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Optimal Grasp Synthesis to Apply Normal and Shear Stresses of Failure in Beams

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Optimal Grasp Synthesis to Apply Normal and Shear Stresses of Failure in Beams

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***Abstract*— This paper investigates the less-studied problem of failing/yielding an object purposefully by a robotic hand. A grasp synthesis capable of using the whole limb surface of the robotic hand is designed based on internal force decomposition. The introduced approach is based on quasistatic assumption and optimization of active internal forces in order to counterbalance the formulated task wrench/load of yielding. As different geometrical constraints are dictated by the manipulation circumstances (*e.g.* metallic sheet shaping or robotic harvesting), the yielding wrench optimization is developed to be not only sufficient for yielding the object but also effective in meeting all motion restrictions on manipulator. Maximum-shear-stress theory is used for yielding analysis of a grasped object. Finite Element Modeling (FEM) simulation results are provided as a validation of our proposed approach.**

I. INTRODUCTION

Robotic hands were designed to enable robots to manipulate objects. As massively reported in literature, robotic manipulation consists of tasks from the simple pick and place robots, to more sophisticated assembly such as circuit chips insertion [1]. Unlike most papers dealing with grasp manipulation planning, our main focus is on an optimal and systematic way of failing an object by means of yielding a tensile object or fracture of a brittle object. To the best of our knowledge, there is no investigation on optimally performing of a failure grasp of an object. Moreover, papers which consider avoiding deflection and/or slippage of a grasped object, hardly study the effect of bending, tension, or torsion on the object [2], [3], [4], [5].

Robotic harvesting can be named as an example of failing an object by means of separating it into pieces. Shaping of a metallic sheet, tailored beam, *etc.* are other applications of yielding in industry. There are cases in which the manipulation environment demands a set of constraints on robot motions. For instance, presence of obstacles causes limitations on applying load/wrench in certain directions (see Fig. 3).

There are different possible ways of manipulating an object in order to break it: applying bending, tension, and torsion [6], [7]. If performing the task of failing an object using a middle sized robot with average actuators is desired, an optimal grasp configuration for applying a combination of all kinds of load is required. In a systematic approach for failing an object, important concerns are such as: How much bending, torsion, and/or tension are needed in which angles to snap a beam? How to avoid exceeding an approximate desired displacement for fracture? How to counterbalance the external load resulted by the object using an optimal grasp?

Our contributions in this paper are as follows:

- Optimized failing wrench. Finding and optimizing minimum wrench which is enough for breaking an object is provided.
- Object motion constraints. Formulating motion constraints on grasped object are considered and it is used to find suitable failure wrench for a grasped object.
- Internal force decomposition. Active internal force corresponds to internal forces caused by active joint displacement. In order to fully exploit the ability of a robotic hand, we considered the more general case of whole-limb manipulation in which using the whole surface of the arm or palm of the robotic hand enables it to handle larger wrench [8]. The introduced method for grasp synthesis decomposes internal forces and then uses active internal forces to meet grasp requirements.
- Maximum, and minimum contact force and Coulomb friction law violence. Constraints on contact forces in order to limit each of them to maintain in a certain range and have certain angles to avoid any slippage are formulated for grasp optimization.

The structure of this paper is as follows: In Section II, a brief background on the problem statement is given. Section III presents the proposed grasp synthesis based on optimal internal force and failure wrench. The validity of the presented approach is investigated via numerical example and Finite Element Modeling (FEM) in Section IV.

II. BACKGROUND

In this paper, failing an object by applying optimal contact forces in a geometrically constrained environment is considered. Failure usually refers to separation of a part into two or more pieces; permanently distortion; geometric ruin; downgrading reliability; or compromised function. Unlike most grasp planning papers, failure by means of controlled separation of the object into pieces is desired. This goal can be limited by the environment. Selection between the amount of bending or torsion can be dictated by environment, since there may not be enough room for either of them.

According to *maximum-shear-stress theory* (MSS theory), *yielding begins whenever the maximum shear stress in any element equals or exceeds the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield [9].*

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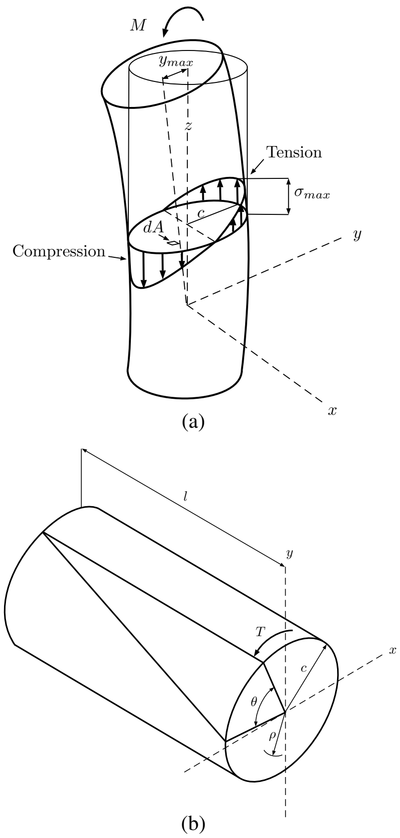


Fig. 1. (a) Bending stress resulted by applying moment M at x. (b) Shear stress resulted by torque T acting about x.

As shown in Fig. 1a the beam subjected to a bending moment M about x axis. z is neutral axis where bending stress varies linearly and it is given by

$$\sigma_z = -\frac{My}{I} \tag{1}$$

where $I = \int y^2 dA$ is the second *moment of area* about the x axis. The *maximum deflection* that moment M causes can be written as

$$y = \frac{Ml^2}{2EI} \tag{2}$$

where l is the length of the beam, and E is called *Young's modulus*.

It can be easily shown that by having c as the radius to outer surface the maximum *magnitude* of the bending stress is

$$\sigma_{max} = \frac{Mc}{I} \tag{3}$$

If force distribution is presented on the length of the beam, moving in the direction of z axis results in larger bending moment which also results the net *force* in the z direction. A shear force is required for equilibrium. This shear force gives rise to a shear stress. However, the bending is usually assumed to be pure in order to eliminate the resulted complications.

As shown in Fig. 1b a beam subjected to a *torque vector* results in object torsion. The shear stress resulted by torque T acting on a round bar with radius ρ is given by

$$\tau_{x,z} = \frac{T\ \rho}{J} \tag{4}$$

where J is polar second moment of area. The *angle of twist*, in radians, for a circular beam can be written as

$$\theta = \frac{Tl}{RJ} \tag{5}$$

where R is modulus of rigidity. Considering c as the radius to outer surface, the maximum shear stress is

$$\tau_{max} = \frac{T\ c}{J} \tag{6}$$

The case of a point undergoing plane stress with only one non-zero normal stress and one shear stress is considered here. Consider S_y as *yield strength*, according to MSS theory, the yielding process starts when

$$\sqrt{(\sigma_{max} + \frac{P}{A})^2 + 4\tau_{max}^2} = S_y \tag{7}$$

where P is the axial tension in z direction. Force and moment applied to the object are combined into the object wrench vector denoted by $w = [f^T, m^T] \in R^6$ where f is contact force vector exchanged by robot link and object and m is the applied moment vector. The wrench vector for applying tension, torsion, and bending is $w = [0\ 0\ P\ M\ 0\ T]^T$ which shows force, P, in z direction, moment, M, about x-axis, and torque, T, about z-axis. Since MSS theory is a conservative criterion for avoiding failure, for our purpose sufficiently larger moment and torque have to be inserted.

Optimal contact forces in a grasp configuration are supposed to be found to meet the criteria (7) for counterbalancing a desired wrench, w_d, while considering the constraints on y from (2) and θ from (5).

III. GRASP ANALYSIS OF FAILURE

Applying suitable wrench in order to fail an object demands the capability of inserting desired contact forces in a grasp configuration. In this paper, *quasistatic assumption* is considered, which is a common assumption in literature [10], [11]. The quasistatic assumption states that there will be no motion for grasped object in case of losing contacts with robotic hand. This should not be confused with the motions of grasped object and robotic manipulator during the performing of the grasp task. Quasistatic model of manipulation system can be expressed as

$$w = -Gf \tag{8}$$

where $w \in R^6$ is the resulted object wrench, $G^T \in R^{3n_c \times 6}$ is grasp matrix, and $f \in R^{3n_c}$ is contact force vector. The general solution of (8) is

$$f = -G^+ w + \mathrm{A}\xi \tag{9}$$

where G^+ is assumed to be right inverse of Grasp matrix, and $\mathrm{A} \in R^{3n_c \times g}$ is a matrix whose column is a basis of null-space of G and $\xi \in R^g$ is a free g-vector which parametrizes the homogeneous solution. The homogeneous part of solution refers to internal forces which have an important role in the stability of grasp.

Not all internal forces are actively controllable in a robotic hand. By definition of Jacobian and Grasp matrices, the infinitesimal motion of a contact point on the robot end-effector and the infinitesimal displacement of the same contact point on the grasped object are $J\delta q$ and $G^T\delta u$, respectively. Corresponding vector of contact forces caused by considering stiffness in each contact points is

$$\delta f = K(J\delta q - G^T\delta u) \tag{10}$$

where $K \in R^{3n_c\times 3n_c}$ is the stiffness matrix of grasp. By differentiating (8) and assuming external load is constant, the following equation is obtained

$$0 = -G\delta f \tag{11}$$

Note that Grasp matrix variation with respect to object displacement can be neglected [12]. Substituting (10) into (11) yields

$$\delta u = (GKG^T)^{-1}GKJ\delta q \tag{12}$$

and the equivalent contact force variation to object displacement (12) is

$$\delta f = (I - KG^T(GKG^T)^{-1}G)KJ\delta q \tag{13}$$

Equation (13) relates the variation of internal contact forces (δf) to the joints activation (δq). Therefore, all active internal contact forces can be expressed as

$$\delta f = \mathrm{B}\upsilon \tag{14}$$

where $\mathrm{B} \in R^{3n_c\times e}$ is a basis matrix of the column space of $(I - KG^T(GKG^T)^{-1}G)KJ$ and $\upsilon \in R^e$ is a free e-vector ($e < g$) that parametrizes the reachable (active or controllable) internal contact forces. Therefore, the general solution (9) is limited to

$$f = -G^+w + \mathrm{B}\upsilon \tag{15}$$

The free vector υ can be used for internal force optimization since it belongs to not only the null space of Grasp matrix but also to column space of Jacobian matrix. This fact physically means that these forces can be actively controlled by displacing the robot joints.

A. GRASP OPTIMIZATION

There can be an infinite number of solutions for the problem (8). Therefore, a cost function can be designed and optimized in order to reach the best solution in certain regards. In addition to applying suitable wrench for counterbalancing the expected task wrench, there are other concerns in a grasp. One of the most important issues in grasp synthesis is actuators saturation which has to be avoided and can be formulated as

$${}^1\lambda_i = \|f_i\| - f_{i,\max} \le 0 \tag{16}$$

where ${}^j\lambda_i$ is jth constraint for ith contact, $f_{i,\max}$ is defined as a maximum contact force which should not be exceeded at contact point i and f_i is actual contact force at contact point i. Duty range of some force sensors requires a minimum amount of force to be applied. Moreover, considering a minimum contact force avoids contact forces chattering and sustaining the continuity of contacts. Minimum force constraint can be written as

$${}^2\lambda_i = -f_i^T n_i + f_{i,\min} \le 0 \tag{17}$$

where $f_{i,\min}$ is defined as desired minimum force to be applied to contact point i and n_i is normal vector at contact point i. Avoiding slippage of contact points which are assumed to obey *Coulomb's friction* law, is also required in a grasp. This law states the following relation between the tangential component of contact force, f_{ti}, and its normal component, f_{ni},

$$\|f_{ti}\| < \mu\,\|f_{ni}\| \tag{18}$$

where μ is friction coefficient. (18) can be rewritten as $\|f_i - f_{ni}\| < \mu\,\|f_{ni}\|$ which implies $\|f_i\| \le \sqrt{1+\mu^2}\,\|f_{ni}\|$. Therefore, the slippage avoidance constraint is

$${}^3\lambda_i = \frac{1}{\sqrt{1+\mu^2}}\|f_i\| - f_i^T n_i \le 0 \tag{19}$$

An optimized grasp is able to efficiently counterbalance external wrenches as well as satisfying the mentioned constraints. Optimization can be done by using the free vector υ in (15) to satisfy constraints while keeping the contact efforts minimum. Simultaneously, external wrenches can be resisted by using the remaining term in contact force solution (15). Hence, the proposed grasp optimization can be expressed as

$$\begin{aligned} &\underset{\upsilon\in R^e}{\text{minimize:}} && \|f(\upsilon)\| \\ &\text{subject to:} && {}^j\lambda_i(\upsilon) \le 0, i \in \{1,\dots,n_c\}, j \in \{1,2,3\} \end{aligned}$$

The provided optimization ensures meeting all contact forces constraints while keeping them at minimum magnitude. This optimization does not need to be real-time and it can be done as a pre-grasp process.

B. OPTIMIZED FAILURE WRENCH

A desired w_d can be considered for the object in order to apply needed torque and moment to fail the object and at the same time avoid violating constraints on the amount of twist and deflection. Defining Y as the maximum allowable deflection and Θ as the maximum allowable twist, from (2) and (5) maximum moment, $M_{max} = \frac{2EI}{l^2}Y$ and maximum torque, $T_{max} = \frac{GJ}{l}\Theta$ will result in the desired wrench $w_d = [0\ 0\ P\ M_{max}\ 0\ T_{max}]^T$. using (7), the optimized wrench for failing the object is

$$\begin{aligned} &\underset{M,T}{\text{minimize:}} && \|w(M,T)\| \\ &\text{subject to:} && \theta(T) - \Theta \le 0 \\ &&& y(M) - Y \le 0 \\ &&& \sqrt{\sigma_{max}^2 + 4\tau_{max}^2} - S_y \le 0 \end{aligned}$$

Note that P is not considered in the proposed optimization since gripper, without loss of generality, is assumed to have enough room for at least one direction to move the grasped

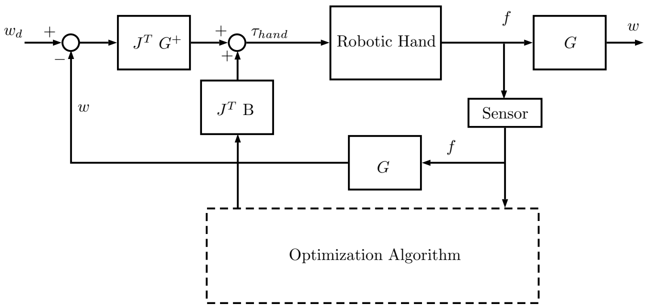


Fig. 2. Block diagram of the proposed approach.

TABLE I
PROPERTIES OF POLYETHYLENE BEAM

Modulus of rigidity (N/m^2)	3.8732×10^8
Young's modulus (N/m^2)	1.1×10^9
Yield strength (N/m^2)	2.5×10^7

part of the failed object. Therefore, it is assumed that the free direction for robotic manipulator movement is toward z direction.

Now the remaining questions are which degrees of freedom of the robot are available to apply torsion and bending about the supported point of the grasped object and also how much torques actuators have to apply. If υ^* considered to be the optimal value provided by the introduced optimization, the desired applied force is $f = -G^+ w_d + \mathrm{B}\upsilon^*$. Robotic hand is shaped and locked itself around the object by applying $\tau_{hand} = J^T f$ and then the manipulator can apply required wrench without losing contact with the grasped object. The grasp analysis static mapping is depicted in Fig. 2.

IV. NUMERICAL RESULTS

Numerical assessments were carried out on grasping of a cantilevered polyethylene beam which is fixed on the ground. As depicted in Fig. 3a there are obstacles which limit the manipulator movements. It is assumed that the locations of the obstacles and the size of the robotic hand limit the twisting angle to less than $3deg$ and displacement along x axis to less than $20cm$. The main properties of the considered polyethylene beam are summarized in Table I. The beam is assumed to have circular shape with radius of $c = 11mm$ and length of $400mm$. According to (2) and (5) maximum moment, $M_{max} = 40Nm$ and maximum torque, $T_{max} = 0.8Nm$ result in the desired wrench $w_d = [0\ 100\ 0\ 0\ 0\ 0.8]^T$. Note that applying desired maximum moment in w_d is done by applying a force in y direction. This way of applying moment is valid, since the grasp does not lose contact with the object and the applied force is always normal to the neutral axis of the object.

Fig. 3b shows the planar projection of the beam grasped by one finger only, with three revolute joints, through three

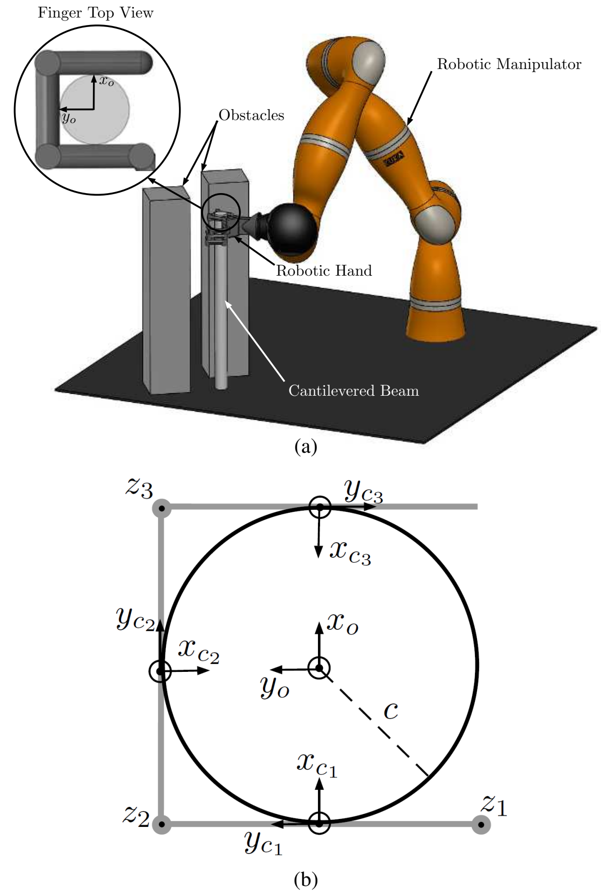


Fig. 3. (a) A robotic hand connected to a manipulator to fail a circular beam without violating constraint caused by obstacles. (b) A grasp configuration for a circular beam grasped by a finger with three revolute joints.

hard contacts. Joints, contacts, and object coordinates are depicted in Fig. 3b. Link lengths are $22mm$ and for sake of simplicity the link diameter are not considered. J_i and G_i correspond to Jacobian matrix and Grasp matrix of contact point i, respectively. The complete Jacobian matrix, $J = [J_1^T, J_2^T, J_3^T]^T$, and Grasp matrix, $G^T = [G_1, G_2, G_3]^T$ are

$$J^T = \begin{bmatrix} .011 & 0 & 0 & .011 & .022 & 0 & .011 & .022 & 0 \\ 0 & 0 & 0 & .011 & 0 & 0 & .011 & .022 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .011 & 0 & 0 \end{bmatrix},$$

$$G^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -.011 \\ 0 & 0 & 1 & 0 & .011 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -.011 \\ 0 & 0 & 1 & .011 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -.011 \\ 0 & 0 & 1 & 0 & -.011 & 0 \end{bmatrix}$$

Null space of Grasp matrix is the subspace of internal forces. From (9) the homogeneous solution is a basis for

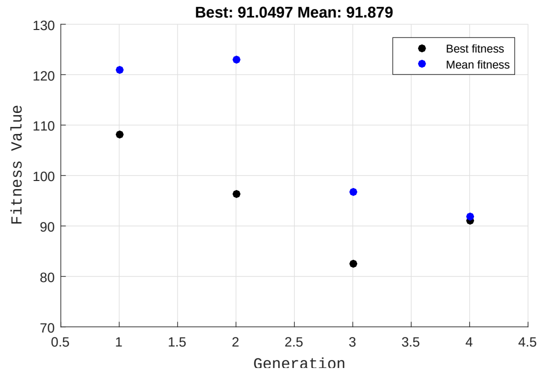


Fig. 4. The best function value versus generation.

this subspace. The null space of Grasp matrix is

$$A = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & -2 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Interpenetration of this matrix can be intuitively done by Fig. 3b. Each column shows opposing forces at different contacts, except the last column which belongs to null space of J^T. This fact can be easily understood after computing active internal force transfer matrix, B

$$B = \begin{bmatrix} -0.5353 & 0.5818 \\ 0.1256 & 0.3305 \\ 0 & 0 \\ 0 & 0 \\ -0.2513 & -0.6609 \\ 0 & 0 \\ -0.7866 & -0.0792 \\ 0.1256 & 0.3305 \\ 0 & 0 \end{bmatrix}$$

which has two columns showing that there is a contact forces corresponds to zero joint action. The column space of matrix B can be used to satisfy the force components constraints. In this example, it is assumed that all contact forces have to have normal component larger than $20N$ to avoid chattering and also maintaining in duty range of sensors. In addition, for avoiding actuators saturation maximum magnitude of $100N$ is considered for each contact forces. If friction coefficient for all contact points is considered to be $\mu = 1$, υ^* can be optimized using genetic algorithm which results $\upsilon^* = [-56.0266 - 1.4190]$. As it was noted before, the optimization process need not to be real-time and can be done as a pre-grasp process. The computation time for this example is 3.62 seconds. The best function value versus generation is plotted in Fig. 4. The resulted contact force vector is

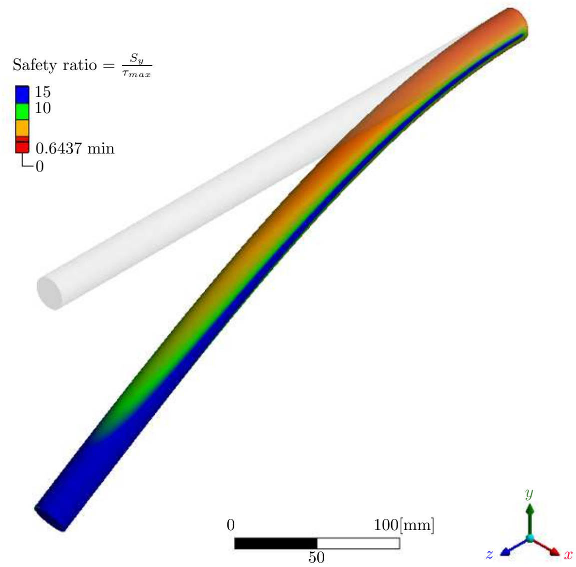


Fig. 5. Safety ratio based on MSS theory after applying $100N$ in y direction and $0.8Nm$ around z axis. The deflection is in true scale.

$$f = -G^+ w_d + \mathrm{B}\upsilon^* = \begin{bmatrix} 20.0747 \\ -13.5691 \\ 0 \\ 33.3333 \\ 33.1988 \\ 0 \\ 53.2735 \\ 53.0976 \\ 0 \end{bmatrix}$$

In this spatial case, all contact forces ($f_1 = [20.0747, -13.5691, 0]^T$, $f_2 = [33.3333, 33.1988, 0]^T$, and $f_3 = [53.2735, 53.0976, 0]^T$) are inside the circular cone, defined by

$$\sqrt{f_y^2 + f_z^2} \leq \mu f_x, f_x \geq 0$$

The torque vector of robotic hand actuators is

$$\tau_{hand} = \begin{bmatrix} -3.0720 \\ -2.1208 \\ -0.5860 \end{bmatrix}$$

The FEM simulation for the specified beam is done by ANSYS®. In this simulation, beam is considered to be isotropic and we set the safety factor in MSS theory to be 1. The resulted displacement and twisted angle are verified to be $2.059deg$ and $16.741cm$, respectively. Figure (5) illustrates the ratio $\frac{S_y}{\tau_{max}}$ which is called *safety ratio* in MSS theory. Red color indicates the ratio of 0.6437 which shows that shear stress is near 1.5 times more than yield strength of the material. According to depicted FEM result the beam fails. Deformations of the beam in x, y, and z are separately shown in Fig. 6. This figure shows that required geometrical constraints are well satisfied.

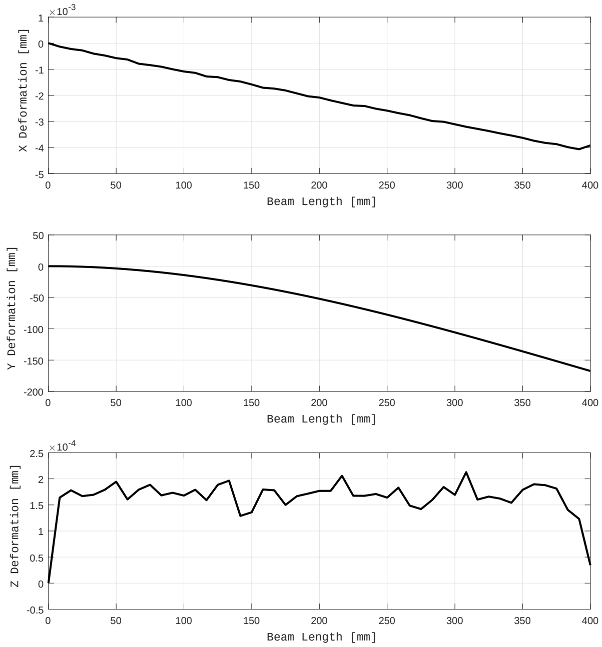


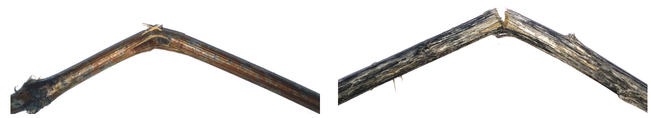
Fig. 6. Deformation of the beam through its length in x,y, and z axes.

V. CONCLUSION AND FUTURE WORK

In this paper, the problem of purposefully failing/yielding an object using an optimal robotic grasp is considered. We took all geometrical restrictions dictated by manipulation environment into account and optimized a minimal external wrench needed to successfully yield the object. A grasp structure capable of applying this failing load was introduced. A generalized grasp problem was considered in which robot can make contact using the whole surface of its body. This fact results in presence of uncontrollable internal forces. We designed a grasp optimization capable of counterbalancing large external wrenches based on internal force decomposition. A numerical example indicating a case of failing an object wrapped by a robotic finger in presence of geometrical constraints was provided. Finite element analysis of failure based on maximum-shear-stress theory was presented in order to validate the proposed grasp synthesis.

The provided analysis is valid for traditional engineering materials such as ceramics, steel or plastic which are uniform, or isotropic, in nature. In other words, material properties, such as strength, stiffness, and thermal conductivity, are independent of both position within the material and the choice of coordinate system. However, there are many applications including robotic harvesting, shaping cardboard, bone implants or any other tasks require yielding anisotropic materials.

The complete bending failure of anisotropic beams such as tree branches or mammals' bones are hard to achieve. There are cases (wood for instance) that the beam is much stronger longitudinally than transversely [13]. Buckling and *green-stick fracture* in biological beams occurs as depicted in Fig. 7. This behavior can be explained by anisotropy between the radial and tangential directions. For instance, 80 per cent of fiber cells in wood are oriented longitudinally which results in higher longitudinal yielding strength [14]. We are excited about addressing this issue in future work.



(a) (b)

Fig. 7. (a) Buckling instead of breaking into pieces. (b) Greenstick fracture rather than breaking.

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