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Reliability Based Design Optimization under Mixed Aleatory/Epistemic Uncertainties: Theory and Applications

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ABSTRACT

Reliability-Based Design Optimization (RBDO) is a well-known design strategy in engineering. However, RBDO usually requires uncertainties to be modelled by statistical distributions. This requires the availability of sufficient sample size so that these variables can be represented accurately by probabilistic distributions. In the design of new systems and structures, usually there is a lack of information about some uncertain variables or parameters and only a reduced set of samples might be available. This prevents their treatment as probability distributions. This type of uncertain is called epistemic uncertainty. This paper proposes two effective multi-objective evolutionary algorithms to solve design problems under both types of uncertainty: aleatory and epistemic. Two objective functions, i.e., the cost of the structures and the probability of failure are considered. The results are Pareto fronts with a trade-off between cost and reliability associated with a specified level of confidence. Pareto fronts show minimum achievable values for the probability of failure for a given cost. The effect of the epistemic uncertainty on the solution is also investigated. An analytical example and two structural examples are solved to show the applicability of the approach and how epistemic uncertainty may affect the results.

INTRODUCTION

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Engineering structures are usually designed following the regulations of construction codes (e.g., Eurocodes 2014; International Building Code 2018). The designer looks for the optimal structure that verifies the constraints imposed by the code. These constraints, usually named limit states, are related with stresses and displacements caused by various loading combinations. Structures are projected to support loads for a specified life cycle. Applied loads, especially loads caused by climatic conditions like wind, snow, ice, etc. are characterized by large variability. In addition, material properties and geometric parameters of structural members are also affected by uncertainties caused by imperfections in manufacture and construction processes. Additional uncertainty comes from the simplifications made in obtaining the mathematical model that represents the behaviour of the structure. This type of uncertainty leads to model uncertainty. These variabilities or uncertainties are unavoidable and must be taken into account in the structural design practice. Structural codes use a semi-probabilistic approach, in which uncertainties regarding materials, loading, and model are considered implicitly using safety factors and characteristic values for loads and strength.

On the other end, the effect of the uncertainty is part of the design in modern structural design methods. Depending on the type of uncertainty, different methods could be applied to determine optimal designs. Researchers have proposed several classifications of uncertainty. The most popular considers two types of uncertainties: aleatory uncertainty and epistemic uncertainty (Der Kiureghian and Ditlevsen 2009).

Aleatory uncertainty, also known as variability or statistical uncertainty, is inherent in any physical variable. It is irreducible and commonly modelled with probability distributions with parameters identified from a relatively large set of samples. It is assumed that the information about aleatory uncertainties is complete and sufficient to characterise their effect on the systems performance in the design optimization process.

On the other hand, epistemic uncertainty represents incomplete or partial information about the uncertainty due to limited or no samples available. Samples are obtained experimentally through tests. For practical or budgetary reasons, it is only possible to carry out a small number of tests. Therefore, there is not enough data available to determine the probabilistic distribution of the uncertain variables and parameters (Der Kiureghian and Ditlevsen 2009; Roccheta *et al.* 2018). Recently, researchers have made great efforts in the field of quantification of uncertainty, reliability analysis and optimal design under uncertainty to deal with problems in which there is not enough information on uncertainties and, therefore, exact probability distributions cannot be assumed to model these uncertainties (Beer and Patelli, 2015). Probabilistic distributions adjusted with such limited data would cause erroneous and unsafe design if they were propagated in design under uncertainty algorithms (Patelli *et al.* 2015). Toft-Christensen and Murotsu (1996) suggested three types of uncertainties in the field of structural reliability analysis: physical uncertainty, statistical uncertainty, and simulation model uncertainty. Physical uncertainty is the randomness inherent to physical observations, which can be described in terms of probability distributions. Statistical uncertainty corresponds to the uncertainty caused by lack of statistical information or limited sample size and is considered as epistemic uncertainty. The uncertainty of the simulation model occurs because of errors and idealizations done in the mathematical model. The uncertainty of the model can also be considered as an epistemic uncertainty. In this work, the uncertainty of the simulation model is not considered, and only statistical uncertainty is taken account as epistemic uncertainty. Figure 1 represents different types of uncertainty according to the available information.

The most adopted methods in the field of engineering design under uncertainty consider random variables represented by probability distributions with known parameters. Two of these design methods that tackle with complete information of uncertainties are well known: Reliability Based Design Optimization (RBDO) (Tu and Choi 1999; Aoues and Chateauneuf 2010; Valdebenito and Schuëller 2010; Celorrio 2010; Celorrio 2012; Hao *et al.* 2019; Zhou *et al.* 2018, Yi *et al.* 2008; Okasha 2016) and Robust Design Optimization (RDO) (Patelli *et al.* 2014; Schuëller and Jensen 2008; Capiez-Lernout and Soize 2008).

RBDO methods consist of finding an optimal design simultaneously with prescribed level of reliability or probability of failure. Generally, a single objective function encoding the cost of the structure is optimized subject to reliability (inequality) constraints. Instead, RDO aims to obtain a design that is insensitive to input variations. This is generally obtained by including the contribution of the variance of the quantity of interest into the objective function. RBDO and RDO approaches can be combined to develop the Reliability-Based Robust Design Optimization (RBRDO) approach. This approach ensures both reliability and robustness during the life cycle of structures. The RBRDO formulation consists of incorporating probabilistic constraints into the RDO formulation. That is, the mean and variation of the system performance function are minimized subjected to probabilistic constraints. This formulation is generally solved by multi-objective optimization algorithms, see e.g., (Lagaros *et al.* 2007; Yadav *et al.* 2010). RBDO methods are classified in three groups: double loop methods, single loop methods and decoupled methods. With respect to reliability analysis, two kinds of methods can be used: approximated methods (e.g., FORM, SORM) and Monte Carlo simulation methods (Angelis *et al.* 2015; Patelli *et al.* 2011). Simulation methods often request prohibitive computational effort while approximate methods, like FORM and SORM, demand less computing resources. However, it is well known that approximate methods experiment convergence difficulties, especially when performance functions are highly nonlinear and random variables are not normally distributed (Valdebenito *et al.* 2010).

Usually, the designer or decision maker is interested in the trade-off between cost and reliability. In other cases, two or more objectives are optimised subject to reliability constraints. The method to solve them is named Multi-Objective Reliability Based Design Optimization (MORBDO) (Sinha 2007). Multi-objective Optimization Evolutionary Algorithms (MOEAs) such as Non-dominated Sorting Genetic Algorithm (NSGA-II) (Deb *et al.* 2002) and Multi-Objective Particle Swarm Optimization (MOPSO) (Coello *et al.* 2004) are the most considered methods to solve MORBDO problems since these methods can handle constraints efficiently. A special MORBDO formulation consists of considering the reliability of the system as additional objective function to maximize. The result of the multi-objective optimisation is therefore a Pareto front that establishes a trade-off between cost and reliability (Celorrio and Patelli 2018).

Recently, new methods have been developed to consider epistemic uncertainty in design optimization under uncertainty. These methods are only applicable with very specific representation of epistemic uncertainty. For example, fuzzy sets are adopted by Du *et al.* (2005) to quantify uncertainties in the proposed Possibility Based Design Optimization (PBDO). Epistemic variables are modelled as membership functions. The sample size is not considered in uncertainty quantification and as consequence the approach usually produces very conservative results. Interval variables are adopted in optimal design by e.g., Rao and Cao (2002) and Penmetsa and Grandhi (2002). The disadvantage of such approach is that the information contained in the available samples is generally not used. Mourelatos and Zhou (2005) apply Evidence Theory in design optimization and propose a method named Evidence Based Design Optimization. Su *et al.* (2016) consider an evidence-based plausibility measure of failure in a multi-objective optimization problem. They applied a differential evolution-based multi-objective optimization algorithm to search for the robust Pareto front. Researchers have proposed methods applying Bayesian inference to solve optimal design problems under epistemic uncertainty (Gunawan and Papalambros 2005, Youn and Wang 2006, Srivastava and Deb 2013, Li and Wang 2020). Bayesian inference based method is able to deal with three different type of uncertainties: 1) model form uncertainty or epistemic uncertainty (model bias and unknown model parameters), 2) data uncertainty due to the lack of training data, and 3) input variation of random variables or aleatory uncertainty. Gaussian calibration (or stochastic model updating) is used to determine model from uncertainty and a hybrid Gaussian process as metamodel (Patelli *et al.* 2017).

This paper proposes the use of two multi-objective evolutionary algorithms to solve a multi-objective reliability-based design optimization of trusses under aleatory and epistemic uncertainty. Epistemic uncertainties are represented by a set of samples with limited size. Bayesian inference provides an efficient method to perform the reliability analysis and compute the probability of failure corresponding to the reliability constraints.

The paper is organised as follows. A short review of Bayesian Inference theory in presented in section 2. Section 3 describes the Bayesian RBDO method, and section 4 studies the Bayesian Multi-objective Reliability-Based Design Optimization (Bayesian MORBDO) problem, where two objective functions are optimised: cost and reliability. Two state of the art Multi-objective Evolutionary Algorithms (MOEAS) are applied to solve Bayesian MORBDO problems: Non-dominated Sorting Genetic Algorithms (NSGA-II) and Multi-objective Particle Swarm Optimization (MOPSO). Although NSGA-II has already been applied to solve Bayesian Multi-objective Reliability-Based Design Optimization problem (Srivanstava and Deb 2013), the MOPSO algorithm is applied for the first time to solve this type of problem according to the authors' knowledge. Finally, section 5 includes the conclusions.

BAYESIAN INFERENCE METHODS

This section describes how to compute the reliability for a probabilistic constraint when there exist aleatory and epistemic random variables.

We can partition the vectors of uncertain variables $\mathbf{X}$ and parameters $\mathbf{P}$ in two sub vectors: $\mathbf{X} = [\mathbf{X}_t, \mathbf{X}_s]$ and $\mathbf{P} = [\mathbf{P}_t, \mathbf{P}_s]$. The vectors $\mathbf{X}_t$ and $\mathbf{P}_t$ are aleatory variables and parameters whose probability density functions (PDFs) are known. In addition, the vectors $\mathbf{X}_s$ and $\mathbf{P}_s$ are epistemic random variables. It is assumed that only a reduced set of samples are known for these variables.

Suppose that we want to compute the reliability for the j^{th} reliability constraint, that is,

$$R_j = Pr\left[g_j(\mathbf{X}, \mathbf{P}) > 0\right] \quad (1)$$

Due to insufficient data for the epistemic random variables, R_j must be uncertain and subjective (Youn and Wang, 2006). A solution is then by modelling reliability using Bayesian inference.

Reliability, as many other outcomes in engineering applications, can be interpreted as the result of a series of trials that can be separated binary into two events: occurrence or non-occurrence, falling in the safe region or falling in the failure region. If, in addition, these events meet the requirements of being statistically independent and the probability of safe event or failure event remains constant, the probability that x reliable or safe results will occur in a total of N trials can be described by a Binomial distribution. Then, if the probability of a safe event is r and the probability of a failure event is $(1-r)$, the probability of x safe events out of a total of *N* trials can be expressed as:

$$Pr(X=x,N|r)=\binom{N}{x}r^x(1-r)^{N-x}. \quad x=0,1,2,\dots,N \tag{2}$$

This is the probability mass function of a Binomial distribution and r is the parameter of this distribution. Considering r as an uncertain parameter and assigning a prior distribution for r, our knowledge of the distribution of r is updated based on the outcomes of the trials (samples), using Bayes' Rule for continuous distributions:

$$f(r|x)=\frac{f(x|r)f(r)}{\int_0^1 f(x|r)f(r)dr} \tag{3}$$

where $f(r)$ is the priori distribution of r, $f(x|r)$ is the likelihood of x for a given r and $f(r|x)$ is the posteriori distribution of r. In this paper, the role of r is the reliability of the j^{th} constraint, called R_j. A priori distribution for the reliability of this performance function is required. If no previous information is available about this reliability, non-informative priori can be considered. We assume that reliability R_j follows a uniform distribution, $U(0,1)$. It is well known that $U(0,1)$ distribution can be viewed as a $Beta(1,1)$ distribution, one of the possible forms of $Beta$ distribution. We compute the reliability for each sample of epistemic variables and parameters as follows:

$$Pr\left[g_j(\mathbf{X}_t, \mathbf{P}_t) > 0 | (\mathbf{X}_s, \mathbf{P}_s)_k\right] \text{ with } k = 1, .., N, \tag{4}$$

where N is the sample size and $g_j(\mathbf{X}, \mathbf{P}) \leq 0$ is the failure region. Repeating this computation for the N samples, we can compute the expected value of the reliability for the j^{th} constraint, $E_j(r)$. The values $E_j(r)$ and N are the parameters of the Binomial likelihood. The expected value $E_j(r)$ of safety realizations for the j^{th} reliability constraint is computed according Srivastava and Deb (2013) as:

$$E_j(r) = \sum_{k=1}^{N} Pr\left[g_j(\mathbf{X}_t, \mathbf{P}_t) > 0 | (\boldsymbol{X}_s, \mathbf{P}_s)_k\right]. \tag{5}$$

The priori distribution is updated with the information given by the likelihood to produce the posteriori distribution of R_j. In summary, the priori distribution is a $Beta(1,1)$ and the likelihood is a Binomial, that is, the Beta-Binomial model. Therefore, the posteriori distribution is a $Beta(\alpha, \beta)$ distribution with parameters α and β, where $\alpha = E_j(r) + 1$ and $\beta = N - E_j(r) + 1$. That is:

$$R_j \sim Beta\left(r_j, E_j(r) + 1, N - E_j(r) + 1\right) \tag{6}$$

This posteriori distribution can be updated every time new samples become available. In this Bayesian framework, R_j is represented by a Beta distribution and not by a crisp value such as in reliability analysis with complete information. There is not enough information to make a precise statement about the reliability of a design. An additional measure, called confidence, is required to decide if a design $\boldsymbol{\mu_X}$ can be considered to satisfy the reliability requirements. The confidence for a design μ_X with respect to j^{th}

reliability constraint is defined as the probability that the probabilistic distribution R_j, will exceed the target reliability, R_j^{target}.

$$\zeta_j(\boldsymbol{\mu_X}) = Pr\left[g_j(\mathbf{X}_t, \mathbf{P}_t) > 0\big|_{\mu_X} \geq R_j^{target}\right] \quad \text{with} \quad j = 1,..,J \tag{7}$$

A $\zeta_j = 0$ means that the design is certainly not reliable, while a $\zeta_j = 1$ means that the design certainly meets or exceeds the target. Since R_j follows a Beta distribution, the confidence can also be written as $\zeta_j(\mu_X) = 1 - \Phi_{Beta_j}(R_j)$, where $\Phi_{Beta_j}(\cdot)$ is the Cumulative Distribution Function (CDF) of the Beta distribution for the j^{th} constraint, with $j = 1,..,J$.

In the case of complete information for uncertain variables and parameters, RBDO methods seek a single design with the best objective value and with reliability greater that o equal to R_j^{target}. In the case on incomplete information, the constraint satisfaction $R_j \geq R_j^{target}$ is only known as a probability.

BAYESIAN RELIBILITY BASED DESIGN OPTIMIZATION

Conventional problem in Reliability-Based Design Optimization (RBDO) consists of computing a design that minimises a cost function subject to reliability constraints. These constraints are formulated as component level reliabilities or as a system-level reliability. Complete information for the uncertainties is considered in conventional RBDO.

Gunawan and Papalambros (2006) proposed to apply Bayesian inference in design optimization under incomplete information. They defined a quantity called the overall confidence of a design, $\zeta_s(\boldsymbol{\mu_X})$. For simplicity, they considered the minimum of all ζ_j as this overall confidence, $\zeta_s(\boldsymbol{\mu_X})$, and proposed a multiobjective problem to solve the reliability-based optimization under incomplete information, given values of R_j^{target}, $j = 1, .., J$:

$$\begin{aligned} &\min_{\boldsymbol{\mu_X}} f(\boldsymbol{\mu_X}, \boldsymbol{\mu_P}) \\ &\max_{\boldsymbol{\mu_X}} \zeta_s(\boldsymbol{\mu_X}) \\ &s.t.\ 0 \leq \zeta_s(\boldsymbol{\mu_X}) \leq 1 \end{aligned} \tag{8}$$

Solving this problem will in general result in a set of Pareto optima instead of a single value. We cannot find a single true optimal-reliable design, that is, the design that corresponds to complete information. Rather, we obtain a set of designs with different values of confidence for a given value of R_j^{target}.

Gunawan and Papalambros (2006) have also shown that the relation between the maximum attainable confidence, ζ_s^{max}, of the Pareto front, the number of samples, N and target reliability, R, with $R_j^{target} = R$, for all $j = 1, \ldots, J$, is:

$$\zeta_s^{max} = 1 - R^{N+1} \tag{9}$$

This relation is a valuable information for a decision maker and establishes a trade-off about how much confidence can be achieved increasing the sample size of epistemic random variables, which increases the tests cost, and by relaxing the reliability target, which could provide designs with less quality.

To make the design optimization under incomplete information more pragmatic, the designer or decision maker is asked to fix a confidence level he or she desires in the design (for example, 0.90 or 0.80). Then, the value of the reliability corresponding to that confidence level can be computed from the reliability distribution estimated by Bayesian inference or other confidence-based reliability assessment method like bootstrapping selection (Moon *et al*, 2018). Here, Bayesian inference has been applied and reliability can also be written in terms of the confidence:

$$R_j(\boldsymbol{\mu_X}) = \Phi^{-1}_{Beta_j}\left(1 - \zeta_j(\boldsymbol{\mu_X})\right) \quad (10)$$

and the probability of failure corresponding to a specified confidence level is:

$$P_{f_j}(\boldsymbol{\mu_X}) = 1 - \Phi^{-1}_{Beta_j}\left(1 - \zeta_j(\boldsymbol{\mu_X})\right) \quad (11)$$

$P_{f_j}(\boldsymbol{\mu_X})$ is not the true probability of failure for the design, $\boldsymbol{\mu_X}$. Rather it is the value of probability corresponding to the confidence-based reliability $R_j(\boldsymbol{\mu_X})$.

A practical formulation of a confidence-RBDO problem consists of minimizing an objective function subject to constraints about the value $P_{f_j}(\boldsymbol{\mu_X})$ for $j = 1, \dots, J$. This formulation is known as Bayesian RBDO and is:

$$\begin{gathered} \min_{\mathbf{d},\boldsymbol{\mu_X}} Cost(\mathbf{d}, \boldsymbol{\mu_X}, \boldsymbol{\mu_P}) \\ s.t.\ P_{f_j}(\boldsymbol{\mu_X}) \le P_{f_{j,target}} \qquad j = 1..J \\ h_k(\boldsymbol{d}) \ge 0,\ \ k = 1,2, \dots, K \\ \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \boldsymbol{\mu}_X^L \le \boldsymbol{\mu_X} \le \boldsymbol{\mu}_X^U \end{gathered} \quad (12)$$

where $P_{f_j} = P\big(G_j(\mathbf{d}, \mathbf{X}, \mathbf{P}\ ) \le 0\big)$ and $\mathbf{X} = [\mathbf{X}_t, \mathbf{X}_s]$ and $\mathbf{P} = [\mathbf{P}_t, \mathbf{P}_s]$ and $G_j(\mathbf{d}, \mathbf{X}, \mathbf{P}\ ) \le 0$ is defined as the failure region. J represents the number of reliability constraints and K, the number of deterministic constraints. As stated in equation (11), $P_{f_j}(\mu_{\boldsymbol{X}})$ is computed for each design and depends on the confidence level requested by the designer. Methods to solve this problem depend on the approaches used to solve the optimization and the reliability analysis. When the sample size increases, the optimum design tends to the “exact” optimum obtained by RBDO under complete information.

BAYESIAN MULTI-OBJECTIVE RELIABILITY-BASED DESIGN OPTIMIZATION

In realistic practice, designers and decision makers prefer to know the various optimal designs for different values of probability of failure, for a determined confidence level established previously (Ben-Haim 2006). Therefore, after setting the level of confidence, a set of optimal solutions can be established, in which there is a compromise between cost and reliability. The set of optimal solutions form the so-called Pareto front which helps the selection of a design in a more practical way. Thus, the designer can see how much the cost increases if more reliable design is required. Similarly to the Bayesian RBDO, an enormous sample size might be required to verify a very low value of probability of failure with a high value of confidence.

The formulation of the MORBDO problem is:

$$\begin{gathered}\min_{\mathbf{d},\boldsymbol{\mu}_{\boldsymbol{X}}}\left[Cost\left(\mathbf{d},\boldsymbol{\mu}_{\boldsymbol{X}},\boldsymbol{\mu}_{\boldsymbol{P}}\right),\ P_{f_S}(\mathbf{d},\boldsymbol{X},\boldsymbol{P})\right]\\ s.t.\ P_{fs}^{l}\leq P_{f_S}<P_{fs}^{u}\\ h_k(\boldsymbol{d})\geq 0,\ k=1,2,\dots,K\\ \mathbf{d}^L\leq\mathbf{d}\leq\mathbf{d}^U,\boldsymbol{\mu}_{\boldsymbol{X}}^{L}\leq\boldsymbol{\mu}_{\boldsymbol{X}}\leq\boldsymbol{\mu}_{\boldsymbol{X}}^{U}\end{gathered} \tag{13}$$

where, $\mathbf{d}$, is the vector of deterministic design variables, $\boldsymbol{\mu}_{\boldsymbol{X}}$ is the vector of uncertain design variables. P_{f_S} is the probability of system failure for the confidence level established by the designer. $\boldsymbol{\mu}_{\boldsymbol{X}}^{L}$ and $\boldsymbol{\mu}_{\boldsymbol{X}}^{U}$ are lower and upper bounds for the mean values of uncertain design variables. P_{fs}^{l} and P_{fs}^{u} are bounds for the probability of system failure. All reliability constraints are combined in a unique system reliability constraint to formulate a bi-objective optimization problem. P_{f_S} is computed as: $P_{f_S}=1-R_S$, where R_S is the reliability of the system. In this work, R_S has been computed as the minimum of the values of the reliabilities of the constraints. That is,

$$R_S=\min_{j=1,..,n_r}R_j \tag{14}$$

where R_j, is the reliability of constraint j^{th} for the confidence level given by the designer and J is the number of reliability constraints. However, more accurate values can be computed considering the configuration of the system (serial, parallel, mixed, etc) and taking into account the correlation between different failure modes (Patelli *et al.* 2011).

The best methods to solve the optimization phase of MORBDO problems are based in multi-objective evolutionary algorithms. In this work, we propose the use of Multi-objective Particle Swarm Optimization (MOPSO) (Coello *et al.* 2004). The results are compared against the results obtained with Non-dominated Sorting Genetic Algorithm (NSGA-II) (Srivastava and Deb 2013).

MOPSO is the multi-objective version of the approach called Pareto Swarm Optimization (PSO) (Kennedy and Eberhart 2001). PSO is a population-based metaheuristic algorithm inspired in the social behaviour of birds within a flock. In PSO each member of the population of potential solutions is named particle and the population of potential solutions is named swarm. The main goal of MOPSO, like other evolutionary algorithms, is to obtain a set of different solutions, called Pareto optimal set and a representation of the values of the objective functions for this optimal set of solutions, called Pareto front. In MOPSO each member of the population of potential solutions is named particle and the population of potential solutions is named swarm. Each particle is featured by a position vector and a velocity vector randomly generated in the first iteration. Each i^{th} particle updates its position at the generation $t+1$ through the formula

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \mathbf{v}_i^{t+1} \tag{15}$$

where $\mathbf{v}_i^{t+1}$ is known as velocity and it is given by:

$$\mathbf{v}_i^{t+1} = \omega \mathbf{v}_i^t + c_1 s_1 \left[\mathbf{x}_{pbest_i}^t - \mathbf{x}_i^t\right] + c_2 s_2 \left[\mathbf{x}_{gbest_i}^t - \mathbf{x}_i^t\right] \tag{16}$$

In these formulas, $\mathbf{v}_i^t$ and $\mathbf{x}_i^t$ represent the current velocity and position of the i^{th} particle in a d −dimensional search space, respectively. s_1 and s_2 are two uniformly distributed random numbers in the range $[0,1]$. $\mathbf{x}_{pbest_i}^t$ is the best position that i^{th} particle has been along its path and $\mathbf{x}_{gbest_i}^t$ is the global best position that the entire swarm has viewed. The parameter ω is the inertia weight, and c_1 and c_2 are positive constants called acceleration constants, which control the effect of the personal and global best particles.

MOPSO, unlike others evolutionary algorithms, has a secondary archive where the no dominated optima, that is, Pareto optima, are recorded. While this secondary archive is not full, all non-dominated solutions with respect to the archived solutions can be entered. After some generations, when the archive is full, if a solution dominates any solution in the archive then this solution can enter the archive and the dominated solution is deleted. The archive works as a grid with subdivisions that helps to redistribute the solutions and to obtain a more uniformly distributed front in the objective functions space. In addition, MOPSO implements a mutation operator and a simple scheme to handle constraints. Figure 2 shows a flowchart of MOPSO algorithm.

A reliability analysis needs to be performed for each candidate solution proposed by the evolutionary optimisation tools. In practical applications 1000 - 10000 reliability analyses are generally required. This makes the adoption of simulation approaches impractical although some recent advanced methods such as Line Sampling (de Angelis *et al.* 2015) requires only a very small number of samples. Therefore, the reliability analysis is carried out by the FORM (First Order Reliability Method), a gradient-based iterative method, to reduce the computational cost of the analysis. A maximum number of allowed iterations equal to 100 was established. It is well known that FORM cannot converge to the most probable point or design point with highly non-linear performance functions or when random variables are non-normally distributed. Therefore, the proposed approach needs to take this eventuality into account.

In the case of non-convergence, the proposed design is discarded from the optimisation method. This is obtained by penalising the proposed solution by assigning a very large numerical values to the objective functions associated with the candidate solution (individual in NSGAII or particle in MOPSO). This penalisation was not necessary to activate in the numerical examples described in the next section since all reliability analyses converged in a few iterations. However, this strategy could be useful for highly non-linear performance functions: for example, when mechanical and geometric non-linearities are considered in the structural analysis and gradients of performance functions are more difficult to compute.

Figure 3 shows a flowchart of this algorithm where the reliability of each performance function, given by eq. (4), is computed by FORM for each combination of candidate solution and sample from the epistemic uncertainty. Then, when all reliabilities are computed for all the combinations based in a candidate solution the parameters of the beta distribution (i.e., posterior distribution) are updated using Bayesian rule. Finally, the probability of failure is computed using eq. (11).

It is important to note that optimization algorithms could also be themselves a source of uncertainty (Su *et al.* 2016). The performance of the evolutionary algorithms NSGAII and MOPSO depends on several tuning parameters and settings of these algorithms: mutation, crossover, crowding, sorting, constraint handling, etc. In addition, the evolution of the population is based on some stochastic process leading to an additional source of uncertainty in the solution process. Thus, the application of several MOEAs permits to compare the results and to detect the uncertainty caused by stochastic optimization algorithm. The Bayesian-MORBDO strategy has been implemented in algorithms using the MATLAB language. The multi-objective optimization part has been implemented enhancing available libraries such as the NSGA-II (Tamilselvi 2020) and the MOPSO algorithm (Martínez-Cagigal 2020). The deterministic analysis of the model has been carried out by the software OpenSees (Mazzoni *et al.* 2006).

CASE STUDIES

1.- ANALYTICAL EXAMPLE

The first example, adapted from (Youn and Wang, 2006), considers two objective functions, two design variables and one epistemic variable. This simple problem permits a graphical representation of the Pareto front in the objective function space and the Pareto set in the space of design variables.

The formulation of the Bayesian MORBDO problem is:

$$\begin{aligned} \min_{\boldsymbol{\mu_X}} \boldsymbol{f}(\boldsymbol{\mu_X}) = \left[f_1(\boldsymbol{\mu_X}) = \mu_{X_1} + \mu_{X_2}, f_2(\boldsymbol{\mu_X}) = P_{f_S}(X_1, X_2, X_3) \right] \\ s.t. \quad 0.0001 \leq P_{fs} \leq 0.1 \\ 0 \leq \mu_{X_1} \leq 10\ ;\ 0 \leq \mu_{X_2} \leq 10 \end{aligned} \tag{17}$$

The first objective function is the cost function while the second objective function represents the probability of the system failure computed as $P_{f_S} = 1 - R_S$. R_S is the reliability of the system. Lower and upper bounds for P_{f_S} are set as 0.0001 and 0.1, respectively, to obtain the optimal design and the Pareto front in the range of interest for the designer. The performance functions of the system are defined as:

$$\begin{gathered} g_1(\boldsymbol{X}) = X_1^2 X_2 X_3/20 - 1 \\ g_2(\boldsymbol{X}) = \frac{(X_1+X_2+X_3-5)^2}{30} + \frac{(X_1-X_2-X_3-12)^2}{120} - 1 \\ g_3(\boldsymbol{X}) = 80/(X_1^2 + 8X_2X_3 + 5) - 1 \end{gathered} \tag{18}$$

In this example, there are two aleatory design variables: X_1 and X_2 distributed according a normal distribution, that is, $X_1 \sim N\left(\mu_{X_1}, CoV = 0.12\right)$ and $X_2 \sim N\left(\mu_{X_2}, CoV = 0.12\right)$. The third variable, X_3, is a parameter with epistemic uncertainty and it is assumed that only a small sample size is available. X_3 is not considered a design variable since none of its statistical properties are known. The samples for X_3 are randomly generated from a (unknown) normal distribution with $X_3 \sim N\left(\mu_{X_3} = 1.0, \sigma_{X_3} = 0.1\right)$. That is, we have sampled a normal distribution as a way to easily obtain samples, however, these samples can come from databases or experimental test in realistic practice. Note that the information about the underlying distribution of the epistemic variable X_3 is not known to the analysist.

The problem is solved using the NSGA-II and MOPSO algorithms, respectively. The problem has been solved for different sample size of the available information about the epistemic uncertainty, i.e. $(N_S = 50, 100, 200)$ and different confidence levels $\left(\zeta_j = 0.8, 0.9\right)$. The population size in NSGA-II and the swarm size in MOPSO are set equal to 100. The number of generations (evolutions) is also set to 100 for both optimizers. As reference solution, the MORBDO problem is also solved considering the availability of a complete information. Figures 4 and 5 show the Pareto fronts in the space of objective functions for different sample sizes, as well for the case of complete information. The figures show how the Bayesian fronts are getting closer to the Pareto front obtained with complete information as the number of samples increases. Given specified confidence level and a limited sample size for the epistemic uncertainty, X_3, there is a minimum value of the probability of system failure that can be achieved. This is a consequence from the relation given in eq. (9).

Table 1 shows the minimum values for the probability of system failure that can be found applying Bayesian MORBDO to the analytical example for various values of sample sizes and confidence levels. These minimum values decrease when sample size increases. Also, they decrease when confidence decreases. Pareto fronts computed with samples sizes equal to 50, 100 and 200 provide, for any cost value, probability of failure values greater than that given by the Pareto front determined with true probabilities of failure. The Bayesian MORBDO algorithm proposed would work improperly if a Pareto front contained a design with a value of probability of failure less than the true probability of failure given by the Pareto front with complete information. If this happened, it could lead to a dangerous design decision by overestimating true reliability. However, this has not happened in any of the runs developed and the Bayesian MORBDO algorithm has provided appropriately conservative Pareto fronts. The true lower bound of probability of failure is attained when complete information is available, that is, when the probability distributions of all uncertain variables are known. Differences between NSGA-II and MOPSO results are practically negligible.

2.- TEN BAR TRUSS EXAMPLE

The second case study considers the minimisation of the amount of steel used in a ten bars truss structure (shown in figure 6) and the minimisation of its probability of failure. The formulation of the Bayesian MORBDO is as follow:

$$
\begin{gathered}
\min_{\mathbf{d},\boldsymbol{\mu}_X}\left[Volume(\mathbf{d},\boldsymbol{\mu}_{\boldsymbol{X}},\boldsymbol{\mu}_{\boldsymbol{P}}),P_{f_S}(\mathbf{d},\mathbf{X},\mathbf{P})\ \right] \\
s.t.\quad 0.001\leq P_{f_{SYS}}\leq 0.1 \\
4\ cm^2\leq \mu_{\mathrm{X}_j}\leq 75\ cm^2,\ \ j=1,2,3 \\
G_1(\mathbf{d},\mathbf{X},\mathbf{P})=1-\left|q_{2,V}(\mathbf{d},\mathbf{X},\mathbf{P})\right|/q^a
\end{gathered} \tag{19}
$$

The first objective function represents the volume of the steel required by the truss. As the steel density is a constant, the minimization of the steel volume is equivalent to the minimization of the steel mass. The second objective function represents the probability of system failure. Bounds are stated for the probability of system failure to find design solutions in the range of interest. Only one displacement constraint is imposed: the vertical displacement of the node 2 must be below 2 cm. That is, the value of the allowable displacement q^a is equal to 2 cm.

The bars of the truss structure are considered in three groups. Group 1 contains horizontal bars; group 2 contains vertical bars and group 3 contains diagonal bars. Bars in the same group have the same cross-sectional area. The mean values of the cross-sectional areas are the design variables of the problem: $\mu_{\mathrm{X}_1},\mu_{\mathrm{X}_2},\mu_{\mathrm{X}_3}$. Therefore, three normal aleatory design variables are assigned to these areas. Two loads are applied to the truss structure in nodes 1, 2 and 4: loads P_1 and P_2. Such loads are assumed to be normally distributed. The elastic modulus E of the bars is an epistemic random parameter with only limited samples available. The samples have been obtained from a normal distribution, $N(\mu=21000\ kN/cm^2,\ \sigma=210\ kN/cm^2)$, as a way to easily obtain samples, however, these samples can come from databases or experimental tests, or simply generated sampling other probability distribution. Note that the information about the underlying distribution of the epistemic variable E is not known to the analysist. The uncertain variables and parameters of the problem are shown in the Table 2. Similarly to the previous example, for small sample sizes and a large value of confidence (0.9), no matter how much the cross section increases, there is a limit of the value of failure probability for a given confidence level that can be reached. This fact is of great importance for the decision maker since it means that a higher cost does not imply greater reliability. Pareto fronts computed with two samples sizes for the epistemic uncertainty, $N = 200$ and $N = 400$ are shown in figure 7 and 8 for NSGA-II and MOPSO, respectively. These fronts become a horizontal line determining the minimum achievable failure probability values. Table 3 shows the minimum values for the probability of system failure that can be found applying Bayesian MORBDO to the ten bars truss example for various values of sample sizes and with a confidence of 0.9. It can be verified that the greater the number of samples, the closer the Pareto front is to the front for the case of complete information for uncertain variables.

From the point of view of computational efficiency, the average execution times of 5 optimisations has been obtained for the different cases considered. A computer with a processor Intel Core i7 – 7500U with 2.90 GHz has been used to perform the analysis. The average runtime for the case with 400 samples was of 153.6 minutes for NSGA II and 102.7 minutes for MOPSO, respectively. Hence, the NSGA II took 50% more time than MOPSO. With 200 samples, the average runtime was 80 minutes for NSGAII and around 60 minutes for MOPSO. Again, NSGA II took 33% more time than MOPSO. Therefore, MOPSO was significantly superior in computational efficiency than NSGA to solve the Bayesian MORBDO problem. However, in the case of complete information available for the uncertain variables, NSGAII was more efficient than MOPSO, 1.9 minutes vs 2.7 minutes.

3.- POWER TRANSMISSION TOWER

The third case study considers a 3D model of a power transmission tower depicted in Figures 9 and 10. The tower height is 16.15 m and supports a power transmission line with single circuit of 66 kV and a ground wire at the top joint. The structure consists of 218 truss elements and 55 nodes. This structural example has been considered previously as a benchmark to apply RDO methods (Lagaros *et al.* 2005, Plevris *et al*. 2005a, Plevris *et al.* 2005b). Loads on conductors and ground wire caused by self-weight, ice and wind are transmitted as nodal loads on joints at the ends of cross-arms and at the top joint, where these cables are attached.

The formulation of the Bayesian MORBDO problem for this example is written below:

$$\begin{gathered}\min_{\mathbf{d},\boldsymbol{\mu}_X}\left[Volume(\mathbf{d},\boldsymbol{\mu}_X,\boldsymbol{\mu}_P),\ P_{f_S}(\mathbf{d},\mathbf{X},\mathbf{P})\ \right]\\ s.t.\quad 0.001\le P_{f_{SYS}}\le 0.1\\ 0.83\ cm^2\le \mu_{X_j}\le 90.59\ cm^2,\ \ j=1,\dots,10\\ G_1(\mathbf{d},\mathbf{X},\mathbf{P})=1-\left|q_{Top\ Joint,X}(\mathbf{d},\mathbf{X},\mathbf{P})\right|/q^a\\ G_2(\mathbf{d},\mathbf{X},\mathbf{P})=1-\left|q_{Top\ Joint,Y}(\mathbf{d},\mathbf{X},\mathbf{P})\right|/q^a\end{gathered}\quad (20)$$

The problem is a bi-objective optimization problem where the first objective is to minimize the steel volume of the structure equivalent to the minimization of the structure weight (because the weights of joint bolts are negligible). The second objective function to minimize represents the probability of system failure. Two displacements constraints are imposed identifying the failure criteria for the power transmission tower: Horizontal displacements of the top joint in two cartesian orthogonal directions must be below the allowable displacement, q^a, equal to 10 cm. As in the previous example, two multi-objective optimization evolutionary algorithms are considered: NSGA-II and MOPSO.

This tower is modeled in OPENSEES software using truss elements, that is, all joints are pinned joints. Because that, elements are supporting axial forces. Structural analysis is linear elastic. The structure is considered as a serial system, where the probability of system failure is computed as the minimum value of the probability of failure of the constraints. The truss elements are grouped in ten groups and the bars in each group are designed with the same steel profile. Figure 9 shows these groups, representing elements of each group with different color. In addition, truss elements are labeled with the group number. Usually, power transmission towers are designed with equal angles. Mean values of the cross-sectional areas of these groups are the design variables. The range of cross-sectional areas is limited by minimum and maximum cross-sectional areas taken from the standards EN 10025-1 and EN 10025-2.

Table 4 shows the values of the parameters used in this problem. Cross-sectional areas follow normal distribution. Five random variables P_1 a P_5 are used to describe the joint loads applied at the end joints of the cross-arms. Figure 11 represents a perspective view of the tower and shows the loads applied on the nodes A, B, C, D and E. These loads are caused by conductors, ground cable self-weight and wind and they are transmitted to the towers as nodal loads. As in the previous examples, a reduced set of samples is assumed to be available for the elastic modulus of steel. Therefore, it is considered an epistemic uncertainty. Since experimental tensile tests have not been done on specimens of the type of steel used in the tower due to budgetary reasons, a set of 50 numerical experimental samples has been randomly generated by sampling a normal distribution $N(\mu = 21000\ kN/cm^2,\ \sigma = 210\ kN/cm^2)$. However, these numerical samples could have been generated other probability distribution. The information about the underlying distribution of the epistemic variable E is not known to the analysist.

The Bayesian MORBDO problem is solved for a value of confidence equal to 0.9. Figures 12 and 13 show the Pareto fronts in the objective space computed using NSGA-II and MOPSO, respectively. The two Pareto fronts represent the case with only 50 samples available for the elastic modulus and the case with complete information. The results have been obtained using a population sizes and maximum number of generations of 50 and 100 for the cases of epistemic uncertainty and complete information, respectively. The minimum achievable probabilities of system failure are shown in Table 5. These values are practically the same in both MOEAs. With a set of 50 samples for the epistemic uncertainty, the minimum value of estimated probability of system failure given a confidence of 0.9 is 0.048299, a very conservative value compared to the true probability of failure. However, this true probability of failure is unknown to the designer.

The structural model of the 3D transmission tower is much more complex respect the previous examples. The model has 218 truss elements and 55 nodes and each call to the finite element analysis software requires significant more runtime. The average runtime obtained repeating the analysis 5 time for the case of 50 is 18 hours and 15 minutes with NSGA-II and 18 hours using MOPSO. In the case of complete information, the average runtime is 2 hours with NSGA-II and 2 hours and 50 minutes with MOPSO. All runs were carried out in a computer with a processor Intel Core i7 – 7500U with 2.90 GHz.

CONCLUSIONS

A Bayesian Multi-objective Reliability Based Design Optimization (MORBDO) is proposed to optimize the structural design under mixed representation of uncertainties. The proposed approach allows to deal with the practical problem of limited sample size available to characterize the uncertainty. It provides a set of optimal designs for different values of estimated probabilities of failures given a confidence level specified by the designer. Bayesian inference is proposed as an efficient approach for estimating the reliability of the system considering a non-informative prior. More research will be carried out by the author about advanced methods to compute less conservative estimates of confidence-based probabilities of failure tackling mixed aleatory/epistemic uncertainties in order to obtain Pareto fronts closer to the Pareto front computed considering complete information.

The proposed approach requires affordable computational costs that can be further reduced by replacing the structural model with surrogate model. However, the use of surrogate model would introduce another type of uncertainty, known as data uncertainty, caused by the lack of training data.

Three examples have been presented to demonstrate the applicability of the proposed strategy for the identification of the Pareto fronts. Due to the epistemic uncertainty, the feasible design options are obtained with a minimum level of probability of failure that depends on the number of samples available to characterize the epistemic uncertainty. The results emphasize the importance of the use of an adequate quality control of the material and the necessary of having complete probabilistic information of the properties of the material in order to obtain optimal reliable designs and avoid obtaining unealistic levels of probability of failure. Thus, increasing the confidence and trust of the design approach.

NOMENCLATURE

CDF	Cumulative Distribution Function
CoV	coefficient of variation
FORM	First Order Reliability Method
MOEA	Multi-objective Evolutionary Algorithm
MOPSO	Multi-Objective Particle Swarm Optimization
MORBDO	Multi-objective Reliability-based Design Optimization
NSGA	Non-dominated Sorting Genetic Algorithm
PDF	Probability Density Function
RBDO	Reliability-based Design Optimization
RBRDO	Reliability-based Robust Design Optimization
RDO	Robust Design Optimization
SORM	Second Order Reliability Method

NOTATION

The following symbols are used in this paper:

c_1, c_2	acceleration constants
$\mathbf{d}$	vector of deterministic design variables
$\mathbf{d}^L, \mathbf{d}^U$	vector of lower and upper bounds for deterministic design variables
$E_j(r)$	expected number of safety realizations
$g_j(\mathbf{X}, \mathbf{P})$	j^{th} performance function
h_k	k^{th} deterministic constraint
J	number of reliability constraints
K	number of deterministic constraints

N_s	number of samples
$\mathbf{P} = [\mathbf{P}_t, \mathbf{P}_s]$	vector of random parameters
$\mathbf{P}_s$	vector of epistemic random parameters
$\mathbf{P}_t$	vector of aleatory random parameters
P_{f_j}	probability of failure for the j^{th} probabilistic constraint
$P_{f_{j,target}}$	target probability of failure for the j^{th} probabilistic constraint
P_{f_s}	probability of failure of the system
q^a	allowable displacement
r_j	number of safety realizations for the j^{th} probabilistic constraint
R_j	reliability of the j^{th} probabilistic constraint
R_j^{target}	target reliability of the j^{th} probabilistic constraint
R_s	reliability of the system
s_1, s_2	uniformly distributed random numbers in the range [0, 1]
$\mathbf{v}_i^t$	velocity vector for the i-th particle of the swarm
$\mathbf{x}_i^t$	position vector for the i-th particle of the swarm
$\mathbf{x}_{pbest_i}^t$	best position for the i-th particle along its path
$\mathbf{x}_{gbest_i}^t$	global best position of all the particles of the swarm
$\mathbf{X} = [\mathbf{X}_t, \mathbf{X}_s]$	vector or random variables
$\mathbf{X}_t$	vector of aleatory uncertain variables
$\mathbf{X}_s$	vector of epistemic uncertain variables
α	parameter of the Beta distribution
β	parameter of the Beta distribution
$\zeta_j(\boldsymbol{\mu}_{\boldsymbol{X}})$	confidence for a design
$\boldsymbol{\mu}_{\boldsymbol{P}}$	vector of mean values of random parameters
$\boldsymbol{\mu}_{\boldsymbol{X}}$	vector of uncertain design variables
$\boldsymbol{\mu}_{\boldsymbol{X}}^L, \boldsymbol{\mu}_{\boldsymbol{X}}^U$	vector of lower and upper bounds for uncertain design variables
$\Phi_{Beta_j}(\cdot)$	cumulative distribution function (CDF) of the j^{th} Beta distribution
ω	inertia weight to update position vector

DATA AVAILABILITY STATEMENTS

Some or all data, models, or code that support the findings of this study are available from corresponding author upon reasonable request.

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Table 1. Values of minimal probability of system failure searchable for the analytical example.

	Confidence = 0,9		Confidence = 0,8	
Sample Size	**NSGA-II**	**MOPSO**	**NSGA-II**	**MOPSO**
$N_s = 50$	0.044394	0.044466	0.031446	0.031430
$N_s = 100$	0.022820	0.022832	0.016024	0.016044
$N_s = 200$	0.011707	0.011677	0.008210	0.008239
COMPLETE INF.	0.000158	0.000170	0.000158	0.000170

Table 2. Uncertain variables in the ten bars truss example

Random variable	Distribution	Mean Value	CoV (%) or σ^2
$X_1 \equiv A_1$	N	μ_{X_1}	5%
$X_2 \equiv A_2$	N	μ_{X_2}	5%
$X_3 \equiv A_3$	N	μ_{X_3}	5%
$X_4 \equiv P_1$	N	100.0 kN	20 kN
$X_5 \equiv P_2$	N	50.0 kN	2.5 kN
E	$Epistemic$		

Table 3. Values of minimum achievable probability of system failure for the ten bars truss example

Sample Size	NSGA-II	MOPSO
$N_s = 200$ (confidence 0.9)	0.011390	0.011390
$N_s = 400$ (confidence 0.9)	0.005725	0.005765
COMPLETE INF.	0.00100	0.00100

Table 4. Uncertain variables in the power transmission tower example

Random variable	Distribution	Mean Value	CoV (%) or σ^2
$A_1, \dots, A_{10}$	$Normal$	$\mu_{X_1}, \dots, \mu_{X_{10}}$	5 %
P_1	$Lognormal$	8.51 kN	10 %
P_2	$Lognormal$	9.77 kN	10 %
P_3	$Lognormal$	10.70 kN	10 %
P_4	$Lognormal$	4.82 kN	10 %
P_5	$Lognormal$	5.36 kN	10 %
E	$Epistemic$		

Table 5. Values of minimum achievable probability of system failure for the 3D power transmission tower

Sample Size	NSGA-II	MOPSO
$N_s = 50$ (confidence 0.9)	0.048635	0.048299
COMPLETE INF.	0.002983	0.002631