Parametric and Non-parametric Mathematical Modelling Techniques: A Practical Approach of an Electrical Machine Identification

Técnicas de modelado matemático paramétrico y no paramétrico: un caso práctico de identificación de una máquina eléctrica

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ABSTRACT

Mathematical modeling is a key feature in analysis and control of dynamic systems. Furthermore, system identification's approach consists in mathematical expressions from experimental data taken from different processes. In this context, this work describes several modeling and identification techniques for determining the behavior of dynamic systems over time. This work emphasizes the main advantages and/or disadvantages of the different mathematical formulations of modeling and identification. This article presents a comprehensive review of the main modeling and identification techniques from a parametric and non-parametric perspective. Parametric and non-parametric models were formulated through their respective equations in order to apply them in a case of study. The experimental data is taken from an electrical machine, a DC motor from a didactic platform in which a set of known inputs are applied to measure the motor speed, then the output data is used as part of the modeling and identification process. The article concludes with the results provided by the comparison of modeling and identification techniques under study where simple solutions such as first order systems are required to model a linear dynamics DC motor over other complex mathematical formulations.

Keywords: Parametric model, Non-parametric model, Experimental data, Modelling, Identification.

RESUMEN

El modelado matemático es una característica muy importante en relación con el análisis y control de sistemas dinámicos. Además, la identificación del sistema es un enfoque para construir expresiones matemáticas a partir de datos experimentales tomados de procesos. En este contexto, este trabajo describe varias técnicas de modelado e identificación que son herramientas poderosas para determinar el comportamiento de los sistemas dinámicos en el tiempo. En Este trabajo se enfatiza las principales ventajas y/o desventajas que tienen las diferentes formulaciones matemáticas de modelación e identificación. También se presenta una revisión exhaustiva de las principales técnicas de modelado e identificación desde una perspectiva paramétrica y no paramétrica. Se formularon los modelos paramétricos y no paramétricos por medio de sus ecuaciones para aplicarlos en un caso de estudio. Los datos experimentales se toman de una máquina eléctrica, un motor de DC de una plataforma didáctica en la cual se aplican un conjunto de entradas conocidas para medir la velocidad del motor y utilizar estos datos como parte del proceso de modelación e identificación. El artículo concluye con las soluciones proporcionadas por la comparación de técnicas de modelación e identificación donde soluciones sencillas como los sistemas de primer orden son precisos para modelar un motor DC de dinámica lineal sobre otras formulaciones matemáticas más complejas.

Palabras clave: Modelo paramétrico, Modelo no paramétrico, Datos experimentales, Modelado, Identificación.

Introduction

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Physical systems describe the environment around us. These systems, regardless of their activity, are represented through a mathematical equation where its dynamics are described (Feldman et al, 2018). This means the current system output depends on a set of previous values that make the plant have a time-domain behavior (Vaidyanathan et al, 2016). The mathematical model is the start point for designing any closed-loop regulator, from key algorithms such as PID control (Proportional-Integral-Derivative) to MPC (Model-based Predictive Control). Other areas in engineering and sciences utilize mathematical models for

dynamic analysis through simulation, considering all the criteria to obtain results as much close as the process in site (Ullrich et al, 2017).

Mathematical equations in dynamic systems are classified in linear and non-linear expressions. In the case of linear schemes, they are desirable by most control system designers due to mathematical simplicity and their flexible implementation through hardware platforms with low-level programming. However, most of the industrial systems are non-linear and the need to be implemented in advanced hardware platforms with programming complexity. Other alternatives such as non-linear modeling or a linear approximation in a process operating point are used to reduce mathematical complexity (Oliveira et al, 2019).

For example, in (Gonzales et al, 2017) the plant variables are temperature and molar concentration. These variables take a considerable time to reach operating points, it means in a period of minutes or hours the set-point is accomplished. This feature of slow dynamics makes certain control system designers use equivalent modelling systems to generate a smooth law of control. Consequently, the use of a first-order response showed the plant performance in a stable operating point, but a deep analysis is not carried out due to the high non-linearities in the process. Other science fields such as, biological structures, use a system-order reduction technique in large-scale models like Snowden et al, 2018, which most of the processes are non-linear systems. In this work, the authors studied the biochemical reaction networks, which are high dimensional systems that are troublesome to study at the simulation level because of the high computational complexity generated in the mathematical expressions. Subsequently, every component is grouped to develop an entire simulation, which generates high-level computational effort so that results take a long time to be produced.

In electrical systems, (Schilders et al, 2018) obtained a simple and smooth model for the pack- aging structure around LED lamps. On the other hand, these lamps are efficient in energy consumption, but there is still a latent problem that refers to heat generation. At a constructive level, it is crucial to have a proper mathematical model of the lamp structure because these expressions generate constructive solutions to mitigate the effects of temperature for long-time work.

In electrical power systems are frequent the fast-dynamic responses that establish several non-linear expressions. Such is the case of (Gonzales et al, 2018), where the power generation of a solid-state transformer is carried out through an analysis of a black box system where a first-order curve is obtained after a step input at the operating point. According to the results, the power consumption regulator is designed in a predefined range of power generation. These criteria offer several advantages for designing a control system due to the mathematical model avoids non-linear phenomena present for the nature of the load.

Complex systems such as quadruple-tank process (Herrera et al, 2018), have several non-linear expressions where the plant is a multivariable array of four tanks and two valves. The control objective is to set a level value at the upper tanks. Apart from having non-linear equations, the process defines a high degree of interaction of its variables. Particularly, the fluid level of the tank number one depends on the control action of the valve number two and vice versa. This aspect requires a reduced-order model for a flexible simulation at operating points and regulation on the state variables is not limited by mathematical model complexity (Sato, 2017).

However, due to the diverse choices in modelling and identification techniques, only elemental models are adopted to represent physical systems. Thus, other valid options give several advantages for system simulation and control design but are set aside for key regulation alternatives. For example, models that consider standard responses, such as approximations to first or second-order systems, may or may not incorporate a delay are used extensively in processes control. Although this method is valid, some control designers do not consider other alternatives that provide more reliable results such as the least-squares algorithm and other similar options. Particularly, in cases where the acquisition of data is accomplished from a small number of samples, a trend curve of the process should be considered, which is performed by non-parametric models. Particularly, in cases where the acquisition of data is accomplished from a small number of samples, a trend curve of the process should be considered, which is performed by non-parametric models (Almeida et al, 2017).

In consequence, this paper reviews the different existing alternatives to develop modelling and identification techniques considering the most used parametric and non-parametric algorithms. The reader will be able to find different preferences to model and identify a plant. Also, this work applied a set of identification techniques through a training educational plant, an electric DC motor on a didactic platform, in which the acquisition of speed data is performed. The present work is shown as follows, in the next section a synthesis of the parametric and non-parametric models is shown, in the next section the mathematical description of the algorithms used in the work and their characteristics is reviewed and finally the respective conclusions of the present work are generated.

Introduction

A DC motor is an electromechanical machine that converts electrical energy y rotational mechanical energy. In Fig. 1, the scheme of a DC motor is presented, where there are two main components: the stator and the rotor. The stator receives the electrical energy, and the rotor performs the rotational movement in order to apply mechanical torque to a load (Gerling, 2016).
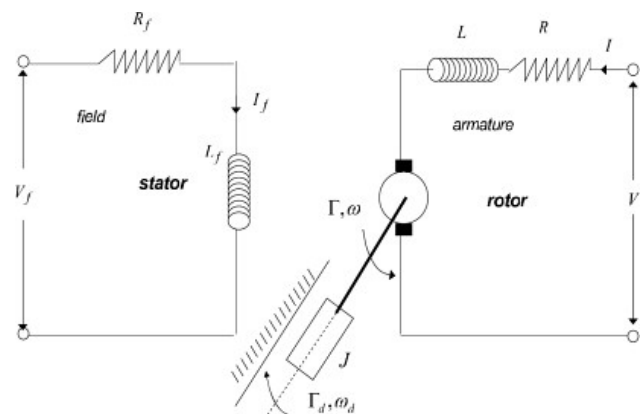

Figure 1. Electromechanical scheme of a DC motor.
Source: Rigatos, 2016

The following parameters define the mathematical model of a DC motor:

- V_f: Voltage of the field circuit.
- R_f: Resistance of the field circuit.

- Lf: Inductance of the field circuit.
- If: Current of the field circuit.
- V: Voltage of the armature circuit.
- R: Resistance of the armature circuit.
- L: Inductance of the armature circuit.
- I: Current of the armature circuit.
- Γ: Momentum of the shaft.
- ω: Rotational velocity of the shaft.
- J: Momentum of inertia of the shaft.
- Γd: Momentum on the mechanical load.
- ωd: Rotational velocity of the mechanical load.

The transfer function is obtained analyzing the motor controlled by only the armature voltage. The mathematical expression that relates the input (applied voltage to the armature) and output (position of the shaft) is displayed below:

$$G(s) = \frac{\Omega(s)}{V(s)} = \frac{k_T}{(Ls + R)(Js + B) + k_B k_T} \tag{1}$$

Where:

- kT: Mechanical constant.

- kB: Electrical constant.

The constants are related with the momentum and the back electromagnetic force through the following relations.

$$\Gamma(s) = k_T I(s) \tag{2}$$

$$E_B(s) = k_B \omega(s) \tag{3}$$

Finally, the angular position can be found depending on the angular velocity values due to the following expression:

$$\Omega(s) = \frac{1}{s} \omega(s) \tag{4}$$

Methods

Parametric methods

The following techniques describe the main-used algorithms for modeling and identification. The first-order response to a step function is a technique described by the exponential response in time that explains dynamic behavior. A step input signal is set in the system and state variables will begin to change until the process stabilizes in an operating value. A time delay factor is added in most processes due to slow-dynamics reaction.

In this method, the time constant τ is taken from the 63.2 % of the output variable at stable state.

The constant of the transfer function where the output and input are correlated is:

$$K = Y_{es}/U \tag{5}$$

Where Y_{es} is the estimated output and U the input of the system. Finally, the transfer function is represented below:

$$G(s) = \frac{Ke^{-ts}}{1 + \mathcal{T}s} \tag{6}$$

Real systems generally contain a dead time zone t denoted by e^{-ts}.

The second-order response is like first-order approximation with dead time, but the transfer function considers two poles at denominator. The main advantage in this technique is the analysis of damped systems based its damping factor. The transfer function is presented as follows:

$$G(s) = \frac{Ke^{-st}}{(1 + \mathcal{T}_1 s)(1 + \mathcal{T}_2 s)} \tag{7}$$

The convolution method (You et al, 2018) uses a mathematical operator to generate a third function from the superposition of two known functions. The convolution equation for a Linear Time Invariant (LTI) discrete system is:

$$h[k] - u[k] = \sum_{i=0}^{n} h[k] * u[i - k] \tag{8}$$

This produces the following expression in discrete time:

$$Y(z) = H(z)U(z) \tag{9}$$

For the present case, the transfer discrete function H(z) is unknown. A de- convolution method is applied in order to obtain the data from the transfer function.

The impulsive response (Ke et al, 2017) method models a system from the output behavior of the process when an impulse input is applied. The impulse response from a signal is found from a step function:

$$ge(k) = \frac{y(k) - y(k - 1)}{\alpha} \tag{10}$$

Using the step data, it is possible to obtain de impulse discrete response. The sinusoidal response data (Faifer et al, 2018) for the system is achieved with a frequency operating point designed from a dynamics analysis. The mathematical relation used in this case is:

$$G(e^{j\omega}) = \sum_{t=-\infty}^{\infty} g(t)e^{-j\omega} \tag{11}$$

The spectrum graphic shows that the fundamental frequency value is located around zero. This is close to the value of frequency used in the tests applied to the plant.

The Fourier transform (Devadasu et al, 2016) is used for analyzing systems in the frequency domain. The transfer function in this kind of systems is:

$$Y(j\omega) = G(j\omega)U(j\omega) \tag{12}$$

Where, the estimated function is represented as follows:

$$G_{est}(e^{j\omega}) = \frac{Y_N(j\omega)}{U_N(j\omega)} \tag{13}$$

The Wiener-Hopf equation (Slavakis et al, 2011) is a representation of an autocorrelated value and its respective transfer function. The mathematical expression in this technique is:

$$g = Ruu^{-1}ruy \tag{14}$$

Where, Ruu is a matrix that contains the autocorrelated values from the input signal, and ruy is the vector with the cross-correlation values from the input and output signal.

The power spectrum (Cho et al, 2016) from a signal is taken from the Wiener-Hopf relation through DFT of the autocorrelation and cross-correlation values.

$$\Phi uu = \sum_{t=-\infty}^{\infty} ruu(l)e^{-j\omega} \quad (15)$$

$$\Phi uu = \sum_{t=-\infty}^{\infty} ruy(l)e^{-j\omega} \quad (16)$$

Non-parametric methods

The main objective in this technique is to minimize a cost function that contains the error between the real data and the estimated data (Choudhary et al, 2016). The parameters estimated are found through the following expression:

$$\theta = (\phi'\phi)^{-1}\phi'y \quad (17)$$

Where, ϕ is a function called regressor and it contains the values of the time of the input/output parameters.

This technique is very similar to the last one. The only difference is a matrix term included in the parametric function that helps to minimize the error. This matrix is diagonal, and a higher value means a great effort in the parametric expression to diminish the error.

$$\theta = (\phi'W\phi)^{-1}\phi'Wy \quad (18)$$

The recursive WLSE is useful when there is data of a process and these values are obtained off-line. The parametric equation changes and other terms are incorporated in the least square minimization in order to add the new data.

$$\theta wrse(k+1) = \theta wrse(k) + K_W(k+1)(y(k+1) - \phi'(k+1)\theta wrse(k)) \quad (19)$$

In this context, the estimated data will add all values that come after the first(s) one(s). It is always necessary to break the inertial feature of the algorithm to avoid problems with convergence.

Lastly, an instrumental variable is a technique that helps to identify a system when the plant has values with cross-correlation between the output, noise and the regressor. In this approach, the parametric expression incorporates a variable x that is an instrumental value. This value helps the system to be not dependent of the cross-correlation between their variables and offer and estimated output using a process like LSE.

$$\theta uv(k) = (X'\Phi)^{-1}XY \quad (20)$$

Results

The plant is a training module from National Instruments that contains sensors like an LM35 and incremental encoder. The actuators are: DC motor, stepper motor and a halogen lamp (Fig. 2).
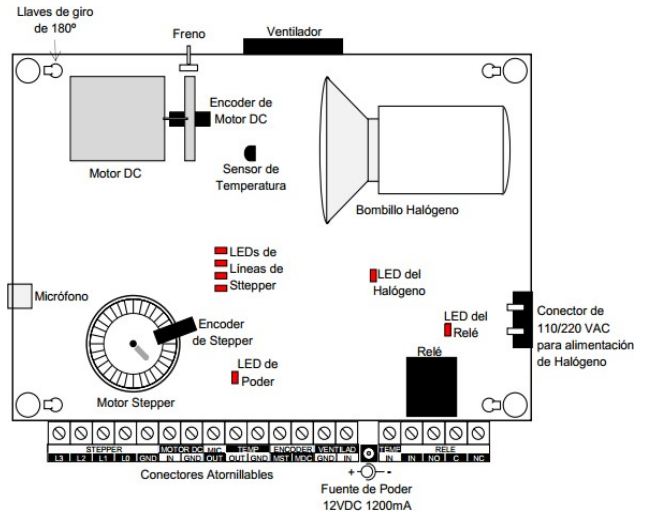

Figure 2. EPC plant from National Instruments.
Source: Chico, 2015

The training module is connected to the PC through a DAQ (data acquisition device) and the software interface is implemented in LABVIEWTM, where the data from the DC motor was acquired.

To analyze the differences from the identification models presented in this paper, the mathematical analysis is performed based on the data of the DC motor. The comparison between parametric and non-parametric models is shown In Fig. 3, where the methods considered are First-Order Response (FOR), Recursive Least Square Estimation (RLSE), Weighted Least Square Estimation (WLSE), Recursive Weighted Least Square Estimation (RWLSE) and Instrumental Variable (IV). The input signal was set at 3[V] to develop in the motor a speed of 556 [rad/s].
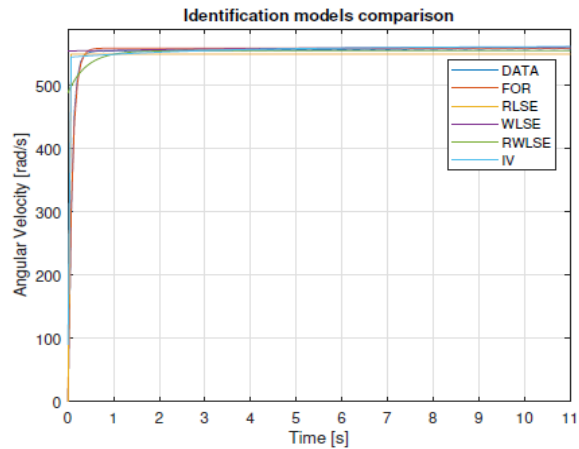

Figure 3. Process identification from different methods.
Source: Author

The data acquisition for angular velocity values was taken through an incremental encoder. The encoder pulses were transformed into velocity values for each one of the proposed methods.

The acquired data were compared with velocity measurements taken from the DAQ device. Fig. 3 shows a comparative visualization of the response of each identification method and thus be

able to evaluate the most reliable options for a dynamic analysis or controller design.

Differences in the rise time of each method are observed. Some identification methods are more accurate in dynamic time than others. Therefore, an error graph is made in Fig. 4.

To quantify the error values in each of the identification processes, the criteria of the Integral of the Absolute value of the Error (IAE) and the Integral of the Time weighted Absolute Error (ITAE) are used as evaluation principles of each of the proposed methods.
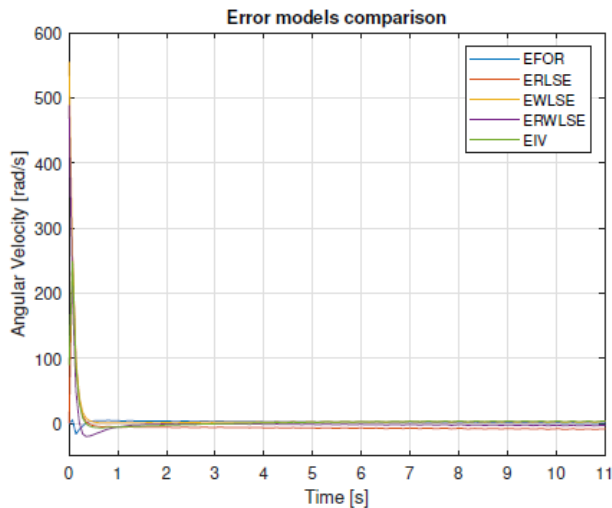

Figure 4. Error curves from identification.
Source: Author

$$IAE = \int_0^t |e(t)|\, dt \tag{21}$$

$$ITAE = \int_0^t t|e(t)|\, dt \tag{22}$$

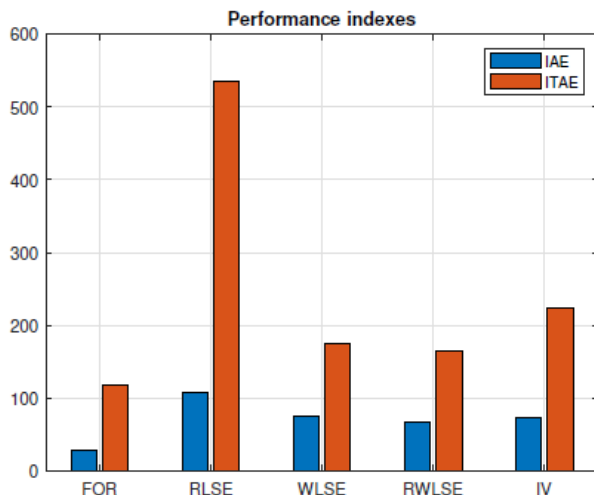The IAE and ITAE values are shown in Fig. 5.

Figure 5. Performance indexes for identification processes.
Source: Author

The data analysis describes the more preferred option, the FOR method. Performance values are: IAE = 28.39 and ITAE = 117.13. Despite its simplicity, this method represents an attractive option for structures with slow dynamics with a delay time. In other cases, approximations to this function can be performed to develop a linear controller with versatility compared to more alternatives. Other methods like the RWLSE (IAE = 66.03 and ITAE = 165.15) or IV (IAE = 56.5 and ITAE = 183.38) are useful for plants with fast or slow dynamics because of the versatility to choose the adequate mathematical expression to model any system with more precision over other methods such as RLSE (IAE = 107.93 and ITAE = 534.82). In this context, for a linear plant such as a DC motor, a simpler model can capture the dynamics and steady state features with high precision. This method is useful for designing complex control algorithms from simpler mathematical models.

Conclusions

Some methods, like classic identification models, are practical proceedings for a lot of cases in the identification world. Also, it is always desirable to achieve a more accurate model when a control algorithm is going to be implemented in any plant. The LSE technique offers more computational process to a controller. However, is more accurate due to the minimizing of the cost function.

Frequency analysis is not accurate when a Bode plot is displayed. Due to the noise and other perturbations, the frequency analysis can be confused presenting a magnitude and phase plots with poles that are not necessary presented in the system.

A priori, it is necessary to take as much data as the process can offer to the control engineer. Furthermore, in the DC motor case, it has a fast dynamic and for a time of more than 1seg achieves a stable state condition that offers redundant information unless a change in the set-point or an external perturbation were implemented in the plant.

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