Determination of ship roll damping coefficients by a differential evolution algorithm

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Abstract. The execution of the so-called extinction tests represents the classical experimental method used to estimate the damping of an oscillatory system. For the specific case of ship roll motion, the roll decay tests are carried out at model-scale in a hydrodynamic basin. During these tests, the vessel is posed in an imbalance condition by an external moment and, after the release, the motion decays to the equilibrium condition. When the damping is far below the critical one, the transient decay is oscillatory. Here a new methodology is presented to determine the damping coefficients by fitting the roll decay curves directly, using a least-square fitting through a differential evolution algorithm of global optimisation. The results obtained with this methodology are compared with the predictions from standard methods. This kind of approach seems to be very promising when the motion model of the system under investigation is established with any level of non-linearities included. The usage of the fitting procedure on the approximate analytic solution of the differential equation of motion demonstrates the flexibility of the method. As a benchmark example, two experimentally measured roll extinction curves have been considered and suitably fitted. The newly predicted results, compared with the ones obtained from standard roll decay analysis, show that the algorithm is capable to perform a good regression on the experimental data.

1. Introduction

A ship which oscillates transversally (rolling) on the free surface of the seaway is subject to the resistance of both water and air. In the absence of wind and for relatively slow motions, the resistance opposed by air is negligible with respect to that caused by the liquid medium. Therefore, in the practical computations on rolling, only the latter resistance is considered.

The engineering importance of roll damping is self-evident as soon as one bears in mind its direct impact on the maximum roll amplitude which can be reached in the most dangerous operative conditions, i.e. those near the resonance of the transversal motion. It is indeed well-known that transversal oscillations present major risks for the safety of the ship since they are those with respect to which the ship has the least reaction capacity.

Though physical processes which generate damping have been known for a long time in their main features [1], and are now universally accepted, the situation is quite different as far as a correct quantitative estimate of their effects is concerned, either on theoretical predictions or on tests on models or on actual ships. One can observe that, although all research workers agree with the non-linear nature of the roll damping [2, 3], many of them still go towards the so-called *roll damping equivalent linearisation methods* [4, 5]. The universally accepted standard procedure to measure the roll damping is still missing.

Since the roll damping represents one of the most discussed subjects of ship hydrodynamics, the present paper investigates the short-cuts and the benefits of roll damping evaluation from free roll decay model tests [6, 7]. Particular emphasis is given to the linear plus quadratic damping [8, 9, 10] by comparing the classical peak methods with more sophisticated and refined procedures, which are based on time domain analysis of the observed free decay record. It has been demonstrated that the application of different approaches leads to different quantitative estimations of the roll damping, the differences increasing in case of large amplitude ship motions.

2. Roll motion equation

To describe the roll behaviour of a ship with external active forces, a single degree of freedom equation can be written (neglecting *sway* and *yaw* couplings) in the following general dimensional form:

$$I_\phi \ddot{\phi} + B_\phi(\dot{\phi}) + C_\phi(\phi) = M_\phi \tag{1}$$

Even if the form is a linear motion equation under small motion amplitudes, the couplings with other motions are present, but to discuss non-linear roll damping the above assumption simplify the mathematical treatment of the problem.

In equation (1), I_ϕ represents the virtual mass moment of inertia (mechanical and hydrodynamic) along a longitudinal baricentric axis, B_ϕ is the damping moment, C_ϕ is the restoring moment and M_ϕ represents the external moment due to waves or other external forces. Equation (1) can be expressed with several orders of non-linearities, depending on the modelling of the damping and of the restoring moment. In case of a decay test, the external moment is not present.

2.1. Damping and restoring moment

The damping moment B_ϕ of a vessel can be expressed as a development series of $\dot{\phi}$ and $|\dot{\phi}|$. It is common practice to consider the development up to the third order, but in case of small amplitudes, the second order of approximation is sufficient. The absolute value is used in the odd velocity powers to ensure that the damping moment will always oppose the motion. Sometimes it is also common to consider other combinations of damping factors, but from practical experience the previous linear plus quadratic approximation is giving the most accurate predictions for intermediate amplitudes.

The restoring moment for roll motion C_ϕ can be expressed as $\Delta\overline{GZ}\,(\phi)$, where Δ is the ship displacement and $\overline{GZ}\,(\phi)$ is the righting arm of the vessel. In this case the restoring moment can be written as a series of various degrees of odd-order polynomials describing the righting arm. Cubic and fifth order expressions are the most common way to approximate it, but even higher orders can be used. In case of small amplitude oscillations, a linear righting arm can be considered, using the metacentric height $\overline{GM}$ instead of $\overline{GZ}$.

2.2. Model equation

Considering a linear restoring arm and a linear plus quadratic damping, the roll equation (1) in still water can be rewritten as:

$$I_\phi \ddot{\phi} + B_{\phi_1}\dot{\phi} + B_{\phi_2}\dot{\phi}|\dot{\phi}| + \Delta\overline{GM}\phi = 0 \tag{2}$$

Dividing (2) by I_ϕ the motion equation takes the standard form, which will be the starting point for the further analysis:

$$\ddot{\phi} + 2\nu\dot{\phi} + w\dot{\phi}|\dot{\phi}| + n^2\phi = 0 \tag{3}$$

where ν is the linear damping coefficient, w the quadratic one and n the natural roll frequency.

3. Standard Roll Decay test analysis

There are several methodologies to perform a roll decay test at model scale. The most common one consists in giving a predetermined pure transversal inclination to the tested model and let it oscillate up to the reach of the equilibrium. Usually the initial inclination magnitude is under 10 deg, in order to avoid non-linearities of the ship righting arm curve. In such a way it is possible to use some specific techniques to perform the analysis of the time series.

3.1. Peak's decay

The most common way to analyse a decay test data is to obtain the logarithmic decrement of the consecutive oscillations in order to highlight the behaviour of the decay process. The method requires that all the peaks (positive and negative) of the given decay record are extracted and analysed from the experimental time-domain record to find the damping coefficients [11, 12]. Such kind of procedure is well established but its application is far from criticism.

Once the peaks have been extracted from the time-domain record, the analysis proceeds by determining the decrement of the successive oscillations. Standard theory refers to linearly damped systems; however, the process can be extended on the time decay model described by formulation (3). Considering the first positive swing, thus imposing $\dot{\phi}|\dot{\phi}| = \dot{\phi}^2$, an algebraic solution can be found by applying the Poisson resolution method:

$$\phi(t) = \left(\phi_0 - \frac{2}{3}w\phi_0^2\right) e^{-\nu t}\cos\omega t + \phi_0^2 w e^{-2\nu t}\left(\frac{1}{2} + \frac{1}{6}\cos 2\omega t\right) \tag{4}$$

where the frequency of oscillation ω is defined as $\sqrt{n^2 - \nu^2}$. Then, the decrease in amplitudes of the first and $i + 1^{th}$ swings are:

$$\Delta\phi_1 = \phi_0\left(1 - e^{-\nu\frac{T_c}{2}}\right) + \frac{2}{3}w\phi_0^2 e^{-\nu\frac{T_c}{2}}\left(1 + e^{-\nu\frac{T_c}{2}}\right) \tag{5}$$

$$\Delta\phi_{i+1} = \phi_i\left(1 - e^{-\nu\frac{T_c}{2}}\right) + \frac{2}{3}w\phi_i^2 e^{-\nu\frac{T_c}{2}}\left(1 + e^{-\nu\frac{T_c}{2}}\right) \tag{6}$$

Here the quantity $\nu T_c/2$ determines the logarithmic decrease of two consecutive amplitudes and is therefore called logarithmic decrement of the oscillation. In such a case it must be also noted that in case of a ship, the decay period T_c can be approximated with the rolling period without resistance $T = 2\pi/n$. By using this kind of notation, all the peaks of the decay are considered, evaluating $|\phi_i|$ from the minima. Once the procedure is applied separately on minima, or on maxima, then the decay period to be considered in the equations is T_c instead of $T_c/2$.

The decrement equation (6) is a polynomial formulation of the initial motion amplitude. Therefore, it can be rewritten in the following form:

$$\Delta\phi_{i+1} = a\phi_i + b\phi_i^2 \tag{7}$$

where:

$$\begin{cases} a = 1 - e^{-\nu\frac{T_c}{2}} \\ b = \frac{2}{3}we^{-\nu\frac{T_c}{2}}\left(1 + e^{-\nu\frac{T_c}{2}}\right) \end{cases} \tag{8}$$

The decrement of single swings can be also determined from the decay record, just making the difference between the consecutive peaks. The computed values can be used to set up a regression of the peaks decrement in quadratic form, or a linear regression of $\Delta\phi_{i+1}/\phi_i$. The regression coefficients obtained from data fitting are then used to determine the damping coefficients.

3.2. Application of standard analysis
To evaluate the procedure used for evaluating the roll damping coefficients, two test cases were run taking into consideration two available decay tests from hydrodynamic basin. The first record (Record#1) refers to a decay test executed starting from a initial heeling angle under 10 degrees. The second one (Record#2) refers to a higher starting heeling angle, leading to possible effects due to non-linearities of the righting arm.

The roll amplitude data for Record#1 are shown in Figure 1, where the time record refers to model scale values. The second roll decay test (Record#2) has been analysed, with the aim to test the algorithm in presence of a higher level of initial heeling. The record is presented in Figure 2, where it is possible to observe that the starting heeling angle is above 40 degrees.

It is common practice to discard the first peaks and consider just the subsequent 6/8 oscillations. The first peaks are usually discarded, because during the first swings the initial condition $\dot{\phi}_0 = 0$ cannot be satisfied. The last swings, where the roll amplitude is significantly small (below 1.5 degrees) can affect the quality of the analysis. For this reason, two separate analyses have been carried out, one considering the whole sequence of peaks (*all peaks*), and another taking into account just a part of the signal for the regression (*selected peaks*).

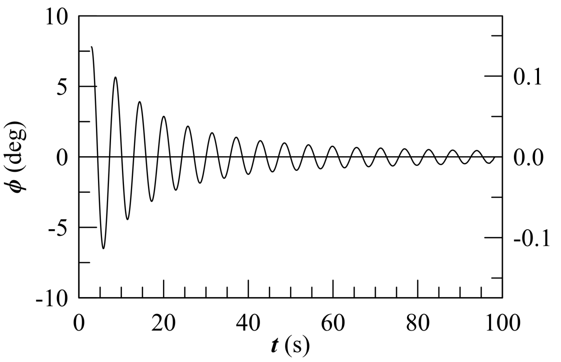

Figure 1. Record#1 roll decay data.

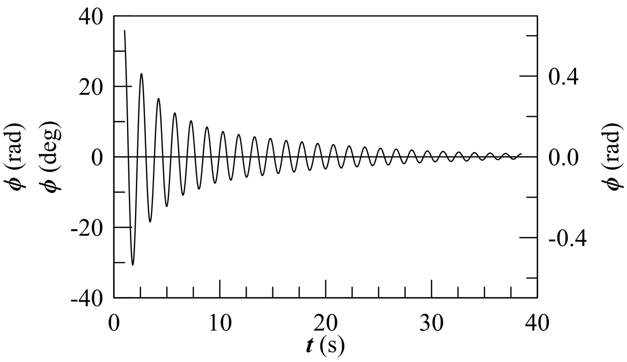

Figure 2. Record#2 roll decay data.

Table 1. Damping coefficients obtained by regression on all peaks for Record#1 and Record#2.

	Record#1				Record#2			
	a	b	ν	w	a	b	ν	w
Peaks Min.	0.0661	0.8452	0.0240	0.7020	0.0232	0.3225	0.0314	0.2505
Peaks Max.	0.0985	0.3577	0.0364	0.3130	0.0323	0.2910	0.0441	0.2292
Peaks Min.+Max.	0.0879	0.5053	0.0323	0.4346	0.0260	0.3145	0.0353	0.2453

There are several methods to carry out the analysis according to the peaks considered. Assuming that it deals with a zero-mean process at least three different options are feasible i.e. one can consider the peaks corresponding to both roll maxima and minima or only to roll

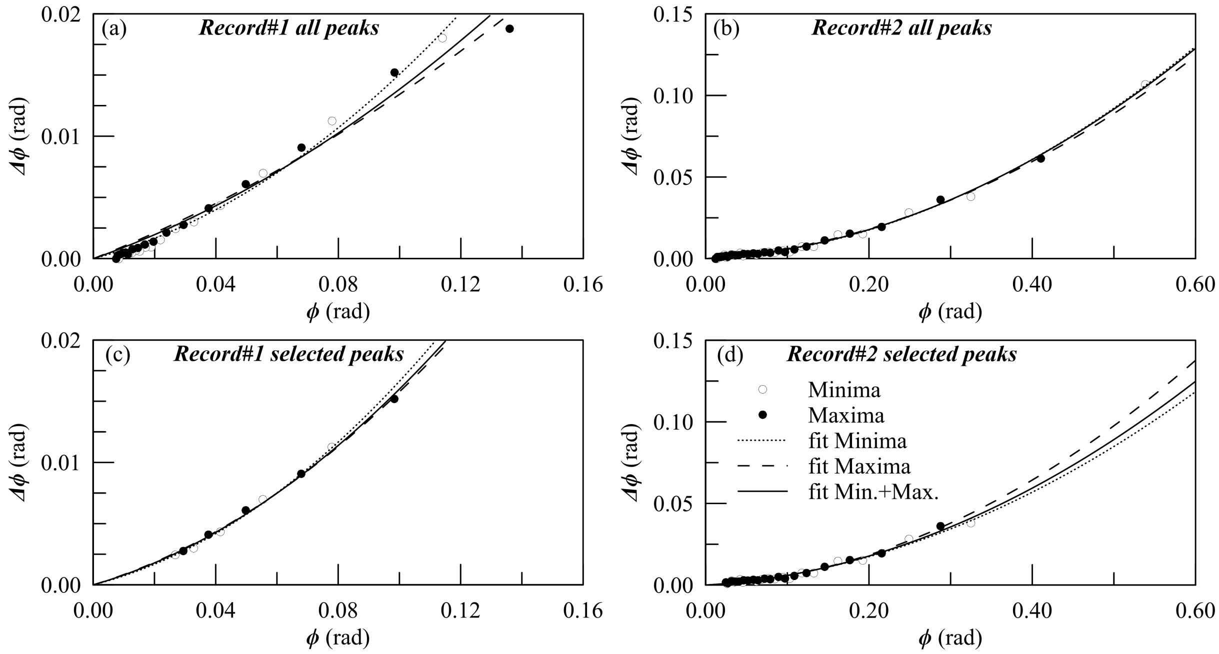

Figure 3. Peaks analysis: (a) Record#1 - all peaks, (b) Record#2 - all peaks, (c) Record#1 - selected peaks, and (d) Record#2 - selected peaks.

Table 2. Damping coefficients obtained by regression on a selected number of peaks for Record#1 and Record#2.

	Record#1				Record#2			
	a	b	ν	w	a	b	ν	w
Peaks Min.	0.0636	1.0275	0.0231	0.8500	0.0318	0.2762	0.0433	0.2174
Peaks Max.	0.0756	0.8210	0.0276	0.6922	0.0237	0.3431	0.0321	0.2667
Peaks Min.+Max.	0.0720	0.8790	0.0262	0.7370	0.0294	0.2979	0.0400	0.2336

maxima or roll minima. Figure 3 shows the obtained results, while Tables 1 and 2 summarise the corresponding coefficients. It can be observed that the selection of the number of peaks to analyse has a substantial impact on the final results. For Record#1, the adoption of the selected set of peaks reduces the spread between the three regressions. However, the effect is opposite for Record#2. Concerning the final value of the damping coefficients ν and w, Record#1 shows a higher sensibility to the peak selection compared to Record#2.

Damping coefficients ν and w have been obtained through the estimation of natural roll period T_c from the decay tests, which resulted of 5.67 s and 1.46 s for Record#1 and Record#2, respectively.

4. Differential evolution algorithm to analyse decay tests

The standard method of analysis for decay tests shows variability in the obtained damping coefficients, depending on the peaks used to fit the data. Therefore, a new methodology has been developed, based on a differential evolution algorithm [13, 14], capable to handle the measured data directly, using as fitting function approximated solutions of the roll equation.

The process requires the user to provide only three parameters: the population number N_{POP}, the scale factor F and the crossover probability p_c. Additional constraints may be given concerning limitations to parameters or stopping criteria for the objective function evaluation.

To fit the decay data with equation (3), the algorithm exposed in the previous section has been used. In order to find a fitting solution, an objective function has been selected. In this case the choice was to maximise the coefficient of determination R^2 of the regression. As the differential evolution algorithm is structured to minimise the objective function, then the quantity $-R^2$ is the true objective of the process.

4.1. Analytic solutions

The fitting procedure can be simplified considering an approximate solution of equation (3), which can be obtained by means of perturbation analysis. The adoption of a linear plus quadratic model for damping requires to find an analytic solution taking into account the $\dot{\phi}|\dot{\phi}|$ term. To do that it is more convenient to adopt an averaging technique, leading to the following approximate analytic solution:

$$\phi(t) = e^{-\nu t} A(t) \cos\left(\omega t + \varphi(t)\right) \tag{9}$$

where:

$$\begin{cases} A(t) = \dfrac{A_0}{1 + A_0 k_A \left(1 - e^{-\nu t}\right)} \\ \varphi(t) = \varphi_0 - \dfrac{k_\varphi}{\nu k_A} \ln\left(1 + A_0 k_A \left(1 - e^{-\nu t}\right)\right) \\ k_A = \dfrac{4}{3\pi} \dfrac{nw}{\nu} \\ k_\varphi = \dfrac{4}{3\pi} \dfrac{nw\nu}{\omega} \end{cases} \tag{10}$$

To fit the decay test with this approximate solution, besides the two unknown damping coefficients ν and w, the following parameters need to be determined: n, A_0 and φ_0. The evolutionary process should estimate n, only in case $\overline{GM}$ and I_ϕ are not known. A_0 and φ_0 derive from the initial conditions of the record, means by ϕ_0 and $\dot{\phi}_0$, solving the following system of equations:

$$\begin{cases} \phi_0 = A_0 \cos\varphi_0 \\ \dot{\phi}_0 = \phi_0 \nu \left(1 + A_0 k_A\right) - A_0 \sin\varphi_0 \left(\omega - k_A \nu^2 \dfrac{A_0}{\omega}\right) \end{cases} \tag{11}$$

The resolution of system (11) is not trivial, as in the evolutionary process it must be solved for each individual using the assigned parameters. Therefore, the fitting process for equation (9) requires the determination of at most three parameters, supposing that the righting arm of the vessel is not known.

5. DE application and comparison with standard methods

The fitting process based on the differential evolution algorithm has been here applied on the two records previously studied with the standard decay analysis method. As mentioned, the whole decay time history is here adopted for both records, thus the only additional inputs to be set are the control parameters of the evolutionary algorithm.

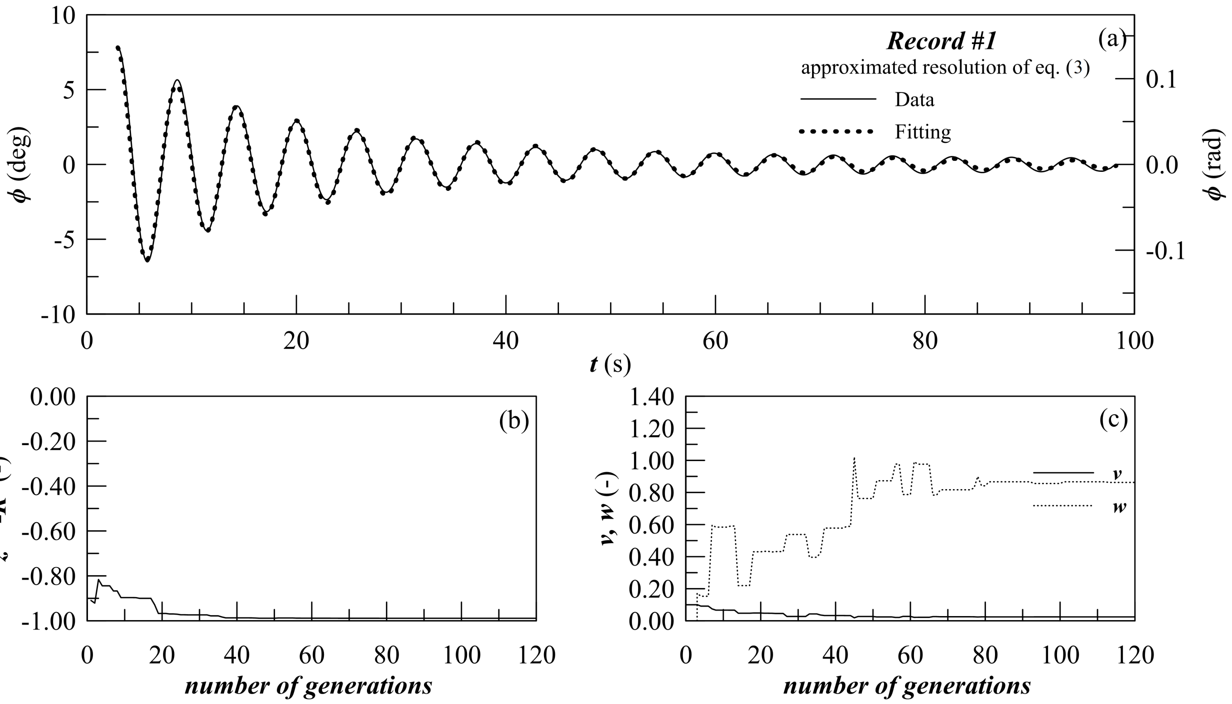

Figure 4. Record#1 fitting with analytic resolution: (a) fitted signal, (b) objective function convergence, (c) damping coefficients.

Table 3. Damping coefficients and T_c obtained by evolutionary regression for Record#1 and Record#2.

Record	ν	w	T_c (s)	R^2
Record#1	0.0242	0.8645	5.9894	0.9888
Record#2	0.2213	0.0141	1.5709	0.8544

5.1. Record#1

The application of the differential evolution algorithm requires to know which are the free variables for the optimisation. For this case, the unknowns are the damping coefficients ν and w, and the oscillation frequency n, as the quantities $\overline{GM}$ and I_ϕ are not known.

Figure 4 reports the results obtained using the approximated solution of equation (3) as fit function. The results show that the process is capable to well reproduce the decay test, achieving a high coefficient of determination R^2=0.9888. The convergence of the objective function is quite fast, as the stopping criterion is reached after 120 iterations. Table 3 reports the damping coefficients obtained from the fitting process, together with the oscillation period T_c, which for this case is 0.3 s higher than the value observed in the standard analysis.

It can be stated that the fitting process according to model derived from equation (3) is adequate for Record#1.

5.2. Record#2

The second decay test has a higher initial heeling angle compared to previous Record#1. Initial conditions (11) have been applied for the fitting procedure. Figure 5 reports the results of

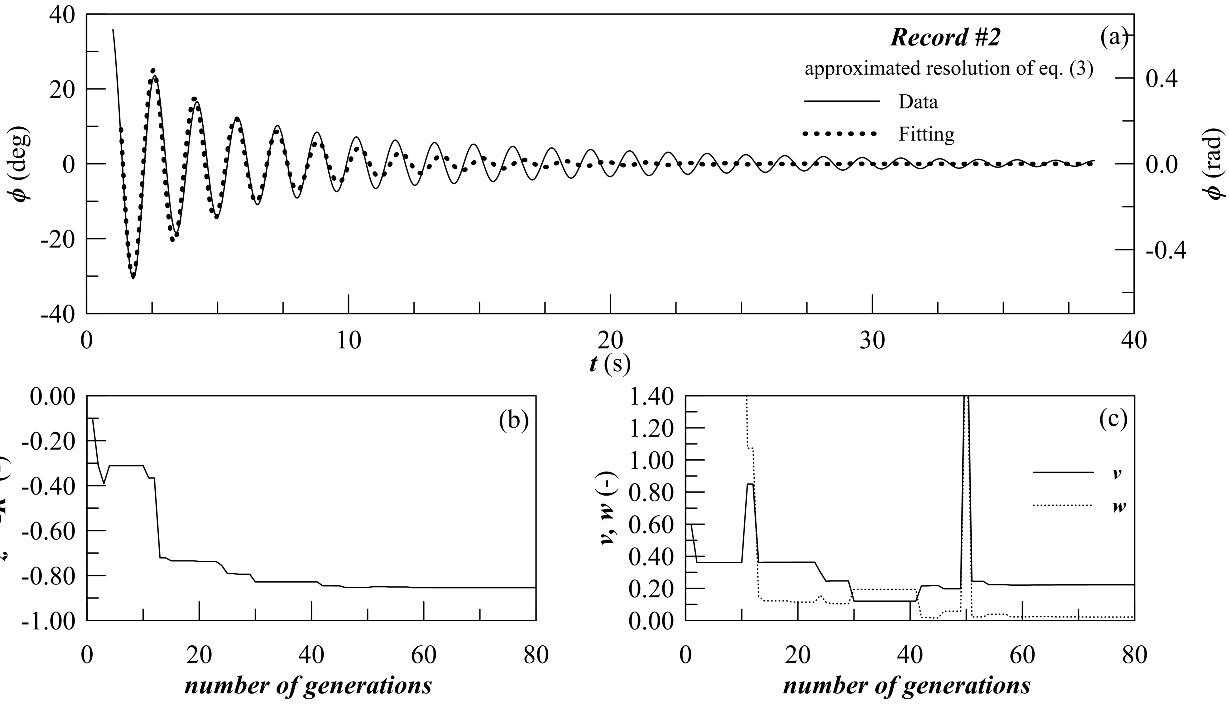

Figure 5. Record#2 fitting with analytic resolution: (a) fitted signal, (b) objective function convergence, (c) damping coefficients.

this new process using the approximated solution of equation (3) as fit function. The results highlight that the fitting process is not capable to achieve a high coefficient of determination R^2 as for the previous case. A value of 0.8544 has been achieved, which is not a satisfactory result for the regression. Observing the fitted function of the decay test, it can be stated that just the first high peaks are well captured, while the small oscillations are not reproduced, as the computed curve decays faster than the original record. Moreover, also the natural frequency of the oscillation is not captured, predicting a higher T_c compared to the standard analysis, resulting in a shift forward of the function after 10 seconds. The obtained damping coefficients are reported in Table 3.

Such a behaviour suggests that the model adopted for the regression is not appropriate for this test, as, for high heeling angles, the non-linearities of the righting arm should be taken into account.

5.3. Comparison with standard analysis

Instead of simply comparing the value of the coefficients, it is physically significant to visualise the effect of the damping coefficients on the decrement curves. The decrement curves, already presented in Figure 3, are a consequence of the standard analysis of the peak decay, but similar curves can be easily obtained by applying the data collected in Table 3 to equations (7) and (8). Figure 6 reports the comparison between the decrement curves obtained with the standard analysis (all peaks condition) and the new proposed fitting procedure.

Considering Record#1, it can be observed that the decrement curve for analytical fitting differs from the decrement curves of the standard method. The decrement curve elaborated from the new fitting process is much steeper than the standard ones. This is mainly due to the quadratic damping coefficient w, which results higher from the enhanced fitting analysis than from the standard one.

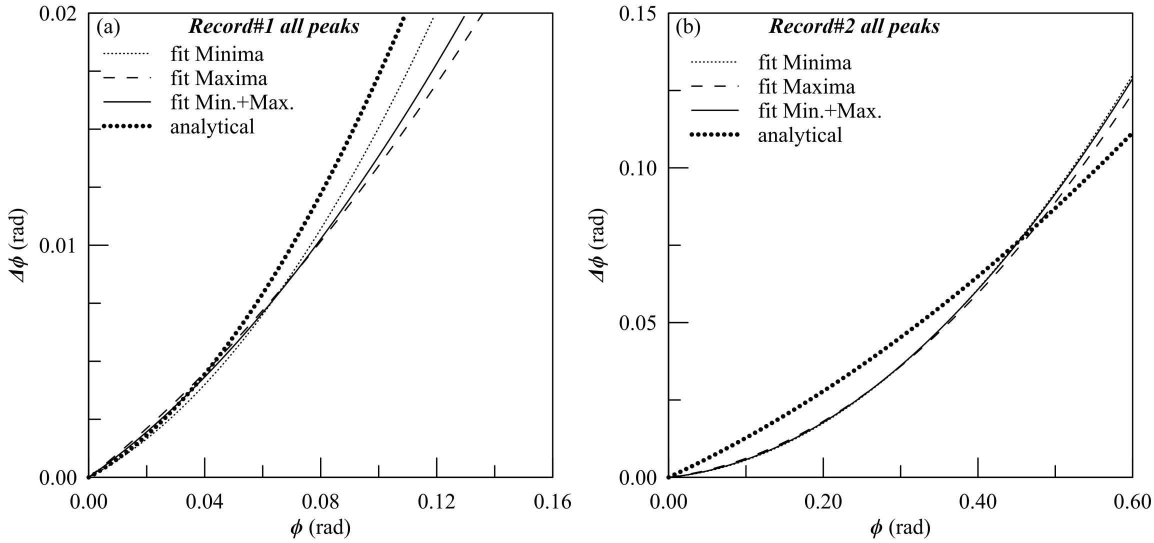

Figure 6. Decrement curves comparison between standard analysis and newly proposed fitting procedure for (a) Record#1 and (b) Record#2.

For Record#2 the situation is different, as the linear arm model is not adequate to fit the complete record. The decrement curve for the analytical solution with linear arm has a totally different trend from the others. This is mainly visible for small amplitudes, where the model was not capable to fit the small oscillations. In fact, the decrement curve is higher than the others at low ϕ values and than matches them at higher ϕ angles, where the fitting of the initial curve was appropriate.

As a general observation, the proposed method is capable to fit the decay curve with a higher level of accuracy compared to standard methods. Moreover, the method does not require a pre-selection of points, avoiding to have variability of results due to the selection of sub-groups of peaks. However, the method is sensible to the theoretical model used for the determination of the fitting function. Therefore, the appropriate model should be selected with care before the analysis.

6. Conclusions

In the present work the topic of ship roll damping coefficients determination from decay test is discussed. Standard state of art procedures shows that there is a variability in the predicted coefficients, depending on the number and type of peaks extracted for the analysis. A novel procedure based on the same theoretical model of roll oscillation has been developed and described to fit decay data by means of a differential evolution algorithm. The algorithm is capable to fit approximated analytical function or manage direct numerical integration of the ship roll motion equation.

The application on two decay test shows that the new method is capable to fit data with a high level of accuracy. However, the proposed procedure is sensible to the theoretical model used to describe the roll motion of the ship. The analysis of a record with a high initial roll amplitude highlights that the hypothesis of a linear arm is not appropriate for such kind of data fitting. Further analyses will be carried out to find a more general model with a dedicated approximate analytical solution to have a fast and complete regression model for any kind of decay test for the roll motion of ships.

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