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RUTH BARCAN MARCUS AND QUANTIFIED MODAL LOGIC

FREDERIQUE JANSSEN-LAURET

ABSTRACT. Analytic philosophy in the mid-twentieth century underwent a major change of direction when a prior consensus in favour of extensionalism and descriptivism made way for approaches using direct reference, the necessity of identity, and modal logic. All three were first defended, in the analytic tradition, by one woman, Ruth Barcan Marcus. But analytic philosophers now tend to credit them to Kripke, or Kripke and Carnap. I argue that seeing Barcan Marcus in her historical context, one dominated by extensionalism and descriptivism, allows us to see how revolutionary she was, in her work and influence on others. I focus on her debate with Quine, who found himself retreating to softened, and more viable, versions of his anti-modal arguments as a result. I make the case that Barcan's formal logic was philosophically well-motivated, connected to her views on reference, and well-matched to her overall views on ontology. Her nominalism led her to reject posits which could not be directly observed and named, such as possibilia. She conceived of modal calculi as facilitating counterfactual discourse about actual existents. I conclude that her contributions ought to be recognized as the first of their kind. Barcan Marcus must be awarded a central place in the canon of analytic philosophy.

Key Words: Ruth Barcan Marcus, Modal Logic, Modality, Direct Reference, W.V. Quine

1. INTRODUCTION

Analytic philosophy in the mid-twentieth century didn't turn out the way it was supposed to. Early analytic philosophy had largely spoken with one voice: adhering strongly to descriptivism about proper names, and equally strongly to extensionalism and anti-modal attitudes. Inspired by the success of extensional, mathematized systems designed by Russell, Whitehead, Tarski, and early Carnap, Quine in particular argued vociferously that extensional, non-modal, logical form was a hallmark of scientific or mathematical objectivity (Quine, *Significance of the New Logic*, 185). But by the mid-analytic period, the distinctive new movements in logic and the philosophy of language centred on the direct-reference theory of names, the necessity of identity, and the many and varied applications of quantified modal logic. For all three of these, the first analytic philosopher to defend them in print was a female logician known as Ruth Barcan Marcus. Ruth Barcan published the first quantified modal logic in the *Journal of Symbolic Logic* (Barcan, 'A Functional Calculus'; Barcan, 'The Deduction Theorem'; Barcan, 'The Identity of Individuals'). She published subsequent work as Ruth Barcan Marcus, appending her married name. But this was not by choice. Around the time when Barcan submitted her Review of Smullyan to the *Journal of Symbolic Logic*, she recalled, 'Church [the editor] informed me testily that he had learned I was married and must heretofore use my "legal" name' (Barcan Marcus,

'A Philosopher's Calling', 82).¹ Out of respect for her preference, I shall call her either 'Barcan' or 'Barcan Marcus', not 'Marcus'.

Barcan Marcus' journal papers are readily available and historical papers or book chapters on her crucial role have been published periodically for the past 25 years (e.g. Smith, 'Marcus, Kripke'; Smith, 'Marcus and the New Theory', Janssen-Lauret, 'Meta-Ontology, Naturalism, Quine-Barcan', Lavine, *Race, Gender, and History*). Yet it remains common among analytic philosophers either to credit quantified modal logic, the necessity of identity, and the direct-reference theory of names to Kripke's *Naming and Necessity*, or to credit the invention of quantified modal logic to Carnap's *Meaning and Necessity* and direct reference and the necessity of identity to Kripke, or even to credit direct reference to Kripke and the necessity of identity to Quine (Burgess, 'On a Derivation'). Around the turn of the millennium, signs appeared that more widespread appreciation of Barcan Marcus might be on the horizon, such as the publication of her Festschrift (Sinnott-Armstrong et al, *Modality, Morality and Belief*), Smith's defence of her, and an approving reference to her in Quine's obituary as 'his great adversary Ruth Barcan Marcus, the inventor of the theory of direct reference' (Lambert, 'Willard Van Orman Quine', 276), which suggested that she may have won the day. But over twenty years later she is still not much read, and students continue to be assigned *Naming and Necessity* to read instead of Barcan Marcus' works. My paper aims to further press the case that she must be awarded her proper place in the canon of analytic philosophy.

Specifically, I build a case that Barcan Marcus should be regarded as a prominent figure in mid-analytic philosophy, even as one of its central thinkers, owing to her invention of symbolic quantified modal logic and her pioneering work on direct reference theory, as well as her demonstrable influence on Quine's philosophy. I also set out to diagnose some of the causes for the continued neglect of Barcan Marcus by mainstream analytic philosophers and historians of philosophy.² Unlike most authors writing on Barcan Marcus, who tend to focus on her work's relationship to Kripke's, I will pursue my line of argument by concentrating primarily on some lesser-known and neglected features of Barcan Marcus' debate with Quine. One component concerns historical narrative. Historians have largely forgotten or under-emphasized how central extensionalism was to Quine's work and generally to early analytic philosophy. To view Barcan Marcus' modal logic in its historical context, against the backdrop of overwhelming extensionalist sentiment, allows us to appreciate how revolutionary she was. A historical perspective also helps us see how different her system was from Carnap's *Meaning and Necessity*. Since Carnap began

¹In the 'List of Officers and Members of the Association for Symbolic Logic', Barcan is listed as 'Barcan, Dr Ruth C (Mrs Jules A. Marcus)', Martha Kneale appears as 'Kneale, Mrs W.' (for 'William'), and multiple female professor's names appear in the form 'Harris, Prof. Marjorie (Miss)'; 'Onderdonk, Prof Virginia (Miss)', 'Swabey, Prof. Marie C. (Mrs W.C.)', appending unnecessary titles serving only to indicate their marital status.

²As whole books could be written on Barcan Marcus' contributions to analytic philosophy and logic, some issues are beyond the scope of this paper; for example, I do not have space to go into detail about Barcan Marcus' reply to Quine's 'number of the planets' argument, and I will not attempt to settle the lengthy Smith-Soames debate or contend in detail with Burgess' (also lengthy) papers on Barcan Marcus, Kripke, and Quine.

to think about intensional metalanguages in late 1937 or early 1938—much to Quine’s dismay (Quine, *Correspondence*, 241)—Hochberg contends that describing Barcan Marcus’ 1946–47 papers as the origin of quantified modal logic is ‘more myth than fact’ (Hochberg, ‘Logic to Ontology’, 288). But this is misleading. Not only does Williamson rightly point out, regarding Barcan Marcus, that ‘it is usual to reckon priority in science by date of publication’ (Williamson, *Modal Logic as Metaphysics*, 31), and Carnap did not publish his modal logic until 1947, but, I will argue, several features of her system, such as the necessity of identity and its lack of reliance on concepts, make it a clearer ancestor of the ones now popular in analytic philosophy. Barcan Marcus’ modal logic lacks several of the disadvantages of Carnap’s, which has contingent identity and unappealing ontological commitments. As I will show, over the years Quine came to appreciate this fact. I make clear that although Quine was rather slow to realize the differences between her system and Carnap’s at first, his debate with Barcan Marcus over modal logic was instrumental in making room for modality in analytic philosophy. The young modal logician Barcan had the highly influential titan of the logic establishment constantly on the back foot, retreating to increasingly softened versions of his anti-modal arguments. Although Quine remained an extensionalist all his life, his eventual position, modal inconstancy, according to which an object’s modal properties are relative to a given enquiry, was much more philosophically fruitful than his original insistence that modal logic was simply confused or mired in abstract intensional posits. Quine’s argument against Carnap’s modal logic—that it implies a distasteful array of abstract objects in the form of individual concepts—had remained unchanged since their correspondence in 1938. By contrast, Quine was challenged to improve and refine his position as a result of his exchange with Barcan Marcus, who like the early Quine, and unlike the anti-metaphysician Carnap, embraced a parsimonious and empiricist meta-ontology. Last but not least, I present Barcan Marcus in her proper context not simply historically but also by considering her whole body of work, presenting her as a systematic philosopher motivated by her classic nominalism and by an empiricist approach to ontology.

There appears to be a pervasive impression among philosophers that Barcan Marcus’ papers are so technical that only dyed-in-the-wool logicians can read them. As a result many analytic philosophers continue not to read her, to read Kripke instead, and to assign *Naming and Necessity* instead of Barcan Marcus’s work to our students, thus perpetuating the impression that modal logic, necessary identity, and direct reference originate with Kripke. In this paper, I aim to remedy this problem by making Barcan Marcus’ results more accessible to readers not expert in formal logic. To achieve this goal, I will generally keep formal treatments to a minimum and explain formal moves in natural language. I set out to explicate Barcan Marcus’ remarkable work first by glossing her modal logic from the 1940s, which is densely formal and contains almost no ordinary-language explication (deliberately, I conjecture), in ordinary-language terms. My ordinary-language gloss is informed by reading her formal logic in the context of her philosophy, specifically the empiricist nominalism which also informed her arguments for taking ordinary proper names’ meanings to be just their bearers, not descriptions of their bearers. Barcan Marcus advocated the direct reference theory because her strict nominalism implied that we have

reason to believe only in the kind of observable individuals to which we can directly assign a proper name, and whose self-identity we straightforwardly assert. Although her philosophical views were published later, I show that there are historically sound reasons to believe that Barcan Marcus already held the direct reference view in the 1940s and that it informed her modal logic, which she viewed as a way of speaking counterfactually of actual existents, thus obviating the need for abstract intensional posits in modality.

2. BARCAN'S INVENTION OF QUANTIFIED MODAL LOGIC AND HER PLACE IN THE HISTORY OF ANALYTIC PHILOSOPHY

Aged just 24, Ruth Barcan published, in March 1946, the first of a series of papers in the *Journal of Symbolic Logic*, the first publications setting out a formal quantified modal logic (Barcan, 'A Functional Calculus'; Barcan, 'The Deduction Theorem'; Barcan, 'The Identity of Individuals'). Barcan, a secular Jewish New Yorker from a socialist background, had been an exceptional undergraduate at NYU, double-majoring in mathematics and philosophy.³ Harvard would have been the obvious choice for her PhD in formal logic, but Barcan's interest in modal logic had rendered an application to Harvard impolitic. At Harvard, Quine reigned supreme. And Quine took modal logic, as Barcan Marcus was later to write, 'to be conceived in sin: the sin of confusing use and mention' (Barcan Marcus, 'Modalities and Intensional Languages', 303). As a result, Barcan went to Yale, to work with the more sympathetic F.B. Fitch. There she devised not just any modal logic, but broke new ground with a logic combining modal operators and quantifiers. Quine, writing a review of the paper according to the custom of the *Journal of Symbolic Logic*, immediately admitted, 'She is scrupulous over the distinction between use and mention of expressions—a virtue rare in the modality literature' (Quine, Review of 'A Functional Calculus', 97).

How had Barcan managed it? In brief, she approached quantified modal logic as an uninterpreted calculus, that is, solely proof-theoretically. And the question of the *use* of symbols to stand for something outside themselves, on which Quine's objection trades, is one which does not arise for uninterpreted calculi. To see why this is, consider that a proof-theoretic system consists in a mathematical theory of strings of symbols, treated merely as a syntactic system. It leaves their interpretation to one side, interpretation being a matter of semantics or model theory. Syntax alone can get a logician quite far: we can list the lexicon of the language, present formation rules which determine the grammar of the language, that is, which combinations of symbols are grammatically well-formed, axioms of the theory, and transformation rules allowing us to derive certain strings of symbols from strings of symbols of a given form. A proof-theoretic approach does not require us to know, nor even to think about, what our symbols stand for—whether they are being *used* to refer to objects other than themselves, or *mentioned* enclosed in quotation marks, considered only as symbols. The rules of inference teach us to derive given strings of symbols from other strings of symbols of a specified form. We do not need to assume

³For brief biographies of Barcan Marcus, see Gendler, 'Ruth Barcan Marcus', and Barcan Marcus, 'A Philosopher's Calling'.

that they are about anything at all. The question of use vs. mention is sidestepped. So if a logician declines to provide an interpretation for her proof theory, the issue whether she has committed the Quinean sin of confusing the use of symbols to denote something with the mention of those symbols in quotation marks cannot arise.

A vital motivation for Quine's heroes—Frege, Russell, Whitehead, Carnap, Tarski—had been to find a philosophy for a new age of mathematical and scientific discovery. The revolution in rigour, the theory of relativity, and their commitment to non-Euclidean geometry, introduced terms and logical forms so novel that they called out for a new logic, and for an attendant new philosophy, to express them in ways the old Aristotelian paradigms could not accommodate. Among such features of the new scientific philosophy were—besides its capacity to account for relational statements, transfinite arithmetic, advances in proof theory, and solutions to the set-theoretic and semantic paradoxes—also its attempts to dispense with modality, intensions, and modal and intensional language in favour of the extensional. Appendix C of *Principia* called an expression in an extensional context 'transparent' and explained this to mean 'nothing is said about it, but by means of it something is said about something else' (Russell and Whitehead, *Principia*, 407). This kind of transparency goes hand-in-hand with intersubstitutivity *salva veritate* of co-referential expressions, as Quine would later characterize referential transparency. In the world's first accessible book on the new logic, Susan Stebbing wrote, 'the intension of a word is commonly said to be all that we intend to mean by it', and objected, first of all, that 'this definition suggests an unfortunate intrusion of psychology into logic'—something the new logicians wished to avoid—and second, that 'what we intend to mean is vague and variable' (Stebbing, *Modern Introduction to Logic*, 28). Some early analytic logicians, like Stebbing, were moderates concerning extensionalism, holding out hope that intensional language could be tamed—systematized and decoupled from the psychological—via quantification over abstract objects. Frege famously proposed that reference to senses occurred in contexts where intersubstitutivity *salva veritate* failed (Frege, 'Ueber Sinn und Bedeutung'). Stebbing allowed that a set might have an intension, in the sense of a property or collection of properties determining membership, as well as an extension, a collection of members (Stebbing, *Modern Introduction to Logic*, 141). But Quine, Tarski, and the early Carnap were radicals, adhering to a stricter kind of extensionalism: 'two concepts with different intensions but identical extensions are logically indistinguishable' (Tarski, 'On the Limitations', 387). According to radical extensionalism, we should pay no heed to intensions, but only extensions, for logical-philosophical purposes. Quine was passionately attached to his extensionalism. When he learnt, in 1938, that Carnap's Principle of Tolerance had led him on the path to intensional metalanguages, Quine castigated his mentor in a letter, writing, 'your principle of tolerance may finally lead you even to tolerate Hitler' (Quine, Correspondence, 241).⁴

⁴An anonymous referee points out that Carnap's Principle of Tolerance had social and political, as well as logico-philosophical, goals (see Carus, *Carnap and Twentieth-century Thought*, 252-261; Creath, 'The Gentle Strength of Tolerance'; Yap, 'Feminism and Carnap's Principle of Tolerance'). Carnap certainly seems to have found Quine's comparison galling, sounding uncharacteristically annoyed in his reply (Carnap, Correspondence, 245; see also Janssen-Lauret, 'Quine's Philosophical Development').

Quine, from his PhD onwards, had made it his mission to purge the *Principia* system of all residual intensional language and propositional entities (Quine, *A System of Logistic*; Quine, 'Russell's Ontological Development', 81–82). His mature view, first expressed in 1953, put the *Principia*, Appendix C characterisation of transparency—of an expression such that nothing is said of it, but by means of it something is said of something else—to work, demanding that a scientific and philosophically sound language must have only transparent contexts (Quine, 'Reference and Modality', 142 n.2). Quine had worked his way up to the 1953 view in several stages, notably making a great leap forward during his visiting professorship in Brazil in 1942, published partly in his 'Notes on Existence and Necessity' and in full in *The Significance of the New Logic*. He suggested that a swathe of common intensional constructions came down to a conflation of use and mention, of words used to say something about something else with words used to say something about language. Quine had long reckoned C.I. Lewis' modal systems, S1, S2, S3, S4 and S5 (C.I. Lewis, *Survey of Symbolic Logic*), among those resting on a use-mention confusion. Lewis had intended his main connective, the fishhook ' \rightarrow ', to be read as 'implies'. Quine took exception to that reading because he maintained that it compounded a mistake made by Russell in the introduction to *Principia*, namely the conflation of 'if .. then', a sentence connective which is part of the object language, with 'implies', which is metalinguistic. Thus Quine explained in 1961 that 'Professor Marcus struck the right note when she represented me as suggesting that modern modal logic was conceived in sin: the sin of confusing use and mention. She rightly did not represent me as holding that modal logic *requires* confusion of use and mention. My point was a historical one' (Quine, 'Reply to Professor Marcus', 323). But read in historical context, this 1961 statement represents a notable climbdown from his early radical anti-modal views ca. 1942. Quine's 1942 Brazilian breakthrough led him to the view that a significant portion of intensional language—modality as well as propositional attitudes—not only ultimately rested on use-mention confusions, but that, being ultimately about language, they were likely eventually to be dispensed with by means of quantification over linguistic entities. By recommending strategies translating quantification over intensional abstracta away in favour of quantification over linguistic entities, Quine thought radical extensionalism was able to triumph over the moderate version of the view, the Frege-Stebbing-late-Carnap line which allowed for quantification over, or reference to, such things as senses, properties, or individual concepts.

Quine gradually came to his 1940s radical extensionalist view as he was making his first thorough study of Frege at the same time as arguing with Carnap about intensional metalanguages (Janssen-Lauret, 'Quine's Philosophical Development', xxixv-xxxvi). Quine admired Frege's approach to propositional attitudes, but found the third realm unpalatable. Confronted with Carnap's venture into intensional metalanguage, Quine began by making syntactic objections. Carnap shot them down easily. Quine was eventually forced to admit that Carnap's view was coherent, and that his real objection was to (what Quine saw as) its attendant ontology of abstract intensions (Quine, *Correspondence*, 326, 371). Carnap's anti-metaphysical attitudes allowed him to write this objection off as irrelevant to his aims. He and Quine reached an unsatisfactory stalemate on the topic. Views remained

stubbornly entrenched on both sides. Some years later, in *Meaning and Necessity*, Carnap continued to appeal to concepts in order to explain contingent identity. Although he took statements with a logical form equivalent to 'a=a' to be analytic (*Meaning and Necessity*, 14), he held that 'the identity sentence "the morning star is the same as the evening star"' is contingent (*Meaning and Necessity*, 134). In other words, his system allowed for the claim that the evening star is, but might not have been, the same as the morning star, this contingency being accounted for by the difference in meaning between the evening star concept and the morning star concept (*Meaning and Necessity*, 134-44). Quine continued to profess distaste for what he viewed as Carnap's positing of abstract intensional concepts. Carnap, the anti-metaphysician, continued to shrug off Quine's ontological objections. When Quine and Carnap met in Harvard to discuss the manuscript of *Meaning and Necessity*, Quine reported that they 'never got past the second page' (Lambert, 'Willard Van Orman Quine', 276). Although that claim might be slightly exaggerated, it is safe to say that there was no rapprochement. Barcan, by contrast, with her modal logic which eschewed both contingent identity and appeal to concepts, managed to shift Quine's attitudes considerably. He took a respectful tone in his exchanges with her, beginning with his reviews of her formal logic (Quine, Review of 'A Functional Calculus'); he referred, for example, to 'Miss Barcan's pioneer papers' (Quine, 'Reference and Modality', 156).

In Brazil in 1942, Quine connected his extensionalism and opposition to abstract intensional posits with use vs. mention—that is, quotation—of words and sentences, and formulated the project of paraphrasing away apparently opaque contexts via quantification over bits of language. Although 'Tully' and 'Cicero' co-refer, Quine noted, we cannot substitute one for the other in "'Cicero" has six letters', or in 'Cicero was so-called because of an association with chickpeas'. (Plutarch claimed that the *cognomen* 'Cicero', meaning chickpea, derived from an ancestor with a chickpea-like bump on his nose. 'Tully' has no leguminous connotations.) It is clear why co-referring names are not intersubstitutable *salva veritate* in the first context, where the name occurs in quotation marks. But we may equally use quotation marks to explain the second failure of substitutivity. We fully spell out the sentence as 'Cicero was called "Cicero" because of an association with chickpeas'. Here we can substitute co-referential names into the referential occurrence of 'Cicero': 'Tully was called "Cicero" because of an association with chickpeas' is also true. Quine extended the same treatment to propositional attitudes. He viewed such attitudes as relations, not to Fregean propositions in the third realm, but to sentences. If Philip knows Marcus Tullius Cicero only as 'Cicero', being unaware of the Anglicized version of his *nomen gentile*, then replacing 'Cicero' in 'Philip believes that Cicero denounced Catiline' with 'Tully' makes a true sentence into a false one (Quine, *Significance of the New Logic* 143). Just like in the 'so-called' case above, if we take the real logical form of 'Philip believes that Cicero denounced Catiline' to be 'Philip believes the sentence "Cicero denounced Catiline"', we should not expect to be able to substitute co-referring names within the belief context, since it is just another kind of quotation context. Lastly, Quine attempted to bring modality under the quotational umbrella. But here Quine had to leave behind his nominalistic comfort zone, in which he quantified only over plausibly physical linguistic entities like expressions, sounds, and their spatiotemporal referents such as Roman

orators. The only way he could see to extend the quotation-context analysis to include modality was to explain away modality in terms of analyticity. Modal statements might be read as meta-linguistic claims about sentences if we hold that ‘the result of applying “necessarily” to a statement is true if, and only if, the original statement is analytic’ (Quine, ‘Notes on Existence’, 121). In the early 1940s, Quine remained willing to entertain the question whether a physicalistically acceptable criterion of sameness of meaning might still be formulated ‘in psychological and linguistic terms’ (Quine, *Significance of the New Logic* 89), a question which he would later declare insoluble.

Enter Barcan’s impeccable proof theory. After the publication of each of her papers, Quine wrote a brief, and not unfavourable, review in the *Journal of Symbolic Logic*, which was then in the habit of publishing reviews of papers. It appears clear given the context above that, simply by being an uninterpreted proof-theoretic calculus, Barcan Marcus’ system succumbed neither to Quine’s objection to C.I. Lewis’s modal logic—the use-mention objection—nor to his objection to Carnap’s modal logic—the objection from abstract intensions. But the latter point was not immediately apparent to Quine. I will argue that, although it took Quine a few years to fully realize that Barcan was not vulnerable to these objections, he learned much more from his exchanges with her on the topic than from those with Carnap. Quine discovered that she was, in this arena, a more interesting adversary to him than Carnap. As we’ll see, this was in part because Barcan’s metaphysical and meta-ontological views were closer to Quine’s than Carnap’s. Both Quine and Barcan Marcus were moderate, empiricist metaphysicians who found versions of nominalism appealing. Carnap considered even such modestly ontological discourse to be composed of metaphysical pseudo-statements. But ontological and meta-ontological questions were to be key to several of the uses of, and debates connected to, modal logic in mid- and late analytic philosophy. As a result of that common ground with Barcan Marcus, and of her incisive arguments, Quine genuinely changed his mind over the course of his debate with her, while his response to Carnap’s modal logic remained fixed.

3. BARCAN’S QUANTIFIED MODAL LOGIC

Barcan’s 1946-47 papers are densely formal. They consist almost entirely in proofs of theorem schemata, after setting out the lexicon, syntax, definition schemata, axiom schemata, and rules of inference for quantified modal logic, or rather, a family of quantified modal logics. In 1946, Barcan introduced first-order quantified S2 and S4, and in 1947, second-order quantified S2 and S4. She then derived lists of theorems, or strictly speaking theorem schemata, from her axiom schemata by means of the rules of inference. Lewis’s simplest propositional modal logic, S1, has the following axioms, where ‘ \cdot ’ is the conjunction operator, and ‘ $p \rightarrow q$ ’ equivalent to ‘ $\Box(p \supset q)$ ’,

$$\begin{aligned} (p \cdot q) \rightarrow (q \cdot p) \\ (p \cdot q) \rightarrow p \\ p \rightarrow (p \cdot p) \end{aligned}$$

$$\begin{aligned} ((p \cdot q) \cdot r) \rightarrow (p \cdot (q \cdot r)) \\ ((p \rightarrow q) \cdot (q \rightarrow r)) \rightarrow (p \rightarrow r) \\ (p \cdot (p \rightarrow q)) \rightarrow p. \end{aligned}$$

S2 adds to the above,

$$\diamond(p \cdot q) \rightarrow \diamond p$$

from which it follows that if ' $p \rightarrow q$ ' is provable, then so is ' $\Box p \rightarrow \Box q$ '. The better known S4 system has the characteristic axiom

$$\Box p \rightarrow \Box \Box p.^5$$

We can then use the inference rules of modus ponens, adjunction, and substitution to derive theorems.

Where Lewis had expressed his axioms in the object language, e.g. the axiom ' $p \rightarrow (p \cdot p)$ ', Barcan used a metalanguage to formulate corresponding axiom schemata, such as ' $A \rightarrow (AA)$ '. (Barcan expressed the conjunction of A and B simply as 'AB', omitting Lewis's '·'.) We can see that Quine's assertion that Barcan was scrupulous over use and mention was justified.

Barcan then added to her metalinguistic statement of S2 a universal quantifier, commonly accepted axiom schemata for the universal quantifier, an inference rule of universal generalisation, as well as her invention, the famous Barcan Formula,

$$\diamond (\exists \alpha)A \rightarrow (\exists \alpha) \diamond A.$$

Like the rest of Barcan's axiom schemata, the Barcan Formula is strictly speaking schematic, not a formula. The real formulae are the instances of the schema. But I will follow the convention of calling the schema the 'Barcan Formula' here. 'The Barcan Formula', in this usage, is not a definite description but a proper name. With the above machinery, Barcan proved more than 80 theorems, of which 60 were quantificational. As Quine pointed out in his review, several of these had no equivalent in non-modal quantification theory (Quine, Review of 'A Functional Calculus'). These include the universally quantified version of the Barcan Formula,

$$(\alpha) \Box A \rightarrow \Box (\alpha) A$$

and the existential and universally quantified versions of the Converse Barcan Formula,

$$(\exists \alpha) \diamond A \rightarrow \diamond (\exists \alpha)A$$

$$\Box (\alpha) A \rightarrow (\alpha) \Box A.$$

Barcan's proof theory appears to have been designed to side-step questions of interpretation and accusations of use-mention confusions, thus avoiding Quine's main argument against modal logic in the 1940s. Her line introducing ' \diamond ' into her lexicon reads simply 'modal operator' ('A Functional Calculus', 1), and does not describe it in terms of possibility. The Barcan system of quantified modal logic had no interpretation in terms of domains at all, even for the quantifiers. In particular, Barcan did not, like Kripke ('Semantical Considerations') was to do, present an interpretation in terms of possible worlds, with the box and the diamond quantifying over all worlds and some worlds respectively.⁶ In sections

⁵Here I use a slightly simplified presentation of Lewis's propositional modal logics based on Lewis and Langford's 1932 *Survey of Symbolic Logic*, rather than the original versions in C.I. Lewis' 1918 *Survey of Symbolic Logic*. The 1918 version has as a consequence that if p is false then p is impossible, but the 1932 version lacks this flaw. For ease of exposition I have used brackets in place of Lewis and Langford's *Principia* dots, and ' $\Box p$ ' instead of ' $\sim \diamond \sim p$ '.

⁶An anonymous referee asks why I do not consider views on which the possible worlds of Kripke's system are 'mere technical devices' not requiring belief in the existence of possible worlds. My answer is twofold:

4-5, I will argue that there is historical evidence that Barcan is likely to have had in mind a parsimonious interpretation of modal logic as speaking counterfactually about actual existents, thus avoiding both possible worlds and Carnapian intensional concepts. Very briefly, Barcan's empiricist epistemology was at odds with belief in objects we cannot even in principle empirically encounter via the senses, such as possible worlds and abstract concepts, but allowed for modal discourse about the concrete individuals we can know empirically and directly.

The absence of a possible-world semantics has repercussions for the interpretation of the Barcan Formula. Formally speaking, the Barcan Formula is a smooth and elegant way to combine quantified logic and modal logic. Logicians who continue to defend formal systems with the Barcan Formula in preference to a Kripke-style interpretation include Linky and Zalta, 'In Defense', and Williamson, *Modal Logic as Metaphysics*. There are interpretive questions to be raised about the Barcan Formula, as is often stressed. But Barcan never endorsed a reading of it as saying, 'if in some possible world, something is ϕ , then there is something in the actual world which is possibly ϕ '. Instead, she treated the Barcan Formula only proof-theoretically at first, and later presented an interpretation which did not posit possible worlds. So the better-known objections raised to systems with the Barcan Formula do not affect Barcan's own system.⁷ It is often forgotten that large interpretive question marks loom over Kripke's interpretation as well, as we shall see in section 5 below. In 1946-47, Barcan sidestepped all of these interpretive issues. The only line in her 1946 paper which might be construed as explicitly philosophical is a brief remark that certain quantified 'theorems corresponding to the so-called paradoxes of material implication' cannot be proved. These so-called paradoxes, which were of interest to C.I. Lewis in developing his modal logic, concern the tension between the intuitive application

first of all, since this is a historical paper, I am concerned primarily with the position of Barcan Marcus' system and philosophy in the mid-20th century debate about modality. This debate was driven in part by the assumption that those who use possible-world models (or other models positing intensional abstracta such as concepts) must believe in the posits of their models, an assumption denied by Carnap for general anti-metaphysical reasons but otherwise widespread, as I show in this paper with respect to Quine's criticisms of Carnap, with respect to the Quine-Barcan debate, and with respect to Quine's objections to Kripke. See also Janssen-Lauret, 'The Quinean Roots', for more on Quine's ontology-focused anti-modal arguments and Lewis's replies. Secondly, I share Quine's view that philosophers should believe in the posits of their best theories, and that if any manner of speaking is considered a mere *façon de parler* or technical device, any apparent ontological commitments which it appears to have must be clearly explained away.

⁷Most advocates of modal systems which presuppose a possible-world semantics and an objectual interpretation of the quantifiers choose systems which validate the Converse Barcan Formula, but not the Barcan Formula. This is because, given these presuppositions, the Barcan Formula is interpreted as saying that if in some possible world, something is ϕ , then there is something in the actual world which is possibly ϕ . Although Linky and Zalta and Williamson defend that reading of the Barcan Formula, most advocates of possible worlds believe in mere *possibilia*, objects which exist in other possible worlds but not in the actual one. As a result, they choose systems which do not validate the Barcan Formula. In her early work, in the 1940s, Barcan did not present an interpretation for her system, nor a preferred reading of the Barcan Formula. By 1961, as we'll see below, her preferred reading appears to be 'if possibly something is ϕ , then there (actually) is something which is possibly ϕ '. Objections to the Barcan Formula which presuppose that it quantifies over possible worlds are therefore not effective against Barcan's own view.

of ‘implies’ and the *Principia* treatment of ‘material implication’ (which Quine thought ought to be called the ‘conditional’ or ‘material conditional’). If implication is identified with the material conditional, a false statement implies any statement whatsoever—since the material conditional ‘ $p \supset q$ ’ is true whenever p is false—and a true statement is implied by any statement, since the material conditional ‘ $p \supset q$ ’ is true whenever q is true as well. Lewis’s strict conditional \rightarrow was intended to mitigate these counterintuitive consequences. Barcan proved that in her system, statements of the form,

$$\sim(\exists\alpha)A \rightarrow (\alpha)(A \rightarrow B),$$

glossed as ‘nothing’s being A implies that necessarily all A s are B s’, are not provable, and so at least some of the so-called paradoxes of material implication do not arise in quantified S4. But although Barcan’s quantified version was new, the general point she made about material implication was one already familiar from Lewis’s treatment of modality. So it did not introduce any new philosophical controversy.

The title of Barcan’s 1947 paper has a more novel and controversial philosophical ring to it: ‘The Identity of Individuals in a Strict Functional Calculus of Second Order’. Here, as the title indicates, Barcan introduced a second-order system, allowing quantification into predicate position. Again she began with quantified S2. Barcan’s proofs largely use quantified S2, adding the resources of S4 only where S2 is insufficient for her purposes. In this paper, Barcan proved the necessity of identity. But the proof was not of the familiar form which ends with ‘ $(a = b) \rightarrow \Box(a = b)$ ’. Barcan’s second-order calculus did not use individual constants, but only variables. Nor did she use primitive identity. Instead she offered up two candidate predicates of defined identity. By modern lights, both would be considered forms of indiscernibility. The first is a relation which she called ‘ I ’, and which can be glossed in ordinary language (though this is not a syntactic, but a semantic, gloss) as necessarily satisfying all the same predicates. The second relation she called ‘ I_m ’, ‘ m ’ for ‘material’, glossed as simply having all the same predicates, whether necessarily or otherwise. Her characterisation of ‘ I ’ is:

$$I =_{df} \hat{\alpha}_1 \hat{\alpha}_2 (\theta)(\theta(\alpha_1) \rightarrow \theta(\alpha_2))$$

Again, a purely syntactic ordinary-language gloss is difficult to give, but the intuitive philosopher’s English gloss is that the relation called ‘ I ’ is defined as the set of pairs $\langle \alpha_1, \alpha_2 \rangle$ such that, necessarily, if α_1 satisfies the predicate θ then α_2 also has the predicate θ . The definition of ‘ I_m ’ is the same except that it has the conditional instead of the strict conditional. Barcan then proved a number of theorems involving the relation ‘ I ’, including that ‘ I ’ is reflexive, symmetric, and transitive. On the final page of her paper, she proved that in second-order quantified S4, ‘ I ’ and ‘ I_m ’ are necessarily equivalent. So if α_1 and α_2 are indiscernible—satisfy all and only the same predicates—then they are necessarily indiscernible. This can be glossed in more natural-sounding ordinary language as saying that statements of identity (strictly speaking, indiscernibility) are never contingently true—if true, they are necessary.

But here we are venturing into interpretation, something not engaged in in Barcan’s 1946-47 papers. Barcan originally couched her work in proof-theoretic terms and declined to provide any interpretation or model theory. This does not mean that she believed that

it should remain uninterpreted. My conjecture is that she chose her austere metamathematical approach precisely because Quine's customary objections to modal logic could have no purchase on it. Nevertheless, she favoured an interpretation according to which modal discourse is not discourse about possible worlds, but counterfactual discourse about actual existents. It has already been made clear that there is no sin of confusing use and mention here. What's more, as we shall see, my reading of Barcan as a systematic philosopher indicates that there is evidence that she already had her intended interpretation in mind in the 1940s, one driven by her underlying empiricist and nominalist philosophy. Past historians have treated men like Quine and Kripke as systematic philosophers, but neglected to do the same for Barcan Marcus, and thus missed not just how rich and interesting her underlying thought was, but how her overall views constitute a worthy rival to Quine's, as Quine himself came to appreciate.⁸

4. BARCAN MARCUS' PHILOSOPHICAL SYSTEM

Traditionally a 'modal' had been a term for any operator which qualified the truth of a judgement. Quine appears to have believed that mathematics and science deal only in unqualified truth, and that the new logic and scientific philosophy ought to be modelled on them in this respect (Quine, *Significance of the New Logic*, 94). But Barcan Marcus held a different view. Although, like Quine (and Carnap) she had empiricist sympathies, Barcan Marcus embraced neither Carnap's anti-metaphysical empiricism nor Quine's approach to ontological commitment. Instead, she took as a point of departure that in empiricist investigations of the world, we encounter concrete individuals: people, animals, plants, rocks, mountains, planets, etcetera. Unlike Quine, she countenanced direct cognitive access to individuals, not mediated by descriptions. Our minds reach out and grasp these concrete objects by acquaintance. Once we have achieved direct, unmediated knowledge of an individual, we may assign it a directly referential name, which Barcan Marcus calls a 'tag' ('Modalities and Intensional Languages', 309-310).

According to Barcan Marcus, tags have an ineliminable role to play in language and logic: to single out an individual, whether or not we have described it. Barcan Marcus considered Russell 'a primary philosophical influence' ('A Philosopher's Calling', 81), and said that her views on knowledge by acquaintance and the possibility of direct reference, which she developed early in her career, were inspired by his ('A Philosopher's Calling', 82).⁹ But, I argue, she was overly modest in attributing to Russell a view which appears original with her. There are significant differences between her doctrine of acquaintance and his. The objects of Russellian acquaintance are very limited, either mental sense

⁸I am grateful to an anonymous referee for encouraging me to stress this point more.

⁹I am grateful to an anonymous referee for the suggestion to say more about Barcan Marcus' relationship to Russell. The referee also contends that views similar to Barcan Marcus' and Russell's are expressed in Smullyan, 'Modality and Description' reviewed by Barcan Marcus (Review of Smullyan) and Fitch, 'The Problem'. Fitch's paper, like Fitch's 'Attribute and Class' which I discuss below, explicitly acknowledges the influence of Barcan Marcus on Fitch's view. Smullyan's pertains to the response to Quine's number of the planets argument which, as I noted in footnote 2 above, is beyond the scope of the present paper but which I intend to discuss in the future.

data or, at best, the surfaces of physical objects, as well as some universals: 'it will be seen that among the objects with which we are acquainted are not included physical objects (as opposed to sense-data)' (Russell, 'Knowledge by Acquaintance', 112). Barcan Marcus's acquaintance is much more expansive, allowing for our minds to reach out directly to whole physical (or otherwise empirically encounterable) individuals. The point of modality, on her view, was to model how empirical science generalizes over such concrete individuals about which our sciences theorize: 'Candidates for essential attributes were, as I understood it, physically necessary properties: those covered by physical or more broadly empirical law' ('A Philosopher's Calling', 85).

Barcan Marcus also gave herself insufficient credit for originality when she ascribed her position that ordinary proper names like 'Scott' are directly referential to the early Russell of 'Knowledge by Acquaintance'. Although some passages in that paper concerning Scott are easily read this way, and were, for example, by Constance Jones, to whose 'Mr Russell's Objections' Russell replied in 'Knowledge by Acquaintance'. Jones objected to what she saw as Russell's position that '*Scott* is merely "a noise or shape" and entirely without intension', that is, without descriptive meaning (Jones, 'New Law' p. 183).¹⁰ But in fact, as we have seen, Russell did not think that middle-sized physical objects such as human bodies were objects of acquaintance. In that same paper he stressed, concerning Bismarck, that although 'Bismarck' might potentially taken as a directly referential name by Bismarck, if acquaintance with the self is possible, by others Bismarck's 'body as a physical object, and still more his mind, were only known as the body and the mind connected with these sense-data [caused in others by Bismarck]' (Russell, 'Knowledge by Acquaintance', 114). Barcan Marcus' much more liberal notion of acquaintance, by contrast, did allow for acquaintance with human organisms, and for assigning them a directly referential tag. According to her, an ordinary proper name is a tag, and the meaning of a tag is just its bearer; it has no sense, no discursive meaning. 'This tag, a proper name, has no meaning. It simply tags' (Barcan Marcus, 'Modalities and Intensional Languages', 310). Although tagging is, by Barcan Marcus' lights, the ur-form whereby we encode knowledge of objects in language, in some cases we may of course know of the existence of objects by inference or by testimony. Other forms of language, by contrast, such as predicates, logical operators, numerals, propositional discourse, and even the quantifiers, do not, on Barcan Marcus' account, have this referential function, but are mere manners of speaking. Their functions are to ascribe characteristics, connect sentences, count, generalize over sentences and sentential functions, and speak with generality. Barcan Marcus took a classic nominalistic attitude according to which all forms of language except names are syncategorematic, meaningful but without standing for something.

To analytic philosophers accustomed to the Quinean line on ontological commitment, the idea of a syncategorematic quantifier, one which is not ontological committing, may appear puzzling. But Barcan Marcus' substitutional theory of quantification managed to relieve the quantifiers of ontological commitment. On her view, it is names which are categorematic, and therefore carry ontological commitments in their wake. Quantifiers,

¹⁰For more on Russell and Jones, see Janssen-Lauret, 'Grandmothers of Analytic Philosophy'.

by contrast, are merely devices allowing us to speak with generality. 'It does not seem to me that the presence there of a quantifier forces an ontology ... If the case is to be made for reference ... it would have to be made, as for proper names, independently' (Barcan Marcus, 'Nominalism and the Substitutional Quantifier', 359). Instead of interpreting the quantifiers as 'there is' and 'all', the objectual reading according to which 'variables ... take objects as values relative to a sequence of objects' (Barcan Marcus, 'Nominalism and the Substitutional Quantifier', 357), she suggested a reading of the particular quantifier as non-temporal 'sometimes' or 'in some cases' or 'it is sometimes the case that' and of the general quantifier as non-temporal 'always', 'in all cases', or 'it is always the case that'. Her substitutional interpretation of the quantifiers rendered variables not as taking objects as values, but as placeholders for substituends: the particular quantifier ' $\exists x\phi x$ ' being explicated as a disjunction of substitution instances ' $\phi a \vee \phi b \vee \phi c \dots$ ' for all terms ' $a, b, c \dots$ ' of the relevant category, e.g. individual terms, and the general quantifier ' $\forall x\phi x$ ' being explicated as a conjunctions of substitution instances ' $\phi a \wedge \phi b \wedge \phi c \dots$ ' for all terms ' $a, b, c \dots$ ' of the relevant category. I say 'of the relevant category' because Barcan Marcus held that there may be different ranges of variables correlated with different terms and featuring in different kinds of substitution instances. For example, she allowed explicitly for sentential quantification, with only sentences as substitution instances and not implying commitment to propositions; "'(p) (pv-p)" need not be paraphrased as "Any proposition bears the excluded middle relation to its negation"' (Barcan Marcus, 'Nominalism and the Substitutional Quantifier', 360).¹¹ Modal operators are also syncategorematic: they are devices for talking counterfactually about empirically encounterable individuals (Barcan Marcus, 'Modalities and Intensional Languages', 319-321).

Unlike Quine, Barcan Marcus thought it plausible that empiricist theorising asks questions and tests hypotheses not just about how individuals are, but about how they might have been; how they might have moved, behaved, been constituted, or changed if placed in different circumstances. It is key to Barcan Marcus' thought that the ineliminable role of direct, immediate, non-descriptive reference is what enables us to formulate and answer such modal questions. A tag is a mere label, whose semantic role is simply to point directly to the individual. Tags manage to perform this role because they lack discursive meaning. If they encoded information in some descriptive manner, then they could not single out their bearers directly. In that case, tags' ability to stand for their bearers would be conditional upon the descriptive information being true of those bearers. From a logical point of view, then, tags are very different from descriptions. It is not a coincidence, but a consequence of the semantic role of names, that once a name has been assigned to an individual, it denotes that individual in any context including the modal. By contrast, it may very well be a coincidence that a description is satisfied by a given individual, especially a concrete individual. Concrete individuals largely satisfy the predicates which

¹¹For more on the uses of Barcan Marcus' substitutional account of quantification and its role in her overall system, see Janssen-Lauret, 'Meta-Ontology, Naturalism, Quine-Barcan', and Janssen-Lauret, 'Committing to an Individual', both of which argue that Barcan Marcus' view amounts to an alternative view of ontological commitment with its own canonical language to rival Quine's.

apply to them as a matter of empirical fact, contingently. So we should not expect to be able to substitute descriptive phrases for names in modal contexts.

It might be objected that there is no direct evidence in Barcan's 1946-7 papers that they had been inspired by such philosophical views. After all, the proof-theoretic system contains no individual constants, the formal analogues of proper names. It is true that Barcan Marcus' proof of the necessity of identity uses variables, not proper names, and uses defined identity. But historical evidence for Barcan Marcus' having held a version of her mature view is found in the works of her PhD supervisor, F.B. Fitch. In 1950, Fitch wrote, 'The system of modal logic developed by Ruth Barcan suggests that the simplest view is that no identities should be regarded as merely contingent and that identified entities should be everywhere intersubstitutable. ... Furthermore, if entities X and Y have been identified with each other, it seems reasonable to suppose that the names of X and Y should also be everywhere intersubstitutable where they are being used as names' (Fitch, 'Attribute and Class', 252, qtd. in Smith, 'Marcus and the New Theory', 219). Corroborating evidence that Barcan held the direct-reference theory of names and sophisticated views on their substitutivity in the 1940s is also found in a paper by Smith, who reports, '[Barcan Marcus] told me in an interview, that she developed this view of names and descriptions in course of writing her thesis in 1943-45 and explained them in conversation to her dissertation adviser Fitch. This is in keeping with the passage from Fitch I just quoted, where Fitch mentioned these ideas and attributes them to [Barcan] Marcus. Notice that he is not advancing these ideas as his own ideas' (Smith, 'Marcus and the New Theory', 219). The importance to Barcan's system of individuals is also apparent from their mention in her 1947 paper title. There she took care to point out that she had 'the purpose of introducing the relation of identity of individuals' in mind and proved that it was necessary (Barcan, 'The Identity of Individuals', 12).¹²

5. THE QUINE-BARCAN MARCUS DEBATE ON QUANTIFIED MODAL LOGIC PHASE I: USE vs. MENTION AND ABSTRACT OBJECTS

The first phase of the Quine-Barcan Marcus debate on the legitimacy of quantified modal logic, then, has a perplexing feature. From 1947 to 1951, Quine appears still to have considered her vulnerable to his two key objections to quantified modal logic, the use-mention argument and the argument from abstract intensions. He even ascribed to her the view that identity is contingent. Given what we know about Barcan Marcus' formal

¹²An anonymous referee raises the objection that even if Barcan's formal system is plausibly inspired by her views on direct reference, the expressive potential of that system without constants remains weaker than the full theory of the direct reference of proper names which the referee contends 'Kripke, Kaplan and others promulgated in the seventies'. This is not quite correct. It is true that a system without constants is weaker in its expressive power than a system with constants, specifically with respect to expressing identity statements, as explained in Barcan Marcus, 'Modalities and Intensional Languages', 310, and Janssen-Lauret, 'Committing to An Individual', 597-599. But Barcan Marcus' 'Modalities and Intensional Languages', published in 1961, nine years before the start of the seventies, does have the full direct reference theory. Although it lacks the causal theory of reference, Barcan Marcus ('A Philosopher's Calling', 85-6) ascribes the causal theory to Geach's 1969 'The Perils of Pauline', not to Kripke.

modal logic, it appears as though Barcan's 1946-47 papers ought to have put paid to these misunderstandings. And yet this first phase took years to resolve—over a decade, if we count up to Quine's first explicit recantation of his claim that Barcan must posit abstract intensions to account for contingent identity (Quine, 'Correction to Review'). The delay resulted, I will argue, from Quine's misreading Barcan Marcus and being slow to realize the full implications of her work.

As we've seen, Barcan Marcus did not succumb to the first objection because she approached quantified modal logic solely proof-theoretically. Her modal logic did not violate Quinean strictures about extensionality in this regard. She never argued for its usefulness based on any argument that the fishhook operator captured the relation of implication, nor did she presume that theorems beginning with ' \square ' were analytic.

Quine admitted that Barcan Marcus was careful about distinguishing use and mention in her modal logic. He never accused her in particular of conflating the two. Yet, strangely, Quine persisted in complaining that modal logic was embroiled in use-mention confusions and analyticity into the 1950s ('Two Dogmas', 22). Perhaps, in the absence of an interpretation for Barcan Marcus' calculus, Quine felt justified in continuing to believe that if we do interpret modal logic, any such interpretation must involve analyticity. Tellingly, he called the paper he wrote after he had seen her papers, but before his 1947 review of her appeared, 'The Problem of Interpreting Modal Logic', and not 'The Problem of Modal Logic' *tout court*.

Barcan Marcus had not left herself open to Quine's second objection either, since she did not posit abstract intensions. Quine at first mistook her view for a version of Carnap's, and assumed that she must have been committed to intensional objects.

'The system is accordingly best understood by reconstruing the so-called individuals as "individual concepts." For example the material planet which is at once the Evening Star and the Morning Star should not be reckoned as a value of the individual variables, lest it turn out not to be identical (in the full sense) with itself; on the other hand, two distinct concepts of Evening Star and Morning Star are available as values of the variables without fear of paradox. See the reviewer's remarks in this volume, p. 47 [i.e. Quine, 'The Problem of Interpreting']' (Quine, Review of 'The Identity of Individuals').

As this quotation reveals, Quine's reasons for thinking so were connected to his mistaken assumption that identity, in Barcan's system, is contingent identity. As we have just seen that Barcan in fact presented a proof best glossed in ordinary language as 'identity is a necessary relation', it is puzzling to see Quine get her so wrong. Again, Quine took some time to realize, or concede, his mistake. It was 11 years after his original review that the puzzle was fully resolved. Quine then finally admitted, in a printed correction in the journal, that he had been in error, and also revealed the reason why. His misreading of Barcan came about because he had never been sent the final page of her manuscript, and had missed her proof of the necessity of identity (Quine, 'Correction', 342).

In later works, Barcan Marcus argued explicitly for nominalism and professed disbelief in meanings as abstract entities (Barcan Marcus, 'Nominalism'). Yet even without that

context, which Quine in 1947 had no access to, it ought to have been clear that her 1946-47 papers do not imply ontological commitment to abstract intensions. Quine's theory of ontological commitment applies only to interpreted theories. Barcan did not provide any interpretation for her 1946-47 proof-theoretic calculus, so it follows that that system had no ontological commitments. *A fortiori*, it had no ontological commitment to abstract intensions. Quine appears to have wanted to press the objection that if her system were to be interpreted, her only option would be to embrace an interpretation with, like Carnap's, 'is committed to an ontology which repudiates material objects (such as the Evening Star properly so-called) and leaves only multiplicities of distinct objects (perhaps the Evening-Star-concept, the Morning-Star-concept, etc.) in their place' (Quine, Review of 'The Identity of Individuals', 47). But, as Quine admitted in his 1958 'Correction to Review', Barcan Marcus' final page was incompatible with that interpretation, since the purpose of concepts is to explain contingent identity. Barcan had proved the necessity of identity, albeit extensionalized identity, or what we would now call indiscernibility. So to a reader who, unlike Quine, had access to the final page of her 'The Identity of Individuals', it would be clear that, if the author had an intended interpretation for her system at all, that interpretation could not resemble Carnap's in this regard. Whether Barcan had an intended interpretation in mind for her modal system would have been unknown to Quine in 1947. Nor could he have been aware, based on her publication record at the time, that Barcan held nominalistic views on ontology, and was in the process of developing meta-ontological views according to which it is directly referential names, not quantifiers, which carry ontological commitment and feature in statements of necessary identity. It was only in 1961 that she published her views on the interpretation of her system.

In 1961, Barcan Marcus revisited the necessity of identity. By this point, she argued for the necessity of identity in terms more familiar to us now. She had gone over to using constants instead of just variables, and felt drawn to the view that her relation 'I', of identity, was a primitive relation, not one defined in terms of indiscernibility. She considered it at least logically possible that there might be two indiscernible objects which are nevertheless distinct, commenting, 'I suppose that at bottom my appeal is to ordinary language, since although it is obviously absurd to talk of two things being the same thing, it seems not quite so absurd to talk of two things being indiscernible from one another' (Barcan Marcus, 'Modalities and Intensional Languages', 305). She thought, rather, that the problem of the morning star and the evening star should be explained in terms of substitutivity restrictions on expressions which were not tags. She also put forward 'modal' arguments of the form now usually attributed to Kripke. Barcan Marcus noted, for example, that it is necessary that Venus is Venus, but that 'the star first seen in the evening might have been different from the star first seen in the morning' ('Modalities and Intensional Languages', 311), and that the latter is therefore contingent. This implies that the statements are different in logical form, not that identity is not necessary.

Barcan Marcus presented a kind of ordinary-language based error theory to account for the sense that 'the morning star is the evening star' is a contingent identity. Where, as we saw above, Carnap's system implicitly allowed for the claim, 'the morning star is, but might not have been, the evening star', Barcan Marcus thought this kind of claim

rested on a mistake, though one made intelligible by an ambiguity in natural language. She maintained that there are two ways of hearing 'the morning star is the evening star'. There is one on which it expresses an identity, but is not contingent, and one on which it is contingent but does not express an identity. The former reading has 'the morning star' and 'the evening star' as directly referential names, flanking the identity sign; on this reading, the sentence is necessary. The latter reading construes 'the morning star' and 'the evening star' as descriptions, and 'is' as expressing a relation weaker than identity. On the latter reading, the sentence is contingent. As these two readings are different disambiguations of the same sentence, it would be fallacious to conjoin them and derive 'the morning star is, but might not have been, the evening star'; so it would be a mistake to admit contingent identity. Barcan Marcus expressed her point by raising the question whether in the statement she labels '(10)', namely 'the morning star eq the evening star', 'eq' is a stand-in for identity or for some weaker equivalence.

If we decide that 'the evening star' and 'the morning star' are names for the same thing, ... then they must be intersubstitutable in every context. In fact it often happens, in a growing, changing language, that a descriptive phrase comes to be used as a proper name – an identifying tag – and the descriptive meaning is lost or ignored. Sometimes we use certain devices such as capitalization and dropping the definite article, to indicate the change in use. 'The evening star' becomes 'Evening Star', 'the morning star' becomes 'Morning Star', and they may come to be used as names for the same thing. Singular descriptions such as 'the little corporal', 'the Prince of Denmark', 'the sage of Concord', or 'the great dissenter', are as we know often used as alternative proper names of Napoleon, Hamlet, Thoreau and Oliver Wendell Holmes. One might even devise a criterion as to when a descriptive phrase is being used as a proper name. Suppose through some astronomical cataclysm, Venus was no longer the first star of the evening. If we continued to call it alternatively 'Evening Star' or 'the evening star' then this would be a measure of the conversion of the descriptive phrase into a proper name. If, however, we would then regard (10) as false, this would indicate that 'the evening star' was not used as an alternative proper name of Venus. (Barcan Marcus, 'Modalities and Intensional Languages', 308-309)

It follows that we cannot in general substitute descriptive expressions for directly referential expressions in all contexts, especially not in modal contexts. When we refer to an individual directly, using a tag, this language form allows us simply to point to the individual without intermediary. Direct reference enables us to ask of that exact individual, which is necessarily itself, how it might have been different or behaved differently. But the expressions which we need to characterize the different conditions which the individual might have met have logical forms which are different from those of tags: descriptive forms, or predicative ones. Concrete individuals are necessarily self-identical, but they

satisfy many of the descriptions which apply to them contingently. It may well be a coincidence that two descriptions correctly describe the same individual. But where two tags refer directly to the same thing, the result of concatenating them with the identity sign yields a logical, necessary truth.

Above, we saw that Barcan Marcus explained that there is a way of hearing ‘the morning star is the evening star’ where the ‘is’ expresses a contingent weaker equivalence, not identity. It expresses that as a matter of empirical fact, the first star visible in the morning is also the first star visible in the evening. But if ‘the morning star’ and ‘the evening star’ function as proper names, then its logical form is a statement of identity. It is equivalent to ‘Venus I Venus’ in her nomenclature, with ‘I’ now standing for primitive identity. In that case, the statement is not contingently true, or true as a matter of empirical fact, but necessary.

‘You may describe Venus as the evening star and I may describe Venus as the morning star, and we may both be surprised that as an empirical fact, the same thing is being described. But it is not an empirical fact that

(17) Venus I Venus

and if ‘a’ is another proper name for Venus

(18) Venus I a.’ (Barcan Marcus, ‘Modalities and Intensional Languages’, 310).

One of Barcan Marcus’ key arguments in this paper concerns the necessity of identity. She argues that true identity statements are logical truths, often called ‘tautologies’ in the mid-twentieth century, and that a logical truth remains a logical truth upon substitution of co-referring names.

‘Consider the claim that

(13) *aIb*

is a true identity. Now if (13) is such a true identity, then *a* and *b* are the same thing. It doesn’t say that *a* and *b* are two things which happen, through some accident, to be one. True, we are using two different names for that same thing, but we must be careful about use and mention. If, then, (13) is true, it must say the same thing as

(14) *aIa*.

But (14) is surely a tautology, and so (13) must surely be a tautology as well.’

(Barcan Marcus, ‘Modalities and Intensional Languages’, 308).

Barcan Marcus also made explicit that her system could be given a model-theoretic interpretation with domains co-extensive with the actual world, and as a result ‘there are no specifically intensional objects’ (Barcan Marcus, ‘Modalities and Intensional Languages’, 320-21). By Quine’s lights, Barcan’s system performed much better on meta-ontological questions than Kripke’s interpretation was to do. She hesitated to admit that the Barcan Formula should be read with objectual quantifiers, and she did not accept an interpretation according to which ‘ \square ’ and ‘ \diamond ’ are to be read as universal and existential quantifiers over possible worlds respectively. But, she explained, even an interpretation of her system according to which the quantifiers are objectual, with a constant domain semantics, implied

that modal logics can be interpreted without positing mere *possibilia*. In such a case, the Barcan formula is read, ‘if possibly something is ϕ , then there (actually) is something which is possibly ϕ ’. In other words, the purpose of her modal logic, thus interpreted, is to speak counterfactually about actual individuals. It systematizes the kind of discourse empiricists engage in when they get an individual in their sight, label it with a tag, and ask modal questions about how it might have behaved differently.¹³

Allegations that the interpretation of the Barcan Formula is perplexing are common in the modality literature. But Kripke-style interpretations, according to which ‘ \Box ’ and ‘ \Diamond ’ range over nodes in a model representing possible worlds, and ‘ \forall ’ and ‘ \exists ’ range locally over domains of discourse representing the contents of each world, carry their own perplexities in their wake. Quine maintained that all quantification must be translatable into standard first-order quantification in order to be able to settle questions of ontology and identity. As Kripke-style interpretations lack a single unified domain of quantification, preferring instead to have multiple domains, modelling worlds, existing side by side, they leave open questions whether an object in a given world can be taken to be identical to an object in another world. In order to identify two posits, it would need to be shown that they have the same criterion of identity. But Kripke’s ‘Semantical Considerations’ provided no criteria of identity for objects in different possible worlds, thus creating a problem David Lewis referred to as ‘the difficulties about inter-world identity’ (D. Lewis, Letter to Quine). Nor did Kripke solve the problem in *Naming and Necessity*. There he objected to the view ‘that it is the properties used to identify the object which ... must be used to identify it in all possible worlds’ that identities across possible worlds are not ‘found out’ but merely ‘stipulated’ (*Naming and Necessity*, 49). By Quine’s lights, this answer was insufficient, since Kripke effectively declined to provide criteria of inter-world identity.¹⁴ By contrast, both David Lewis and Ruth Barcan Marcus provided satisfactory answers to the problem of inter-world identity. Objects in two different Lewis-worlds are simply spatiotemporal objects, whose criteria of identity imply that they are distinct if they have distinct spatiotemporal locations. Lewis’s concrete possible worlds never overlap, so objects in distinct Lewis-worlds are never identical (Janssen-Lauret, ‘The Quinean Roots’, 259). Barcan Marcus’ system, with constant domains and the Barcan Formula, also clearly settles the question of inter-world identity: all domains of discourse are co-extensive.

¹³Later Barcan Marcus viewed modal discourse about *actualia* as counterfactual descriptions of actual objects, and modal discourse purporting to be about mere *possibilia* as consisting in false Russellian descriptions (Barcan Marcus, ‘Quine’s Animadversions’), so her view remained ontologically parsimonious.

¹⁴An anonymous referee suggests that Kripke might answer the question by taking the union of all of his domains of discourse. But this answer does not lay to rest the difficulties of inter-world identity. If we do not know whether a and b , the members of two different sets representing possible worlds, are identical or distinct, we do not come to know whether they are identical by taking the union of the two sets. It is the case that a and b are either identical or distinct, and so it is the case that there are either two things, a and b , in the union set or just one thing. But we cannot come to know which one of these alternatives is the case unless we first determine whether a and b are identical, and that is precisely the problem of inter-world identity. Criteria of inter-world identity must be given before it can be determined what is in the union set of the domains of all possible worlds (provided that this collection is not too large to be set-sized).

6. THE QUINE-BARCAN MARCUS DEBATE ON QUANTIFIED MODAL LOGIC PHASE II: ESSENTIALISM ARGUMENT

A second phase of Quine's campaign against modal logic, which to an extent overlapped with the first, turns on the argument that the interpretation of modal logic requires an assumption of essentialism, and that essentialism is incomprehensible. Quine's initial versions of this argument clung on to his old presupposition that modality was explicable in terms of analyticity. According to his first presentation of what came to be called the 'mathematical cyclist argument', statements of the form ' x is essentially F ' have an underlying form which attributes analyticity to sentences ascribing F -ness to x . Even if 'rationality is involved in the meaning of the word "man" while two-leggedness is not; but two-leggedness may at the same time be viewed as involved in the meaning of "biped" while rationality is not, it makes no sense to say of the actual individual, who is at once a man and a biped, that his rationality is essential' (Quine, 'Two Dogmas', 22). A later version of this argument is called the 'mathematical cyclist' argument. The *Word and Object* version no longer assumes that necessity must be defined away in terms of analyticity.

In the 1951 version of the argument, Quine assumes that modally-inclined logicians think that it is necessary that all humans are rational, because the meaning of the word 'human' includes the word 'rational', being something like 'rational animal'. He assumes that they also think (wrongly, because there are disability-adapted bicycles) that it is necessary that all cyclists are bipedal, because the meaning of the word 'cyclist' includes the word 'bipedal', being something like 'bipedal person who propels a bicycle using a pedal for each foot'. Now, Quine contends, take a particular person who is both human and a cyclist: it is simply a mistake to ask whether the meaning of this person includes 'rational' or 'bipedal'. That is a use-mention confusion; she is a person, not an expression, and does not have a meaning, or include a word.

In the *Word and Object* version of the argument, Quine began by stating that in Barcan's modal logic, 'we must somehow distinguish between necessary and contingent ways of uniquely specifying one and the same object' (*Word and Object*, 198). He then argued that it is senseless to distinguish between necessary and contingent ways of specifying an object, attributing to the modal logician claims of the form 'mathematicians are necessarily rational but not necessarily bipedal' and 'cyclists are necessarily bipedal but not necessarily rational' and concluding, 'Talking referentially of the object with no special bias toward a background grouping of mathematicians as against cyclists ... there is no semblance of sense in rating some of his attributes as necessary and others as contingent' (Quine, *Word and Object*, 199–200).

Barcan Marcus swiftly proved that Quine's argument was fallacious and her quantified modal logic did not entail essentialism of the type Quine disapproved of. First, she explained, her modal logic indeed contained some predicates of which we can prove that they are necessarily satisfied by anything. So, in a very weak sense, quantified modal logic is essentialist: there are some attributes which objects cannot fail to have. Or, as Barcan might have preferred to say: there are some predicates which objects cannot fail to satisfy. But these predicates, in Barcan's quantified modal logic, were just the logical predicates,

such as ' $\phi x \vee \sim \phi x$ ' and ' xIx ' (Barcan Marcus, 'Modalities and Intensional Languages', 318). A few years later, she added to this list of predicates that, for an object named ' a ', ' xIa ' and ' aIx ' also hold of a necessarily. So there is at least one way of necessarily, rather than contingently specifying an individual object: calling it by its directly referential name, ' a ', or specifying it as the object identical to a . But, Barcan Marcus argued, logical attributes and identity attributes, although they do hold of objects necessarily, do not have any of the baffling features which Quine claims necessary attributes have. Similarly, specifying an object as 'the object identical to a ' is to give a non-contingent specification of it, but is in no way senseless (Barcan Marcus, 'Essentialism in Modal Logic', 94).

As for Quine's mathematical cyclist argument, Barcan Marcus proved that the 1951 and 1960 versions appear to trade on a fallacy. Let us take as an example the mathematical runner—a more plausibly bipedal category than cyclists, because human runners need two legs to run on (whether flesh and blood or prosthetic)—and focus on one specific mathematical runner, Claire. Quine's argument relies on an inference from 'necessarily, if someone is a runner then she is bipedal' and 'Claire is a runner' to 'Claire is necessarily bipedal'. That inference is invalid in Barcan's system, as well as in the now more standard quantified S5. It is invalid independently of the question of paraphrasing modality away in virtue of analyticity. Barcan's formal argument translates into ordinary language roughly as follows. 'Runners are necessarily bipedal' has the form ' $\Box(Rx \supset Bx)$ ', that is, 'Necessarily, if x is R then x is B '. 'Claire is a runner' has the form ' Rc '. Barcan Marcus explained that from those modal premises, we may draw several conclusions, including ' $\Box(Rc \supset Bc)$ ' ('Necessarily, if Claire is a runner then she is bipedal'), ' $\sim \Diamond(Rc \cdot \sim Bc)$ ' ('It is not possible for Claire to be a runner but not bipedal') and ' Bc ' ('Claire is bipedal'), but not ' $\Box Bc$ ' ('Claire is necessarily bipedal'). Barcan Marcus put it as follows,

'it would appear that Professor Quine is assuming
(55) $(p \supset q) \supset (p \supset \Box q)$
is provable in QS4, but it is not' (Barcan Marcus, 'Modalities and Intensional Languages', 318).

Her point is that the assumption that 'necessarily, if x is F then x is G ' does not entail 'necessarily, x is G ' even if x is in fact F . Philosophically, we might explain as follows why this is so. 'Necessarily, if x is F then x is G ' asserts some necessary relation between (the extensions of) being F and being G —perhaps plausible for running and having two legs. But ' x is necessarily F ' is very different in logical form. It says of that individual that she could not exist, or not be the same individual, without being F . Claire could continue to exist or be herself without two legs. So, in modal logics, we cannot in general move from constructions which say, approximately, 'necessarily, if Claire is a runner then Claire has two legs to run on' to constructions which say, approximately, 'Claire could not be Claire without having two legs'.¹⁵ In short, according to Barcan

¹⁵An anonymous referee suggests that this implies that philosophers of disability and opponents of disablism should take an interest in modality. I think that the referee's suggestion is plausible; see, for example, Janssen-Lauret, 'Anti-Essentialism, Modal Relativity' §3 for a case against Kripkean origin essentialism which considers first-personal statements by speakers discussing their own genetic illnesses or disabilities.

Marcus, her modal logic does distinguish between ways of specifying an object which are necessary and contingent, and predicates which are satisfied necessarily and contingently, but these are very different from the way Quine assumes they are. It is a further option to admit essentialist assumptions such as 'Claire is necessarily rational', that is, to attribute a necessary property to an individual which goes beyond the logical properties and identity properties she described in 'Essentialism in Modal Logic'. But to do so, we must add something of the form ' $\Box Rc$ ' as a premise. We cannot derive it from generalizations about, for example, mathematicians and rationality, even if Claire is indeed a mathematician and the generalization itself is necessary.

7. THE QUINE-BARCAN MARCUS DEBATE ON QUANTIFIED MODAL LOGIC PHASE III: MODAL INCONSTANCY ARGUMENT

After Barcan Marcus' decisive refutation of the 1960 version, Quine softened his mathematical cyclist argument to the position I'll call 'modal inconstancy'. This term derives from David Lewis but the basic thought was Quine's: essential attributions are comprehensible, but only relative to a background of assumptions, or relative to a given enquiry. Quine gave up the bad argument that modal logic required describing someone as, for example, necessarily rational. He shifted his argument away from expressions of incomprehension and towards a more positive proposal: given a description of an object, it is possible to hold fixed some of the predicates comprising that description, as though they were immutable, and let others vary (Beebe and MacBride, 'De Re Modality', 224-7). For present purposes the predicates held fixed count as essential, others as accidental. For example, the question whether Hillary Clinton would have won the US election if she had been a cisgender man holds fixed her political and mental attributes and coherently construes her biological sex as accidental, mutable for the purposes of the present inquiry (Janssen-Lauret, 'Anti-Essentialism, Modal Inconstancy'). The question whether Hillary Clinton might have given birth to more than one child, by contrast, keeps her biological sex fixed, considering it essential for the present inquiry. As Quine put the point, 'the respectable vestige of essentialism [which] consists in picking out those minimum distinctive traits of a chemical, or a species, or whatever, that link it most directly to the central laws of the science' is 'of a piece rather with the chemical or biological theory itself' (Quine, 'Vagaries of Definition', 52).

A version of this view is defended by David Lewis under the guise of counterpart theory (D. Lewis, *Plurality of Worlds*, 248-263). Some twenty-first century anti-essentialists defend more ontologically parsimonious versions of modal inconstancy (Janssen-Lauret, 'Anti-Essentialism, Modal Relativity', MacBride and Janssen-Lauret, 'Meta-Ontology, Epistemology, and Essence'). Where Barcan Marcus showed that several of Quine's other arguments against modal logic were simply fallacious or based on misunderstandings, in the 1970s, with the arrival on the scene of modal inconstancy, they reached a satisfactory rapprochement. At this point, there was still some disagreement between them. Quine remained anti-essentialist, while Barcan Marcus had moved towards a moderate form of essentialism, arguing that, for example, 'Hesperus might not have been a planet of earth

but it is essentially a physical object' (Barcan Marcus, 'Dispensing with Possibilia', 44). Yet she never explicitly endorsed the essentiality of origins, calling material-origin essentialism and the proposal that a person's sex is an essential property 'debatable' (Barcan Marcus, 'Dispensing with Possibilia', 44).¹⁶ But Quine's modal inconstancy version of anti-essentialism, unlike the previous versions, was intelligible and non-fallacious. In another respect they had also reached a rapprochement: Quine remained opposed to positing mere *possibilia*, but so did Barcan Marcus (Barcan Marcus, 'Dispensing with Possibilia').

8. CONCLUSION

Ruth Barcan's contributions to logic and analytic philosophy—the first quantified modal logic, the first detailed defence of direct reference in the analytic tradition, and the first formal proof of the necessity of identity—ought to be recognized as truly innovative and the first of their kind. My ordinary-language gloss on her formal work connects the first quantified modal logic with Barcan Marcus' nominalist ontological-epistemological system, which gives pride of place to the direct reference of proper names, concrete objects which we encounter in our dealings with the world, and about which we can ask modal questions: how might they have been different? What might they have done differently? Quine, I argue, noticeably shifted his views over the years as a result of his debate with Barcan Marcus. Under her influence he moved away from the misguided arguments that modality was largely disguised analyticity or committed to an unpalatable essentialism, landing on an interesting and fruitful kind of anti-essentialism still viable today. Accordingly, Barcan Marcus must be awarded her proper place in the canon of analytic philosophy.

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¹⁶An anonymous referee suggests that Barcan Marcus' reluctance to take biological sex as essential may be connected to her feminist views. Barcan Marcus did profess feminist views in her 'Philosopher's Calling', 80-1, 86-8. For example, she pointed out the absurdity of the objection made to her appointment as chair of philosophy at the University of Illinois that there had never previously been a female chair at that university. Barcan Marcus did not, as far as I am aware, explicitly connect her feminism to her views on essentialism. It does seem plausible to speculate, as the referee does, that a feminist in the 1970s might find it easier to see how biological sex might not be an essential property. I conjecture, for example, that familiarity with the distinction between biological sex and gender as a social and psychological category, popularized by feminists in the 1960s and '70s, might make it easier for a philosopher to consider it possible that one and the same person in the social-psychological sense might have had a different biological sex.

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