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Determining the banking solvency risk in times of COVID-19 through Gram-Charlier expansions

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Abstract

This paper proposes risk measures for bank solvency by accurately measuring the solvency risk components. These measures consider the minimum regulatory solvency levels and banks' risk appetite level and risk profile. For this purpose, we used semi-nonparametric statistics to model stylized facts of the risk distribution, particularly the high-order moments of the *Solvency Decline Rate*, the *Tier Decline Rate*, and the *Portfolio Growth Rate* variables. Additionally, these risk measures can be used to measure the risk of regulatory intervention and to define policies that establish the minimum solvency levels required by banking regulators by estimating the *Quantile Risk Metrics*. As a case study, we collected data on the solvency indicators of the Colombian banking system, which adapts to the standards established by the Basel Committee. According to the results, the liquidity injection measures implemented in response to the needs generated by the COVID-19 pandemic led to an increase in the levels of the risk portfolio in the Colombian banking system, which exceeded the 99th percentile of the probability distribution of monthly portfolio value changes.

Keywords:

Solvency risk; Quantile Risk Metrics; Semi-nonparametric approach; Gram-Charlier expansions; COVID-19.

JEL Classification: C14, C22, C54, G21, G28

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1. Introduction

With more than 25 million confirmed cases and over 800,000 deaths around the world as of August 2020, the COVID-19 pandemic was not only a global tragedy, but also raised fears of an imminent recession and economic crisis (Nicola et al., 2020). As a response, most central banks injected liquidity into their economies to recover economic stability, ensuring an adequate supply of credit for households and businesses and preventing banks and taxpayers from being forced to draw on their reserves and savings (Balthazar, 2006). This intervention, however, implied higher financial and reputational risks for banks, as it increased (reduced) their level of leverage (solvency), and unsustainable debt amounts – International Monetary Fund, IMF (2020). In light of this situation, banking regulators in Europe had to relax the capital and liquidity requirements levels, as described by Agnès & Mauro (2020), and thus solvency positions in the insurance industry could quickly become complicated, as asserted by an article published by KPMG (2020). This scenario raises different uncertainties that should be faced in the near future, particularly for the banking industry: Will solvency ratios plunge to the point that some companies would require regulatory actions? What will be the impact of these decreases? How will the decline in the equity ratios affect rating agencies' opinions at the company and industry levels?

At the international level, the Basel committee has established a regulatory and supervisory framework for banking financial risks by publishing in 1988 the *Basel I* agreement in which the *Capital Adequacy* is established as the main element to cover the materialization of losses that destabilize a bank and the financial system –Banking Regulations & Supervisory Practices (Basel, 1988). The indicator used to measure capital adequacy is the *Solvency Ratio (SR)*, which is calculated by dividing *Tier Capital* by *Risk-Weighted Assets (RWAs)* and should not be below a certain value set by regulators.[4] The main objective of the Basel I agreement was to strengthen the stability of the international banking system while not creating competitive inequalities among international banks.

This objective leads to *SR* being calculated using standard methods that employ arbitrary models and parameters. Although the measurement of the bank's risk portfolio is imprecise, it is simple and is presumed to comply with not creating competitive inequalities (Banking Supervision, 1998). However, the implementation of this agreement revealed its weaknesses and the need to incorporate measurement models that are more sensitive to variations in the risks associated with banking assets. Therefore, in 2004, a new agreement, known as *Basel II*, was released. It was founded on three pillars (capital requirements, risk management and supervision, and market discipline) and sought to improve risk measurements. The first pillar entails *SR* to be measured in a standard manner so that different agents can be compared and aggregated. The second pillar seeks that banks develop more precise risk management techniques that consider the relationship of these risks with banks' risk profile and environment. This pillar, in turn, requires banks to measure their capital requirements using regulatory models and rigorous models to calculate *Economic Capital (EC)*.[5] The third pillar is associated with market discipline and complements the other two pillars by allowing market players to assess the bank's capital adequacy. Nonetheless, this agreement was insufficient to protect the banking system from the 2008 crisis, which revealed the need to re-evaluate policies, business models, and financial risk management systems (Borio, 2008). Authors such as Connolly (2009), Ng & Roychowdhury (2010), Rötheli (2010), Fahlenbrach et al. (2012), Huang et al. (2012) and, Macey (2017) have studied the causes of the 2008 crisis regarding deficiencies in bank risk management, notably failures in regulation, banks' bounded rationality, and risk misperception. Based on

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[4] This value is different for the *Tier 1* capital and the *total capital ratio*. According to the Basel I and Basel II agreements, the minimum value for the total capital ratio is 8%, i.e. $total\ capital\ ratio = \frac{Tier1+Tier2}{RWAs} \geq 8\%$

[5] *EC* and *RC* are terms used in the framework for bank capital regulation proposed by the Basel II Committee. Elizalde & Repullo (2004), Tiesset & Troussard (2005) define the *RC* as the minimum capital required by the regulator, and the *EC* is defined as the level of capital required to cover the bank's losses (due to risk materialization) with a certain probability (or confidence level) for a given horizon.

the lessons learned from this crisis, the Basel Committee implemented the *Basel III* agreement This agreement includes amendments mainly focused on ensuring financial intermediaries' capital and liquidity adequacy to absorb economic and financial shocks and avoid contagion effects in the real economy. The capital of financial institutions must be able to absorb the losses generated by the materialization of credit, market, liquidity, and operational risks.

Balthazar (2006) stresses the importance of pillar 2 in the evolution of the regulatory framework. It promotes *EC*, instead of *Regulatory Capital (RC)*, as the capital necessary to cover the losses of a risk portfolio. The reason for this is that *EC* is calculated using internal models, adapts to the risk profile of each bank, and considers their risk appetite, as it is based on *Quantile Risk Metrics (QRMs).* To calculate *QRMs* –see Section 3– excess skewness and kurtosis must be considered a characteristic, commonly described in the financial literature, of the banks' risk portfolio components so that risks are not underestimated. Furthermore, the financial mechanisms used to inject capital to cover the liquidity needs of the different economic agents, e.g. the way in which the banking system is used to provide liquidity through the granting of credit, is another source affecting banking stability, which must, therefore, be regulated. Both international financial regulations and the work carried out by researchers in the field of banking risks point to the need to define regulatory restrictions on the banking system in terms of *EC* using accurate risk measures that reflect the real vulnerability of banks. These restrictions should be established based on the probability distribution of variations in the bank's risk portfolio and the capital supporting it, since these parameters reflect its risk profile. In addition, they should consider stylized facts of its *probability density function (pdf)*, including asymmetry and excess kurtosis observed in the sample data, and its risk appetite by setting $1 - \alpha$ probabilistic confidence level that EC should cover. The purpose is to facilitate the adequate application of monetary interventions to prevent economic crises triggered by COVID-19. This, in turn, would prevent these interventions from having negative impacts on the stability of the financial system when used as a vehicle to implement intervention measures.

This study contributes to the existing literature by proposing a methodology to measure solvency risk and establish *EC* through policies that consider each bank's risk profile and appetite. As a starting point, the *pdf* of the variations in the *SR*, the risk portfolio, and the capital supporting such portfolio is calculated using *semi-nonparametric (SNP)* techniques based on *Gram-Charlier (GC)* expansions. The *SNP* distribution allows capturing stylized facts in the tails of the probability distribution, such as skewness, leptokurtosis, and others as the multimodality in extreme values of the distribution, which is not possible under the assumption of normality and other typical parametric specifications. The methodology proposed in this study can be applied at all levels of aggregation of the risk portfolio and the capital supporting it. It can also be used for more straightforward or more complex models. If the components of creditworthiness are disaggregated, multivariate approaches can provide important information regarding the dynamics of the correlations between these components, which can be used for the optimal allocation of resources to the bank's risk portfolio, considering regulatory constraints.

An application of this methodology is presented with data from the following variables: *Solvency Decline Ratio (SDR)*, which is calculated as the first logarithmic difference of *SR*; *Portfolio Growth Rate (PGR)*, which is calculated as the first logarithmic difference of the value of the risk portfolio; and *Tier Decline Rate (TDR)*, which is calculated as the first logarithmic difference of the value of the *Tier* capital supporting the risk portfolio. Additionally, these risk measures can be used to measure the risk of regulatory intervention and to define policies that establish the minimum solvency levels required by banking regulators by estimating the QRMs. As a case study, we collected data on the solvency indicators of the Colombian banking system, which adapts to the standards established by the Basel Committee. Regarding the findings, the frequency distributions of *SDR* and its components (*TDR* and *PGR*) were found to have time-varying patterns components in the mean and variance, which can be captured using ARMA and GARCH models, respectively. Concerning the higher moments of the probability distributions, we observed that the frequency distributions of the variables under analysis are leptokurtic. The *PGR* has a marked skewness to the right. After modeling the pdf of the *PGR*, the shock suffered by the increase in the risk portfolio in March 2020 was

found to exceed the 99th percentile under *GC* and normal distribution. In addition, this shock increased the probability of regulatory intervention for April 2020.

The rest of this paper is structured as follows. Section 2 analyzes the components of the *SR* to understand the behavior of the different sources affecting banks' risk portfolio and the capital allocated to cover their portfolio risk. Section 3 proposes risk measures on bank solvency and the models to estimate them. Section 4 describes the data set used as the case study; in addition, the parameters for this data set are estimated in this section. Section 5 presents the empirical results and Section 6 draws the conclusions and provides some practical recommendations.

2. Capital Adequacy Requirements (CARs)

CARs include both regulatory and economic capital. In particular, Basel II aims to establish more risk-sensitive minimum capital requirements so that regulatory capital is closer to a bank's economic capital (Caruana, 2005). According to the Basel Committee on Banking Supervision (1998), *Tier regulatory* is divided into two components: Tier 1, which refers to *core capital* that includes equity capital and disclosed reserves; and Tier 2, which refers to *supplementary capital* that includes revaluation reserves, general provisions, hybrid capital instruments, and subordinated debt. *RC* and *EC* requirements are determined as a function of the portfolio risk. As a matter of fact, accurately calculating portfolio risks is not easy from a modeling and computational point of view (Wason et al., 2004). One can measure the risk of each module that composes the risk portfolio and aggregate these modules at different levels by dividing each main risk module into risk submodules. A higher degree of unbundling indicates a more accurate but less simple measurement (Sandström, 2007). Assuming a multivariate normal distribution and a linear correlation between the risk modules for aggregation, the solvency capital requirement to cover the portfolio can be estimated as the α percentile. This method coincides, as described by Wason et al. (2004), with that used under the Solvency II guidelines of the *European Union law* where the solvency capital requirement must be sufficient to survive extreme losses over a one-year horizon with a minimum confidence level of 99.5%. The solvency capital requirement incorporates insurance, market, credit, operational, and counterparty risks and must be recalculated at least once a year. From the regulatory point of view of the Basel Framework, the *CAR* establishes the proportion of *RC* required to support a certain amount of *RWA,* which determines the value of the risk portfolio made of credit risk, market risk, and operational risks as expressed in Eq. (1)

$$RWA = \sum_{i=1}^{n} w_i * asset_i. \tag{1}$$

Under the standard method: each source of risk *(i)* is multiplied by a standardized factor (w_i), which is expected to be set conservatively in each jurisdiction. In this weighted aggregation of the risk portfolio, correlations between assets are not considered, and relative weights (w_i) are assigned as arbitrary constants. Basel's *Internal Models Approach* allows banks to develop their internal estimates of risk components to determine the capital requirement for that position, subject to certain minimum requirements and disclosure obligations. In some cases, banks will have to use a supervisory value instead of an internal estimate for one or more risk components (Banking Supervision, 2004). The *Internal Models Approach* assumes that the loss distributions are close to the normal distribution and consider correlations between assets. For certain groups of assets, these correlations are defined in a regulatory manner. The assumption of a normal distribution of the risk portfolio components in finance and insurance is an implausible condition due to the high frequency of outliers and asymmetry (Wason et al., 2004). Balthazar (2006), for instance, highlights the presence of heavy tails to the right of the loss distribution. The solvency risk measurement of a bank's portfolio depends on the risk measurement of each of the portfolio components, where determining the loss probability distribution is particularly important. In addition, it is common to find in the literature models that assume Gaussian distributions such as those proposed by Merton (1973; 1974); Vasicek (2002); Jiménez & Mencía (2009), Chava et al. (2011), Belkin et al. (1998), Frachot et al. (2001); and Shevchenko (2010), which, in most cases, underestimate the risks faced by not

taking into account the frequency of occurrence of extreme events that cause distortions in probability distribution tails. Different studies have demonstrated that the assumption of normality is significantly different from the distributions observed in the variables related to banks' financial risks.

Regarding deviations from normality, Sandström (2007) analyzes the skewness of the probability distributions of the different components of a bank's portfolio risk and the effect of not parameterizing it in some underlying distribution. This author proposes its parameterization using a *Cornish–Fisher* expansion and finds that if a normal multivariate risk distribution is assumed (without considering module skewness), the capital requirement can be well below the risk when skewness is omitted. Bølviken & Guillen (2017) argue that the accuracy of risk aggregation in solvency can be improved by recursively updating the skewness in the risk measurement of specific instruments. Le Maistre & Planchet (2013) show that the standard approach used in the Basel Framework to assess interest rate risk leads to a biased risk measurement. In the contexts of operational risk measurement Dutta & Perry (2006); De Fontnouvelle et al. (2003); Feria-Domínguez et al. (2015) reveal that the loss distributions due to the materialization of operational risks exhibit skewness and heavy-tailed distributions. Kretzschmar et al. (2010); Bateni et al. (2014); Madan (2009); and Lynn Wirch & Hardy (1999) have studied probability distributions in the estimation of both aggregated and disaggregated solvency risks and reported that skewness and kurtosis do not correspond to the parameters of a normal distribution. In order to correct the distortions between the loss frequency distributions of the components of a bank's risk portfolio and the normal distribution, recent studies have proposed using *Gram–Charlier (GC)* expansions. These expansions were introduced by Edgeworth (1896) and have been widely studied and applied to approximate the probability curves of random variables related to various scientific fields. Sargan (1975) first introduces this methodology in semi-nonparametric (SNP) econometrics to approximate the confidence intervals of *t ratios* and concludes that these intervals are more accurate than the usual asymptotic confidence intervals for large samples. After this, the use of *GC* expansions in econometrics has expanded to model random variables that present significant deviations from the normal distribution. Jarrow & Rudd (1982); Lee (1984); Corrado & Su (1996); Mauleon & Perote (2000); Jondeau & Rockinger (2001) and Del Brio & Perote (2012) are examples of authors that have used *GC* expansions in econometrics. From a multivariate approach Del Brio et al. (2009) demonstrate how Pearson's correlation coefficients are different when estimated under the assumption of normality and when estimated under SNP approaches.

In addition to the problems of asymmetry and excess kurtosis that arise when measuring financial portfolio risks, banking regulations can negatively impact solvency risk. Some studies have focused on evaluating *RC* as an indicator of a bank's capital adequacy to cover the losses caused by the materialization of financial risks and found weaknesses in the measurement. Among the deficiencies of *RC*, *regulatory arbitrage* stands out since it allows banks to take greater financial risks without increasing their capital levels, as pointed out by Jones (2000); Ward (2002); Houston et al. (2012); Karolyi & Taboada (2015); and Boyer & Kempf (2020). Kim & Santomero (1988) investigate the role of bank capital in risk control using a mean-variance model and report that one of the drivers for banks to select high-risk portfolios is the role of bank capital regulation. In addition, they find that the mere use of *RC* is not enough and effective in limiting the risk of bank failure. Another problem raised by (Drumond, 2009) is that regulatory capital requirements have accentuated procyclicity in solvency risk. Other studies into *CAR* focus on analyzing the behavior of this indicator. For instance, Abou-El-Sood (2016) examines the relationship between the *CAR* required by regulators and bank failures and whether this relationship depends on the proximity of the *CAR* to the minimum required regulatory levels.

3. Description of the risk measures and the estimation methodology

The methodology is based on the need to provide accurate probability measures for the loss distribution determination to measure solvency risk. This measurement can be made at different levels of aggregation of the solvency risk components. The highest aggregation level corresponds to the *SDR*, a variable that groups all the components of the risk portfolio and the capital it supports. The loss distribution is

established on the relative changes of *SR* per unit of time, which corresponds to *SDR*. At the first level of disaggregation, it is separated into two classes of components: The *Tier* capital components (corresponding to the numerator of *SR*) and the *RWA* risk portfolio components (corresponding to the denominator of *SR*). The loss distribution for each component is based on the relative changes of the capital and portfolio value over time corresponding to *TDR* and *PGR*, respectively. Higher levels of disaggregation on the risk portfolio allow the analysis of correlations between different risk modules and sub-modules, facilitating the rebalancing and optimization of the portfolio allocation, under capital constraints, given a minimum level of solvency. In this document only, an aggregated analysis of solvency risk is made on *SDR*, *TDR*, and *PGR*. For multivariate analysis of the risk portfolio, it is necessary to disaggregate the risk portfolio by sub-modules. These measures make it is possible to determine *regulatory intervention* probabilities, which are defined as the likelihood that a bank's solvency ratio is below a certain minimum solvency level (η) established by the banking regulator. In addition, they enable setting policies based on *Quantile Risk Measures (QRMs)*, which helps establish *EC* restrictions, considering the risk appetite and the stylized facts about the distribution tails such as asymmetry and excess kurtosis.

3.1. Regulatory intervention probability

Let η be a minimum bank solvency level at time t defined by a banking regulator to cover unexpected losses in the portfolio risks. A bank must maintain a SR_t level equal to or greater than η in order not to be intervened ($SR_t \geq \eta$).

Since SR_t is calculated by dividing $Tier_t$ by RWA_t, it can be expressed in $t + 1$ by means of Eq. (2).

$$SR_{t+1} = \frac{Tier_t * exp(-TDR_{t+1})}{RWA_t * exp(PGR_{t+1})} = \frac{Tier_t}{RWA_t * exp(PGR_{t+1} + TDR_{t+1})}, \tag{2}$$

where *TDR* is calculated as the first logarithmic difference of the value of the *Tier* capital supporting the risk portfolio.

Given that $SDR_{t+1} = PGR_{t+1} + TGR_{t+1}$ to avoid regulator's intervention, $SR_t * e^{-SDR_{t+1}} \geq \eta$ must be satisfied.

Therefore, the maximum value that SDR_{t+1} can take, given a regulatory solvency level of η, is given by Eq. (3).

$$SDR^{\eta}_{t+1} = ln\left[\frac{SR_t}{\eta}\right], \tag{3}$$

i.e., SDR^{η}_{t+1} depends on the distance between the logarithm of the observed solvency ratio level and the minimum regulatory level. Thus, the *regulatory intervention probability* can be expressed using Eq. (4).

$$p\left(SDR_{t+1} > SDR^{\eta}_{t+1}\right) = 1 - F_{SDR}\left(ln\left[\frac{SR_t}{\eta}\right]\right), \tag{4}$$

F_{SDR} being the cumulative probability function (*cdf*) of the *SDR*. Analogously, in the event that the decline in the solvency ratio is only due to an increase in the risk portfolio or Tier, the intervention probability will be $1 - F_{PGR}(ln[SR_t/\eta])$ and $1 - F_{TDR}(ln[SR_t/\eta])$, respectively. In these latter cases, F_{PGR}[6] and F_{TDR} are the cumulative probability functions of the *PGR* and the *TDR*, respectively, and *regulatory intervention probability* depends on the distance between the logarithm of the observed solvency ratio level and the minimum regulatory level.

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[6] F_{PGR} can be determined as the joint distribution of different modules and submodules of the risk portfolio.

3.2 Policies based on Quantile Risk Measures (QRMs)

The second pillar established by the Basel Committee requires the development of risk management policies through measures that reflect banks' risk profile and appetite. In this framework, the EC should be estimated as a quantile of the *pdf* of losses of the banks' portfolio risk.

The estimation of such a quantile considers the bank's risk profile, which is reflected in the different parameters of the *pdf* (e.g., variance, skewness, and kurtosis), as well as its risk appetite, by determining the probability α associated to the quantile, which depends on the risk appetite in the decision-making process.

Alexander (2008) defines *Quantile Risk Metrics (QRMs),* for any α between 0 and 1, as the x_α quantile of the distribution of a continuous random variable X such that $P(X < x_\alpha) = \alpha$. *QRMs* can be calculated using the quantile function F_X^{-1}, associated to a *cdf* F_X, as defined in Eq. (5).

$$QRM_\alpha = F_X^{-1}(\alpha) = inf\{x \in R: \alpha \leq F_X(x)\}. \tag{5}$$

These QRM_α allow the incorporation of the risk appetite by establishing the probability of obtaining positive or negative variations of X, that are greater than the expected maximum value. Policies based on *QRMs* make it possible to set the minimum solvency level that a bank should have in order to withstand the maximum expected shock, $F_X^{-1}(\alpha)$, that would deteriorate solvency in case of falling below a minimum regulatory level (η). In general terms, random variable X is any source of risk on which bank solvency depends such as PGR, TDR or SDR. Let $SR_t^{\eta,\alpha}$ be the solvency ratio level required to withstand the maximum expected shock, $F_X^{-1}(\alpha)$, given a confidence level of $1 - \alpha$, it can be expressed by means of Eq. (6).

$$SR_t^{\eta,\alpha} = \eta * e^{F_X^{-1}(\alpha)}. \tag{6}$$

In the event that the portfolio risk cannot be rebalanced, the adjustment of the solvency ratio will depend on an economic capital readjustment. Therefore. The economic capital that must be held at the beginning of the period $t + 1$ to support a $F_X^{-1}(\alpha)$ shock must meet the following condition:$Tier \geq \eta * RWA_t * e^{F_X^{-1}(\alpha)}$. Thus, economic capital does not only depend just on the value of the portfolio risk and the minimum regulatory level (η), but also depends on the risk appetite (α) and the risk profile which is reflected in $F_X^{-1}(\alpha)$.

3.3. Determination the probability density functions of the SDR , the TDR, and the PGR

To measure the regulatory intervention probability and establish policies based on QRM_α, the *pdf* of the sources of solvency risk must be determined. In this paper, we propose the modeling of such *pdf* using *GC expansions*. These expansions allow to model the heavy tails observed in the sources of bank solvency risk. Given the conditional dynamics of the *mean* and *variance* of the sources of solvency risk, we propose the use of Autoregressive Moving Average (ARMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, respectively, to estimate these parameters. However, for the sake of notational simplicity, random variable X representing any source of solvency risk denotes a standard variable with location and scale parameter 0 and 1 respectively.

3.3.1. Mean-variance model

To calculate the mean of the random variable ω, we propose the use of *ARMA* because they are sensitive to short-term variations and capture the time-varying dependence patterns observed in the series under analysis. Random variable ω can be any source of solvency risk, such as *PGR*, *TDR* or *SDR* with no standardization. It is expressed in t as follows:

$$\omega_t = \phi_0 + \sum_{i=1}^{p} \phi_i \, \omega_{t-i} + \sum_{j=1}^{q} \theta_j \, a_{t-q} + a_t, \tag{7}$$

where ϕ denote the Autoregressive (AR) parameters; θ, the Moving Average (MA) parameters; and a, the model errors. a can be expressed in t as

$$a_t = \sigma_t \xi_t, \tag{8}$$

where ξ_t is a random white noise variable; and σ_t is the conditional variance of ω, which follows a GARH

Bollerslev ([1986](#)) process as illustrated in Eq. (9).

$$\sigma_t = \sqrt{\alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_i \sigma_{t-j}^2}. \tag{9}$$

3.3.2. Gram-Charlier expansion

To estimate the density function of the random variable X, reference is made to Davis ([1976](#)) and Kolassa ([2006](#)), who propose a model that considers higher order approximations of the density f_X of random variable X, from a reference density f_Y, which corresponds to a random variable Y. We define $X = Z + Y$, where Z is a variable with mean and zero variance and with the same higher order cumulants ($k_3, k_4, ...$) as X, which contains the corresponding information of the distortions of f_X with respect to the normal distribution. Z and Y are orthogonal, which implies that they are linearly independent.

Let $Z = \frac{\sum_{j=1}^{n} Y_j}{\sqrt{n}}$ be standardized sum of n independent and identically distributed variables $(Y_1, Y_2, ... Y_n)$. Then, its characteristic function $\zeta(u)$ defined as Fourier's inverse transform of a μ probability measure into R^n can be written as

$$\zeta(u) = \int e^{i(u,\vartheta)} \mu(d\vartheta), \tag{10}$$

where i is the imaginary unit. Hence, the characteristic function of Z is $\varphi_Z(u) = E[e^{i(u,Z)}] = \int e^{i(u,z)} f_Z(dz)$, where f_Z is the *pdf* of Z. Characteristic functions always exist because they are equal to the Fourier transforms of the probability measures that always exist (Jacod & Protter, [2012](#)). By conditioning on $Z = z$, X has a *pdf* and by expanding f_Y as a Taylor series, $f_Y(x - z) = \sum_{j=0}^{\infty} f_Y^{(j)}(x)(-z)^j/j!$. Thus, according to Kolassa ([2006](#)) the unconditional density of X is given by

$$f_X(x) = \sum_{j=0}^{\infty} f_Y^{(j)}(x) \frac{(-1)^j \mu_j^*}{j!}, \tag{11}$$

where μ_j^* denote the moments of Z that need to be added to Y to get X. The order cumulant j, k_j^*, associated with Z, is the cumulant j of X minus the corresponding cumulant of Y. Multiplying $f_X(x)$ by $f_Y(x)$ in the numerator and the denominator we get the following expression:

$$f_X(x) = \frac{f_Y(x) \sum_{j=0}^{\infty} f_Y^{(j)}(x)(-1)^j \mu_j^*}{j! f_Y(x)}.$$

By defining $h_j = \frac{(-1)^j f_Y^{(j)}(x)}{f_Y}$, it can be expressed as

$$f_X(x) = f_Y(x) \sum_{j=0}^{\infty} h_j(x) \mu_j^*/j!. \tag{12}$$

Function h_j are the ratios between $f_Y^{(j)}$, which is the order derivative j of weight function f_Y. If weight function f_Y is the normal density, $\phi(x)$, then h_j corresponds to the polynomial functions known as *Hermite Polynomials (HPs)*, which are orthogonal to $\phi(x)$. The infinite series in terms of HPs express a function $\theta(x)$, such that $\theta(x) = \sum_{j=1}^{\infty} \delta_j h_j$, where according to Eq. (13),

$$\delta_j = \frac{1}{j!} \int_{-\infty}^{\infty} h_j(x) \phi(x) dx. \tag{13}$$

In addition, f_X and $\phi(x)$ will have the same mean and variance and function h_j is given by

$$h_j = \frac{(-1)^j \left[\frac{d^j}{dx^j} e^{\frac{-x^2}{2}} \right]}{e^{\frac{-x^2}{2}}}.$$
(14)

The orthogonality condition is satisfied in such a way that

$$\int_{-\infty}^{\infty} h_j(z) h_i(z) \phi(x) = 0, \quad \forall j \neq i.$$
(15)

Eq. (15) indicates that the *HPs* represent an orthogonal base with respect to the weight function $\phi(x)$. For the empirical application of the model, this property of orthogonality with respect to the weight function makes it possible to truncate the HP series to an order n, thus defining the family of functions in Eq. (16). This family integrates one in virtue of the orthogonality property and thus define the Gram-Charlier *pdf*s in the regions recently described by Lin & Zhang (2020).

$$f_{X,n}(x) = \phi(x) \sum_{j=0}^{n} h_j(x) \mu_{j,n}^* / j!.$$
(16)

On the other hand, this expansion density may be also characterized in terms of *cdf*s. In particular, F_Y and F_X are the *cdf*s associated with f_Y, and f_X, respectively, then F_X can be approximated as follows:

$$F_X = F_Y(x) - f_Y(x) \sum_{j=1}^{\infty} h_{j-1}(x) \mu_j^* / j!.$$
(17)

If the weight function is the normal *pdf* $\phi(x)$ with *cdf* denoted by $\Phi(x)$, the *cdf* is given by:

$$F_{X,n} = \Phi(x) - \phi(x) \sum_{j=1}^{n} h_{j-1}(x) \mu_{j,n}^* / j!.$$
(18)

For convenience, the moments μ_j^* are replaced with cumulants k_j^* and usually it is assumed $\mu_{0,n}^* = 1, \mu_{1,n}^* = \mu_{2,n}^* = 0$, thus expressing $f_{X,n}(x) = g(x; d)$ as in Eq. (19), as stated by Cortés et al. (2016), among others,

$$g(x; d) = \left[1 + \sum_{j=3}^{n} d_j h_j(x) \right] \phi(x).$$
(19)

Here, d is a vector of parameters $(d_1, d_2, \ldots, d_n,)$ that contains the corresponding information of the distortions of f_X with respect to the normal distribution $\phi(x)$ and that guarantees that $g(x; d) \geq 0, \forall x \in \mathbb{R}$. The *Gram-Charlier* series can accurately approximate the sample distribution to f_X, because $\lim_{n} g(x; d) = f_X$. In practice, most applications of this distribution only include third- and fourth-order *HPs* related to the skew and excess kurtosis (Del Brio & Perote, 2012) so that

$$g(x; d_3, d_4) = [1 + d_3(x^3 - 3x) + d_4(x^4 - 6x^2 + 3)] \phi(x).$$
(20)

3.3.3. Estimating Gram-Charlier parameters

In most applications of *Gram-Charlier* expansions parameters are estimated using the *Maximum Likelihood (ML)* method which, assuming that the first two moments of the distribution are well specified, the global optima guarantee that $g(x; d)$ is positive. Del Brio & Perote (2012) compare estimation via the *ML* method with that of the *Method of Moments (MM)* and conclude that both provide similar results. However, the *MM* estimation can only guarantee positive values for $g(x; d)$ in the asymptotic expansion and does not ensure positivity when the series is truncated with few terms. Therefore, we implement *ML* estimation and expand the series until the fourth moment, as is usually done to capture skewness and kurtosis. Thus, for a sample size T the log-likelihood function $\log(L)$ is given by

$$\log(L) = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \log(x_t^2) + \sum_{t=1}^{T} \log\left(\left[1 + \sum_{j=3}^{n} d_j h_j(x_t) \right] \right).$$
(21)

4. Sample description

As a case study, we collected data on the solvency of banks in Colombia by month from January 2002 to May 2020.[7] In the sample, 60% of the institutions correspond to international banks that concentrate more than 80% of the banking assets. Colombian banking regulation is currently transitioning from Basel II to Basel III. According to the calculations by the International Monetary Fund (IMF) for 2011, the solvency of the financial system in countries such as Colombia, Chile, Brazil, Mexico, and Peru partially comply with the requirements set by Basel III, considering that, in general terms, the quality of capital was good and the average regulatory margin regarding SR was above 10% (the minimum regulatory is $SR \geq 9\%$). According to Clavijo et al. (2012) Colombia must adjust its capital to comply with the Basel III requirements. These adjustments (or capital cleaning) could imply decreases in the solvency ratio in the range of 2.2% to 2.5%. For the solvency analysis in Colombia, the values of $Tier$ and RWA are calculated by adding the value of the capital and portfolio of all banks, respectively. $Tier = \sum_{j=1}^{K} Tier_j$ and $RWA = \sum_{j=1}^{K} RWA_j$, where $Tier_j$ and RWA_j are the value of the capital and portfolio risk of bank j and K is the number of banks being aggregated.
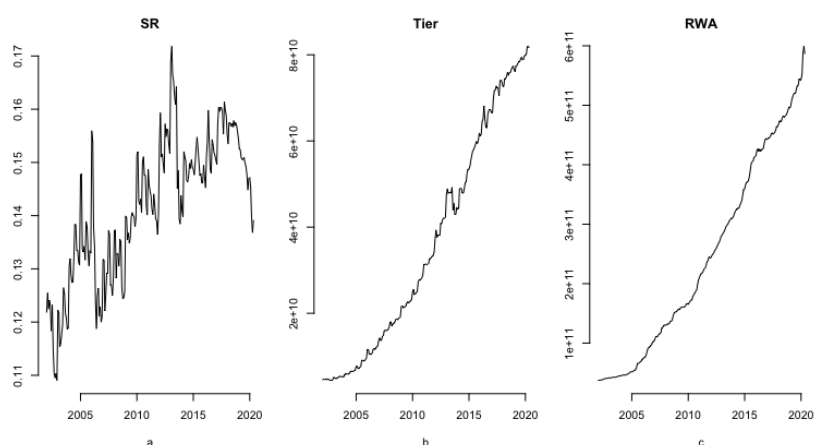

Fig. 1. Monthly time series of Solvency Ratio (a), Tier (b), and Risk-Weighted Assets (RWA) (c) of the Colombian banking system.

According to Fig. 1a, SR shows an upward trend and reaches its highest value (17%) in February 2013. In 2017, SR reaches values close to 16% and then begins to decrease until reaching 14.5% in February 2020. In March 2020, the central bank of Colombia (*Banco de la República de Colombia*) is required to inject permanent liquidity into the economy to facilitate the proper functioning of the financial markets, which causes the solvency of the banking system to fall to 13.6%. These levels had not been observed since the end of 2013. In total, from January to April 2020, the solvency ratio presented a higher decrease than 7%. Tier capital (Fig. 1b) shows no significant changes at the beginning of the COVID-19 pandemic, while the RWA value exhibits an increase of nearly 6% in March 2020, caused by the liquidity injection measures implemented to withstand the effects of COVID-19 (Fig. 1c). The increased RWA value without an increase in the Tier causes leads to marked decline in the solvency levels.

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[7] The database is published on the website of the *Superintendencia Financiera de Colombia* (Financial Superintendence of Colombia), which is the entity in charge of regulating the Colombian banking system.
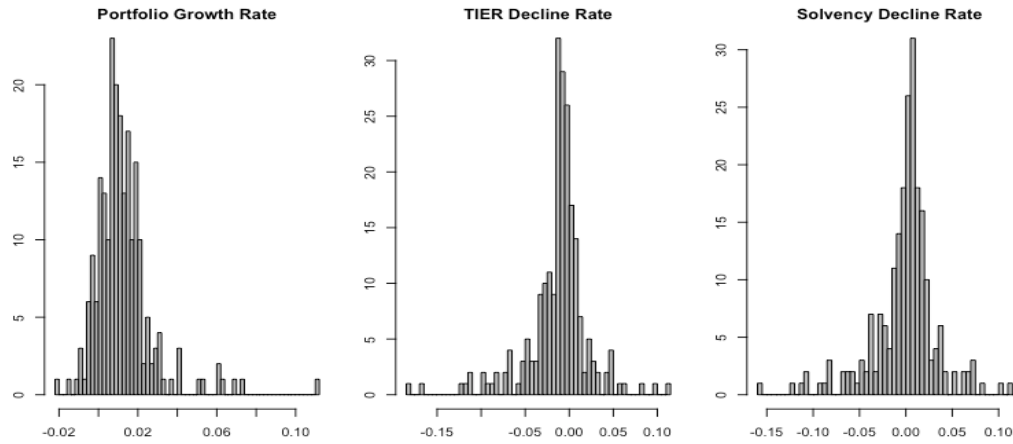
Fig. 2. Frequency distributions of the monthly Portfolio Growth Rate, Tier Decline Rate, and Solvency Decline Rate.

As depicted in Fig. 2, the *PGR* is skewed to the right, which means that its observed increases are more extreme than its decreases. The distribution of the *TDR* is more symmetric. The observed distribution of the *SDR* corresponds to the sum of the *PGR* and the *TDR* and presents asymmetry in the right tail.

Table 1. Correlation matrix among variations in the components of solvency.

	PGR	TDR	SDR
PGR	1.000	-0.150	0.267
TDR	-0.150	1.000	0.913
SDR	0.267	0.913	1.000

Table 1 shows the correlations between *PGR*, *TDR*, and *SDR*. As observed, the correlation between *SDR* and *TDR* is much greater than that between *SDR* and *PGR*, which implies that solvency is more sensitive to variations in the Tier than to variations in the risk portfolio. The low correlation between *PGR* and *TDR* (negative sign) suggests that when the value of the portfolio risk increases, so does the Tier capital (which corresponds to countercyclicality). However, this low correlation indicates that this adjustment is not made every time in the same period.[8]

Table 2. Descriptive statistics of the PGR, TDR and SDR.

	Min	Max	Mean	Std. Deviation	Skewness	Excess Kurtosis	q5	q10	q90	q95
PGR	-0.022	0.112	0.013	0.015	2.377	10.356	-0.004	-0.002	0.026	0.036
TDR	-0.180	0.113	-0.013	0.036	-1.018	4.851	-0.081	-0.049	0.018	0.031
SDR	-0.159	0.124	-0.001	0.036	-0.650	3.309	-0.066	-0.039	0.032	0.055

Table 2 presents basic descriptive statistics of the *PGR*, *TDR* and *SDR*. It is noteworthy that *PGR* and *TDR* present an average growth rate of 1.3% per month, which indicates that the Tier has grown in proportion to the risks. Nevertheless, the standard deviation of *TDR* is greater than that of *PGR*. The average *SDR* is close to zero, which means that, for the period under analysis, solvency has remained at around an average level (close to 14%). The positive excess kurtosis in all the series suggests the presence of heavy tails in all components and their aggregation. The *PGR* shows a marked asymmetry towards the right tail.

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[8] The cross-correlation diagram presented in Annex 1 shows that the highest correlation between *PGR* and capital *CDR* occurs at lags 5 and 10, which means that TIER adjustments are five months ahead of the variations of this component. However, a high correlation is also observed in period -2, which means that certain increases in the portfolio risk are compensated by capital increases two months later.
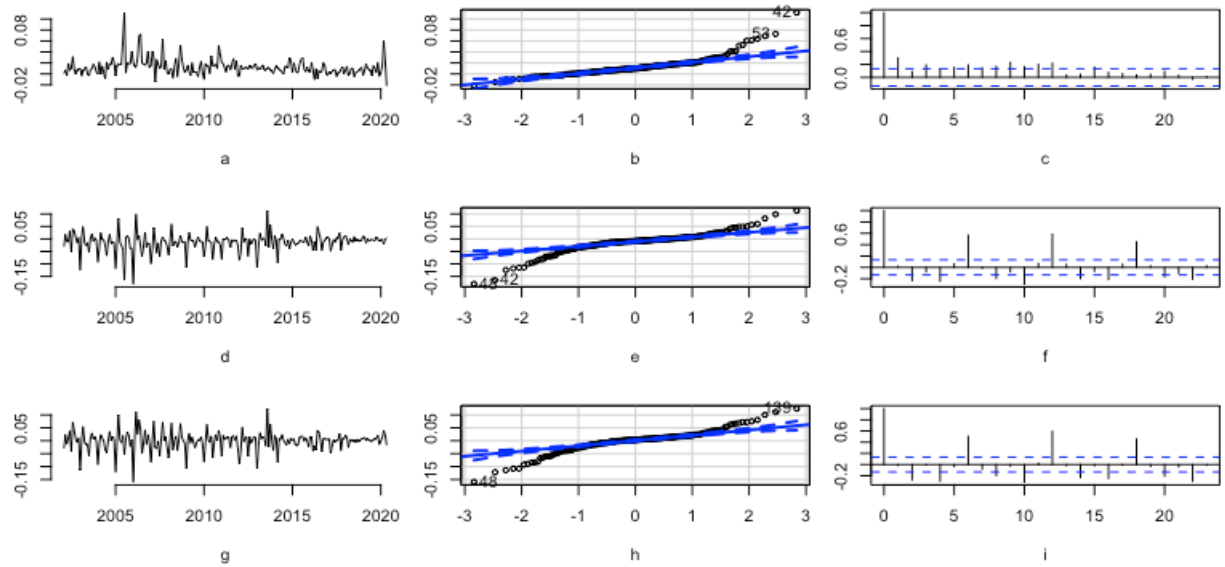
Fig. 3. Monthly time series, Q-Q plot, and correlogram of the Portfolio Growth Rate (a, b, c), Tier Decline Rate (d, e, f), and Solvency Decline Rate (g, h, i).

As observed in Fig. 3, there are time-varying patterns of mean (given the frequency in which the financial statements are reported) and clusters of volatility in the time series. In March and April 2020, a positive shock on *RWA* is observed, which represents an increase of more than 8% with respect to February levels, followed by a fall of 2.18%, which becomes the maximum negative variation in the series. According to the Q-Q plot of *PGR* (Fig. 3b), in the left tail and in the center, the quantile of the normal distribution is close to the frequency distribution, while, in the right tail, the frequency distribution is heavier than the normal distribution. The Q-Q plots of *TDR* and *SDR* (Figs. 3e and 3h, respectively) show that the frequency distribution does not approximate the normal quantile in either tail.

In relation to the Autocorrelation Function (ACF) correlograms (Figs. 3c, 3f, and 3i), there are cuts of the confidence interval in order 1 and the autocorrelation grows for lags 3, 6, and 9. The time series of the SDR (Fig. 3d) exhibits volatility clusters. The time series of the SDR (Fig. 3d) exhibits volatility clusters.

Table 3. Dickey-Fuller and Box-Ljung tests on variations in Portfolio Growth Rate, Tier Decline Rate, and Solvency Decline Rate.

test	p-value PGR	p-value TDR	p-value SDR
Dickey-Fuller	0.01	0.01	0.01
Box-Ljung	0.00	0.00	0.00

The results of the Dickey–Fuller test for the *PGR*, the *TDR*, and the *SDR* indicate that these variables are stationary since the null hypothesis of non-stationarity is rejected; and those of the Box–Ljung test reveal the presence of autocorrelation in the three series (Table 3).

5. Results

5.1. Fitted distribution

Table 4. Estimated parameters of the moments of the *GC* probability distributions for the Portfolio Growth Rate (PGR), Tier Decline Rate (TDR), and Solvency Decline Rate (SDR).

	PGR parameters					TDR parameters					SDR parameters			
	Estimate	Std. Error	t value	Pr(>|t|)		Estimate	Std. Error	t value	Pr(>|t|)		Estimate	Std. Error	t value	Pr(>|t|)
ar1	0.249	0.064	3.877	0.000	ar6	0.186	0.055	3.388	0.001	ar6	0.265	0.057	4.649	0.000
ar3	0.113	0.065	1.740	0.082	ar12	0.712	0.065	11.019	0.000	ar12	0.461	0.056	8.166	0.000
ar9	0.147	0.065	2.267	0.023	ma12	-0.430	0.086	-5.021	0.000	a0	0.000	0.000	7.425	0.000
ar12	0.118	0.066	1.805	0.071	a0	0.000	0.000	8.150	0.000	a1	0.145	0.062	2.361	0.018
intercept	0.005	0.002	2.964	0.003	a1	0.444	0.078	5.692	0.000	a2	0.232	0.084	2.775	0.006
a0	0.000	0.000	17.312	0.000	a2	0.174	0.062	2.794	0.005	d4	0.093	0.017	5.385	0.000
a1	0.189	0.042	4.545	0.000	d4	0.086	0.016	5.493	0.000					
d3	0.124	0.039	3.193	0.001										
d4	0.102	0.016	6.433	0.000										

Table 4 presents the estimated parameters of the moments of the probability distributions for *PGR*, *TDR*, and *SDR*. In the three cases, the mean shows time-varying patterns effects of quarterly multiples (3, 6, 9, and 12 months). Additionally, *PGR* exhibits an autoregressive effect of order 1. The variances are the ARCH 1 process for the three variables under analysis (parameters a0 and a1). With respect to the *GC* parameters, the kurtosis parameters are significant for the three series, while skewness is only significant for *PGR*.
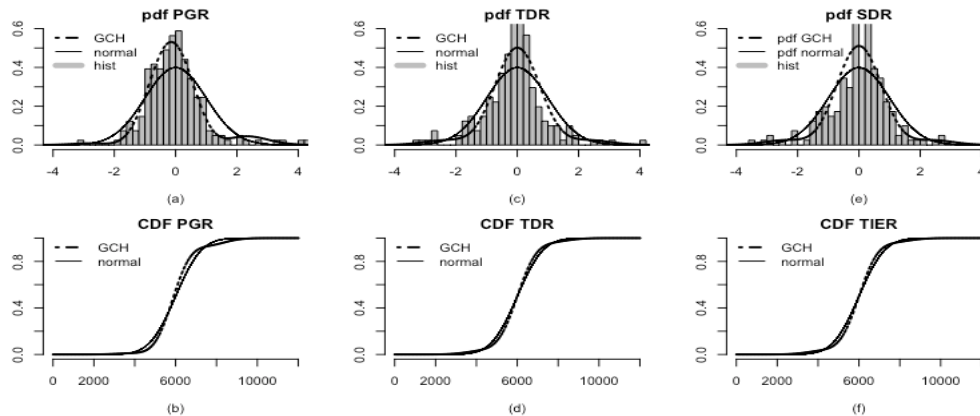

Fig. 4. a, c, and e compare the frequency histograms of the standardized residuals of the mean model with the normal (continuous line) and *GC* (dotted line) probability density functions of the Portfolio Growth Rate (PGR), Tier Decline Rate (TDR), and Solvency Decline Rate (SDR), respectively. b, d, and f present the cumulative distribution functions for the *GC* and normal and the three-time series.

Figure 4 shows how the Gram-Charlier *pdf* (Figs. 4a, 4c, and 4e) fits the deviations observed in the frequency histograms of the standardized residuals of the mean model compared to the normal fit and for the *PGR*, the *TDR*, and the *SDR*, respectively. The *pdf* Gram-Charlier captures the heavy tails of the frequency histograms and the right-tail asymmetry of *PGR*. Figs. 4b, 4d, and 4f compare the *GC cdfs* with that of normal *cdfs*. The differences between the *cdf GC* and its normal counterpart for *PGR* in the left tail are less marked than those observed in the right tail, where the *GC* quantiles are greater than the normal ones, whilst for *TDR* and *SDR*, this difference between the right and left tails is not marked.

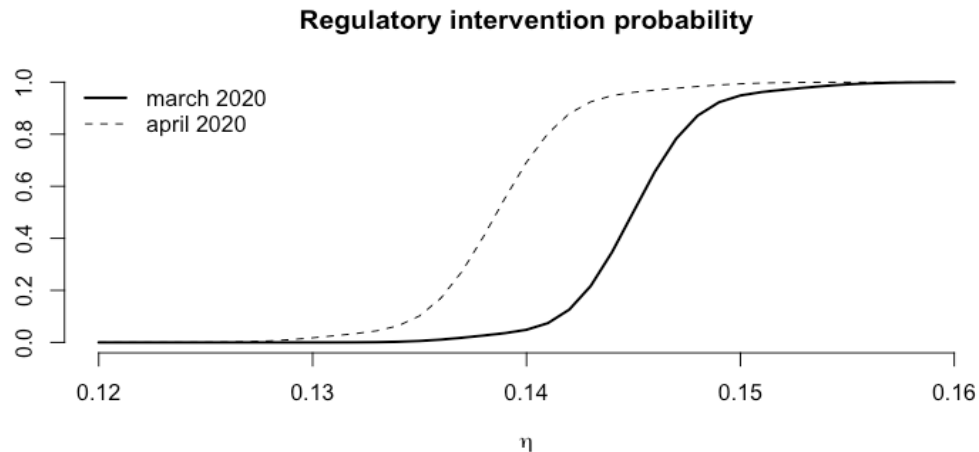### 5.2. Regulatory intervention probability

Fig. 5. Probability of regulatory intervention for different minimum regulatory solvency levels in March and April 2020.

As observed in Fig. 5, the probability of regulatory intervention increases for all the different levels of minimum regulatory solvency levels due to the liquidity injection undertaken in the economy to solve the needs generated by the COVID-19 pandemic in March 2020. In the case of Colombia, if we take into account the adjustment proposed by Clavijo et al. (2012) (from 2% to 2.5%) and the higher required levels behind the transition from Basel II to Basel III, we could consider an η higher than 14% (considering the adjustment over the minimum level, η). Above this 14% level, the probability of regulatory intervention rose from 4.9% in March 2020 to 69.2% in April 2020 after the COVID-19 shock. The increase in the probability of insolvency from March to April 2020 is smaller for the extreme values of η.

5.3. Quantile Risk Measures over the PGR, the TDR and the SDR.

Based on the parameters estimated for the probability distributions of the *PGR*, the *TDR*, and the *SDR*, we calculated the *QRMs*, which provide estimated scenarios involving loss of bank solvency, given a risk appetite level of α.
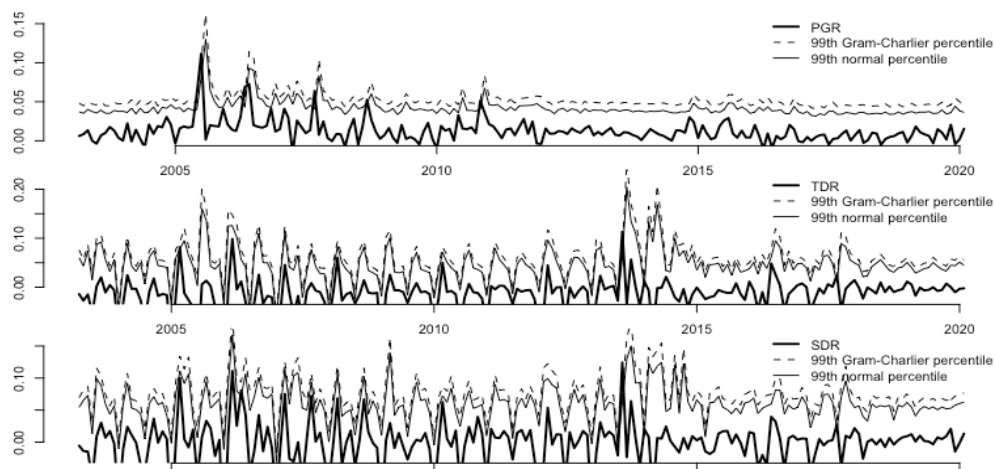

Fig. 6. Probability interval backtesting for the Portfolio Growth Rate (PGR), Tier Decline Rate (TDR), and Solvency Decline Rate (SDR).

Fig. 6 compares the time series under analysis with the estimated *QRMs* assuming a *GC pdf* and a normal *pdf*. The interval for the Gram-Charlier is found to be wider than the normal one, which implies that the risk measurement under the Gaussian assumption underestimates the risk when compared to the measurement obtained using a *GC pdf* (asymptotically the true distribution).

5.3.1. Backtesting

To measure and compare the performance of the *QRMs* estimated under the assumption of either a Gram-Charlier or a normal *pdf*, we used the Kupiec's and Lopez's tests.

Table 5. Comparison of backtesting results under a Gram-Charlier and normal setting using Kupiec's and Lopez's tests on the Portfolio Growth Rate (*PGR*), Tier Decline Rate (*TDR*), and Solvency Decline Rate (*SDR*).

	p-value Kupiec PGR	p-value Kupiec TDR	p-value Kupiec SDR	Score Lopez PGR	Score Lopez TDR	Score Lopez SDR
Gram-Charlier	0.151	0.182	0.303	5.001	2.001	1.002
normal	0.321	0.051	0.321	6.003	4.003	4.004

Table 5 presents the results of the Kupiec's and Lopez's tests used in this study to measure the performance of the *GC* and normal quantile measures for the *PGR*, *TDR*, and *SDR*. According to these results, the Kupiec's test, which compares the theoretical quantile with the empirical one through a ratio comparison hypothesis test, only generates rejection for the normal model applied to the TDR. For its part, the Lopez's test, which considers the distance between the *QRMs* and the observed exceptions, reveals that, for all the series, the score is lower under *GC* settings than under normal settings. This means that the intensities of the exceptions are lower under the *GC* setting.
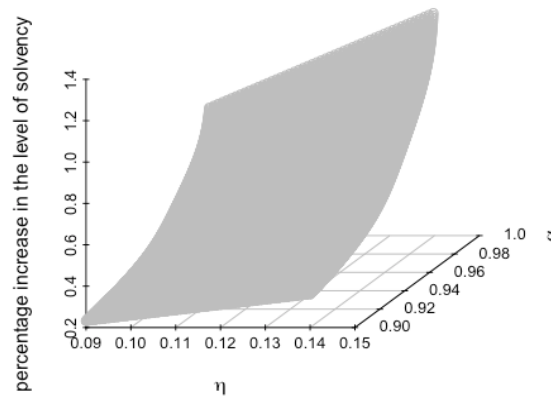

Fig. 7. Percentage increase in the solvency level for different minimum regulatory solvency levels and different risk appetite levels.

Fig. 7 illustrates the necessary increase in the solvency levels to support the maximum expected shock, $F_X^{-1}(\alpha)$, on solvency under the assumption of a Gram-Charlier *pdf*, for different minimum regulatory solvency levels (η) and different risk appetite levels (α). *RWA* of the Colombian banking system in March 2020 increased by 6.04% with respect to February 2020. This suggests that said increase is greater than the 99th percentile of the probability distribution of *PGR*, which was measured under the assumption of either a Gram-Charlier or a normal *pdf* with the 99th percentile being 4.8% and 3.9%, respectively. Thus, the increase in the *RWA* in March 2020 exceeds the 99th Gram-Charlier (normal) percentile by 1.2% (2.2%).

Conclusions.

The *QRMs* based on Gram-Charlier *pdfs* facilitate the solvency risk management and promote a better convergence of regulatory capital to the economic capital. This makes it possible to incorporate risk appetite levels by establishing the levels for the parameter α required to estimate the *QRMs*. In addition, it allows the incorporation of parameters that depend on each bank's risk profile, such as those provided by the *GC* *pdf*, which, compared to the normal *pdf*, captures the distortions of the higher moments of the loss *pdf*. *QRMs* follow the guidelines established by the Basel Committee, which consider banks' risk appetite and profile.

Also, the distance between the observed solvency level and the minimum required by the regulator is considered.

Moreover, measuring solvency risk using *pdfs* provides more risk-sensitive measures that consider the dynamics of a bank's risk portfolio and the capital supporting this portfolio. This facilitates decision making about the different factors that affect bank solvency (e.g., measures of liquidity injection implemented by governments to cover the needs of liquidity generated by the COVID-19 crisis), considering the restrictions of the banking system has in terms of solvency. The assumption of normality does not allow the estimation of the higher moments of the *pdf*, thus undervaluing banks' risk profiles. This methodology based on *QRMs* is Basel-compliant, in the sense that it considers each bank's risk appetite and profile.

The shock suffered by the risk portfolios at the beginning of the crisis caused by the injection of liquidity into the different economies shows that this type of event must be considered in allocating economic capital to support portfolio risk. The methodology proposed in this paper allows the estimation of the necessary increases in the solvency levels to absorb these events. Regarding the findings from the sample under analysis, the frequency distributions of *SDR* and its components (*TDR* and *PGR*) were found to have time-varying patterns components in the mean and variance, which can be captured using ARMA and GARCH models, respectively. Concerning the higher moments of the probability distributions, we observed that the frequency distributions of the variables under analysis are leptokurtic and, *PGR* has a marked skewness to the right. After modeling the *pdf* of the *PGR*, the shock suffered by the increase in the portfolio risk in March 2020 in Colombia was found to exceed the 99th percentile under both *GC* distribution and a normal distribution. In addition, this shock increased the probability of regulatory intervention for April 2020. Considering the higher minimum regulatory solvency levels behind the transition of the Colombian banking system from Basel II to Basel III, the probability of regulatory intervention is in a more critical position considering that the levels of the economic capital requirement to cover the maximum expected solvency declines (estimated as *QRMs*) increase exponentially with higher minimum regulatory solvency levels and higher risk appetite levels.

In a nutshell, increases in a bank's risk portfolio without a capital increase cause a decline in solvency and, thus, increase the risk of insolvency. These falls in solvency lead to a decreased confidence among market agents and can trigger reductions in credit ratings. After backtesting, the Gram-Charlier *pdf* was proven to have a better performance than the normal *pdf* according to the lower magnitude of the exceptions found at a 99%. This evidence at the Colombian economy is expected to happen in most economies after the global Covid19 pandemic. Thus, revising the methodologies accounting to solvency risk to incorporate flexible distributions as the Gram-Charlier would be worthwhile.

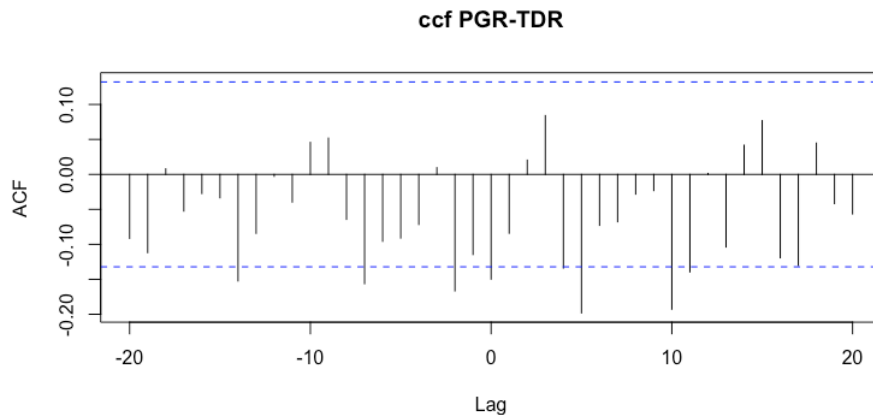## Declaration of Competing Interest
None

Acknowledgements

Annex 1: Tier cross-correlation diagram with risk portfolio components

Fig. 8. Cross-correlation between changes in the risk portfolio and changes in Tier capital.

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