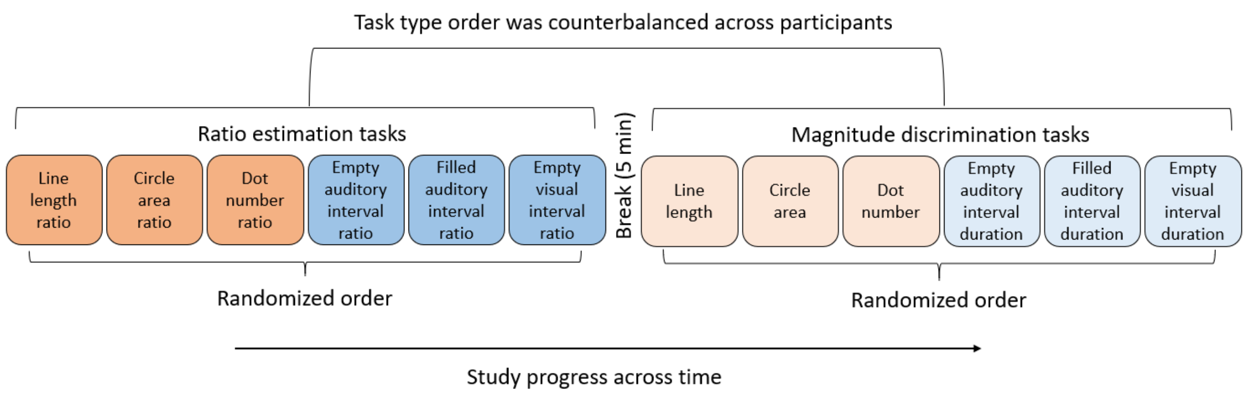

Figure 2. Task counterbalance and randomization

Ratio estimation tasks

The ratio estimation task was a variation on the number line task commonly used in research on numerical cognition (Siegler & Opfer, 2003). There were three visuospatial, hereinafter spatial, ratio estimation tasks (i.e., ratio estimation between pairs of dot arrays, line lengths and circle areas) and three temporal ratio estimation tasks (i.e., ratio estimation of auditory and visual durations with ‘empty’ time intervals, and auditory duration with ‘filled’ intervals). Thus, all spatial ratio estimation tasks were visual tasks, and two temporal ratio estimation tasks were auditory and one was visual.

Spatial ratio estimation tasks. For the spatial ratio estimation tasks, participants were presented with a pair of stimuli: one of the stimuli represented the part, while the other represented the whole (Figure 3). The participants’ task was to represent the *part*:*whole* ratio on a bounded line (adapted from Meert, Grégoire, Seron, & Noël (2012) &

Möhring, Newcombe, Levine, & Frick (2016)). For example, if the stimulus corresponding to the *part* was half the size of the stimulus corresponding to the *whole*, then the participant would respond by marking the middle of the line. At each end of the line was a figure showing either a ratio of 0:1 on the left and 1:1 on the right. In each trial, the visual stimuli were presented for 1500 ms. Participants were instructed to try to use the entire response line. In each trial, participants could click anywhere on the line and subsequently adjust their estimation if needed. Participants then pressed on the space bar to continue to the next trial.

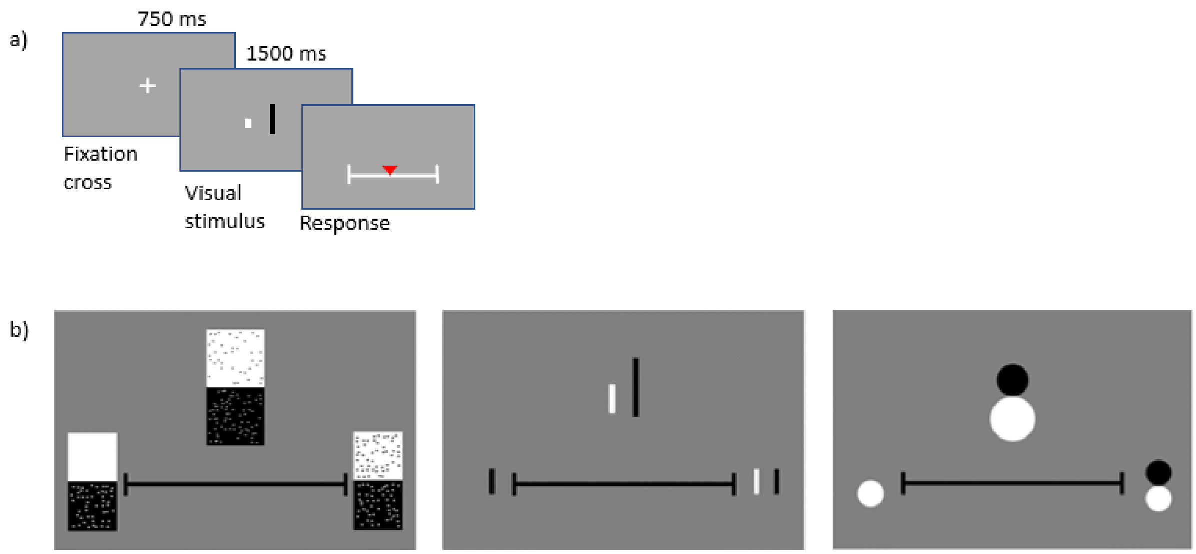

Figure 3. a) Spatial ratio estimation trial and b) stimuli for the dot array, line length and circle area tasks respectively. Note: The stimuli and response screens were presented sequentially (3a), and not shown in the same frame as depicted in figure (3b).

Temporal ratio estimation tasks. For the temporal ratio estimation tasks, participants were presented a divided interval. These divided intervals were denoted either by three empty or filled tones, or three brief flashes (Figure 4). The task was to represent the ratio of the divided interval using the same bounded line as previously described (adapted from Nakajima, 1987). Participants were instructed to estimate the occurrence of the second tone/flash in relation to the first and third tone. For example, if the second tone/flash was presented halfway between the first and third tone/flash, then the participant would respond by marking the middle of the line. Again, participants were instructed to try and use the entire response line.

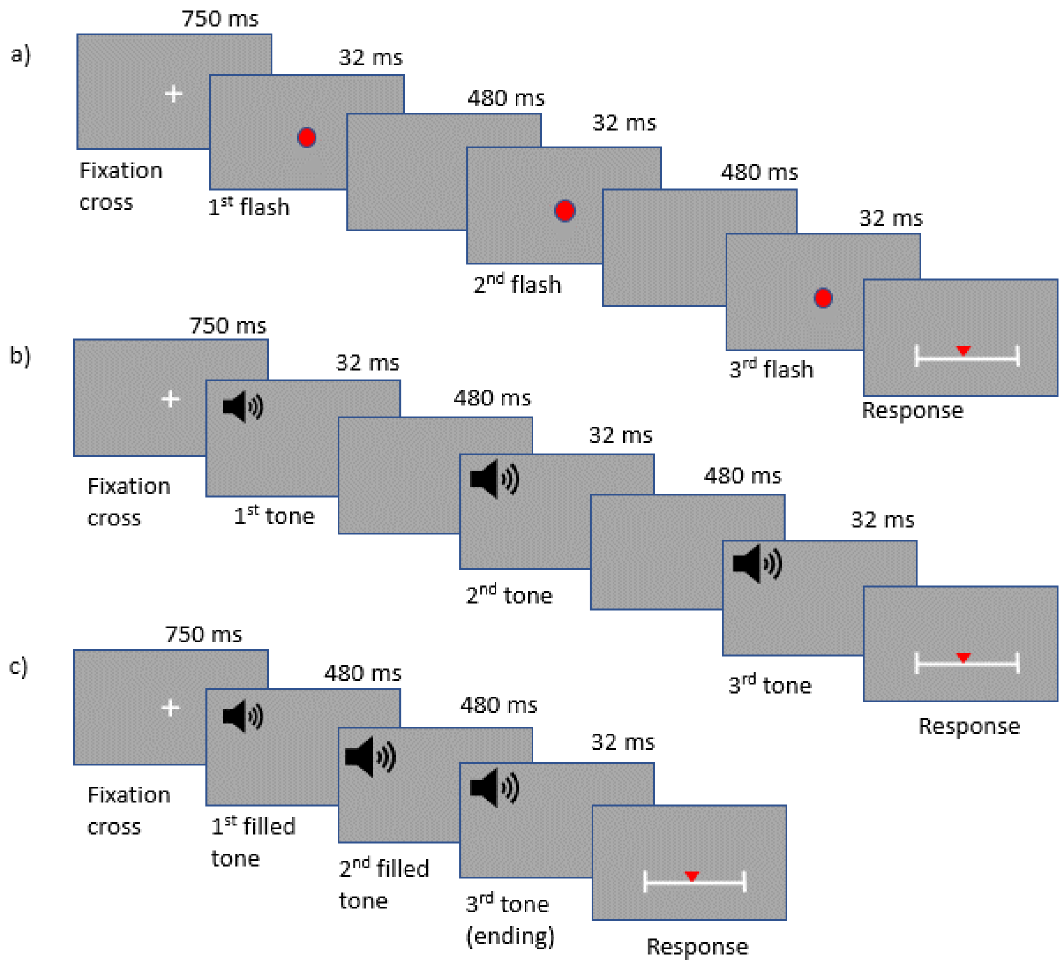

Figure 4. Temporal ratio estimation trials for a) empty visual intervals, b) empty auditory intervals and c) filled auditory intervals. Examples are for a ratio of 0.5 and a total duration of 960 ms. A blank screen lasting 750 ms immediately preceded and followed the first and last flash/tone respectively (not depicted in figure).

Ratio estimation stimuli. Each "whole" stimulus in the part-whole pair had three total magnitude sizes. Table 1 lists the three magnitude sizes used for each type stimulus. The three sizes were randomized throughout the task. Each "part" in the part-whole pair was created from 11 possible ratios (1/12 to 11/12). This resulted in a total of 33 trials (3 total magnitudes x 11 ratios) per task. All spatial stimuli were adapted from Matthews, Lewis, & Hubbard (2015), Park & Matthews (n.d.) & Park, Viegut, & Matthews (2021). Dimensions for the various spatial stimuli can be found in Table 1. The center of the left line was aligned with the center of the right line ± 15 pixels on every trial. For temporal stimuli, durations were measured from the onset of one flash or tone to the onset of the subsequent flash or tone (inter-onset interval). For visual stimuli, flash duration was 2 frames with a refresh rate of 60 Hz (~32 ms). For auditory stimuli, the tone duration for empty intervals was matched to the flash duration (~32 ms). For filled tones, the duration of each tone was equal to the length of the specified duration followed by a silence of 32 ms (to demarcate the onset of the next tone). Thus, inter-onset intervals were matched across stimuli. Tones of 500 Hz were used in both the empty and filled tasks and had 10 ms linear onset/offset ramps.

Magnitude discrimination tasks

To account for absolute magnitude processing ability for both spatial and temporal magnitudes, participants completed six magnitude discrimination tasks, each created to be analogous to the six ratio estimation tasks. For all tasks, participants were presented two stimuli and indicated which of the two was the largest/longest. They were instructed to press the ‘f’ key if the first/stimulus on the left was larger/longer, or the ‘j’ key if the second/stimulus on the right was larger/longer. They were also instructed to respond as quickly as possible.

Spatial magnitude discrimination task. In spatial discrimination tasks, participants were presented a pair of visual stimuli and asked to indicate which of the two was the largest (i.e., circle area), longest (i.e., line length) or had the greatest quantity (i.e., number of dots). For example, in the line length discrimination task, two lines were presented, and the participant indicated which of the two lines was the longest (Figure 5). The pair of stimuli were presented simultaneously for 1000 ms. Participants’ response immediately triggered the start of the next trial.

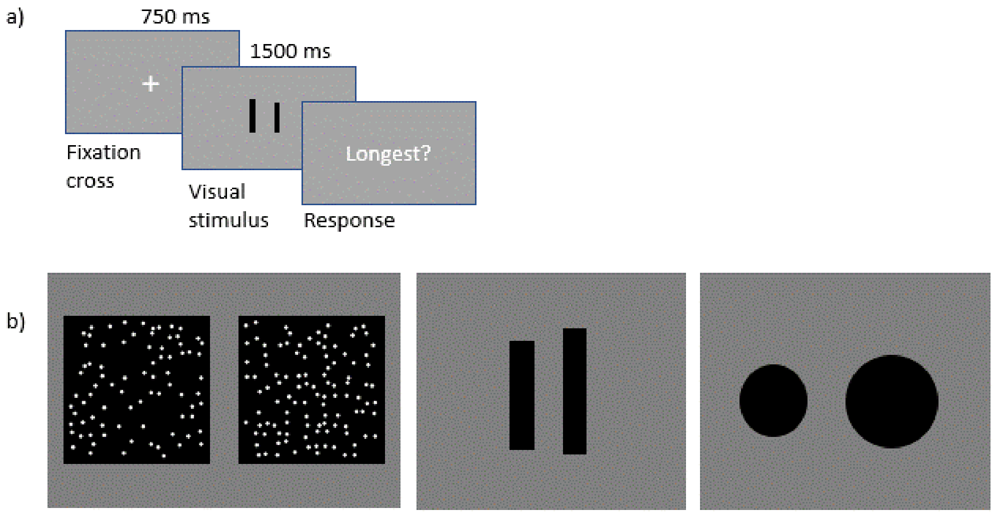

Figure 5. a) Magnitude discrimination trial and b) stimuli for the dot array, line length and circle area tasks.

Temporal magnitude discrimination task. In a temporal discrimination task, two intervals were presented, and participants indicated which of the two intervals lasted the longest (Figure 6). The interval pair were presented serially, separated by an interval of approximately 2400 ± 150 ms (700 ±150 ms blank screen + 1000 ms message + 700 ms blank screen). Participants' responses immediately triggered the start of the next trial.

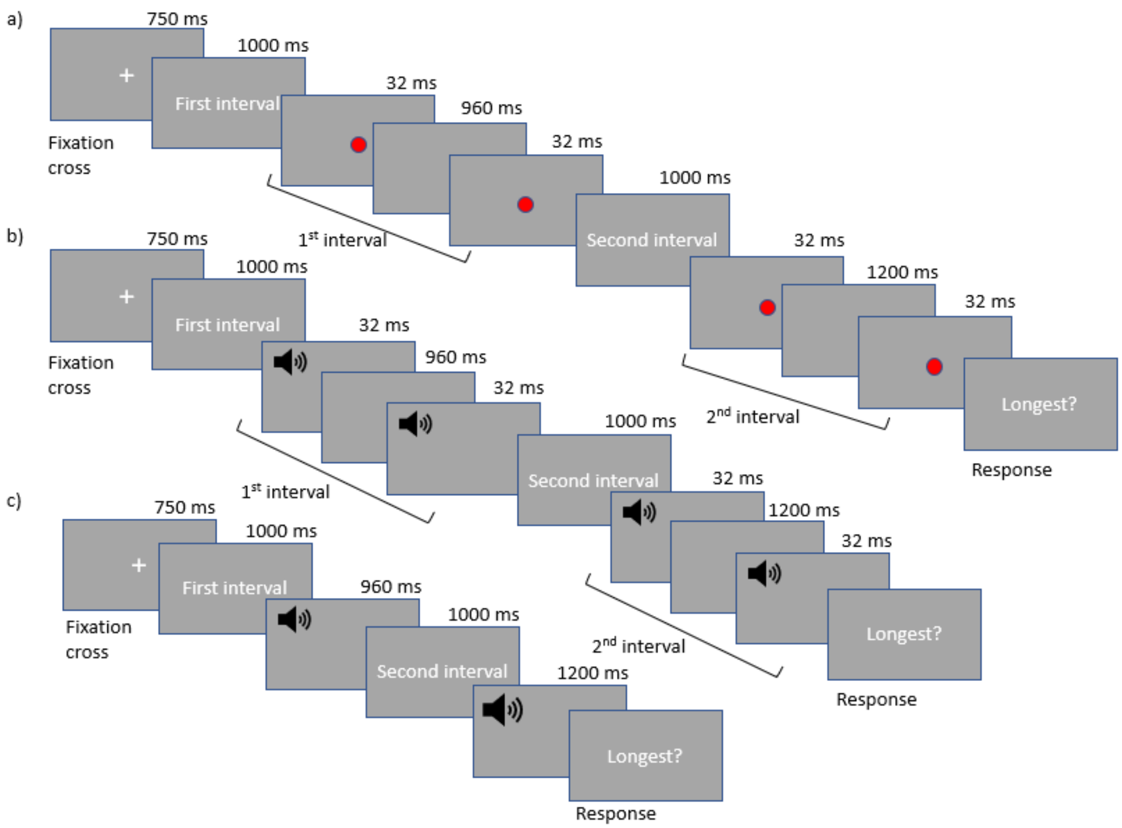

Figure 6. Temporal ratio estimation trials for a) empty visual, b) empty auditory and c) filled auditory intervals. Examples are for a deviant ratio of 1.25 and a total duration of 960 ms. A blank screen lasting 750 ms immediately preceded and followed the first and last interval respectively (not depicted in figure).

Discrimination task stimuli. Stimuli for the discrimination task were created using eight standard magnitudes and five deviant (comparison) ratios for each standard. For example, in the discrimination task with empty time intervals, participants were presented a standard and a comparison interval (i.e., the product of the standard and deviant ratio). For instance, given a standard of 1 second and the five ratio bins 1:1.20, 1:1.25, 1:1.30, 1:1.40 and 1:1.60, participants were presented the standard-comparison duration pairs 1 s and 1.20 s intervals, 1 s and 1.25 s intervals, etc. Deviant ratios were determined based on previous piloting. We chose deviant ratios on which participants were above chance on average without producing ceiling effects. Deviant ratios varied across tasks (e.g., they were different for the line length and the circle area discrimination tasks) but remained constant across participants. Standards and comparison magnitudes spanned the range of the magnitudes presented in the ratio estimation tasks. The range of standards for each task is listed in Table 1.

As a result, each task was composed of 40 trials (5 deviant ratios x 8 standards). The side on which the correct response was presented (or order in the case of temporal stimuli) was controlled so that an equal number of larger/longer trials were presented on both sides (or in both orders in the case of temporal tasks). The side/order of presentation of the stimulus pairs was also counterbalanced across participants.

Lastly, stimuli for the discrimination tasks were modeled after the stimuli used in the ratio tasks, except for a few notable differences. Similar to the ratio estimation, the center of the left line was aligned with the center of the right line ±10 pixels on every trial. Contrary to the ratio task, dots and circles were presented side by side similarly to the line discrimination task. The enter of the circle on the left was aligned with position of the circle on the right ± 30 pixels on every trial.

Table 1. Task parameters for ratio estimation and magnitude discrimination tasks.

<table>
<tr><td rowspan="7">Ratio estimation tasks</td><td>Tasks (6)</td><td>Ratios (11)</td><td>Total magnitudes (3)</td><td rowspan="7">6 tasks x
11 ratios x
3 total magnitudes
= 198 trials</td></tr>
<tr><td>Dot number</td><td rowspan="6">11 ratios (1/12, 2/12, 3/12, etc.)</td><td>75, 100, 125 dots</td></tr>
<tr><td>Line length</td><td>75, 100, 125 px</td></tr>
<tr><td>Circle area</td><td>50, 75, 100 px (radius)</td></tr>
<tr><td>Empty auditory intervals</td><td rowspan="2">480, 960, 1440 ms</td></tr>
<tr><td>Filled auditory intervals</td></tr>
<tr><td>Empty visual intervals</td><td>960, 1200, 1440 ms</td></tr>
<tr><td rowspan="4">Magnitude discrim-ination tasks</td><td>Tasks (6)</td><td>Deviant ratios (5)</td><td>Standards (8)</td><td rowspan="4">6 tasks x
5 ratio bins x
8 total magnitudes
= 240 trials</td></tr>
<tr><td>Dot number</td><td>1.09(12:11),
1.10(11:10),
1.12(9:8),
1.14(8:7),
1.25(5:4)</td><td>Range: 48 – 133 dots</td></tr>
<tr><td>Line length</td><td>1.01, 1.02, 1.03, 1.06, 1.12</td><td>Range: 75 – 125 px</td></tr>
<tr><td>Circle area</td><td>1.02, 1.04, 1.06, 1.08, 1.18</td><td>Range: 50 – 100 px (radius)</td></tr>
</table>

	Empty auditory intervals	1.20, 1.25, 1.3 1.4, 1.6	Range: 200 – 900ms	
	Filled auditory intervals	1.20 1.25,1.3, 1.4, 1.6		
	Empty visual intervals	1.20, 1.25, 1.3, 1.4, 1.6	Range: 400 – 900ms	

Practice trials

Participants completed 3 practice trials for each ratio estimation task. In these practice trials, participants were shown a stimulus pair and asked to estimate the ratio for that pair using the response line. After they responded, a green line appeared on the response line indicating the correct answer. The same three ratios were given for every practice trial set (i.e., 0.25, 0.5, 0.75). Practice trials were done on the same total magnitude across participants.

Participants also completed 3 practice trials at the beginning of each magnitude discrimination task. In these practice trials, participants were shown a pair of stimuli (i.e., the standard and a comparison) and indicated which was the largest/longest. After they responded, feedback was given indicating correctness (i.e., “Correct” or “Incorrect”).

Attention checks

Given that the study was conducted online, each task included one attention trial to verify that participants were not simply clicking through, instead of paying attention to the task. For all attention trials, participants saw a screen after the stimulus presentation displaying “Attention check!” which lasted 1 second. For ratio estimation tasks, participants were then instructed to either place their cursor to the extreme left or right of the response line. The attention trial stimulus ratio for ratio tasks was always 0.5 so that the attention check response would not be confounded by actual estimations. The side of the correct response (i.e., left or right) was decided randomly for each trial.

For all magnitude discrimination task attention checks, participants were instructed to press either the ‘f’ or ‘j’ key, regardless of the stimulus presented for that trial. The attention trial stimulus for the magnitude discrimination tasks were drawn from the easiest ratio bin, for which the difference between the stimulus pair was the largest and easiest to identify. The specific key participants were instructed to respond (i.e., ‘f’ or ‘j’) corresponded to the incorrect answer for the stimulus pair presented.

Demographics

Once participants completed all ratio estimation and discrimination tasks, they completed a demographics questionnaire. Information such as gender, age, years of education, hearing and music experience (e.g., years of formal music training and years of music practice) were collected. Participants were also asked whether they understood how to perform the tasks, how difficult they perceived the tasks to be using a 3-point Likert scale (easy, neutral, or difficult) and whether they experienced any technical difficulties with either the auditory or visual stimuli during the experiment. This information was collected to support decisions regarding data exclusion during data preprocessing.

A priori power analyses

Using Mplus, simulations were conducted for all four proposed models. However, sample size was decided based on the results of the power analysis for the third model (Figure 1c). Results from the simulations showed that a sample size of 275 was appropriate to detect a medium effect size for the relationship between spatial and temporal ratio processing with a power of .8 at the standard .05 alpha error probability. As there was no previous literature on the relationship between spatial and temporal ratio processing, we set the value to the smallest effect size of interest (i.e., a correlation of 0.25 between the residuals of the spatial and temporal ratio latent factors). Other relationships, such as spatial ratio-magnitude processing and spatial-temporal magnitude processing, were estimated based on previous literature (see Appendix A). Power analyses for this model can be found at https://osf.io/374zu/ .

Preprocessing

In the following section, the term *response* will refer to the set of trials for a given task for a participant (i.e., 275 participants x 12 tasks = 3300 responses). First, ratio estimation trials with reaction times greater than 30 seconds and discrimination trials with reaction times greater than 10 seconds were identified and excluded. This resulted in the exclusion of 1 to 5 trials in 132/3300 responses. This also resulted in the exclusion of 12/40 trials in one discrimination task response for one participant. Because multiple trials in the participant’s response for that specific task were above 50 seconds, the participant’s response for that specific task was excluded from the analysis. We then identified and excluded ratio estimation trials that were statistical outliers relative to the participant’s overall response. To do this, a linear regression was fit to each ratio response. The stimulus ratio, which ranged from 1/12 to 11/12, was set as an independent variable (plotted on the x-axis of Figure 7a), and the estimated ratio, which ranged range from 0 to 1, was set as a dependent variable (plotted on the y-axis in Figure 7a). Using the regression coefficients, we calculated the residuals for each trial by subtracting the real estimation from the predicted estimation. Statistical outliers were identified as trials with a residual greater than ± 3 SD (dotted line on Figure 7a).

Once trials with long reaction times and statistical outliers were excluded, we identified non-typical response patterns for ratio estimation tasks. A typical response for the ratio estimation task is expected to have a slope of 1 and an intercept of 0 (Figure 7b). Using the regression coefficients obtained in the previous step, we first identified and visually inspected responses with a negative slope which is an indicator of systematic response line inversion (Figure 7c and e). Out of 30 responses with negative slopes, a total of 14 responses (0.4% of all responses) were identified as showing evidence of systematic scale inversion. These were corrected by subtracting the estimated ratio from 1.

Next, half slope response patterns were identified. Half slope response patterns are responses in which participants used the line mid-point as a reference (i.e., representing a ratio of 1:1) instead of the right end of the line (Figure 7d and e). A typical half slope response pattern would have all (or almost all) ratio estimations at or below 0.5, a slope close to 0.5 and an intercept close to 0. In practice, these response patterns were identified using the following criteria: (i) a response with a slope between 0.45 and 0.55 as well as an intercept below 0.1 (or above 0.9 in the case of systematic scale inversion), or (ii) a slope greater than 0.2 and the maximum estimation is inferior to 0.65 for ratios greater than 0.5. A total of 39 responses across 31 participants were identified as having a half slope response pattern. These were corrected by dividing the estimations across all trials by the maximum estimation used to identify these responses. The mean maximum estimation was 0.52 ± 0.04.

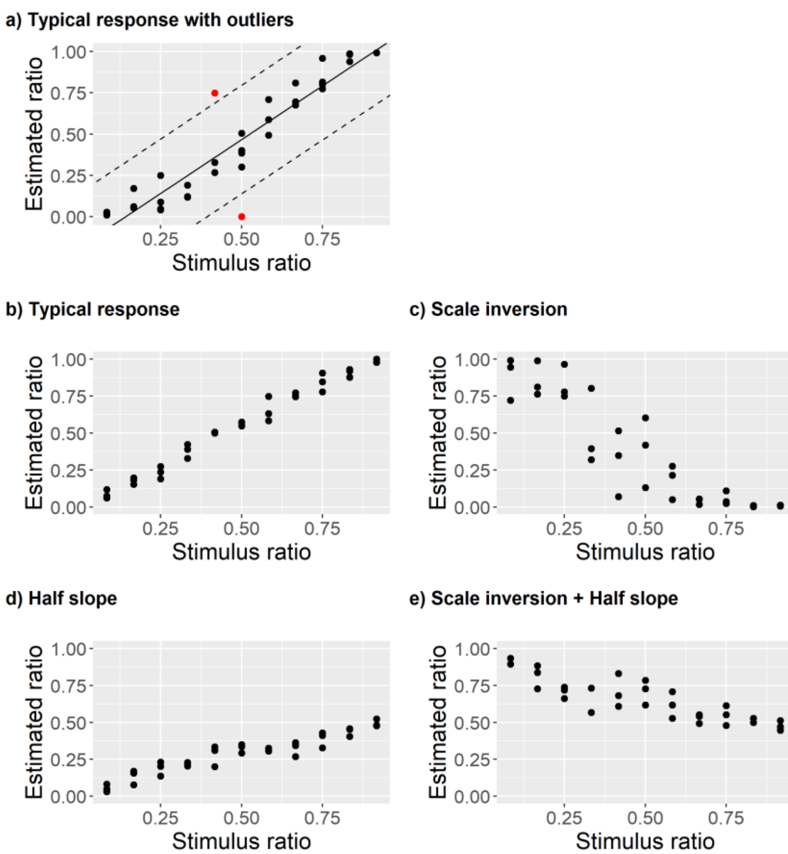

Figure 7. Examples of ratio estimation responses. Each dot represents a ratio estimation.

Next, responses with low accuracy for both ratio estimation and magnitude discrimination were identified and visually inspected for any potential errors. Low accuracy for ratio estimation tasks was identified as responses with a slope inferior to 0.6. For discrimination tasks, low accuracy was identified as responses with a proportion correct of less than 0.55 (i.e., near chance performance). Once low accuracy responses had been identified, these were excluded if participants indicated they did not understand the task, had technical difficulties (e.g., did not hear all tones) or if participants failed the attention check. This resulted in the exclusion of 20 responses across 14 participants. Excluded tasks were treated as missing data.

Ratio estimation responses with low slopes were also visually inspected for other non-typical response patterns causing a low slope. From this inspection, 12 responses were identified as exhibiting a half slope response pattern, though they did not fit the criteria described above due to non-statistical outliers. These responses were corrected by dividing all the estimations by the maximum estimation. For three participants, this maximum estimation excluded visual outliers (i.e., 1-3 estimations above .70 that were not visually part of the response trend). Estimations greater than 1 after half slope correction were excluded from the analysis. Another three responses were identified as exhibiting non-systematic scale inversion. In other words, part of the data points showed a clear downward trend while another group of datapoints showed a clear upwards trend. These responses were excluded from the analysis. Two responses were excluded because participants estimated values were similar/constant across trials or limited to values 0 and 1. Finally, three participants were entirely excluded from the analysis: one participant did not execute any of the ratio estimation tasks properly (i.e., only estimated values of 0, 0.5 and 1, but none in between), two participants performed with low accuracy on most tasks and provided questionnaire responses suggesting that they were not invested in the study. One of the two latter participants also had extremely long reaction times for multiple tasks.

Once all non-typical response patterns were identified and corrected, the error (*stimulus ratio – estimated ratio*) was calculated for each ratio estimation trial. For ratio estimation tasks, responses with reaction times between 15 and 30 seconds with an error greater than ± 0.2 were excluded. This cut-off was decided based on previous piloting in order to remove trials in which participants might have been distracted without excluding data points from trials in which participants might have used a time-consuming strategy like reproduction (e.g., tapping) to make an estimation. We then excluded ratio estimations with an error greater than ± 3 SD away from the mean error for a given response, and ratio estimations greater than 0.9 for ratios smaller than 0.5 and vice versa (i.e., responses smaller than 0.1 for ratios greater than 0.5). These were excluded as they might have been caused by inattention (e.g., lack of attention on a particular trial), spontaneous scale inversion (e.g., responding 0.9 when the target ratio was actually 0.1) or software error (e.g., not perceiving all three tones or flashes in the empty interval temporal ratio estimation tasks). Finally, the average absolute error (average |*error*|) across trials for each response was calculated. For discrimination tasks, the proportion of correct trials in given response was calculated. This score was then reversed prior to conducting the CFA and SEM analyses so that both aggregate scores would have the same direction (i.e., lower indicate better performance). All visualizations from the preprocessing steps described above as well as a summary of trial and response exclusions are available at (https://osf.io/ur6a4/).

Univariate and multivariate outliers. After the implementing the preprocessing steps described above, we proceeded to inspecting the data for univariate and multivariate outliers. In the this study's preregistered report, we had planned to treat univariate outliers as missing and exclude multivariate outliers, in part to handle problems related to multivariate non-normality. However, Aguinis, Gottfredson, & Joo (2013) provide alternative ways of handling outliers instead of completely excluding them from the analysis (which could bias results). Therefore, the analyses described in the next section follow the best-practices recommendations listed in Aguinis, Gottfredson, & Joo (2013). In the aim of transparency, results from the original analysis plan are also available at this link: (https://osf.io/ekzm8/).

Once the aggregate score for each response was calculated, univariate outliers were identified as responses with aggregate scores ± 3 median absolute deviations (MAD) for a given task. We identified between 3 and 22 univariate outliers for each task (total of

137). Each of these univariate outliers were inspected for potential errors (e.g., illegitimate observations due to technical issues or coding errors), though none were found. Multivariate outliers were identified using the Mahalanobis distance test using a threshold of $p < 0.001$ (Kline, 2015). Fifteen outliers were identified and visually inspected for potential errors. After this inspection, one participant was excluded as 4/12 tasks had already been excluded because the participant did not understand the instructions, and five out of the remaining eight tasks had low accuracy. This resulted in a final sample of 271 participants. Of these 271 participants, 258 participants had complete datasets. Visualizations of responses identified as univariate and multivariate outliers are also available in the preprocessing document mentioned above (https://osf.io/ur6a4/). Visualizations of responses for the retained sample are available at (https://osf.io/dftgh/).

The remaining univariate and multivariate outliers were retained in the main analyses and considered as "interesting outliers", data points that are identified as statistical outliers but that cannot be *confirmed* as errors (Aguinis et al., 2013). Sensitivity analyses were later conducted on the final model to (i) assess the stability of the model fit and parameter estimates, (ii) identify "influential outliers" (i.e., data points that significantly alter model fit or parameter estimates), and (iii) assess the influence of "interesting outliers" on the model fit and parameter (Aguinis et al., 2013; Pek & MacCallum, 2011). The influence of individual data points was measured on three aspects of the model: global model fit using the χ^2 difference ($\Delta\chi^2_i$), global parameter estimates using generalized Cook's distance (gCD_i), and specific parameter estimates using the single parameter influence ($\Delta\hat{\theta}_{ji}$). All three influence measures involve the same procedure: (1) each data point is deleted, (2) the model is refit to the remaining data, and (3) the parameters before and after the deletion are compared. Sensitivity analyses were conducted using the R package influence.SEM version 2.2 (Pastore & Altoe, 2018).

Main analyses

Model specification and identification

The first step in SEM analyses consisted of specifying the models to be estimated and verifying that the models were identified. A model is identified if it has a unique solution. This can be verified in two ways: using matrix algebra or identification rules. In this case, measurement models (i.e., 1-factor, 2-factor, and 4-factor CFA models) were identified according to the t-rule, the 3-indicator rule (Kline, 2015). The structural part of 4-factor SEM model was identified according to the recursive rule (Kline, 2015). All models were scaled by fixing the variance to 1.

Although a model may be theoretically identified, there are situations in which a model may be empirically underidentified, meaning that the maximum likelihood (ML) estimation cannot arrive at a single solution due to the sample characteristic. To verify this, we conducted an empirical identification check for each final model by estimating the model on the model implied covariance matrix (T. Jorgensen, personal communication, June 13, 2021). If the analysis produces the same estimates, this indicates that the model is likely identified (though it does not exclude the possibility that it is not). If, however, the analysis produces different estimates, this indicates that the model is empirically underidentified and parameter estimates are not admissible.

Model estimation

All analyses were conducted using the free software R (version 1.4.1106), and the lavaan R package version 0.6.8 (Rosseel, 2012). Models were estimated using a robust maximum likelihood (MLR) estimator. This method provides robust standard errors (Huber-White) and scaled fit statistics for data with slight deviations from multivariate normality (Savalei, 2014; Savalei & Falk, 2014; Yuan, Tong, & Zhang, 2015). Once models were estimated, we verified that solutions were admissible (e.g., all standardized covariances were below 1, no negative variances). Missing data was managed by using full information maximum likelihood (FIML). This method has been shown to give unbiased estimates when missing data is either missing completely at random (MCAR, i.e., there is no link between missing data and any other variable) or missing at random (MAR, i.e., missing data is related to an auxiliary variable; Enders, 2001). In the case of this study, missing data were due to technical issues, non-compliance or misunderstanding of the tasks. We do not believe these missing data to be related to any other auxiliary variable, thereby satisfying the MCAR/MAR assumption.

Model evaluation

Each model was fit and assessed individually using global and local fit indices. Four global fit indices were considered: chi-squared test, CFI, RMSEA and SRMR. The chi-squared test of independence evaluates whether the data covariance matrix significantly differs from the model implied covariance matrix. A significant result ($p<0.05$) indicates that the model significantly differs from the data, and therefore fits the data poorly. Given that this statistical test is sensitive to sample size, large samples can result in rejecting the model for small discrepancies between the model and data. That is why we considered the chi-squared statistic in combination with three other fit indices. CFI is a relative goodness of fit index which evaluates the improvement of the tested model over the baseline model (model in which none of the variables are correlated). RMSEA is an absolute fit index which measures model misfit while taking model parsimony into account. Finally, SRMR is an absolute fit index which is the sum of the residual correlations that cannot be explained by the model. CFI values greater than 0.95, RMSEA values lower than 0.10 and SRMR values lower than 0.08 were used as thresholds indicating a model with reasonable fit (Kline, 2015). Local fit was analyzed by looking at the residual correlation matrix (i.e., the difference between model correlation matrix and the data correlation matrix). As a rule of thumb, absolute residual correlations greater than 0.10 may indicate poor local fit (Kline, 2015).

Chapter 3

Results

Descriptive statistics

Before estimating the various models, we inspected the data for evidence of multivariate non-normality. Although the assumption of multivariate normality of the residuals cannot be directly tested, univariate and bivariate non-normality can be taken as indirect indicators of multivariate non-normality (Kline, 2015). Descriptive statistics for each task are listed in Table 2. Empty visual ratio estimation had a skewness greater than ± 2 and a kurtosis greater than 4. Four other tasks, empty auditory ratio estimation and duration discrimination as well as circle and line ratio estimation, had a skewness lesser than ± 2 but a kurtosis greater than 4. These indices indicate that scores on these five tasks deviate substantially from univariate normality. In addition to inspecting univariate normality, we visually inspecting the bivariate scatterplots and QQ plots for all task pairs for evidence of bivariate normality, linearity, and homoscedasticity of the residuals. Whereas all scatterplots supported bivariate linearity and homoscedasticity of the residuals, residuals were not normally distributed for approximately half of the task pairs. Altogether, these substantial deviations from univariate and bivariate normality indicate that the assumption of multivariate normality is likely violated. To address the violation of this assumption, a robust maximum likelihood estimator was used to fit the various models. Robust maximum likelihood corrects standard errors and model fit statistics in the case of deviation from multivariate normality even in the presence of missing data (Savalei, 2014). Although it does not correct parameter estimates, simulation studies have shown that non-normality produces little bias in the parameter estimates (Finch, 1992; Lei & Lomax, 2005).

Table 2. Descriptive statistics for average absolute error (ratio estimation tasks) and proportion incorrect (magnitude discrimination tasks).

Tasks	N	Mean (SD)	Median	Range (min-max)	Skewness	Kurtosis	VIF

Spatial ratio estimation							
Circle-R	270	.116 (.035)	.112	.052 - .249	0.93	4.15	1.28
Dot-R	270	.126 (.038)	.118	.055 - .265	0.91	3.98	1.55
Line-R	267	.098 (.04)	.089	.034 - .314	1.39	6.44	2.06
Temporal ratio estimation							
EA-R	266	.09 (.037)	.080	.042 - .281	2.00	8.31	2.22
FA-R	269	.115 (.052)	.103	.038 - .294	1.20	3.93	1.91
EV-R	269	.094 (.047)	.081	.036 - .344	2.32	9.75	1.94
Spatial magnitude discrimination							
Circle-M	271	.164 (.071)	.150	.025 - .425	0.62	3.87	1.28
Dot-M	271	.158 (.071)	.150	0 - .4	0.68	3.44	1.17
Line-M	271	.162 (.071)	.150	.025 - .375	0.50	2.88	1.17
Temporal magnitude discrimination							
EA-M	269	.177 (.11)	.150	0 - .575	1.06	4.29	1.89
FA-M	269	.138 (.081)	.125	0 - .4	0.74	3.25	1.55
EV-M	270	.18 (.107)	.150	0 - .575	1.02	3.81	1.66

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation; VIF = variance inflation factor. The mean score for discrimination tasks refers to the proportion of incorrect responses. These means were transformed from the proportion of correct responses to make the direction of scores constant across the ratio estimation tasks and the magnitude discrimination tasks (lower is better). The mean score for ratio estimation tasks refers to the averaged absolute error.

In addition to verifying the assumption of multivariate normality, the data were screened for extreme bivariate and multivariate collinearity. Table 3 displays the zero-order correlation matrix for all 12 tasks. All tasks had low to moderate correlation coefficients (range: .143 - .628). There was neither evidence of extreme bivariate collinearity (all correlations were below .85; Brown, 2006), nor multivariate collinearity (all variance inflation factors (VIF) were below 10; Kline, 2015).

Table 3. Zero-order correlations for all tasks from the FIML observed covariance matrix.

Tasks	1	2	3	4	5	6	7	8	9	10	11	12
Spatial ratio estimation												
1. Circle-R	1											
2. Line-R	.405	1										
3. Dots-R	.336	.564	1									
Temporal ratio estimation												
4. EA-R	.311	.568	.388	1								
5. FA-R	.245	.462	.353	.606	1							
6. EV-R	.270	.482	.434	.628	.586	1						

Spatial magnitude discrimination												
7. Circle-M	.255	.377	.250	.270	.225	.268	1					
8. Line-M	.143	.280	.223	.269	.265	.266	.274	1				
9. Dots-M	.243	.206	.247	.281	.162	.232	.225	.148	1			
Temporal magnitude discrimination												
10. EA-M	.314	.433	.334	.582	.506	.454	.331	.350	.282	1		
11. FA-M	.252	.388	.356	.425	.431	.481	.295	.272	.231	.594	1	
12. EV-M	.269	.383	.397	.487	.487	.561	.311	.301	.272	.548	.497	1

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation. All correlations were statistically significant at $p < .001$, except for the correlations between FA - R and Dot - M ($p < .01$), Line - M and Circle - R ($p < .01$), and Line - M and Dot - M ($p < .01$). These exceptions are underlined in the table. Colors do not have a gradient scale but were included to help visualize correlation clusters (darker shade indicates a higher correlation).

Higher-Order Model

Measurement Model Fit

Table 4 summarizes goodness-of-fit statistics for each model estimated. Though the 4-factor model was the model of interest, we also tested three alternative models. All CFA and SEM related to the higher-order model were shown to be empirically identified. Fully standardized parameter estimates are reported in the path diagrams and can therefore be interpreted as correlations. Complete standardized and unstandardized solutions for all models can be found in Appendix B.

Table 4. Goodness-of-fit statistics for all models

Models	χ^2 (df)	rRMSEA (90% CIs)	rCFI	rSRMR	AIC	BIC
1-Factor CFA	127.61 (48) $p < .001$	0.087 (0.071 - 0.103)	0.909	**0.047**	-10094.55	-9943.257
2-Factor CFA (Magnitude-Ratio)	82.41 (47) $p = .001$	**0.061 (0.043 - 0.079)**	**0.956**	**0.039**	-10143.92	-9989.025
2-Factor CFA (Spatial - Temporal)	93.64 (47) $p < .001$	**0.068 (0.051 - 0.086)**	0.945	**0.040**	-10132.16	-9977.264
4-Factor CFA and SEM	**40.77 (42) $p = .525$**	**0.022 (0 - 0.048)**	**0.995**	**0.027**	-10181.19	-10008.284
4-Factor SEM (trimmed)	**41.51 (44) $p = .579$**	**0.017 (0 - 0.045)**	**0.997**	**0.027**	-10184.94	-10019.247

Note: r = robust; Bolded values are fit statistic values indicating good fit according the criteria described in the methods sections. The chi-squared statistic was corrected using the Mplus variant of the Yuan-Bentler correction factor. Chi-squared scaling factors were between 1.136 and 1.162.

1-factor CFA. We first tested the theory that all tasks are explained by a single general magnitude processing factor (Figure 8). This model yielded a poor fit according to the chi-squared statistic and CFI index. The chi-squared test was significant indicating that the model-implied covariance matrix significantly differed from the observed covariance matrix. RMSEA and SRMR were at the limit of what is considered reasonable fit (Hancock & Mueller, 2008). Finally, local fit testing showed several instances of poor local fit in which the residual correlation was greater than ± .10. Thus, the 1-factor model could not adequately explain the participants' performance on the various tasks.

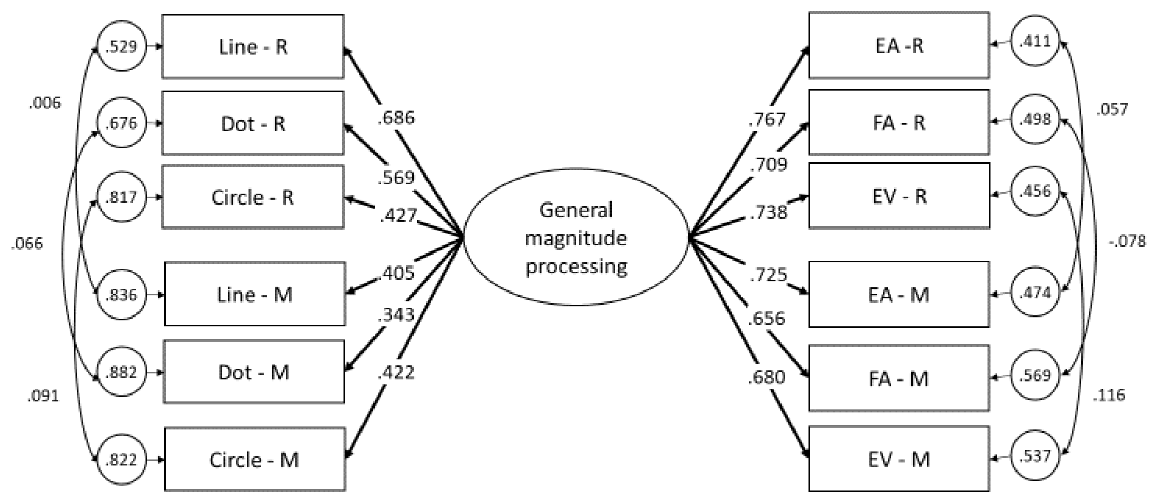

Figure 8. 1-factor CFA model. EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation. Parameter estimates are fully standardized.

2-Factor CFA. The second model tested whether the data could be explained by two underlying factors: a general ratio processing factor and a general (absolute) magnitude processing factor (Figure 9a). Like for the previous model, this 2-factor model showed poor fit according to the chi-squared statistic. RMSE, CFI and SRMR all indicated adequate fit. In terms of local fit, there were again multiple instances of poor local fit. Finally, when compared to the previous 1-factor model, the nested chi-squared difference test indicated that the 2-factor model fit the data significantly better than the 1-factor model (χ^2 (1) = 54.34, p < .001).

Although it was not planned in the original analysis plan, we also tested whether the data could be modeled using two different factors: general spatial and general temporal processing (Figure 9b). In this case, the model assumes that performance on the set of tasks can be explained by domain related factors. This model yielded slightly worse model fit than the previous 2-factor model. However, it still fit the data significantly better than the 1 factor model (χ^2 (1) = 26.108, p < .001).

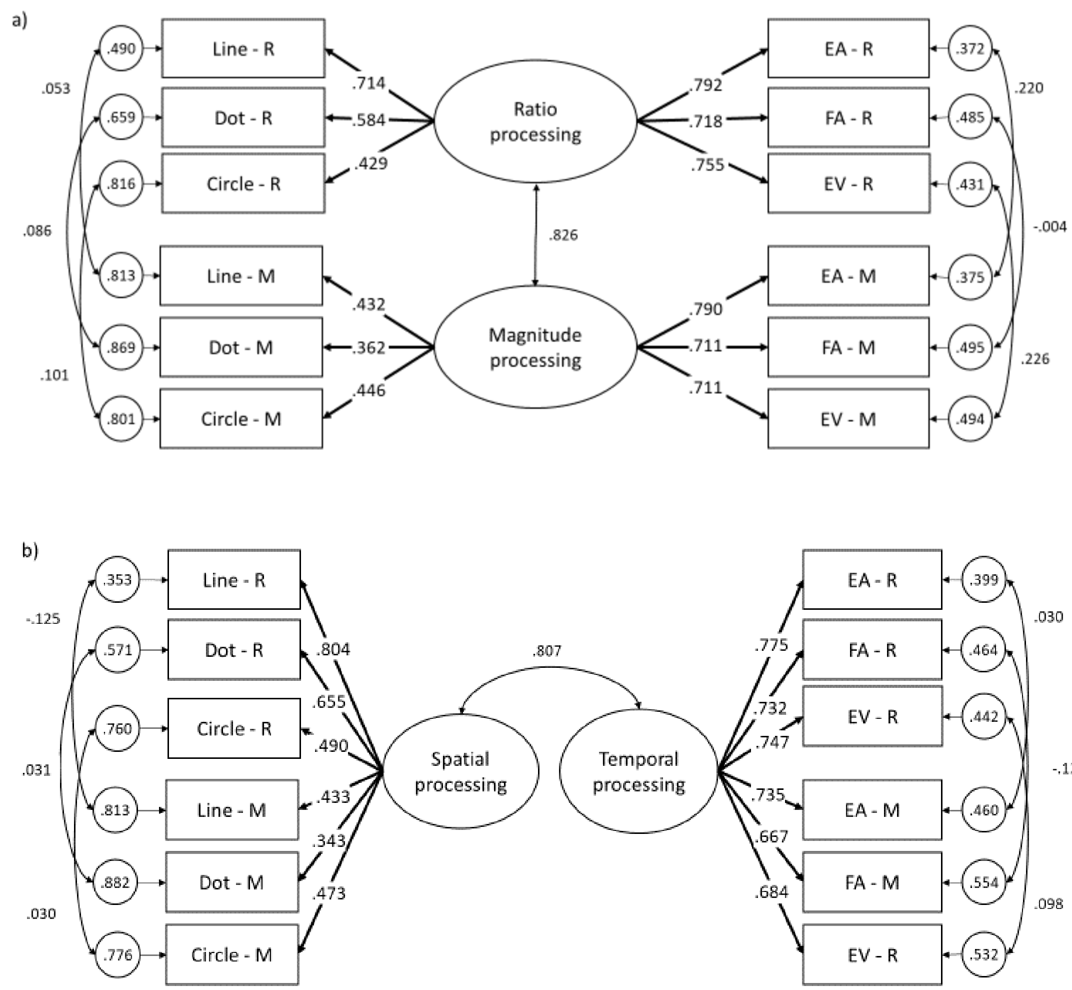

Figure 9. 2-factor CFA models with a) ratio and absolute magnitude factors and b) spatial and temporal factors. EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation. Parameter estimates are fully standardized.

4-Factor CFA. Contrary to previous models, the 4-factor model showed adequate model fit according to all fit indices. The 4-factor model also had significantly better fit than both 2-factor models (ratio and magnitude factors: χ^2 (5) = 45.75, $p < .001$; spatial and temporal factors: χ^2 (5) = 62.25, $p < .001$). Residual correlations indicate good local fit except for one task pair, circle ratio estimation and dot magnitude discrimination, which had a residual correlation of .10. Figure 10 shows the parameter estimates for this model.

All factor loadings were significantly different than 0 ($p < 0.001$), meaning that all latent factors explained a significant amount of variance in their respective indicator variables (i.e., the tasks). Loadings on temporal ratio and magnitude factors were high (.747-.809 and .711-.799 respectively) indicating that the factor indicators (i.e., temporal ratio estimation and discrimination tasks) were reliable measures of temporal ratio and magnitude processing. Construct reliability scores were greater than 0.7 (0.821 and 0.788 for the temporal ratio and magnitude factors respectively) indicating that these indicators were reliable measures of the latent construct (Morrison, Morrison, & McCutcheon, 2017). Next, we evaluated convergent validity of the temporal factors by computing the average variance explained (AVE) of each factor. AVE scores greater than 0.5 (i.e., latent factors that explain on average more than 50% of indicator variance) are considered to have adequate convergent validity (Morrison et al., 2017). Temporal ratio and magnitude factors showed adequate convergent validity as their AVE were 0.605 and 0.554 respectively.

In contrast, loadings on the spatial factors were lower and more variable (.485-.845 for spatial ratio processing and .383-.524 for spatial magnitude). Whereas the spatial ratio factor showed adequate construct reliability (0.782), the spatial magnitude factor showed poor construct reliability (0.472). Accordingly, both spatial factors showed poor convergent validity (AVE = 0.464 for spatial ratio and AVE = 0.225 for spatial magnitude). The model also revealed several large correlations between the latent factors (all statistically significant, $p < 0.001$) which may indicate low divergent validity. To verify divergent validity, we squared the correlation coefficient for each latent variable pair and verified whether the AVE of either variable in the pair was lower than the squared correlation coefficient (Morrison et al., 2017). Adequate divergent validity was only shown between spatial ratio and temporal magnitude latent factors.

Finally, tolerance and VIF values for both independent latent variables, spatial magnitude and temporal magnitude, revealed potential collinearity issues (Tolerance = 0.1279, VIF = 7.8168; Hair, Sarstedt, Ringle, & Mena, 2012). Despite this potential issue, we decided to keep the current specification for theoretical reasons and because the collinear independent variables are control variables (Allison, 2012). In the end, the 4-factor measurement model was retained for the structural analyses.

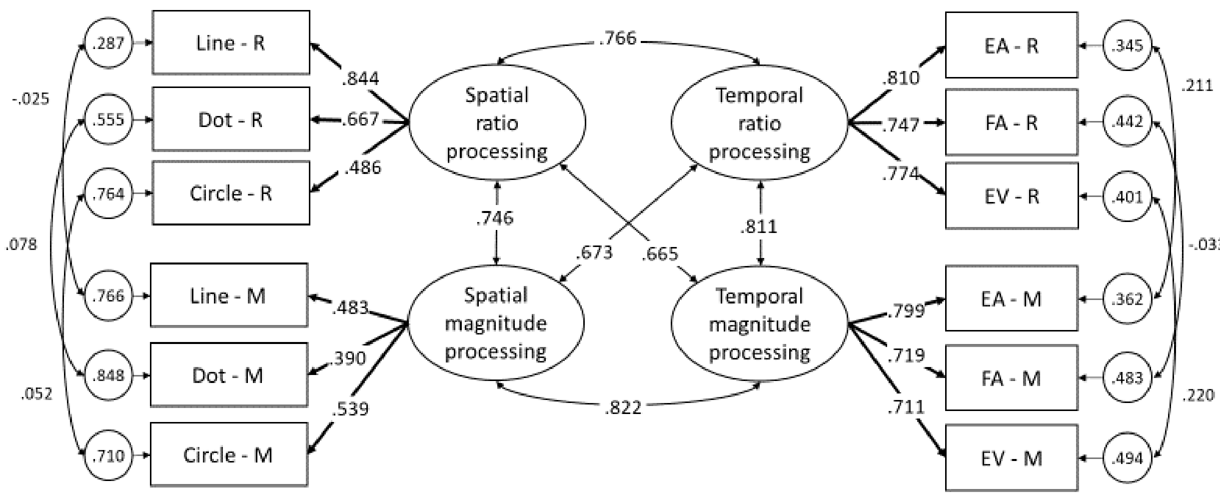

Figure 10. 4-factor CFA model. EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation. Parameter estimates are fully standardized.

Structural Model Fit

After determining that the 4-factor model was an appropriate measurement model, we added the regression paths between the latent variables and estimated the structural model. Given that this 4-factor SEM had the same number of paths between the latent variables as the measurement model, the model fit is the same between the CFA and SEM. We then tested the significance of the paths going across domains and magnitude type (i.e., from temporal magnitude to spatial ratio processing, and spatial magnitude to temporal ratio processing). To do this, we estimated a trimmed model in which these paths were constrained to zero. The resulting model had equivalent fit to the previous 4-factor SEM; the fit of the trimmed model was not significantly worse than the model with all paths included (χ^2 (2) = 0.288, p = .866). Other fit indices such as SRMR showed equivalent fit or, in the case of CFI and RMSEA, slightly better fit for the trimmed model. In addition, all residual correlations for the trimmed model were below .10, indicating good local fit. Because the fit indices indicate that these two models are equivalent and the two independent variables (i.e., spatial and temporal magnitude) were determined to be colinear, we retained the most parsimonious model. Figure 11 depicts the parameter estimates for the retained structure model.

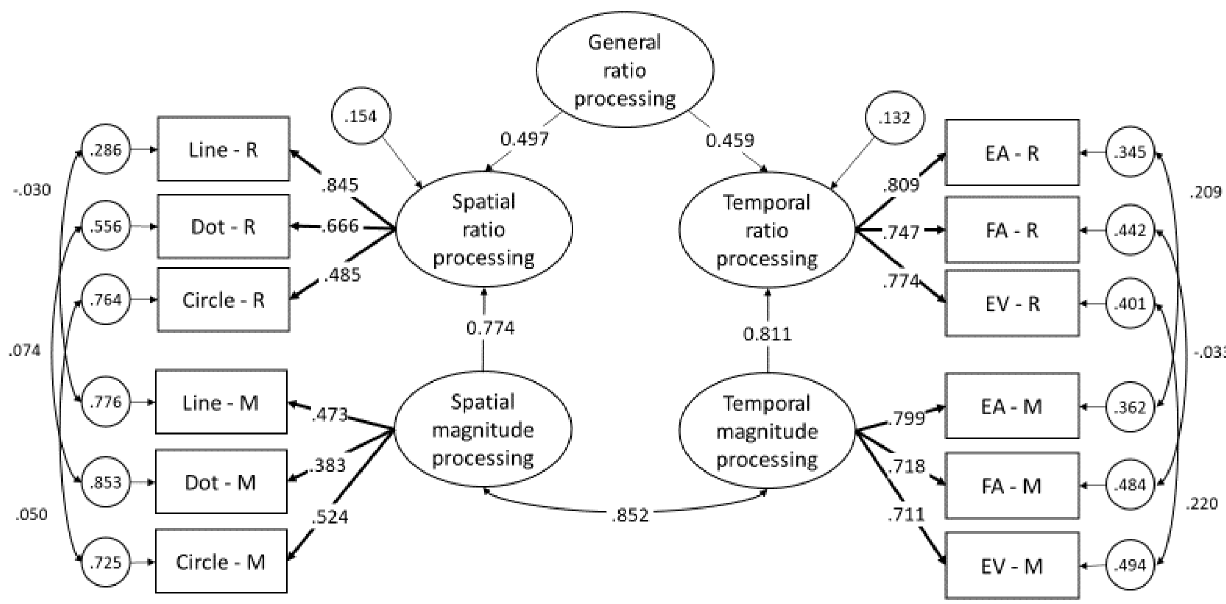

Figure 11. Retained structure model. EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation. All parameter shown in the figure are significant (p < .001), except for the loading between circle ratio and spatial ratio processing (p = 0.001). Values in circles are indicators residual variance.

Model Evaluation and Sensitivity Analyses

Overall, the model explains 85% of the variance for the spatial ratio processing and 87% of the variance for temporal ratio processing. Spatial magnitude processing significantly predicted spatial ratio processing (β=0.774, SE=0.053, $p < .001$, 95% CI [0.670, 0.878], 99% CI [0.637, 0.911]). This means a 1-unit standard deviation change in spatial magnitude processing was related to a 0.774 standard deviation unit change in spatial ratio processing. Temporal magnitude significantly predicted temporal ratio processing (β=0.811, SE=0.044, $p < .001$, 95% CI [0.725, 0.897], 99% CI [0.698, 0.924]). This means a 1-unit standard deviation change in temporal magnitude processing was related to a 0.811 standard deviation unit change in temporal ratio processing. Spatial and temporal magnitude processing were significantly correlated (.852, $p < .001$). Spatial and temporal ratio processing were significantly related by the general ratio processing factor (β=0.497, SE=0.061, $p < .001$, 95% CI [0.378, 0.616], 99% CI [0.340, 0.653], and β=0.459, SE=.070, $p < 0.001$, 95% CI [0.322, 0.597], 99% CI [0.278, 0.640], respectively). Given that these factor loadings were constrained to equality for model identification purposes, another way to look at the relationship between the two ratio processing factors is by looking at the correlation between the factors after controlling for absolute magnitude latent variables. This correlation is high (.615, $p < .001$), indicating that there is a significant correlation between spatial and temporal ratio processing once we control for absolute magnitude processing (see Figure B1 for model diagram with the relationship specified as a correlation instead of higher-order factor).

Sensitivity analyses were conducted on the 4-factor CFA model to investigate the influence of individual data points. Figure 12 shows index plots of the influence each participant exerted on the model fit and global parameter estimates. One participant was found to have a lot of influence on the global model (Figure 12a). This participant was not a previously detected outlier. Two participants, which were previously identified as multivariate outliers, had extreme influence on global parameter estimates (Figure 12b). Another 10 participants previously identified as multivariate outliers had moderate influence on global parameter estimates. We also analyzed each participant's influence on all factor loadings as well as the relationship of interest.

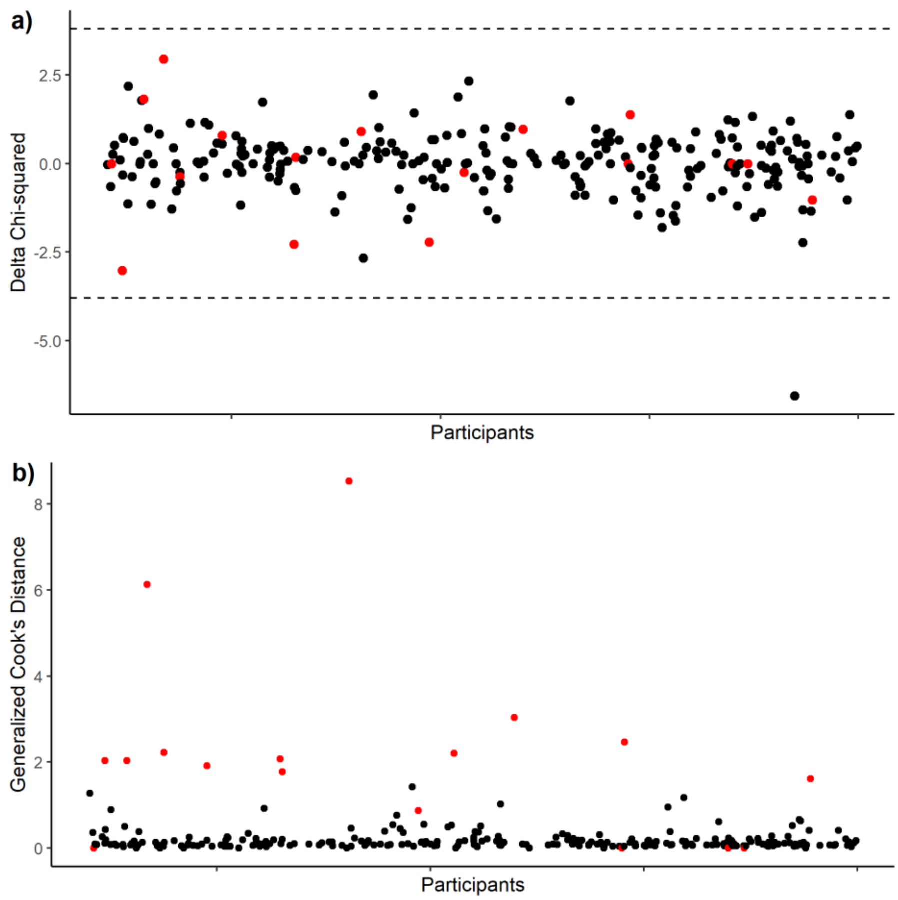

Figure 12. Index plots showing the influence individual participants on a) model fit and b) global parameter estimates. Dashed lines in the top plot indicate a significant difference in chi-square model fit (df = 1, p = 0.05). Each point represents a participant. Multivariate outliers are depicted in red.

Put together, 18 participants were identified as influential outliers, 14 of which were previously identified as multivariate outliers. These outliers were identified as participants causing a significant difference in model chi-square ($\Delta\chi^2_i$ >3.84), a $gCD_i > 2$ and data points exerting influence on most parameter estimates analyzed (more than 8/12 of the factor loadings). We refitted the retained 4-factor SEM model to the data without these influential outliers to verify that the previous findings were not driven by these outliers. Model fit remained adequate ($\chi^2(44) = 52.268$, $p = 0.18$, CFI = 0.989, RMSEA = 0.027 90% CI [0.00 – 0.051], SRMR = 0.034, AIC = -10054.813, BIC = -9891.377). Parameter estimates from the refitted model did not substantially change the conclusions of the previous analysis (see Appendix C for full solutions). In summary, these analyses indicate that the model is stable despite the presence of some influential data points and that these influential data points do not alter the conclusions drawn in the previous analysis. Full outputs from the sensitivity analyses are available at (https://osf.io/hw7kq/), including index plots showing the influence of each data point on individual parameter estimates.

Bifactor Model

The aim of estimating the bifactor model was to examine the amount of variance explained by a general ratio processing factor separately from specific ratio processing factors. Similar to the analysis of the higher-order 4-factor model, the bifactor model was fitted in two stages. The first stage consisted in fitting a bifactor CFA model in which the three ratio processing variables are orthogonal, and each ratio processing variable is correlated to both spatial and temporal ratio processing. The resulting model showed adequate fit (χ^2 (41) = 50.37, $p = 0.15$, CFI = 0.990, RMSEA = 0.031 90% CI [0.00 – 0.057], SRMR = 0.026, AIC = -10168.65, BIC = -10147.51). Unfortunately, the model did not pass the empirical identification check, indicating that the model is not identified (i.e. there is not a unique solution). Sensitivity analyses confirmed the instability of the bifactor model. Given that the solution is inadmissible, results for this analysis will not be presented to avoid erroneous interpretation.

Exploratory analyses

Reliability of the discrimination tasks

In the previous section on reliability and validity of the measures, indicators of the spatial magnitude factor were shown to have poor reliability. As a first set of exploratory analyses, we explored whether this poor reliability could be explained by the deviant ratios used in this study. When designing the study, we chose sets of deviant ratios on which participants performed above chance, but difficult enough to avoid ceiling effects. However, we did not verify that these deviant ratios had adequate reliability. Therefore, we used confirmatory factor analyses to examine the reliability of the deviant ratios. Descriptive statistics for the deviant ratios are shown in Table 5. Participants were above chance on all deviant ratios ($p < 0.001$).

Table 5. Descriptive statistics on proportion correct for the deviant ratios used in each discrimination tasks.

Discrimination Task	Deviant ratio	N	Mean (SD)	Range (min-max)	Skewness	Kurtosis
Empty auditory interval (EA-M)	1.20	269	.744 (.178)	0.167 - 1	-0.567	2.881
	1.25	269	.782 (.156)	0.375 - 1	-0.661	2.988
	1.30	269	.811 (.164)	0.25 - 1	-0.765	3.126
	1.40	269	.863 (.149)	0.25 - 1	-1.321	4.700
	1.60	269	.913 (.136)	0.25 - 1	-2.224	9.368
Filled auditory interval (FA-M)	1.20	269	.781 (.152)	0.25 - 1	-0.673	3.256
	1.25	269	.818 (.143)	0.125 - 1	-0.911	4.530
	1.30	269	.862 (.126)	0.375 - 1	-0.952	3.960
	1.40	269	.903 (.117)	0.375 - 1	-1.323	4.924
	1.60	269	.946 (.092)	0.5 - 1	-2.086	8.014
Empty visual interval (EV-M)	1.20	270	.744 (.167)	0.143 - 1	-0.647	3.262
	1.25	270	.779 (.167)	0.125 - 1	-0.862	3.977
	1.30	270	.81 (.164)	0.286 - 1	-0.770	3.099
	1.40	270	.857 (.152)	0.375 - 1	-1.128	3.800
	1.60	270	.908 (.135)	0.25 - 1	-1.968	7.389
Circle area (Circle-M)	1.02	271	.663 (.156)	0.25 - 1	-0.202	2.860
	1.04	271	.768 (.16)	0.375 - 1	-0.296	2.271
	1.06	271	.851 (.15)	0.375 - 1	-1.021	3.545
	1.08	271	.912 (.107)	0.5 - 1	-1.078	3.758
	1.18	271	.987 (.054)	0.5 - 1	-5.355	37.930
Dot number (Dots-M)	1.09	271	.78 (.149)	0.375 - 1	-0.467	2.831
	1.10	271	.764 (.144)	0.25 - 1	-0.569	3.129

	1.12	271	.858 (.122)	0.5 - 1	-0.399	2.219
	1.14	271	.852 (.123)	0.375 - 1	-0.913	3.951
	1.25	271	.959 (.077)	0.625 - 1	-1.887	6.028
Line length (Line-M)	1.01	271	.695 (.163)	0.125 - 1	-0.181	2.828
	1.02	271	.721 (.155)	0.375 - 1	-0.228	2.485
	1.03	271	.837 (.147)	0.375 - 1	-0.683	2.814
	1.06	271	.947 (.091)	0.5 - 1	-1.885	6.589
	1.12	271	.988 (.052)	0.5 - 1	-5.736	42.218

We then estimated a 2-factor model with spatial magnitude and temporal magnitude as latent variables. Each factor was composed of three latent subfactors (i.e., the tasks): line length, circle area and dot number discrimination for the spatial magnitude factor, and empty auditory, full auditory and empty visual interval discrimination for the temporal magnitude factor. Each subfactor was composed five indicators corresponding to the deviant ratios. The model was estimated using full information maximum likelihood to handle missing data and robust maximum likelihood to handle the non-normality of the factor indicators. The model yielded adequate fit (χ^2 (398) = 399.828, p = 0.47, CFI = 0.998, RMSEA = 0.004 90% CI [0.00 – 0.023], SRMR = 0.047, AIC = -10168.65, BIC = -10147.51). The model along with fully standardized parameter estimates are shown in Figure 13. From this figure, we observed two deviant ratios that are potentially problematic, that is the hardest deviant ratio for both the line and circle discrimination task. These two loadings were not significantly different from zero (β=0.110, SE=0.072, p = 0.123, 95% CI [-0.030, 0.250], 99% CI [-0.074, 0.295], and (β=0.112, SE=0.064, p =0.08, 95% CI [-0.013, 0.238], 99% CI [-0.053, 0.277]), respectively). This indicates that these deviant ratios might not be appropriate or reliable measures of magnitude discrimination even though participants are, on average, above chance.

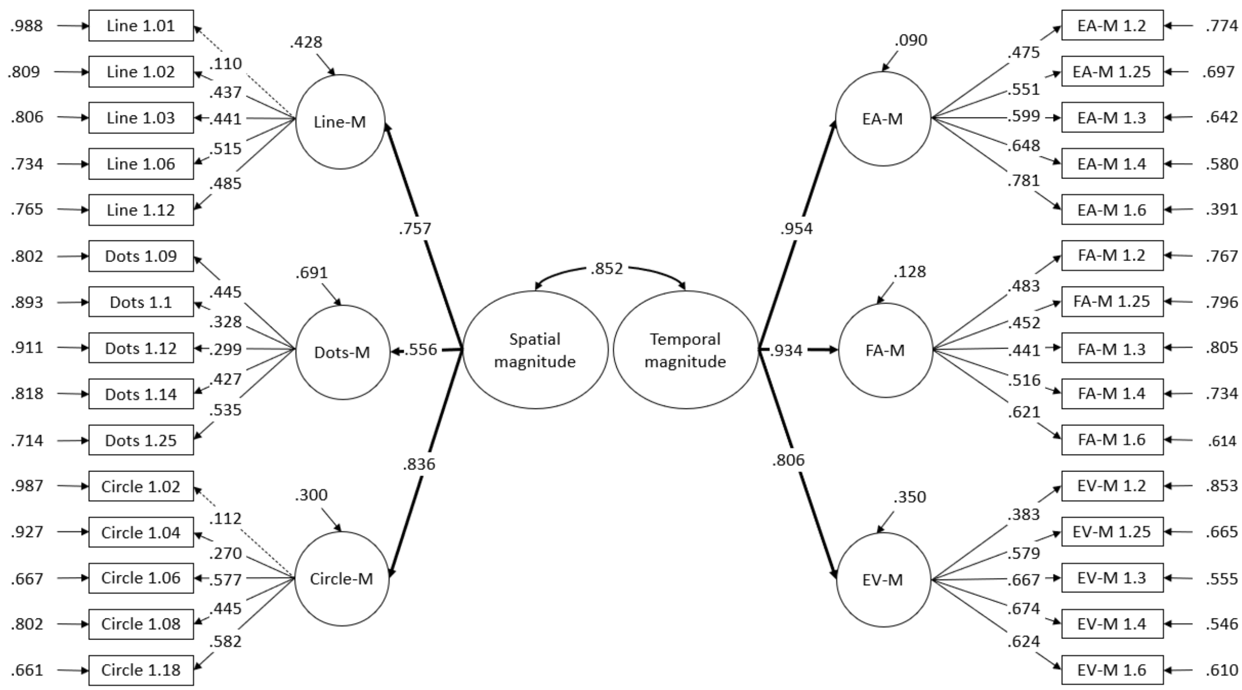

Figure 13. Deviant ratio CFA model. EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination. Parameter estimates are fully standardized. Dashed lines indicate that the parameter estimate was not significantly different from 0 (p >0.05).

We then wanted to investigate whether these poor deviant ratios impacted the results found in the previous SEM analysis. We recalculated the aggregate scores excluding the problematic deviant ratios (Line 1.01 and Circle 1.02) and re-estimated the high-order model. Model fit remained adequate ($\chi^2(44) = 39.896$, p = 0.648, CFI = 1.000, RMSEA = <0.001 90% CI [0.00 – 0.037], SRMR = 0.026, AIC = -10145.147, BIC = -10125.302) and most parameter estimates showed little change, qualitatively speaking. The factor loading for line discrimination indicator improved slightly ($\beta = 0.473$ to $\beta = 0.530$) and the correlation between spatial and temporal ratio slightly decreased (.615 to .564). However, these changes did not impact our conclusions. Full solutions can be found in Appendix D.

Number line estimation response bias

One possible explanation for the correlation between the spatial and temporal ratio factors is that participants have a similar response bias that is unrelated to their ratio processing ability. For example, a person might have a bias away from the response line extremities thereby causing them to have a lower slope while being highly precise (Figure 14 b). This leads to a worse average absolute error score even though they are just as precise as an individual with a slope near 1 (Figure 14a). If people have consistent biases, the use of the number line for all ratio tasks may inflate correlations between the two ratio processing factors. To test whether the results found in the previous sections may be influenced by response bias, we refit the final model using R^2 as an accuracy measure instead of average absolute error. R^2 was extracted from a linear model fit to each participant's response in every task following data cleaning. The advantage of using R^2 as a measure of ratio estimation accuracy is that it measures precision relative to the slope for each task and participant and it is also robust to problems created to non-typical response patterns. The disadvantage is that this measure might not capture the accuracy of individual's ratio estimation relative to the true stimulus ratio. Also, R^2, like average absolute error, does not differentiate response biases when the participant as has low precision (Figure 14 c and d).

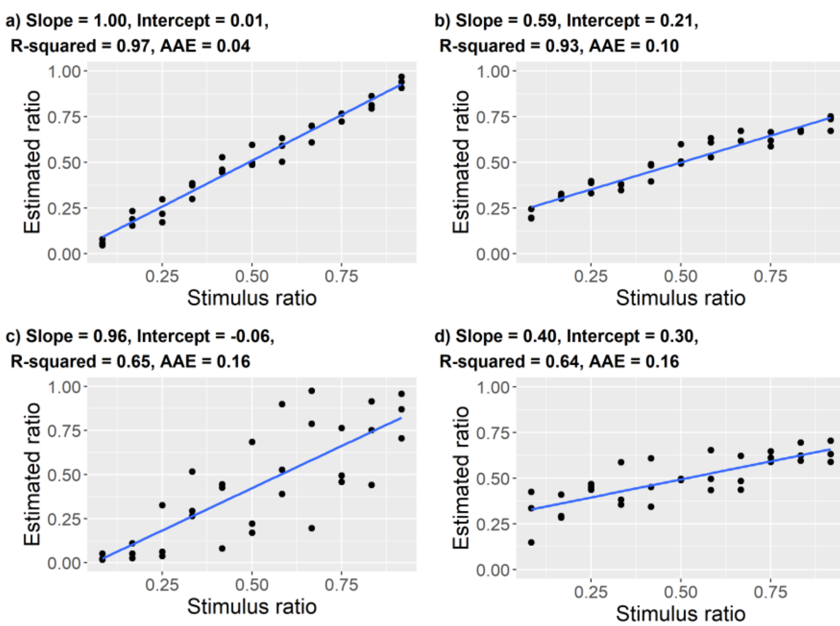

Figure 14. Examples of response bias influencing average absolute error (AAE). a) Perfect slope with high precision. b) Low slope with high precision. c) Near perfect slope with low precision. d) Low slope with low precision.

Model fit remained adequate ($\chi^2(44) = 53.013$, $p = 0.166$, CFI = 0.990, RMSEA = 0.031 90% CI [0.00 – 0.058], SRMR = 0.031, AIC = -5518.733, BIC = -5353.036). Parameter estimates were similar to the ones previously reported, with two notable exceptions. First, the loading for the circle ratio estimation task went from $\beta = .485$ (SE = .078) to $\beta = .727$ (SE = .078). The spatial ratio factor now showed adequate construct reliability (.844) and convergent validity (.594), but divergent validity did not change compared to the original model. Second, the residual correlation between spatial and temporal ratio processing went from .615 (SE = .120) to .723 (.116). Complete unstandardized and standardized solutions can be found in Appendix E.

Effect of education and music experience

Finally, we tested whether task performance was related to previous experience. For example, musicians have been shown to perform better on certain temporal tasks (Banai, Fisher, & Ganot, 2012; Rammsayer & Altenmuller, 2006; Vibell, Lim, & Sinnett, 2021). If populations with different prior experience perform differently on the tasks used in this study, a deeper study of measurement invariance, that is, how measures vary across different populations, would be warranted. To examine the effect of prior experience, we analyzed the correlations between each task, and years of education, years of music training and years of music playing experience. None of the correlations were statistically significant (range -0.081 to .118), except for one marginally significant negative correlation between education and dot discrimination (-.139, $p = 0.02$; see Appendix F for correlation matrix). This correlation was not found to be significant when multiple comparisons corrections were applied.

Chapter 4

Discussion

The aim of the current study was to examine whether ratios across spatial and temporal domains are processed by a common ratio processing system (RPS). If ratios in space and time are processed by a common mechanism, then individuals' ability to process ratios in space (either in length, area or numerosity) should correlate with their ability to process ratios in time. To test this hypothesis, we measured people's ability to reproduce spatial and temporal ratios on a bounded line and modeled their relationships using SEM. In the first analysis, our higher-order model revealed a significant relationship between spatial and temporal ratio processing abilities when controlling for absolute magnitude processing, indicating that spatial and temporal ratios might be processed by a common mechanism. Whereas previous studies have shown that spatial ratios are processed by similar mechanisms across different symbolic formats (Jacob & Nieder, 2009a) as well as different non-symbolic magnitudes, such as length and numerosity (Jacob & Nieder, 2009b; Matthews et al., 2015), this is the first study to show a relationship between ratio processing across spatial and temporal domains. The findings in the current study therefore support the existence of the RPS proposed by Matthews et al. (2015) and extend the theory beyond symbolic and non-symbolic (spatial) ratios to spatial and temporal ratios.

Our model also revealed that people's ability to discriminate absolute magnitudes is an important predictor of their ability to process ratios in both the spatial and temporal domain. This finding replicates the relationship found in a secondary analysis on data from Park et al. (2021; see Appendix A) in the spatial domain, and aligns with findings by Jacob & Nieder (2009b) who showed that absolute magnitude and ratio processing have overlapping neural correlates. The strong relationship between absolute magnitude and ratio processing factors was also extended to the magnitudes and ratios in the temporal domain which is a novel finding.

Finally, spatial discrimination tasks were found to load poorly on the spatial magnitude factor. This suggests that absolute magnitude processing between different spatial magnitudes only partially overlap. However, we also found a strong relationship between spatial and temporal magnitude factors, which supports the existence of a common magnitude system. We therefore suggest that the mechanism associated to the partial overlap between the spatial magnitude tasks is the same mechanism responsible for magnitude discrimination in the temporal domain. More specifically, the common magnitude system might be, as other authors have suggested, a higher-order mechanism responsible for magnitude comparison that operates only after magnitudes have been encoded by neuronal populations tuned to those specific types of magnitude (Cohen Kadosh, Lammertyn, & Izard, 2008; Holloway & Ansari, 2008; Pinel et al., 2004). This would align with previous single cell studies showing that separate but overlapping neuronal populations encode information for different types of visual magnitudes, while other neuronal populations, possibly part of larger fronto-parietal network responsible for general magnitude processing, encode magnitude information across different types of visual magnitudes (Eiselt & Nieder, 2013; Nieder, Diester, & Tudusciuc, 2006; Tudusciuc & Nieder, 2007b).

In our second analysis, we attempted to estimate a bifactor model to examine how much variance in each task was common across all the ratio tasks (domain general), or common to only the spatial or temporal ratio tasks (domain specific). Unfortunately, the bifactor model did not yield an admissible solution. Therefore, although we showed in the first analysis that there is a relationship between spatial and temporal latent factors, we cannot make any specific conclusions about how much of the performance in ratio estimation tasks is explained by a domain general ratio processing factor vs domain specific ratio processing factors.

The measurement models

In the first part of the analysis, four measurement models were estimated. The 1-factor model tested the hypothesis that all tasks could be explained by a single general factor.

This model showed the worst fit out of the four models. Next, two 2-factor models tested whether the data could be modeled using either a spatial and temporal factor, or a magnitude and ratio factor. Although these two models fit the data better than the previous 1-factor, other model indices, such as CFI and the chi-squared test statistic, indicated that both 2-factor models poorly fitted to the data. Out of all the measurement models estimated, the 4-factor CFA model had the best fit according to all fit indices used, indicating that the performance on the tasks can be modeled by four separable constructs: spatial magnitude, spatial ratio, temporal magnitude, and temporal ratio processing.

For temporal processing, auditory and visual tasks related to both ratio and magnitude processing loaded highly on their respective factors. They also had adequate construct reliability and convergent validity, indicating that the tasks were tapping into general timing ability, rather than modality specific timing ability. This is consistent with previous literature showing that, although individuals tend to have a higher temporal resolution in the auditory modality than the visual modality, both use the same underlying timing mechanism when longer stimuli (near the 1-second range) are used (Rammsayer, Borter, & Troche, 2015; Stauffer, Haldemann, Troche, & Rammsayer, 2012).

Tasks related to spatial ratio processing had more variable loadings. Although they showed adequate construct reliability, we failed to show convergent validity because one of the tasks, circle ratio estimation, loaded poorly on the spatial ratio factor. These loadings are similar, though lower, to loadings obtained in a secondary analysis conducted on existing data from Park et al. (2021). This secondary analysis had initially been conducted in order to do the a-priori power analysis for the current project. One important difference between the current study and the one conducted by Park and colleagues is that the current study used ratio estimation tasks while Park et al. used ratio discrimination tasks. In ratio discrimination tasks, participants are shown two pairs of stimuli, and must determine which has the largest ratio. The difference between the ratio task loadings found in previous studies and the current study could therefore be explained by the type of the ratio task used. More specifically, the Park et al. ratio discrimination tasks measure ratio perception and comparison whereas the current ratio estimation tasks measure ratio perception combined with ratio estimation.

Tasks related to spatial magnitude had low loadings and did not have adequate construct reliability or convergent validity. This is surprising because the secondary analysis from Park et al. (2021), which used similar discrimination tasks, yielded factor loadings between .74 and .81, which is far greater than the loadings found in the current study. Though the tasks were closely modeled after the ones used in Park et al. (2021) as well as other studies by that research group (Matthews et al., 2015; Park & Matthews, n.d.), the difference in factor loadings could be partially explained by the difference in sample populations. The sample used by Park et al. consisted of 2nd graders, 5th graders and undergraduate students whereas the sample of the current study consisted of adults in the general population.

Another possible explanation for low loadings is that some of the task parameters chosen were not themselves reliable, thereby introducing noise into the measures. For example, the smallest deviant ratio (i.e., the ratio between the standard and comparison stimuli) might have been too small for participants to discriminate the difference without partially guessing. To test this hypothesis, we estimated a CFA using the deviant ratios as indicators and examined how well each deviant ratio loaded onto its respective task. This analysis showed that the smallest deviant ratio in the line and circle discrimination tasks loaded poorly, indicating that they are perhaps not reliable measures of line length and circle area discrimination, even though participants performed above chance on average on these deviant ratios. To further examine the influence of these unreliable deviant ratios, we estimated the final structural model without the problematic deviant ratios and examined the change in parameter estimates. Although the reliability of line discrimination task slightly improved, other parameter estimates remained stable, suggesting that the reliability of the deviant ratios did not have an important effect on the final higher order model.

As discussed above, low factor loadings for the spatial magnitude factor could also indicate that the mechanisms related to length, number and area discrimination only partially overlap. The low correlations between the spatial magnitude tasks align with other studies that have investigated relationships between different non-symbolic (spatial) discrimination tasks. Although these tasks are assumed to measure non-symbolic magnitude ability, they were often shown to correlate poorly (Clayton, Gilmore, & Inglis, 2015; DeWind & Brannon, 2016; Gilmore, Attridge, & Inglis, 2011).

In addition to testing construct reliability and convergent validity, we examined the divergent validity of the latent variables. Divergent validity gives insight into whether the latent factors are truly different constructs given their reliability. Although we consider latent factors to be theoretically divergent (e.g., spatial magnitude discrimination tasks are assumed to measure a different construct from temporal magnitude discrimination tasks), we could only show divergent validity between the spatial ratio and temporal magnitude factors, but not between any of the other pairs of factors. Although this seems to contradict our earlier statement that the data can be modeled by four separable factors, neither of the other measurement models with one or two factors showed adequate model fit. Therefore, we suggest that the data can be modeled by four separable constructs that are also highly related.

Relationships between spatial and temporal ratio and magnitude latent factors

After showing that the 4-factor model was the best measurement model, two 4-factor structural models were estimated and compared. The first model controlled for within domain magnitude factors (e.g., spatial magnitude and spatial ratio) as well as between domain magnitude factors (e.g., temporal magnitude and spatial ratio). The between domain paths (i.e., temporal magnitude and spatial ratio, and spatial magnitude and temporal ratio) were included to control for leftover variance related to general cognitive ability. Conversely, we had little theoretical reason to believe that these paths would be significantly different from zero. The second model tested this hypothesis by removing the between domain paths and controlling only for within domain magnitude factors.

Although the second model had two paths less than the first model, it was retained as it fit the data equally well to the first model and was the most parsimonious. This suggests there is no evidence for additional mechanisms that are involved in linking magnitude and ratio factors across domains other than the common mechanisms relate to absolute magnitude perception and ratio processing. Additionally, the second model eliminated a collinearity issue present in the first model due to the high correlation between the two magnitude factors.

The retained model revealed a significant relationship between people's ability to estimate spatial and temporal ratios when controlling for people's ability to discriminate absolute magnitudes. This supports the hypothesis that ratio processing in the spatial and temporal domain are related. These results do not, however, answer the question as to how they are related. More specifically, each ratio processing factor has two sources of common variance: the ability to perceive the stimulus ratio and the ability to estimate a given ratio. The current analysis does not allow us to determine whether the common variance is related to ratio perception ability across different types of magnitude, or both ratio perception and estimation abilities (though the latter option seems the most likely). It also seems unlikely that results are solely a product of task demands as the precision of ratio estimation is limited by the precision of ratio perception by principle of precedence.

In addition to finding a significant relation between spatial and temporal ratio processing, magnitude processing factors were also found to be significant predictors of ratio processing factors. This finding indirectly replicates the relationship found in the secondary analyses conducted on data by Park et al. (2021). Though the replication is indirect because Park et al. (2021) used ratio discrimination tasks instead of estimation tasks, it also provides some validity to the relationships between latent factors found in the current study despite the poor measurement qualities of the spatial magnitude factor. Our results also extend the ratio-magnitude relationship found in the spatial domain to temporal domain. Lastly, these significant relations between magnitude and ratio processing factors are interesting in themselves because it means that most of the variance in ratio processing factors is explained by participants' ability to discriminate absolute magnitudes. For example, the spatial magnitude factor explained 60% of the variance in spatial ratio processing factor, while the general ratio processing factor only explained 25% of the variance in the same factor. Similarly, the temporal magnitude factor explained 66% of the variance on the temporal ratio processing factor, while the general ratio processing factor only explained 21% of the variance in the same factor. This strong relationship between within-domain magnitude and ratio factors may indicate a source of domain specific processes. Namely, individuals' performance on ratio estimation tasks is largely explained by absolute magnitude processes that are related to the magnitude type, and thereby are specific to either the spatial or temporal domain. These findings align with previous neuroimaging studies on ratio processing and absolute magnitude processing in both humans and monkeys. Jacob & Nieder (2009b) found that neural correlates associated with non-symbolic (spatial) ratio processing were similar to the correlates associated with non-symbolic absolute magnitude processing in the prefrontal and parietal cortices in both humans and monkeys (Jacob & Nieder, 2009b; Vallentin, Jacob, & Nieder, 2012; Vallentin & Nieder, 2010).

Conversely, our ability to make conclusions about domain-specific mechanism is limited. The high correlation between the two magnitude factors indicates that they are controlling for much of the same variance in each ratio factor. Therefore, we cannot say with certainty that the variance explained by the magnitude factors is purely domain specific. Statistically, this is a problem of collinearity and can be illustrated using a multiple regression framework. Imagine a regression with two independent variables and one dependent variable. If the independent variables are uncorrelated, then each independent variable will account for unique variance. However, the closer the correlation between the two independent variables is to 1, the more the independent variables account for the same variance, and the two independent variables can be seen as indicators of the same underlying construct. In the context of the current results, both magnitude factors could be indicators of a general magnitude system and therefore control for general magnitude processing in addition to some within domain magnitude processing (specific to temporal or spatial magnitudes).

Finally, the high correlation between spatial and temporal magnitude factors was unexpected. We can think of two plausible explanations for this relationship. The first is that the spatial magnitude factor, which was shown to have low convergent validity, is measuring general processes (e.g., working memory and decision making) needed to successfully complete the discrimination task rather than (or in addition to) spatial magnitude acuity. This explanation would align with findings by Marcos et al. (2016) showing that discrimination for different type of magnitudes might share neuronal populations related to decision making, but not magnitude processing. However, to obtain a high correlation of 0.85 between spatial and temporal magnitude factors, the temporal magnitude factor would also have to be measuring general cognitive processes related to magnitude comparison rather than temporal magnitude acuity. This seems unlikely given that the correlations between domains (i.e., spatial magnitude with temporal ratio, and temporal magnitude with spatial ratio) were lower than the correlations within domains (i.e., spatial magnitude with spatial ratio, and temporal magnitude with temporal ratio), indicating that there is some domain specificity to the magnitude factors (i.e., they are not solely measuring domain general processes).

Another possibility is that spatial and temporal magnitudes are processed by a common magnitude system, as suggested by the ATOM theory (Bueti & Walsh, 2009; Walsh, 2003). In contrast to the possibility outlined in the previous paragraph, this domain general process would be specific to magnitude encoding (e.g., magnitude comparison) rather than unrelated to magnitude (e.g., general decision making). As stated previously, discrete and continuous spatial magnitudes such as numerosity and length have been shown to be encoded by distinct but overlapping neuronal populations as well as by neurons encoding both types of magnitudes (Tudusciuc & Nieder, 2007a). Cona, Wiener, & Scarpazza (2020) have also shown in a meta-analysis that neuronal populations encoding spatial and temporal magnitudes might be organized in a gradient like way in the same brain areas, such as the insula, (pre-) supplementary motor area (SMA) and the intraparietal sulci (Cona et al., 2020). Therefore, spatial and temporal magnitudes may be processed similarly to how different types of spatial magnitudes are processed: although neurons encoding spatial and temporal magnitude partially overlap, there may be distinct populations encoding spatial and temporal magnitudes as well as neuronal populations processing higher order magnitude information across the two domains.

Model robustness and effect of prior experience

Multiple exploratory analyses were conducted to determine the robustness of the results obtained in the higher order model. First, sensitivity analyses were conducted to test whether the current results were due to the presence of influential outliers or to the violation of the assumption of multivariate normality. Though we did detect the presence of some influential outliers, exclusion of the outliers did not cause a qualitatively significant change in the model parameter estimates. This indicates that results were not due to influential outliers or the violation of the multivariate normality assumption.

Second, we examined the effect of unreliable deviant ratios in the spatial magnitude tasks on the model parameter estimates. Removing unreliable deviant ratios in two of the three spatial discrimination tasks resulted in the slight increase of one of the spatial discrimination tasks. However, the other model parameter estimates remained qualitatively similar to the original model parameter estimates.

Finally, we refit the model using an alternative measure of ratio processing accuracy (R^2). Whereas average absolute error is based on the absolute difference between the stimulus ratio and the estimated ratio, R^2 is based on how linearly precise estimations are across stimulus ratios. Using R^2 over average absolute error has an advantage in that it is less vulnerable to individuals' estimation bias, such as avoiding extremities, which would lead to a worse average absolute error despite having precise estimations. R^2 also has a disadvantage as it is not a measure of ratio estimation accuracy per se (i.e., how far away is an individual estimation to the true ratio), but a measure of relative precision (i.e., how consistent is an individual in their estimation of one ratio relative to other ratios, regardless of the true stimulus ratio).Though most parameter estimates did not vary from the original model fit using average absolute error, the factor loading for the circle ratio task greatly improved leading to increased construct reliability and convergent validity of the spatial ratio factor but same lack of divergent validity. This also led to a higher correlation between spatial and temporal ratio processing factors. Taken together, this indicates that findings are robust to the accuracy measure used and somewhat robust to linear response biases, though the impact of non-linear response biases remains unknown. Furthermore, this suggests R^2 might also be a more reliable measure of ratio processing than average absolute error given that it is less vulnerable to certain types of response bias.

In addition to assessing the robustness of the higher-order model, we tested whether there was a relationship between prior experience such as number of years of education, music experience or music training and any of the tasks used in the current study. Neither music experience or training, nor years of education were correlated with any of the spatial or temporal tasks. These findings suggest that performance on these tasks is little or not affected by prior experience, perhaps due to their perceptual nature. While there has been little literature on the relationship between temporal ratio estimation and music experience, the current results do not align with other studies that have found musicians to be better at duration discrimination than non-musicians (Banai et al., 2012; Ehrlé & Samson, 2005; Rammsayer & Altenmuller, 2006).

Strengths and Limitations

The current study has strengths and limitations. First, the use of a ratio estimation task is considered a strength, but also presents some limitations. The advantage of using a ratio estimation task with the same response line across all spatial and temporal tasks is that participants are doing the same operation in all tasks: they are transforming a ratio from one (spatial or temporal) format to a representation of that ratio on a line. This limits differences between domains that may be induced by a discrimination task. For example, a temporal discrimination task in which two divided intervals are presented sequentially is probably more demanding on working memory than a spatial ratio discrimination task in which two ratios are presented side by side simultaneously. However, this might also induce elevated correlations between the two domains because the response format is the same (i.e., the common method is contributing to the shared variance). We provide two counter arguments. First, though the common method may be problematic for studies in which the method is orthogonal to the subject of interest, the response format (ratio estimation) is directly related to the subject of interest in the current study (ratio processing). Second, one might argue that, even though the response format is directly related to the subject of interest, participants may still have response biases unrelated to their ratio processing ability (e.g., avoiding making estimations at either end of the response line). We acknowledge this limitation, but simultaneously show that using another measure of estimation accuracy less susceptible to linear biases (R^2) leads to the similar results and conclusions.

Second, one strength of the study is that the sample consisted of a wide range of individuals across 11 countries with varying levels of education, making results more representative of the general population than is typically the case. However, there might still be some self-selection bias which limits generalizability. For example, participants who choose to do hour long studies instead of multiple short studies might have a longer attention span. Participants who chose to participate in this study might also be more interested in doing numerical tasks than participants who chose not to participate in this study.

Third, the study was conducted online which may have impacted data quality. Although most participants performed well on all tasks, some participants may have performed poorly for various reasons such as fatigue or interruptions. For example, some participants performed very well on all but one task, but we were unable to determine with certainty whether the poor performance was a true reflection of their ability or caused by an unrelated external factor. Some participants might have also benefited from going over instructions with an experimenter or from having more practice trials to understand and perform well on the tasks.

Finally, the low reliability and convergent validity associated to the spatial magnitude tasks is problematic. As mentioned before, this might indicate that the tasks lacked reliability. Despite the low factor loadings, the spatial magnitude factor still showed strong relationships with the other more reliable latent variables and replicated findings from a secondary analysis on similar tasks (Park et al., 2021). Given that the correlations between the latent variables seem reasonable, we think we are measuring absolute magnitude processing to some degree, along with domain general processes related to magnitude comparison.

In a related issue, the strength SEM, which is the ability to estimate the true measures of latent constructs, also has limitations. SEM allows the researcher to parse the "true score" of an underlying construct from the error variance. Assuming observed measures are reliable and valid, this makes relations between constructs more generalizable as they are less vulnerable to the measurement error associated to specific tasks. However, the common variance (i.e., true score) estimated by the factor analysis is data driven and might not be a pure measure of the hypothesized construct.

Future directions

We propose three avenues for future research. First, future directions should include replicating the current findings as well as extending them to other populations. For example, studying individuals with either numerical, spatial or temporal deficits might give insight as to how absolute vs relative magnitude processing are related across different types of magnitude. Studying the developmental trajectories related to ratio processing might also give insight into the underlying mechanisms related to absolute and relative magnitude processing (Park et al., 2021). Given that all the factors are highly correlated with each other, it could be interesting to see if absolute magnitude and relative magnitude processing diverge, converge, or develop in parallel throughout development. This might give on insight on how different or similar magnitude comparison mechanisms are to relative magnitude mechanisms (i.e., mechanisms that allow us to quantify relationships between magnitudes). Lastly, future studies could also extend the current findings by including ratio discrimination tasks to parse out the mechanisms related to ratio perception vs. ratio estimation.

Second, future research should investigate neural mechanisms associated with ratio processing. As previously mentioned, behavioural studies can help uncover associations based on the study of variance in performance. However, the ultimate goal is to understand the neural mechanisms responsible for ratio processing which requires going beyond behavioural paradigms. Although some neuroimaging studies of ratio processing have already been conducted, they are limited to numerical and spatial magnitudes. Therefore, future studies should investigate ratio processing using neuroimaging techniques to understand how ratios in different domains are processed in the brain.

Third, future directions could involve studying the ratio estimation patterns themselves. Individuals may have different prototypical estimation patterns such as a categorical pattern, a bias against extremities, or patterns of bias predicted by the cyclical power model (Hollands & Dyre, 2000; Spence, 1990). Although this study found no relationship between demographic variables such as music training and performance on temporal task, the estimation patterns, rather than overall accuracy, may be related to certain demographic characteristic (e.g., music training) or deficits related to ratio perception or estimation.

Conclusion

From the progress bars on our screens to eighth notes in music, ratios are present everywhere: we cannot help but think in a relative way. The current study has important implications as it is the first, to our knowledge, to investigate how ratio processing in space and time are related, thereby extending the literature on ratio processing beyond the numerical and visuo-spatial domain. Results indicate that spatial and temporal ratio processing are related even when controlling for absolute magnitude processing ability, and that absolute magnitude processing is a significant predictor of ratio processing in both spatial and temporal domains. This supports the idea of a common ratio processing system operating across different domains. We also show that spatial magnitude discrimination is highly related to temporal magnitude discrimination, which supports the idea absolute magnitudes might also be somewhat processed by a common magnitude system across domains. However, more research is needed to uncover the underlying neural mechanisms related to ratio processing across these two domains. Taken together, extending the scope of ratio processing ability from a domain specific ability associated with numerical cognition to a domain general framework has important implications as it may eventually allow us to understand more complex behavior such as how people make relative judgments on stimuli with no clear magnitude, a common practice in psychology research.

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Appendices

Appendix A: Secondary analysis on the relation between spatial magnitude and ratio processing and power analysis

This appendix reports results of a secondary analyses conducted on data from Park, Viegut, & Matthews (2021). The goal of the analyses was to obtain plausible parameter estimates for the power analysis conducted when planning the current study. We also contrast results found in the current study to the results found in these secondary analyses (see discussion section).

Secondary analyses consisted of a confirmatory factor analyses on the covariance matrix reported in the article using the default maximum likelihood estimator (CFA).Park, Viegut, & Matthews (2021) examined the developmental trajectories of non-symbolic ratio processing in preschoolers, 2nd graders, 5th graders and adults. Participants completed four ratio and four absolute magnitude comparison (discrimination) tasks: line length, circle area, blob area, and dot number. In the ratio comparison task, participants had to identify which of two stimulus pairs had the largest ratio. In the magnitude comparison task, participants had to identify which of two stimuli was the largest/longest. Table A1 shows the correlation matrix used in the analysis (N = 79, 22 2nd graders, 26 5th graders and 31 adults). Given that the sample is heterogeneous, results must be interpreted with caution as measurement invariance, how measures differ across different populations (e.g., across age), was not analyzed.

Table A1. Bivariate correlation between variables.

Tasks	2	3	4	5	6	7	8	9
1.Age	.34	.46	.29	.40	.50	.46	.63	.54
Ratio comparison								
2. Line-R	1							
3. Circle-R	.52	1						
4. Blob-R	.38	.38	1					
5. Dot-R	.35	50.	.39	1				
Magnitude comparison								

6. Line-M	.45	.49	.40	.41	1			
7. Circle-M	.38	.41	.36	.28	.59	1		
8. Blob-M	.37	.39	.37	.28	.60	.65	1	
9. Dots-M	.39	.41	.42	.30	.51	.58	.62	1

Note: All correlations were significant $p < 0.01$. M = magnitude comparison; R = ratio comparison.

Model fit was adequate ($\chi^2(19) = 11.311$, $p = 0.913$, CFI = 1.000, RMSEA = 0.000 90% CI [0.00 – 0.038], SRMR = 0.042, AIC = -3800.136, BIC = -3759.855). Figure A1 shows the standardized solution along with the path diagram. All factor loadings were significantly greater than zero ($p < 0.001$).

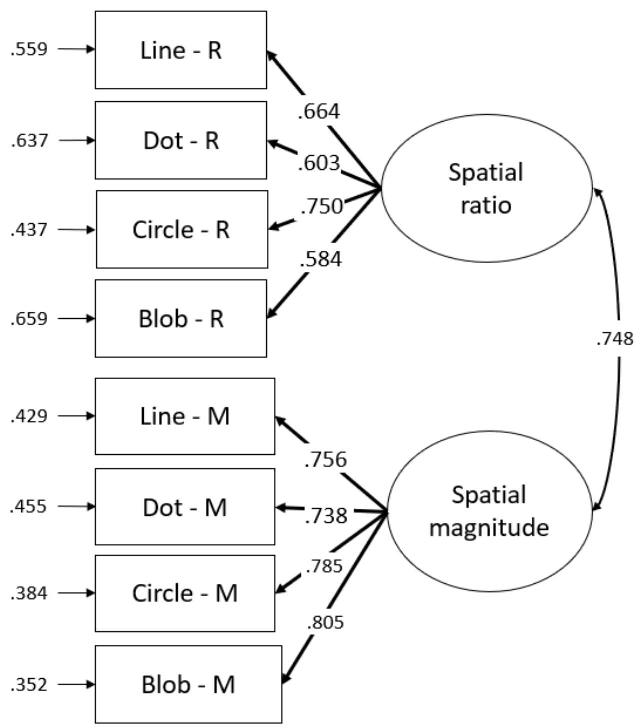

Figure A1. CFA on data from Park, Viegut & Matthews (2021). M = magnitude comparison; R = ratio comparison. Parameter estimates are fully standardized and are all significant ($p < 0.001$).

Based on these values, a power analysis was conducted on the following model (Figure A2).

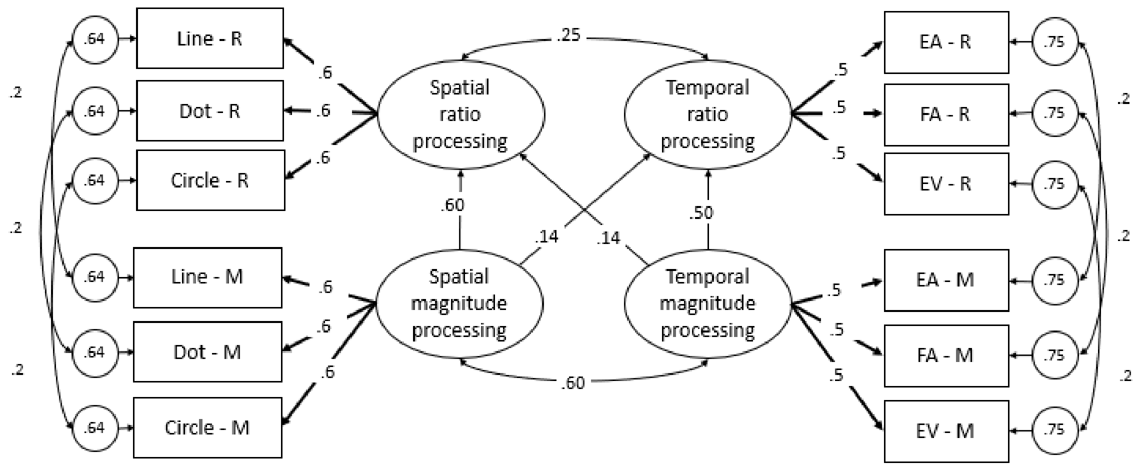

Figure A2. Model used in power analysis. EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation.

Appendix B: Complete solutions for all measurement and structure models

Table B1. Unstandardized and standardized parameter estimates for the 1-factor CFA model

Parameter	b (SE)	β (SE)	*p*	R^2
General factor				
Circle-R	.015 (.003)	.427 (.061)	<.001	.183
Line-R	.028 (.004)	.686 (.049)	<.001	.471
Dots-R	.021 (.003)	.569 (.053)	<.001	.324
EA-R	.029 (.004)	.767 (.043)	<.001	.589
FA-R	.037 (.003)	.709 (.044)	<.001	.502
EV-R	.035 (.004)	.738 (.045)	<.001	.544
Circle-M	.030 (.005)	.422 (.064)	<.001	.178
Line-M	.029 (.005)	.405 (.061)	<.001	.164
Dots-M	.024 (.005)	.343 (.062)	<.001	.118
EA-M	.08 (.008)	.725 (.045)	<.001	.526
FA-M	.053 (.006)	.656 (.05)	<.001	.431
EV-M	.074 (.007)	.680 (.044)	<.001	.463
Residual covariances				
Circle-R ~~ Circle-M	0 (0)	.091 (.072)	.21	
Line-R ~~ Line-M	0 (0)	.006 (.074)	.94	
Dots-R ~~ Dots-M	0 (0)	.066 (.071)	.35	
EA-R ~~ EA-M	0 (0)	.057 (.086)	.50	
FA-R ~~ FA-M	0 (0)	-.078 (.099)	.43	
EV-R ~~ EV-M	0 (0)	.116 (.082)	.16	

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations.

Table B2. Unstandardized and standardized parameter estimates for the 2-factor CFA model (Ratio and Magnitude Factors)

Parameter	b (SE)	β (SE)	*p*	R^2
Ratio factor				
Circle-R	.015 (.003)	.429 (.06)	<.001	.184
Line-R	.029 (.004)	.714 (.05)	<.001	.510
Dots-R	.022 (.003)	.584 (.055)	<.001	.341
EA-R	.030 (.004)	.792 (.042)	<.001	.628
FA-R	.038 (.003)	.718 (.047)	<.001	.515
EV-R	.036 (.004)	.755 (.046)	<.001	.569
Magnitude factor				
Circle-M	.031 (.005)	.446 (.064)	<.001	.199
Line-M	.030 (.005)	.432 (.062)	<.001	.187
Dots-M	.026 (.005)	.362 (.060)	<.001	.131
EA-M	.088 (.008)	.790 (.038)	<.001	.625
FA-M	.057 (.005)	.711 (.045)	<.001	.505
EV-M	.077 (.007)	.711 (.044)	<.001	.506
Residual covariances				
Circle-R ~~ Circle-M	0 (0)	.101 (.071)	.16	
Line-R ~~ Line-M	0 (0)	.053 (.074)	.47	
Dots-R ~~ Dots-M	0 (0)	.086 (.07)	.22	
EA-R ~~ EA-M	0 (0)	.22 (.086)	.01	
FA-R ~~ FA-M	0 (0)	-.004 (.098)	.97	
EV-R ~~ EV-M	.001 (0)	.226 (.084)	.01	
Factor covariances				
Ratio factor ~~ Magnitude factor	.826 (.037)	.826 (.037)	<.001	

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations.

Table B3. Unstandardized and standardized parameter estimates for the 2-factor CFA model (Spatial and Temporal Factors)

Parameter	b (SE)	β (SE)	*p*	R^2
Spatial factor				
Circle-R	.017 (.003)	.490 (.063)	<.001	.240
Line-R	.032 (.004)	.804 (.046)	<.001	.647
Dots-R	.025 (.002)	.655 (.049)	<.001	.429
Circle-M	.033 (.005)	.473 (.063)	<.001	.224
Line-M	.030 (.005)	.433 (.068)	<.001	.187
Dots-M	.024 (.006)	.343 (.071)	<.001	.118
Temporal factor				
EA-M	.081 (.008)	.735 (.045)	<.001	.540
FA-M	.054 (.006)	.667 (.050)	<.001	.446
EV-M	.074 (.007)	.684 (.043)	<.001	.468
EA-R	.029 (.004)	.775 (.044)	<.001	.601
FA-R	.039 (.003)	.732 (.041)	<.001	.536
EV-R	.036 (.004)	.747 (.044)	<.001	.558
Residual covariances				
Circle-R ~~ Circle-M	0 (0)	.030 (.073)	.68	
Line-R ~~ Line-M	0 (0)	-.125 (.099)	.21	
Dots-R ~~ Dots-M	0 (0)	.031 (.079)	.70	
EA-M ~~ EA-R	0 (0)	.030 (.095)	.75	
FA-M ~~ FA-R	0 (0)	-.126 (.106)	.23	
EV-M ~~ EV-R	0 (0)	.098 (.084)	.25	
Factor covariances				
Spatial factor ~~ Temporal factor	.807 (.039)	.807 (.039)	<.001	

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations.

Table B4. Unstandardized and standardized parameter estimates for the 4-factor CFA model

Parameter	b (SE)	β (SE)	*p*	R^2
Spatial ratio (SR)				
Circle-R	0.017 (0.003)	0.486 (0.066)	<.001	.236
Line-R	0.034 (0.004)	0.844 (0.048)	<.001	.713
Dots-R	0.025 (0.003)	0.667 (0.052)	<.001	.445
Temporal ratio (TR)				
EA-R	0.03 (0.004)	0.81 (0.048)	<.001	.655
FA-R	0.04 (0.003)	0.747 (0.043)	<.001	.558
EV-R	0.037 (0.004)	0.774 (0.042)	<.001	.599
Spatial magnitude (SM)				
Circle-M	0.038 (0.006)	0.539 (0.067)	<.001	.290
Line-M	0.034 (0.006)	0.483 (0.071)	<.001	.234
Dots-M	0.028 (0.005)	0.39 (0.067)	<.001	.152
Temporal magnitude (TM)				
EA-M	0.089 (0.008)	0.799 (0.038)	<.001	.638
FA-M	0.058 (0.005)	0.719 (0.045)	<.001	.517
EV-M	0.077 (0.007)	0.711 (0.044)	<.001	.506
Residual covariances				
Circle-R ~~ Circle-M	0 (0)	0.052 (0.076)	0.49	
Line-R ~~ Line-M	0 (0)	-0.025 (0.113)	0.83	
Dots-R ~~ Dots-M	0 (0)	0.078 (0.078)	0.32	
EA-R ~~ EA-M	0 (0)	0.211 (0.108)	0.05	
FA-R ~~ FA-M	0 (0)	-0.033 (0.105)	0.75	
EV-R ~~ EV-M	0.001 (0)	0.22 (0.088)	0.01	
Factor covariances				
Spatial ratio ~~ Temporal ratio	0.766 (0.046)	0.766 (0.046)	<.001	
Spatial ratio ~~ Spatial magnitude	0.746 (0.07)	0.746 (0.07)	<.001	
Spatial ratio ~~ Temporal magnitude	0.665 (0.053)	0.665 (0.053)	<.001	
Temporal ratio ~~ Spatial magnitude	0.673 (0.084)	0.673 (0.084)	<.001	
Temporal ratio ~~ Temporal magnitude	0.811 (0.045)	0.811 (0.045)	<.001	
Spatial magnitude ~~ Temporal magnitude	0.822 (0.078)	0.822 (0.078)	<.001	

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations.

Table B5. Unstandardized and standardized parameter estimates for the both 4-factor SEM model (all paths included and trimmed)

Parameter	All paths			Trimmed		
	b (SE)	β (SE)	*p*	b (SE)	β (SE)	*p*
Spatial Ratio (SR)						
Circle-R	0.011 (0.002)	0.486 (0.134)	<.001	0.011 (0.002)	0.485 (0.078)	<.001
Line-R	0.022 (0.003)	0.844 (0.158)	<.001	0.022 (0.003)	0.845 (0.072)	<.001
Dots-R	0.016 (0.002)	0.667 (0.147)	<.001	0.016 (0.002)	0.666 (0.072)	<.001
Temporal Ratio (TR)						
EA-R	0.018 (0.003)	0.81 (0.173)	<.001	0.018 (0.003)	0.809 (0.076)	<.001
FA-R	0.023 (0.003)	0.747 (0.087)	<.001	0.023 (0.003)	0.747 (0.05)	<.001
EV-R	0.022 (0.003)	0.774 (0.107)	<.001	0.021 (0.003)	0.774 (0.053)	<.001
Spatial Magnitude (SM)						
Circle-M	0.038 (0.006)	0.539 (0.072)	<.001	0.037 (0.006)	0.524 (0.065)	<.001
Line-M	0.034 (0.006)	0.483 (0.075)	<.001	0.033 (0.005)	0.473 (0.067)	<.001
Dots-M	0.028 (0.005)	0.39 (0.07)	<.001	0.027 (0.005)	0.383 (0.066)	<.001
Temporal Magnitude (TM)						
EA-M	0.089 (0.008)	0.799 (0.039)	<.001	0.088 (0.008)	0.799 (0.038)	<.001
FA-M	0.058 (0.005)	0.719 (0.053)	<.001	0.058 (0.005)	0.718 (0.045)	<.001
EV-M	0.077 (0.007)	0.711 (0.044)	<.001	0.077 (0.007)	0.711 (0.044)	<.001
Regression coefficients						
SR ~ SM	0.935 (0.451)	0.617 (0.252)	0.01	1.222 (0.209)	0.774 (0.053)	<.001
TR ~ TM	1.356 (0.377)	0.794 (0.171)	<.001	1.385 (0.219)	0.811 (0.044)	<.001
SR ~ TM	0.239 (0.369)	0.158 (0.251)	0.53	0 (0)	0 (0)	NA
TR ~ SM	0.034 (0.324)	0.02 (0.19)	0.92	0 (0)	0 (0)	NA
Residual covariances						
Circle-R ~~ Circle-M	0 (0)	0.052 (0.33)	0.87	0 (0)	0.05 (0.134)	.71
Line-R ~~ Line-M	0 (0)	-0.025 (0.449)	0.96	0 (0)	-0.03 (0.188)	.87
Dots-R ~~ Dots-M	0 (0)	0.078 (0.328)	0.81	0 (0)	0.074 (0.133)	.58
EA-R ~~ EA-M	0 (0)	0.211 (0.426)	0.62	0 (0)	0.209 (0.18)	.24
FA-R ~~ FA-M	0 (0)	-0.033 (0.303)	0.91	0 (0)	-0.033 (0.132)	.80
EV-R ~~ EV-M	0.001 (0)	0.22 (0.271)	0.42	0.001 (0)	0.22 (0.117)	.06
Factor covariances						
SM ~~ TM	0.822 (0.078)	0.822 (0.078)	<.001	0.852 (0.05)	0.852 (0.05)	<.001
SR ~~ TR	0.578 (0.136)	0.578 (0.136)	<.001	0.615 (0.12)	0.615 (0.12)	<.001

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations. The relationship of interest SR ~~ TR (highlighted in grey) is specified as a correlation in this table to make the change in coefficient easier to interpret (see Figure B1 for path diagram). Paths that were fixed to zero in the trimmed model are framed by dotted lines.

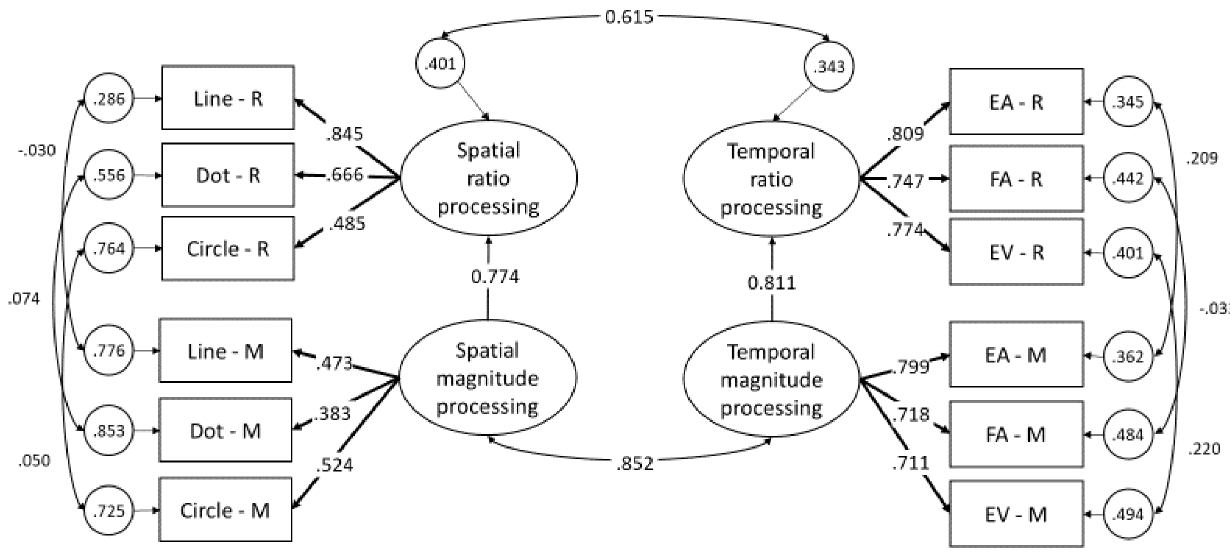

Figure B1. Equivalent trimmed 4-factor SEM model with correlated residuals between the two ratios factors instead of a general factor with regression paths. EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation. Parameter estimates are fully standardized.

Appendix C: Complete solutions for models with and without influential outliers

Table C1. Unstandardized and standardized parameter estimates for the both 4-factor SEM models including and excluding influential outliers

	Including influential outliers			Excluding influential outliers		
Parameter	b (SE)	β (SE)	*p*	b (SE)	β (SE)	*p*
Spatial Ratio (SR) factor						
Circle-R	0.011 (0.002)	0.485 (0.078)	<.001	0.011 (0.002)	0.51 (0.082)	<.001
Line-R	0.022 (0.003)	0.845 (0.072)	<.001	0.019 (0.003)	0.849 (0.094)	<.001
Dots-R	0.016 (0.002)	0.666 (0.072)	<.001	0.014 (0.002)	0.61 (0.082)	<.001
Temporal Ratio (TR) factor						
EA-R	0.018 (0.003)	0.809 (0.076)	<.001	0.017 (0.002)	0.796 (0.106)	<.001
FA-R	0.023 (0.003)	0.747 (0.05)	<.001	0.024 (0.003)	0.752 (0.057)	<.001
EV-R	0.021 (0.003)	0.774 (0.053)	<.001	0.021 (0.003)	0.777 (0.069)	<.001
Spatial Magnitude (SM) factor						
Circle-M	0.037 (0.006)	0.524 (0.065)	<.001	0.031 (0.005)	0.478 (0.066)	<.001
Line-M	0.033 (0.005)	0.473 (0.067)	<.001	0.029 (0.006)	0.425 (0.077)	<.001
Dots-M	0.027 (0.005)	0.383 (0.066)	<.001	0.022 (0.004)	0.328 (0.058)	<.001
Temporal Magnitude (TM) factor						
EA-M	0.088 (0.008)	0.799 (0.038)	<.001	0.074 (0.007)	0.764 (0.047)	<.001
FA-M	0.058 (0.005)	0.718 (0.045)	<.001	0.052 (0.005)	0.678 (0.05)	<.001
EV-M	0.077 (0.007)	0.711 (0.044)	<.001	0.069 (0.007)	0.68 (0.045)	<.001
Regression coefficients						
SR ~ SM	1.222 (0.209)	0.774 (0.053)	<.001	1.242 (0.256)	0.779 (0.063)	<.001
TR ~ TM	1.385 (0.219)	0.811 (0.044)	<.001	1.178 (0.197)	0.762 (0.053)	<.001
SR ~ TM	0 (0)	0 (0)	NA	0 (0)	0 (0)	NA
TR ~ SM	0 (0)	0 (0)	NA	0 (0)	0 (0)	NA
Residual covariances						

Circle-R ~~ Circle-M	0 (0)	0.05 (0.134)	0.71	0 (0)	0.017 (0.161)	0.91
Line-R ~~ Line-M	0 (0)	-0.03 (0.188)	0.87	0 (0)	-0.076 (0.235)	0.75
Dots-R ~~ Dots-M	0 (0)	0.074 (0.133)	0.58	0 (0)	0.098 (0.151)	0.52
EA-R ~~ EA-M	0 (0)	0.209 (0.18)	0.24	0 (0)	0.107 (0.222)	0.63
FA-R ~~ FA-M	0 (0)	-0.033 (0.132)	0.80	0 (0)	-0.104 (0.156)	0.50
EV-R ~~ EV-M	0.001 (0)	0.22 (0.117)	0.06	0 (0)	0.243 (0.147)	0.10
Factor covariances						
SM ~~ TM	0.852 (0.050)	0.852 (0.050)	<.001	0.764 (0.067)	0.764 (0.067)	<.001
SR ~~ TR	0.615 (0.120)	0.615 (0.120)	<.001	0.608 (0.118)	0.608 (0.118)	<.001

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; R = ratio estimation; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations. The relationship of interest SR ~~ TR (highlighted in grey) is specified as a correlation in this table to make the change in coefficient easier to interpret (see Figure B1 for path diagram).

Appendix D: Deviant ratio CFA

Table D1. Unstandardized and standardized parameter estimates for the deviant ratio CFA model

Parameter	b (SE)	β (SE)	*p*
Deviant ratio factor loadings			
Empty auditory (EA-M)			
1.2	0.025 (0.011)	0.475 (0.053)	<.001
1.25	0.026 (0.011)	0.551 (0.057)	<.001
1.3	0.029 (0.012)	0.599 (0.049)	<.001
1.4	0.029 (0.011)	0.648 (0.05)	<.001
1.6	0.032 (0.013)	0.781 (0.047)	<.001
Filled auditory (FA-M)			
1.2	0.026 (0.011)	0.483 (0.066)	<.001
1.25	0.023 (0.01)	0.452 (0.065)	<.001
1.3	0.02 (0.007)	0.441 (0.062)	<.001
1.4	0.022 (0.008)	0.516 (0.063)	<.001
1.6	0.02 (0.008)	0.621 (0.082)	<.001
Empty visual (EV-M)			
1.2	0.038 (0.009)	0.383 (0.072)	<.001
1.25	0.057 (0.01)	0.579 (0.066)	<.001
1.3	0.065 (0.011)	0.667 (0.048)	<.001
1.4	0.061 (0.009)	0.674 (0.059)	<.001
1.6	0.05 (0.009)	0.624 (0.062)	<.001
Circle area (Circle-M)			
1.02	0.01 (0.007)	0.112 (0.064)	0.08
1.04	0.024 (0.01)	0.27 (0.062)	<.001
1.06	0.047 (0.022)	0.577 (0.085)	<.001
1.08	0.026 (0.014)	0.445 (0.108)	<.001
1.18	0.017 (0.006)	0.582 (0.173)	<.001
Line length (Line-M)			
1.01	0.012 (0.008)	0.11 (0.072)	0.12
1.02	0.044 (0.011)	0.437 (0.068)	<.001
1.03	0.042 (0.011)	0.441 (0.102)	<.001
1.06	0.031 (0.011)	0.515 (0.107)	<.001
1.12	0.017 (0.006)	0.485 (0.166)	0
Dot number (Dots-M)			
1.09	0.055 (0.011)	0.445 (0.075)	<.001
1.1	0.039 (0.011)	0.328 (0.086)	<.001
1.12	0.03 (0.009)	0.299 (0.091)	<.001
1.14	0.043 (0.011)	0.427 (0.089)	<.001
1.25	0.034 (0.007)	0.535 (0.112)	<.001
Subfactor loadings			

Temporal magnitude (TM)			
EA-M	3.175 (1.444)	0.954 (0.039)	<.001
FA-M	2.611 (1.068)	0.934 (0.049)	<.001
EV-M	1.363 (0.268)	0.806 (0.055)	<.001
Spatial magnitude (SM)			
Circle-M	1.526 (0.77)	0.836 (0.127)	<.001
Line -M	1.157 (0.381)	0.757 (0.107)	<.001
Dots -M	0.669 (0.169)	0.556 (0.097)	<.001
Factor covariance			
TM~~SM	0.852 (0.086)	0.852 (0.086)	<.001

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; b = unstandardized coefficients; β = standardized coefficients.

Table D2. Unstandardized and standardized parameter estimates for the final 4-factor SEM models including all deviant ratios and excluding problematic deviant ratios

	Including all deviant ratios			Excluding problematic deviant ratios		
Parameter	b (SE)	β (SE)	*p*	b (SE)	β (SE)	*p*
Spatial Ratio (SR) factor						
Circle-R	0.011 (0.002)	0.485 (0.078)	<.001	0.011 (0.002)	0.486 (0.079)	<.001
Line-R	0.022 (0.003)	0.845 (0.072)	<.001	0.022 (0.003)	0.844 (0.074)	<.001
Dots-R	0.016 (0.002)	0.666 (0.072)	<.001	0.016 (0.002)	0.665 (0.073)	<.001
Temporal Ratio (TR) factor						
EA-R	0.018 (0.003)	0.809 (0.076)	<.001	0.017 (0.003)	0.809 (0.079)	<.001
FA-R	0.023 (0.003)	0.747 (0.05)	<.001	0.022 (0.003)	0.747 (0.05)	<.001
EV-R	0.021 (0.003)	0.774 (0.053)	<.001	0.021 (0.003)	0.777 (0.054)	<.001
Spatial Magnitude (SM) factor						
Circle-M	0.037 (0.006)	0.524 (0.065)	<.001	0.043 (0.006)	0.559 (0.062)	<.001
Line-M	0.033 (0.005)	0.473 (0.067)	<.001	0.039 (0.006)	0.53 (0.063)	<.001
Dots-M	0.027 (0.005)	0.383 (0.066)	<.001	0.027 (0.005)	0.378 (0.065)	<.001
Temporal Magnitude (TM) factor						
EA-M	0.088 (0.008)	0.799 (0.038)	<.001	0.087 (0.008)	0.789 (0.038)	<.001
FA-M	0.058 (0.005)	0.718 (0.045)	<.001	0.058 (0.005)	0.718 (0.045)	<.001
EV-M	0.077 (0.007)	0.711 (0.044)	<.001	0.077 (0.007)	0.713 (0.043)	<.001
Regression coefficients						
SR ~ SM	1.222 (0.209)	0.774 (0.053)	<.001	1.192 (0.201)	0.766 (0.053)	<.001
TR ~ TM	1.385 (0.219)	0.811 (0.044)	<.001	1.451 (0.236)	0.823 (0.043)	<.001
SR ~ TM	0 (0)	0 (0)	NA	0 (0)	0 (0)	NA
TR ~ SM	0 (0)	0 (0)	NA	0 (0)	0 (0)	NA
Residual covariances						
Circle-R ~~ Circle-M	0 (0)	0.05 (0.134)	0.71	0 (0)	0.035 (0.133)	0.79

Line-R ~~ Line-M	0 (0)	-0.03 (0.188)	0.87	0 (0)	-0.019 (0.196)	0.92
Dots-R ~~ Dots-M	0 (0)	0.074 (0.133)	0.58	0 (0)	0.081 (0.137)	0.56
EA-R ~~ EA-M	0 (0)	0.209 (0.18)	0.24	0 (0)	0.204 (0.181)	0.26
FA-R ~~ FA-M	0 (0)	-0.033 (0.132)	0.8	0 (0)	-0.033 (0.136)	0.81
EV-R ~~ EV-M	0.001 (0)	0.22 (0.117)	0.06	0 (0)	0.207 (0.121)	0.09
Factor covariances						
SM ~~ TM	0.852 (0.05)	0.852 (0.05)	<.001	0.878 (0.048)	0.878 (0.048)	<.001
SR ~~ TR	0.615 (0.12)	0.615 (0.12)	<.001	0.564 (0.12)	0.564 (0.12)	<.001

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations. The relationship of interest SR ~~ TR (highlighted in grey) is specified as a correlation in this table to make the change in coefficient easier to interpret (see Figure B1 for path diagram).

Appendix E: Complete solutions for models with R^2 as the ratio accuracy measure

Table E 1. Unstandardized and standardized parameter estimates for the final 4-factor SEM model using R^2 as ratio estimation accuracy measure

	Using average absolute error			Using R^2		
Parameter	**b (SE)**	**β (SE)**	***p***	**b (SE)**	**β (SE)**	***p***
Spatial Ratio (SR) factor						
Circle-R	0.011 (0.002)	0.485 (0.078)	<.001	0.067 (0.013)	0.727 (0.057)	<.001
Line-R	0.022 (0.003)	0.845 (0.072)	<.001	0.091 (0.017)	0.878 (0.044)	<.001
Dots-R	0.016 (0.002)	0.666 (0.072)	<.001	0.072 (0.013)	0.695 (0.063)	<.001
Temporal Ratio (TR) factor						
EA-R	0.018 (0.003)	0.809 (0.076)	<.001	0.089 (0.013)	0.839 (0.04)	<.001
FA-R	0.023 (0.003)	0.747 (0.05)	<.001	0.109 (0.013)	0.714 (0.046)	<.001
EV-R	0.021 (0.003)	0.774 (0.053)	<.001	0.095 (0.012)	0.731 (0.052)	<.001
Spatial Magnitude (SM) factor						
Circle-M	0.037 (0.006)	0.524 (0.065)	<.001	0.036 (0.006)	0.508 (0.067)	<.001
Line-M	0.033 (0.005)	0.473 (0.067)	<.001	0.033 (0.005)	0.469 (0.066)	<.001
Dots-M	0.027 (0.005)	0.383 (0.066)	<.001	0.027 (0.005)	0.385 (0.067)	<.001
Temporal Magnitude (TM) factor						
EA-M	0.088 (0.008)	0.799 (0.038)	<.001	0.09 (0.008)	0.809 (0.037)	<.001
FA-M	0.058 (0.005)	0.718 (0.045)	<.001	0.058 (0.005)	0.714 (0.045)	<.001
EV-M	0.077 (0.007)	0.711 (0.044)	<.001	0.076 (0.008)	0.703 (0.045)	<.001
Regression coefficients						
SR ~ SM	1.222 (0.209)	0.774 (0.053)	<.001	1.118 (0.21)	0.745 (0.062)	<.001
TR ~ TM	1.385 (0.219)	0.811 (0.044)	<.001	1.441 (0.237)	0.822 (0.044)	<.001
SR ~ TM	0 (0)	0 (0)	NA	0 (0)	0 (0)	NA
TR ~ SM	0 (0)	0 (0)	NA	0 (0)	0 (0)	NA
Residual covariances						
Circle-R ~~ Circle-M	0 (0)	0.05 (0.134)	0.71	0 (0.001)	-0.029 (0.089)	0.75

Line-R ~~ Line-M	0 (0)	-0.03 (0.188)	0.87	0 (0)	0.02 (0.099)	0.84
Dots-R ~~ Dots-M	0 (0)	0.074 (0.133)	0.58	0.001 (0.001)	0.076 (0.073)	0.29
EA-R ~~ EA-M	0 (0)	0.209 (0.18)	0.24	0.001 (0.001)	0.161 (0.113)	0.16
FA-R ~~ FA-M	0 (0)	-0.033 (0.132)	0.8	0 (0.001)	-0.033 (0.099)	0.74
EV-R ~~ EV-M	0.001 (0)	0.22 (0.117)	0.06	0.003 (0.001)	0.284 (0.079)	<.001
Factor covariances						
SM ~~ TM	0.852 (0.05)	0.852 (0.05)	<.001	0.872 (0.048)	0.872 (0.048)	<.001
SR ~~ TR	0.615 (0.12)	0.615 (0.12)	<.001	0.723 (0.116)	0.723 (0.116)	<.001

Note: EA = empty auditory interval; FA = filled auditory interval; EV = empty visual interval; M = magnitude discrimination; b = unstandardized coefficients; β = standardized coefficients. Standardized covariances (~~) can be interpreted as correlations. The relationship of interest SR ~~ TR (highlighted in grey) is specified as a correlation in this table to make the change in coefficient easier to interpret (see Figure B1 for path diagram).

Appendix F: Correlations between task performance and demographic variables

Table F1. Bivariate correlations between tasks and demographic variables

	Education	Music training	Music playing
Temporal tasks			
Empty auditory interval magnitude	-.056	-.012	.086
Empty auditory interval ratio	-.086	.102	.044
Filled auditory interval magnitude	.081	.003	.094
Filled auditory interval ratio	.010	.055	.110
Empty visual interval magnitude	.024	-.037	.061
Empty visual interval ratio	.005	.055	.068
Spatial tasks			
Circle magnitude	-.045	.073	.118
Circle ratio	-.078	-.001	.042
Dots magnitude	-.139 *	.018	.037
Dots ratio	-.014	-.041	.068
Line magnitude	.040	.094	.070
Line ratio	-.018	.035	.118

* $p < .05$

Appendix G: Ethics Approval

Date: 5 February 2021

To: Prof. Daniel Ansari

Project ID: 117912

Study Title: Behavioral Studies on Numerical, Spatial and Temporal Ratio Processing

Short Title: Behavioral Studies on Ratio Processing

Application Type: NMREB Initial Application

Review Type: Delegated

Full Board Reporting Date: 05/Mar/2021

Date Approval Issued: 05/Feb/2021 16:11

REB Approval Expiry Date: 05/Feb/2022

Dear Prof. Daniel Ansari

The Western University Non-Medical Research Ethics Board (NMREB) has reviewed and approved the WREM application form for the above mentioned study, as of the date noted above. NMREB approval for this study remains valid until the expiry date noted above, conditional to timely submission and acceptance of NMREB Continuing Ethics Review.

This research study is to be conducted by the investigator noted above. All other required institutional approvals must also be obtained prior to the conduct of the study.

Documents Approved:

Document Name	Document Type	Document Date	Document Version
Ratio_processing_end_of_study_questionnaire_Nov30	Online Survey	30/Nov/2020	
End_of_study_questionnarie_Nov30_2020	Online Survey	30/Nov/2020	
Task and instruction examples_Nov_30_2020	Other Data Collection Instruments	01/Dec/2020	
DebriefingForm_Nov30_2020	Debriefing document	01/Dec/2020	
Survey_panel_recruitment_Nov30_2020	Recruitment Materials	01/Dec/2020	
LOI_Panel_half-hour_Jan19_2021_Clean	Implied Consent/Assent		
LOI_SONA_half-hour_Jan19_2021_Clean	Implied Consent/Assent		
SONA_recruitment_Jan19_2021_Cleaned	Recruitment Materials		
LOI_Panel_full-hour_Feb5_2021_Clean	Implied Consent/Assent		
LOI_SONA_full-hour_Feb5_2021_Clean	Implied Consent/Assent		

No deviations from, or changes to the protocol should be initiated without prior written approval from the NMREB, except when necessary to eliminate immediate hazard(s) to study participants or when the change(s) involves only administrative or logistical aspects of the trial.

The Western University NMREB operates in compliance with the Tri-Council Policy Statement Ethical Conduct for Research Involving Humans (TCPS2), the Ontario Personal Health Information Protection Act (PHIPA, 2004), and the applicable laws and regulations of Ontario. Members of the NMREB who are named as Investigators in research studies do not participate in discussions related to, nor vote on such studies when they are presented to the REB. The NMREB is registered with the U.S. Department of Health & Human Services under the IRB registration number IRB 00000941.

Please do not hesitate to contact us if you have any questions.

Sincerely,

Ms. Katelyn Harris , Research Ethics Officer on behalf of Dr. Randal Graham, NMREB Chair

Curriculum Vitae

Name:	Rebekka Lagacé-Cusiac
Post-secondary Education and Degrees:	Collège Lionel-Groulx (Natural Sciences and Classical Music) Ste-Thérèse, Québec, Canada 2013-2016 Double D.E.C.
	Université de Montréal (Cognitive Neurosciences) Montréal, Québec, Canada 2016-2019 B.Sc.
	University of Western Ontario (Psychology) London, Ontario, Canada 2019-present M.Sc.
Honours and Awards:	Dean's honor list 2017-2019
Related Work Experience	Teaching Assistant University of Western Ontario 2019-2021
	Research Assistant University of Western Ontario and Memorial University 2020-present
	Honor's Thesis Supervision University of Western Ontario 2020-2021
	Teaching Assistant Université de Montréal Fall 2018
	Math and Music Help Center Tutor Collège Lionel-Groulx 2014-2016

Publications:

Lagacé-Cusiac, R., Grahn, J., Ansari, D., Tremblay, P.F. (2021) A Sense of Proportion: How Humans Represent Relative Magnitudes in Space and Time. [Preregistered report on OSF]. https://osf.io/kvh9z/

Lau, N., Wilkey, E., Soltanlou M., Lagacé Cusiac, R., Peters L., Tremblay P., Goffin C., …, Ansari, D. (2020). Numeracy and COVID-19: examining interrelationships between numeracy, health numeracy and behaviour. Royal Society of Open Science. (Accepted as Registered Report)

Conferences and Presentations:

Kingdom, E., Lagacé-Cusiac, R., Carter, C., De Sousa, J., Grahn, J. (2021). The effects of interleaved and blocked practice on music style recognition. University of Nottingham. International Conference on Music Perception and Cognition (International; Talk)

Nadon, E., Lagace-Cusiac, R., Pilon, J., Gosselin, N. (2018). The effect of the emotional characteristics of different sound environments on selective attention of adults. Université de Québec à Montréal. CRBLM Scientific Day (Regional; Poster)

Oswald, V., Lagacé-Cusiac, R., Létourneau, A., Trahan, H., Toupin, G., Younes, Z., Rabaey, P., Jerbi, K. (2017). À l'état de repos, la puissance spectrale des oscillations cérébrales prédit la performance de la vitesse de traitement. Université de Montréal. 10e Journée scientifique du Département de Psychologie (Institutional; Poster)