Taming the Housing Crisis: An LTV Macroprudential Policy

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Abstract

This paper develops a DSGE framework featuring heterogeneous housing markets, endogenous mortgage defaults, and a banking sector. We find that the idiosyncratic mortgage risk shock plays an important role in explaining the fluctuations of house prices during the 1980s and the years leading up to the financial crisis. The same shock is also an important driving force of household loans. By placing an occasionally binding constraint on the loan-to-value ratio via a counterfactual analysis, we find that the overheating of the housing economy in the early 2000s and the subsequent crash could have been alleviated, if authorities had adopted such a macroprudential policy measure. A welfare comparison suggests that such a maximum loan-to-value ratio policy is preferable over an augmented Taylor rule that responds to house price growth.

Keywords: DSGE Model, Endogenous Default, Housing, Macroprudential Policy, Loan-to-Value

JEL Classification: E32, E44, E58, G21

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1 Introduction

As we know today, the roots of the financial crisis can be traced back to the U.S. housing market. Increasing property prices and a sharp rise in household borrowing characterized the years leading up to the crisis. Eventually, the burst of the housing bubble was followed by a large wave of mortgage defaults and a severe contraction of economic activity. Figure 1 illustrates the dramatic expansion of household debt in the early 2000s and the rise of the delinquency rate following the burst of the bubble.

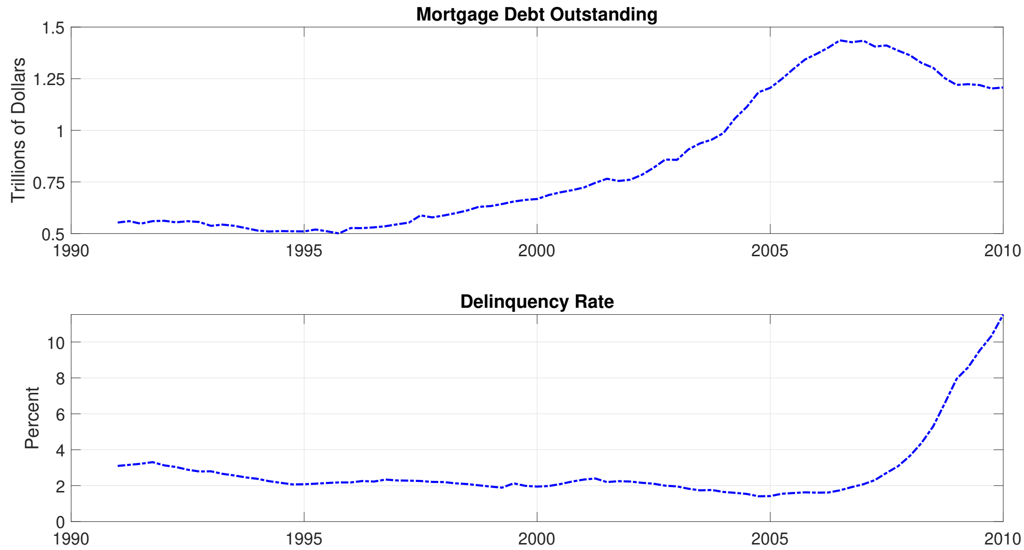

Figure 1: Top panel: Mortgage Debt Outstanding (MDOTHIOH, FRED). Bottom panel: Delinquency Rate on Single-Family Residential Mortgages (DRSFRMACBS, FRED).

Our paper therefore explores three research questions. First, to what extent are house and rental prices affected by mortgage default? Second, could the dramatic pre-crisis expansion of the housing economy and the subsequent crash have been prevented, if authorities had followed a macroprudential loan-to-value (LTV) ratio? Third, can an augmented Taylor rule outperform a preemptive LTV ratio policy in terms of its welfare effects? In order to answer these questions, we construct a comprehensive Dynamic Stochastic General Equilibrium (DSGE) framework with various frictions and features, and estimate the model with the help of Bayesian techniques on U.S. data.

The underlying framework in this study is a general equilibrium model, similar to those presented in Sun and Tsang (2017), Iacoviello (2015), and Iacoviello and Neri (2010). The economy is inhabited by three types of agents: households, entrepreneurs, and banks. There is heterogeneity in households, which is represented by impatient and patient agents. The difference between these two types of households is their discount factors. Impatient agents (net borrowers) discount the future more heavily than patient households (net savers), which creates an incentive to borrow from banks. Savers supply deposits to banks and provide borrowers with rental services. Impatient households are credit-constrained and decide between renting and owning a home. The supply side of the economy consists of two sectors: consumption and housing. Entrepreneurs face a liquidity constraint and produce the final good and new housing. The central bank sets interest rates at which banks collect deposits. Therefore, changes in the deposit rates either contract or loosen the credit supply. Banks act as an intermediary between savers and borrowers and are exposed to a capital adequacy constraint. Finally, the model allows borrowers to default endogenously on their loans. We follow the approach described in Lambertini, Nuguer, and Uysal (2017) to incorporate endogenous mortgage default into our model. In this setting, housing investment is risky. An idiosyncratic risk shock determines the ex-post value of the house and, once the shock is realized, borrowers decide whether to default on the mortgage or not based on a comparison between the value of the house and the mortgage payment.[1] To the best of our knowledge, we are the first to investigate endogenous default and macroprudential measures in a DSGE framework equipped with a banking and rental sector.

We explicitly model a banking sector due to the following reasons. First, banks were at the heart of the financial crisis and therefore played a key role in providing households with the necessary funds to fuel their consumption. Second, allowing for banks adds another layer of frictions into the model economy. As the financial sector faces a capital adequacy constraint, it will ensure that its assets side matches its liabilities. Disruptions, caused by shocks, alter the loan supply regulated by banks and therefore have an effect on the propagation mechanism of shocks, as shown by Iacoviello (2015) and Ge, Li, and Zheng (2020). For this reason, combining conventional financial frictions with banking frictions intensifies the shock responses. Iacoviello (2015) also shows, with the help of Bayesian methods, that a model with a banking sector is preferred to one without. Third, studying LTV requirements and policies hold important implications for the banking sector. Changing the LTV ratio, from a policy perspective, alters the loan supply provided by banks and affects home buyers through an either increased or reduced down payment. Thus, equipping our model with a banking sector is a crucial feature in order to analyze the implications of a maximum LTV policy. As mentioned above, we also incorporate a rental market into our model setup. Relaxing the assumption of a homogeneous housing market allows borrowers to opt for rental and owner-occupied housing. This implies important substitution effects when it comes to studying the model responses and the counterfactual analysis, as households can choose between the two housing types.

The results of the estimated model indicate that the idiosyncratic risk shock drives 17 percent of the variance of house prices and more than 20 percent of the variation in household loans over the business cycle. A historical shock decomposition reveals that most of the movements of house prices and household loans are determined by investment technology, housing technology, and intertemporal preference shocks.

[1] Note that our analysis abstracts from factors such as unemployment, income distributions, education and other socio-economic variables which also influence somebody's ability to buy a home.

The idiosyncratic risk shock increases its importance during the 1980s and the years leading up to the financial crisis. To assess the effectiveness of a preemptive LTV policy, we perform counterfactual simulations of key model variables by imposing a maximum value on the LTV ratio. This is achieved by placing an occasionally binding constraint on the LTV ratio which is now set endogenously as a byproduct of introducing mortgage default into the model. The simulation results show that house and rental prices would have been much lower in the mid-2000s. Furthermore, the subsequent sharp drop in house prices would have been less severe under the preemptive LTV policy. The same holds for the rise in household loans and residential investment, as the maximum LTV ratio causes a smaller expansion of both time series. Finally, our counterfactual welfare comparison shows that a maximum LTV ratio policy is preferable to an augmented Taylor rule which responds to house price growth.

This paper is linked to two fields in the literature, where the first consists of DSGE housing studies and the other is comprised of articles analyzing the effectiveness of macroprudential policy tools. The articles by Sun and Tsang (2017), Iacoviello (2015), and Lambertini, Nuguer, and Uysal (2017) are the most closely related to ours. Sun and Tsang (2017) develop and estimate a DSGE model to investigate how house and rental prices respond to various shocks. Their model economy is based on the Iacoviello and Neri (2010) framework and is complemented by a rental housing sector. The difference between this paper and the study by Sun and Tsang (2017) is that we explicitly model a banking sector and allow for endogenous mortgage default of borrowers. We also relax the assumption made in Iacoviello (2015) where default is characterized by an exogenous process. Lambertini, Nuguer, and Uysal (2017) estimate a DSGE model with housing and endogenous default to shed new light on the subprime crisis and the Great Recession. The paper studies how an increase in the rate of default and interest rate spreads, caused by a rise in risk in the mortgage market, can trigger a recession. Modeling endogenous default explicitly, as shown in Lambertini, Nuguer, and Uysal (2017), introduces a different transmission mechanism into the model, as opposed to the exogenous approach taken by Iacoviello (2015), where default is represented by a wealth redistribution shock to the budget constraints of entrepreneurs, bankers and borrowers. In comparison to Lambertini, Nuguer, and Uysal (2017), we relax the assumption of a homogeneous housing sector and explicitly introduce banks into our model. Also, our policy analysis focuses specifically on a maximum LTV ratio and its comparison to an augmented Taylor rule that responds to house price growth. Therefore, allowing for a rental housing market gives borrowers an alternative to owner-occupied housing which in turn holds utility and social welfare effects.

The counterfactual policy analysis of this paper shows that the dangerous pre-crisis build-up in the U.S. housing sector could have been effectively slowed down by implementing a maximum LTV rule. We pick this particular rule for two reasons. First of all, it is an effective policy tool when it comes to regulating the loan supply. The Durante et al. (2017)[2] Well-Being of U.S. Households report reveals an interesting fact that 50 percent of renters cannot afford the down payment in order to buy their own home. In other words, half of the renters questioned in the survey already fail at the first hurdle towards owning a house. However, this in turn means that adjusting the down payment (i.e. the LTV ratio), has a severe effect on the credit supply and homeownership. Second, a maximum LTV rule could have acted as an alternative safeguard in the years leading up to the financial crisis. Markets like Hong Kong and Canada have already adopted such an approach. In fact, countercyclical LTV policies in Hong Kong have been established for a relatively long time in order to cool down an overheating of the property markets. Between 1990 and 2010 the Hong Kong monetary authority stepped in and adjusted the LTV limits several times. For example, during the global crisis periods 2008 and 2009, LTV ratios of higher priced properties were lowered by 10 percent[3] in order to bring house prices down (see Funke and Paetz (2012)).

Canada is another famous example of using the LTV ratio in a macroprudential fashion. First of all, it is important to mention that loans with LTV ratios greater than 80 percent have to be insured. Mortgage insurance is provided by the government-owned Canadian Mortgage and Housing Corporation (CMHC) and two private companies. However, CMHC is by far the largest insurer with a market share of three quarters. The mortgage insurance applies to all governmentally regulated lenders and covers the entire Canadian banking sector. Banks are responsible for the majority of mortgage lending. In 2013, 74 percent of the mortgage supply originated from banks (see Krznar and Morsink (2014)). In the past and very recent years, Canadian authorities have adjusted LTV ratios in a countercyclical way. Due to the recession in 1991, and to stimulate residential investment, maximum LTV ratios of mortgages were raised in 1992 from 90 to 95 percent. This pilot project was specifically introduced for first-time home buyers. Regulations changed in 1998, which meant that mortgages with a LTV ratio of 95 percent could now be given to all home buyers within the regional price boundaries. In the years before the global financial crisis unfolded, macroprudential tools were substantially loosened. In 2006 it was decided that limits on LTV ratios were allowed to climb up to 100 percent before they were changed back to 95 percent in 2008 with the outbreak of the crisis. Therefore, Canadian authorities decreased LTV ratios and tightened the access to mortgages in response to the onset of the global financial crisis (see Allen et al. (2020)).

The policy analysis carried out in this paper is related to the macroprudential exercises performed by Funke and Paetz (2012), Lambertini, Mendicino, and Punzi (2013), Brzoza-Brzezina, Kolasa, and Makarski (2015), Bruneau, Christensen, and Meh (2016), and Ferrero, Harrison, and Nelson (2018).

[2]DCCA stands for Consumer and Community Development Research Section of the Federal Reserve Board's Division of Consumer and Community Affairs.

[3]During that time, the LTV ratio was decreased from 60 to 50 percent for properties with a market value $\geq$ HK\$ 12 million and declined from 70 to 60 percent for properties with values of HK\$ 12 million $>$ HK\$ 8 million. For more details on the history see Funke and Paetz (2012) or Wong et al. (2011).

The difference between the literature and this study, is the model environment and the methodological approach. Financial frictions arise not only at the household level but also from the banking sector. Furthermore, we combine the choice of households between renting and owning a home with the possibility of endogenous default of household borrowers. This provides us with a realistic set-up to study the effects of macroprudential policies. In order to perform our counterfactual analysis, we impose a maximum LTV ratio on the impatient households' side. Therefore, this type of agents face an occasionally binding constraint when it comes to the maximum amount they can borrow. All other constraints in the model, including entrepreneurs' borrowing constraint and bankers' capital adequacy constraint, are assumed to be binding following the literature. This is not only consistent with the evidence of a sharp increase in household borrowing in the years leading up to the financial crisis, but also keeps the model environment tractable and allows us to disentangle the effect of a maximum LTV policy.

The outline of this paper is as follows. Section 2 outlines the model economy. The data and estimation output are described in section 3. Section 4 discusses the impulse response functions, the variance and historical shock decomposition. The macroprudential policy analysis with the counterfactual analysis and welfare analysis is illustrated in section 5. Section 6 concludes.

2 The Model

The model economy is related to those outlined in Sun and Tsang (2017), Iacoviello (2015), and Iacoviello and Neri (2010). However, this paper relaxes the assumption of exogenous default. Mortgage default is introduced as outlined in Lambertini, Nuguer, and Uysal (2017). The model accommodates three different types of agents: households, banks, and entrepreneurs. Furthermore, nominal price rigidities are introduced at the retail level and a central bank sets interest rates according to a Taylor rule. Patient households transform their owner-occupied housing units into rental housing services and lease them to impatient households. Each economic agent is represented by a continuum of measure one.

2.1 Patient Households

Patient households discount at rate β_H. They choose consumption $C_{H,t}$ and housing $H_{H,t}$, and derive disutility from working. $N^c_{H,t}$ and $N^h_{H,t}$ are the hours supplied to the consumption and construction sectors. They maximize their lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta_H^t \Big\{ A_{p,t}(1-\eta)\log(C_{H,t}-\eta C_{H,t-1}) + jA_{j,t}A_{p,t}\log(H_{H,t}) - \\ - \frac{\tau}{1+\chi^H}\Big[(N^c_{H,t})^{1+\kappa^N_H} + (N^h_{H,t})^{1+\kappa^N_H}\Big]^{\frac{1+\chi^H}{1+\kappa^N_H}}\Big\}, \tag{1}$$

subject to the following budget constraint:

$$C_{H,t} + \frac{K^c_{H,t}}{A_{K,t}} + K^h_{H,t} + D_t + q_t\big\{[H_{H,t} - (1-\delta_H)H_{H,t-1}] + [H_{r,t} - (1-\delta_{Hr})H_{r,t-1}]\big\} + ac^c_{KH,t} + ac^h_{KH,t} +$$
$$+ ac_{DH,t} = \Big(R^c_{M,t}z^c_{KH,t} + \frac{1-\delta^c_{KH,t}}{A_{K,t}}\Big)K^c_{H,t-1} + (R^h_{M,t}z^h_{KH,t} + 1 - \delta^h_{KH,t})K^h_{H,t-1} + \frac{R_{H,t-1}D_{t-1}}{\pi_t} +$$
$$+ W^c_{H,t}N^c_{H,t} + W^h_{H,t}N^h_{H,t} + q_{r,t}\Omega_r H_{r,t} + DIV_t. \tag{2}$$

External habit formation in consumption is represented by the parameter η. Two shocks enter the utility function of the patient household: the intertemporal preference (or aggregate spending) shock $A_{p,t}$ and the housing demand shock $A_{j,t}$, both follow an AR(1) process. The aggregate spending shock simultaneously effects the saver's choices of consumption and housing. j determines the preference share in housing and τ stands for the labor supply parameter. The way the disutility of labor is defined (κ^N_H, $\chi^H \geq 0$) allows for less than perfect mobility between sectors. Turning to the budget constraint, savers deposit D_t and receive a predetermined gross return of $R_{H,t}$. Patient households accumulate owner-occupied housing $H_{H,t}$ and rental housing $H_{r,t}$ priced at q_t. The term DIV_t refers to the lump-sum dividends paid by retailers. $W^c_{H,t}$ and $W^h_{H,t}$ are real wages patient households earn in the consumption and housing sectors, respectively. π_t is the gross money inflation. Patient households rent capital to entrepreneurs, which is used to produce the final good and new homes. $K^c_{H,t}$ and $K^h_{H,t}$ represent therefore the capital stock in the consumption and construction sectors with their respective utilization rates of $z^c_{KH,t}$ and $z^h_{KH,t}$. $A_{K,t}$ is an AR(1) investment-specific technology shock that measures the marginal cost of producing capital used in the consumption sector. Patient agents receive a rental rate of capital denoted by $R^c_{M,t}$ and $R^h_{M,t}$. They convert their rental property into rental services Z_t, which they then lease to borrowers. This transformation process is captured by the production function $Z_t = \Omega_r H_{r,t}$. The parameter Ω_r measures the efficiency in converting rental homes into rental services. Patient households receive rental income according to $q_{r,t}\,\Omega_r H_{r,t}$ at a rental rate $q_{r,t}$. The terms $ac^c_{KH,t}$, $ac^h_{KH,t}$, and $ac_{DH,t}$ are the respective (quadratic and convex) external adjustment costs for capital and deposits. As habits, adjustment costs are assumed to be external. Owner-occupied and rental housing depreciates at rates δ_H and δ_{Hr}. The capital depreciation functions are given by $\delta^c_{KH,t}$ and $\delta^h_{KH,t}$. The exact specifications of adjustment costs, capital depreciation functions, marginal utilities, and the resulting first order conditions can be found in the appendix.

2.2 Impatient Households

Impatient households are credit-constrained and discount the future at a rate $\beta_S < \beta_H$. Furthermore, they have access to the loan market and use their housing stock $H_{S,t}$ as collateral. As a share of borrowers face the default on their mortgages, lenders pay a monitoring cost μ and seize a fraction $G_t(\bar{\omega}_t)$ of the borrowers' housing stock. The share of repaid loans to lenders is represented by the expression $1-F_t(\bar{\omega}_t)$ and $\Gamma_{t+1}(\bar{\omega}_{t+1})$ stands for the expected share of housing value, including monitoring costs, that lenders receive after default. The decision of borrower households to default on their mortgages is determined by the threshold value, $\bar{\omega}_t$, of an idiosyncratic risk shock ω_t. The exact mechanism of endogenous default is clearly outlined in the subsequent paragraphs. Impatient households maximize their lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta_S^t \Bigg\{ A_{p,t}(1-\eta)\log(C_{S,t}-\eta C_{S,t-1}) + jA_{j,t}A_{p,t}\log(\widetilde{H}_{S,t}) - \\ - \frac{\tau}{1+\chi^S}\left[(N_{S,t}^c)^{1+\kappa_S^N} + (N_{S,t}^h)^{1+\kappa_S^N}\right]^{\frac{1+\chi^S}{1+\kappa_S^N}} \Bigg\}, \tag{3}$$

where

$$\widetilde{H}_{S,t} = \left[\theta_S^{1/\kappa_s}(H_{S,t})^{\frac{\kappa_S-1}{\kappa_S}} + (1-\theta_S)^{1/\kappa_s}(Z_t)^{\frac{\kappa_S-1}{\kappa_S}}\right]^{\frac{\kappa_S}{\kappa_S-1}},$$

subject to

$$C_{S,t} + q_{r,t}Z_t + q_tH_{S,t} + \frac{R_{S,t-1}L_{S,t-1}}{\pi_t} + ac_{SS,t} = L_{S,t} + (1-\delta_H)[1-\mu G_t(\bar{\omega}_t)]q_tH_{S,t-1} + \\ + W_{S,t}^cN_{S,t}^c + W_{S,t}^hN_{S,t}^h. \tag{4}$$

The CES housing aggregator $\widetilde{H}_{S,t}$ in the borrower's utility function captures the assumption that owner-occupied and rental homes are substitutes. In other words, the impatient agent's demand for housing is a composite index consisting of owner-occupied and rental housing. The constant elasticity of substitution between both housing types is represented by the parameter κ_S. θ_S is the preference share of mortgaged housing, and $1-\theta_S$ the weight on rental services. $N_{S,t}^c$ and $N_{S,t}^h$ are hours supplied to the consumption and construction sectors. The terms $W_{S,t}^c$ and $W_{s,t}^h$ are real wages impatient households earn in the consumption and housing sectors, respectively. The expenditure side of the budget constraint includes consumption $C_{S,t}$, the accumulation of mortgaged housing $H_{S,t}$, payments for rental services Z_t priced at $q_{r,t}$, and loan payments $L_{S,t}$ at a predetermined gross return $R_{S,t}$ under a participation constraint of the lenders. The term $ac_{SS,t}$ reflects the loan adjustment costs of the impatient households.

As outlined in Lambertini, Nuguer, and Uysal (2017), impatient households face a threshold of default, denoted by $\bar{\omega}_t$, and total housing investment is equally distributed across the members of the household. Each borrower household enters a contract with the lender before the idiosyncratic shock ω_{t+1}^i materializes. The shock determines the ex-post housing value $\omega_{t+1}^i q_{t+1} H_{S,t}$, which captures the risk of investing into a house. The shock ω_{t+1}^i is i.i.d. across all household members and follows a log-normal distribution described by a cumulative distribution function $F_{t+1}(\omega_{t+1}^i)$. In order to rule out aggregate uncertainty, the expected value of the idiosyncratic shock is 1 in every period. We assume that the riskiness of housing investment can adjust over time, implying that the standard deviation $\sigma_{\omega,t}$ of $\log \omega_t^i$ is exposed to an exogenous shock. Once the idiosyncratic shock has materialized, borrowers choose either to default on their mortgages or to fulfill their repayment obligations. The threshold $\bar{\omega}_t$ is implicitly defined as:

$$R_{Sz,t+1} L_{S,t} = \bar{\omega}_{t+1}(1-\delta_H) q_{t+1} H_{S,t} \pi_{t+1}, \tag{5}$$

where $R_{Sz,t+1}$ is the state-contingent interest rate that borrowers pay at time $t+1$ on the loans $L_{S,t}$ taken at time t if they choose not to default on their mortgages. The idiosyncratic risk shock ω is distributed log-normally:

$$\log \omega_t \sim N\left(-\frac{\sigma_{\omega,t}^2}{2}, \sigma_{\omega,t}^2\right), \tag{6}$$

where $\sigma_{\omega,t} = \bar{\sigma}_\omega A_{\omega,t}$, $\bar{\sigma}_\omega$ is the steady-state standard deviation, and $A_{\omega,t}$ is an AR(1) process.

The collateral constraint (7) shows that impatient agents borrow against a fraction of the expected future value of their homes. The inertia parameter ρ_S accounts for a slow adjustment of the constraint (see Iacoviello (2015)) and $\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})$ represents the LTV ratio. This specification of the borrowing constraint is consistent with the empirical evidence that aggregate debt measures tend to lag changes in house prices:

$$L_{S,t} \leq \rho_S L_{S,t-1} + (1-\rho_S)[\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})](1-\delta_H) E_t\left[\frac{\pi_{t+1}}{R_{S,t}} q_{t+1} H_{S,t}\right]. \tag{7}$$

Note that the above formulation of the impatient agent's housing choice does not imply that borrowers live simultaneously in a mortgaged and a rented house. Instead, we assume that some fraction of borrower-type households choose to live in a rental house and the rest in an owner-occupied home. For this reason, the composite index $\widetilde{H}_{S,t}$ represents the aggregate preferences of all household members with respect to each type of housing services. This is equivalent to the "within a family" approach of Gertler and Karadi (2011). As before the borrower's adjustment costs, marginal utilities, and equilibrium conditions can be found in the appendix.

2.3 Bankers

Bankers with a discount rate β_B play an important role in the economy as they collect deposits from patient households on which they pay the interest rate $R_{H,t}$ set by the central bank. In addition to this, bankers issue loans to entrepreneurs and impatient households, denoted by $L_{E,t}$ and $L_{S,t}$. Bankers maximize their lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta_B^t (1-\eta) \log(C_{B,t} - \eta C_{B,t-1}), \tag{8}$$

subject to the budget constraint:

$$C_{B,t} + \frac{R_{H,t-1} D_{H,t-1}}{\pi_t} + L_{E,t} + L_{S,t} + ac_{DB,t} + ac_{EB,t} + ac_{SB,t} = D_t + \frac{R_{E,t} L_{E,t-1}}{\pi_t} + \frac{R_{S,t-1} L_{S,t-1}}{\pi_t}, \tag{9}$$

and the participation constraint of lenders:

$$R_{S,t} L_{S,t} = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} (1-\mu)(1-\delta_H) q_{t+1} H_{S,t} \pi_{t+1} f_{t+1}(\omega) d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} R_{Sz,t+1} L_{S,t} f_{t+1}(\omega) d\omega. \tag{10}$$

Bankers consume $C_{B,t}$ and hold assets and liabilities in the form of deposits and loans. Following Lambertini, Nuguer, and Uysal (2017) and Bernanke, Gertler, and Gilchrist (1999), we assume a one-period mortgage contract. Banks demand the predetermined interest rate $R_{S,t}$. The shock ω is described by the probability density function $f(\omega)$. Recall that the standard deviation of ω is subject to an exogenous shock, making it change over time. The return on total loans can be split up into two parts: the housings stock of borrowers adjusted for monitoring costs and the depreciation of defaulting household borrowers (first term on the right-hand side of Equation (10)), and the loan repayment of non-defaulting household borrowers (second term on the right-hand side of Equation (10)). Once the idiosyncratic and aggregate shocks have materialized, the state-contingent mortgage rate $R_{Sz,t}$ and the threshold value $\bar{\omega}_t$ are set. Lambertini, Nuguer, and Uysal (2017) define the expected value of the idiosyncratic shock for defaulting household borrowers multiplied by the probability of default as

$$G_{t+1}(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f_{t+1}(\omega) d\omega, \tag{11}$$

and as mentioned above, the expected fraction of housing value as

$$\Gamma_{t+1}(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} \omega_{t+1} d\omega + G_{t+1}(\bar{\omega}_{t+1}). \tag{12}$$

The exact specification of the quadratic adjustment costs for deposits (D_t) and loans ($L_{S,t}$, $L_{E,t}$) can be found in the appendix. Beside the budget constraint, bankers face a capital adequacy constraint, which is defined as:

$$L_t - D_t \geq \rho_D \big(L_{t-1} - D_{t-1} \big) + (1-\gamma)(1-\rho_D) L_t. \tag{13}$$

The total level of assets is given by the sum $L_t = L_{E,t} + L_{S,t}$. The left hand side of the capital adequacy constraint shows the net equity of banks. This expression has to be equal or greater than last periods equity plus some fraction of bank assets. The non-zero inertia parameter ρ_D ensures a partial adjustment of bank capital and a deviation from its capital-to-asset ratio (long-run) target $1-\gamma$.

2.4 Entrepreneurs

Entrepreneurs with a discount rate β_E produce the final good Y_t and new homes IH_t. The factors of input are labor and capital supplied by households, land ℓ_t, intermediate inputs $K_{B,t}$, and capital $K_{E,t}$ produced by entrepreneurs themselves. They maximize:

$$\sum_{t=0}^{\infty} \beta_E^t (1-\eta) \log(C_{E,t} - \eta C_{E,t-1}), \tag{14}$$

subject to the budget constraint:

$$\begin{aligned} C_{E,t} + \frac{K_{E,t}}{A_{K,t}} + \frac{R_{E,t} L_{E,t-1}}{\pi_t} + K_{B,t} + W^c_{H,t} N^c_{H,t} + W^h_{H,t} N^h_{H,t} + W^c_{S,t} N^c_{S,t} + W^h_{S,t} N^h_{S,t} + p_{\ell,t}(\ell_t - \ell_{t-1}) + \\ + R^c_{M,t} z^c_{KH,t} K^c_{H,t-1} + R^h_{M,t} z^h_{KH,t} K^h_{H,t-1} + ac_{KE,t} + ac_{EE,t} = \frac{Y_t}{X_t} + q_t IH_t + \frac{1-\delta_{KE,t}}{A_{K,t}} K_{E,t-1} + L_{E,t}, \end{aligned} \tag{15}$$

and a borrowing constraint of the form:

$$L_{E,t} \leq \rho_E L_{E,t-1} + (1-\rho_E) A_{ME,t} E_t \Big[m_K K_{E,t} - m_N \Big(\sum_{i=c,h} W^i_{H,t} N^i_{H,t} + \sum_{i=c,h} W^i_{S,t} N^i_{S,t} \Big) \Big]. \tag{16}$$

As we can see from the budget constraint, entrepreneurs pay households the sector-specific real wages $W^c_{H,t}$, $W^h_{H,t}$, $W^c_{S,t}$ and $W^h_{S,t}$. Inflation in the consumption sector is denoted by π_t and $p_{\ell,t}$ is the price of land. The terms $ac_{KE,t}$ and $ac_{EE,t}$ are the respective adjustment costs for capital and loans. $\delta_{KE,t}$ is the capital depreciation rate. Retailers purchase consumption goods from entrepreneurs and sell them at a markup X_t. The term $L_{E,t}$ denotes the loans that banks extend to entrepreneurs, with a gross return $R_{E,t}$. As impatient households, entrepreneurs face a borrowing constraint and borrow against a fraction of their capital and have to pay their workers upfront. The parameter ρ_E is the inertia in the entrepreneurs liquidity constraint. The term $A_{ME,t}$ is an AR(1) process that captures the entrepreneurs' LTV ratio and m_K stands for the LTV requirement on capital. The parameter m_N captures the fraction of wage bills that must be paid in advance. While we model the LTV ratio of the borrower-type households in an endogenous manner, we impose an exogenous process on the entrepreneurs' LTV ratio, which is not the main focus of this paper. The production functions of the consumption and construction sectors are:

$$Y_t = A_{Z,t}(z^c_{KH,t}K^c_{H,t-1})^{\alpha(1-\mu_c)}(z_{KE,t}K_{E,t-1})^{\alpha\mu_c}(N^c_{H,t})^{(1-\alpha)(1-\sigma)}(N^c_{S,t})^{(1-\alpha)\sigma}, \tag{17}$$

$$IH_t = A_{H,t}(z^h_{KH,t}K^h_{H,t-1})^{\mu_h}(N^h_{H,t})^{(1-\mu_h-\mu_b-\mu_l)(1-\sigma)}(N^h_{S,t})^{(1-\mu_h-\mu_b-\mu_l)\sigma}K^{\mu_b}_{B,t}\ell^{\mu_l}_{t-1}, \tag{18}$$

where $A_{Z,t}$ and $A_{H,t}$ are AR(1) processes that capture the technology shock in the consumption and housing sectors.

2.5 Nominal Rigidities and Monetary Policy

The existence of retailers, who operate under monopolistic competition, allows for sticky prices in the consumption sector. Nominal price adjustments in the retail sector entail implicit costs, which follow Calvo-style contracts (see Calvo (1983)). Consistent with the literature we assume that house prices are flexible.[4] Patient households own retailers and receive dividends in the form of $DIV_t = \frac{X_t - 1}{X_t}Y_t$. The resulting Phillips curves takes the form:

$$\log(\pi_t) - \iota_\pi \log(\pi_{t-1}) = \beta_H\{E_t\left[\log(\pi_{t+1})\right] - \iota_\pi \log(\pi_t)\} - \frac{(1-\Theta_\pi)(1-\beta_H\Theta_\pi)}{\Theta_\pi}\log\left(\frac{X_t}{X}\right) + \upsilon_\pi. \tag{19}$$

As described in Smets and Wouters (2003), Equation (19) implies partial indexation to lagged inflation of prices which cannot be re-optimized. Therefore, setting the elasticity ι_π equal to zero leaves us with the standard forward looking Phillips curve. Each period a fraction of retailers Θ_π cannot reset their prices optimally and υ_π is an identically and independently distributed cost-push shocks with zero mean and variance σ^2_π. In order to close the model, we assume that the central bank sets interest rates $R_{H,t}$ according to the following Taylor rule:

$$\begin{aligned}\log(R_{H,t}) = \Psi_R \log(R_{H,t-1}) + (1-\Psi_R)\log\left(\frac{1}{\beta_H}\right) + (1-\Psi_R)\Psi_\pi \log(\pi_t) + \\ + (1-\Psi_R)\Psi_Y \log\left(\frac{GDP_t}{GDP_{t-1}}\right) + \upsilon_{R,t} - A_{S,t},\end{aligned} \tag{20}$$

where interest rates react to inflation and GDP growth. $\frac{1}{\beta_H}$ is the steady-state real interest rate on deposits; $\upsilon_{R,t}$ stands for an identically and independently distributed monetary policy shock with zero

[4] See for example the discussion by Barsky, House, and Kimball (2007).

mean and variance σ_r^2; $A_{S,t}$ is a highly persistent AR(1) shock process, which measures long-lasting inflation deviations from its steady state level. This could be due to changes in the central bank's inflation target.

2.6 Market Clearing

The central clearing conditions of the goods and housing markets are:

$$C_t + \frac{IK_{c,t}}{A_{K,t}} + IK_{h,t} + K_{B,t} = Y_t - AC_t, \tag{21}$$

$$IH_t = H_{H,t} - (1-\delta_H)H_{H,t-1} + H_{S,t} - (1-\delta_H)[1-\mu G_t(\bar{\omega}_t)]H_{S,t-1} + H_{r,t} - (1-\delta_{Hr})H_{r,t-1}, \tag{22}$$

where IH_t is the sum of the individual housing stocks of the impatient and patient households. The goods sector produces (aggregate) consumption given by $C_t = C_{H,t} + C_{S,t} + C_{B,t} + C_{E,t}$. Business and residential investment is obtained by adding up the respective capital components accumulated by savers and entrepreneurs. Hence, business investment is defined as $IK_{c,t} = K^c_{H,t} - (1-\delta^c_{KH,t})K^c_{H,t-1} + K_{E,t} - (1-\delta_{KE,t})K_{E,t-1}$ and residential investment as $IK_{h,t} = K^h_{H,t} - (1-\delta^h_{KH,t})K_{H,t-1}$, and intermediate inputs $K_{B,t}$. In this study land is fixed and normalized to one. The term AC_t represents the aggregate adjustment costs in the economy.

2.7 AR(1) Shock Processes

There are in total eight AR(1) structural shocks, which are: the housing preference shock $A_{j,t}$, the LTV ratio shock of entrepreneurs $A_{ME,t}$, the technology shock of capital $A_{K,t}$, the intertemporal preference (or aggregate spending) shock $A_{p,t}$, the technology shocks in the consumption sector $A_{Z,t}$ and housing sector $A_{H,t}$, the inflation target shock $A_{S,t}$, and the idiosyncratic risk shock $A_{\omega,t}$:

$$\log A_{j,t} = \rho_j \log A_{j,t-1} + \upsilon_{j,t} \text{ with } \upsilon_{j,t} \sim N(0,\sigma_j^2), \tag{23}$$

$$\log A_{ME,t} = \rho_m \log A_{ME,t-1} + \upsilon_{m,t} \text{ with } \upsilon_{m,t} \sim N(0,\sigma_m^2), \tag{24}$$

$$\log A_{K,t} = \rho_k \log A_{K,t-1} + \upsilon_{k,t} \text{ with } \upsilon_{k,t} \sim N(0,\sigma_k^2), \tag{25}$$

$$\log A_{p,t} = \rho_p \log A_{p,t-1} + \upsilon_{p,t} \text{ with } \upsilon_{p,t} \sim N(0,\sigma_p^2), \tag{26}$$

$$\log A_{Z,t} = \rho_c \log A_{Z,t-1} + \upsilon_{c,t} \text{ with } \upsilon_{c,t} \sim N(0,\sigma_c^2), \tag{27}$$

$$\log A_{H,t} = \rho_h \log A_{H,t-1} + \upsilon_{h,t} \text{ with } \upsilon_{h,t} \sim N(0, \sigma_h^2), \tag{28}$$

$$\log A_{S,t} = \rho_s \log A_{S,t-1} + \upsilon_{s,t} \text{ with } \upsilon_{s,t} \sim N(0, \sigma_s^2), \tag{29}$$

$$\log A_{\omega,t} = \rho_\omega \log A_{\omega,t-1} + \upsilon_{\omega,t} \text{ with } \upsilon_{\omega,t} \sim N(0, \sigma_\omega^2). \tag{30}$$

3 Parameter Estimates

3.1 Data Description

We estimate the model with U.S. quarterly data on twelve observable variables from 1975 to 2008.[5] The observables include real consumption, real nonresidential fixed investment, real residential fixed investment, loans to businesses, loans to households, real house prices, real rental prices, hours in the consumption sector, hours in the housing sector, nominal interest rates, inflation, and total factor productivity. Figure 2 plots the data series.

Personal consumption expenditures, nonresidential fixed investment, and residential fixed investment are collected from the Bureau of Economic Analysis (BEA), and then log transformed and detrended with a quadratic trend. Real house prices and real rental prices are constructed from the Federal Housing Finance Agency (FHFA)'s all-transactions index and the Bureau of Labor Statistics (BLS)'s rent of primary residence index. Both indices are seasonally adjusted, deflated with the consumer price index (all items less shelter), log transformed, and detrended with a quadratic trend.

We obtain hours in the consumption sector by multiplying average weekly hours of production workers to total nonfarm payrolls less all employees in the construction sector and then dividing by the civilian noninstitutional population. Hours in the housing sector are obtained by multiplying average weekly hours of construction workers to all employees in the construction sector and then dividing by the civilian noninstitutional population. All data series are collected from the Federal Reserve Economic Data (FRED). Hours in both sectors are then log transformed. Nominal interest rates are the secondary market rate of 3-month treasury bill from FRED. Inflation is the percentage change in the nonfarm business sector implicit price deflator from FRED. Both nominal interest rates and inflation rates are demeaned.

We follow Iacoviello (2015) and construct the measure of total factor productivity from the utilization-

[5]We choose to end our sample at 2008 for two reasons. First, our focus in this paper is on the overheating of the housing mortgage market preceding the Great Recession. Second, the policy interest rate was cut to near zero at the end of 2008, which further complicates the model. The zero lower bound of the policy rate is an interesting topic that has been studied by Guerrieri and Iacoviello (2017), Bianchi and Melosi (2017), and Kulish, Morley, and Robinson (2017), among others, but is beyond the scope of this paper.

adjusted quarterly growth rate of TFP of Fernald (2014) by integrating the growth rates back to levels, and then applying the log transformation and the detrending with a quadratic trend.

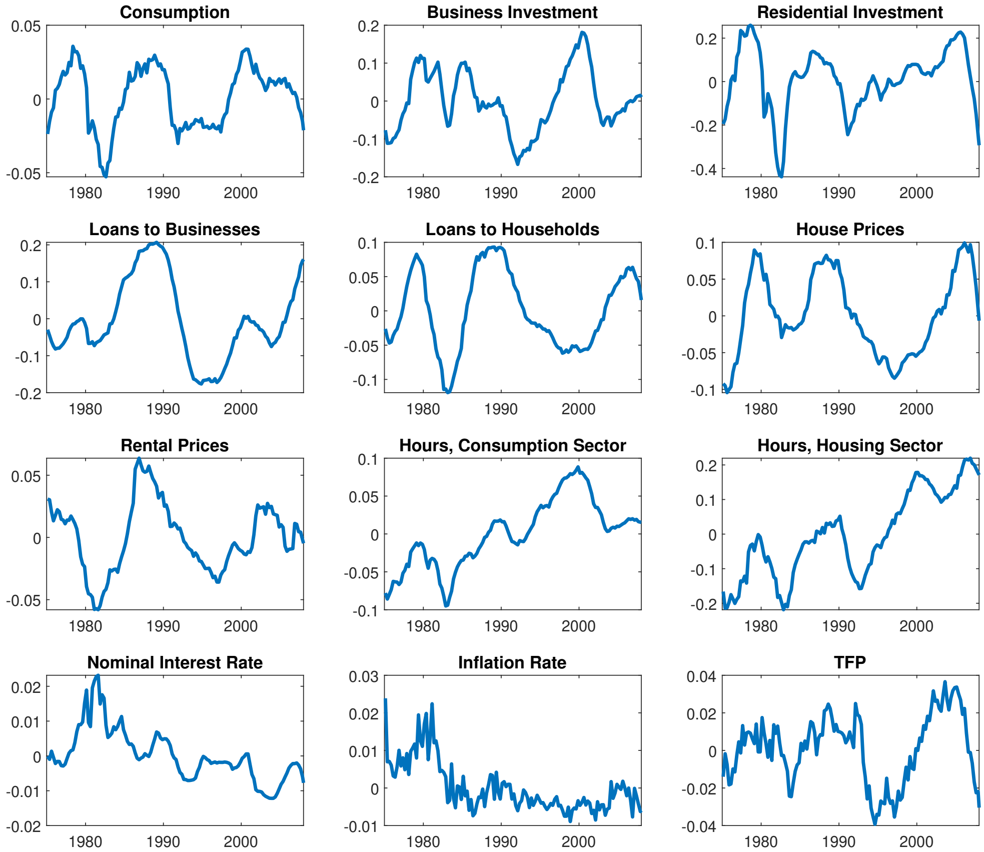

Figure 2: Data

3.2 Calibration

We estimate the model parameters using Bayesian techniques. Given that our data are demeaned, a subset of the parameters need to be calibrated in the estimation procedure. Table 1 summarizes the calibrated parameters. The discount factor β_H is fixed at 0.9925 to pin down the 3 percent steady-state annual return on deposits. The other three discount factors β_B, β_E, and β_S are set at 0.945, 0.94, and 0.94 respectively, implying a 5 percent steady-state annual return on loans. This assumption ensures that for small shocks the collateral constraint binds in the neighbourhood of the steady state. Capital depreciation rates are set at $\delta_{KE} = \delta^c_{KH} = 0.035$ in the consumption sector and $\delta^h_{KH} = 0.040$ in the housing sector. The leverage parameter on capital is set at $m_K = 0.9$. We assume that labor in both sectors needs to be fully paid in advance and choose $m_N = 1$. Bankers' liabilities to assets ratios are set at $\gamma_E = \gamma_S = 0.9$ (see Iacoviello (2015)). Depreciation rates of owner-occupied housing and rental housing are assumed to be $\delta_H = 0.01$ and $\delta_{Hr} = 0.02$. In other words, housing occupied by a renter depreciates more rapidly than housing occupied by an owner, due to moral hazard. For simplicity, we assume a one-to-one conversion of rental homes into rental services, i.e., $\Omega_r = 1$.

We set the capital share in production at $\alpha = 0.42$ and the weight of housing in the utility function at $j = 0.18$ to pin down the shares of consumption (67%), business investment (27%), and housing investment (6%) in GDP and the steady-state price-rent ratio of about 36. The labor supply parameter is set at $\tau = 2$ (see Iacoviello (2015)). The input share parameters in the production function are set at $\mu_c = 0.35$ and $\mu_h = \mu_b = \mu_l = 0.10$ (see Iacoviello and Neri (2010) and Davis and Heathcote (2005)). The markup parameter is set at its typical value $X = 1.15$ (see Corsetti et al. (2013)). The inflation target is highly persistent but its persistence is found hard to estimate in previous studies. We follow the usual practice to set the persistence parameter at $\rho_s = 0.95$. The steady-state standard deviation of the idiosyncratic risk shock is set at $\bar{\sigma}_\omega = 0.1$ (see Lambertini, Nuguer, and Uysal (2017)).

We choose to set the monitoring cost parameter at $\mu = 0.04$. This value, together with other parameters, implies a steady-state default rate of residential mortgages of 5 percent and a steady-state LTV ratio of about 0.84. The implied default rate is specific to borrowers in our model and is thus slightly higher than the observed delinquency rate over our sample period.

Table 1: Calibrated Parameters

Parameter		Value
Discount factor Saver (S)	β_H	0.9925
Discount factor Borrower (B)	β_S	0.94
Discount factor Banker	β_B	0.945
Discount factor Entrepreneur (E)	β_E	0.94
Total capital share in production	α	0.42
Capital LTV ratio, E	m_K	0.9
Wage bill paid in advance	m_N	1
Bankers' liabilities to assets ratios	γ_E, γ_S	0.9
Housing preference share	j	0.18
Capital depreciation rates consump. sector	δ_{KE}, δ^c_{KH}	0.035
Capital depreciation rates housing sector	δ^h_{KH}	0.040
Depreciation owner-occupied housing	δ_H	0.01
Depreciation rental housing	δ_{Hr}	0.02
Rental home conversion efficiency	Ω_r	1
Labor supply parameter	τ	2
Monitoring cost parameter	μ	0.04
Input share parameters	μ_h, μ_b, μ_ℓ	0.10
	μ_c	0.35
Markup	X	1.15
Persistence of inflation target	ρ_s	0.95
Steady-state SD of idiosyncratic risk	$\bar{\sigma}_\omega$	0.1

3.3 Prior and Posterior Distributions

Tables 2 and 3 present the prior and posterior distributions of remaining parameters in the structural model and shock processes. The posterior statistics are based on 1,000,000 draws from the posterior distribution.

The prior distributions for the habit formation parameter (η), adjustment cost parameters (ϕ's), inertia parameters (ρ's), wage share parameter (σ), and curvature parameters (ζ's) are taken from Iacoviello (2015). For the labor disutility parameters (κ's and χ's), Taylor rule parameters (Ψ's), and price rigidity parameters (Θ_π and ι_π), the prior distributions are taken from Iacoviello and Neri (2010). The prior distributions for parameters concerning the substitution between owner-occupied housing and rental housing (θ_s and κ_S) are obtained from Sun and Tsang (2017) with the exception that we impose a tighter prior on the parameter θ_s.[6] The persistence of AR(1) shocks is assumed to have a Beta prior with mean 0.8 and standard deviation 0.1. The standard deviation of structural shocks and measurement errors follows an Inverse Gamma distribution with mean 0.005 and standard deviation 0.025. These specifications are largely consistent with previous studies.

[6]The parameter θ_s turns out to be a key determinant of the relative steady-state values of H_H, H_S, and H_r. While the data that we use for estimation are aggregated and do not provide any information about the relative size of the housing stock owned by the two types of households, we try to use data from other sources to match the relative values of H_H, H_S, and H_r. First, we match the model-implied homeownership rate, i.e., $(H_H + H_S)/(H_H + H_S + H_r)$, with imputed rental of owner-occupied housing divided by its sum with rental of tenant-occupied housing, where both variables are published by the U.S. Bureau of Economic Analysis and available at https://fred.stlouisfed.org/series/A2013C1A027NBEA and https://fred.stlouisfed.org/series/DTENRC1A027NBEA. Second, we compute the model-implied households' equity share as $1 - L_S/q/(H_H + H_S)$ and match it with the share of homeowner equity in real estate property in the U.S. published by Statista Research Department (available at https://www.statista.com/statistics/375884/share-of-homeowner-equity-in-real-estate-usa/). By imposing a relatively tight prior on the parameter θ_s, we are able to match the model-implied homeownership rate and equity share with their data counterparts fairly well. Our estimated model implies a steady-state homeownership rate of 86% (compared to about 78% in the data) and a steady-state equity share of 74% (compared to 68% in the data).

Table 2: Prior and Posterior Distributions, Structural Parameters

Parameter		Prior Distribution			Posterior Distribution			
		Density	Mean	Std	Mean	5%	Median	95%
Habit in consumption	η	Beta	0.500	0.100	0.6965	0.6587	0.6972	0.7334
Deposit adj. cost, Banks	ϕ_{DB}	Gamma	0.250	0.100	0.0446	0.0189	0.0429	0.0698
Deposit adj. cost, S	ϕ_{DH}	Gamma	0.250	0.100	0.1877	0.1240	0.1854	0.2540
Capital adj. cost consum. sector, E	ϕ_{KE}	Gamma	1.000	0.500	2.2396	1.8134	2.2195	2.6807
Capital adj. cost consum. sector, S	ϕ_{KC}	Gamma	1.000	0.500	3.8722	3.5309	3.8678	4.2474
Capital adj. cost housing sector, S	ϕ_{KH}	Gamma	1.000	0.500	0.6661	0.4780	0.6548	0.8541
Loans to E adj. cost, Banks	ϕ_{EB}	Gamma	0.250	0.100	0.0623	0.0274	0.0604	0.0958
Loans to E adj. cost, E	ϕ_{EE}	Gamma	0.250	0.100	0.5294	0.3171	0.5270	0.7466
Loans to B adj. cost, Banks	ϕ_{SB}	Gamma	0.250	0.100	0.1360	0.0501	0.1300	0.2139
Loans to B adj. cost, B	ϕ_{SS}	Gamma	0.250	0.100	0.3766	0.2251	0.3743	0.5353
Inertia in capital adequacy constraint	ρ_D	Beta	0.250	0.100	0.7936	0.7612	0.7946	0.8266
Inertia in borrowing constraint, E	ρ_E	Beta	0.250	0.100	0.4602	0.3927	0.4601	0.5253
Inertia in borrowing constraint, B	ρ_S	Beta	0.250	0.100	0.8234	0.7746	0.8258	0.8754
Wage share, B	σ	Beta	0.300	0.100	0.7365	0.6628	0.7381	0.8094
Curvature for utilization function, E	ζ_E	Beta	0.200	0.100	0.3444	0.2548	0.3401	0.4306
Curvature for utilization function, S	ζ_H	Beta	0.200	0.100	0.2502	0.1957	0.2498	0.3010
Inverse elast. of subst. across hours, S	κ_H^N	Beta	0.500	0.075	0.5788	0.4761	0.5800	0.6844
Inverse elast. of subst. across hours, B	κ_S^N	Beta	0.500	0.075	0.7352	0.6609	0.7374	0.8138
Elasticity of hours, S	χ_H	Gamma	0.500	0.100	0.4047	0.2587	0.3978	0.5490
Elasticity of hours, B	χ_S	Gamma	0.500	0.100	0.3232	0.2241	0.3186	0.4200
Taylor rule, inflation parameter	Ψ_π	Normal	1.500	0.100	1.4238	1.2855	1.4137	1.5496
Taylor rule, interest rate parameter	Ψ_R	Beta	0.750	0.100	0.6442	0.5839	0.6461	0.7046
Taylor rule, output parameter	Ψ_Y	Normal	0.000	0.100	0.3733	0.2767	0.3737	0.4677
Price indexation	ι_π	Beta	0.500	0.200	0.0735	0.0095	0.0659	0.1347
Calvo price	Θ_π	Beta	0.667	0.050	0.5617	0.5049	0.5632	0.6169
Weight owner-occup. housing	θ_S	Beta	0.500	0.050	0.7261	0.6778	0.7269	0.7770
Elasticity of substitution	κ_S	Normal	2.000	0.500	1.1498	1.0010	1.1335	1.2979

The estimate of η is 0.7, which indicates a high degree of habit formation across agents. The wage share of credit-constrained households σ is slightly above 0.7, which is higher than most estimates reported in previous studies. For example, this value is around 0.2 in Jappelli (1990) and Iacoviello and Neri (2010). The reason that we find a higher fraction of net borrowers in our setting might be that we have separated entrepreneurs and bankers from net savers and modeled them independently. Nevertheless, our estimate is consistent with Guerrieri and Iacoviello (2017). Both labor supply parameters κ_N^H and κ_N^S are significantly different from zero, which indicates less than perfect labor mobility across sectors. The Taylor rule parameter estimates are in line with previous studies. The Calvo price parameter Θ_π is close to 0.6, which means that about 40 percent of retailers are able to re-optimize prices in each period and 60 percent are not able to do so. Credit-constrained households put a relatively high weight, about 73 percent, on owner-occupied housing in their utility function. The elasticity of substitution between owner-occupied housing and rental housing is lower than its prior mean. All of the AR(1) shocks are estimated to be highly persistent.

Table 3: Prior and Posterior Distributions, Shock Processes

Parameter		Prior Distribution			Posterior Distribution			
		Density	Mean	Std	Mean	5%	Median	95%
Autocor. housing demand shock	ρ_j	Beta	0.800	0.100	0.8362	0.8008	0.8368	0.8703
Autocor. investment shock	ρ_k	Beta	0.800	0.100	0.7489	0.7056	0.7499	0.7925
Autocor. LTV shock, E	ρ_{me}	Beta	0.800	0.100	0.9780	0.9727	0.9782	0.9835
Autocor. preference shock	ρ_p	Beta	0.800	0.100	0.9992	0.9985	0.9993	0.9999
Autocor. tech. shock, consum.	ρ_{zc}	Beta	0.800	0.100	0.9774	0.9618	0.9786	0.9938
Autocor. tech. shock, housing	ρ_{zh}	Beta	0.800	0.100	0.9749	0.9685	0.9753	0.9813
Autocor. Idiosyncratic risk shock	ρ_ω	Beta	0.800	0.100	0.9421	0.9070	0.9427	0.9762
Std housing demand shock	σ_j	Inv.gamma	0.005	0.025	0.0243	0.0213	0.0242	0.0272
Std investment shock	σ_k	Inv.gamma	0.005	0.025	0.0204	0.0173	0.0203	0.0236
Std LTV shock, E	σ_{me}	Inv.gamma	0.005	0.025	0.0167	0.0145	0.0166	0.0188
Std preference shock	σ_p	Inv.gamma	0.005	0.025	0.0364	0.0313	0.0362	0.0412
Std tech. shock, consum.	σ_{zc}	Inv.gamma	0.005	0.025	0.0074	0.0067	0.0074	0.0081
Std tech. shock, housing	σ_{zh}	Inv.gamma	0.005	0.025	0.0193	0.0172	0.0192	0.0213
Std monetary policy shock, (iid)	σ_r	Inv.gamma	0.005	0.025	0.0033	0.0028	0.0033	0.0037
Std monetary policy shock	σ_s	Inv.gamma	0.005	0.025	0.0008	0.0007	0.0008	0.0010
Std cost-push shock, (iid)	σ_π	Inv.gamma	0.005	0.025	0.0081	0.0062	0.0079	0.0099
Std idiosyncratic risk shock	σ_ω	Inv.gamma	0.005	0.025	0.0498	0.0408	0.0494	0.0587
Std measurement error	σ_{NC}	Inv.gamma	0.005	0.025	0.0312	0.0278	0.0311	0.0344
Std measurement error	σ_{NH}	Inv.gamma	0.005	0.025	0.1229	0.1096	0.1225	0.1356

Note: To make sure that the number of shocks is more than or equal to the number of data series, we impose independently and identically distributed measurement errors on hours in the consumption and housing sectors, with mean zero and standard deviations σ_{NC} and σ_{NH}.

4 Results

In this section we present the properties of the estimated model, including impulse response functions (IRFs), variance forecast error decomposition, and historical shock decomposition.

4.1 Impulse Response Functions

We plot the IRFs of key model variables in Figures 3 to 7. All IRFs are computed in percent deviations from their respective steady states.

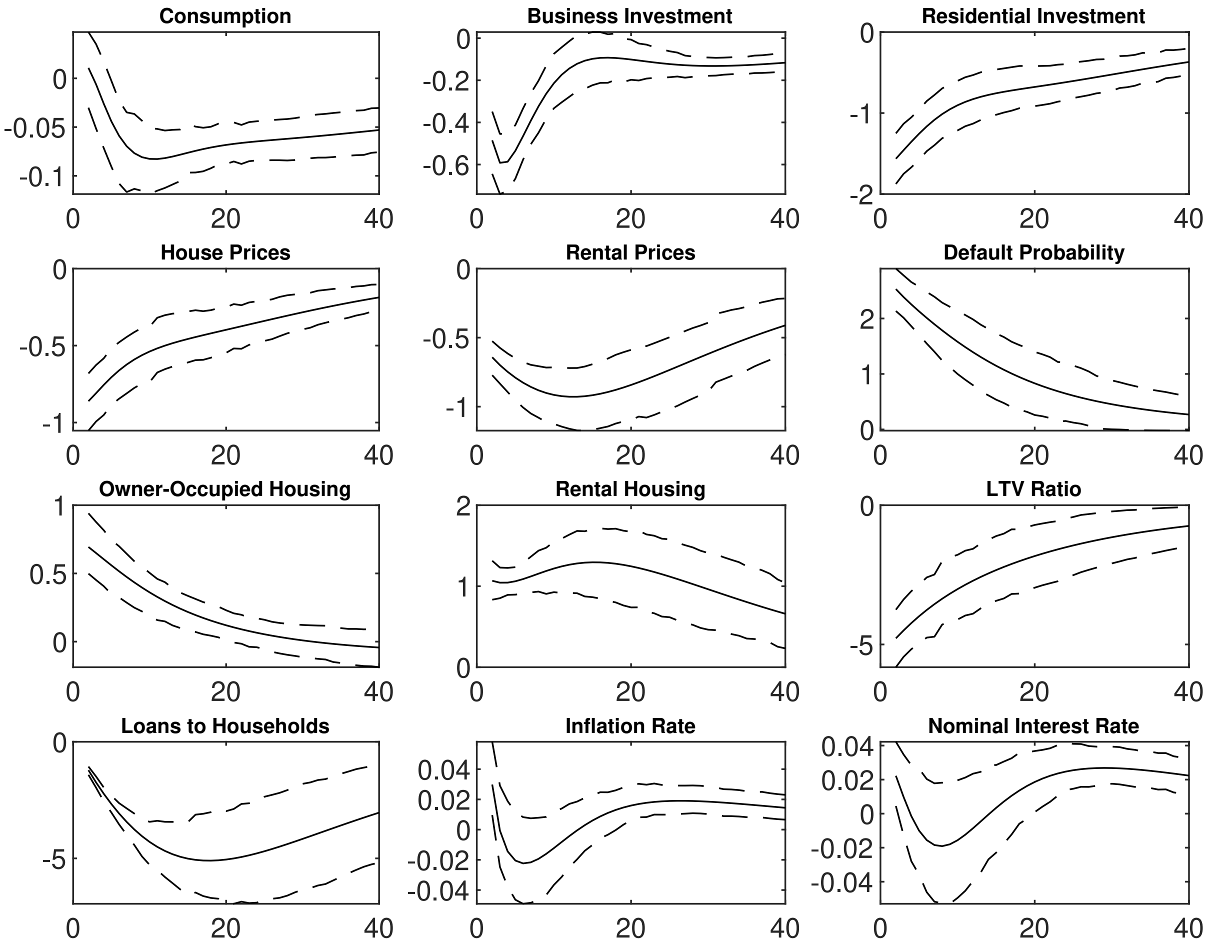

Figure 3: Impulse response to a one standard deviation idiosyncratic risk shock

The idiosyncratic risk shock is one of the key structural shocks in our model. It determines the ex-post value of housing and affects net borrowers' decision on whether or not to default on their mortgages. Figure 3 shows the responses of nine key model variables to a one standard deviation idiosyncratic risk shock. When such a shock hits the economy, credit-constrained households are more likely to choose to default on their mortgages. Loans to households decline and the loan-to-value ratio also decreases. Both house prices and rental prices tend to decrease. Credit-constrained households would choose more rental housing. Due to the drop in house prices, unconstrained households would opt to own more housing. All three components of aggregate demand decrease, with the largest drop in residential investment, followed by business investment and consumption. The impact on inflation and nominal interest rate is small.

Figure 4 shows the responses of variables to a one standard deviation housing preference shock. Following a positive housing preference shock, residential investment increases. Credit-constrained households face a higher demand for both types of housing and unconstrained households face a higher demand for owner-occupied housing. The higher demand drives up both house prices and rental prices. Due to the wealth effect, consumption expands and inflation also increases. The central bank adjusts the nominal interest rate up as a response to the increase in GDP and inflation. However, we notice that the impact of a housing preference shock is very small in magnitude, compared to previous studies. For example, in Iacoviello and Neri (2010), the housing demand shock explains a quarter of the volatility of housing investment and house prices. As Iacoviello and Neri (2010) point out, the housing preference shock is hard to interpret; it might capture all the disturbances that the model fails to identify. Interestingly, our model does not rely on the housing preference shock to explain the business cycle fluctuations of housing investment and house prices. This may be due to the richness of our model economy.

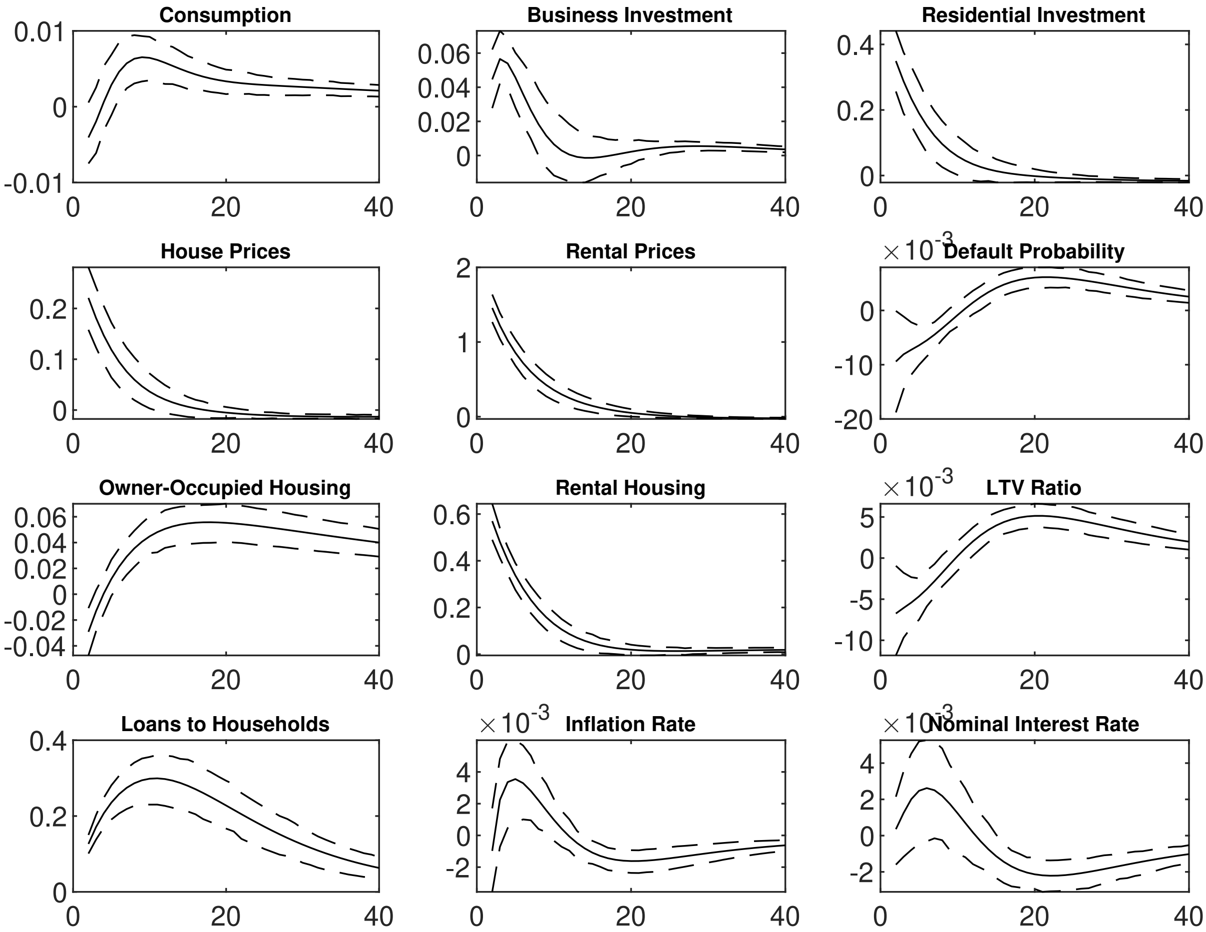

Figure 4: Impulse response to a one standard deviation housing preference shock

Figure 5 illustrates the impact of a one standard deviation housing technology shock. A technological advance in the housing sector drives residential investment up significantly and makes both types of housing less expensive. Households choose to have more of both owner-occupied housing and rental housing. The impact on other variables is limited.

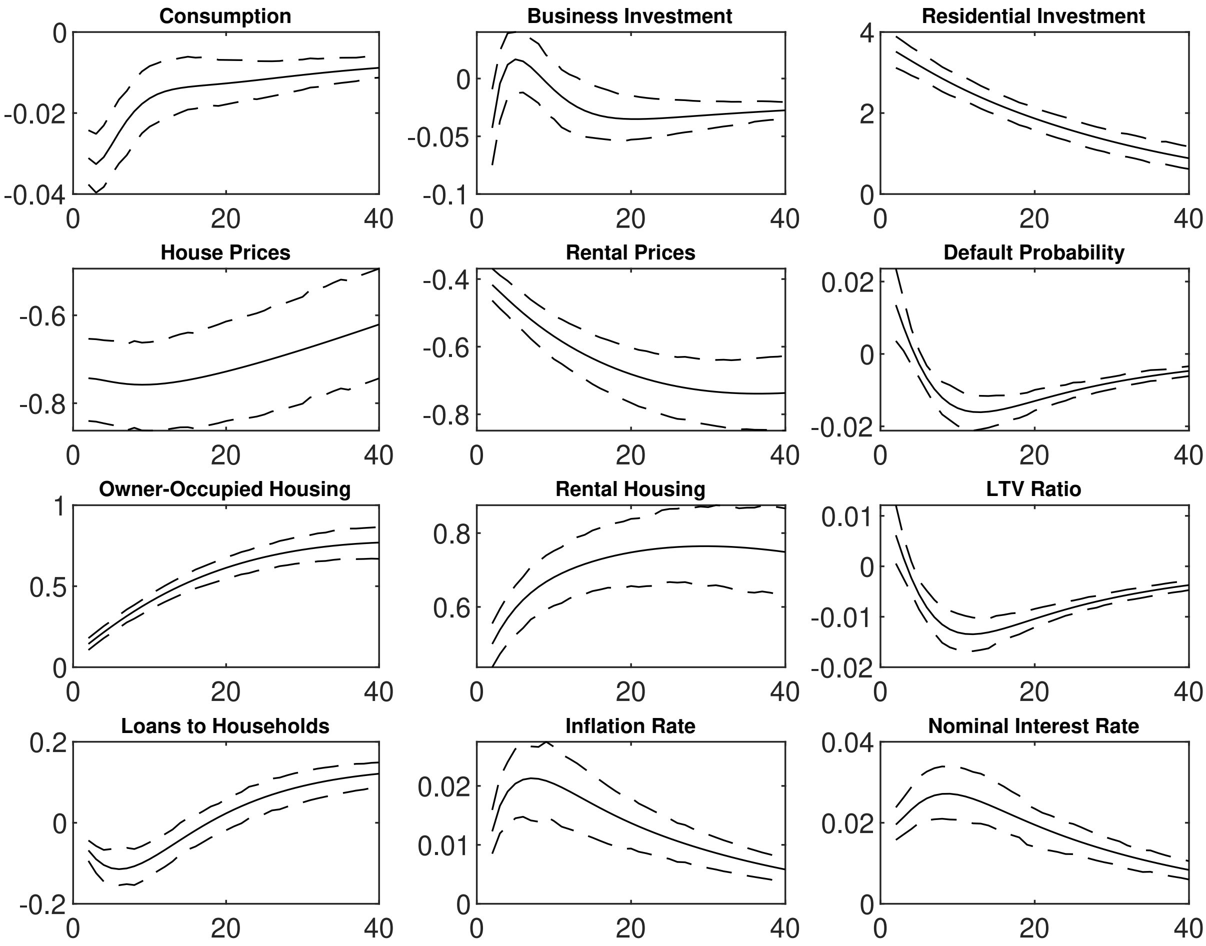

Figure 5: Impulse response to a one standard deviation housing technology shock

Figure 6 presents the responses of model variables to a one standard deviation intertemporal preference shock. Following a positive intertemporal preference shock, households' aggregate spending increases. They demand more consumption and housing. Both types of housing become more expensive. Due to the increase in housing value, credit-constrained households have access to more loans and they are less likely to default on their mortgages. The loan-to-value ratio decreases. Inflation increases and the central bank raises the nominal interest rate as a response to the increase in GDP and inflation.

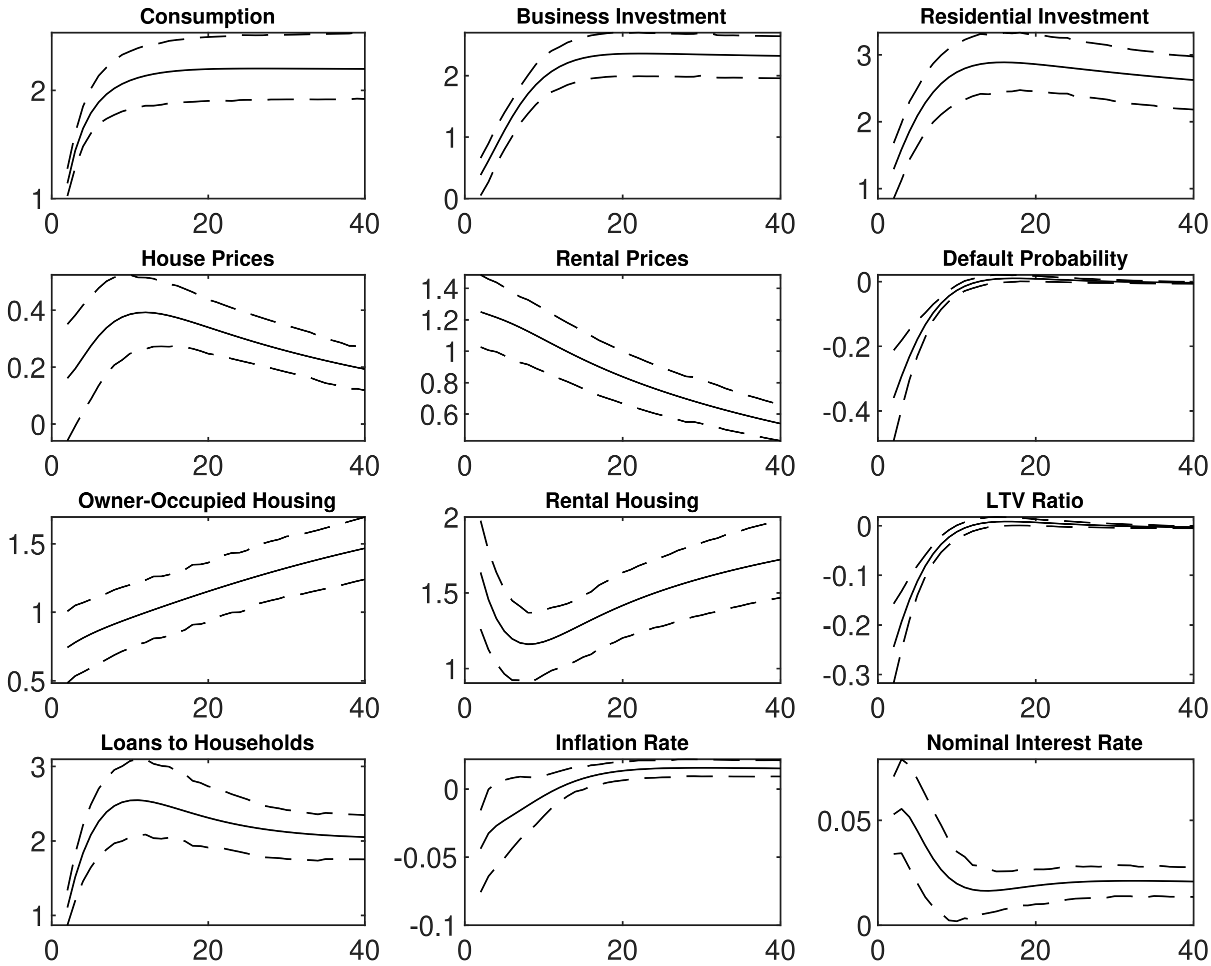

Figure 6: Impulse response to a one standard deviation intertemporal preference shock

Figure 7 demonstrates the impact of a one standard deviation monetary policy shock. As a result of such a shock, the nominal interest rate increases, which dampens all three components of aggregate demand and drives both house prices and rental prices down. Inflation follows the same behavior and experiences a decrease. Due to the higher return on deposits, unconstrained households are more willing to lend to constrained households through the banking sector. Given higher loans to households, credit-constrained households choose to own more housing and therefore the loan-to-value ratio decreases.

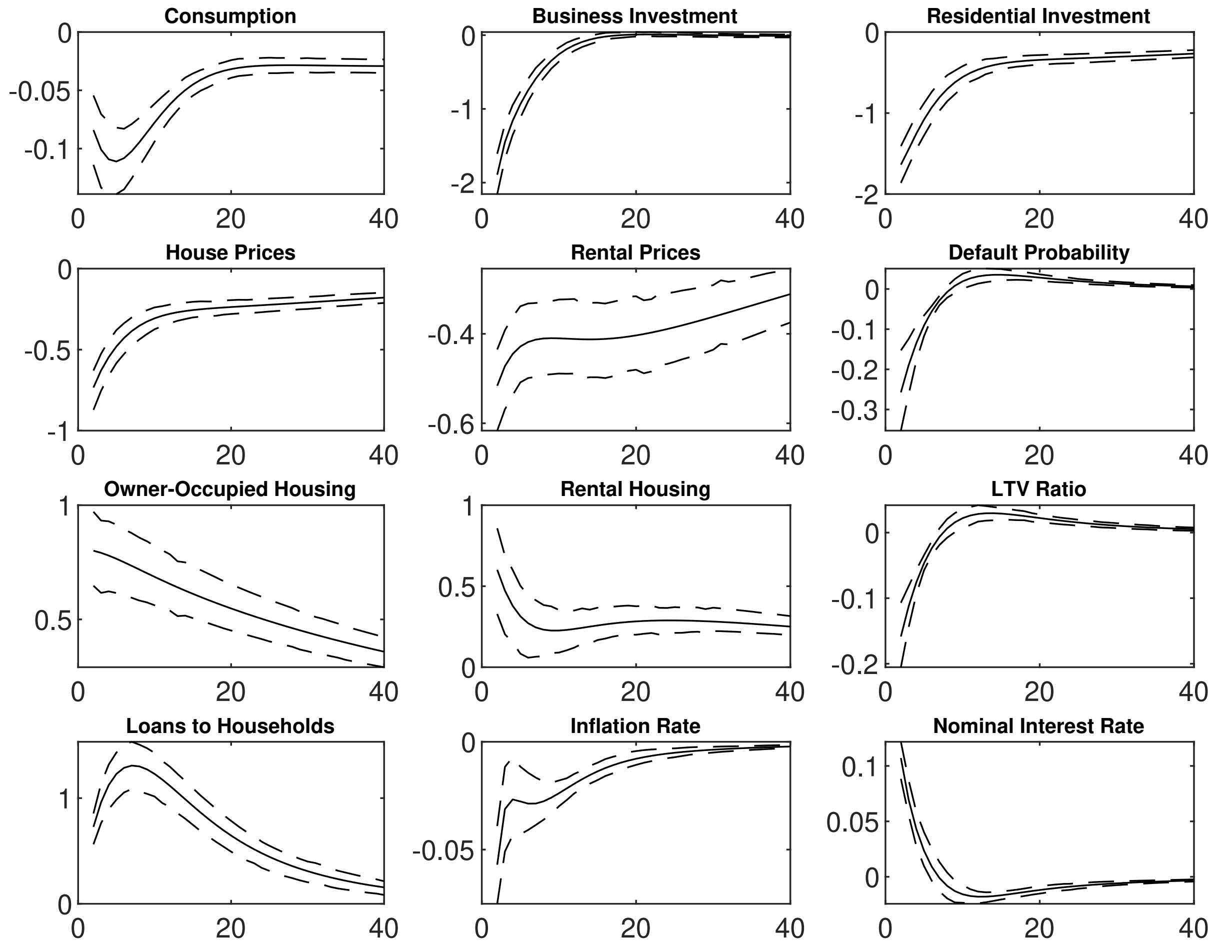

Figure 7: Impulse response to a one standard deviation monetary policy shock

4.2 Variance and Historical Shock Decomposition

Table 4 presents the variance forecast error decomposition for a set of key model variables. A notable feature of the variance decomposition result is that the housing preference shock does not explain much of the variation in housing investment or house prices. Instead, the shock plays an important role in the rental housing market; it explains 44 percent of the variation in rental prices. More than two thirds of the variation in real consumption is driven by the intertemporal preference shock. More than 40% of the fluctuations in real business investment is explained by the investment technology shock. One third of the variation in real residential investment is driven by the technology shock in the housing sector. The idiosyncratic risk shock determines the endogenous loan-to-value ratio and explains 13 percent of the changes in the default rate. It also explains 17 percent and 13 percent of the variation in house prices and the size of the owner-occupied housing market, respectively. The highly persistent inflation target shock plays an important role in our model. It explains one third of the variation in loans to households and accounts for around 40% of the fluctuations of the default rate, and both the price and the size of the owner-occupied housing market. We can see that the idiosyncratic risk shock plays only a minor role in explaining the variations in consumption and business investment. This is consistent with the findings of Lambertini, Nuguer, and Uysal (2017). As we do not include the crisis period after the housing market collapse, our estimation is therefore unable to capture the transmission of shocks from the mortgage sector to the rest of the economy. Furthermore, once the idiosyncratic risk shock has materialized, households will opt to default on their mortgages in order to remain on their consumption path.

Table 4: Variance Forecast Error Decomposition

	Housing pref. υ_j	IK tech υ_k	LTV υ_m	Inter. pref. υ_p	Consum. tech υ_c
Consumption	0.00	3.22	4.66	70.70	15.13
Buss. Investment	0.00	41.87	13.15	0.08	0.04
Resi. Investment	0.42	0.88	15.80	2.36	3.40
House Prices	1.39	0.08	6.43	0.48	3.82
Rental Prices	44.07	0.18	3.18	23.55	6.07
Default Prob.	0.04	4.09	7.08	13.63	3.33
Owner-Occ. Housing	0.06	4.41	9.57	11.09	2.39
Rental Housing	3.59	2.35	0.10	27.40	6.53
LTV Ratio	0.00	0.08	0.01	0.34	0.12
Loans to Households	0.25	4.33	4.54	18.13	6.29
Inflation	0.14	0.63	32.72	9.86	4.01
Interest Rate	0.00	25.92	2.75	2.20	0.02
	Housing tech υ_h	**Monetary υ_r**	**Infl. target υ_s**	**Cost-push υ_π**	**Idio. Risk υ_ω**
Consumption	0.08	0.52	0.41	5.24	0.03
Buss. Investment	0.04	20.15	4.20	20.44	0.03
Resi. Investment	31.17	7.32	22.92	8.73	7.00
House Prices	11.26	16.71	41.39	1.40	17.05
Rental Prices	2.36	5.65	9.63	0.11	5.20
Default Prob.	0.25	13.28	41.09	3.77	13.43
Owner-Occ. Housing	0.28	15.27	41.83	1.62	13.49
Rental Housing	1.66	4.54	27.88	15.68	10.26
LTV Ratio	0.00	0.18	0.82	0.24	98.21
Loans to Households	0.07	8.65	31.12	5.28	21.32
Inflation	0.17	9.27	22.19	1.43	19.59
Interest Rate	0.58	47.34	2.57	17.07	1.55

We plot the historical shock decompositions of house prices and loans to households in Figures 8 and 9. Most of the historical movements in house prices are driven by the investment and housing technology shocks. The idiosyncratic risk shock matters during the 1980s and over the few years right before the Great Recession.

The combination of idiosyncratic risk, monetary policy, investment technology, and intertemporal preference shocks dominates the historical movements in loans to households. During the 1980s and over the few years right before the Great Recession, the idiosyncratic risk shock plays an important role. From the mid-1980s to the mid-1990s, the other shocks become more important instead.

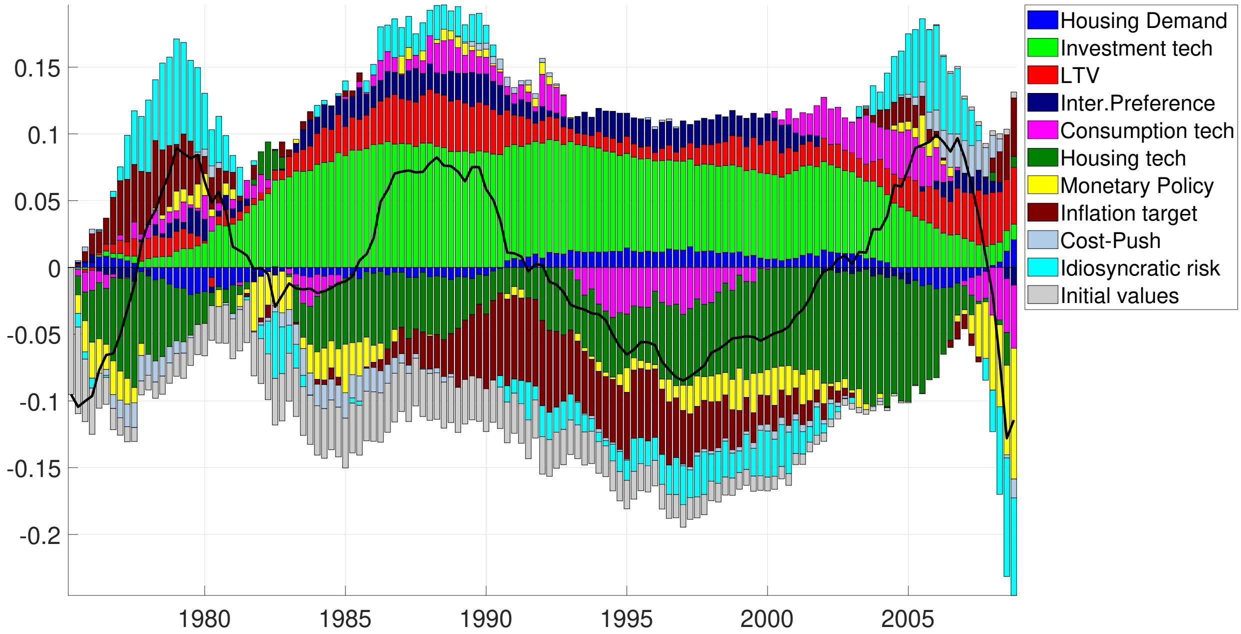

Figure 8: Historical shock decomposition of house prices

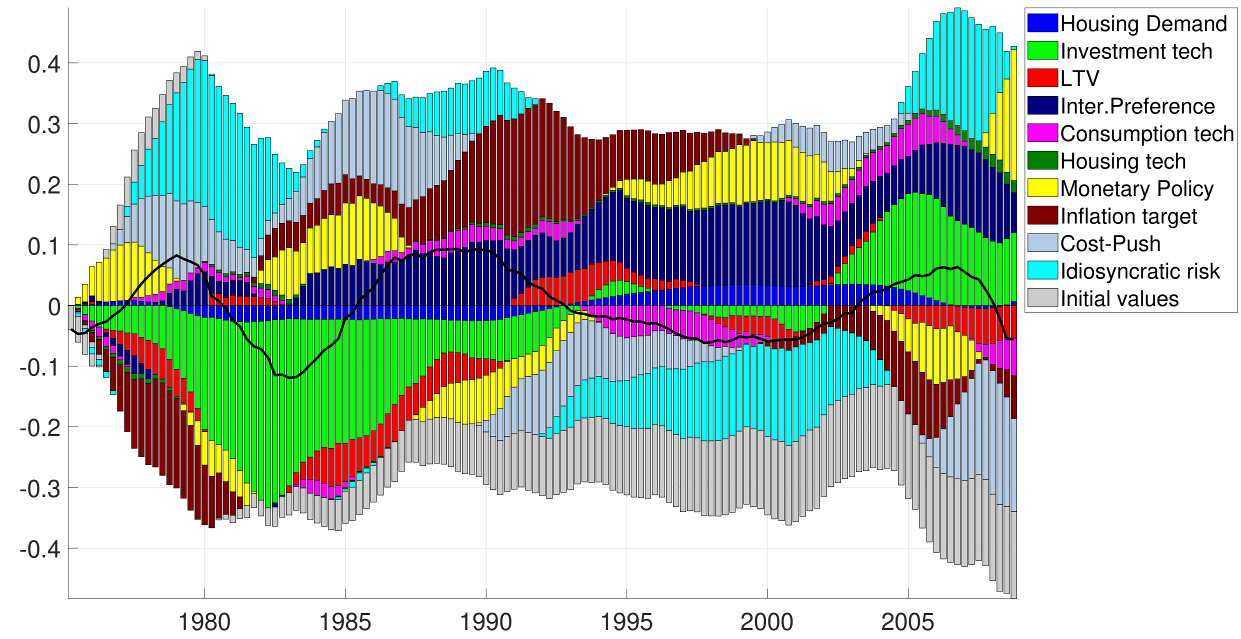

Figure 9: Historical shock decomposition of loans to households

5 Macroprudential Policy Analysis

Unlike in Iacoviello and Neri (2010) and Iacoviello (2015) where the LTV ratio is assumed to be a constant, we model credit-constrained households' LTV ratio endogenously in this paper. While there is no data available to match the LTV ratio specific to credit-constrained households, its counterpart implied by our DSGE model shows significant variation between 1975 and 2008. As the upper left panel of Figure 11 shows, the model-implied LTV ratio spikes around 1979 and 2005 and falls sharply before the Great Recession. It is widely agreed that the dangerous excess of unregulated mortgage lending is the primary driver of the rapid growth in house prices in the early 2000s and the subsequent housing crisis. In this section, through a counterfactual analysis, we examine whether the house price run-up and the subsequent crash could have been alleviated by a maximum LTV ratio policy. We also compare the maximum LTV ratio policy to a Taylor rule that responds to house price growth.

5.1 A Maximum LTV Ratio Policy

We impose a maximum value on the LTV ratio of credit-constrained households at 0.85. Since the model-implied LTV ratio, as Figure 11 shows, is higher than 0.85 only around 1979, in the late 1980s, and in the mid-2000s, this constraint will be occasionally binding. We adopt the OccBin toolkit developed by Guerrieri and Iacoviello (2015) to simulate the model variables from the estimated smoothed shocks and model parameters, assuming that the LTV ratio cannot exceed the maximum value.[7] The true model variables and their counterfactual counterparts are plotted in Figures 10 and 11 with solid and dashed lines respectively.

Among the twelve observable variables, residential investment, loans to households, house prices, rental prices, and hours in the housing sector are heavily affected by the maximum LTV ratio policy. If such a maximum LTV ratio policy had been imposed, both house prices and rental prices would have been much lower in the mid-2000s. For example, when the actual house prices reach the peak (10 percent higher than the steady-state level) in the first quarter of 2006, the counterfactual house prices would have been only 1 percent higher than the steady-state level. The subsequent drop in house prices would therefore have been less deep. The rental prices would have experienced a mild increase rather than a decrease after 2005.

[7] OccBin relies on different regime specifications of the underlying model. In one regime, the constraint of interest is binding and in the other the occasionally binding constraint is slack. A piecewise linear perturbation approach is then applied to obtain the solution to the occasionally binding constraint problem. The solution algorithm rests on two central requirements. First, the Blanchard-Kahn conditions of the existence of a rational expectations solution have to be satisfied under the reference regime (see Blanchard and Kahn (1980)). Second, once a shock shifts the economy away from the reference regime towards the alternative regime, it is assumed that agents do not expect any more shocks in the future. This assumption is therefore similar to the concept of a MIT shock. See Guerrieri and Iacoviello (2017) for more details and a discussion of the the accuracy of the solution method.

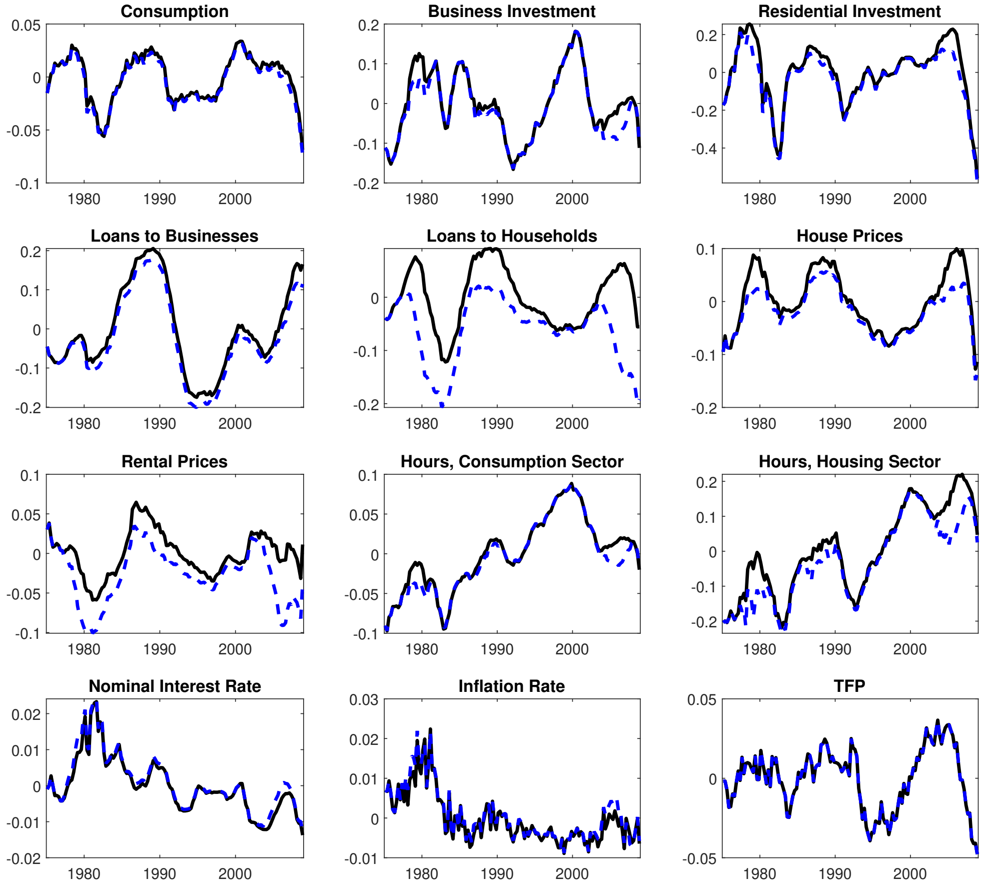

Figure 10: Counterfactual observable variables with maximum LTV ratio at 0.85

While variables presented in Figure 10 are observables that we use to estimate the model parameters, those in Figure 11 are not directly observable but are implied by the estimated model. If a maximum LTV ratio policy had been imposed, net savers or unconstrained households in the economy would have chosen to own more housing and net borrowers would have the option to rent more housing in the mid-2000s. Interestingly, neither savers' nor borrowers' utility would have been negatively impacted over the entire sample period. The availability of the rental housing market makes it possible for credit-constrained households to maintain the same level of utility in the presence of a restriction imposed on their access to credit.

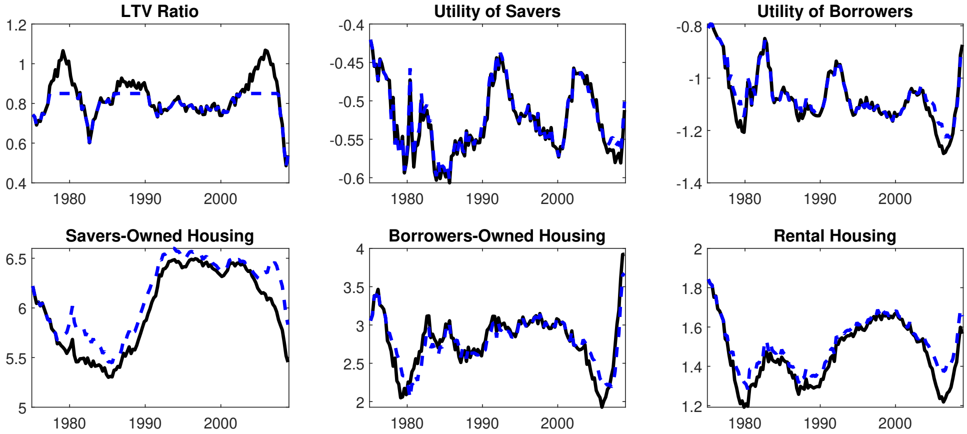

Figure 11: Counterfactual model-implied variables with maximum LTV ratio at 0.85

5.2 LTV Ratio Policy versus Augmented Taylor Rule

We now compare the maximum LTV ratio policy, in the form of an occasionally binding constraint affecting the impatient household's borrowing capacity, to an augmented Taylor rule with response to house price growth in the following form:

$$\log(R_{H,t}) = \Psi_R \log(R_{H,t-1}) + (1-\Psi_R)\log\left(\frac{1}{\beta_H}\right) + (1-\Psi_R)\Psi_\pi \log(\pi_t) + \\ + (1-\Psi_R)\Psi_Y \log\left(\frac{GDP_t}{GDP_{t-1}}\right) + (1-\Psi_R)\Psi_Q \log\left(\frac{q_t}{q_{t-1}}\right) + \upsilon_{R,t} - A_{S,t}, \quad (31)$$

where Ψ_Q measures the responsiveness of nominal interest rates to the percent change in real house prices. Sun and Tsang (2014) show that the value of this parameter estimated with U.S. data is around zero. In other words, there is no evidence that the U.S. monetary policy has been responding to house price growth. There has been, however, considerable debate on the role of asset prices in the formulation of monetary policy; see Bernanke and Gertler (2001). While many studies are against monetary policy responding to asset prices, many others are in favor of it. Kontonikas and Ioannidis (2005) suggest that taking into account stock and house price misalignments helps improve the overall macroeconomic stability. Finocchiaro and Von Heideken (2013) find that it is optimal for monetary policy to respond to house price inflation based on their estimation of the model of Iacoviello (2005) for the U.S., U.K., and Japan.

Our objective in this section is not to argue whether or not monetary policy should respond to house price growth or conduct a fully fledged Ramsey optimal monetary policy exercise. Instead, our goal is to examine how an augmented monetary policy would have affected households' lifetime utility, as defined in Equations (1) and (3), and social welfare, compared to a hypothetical maximum LTV ratio policy in the environment of our model economy. Interest rates are set according to the usual Taylor rule represented by condition (20) when limiting the LTV ratio. Following Rubio and Carrasco-Gallego (2015), we define social welfare as a weighted sum of the individual welfare for patient and impatient households, with weights being equal to $1 - \beta_H$ and $1 - \beta_S$, respectively.[8]

Given the estimated smoothed shocks and model parameters, we solve the framework at first order and simulate the model variables, assuming a maximum LTV ratio policy or an augmented Taylor rule had been implemented. We consider the maximum LTV ratio at 0.85 and alternative values of the Taylor rule parameter Ψ_Q from 0.1 to 0.5. We report the percent changes in households' lifetime utility and social welfare, compared to their baseline values, in Table 5.

Other things being equal, with monetary policy responding to house price growth, both credit-constrained and unconstrained households' would have experienced a decrease in their lifetime utility, even though the corresponding utility losses are very small. The more aggressive monetary policy responds to house price growth, the larger the utility loss. A maximum LTV ratio policy, instead, would have increased the lifetime utility of unconstrained and constrained households by 0.5% and 1.5%, respectively, and social welfare by 1.23% over the sample period of 1975 to 2008.

Table 5: Counterfactual Welfare Comparison

	$\%\Delta U_H$	$\%\Delta U_S$	$\%\Delta U$
Maximum LTV ratio at 0.85	0.4997	1.4999	1.2296
Aug. Taylor rule with $\Psi_Q = 0.1$	-0.0743	-0.0743	-0.0743
Aug. Taylor rule with $\Psi_Q = 0.2$	-0.1432	-0.1424	-0.1426
Aug. Taylor rule with $\Psi_Q = 0.3$	-0.2073	-0.2050	-0.2056
Aug. Taylor rule with $\Psi_Q = 0.4$	-0.2671	-0.2628	-0.2639
Aug. Taylor rule with $\Psi_Q = 0.5$	-0.3230	-0.3161	-0.3179

U_H and U_S denote the lifetime utility of savers and borrowers, discounted back to the start of the sample period. U stands for social welfare and equals $(1 - \beta_H)U_H + (1 - \beta_S)U_S$.

Table 5 provides strong evidence that a maximum LTV ratio policy is preferable to an augmented Taylor rule that responds to house price growth in terms of maintaining the lifetime utility of households and social welfare. The intuition behind our results is quite straightforward. Having monetary policy respond to house price growth dampens the fluctuations of house price. The central bank raises the interest rate, which benefits the lenders and hurts the borrowers, when house prices are on the way up and lowers the interest rate, which benefits the borrowers and hurts the lenders, on the way down. Unlike an augmented Taylor rule, limiting the maximum LTV ratio only creates downward pressure on house and rental prices

[8] We exclude the consumption components of entrepreneurs and bankers from the welfare analysis due to following two reasons. First, the banker's and entrepreneur's utility maximization problem is equivalent of maximizing profits (see for example Iacoviello (2015)). Second, impatient and patient households are linked through their accumulation of rental and owner-occupied housing which makes it interesting to study the effects of counterfactual policies such as a limit on the LTV requirement.

during a housing boom and reduces default risk, and therefore benefits both types of households.

6 Conclusion

This study has developed a DSGE model of the U.S. which accounts for three important features: rental and owner-occupied housing, endogenous default, and a banking channel. Allowing for these important components enables us to study in detail the effects of mortgage default on borrowing, rental and house prices. Furthermore, the rich framework presented in this paper provides a suitable environment to evaluate macroprudential policies. For this reason, we perform a counterfactual analysis by placing an occasionally binding constraint on the LTV ratio. Our analysis clearly shows that the dangerous expansion of the housing sector in the early 2000s could have been offset, if authorities had followed the LTV ratio policy studied in this paper. Finally, comparing an augmented Taylor rule that responds to house price growth with the maximum LTV policy, we find that the latter is preferable in terms of its effects on social welfare.

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A Technical Appendix (not for publication)

A.1 First Order Conditions, Adjustment Costs and Capital Utilization

A.1.1 Patient Households

The problem:

$$E_0 \sum_{t=0}^{\infty} \beta_H^t \Big\{ A_{p,t}(1-\eta)\,\log(C_{H,t}-\eta C_{H,t-1}) + jA_{j,t}A_{p,t}\log(H_{H,t}) - \frac{\tau}{1+\chi^H}\Big[(N_{H,t}^c)^{1+\kappa_H^N} + (N_{H,t}^h)^{1+\kappa_H^N}\Big]^{\frac{1+\chi^H}{1+\kappa_H^N}}\Big\} \tag{A.1}$$

subject to

$$\begin{aligned} &C_{H,t} + \frac{K_{H,t}^c}{A_{K,t}} + K_{H,t}^h + D_t + q_t\big\{[H_{H,t}-(1-\delta_H)H_{H,t-1}] + [H_{r,t}-(1-\delta_{Hr})H_{r,t-1}]\big\} + ac_{KH,t}^c + ac_{KH,t}^h + \\ &+ ac_{DH,t} = \Big(R_{M,t}^c z_{KH,t}^c + \frac{1-\delta_{KH,t}^c}{A_{K,t}}\Big)K_{H,t-1}^c + (R_{M,t}^h z_{KH,t}^h + 1 - \delta_{KH,t}^h)K_{H,t-1}^h + \frac{R_{H,t-1}D_{t-1}}{\pi_t} + W_{H,t}^c N_{H,t}^c + \\ &+ W_{H,t}^h N_{H,t}^h + q_{r,t}\Omega_r H_{r,t} + DIV_t \end{aligned} \tag{A.2}$$

The Lagrangian:

$$\begin{aligned} \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta_H^t \Big\{ &A_{p,t}(1-\eta)\log(C_{H,t}-\eta C_{H,t-1}) + jA_{j,t}A_{p,t}\log(H_{H,t}) - \frac{\tau}{1+\chi^H}\Big[(N_{H,t}^c)^{1+\kappa_H^N} + (N_{H,t}^h)^{1+\kappa_H^N}\Big]^{\frac{1+\chi^H}{1+\kappa_H^N}} + \\ &+ \lambda_{H,t}\Big[\Big(R_{M,t}^c z_{KH,t}^c + \frac{1-\delta_{KH,t}^c}{A_{K,t}}\Big)K_{H,t-1}^c + \frac{R_{H,t-1}D_{t-1}}{\pi_t} + W_{H,t}^c N_{H,t}^c + W_{H,t}^h N_{H,t}^h + q_{r,t}\Omega_r H_{r,t} + DIV_t + \\ &+ (R_{M,t}^h z_{KH,t}^h + 1 - \delta_{KH,t}^h)K_{H,t-1}^h - C_{H,t} - \frac{K_{H,t}^c}{A_{K,t}} - K_{H,t}^h - D_t - q_t\big\{[H_{H,t}-(1-\delta_H)H_{H,t-1}] + \\ &+ [H_{r,t}-(1-\delta_{Hr})H_{r,t-1}]\big\} - ac_{KH,t}^c - ac_{KH,t}^h - ac_{DH,t}\Big]\Big\} \end{aligned} \tag{A.3}$$

where

$$Z_t = \Omega_r H_{r,t} \qquad u_{HH,t} = \frac{jA_{j,t}A_{p,t}}{H_{H,t}} \qquad DIV_t = \frac{X_t-1}{X_t}Y_t$$

and

$$u_{NH,t}^c = \tau\Big[\Big((N_{H,t}^c)^{1+\kappa_H^N} + (N_{H,t}^h)^{1+\kappa_H^N}\Big)^{\frac{\chi^H-\kappa_H^N}{1+\kappa_H^N}}\Big](N_{H,t}^c)^{\kappa_H^N}$$

$$u^h_{NH,t} = \tau\left[\left((N^c_{H,t})^{1+\kappa^N_H} + (N^h_{H,t})^{1+\kappa^N_H}\right)^{\frac{\chi^H - \kappa^N_H}{1+\kappa^N_H}}\right](N^h_{H,t})^{\kappa^N_H}$$

The first order conditions are:

$$C_{H,t}: \qquad \lambda_{H,t} = u_{CH,t} = \frac{A_{p,t}(1-\eta)}{C_{H,t} - \eta C_{H,t-1}} \tag{A.4}$$

$$D_t: \qquad u_{CH,t}\left(1 + \frac{\partial ac_{DH,t}}{\partial D_t}\right) = \beta_H E_t\left(\frac{u_{CH,t+1} R_{H,t}}{\pi_{t+1}}\right) \tag{A.5}$$

$$N^c_{H,t}: \qquad u_{CH,t} W^c_{H,t} = u^c_{NH,t} \tag{A.6}$$

$$N^h_{H,t}: \qquad u_{CH,t} W^h_{H,t} = u^h_{NH,t} \tag{A.7}$$

$$K^c_{H,t}: \qquad \beta_H E_t\left[u_{CH,t+1}\left(R^c_{M,t+1} z^c_{KH,t+1} + \frac{1 - \delta^c_{KH,t+1}}{A_{K,t+1}}\right)\right] = u_{CH,t}\left(\frac{1}{A_{K,t}} + \frac{\partial ac^c_{KH,t}}{\partial K^c_{H,t}}\right) \tag{A.8}$$

$$K^h_{H,t}: \qquad \beta_H E_t\left[u_{CH,t+1}\left(R^h_{M,t+1} z^h_{KH,t+1} + 1 - \delta^h_{KH,t+1}\right)\right] = u_{CH,t}\left(1 + \frac{\partial ac^h_{KH,t}}{\partial K^h_{H,t}}\right) \tag{A.9}$$

$$H_{H,t}: \qquad q_t u_{CH,t} = u_{HH,t} + (1 - \delta_H)\,\beta_H E_t(q_{t+1} u_{CH,t+1}) \tag{A.10}$$

$$H_{r,t}: \qquad (1 - \delta_{Hr})\,\beta_H E_t(u_{CH,t+1} q_{t+1}) = u_{CH,t}(q_t - q_{r,t}\,\Omega_r) \tag{A.11}$$

$$z^c_{KH,t}: \qquad R^c_{M,t} = \frac{1}{A_{K,t}} \frac{\partial \delta^c_{KH,t}}{\partial z^c_{KH,t}} = \frac{\partial \delta^c_{KH,t}}{\partial z^c_{KH,t}} \tag{A.12}$$

$$z^h_{KH,t}: \qquad R^h_{M,t} = \frac{\partial \delta^h_{KH,t}}{\partial z^h_{KH,t}} \tag{A.13}$$

The adjustment costs of both capital types and deposits are defined as follows:

$$ac^c_{KH,t} = \frac{\phi_{KC}}{2} \frac{(K^c_{H,t} - K^c_{H,t-1})^2}{K^c_H}, \tag{A.14}$$

$$ac^h_{KH,t} = \frac{\phi_{KH}}{2} \frac{(K^h_{H,t} - K^h_{H,t-1})^2}{K^h_H}, \tag{A.15}$$

$$ac_{DH,t} = \frac{\phi_{DH}}{2}\frac{(K_{D,t} - K_{D,t-1})^2}{D}. \tag{A.16}$$

K^c_H, K^h_H and D are the respective steady state expressions for capital and deposits. The depreciation functions $\delta^c_{KH,t}$ and $\delta^h_{KH,t}$ take the following form:

$$\delta^c_{KH,t} = \delta^c_{KH} + b^c_{KH}\big[0.5\,\zeta'_H(z^c_{KH,t})^2 + (1-\zeta'_H)z^c_{KH,t} + (0.5\,\zeta'_H - 1)\big], \tag{A.17}$$

$$\delta^h_{KH,t} = \delta^h_{KH} + b^h_{KH}\big[0.5\,\zeta'_H(z^h_{KH,t})^2 + (1-\zeta'_H)z^h_{KH,t} + (0.5\,\zeta'_H - 1)\big]. \tag{A.18}$$

The curvature of the depreciation function is determined by $\zeta'_H = \frac{\zeta_H}{1-\zeta_H}$. Defining $b^c_{KH} = \frac{1}{\beta_H} + 1 - \delta^c_{KH}$ implies a steady-state utilization rate z^c_{KH} of one. Symmetrically, this result also holds for z^h_{KH}.

$$u^c_{NH,t} = \tau\Bigg[\Big((N^c_{H,t})^{1+\kappa^N_H} + (N^h_{H,t})^{1+\kappa^N_H}\Big)^{\frac{\chi^H-\kappa^N_H}{1+\kappa^N_H}}\Bigg](N^c_{H,t})^{\kappa^N_H},$$

$$u^h_{NH,t} = \tau\Bigg[\Big((N^c_{H,t})^{1+\kappa^N_H} + (N^h_{H,t})^{1+\kappa^N_H}\Big)^{\frac{\chi^H-\kappa^N_H}{1+\kappa^N_H}}\Bigg](N^h_{H,t})^{\kappa^N_H}.$$

A.1.2 Impatient Households

$$E_0\sum_{t=0}^{\infty}\beta^t_S\Bigg\{A_{p,t}(1-\eta)\log(C_{S,t}-\eta C_{S,t-1}) + jA_{j,t}A_{p,t}\log(\widetilde{H}_{S,t}) - \frac{\tau}{1+\chi^S}\Big[(N^c_{S,t})^{1+\kappa^N_S} + (N^h_{S,t})^{1+\kappa^N_S}\Big]^{\frac{1+\chi^S}{1+\kappa^L_S}}\Bigg\} \tag{A.19}$$

where

$$\widetilde{H}_{S,t} = \Big[\theta_S^{1/\kappa_s}(H_{S,t})^{\frac{\kappa_S-1}{\kappa_S}} + (1-\theta_S)^{1/\kappa_s}(Z_t)^{\frac{\kappa_S-1}{\kappa_S}}\Big]^{\frac{\kappa_S}{\kappa_S-1}},$$

subject to

$$C_{S,t} + q_{r,t}Z_t + q_tH_{S,t} + [1-F_t(\bar\omega_t)]\frac{R_{Sz,t}L_{S,t-1}}{\pi_t} + ac_{SS,t} = L_{S,t} + (1-\delta_H)[1-G_t(\bar\omega_t)]q_tH_{S,t-1} + \\ + W^c_{S,t}N^c_{S,t} + W^h_{S,t}N^h_{S,t}, \tag{A.20}$$

$R_{Sz,t}$ is the state-contingent interest rate paid by non-defaulting borrowers in period t on loans $L_{S,t-1}$ taken in period $t-1$. The state-contingent interest rate, which satisfies the lenders' participation constraint, is adjustable and is set after the realization of the shocks. The budget constraint above can be rewritten as:

$$C_{S,t}+q_{r,t}Z_t+q_tH_{S,t}+\frac{R_{S,t-1}L_{S,t-1}}{\pi_t}+ac_{SS,t}=L_{S,t}+(1-\delta_H)[1-\mu G_t(\bar{\omega}_t)]q_tH_{S,t-1}+ \\ +W^c_{S,t}N^c_{S,t}+W^h_{S,t}N^h_{S,t}, \tag{A.21}$$

and

$$L_{S,t}\leq\rho_S L_{S,t-1}+(1-\rho_S)[\Gamma_{t+1}(\bar{\omega}_{t+1})-\mu G_{t+1}(\bar{\omega}_{t+1})](1-\delta_H)E_t\Big[\frac{\pi_{t+1}}{R_{S,t}}q_{t+1}H_{S,t}\Big]. \tag{A.22}$$

The threshold $\bar{\omega}_t$ is defined as:

$$R_{Sz,t+1}L_{S,t}=\bar{\omega}_{t+1}(1-\delta_H)q_{t+1}H_{S,t}\pi_{t+1}. \tag{A.23}$$

The Lagrangian:

$$\mathcal{L}=E_0\sum_{t=0}^{\infty}\beta_S^t\Big\{A_{p,t}(1-\eta)\log(C_{S,t}-\eta C_{S,t-1})+jA_{j,t}A_{p,t}\log(\widetilde{H}_{S,t})-\frac{\tau_t}{1+\chi^S}\Big[(N^c_{S,t})^{1+\kappa^N_S}+(N^h_{S,t})^{1+\kappa^N_S}\Big]^{\frac{1+\chi^S}{1+\kappa^L_S}}+ \\ +\lambda^*_{S,t}\Big[L_{S,t}+(1-\delta_H)[1-\mu G_t(\bar{\omega}_t)]q_tH_{S,t-1}+W^c_{S,t}N^c_{S,t}+W^h_{S,t}N^h_{S,t}-C_{S,t}-q_{r,t}Z_t-q_tH_{S,t}-\frac{R_{S,t-1}L_{S,t-1}}{\pi_t}- \\ -ac_{SS,t}\Big]+\psi_{S,t}\Big[\rho_S L_{S,t-1}+(1-\rho_S)[\Gamma_{t+1}(\bar{\omega}_{t+1})-\mu G_{t+1}(\bar{\omega}_{t+1})]E_t\Big(\frac{\pi_{t+1}}{R_{S,t}}(1-\delta_H)q_{t+1}H_{S,t}\Big)-L_{S,t}\Big]\Big\} \tag{A.24}$$

The first order conditions are:

$$C_{S,t}:\qquad \lambda^*_{S,t}=u_{CS,t}=\frac{A_{p,t}(1-\eta)}{C_{S,t}-\eta C_{S,t-1}} \tag{A.25}$$

$$L_{S,t}:\qquad u_{CS,t}\Big(1-\lambda_{S,t}-\frac{\partial ac_{SS,t}}{\partial L_{S,t}}\Big)=\beta_S E_t\Big[u_{CS,t+1}\Big(\frac{R_{S,t}}{\pi_{t+1}}-\rho_S\lambda_{S,t+1}\Big)\Big] \tag{A.26}$$

$$N^c_{S,t}:\qquad u_{CS,t}W^c_{S,t}=u^c_{NS,t} \tag{A.27}$$

$$N^h_{S,t}:\qquad u_{CS,t}W^h_{S,t}=u^h_{NS,t} \tag{A.28}$$

$$H_{S,t}: \qquad u_{HS,t} + (1-\delta_H)[1-\mu G_{t+1}(\bar{\omega}_{t+1})]\beta_S E_t(u_{CS,t+1}q_{t+1}) = u_{CS,t}\Big[q_t - \lambda_{S,t}(1-\rho_S)[\Gamma_{t+1}(\bar{\omega}_{t+1}) - \mu G_{t+1}(\bar{\omega}_{t+1})](1-\delta_H)E_t\left(\frac{\pi_{t+1}q_{t+1}}{R_{S,t}}\right)\Big] \tag{A.29}$$

$$Z_t: \qquad u_{ZS,t} = q_{r,t}u_{CS,t} \tag{A.30}$$

$$\bar{\omega}_{t+1}: \qquad \beta_S u_{CS,t+1}\mu\frac{\partial G_{t+1}(\bar{\omega}_{t+1})}{\partial\bar{\omega}_{t+1}} = \lambda_{S,t}u_{CS,t}(1-\rho_S)\left[\frac{\partial\Gamma_{t+1}(\bar{\omega}_{t+1})}{\partial\bar{\omega}_{t+1}} - \mu\frac{G_{t+1}(\bar{\omega}_{t+1})}{\bar{\omega}_{t+1}}\right]\frac{\pi_{t+1}}{R_{S,t}} \tag{A.31}$$

Let the borrower's marginal utilities of housing and labor be:

$$u_{ZS,t} = A_{j,t}A_{p,t}\frac{j}{\widetilde{H}_{S,t}}\left[\frac{(1-\theta_S)\widetilde{H}_{S,t}}{Z_t}\right]^{\frac{1}{\kappa_S}},$$

$$u_{HS,t} = A_{j,t}A_{p,t}\frac{j}{\widetilde{H}_{S,t}}\left[\frac{\theta_S\widetilde{H}_{S,t}}{H_{S,t}}\right]^{\frac{1}{\kappa_S}},$$

$$u^c_{NS,t} = \tau\left[\left((N^c_{S,t})^{1+\kappa^N_S} + (N^h_{S,t})^{1+\kappa^N_S}\right)^{\frac{\chi^S-\kappa^N_S}{1+\kappa^N_S}}\right](N^c_{S,t})^{\kappa^N_S},$$

$$u^h_{NS,t} = \tau\left[\left((N^c_{S,t})^{1+\kappa^N_S} + (N^h_{S,t})^{1+\kappa^N_S}\right)^{\frac{\chi^S-\kappa^N_S}{1+\kappa^N_S}}\right](N^h_{S,t})^{\kappa^N_S}.$$

The loan adjustment costs of loans take same functional form as above and can be written as:

$$ac_{SS,t} = \frac{\phi_{SS}}{2}\frac{(L_{S,t}-L_{S,t-1})^2}{L_S}. \tag{A.32}$$

A.1.3 Bankers

Bankers maximize their lifetime utility according to:

$$E_0\sum_{t=0}^{\infty}\beta_B^t(1-\eta)log(C_{B,t}-\eta C_{B,t-1}) \tag{A.33}$$

subject to

$$C_{B,t} + \frac{R_{H,t-1}D_{H,t-1}}{\pi_t} + L_{E,t} + L_{S,t} + ac_{DB,t} + ac_{EB,t} + ac_{SB,t} = D_t + \frac{R_{E,t}L_{E,t-1}}{\pi_t} + \frac{R_{S,t-1}L_{S,t-1}}{\pi_t} \tag{A.34}$$

and the participation constraint of lenders is:

$$R_{S,t}L_{S,t} = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1}(1-\mu)(1-\delta_H)q_{t+1}H_{S,t}\pi_{t+1}f_{t+1}(\omega)d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} R_{Sz,t+1}L_{S,t}f_{t+1}(\omega)d\omega \tag{A.35}$$

where

$$ac_{DB,t} = \frac{\phi_{DB}}{2}\frac{(D_t - D_{t-1})^2}{D} \tag{A.36}$$

$$ac_{EB,t} = \frac{\phi_{EB}}{2}\frac{(L_{E,t} - L_{E,t-1})^2}{L_E} \tag{A.37}$$

$$ac_{SB,t} = \frac{\phi_{SB}}{2}\frac{(L_{S,t} - L_{S,t-1})^2}{L_S} \tag{A.38}$$

$$L_t - D_t \geq \rho_D\big(L_{t-1} - D_{t-1}\big) + (1-\gamma)(1-\rho_D)L_t \tag{A.39}$$

and $L_t = L_{S,t} + L_{E,t}$

The optimality conditions are:

$$C_{B,t}: \qquad u_{CB,t} = \frac{A_{p,t}(1-\eta)}{C_{B,t} - \eta C_{B,t-1}} \tag{A.40}$$

$$L_{E,t}: \qquad u_{CB,t}\left\{1-\lambda_{B,t}\big[\rho_D+\gamma_E(1-\rho_D)\big]+\frac{\partial ac_{EB,t}}{\partial L_{E,t}}\right\} = \beta_B E_t\left[u_{CB,t+1}\left(\frac{R_{E,t+1}}{\pi_{t+1}}-\lambda_{B,t+1}\rho_D\right)\right] \tag{A.41}$$

$$L_{S,t}: \qquad u_{CB,t}\left\{1-\lambda_{B,t}\big[\rho_D+\gamma_S(1-\rho_D)\big]+\frac{\partial ac_{SB,t}}{\partial L_{S,t}}\right\} = \beta_B E_t\left[u_{CB,t+1}\left(\frac{R_{S,t}}{\pi_{t+1}}-\lambda_{B,t+1}\rho_D\right)\right] \tag{A.42}$$

$$D_t: \qquad u_{CB,t}\left(1-\lambda_{B,t}-\frac{\partial ac_{DB,t}}{\partial D_t}\right) = \beta_B E_t\left[u_{CB,t+1}\left(\frac{R_{H,t}}{\pi_{t+1}}-\lambda_{B,t+1}\rho_D\right)\right] \tag{A.43}$$

The banker's adjustment costs of deposits, household and corporate loans are summarized below:

$$ac_{DB,t} = \frac{\phi_{DB}}{2}\frac{(D_t - D_{t-1})^2}{D}, \tag{A.44}$$

$$ac_{EB,t} = \frac{\phi_{EB}}{2}\frac{(L_{E,t} - L_{E,t-1})^2}{L_E}, \tag{A.45}$$

$$ac_{SB,t} = \frac{\phi_{SB}}{2}\frac{(L_{S,t} - L_{S,t-1})^2}{L_S}. \tag{A.46}$$

A.1.4 Entrepreneurs

The production function of the consumption sector:

$$Y_t = A_{Z,t}(z^c_{KH,t}K^c_{H,t-1})^{\alpha(1-\mu_c)}(z_{KE,t}K_{E,t-1})^{\alpha\mu_c}(N^c_{H,t})^{(1-\alpha)(1-\sigma)}(N^c_{S,t})^{(1-\alpha)\sigma} \tag{A.47}$$

The production function of the housing sector:

$$IH_t = A_{H,t}(z^h_{KH,t}K^h_{H,t-1})^{\mu_h}(N^h_{H,t})^{(1-\mu_h-\mu_b-\mu_l)(1-\sigma)}(N^h_{S,t})^{(1-\mu_h-\mu_b-\mu_l)\sigma}K^{\mu_b}_{B,t}\ell^{\mu_l}_{t-1} \tag{A.48}$$

$$IH_t = H_{H,t} - (1-\delta_H)H_{H,t-1} + H_{S,t} - (1-\delta_H)[1-\mu G_t(\bar{\omega}_t)]H_{S,t-1} + H_{r,t} - (1-\delta_{Hr})H_{r,t-1} \tag{A.49}$$

Entrepreneurs maximize their lifetime utility

$$\sum_{t=0}^{\infty}\beta^t_E\,(1 - \eta)\,\log(C_{E,t} - \eta C_{E,t-1}) \tag{A.50}$$

subject to

$$C_{E,t} + \frac{K_{E,t}}{A_{K,t}} + \frac{R_{E,t}L_{E,t-1}}{\pi_t} + K_{B,t} + W^c_{H,t}N^c_{H,t} + W^h_{H,t}N^h_{H,t} + W^c_{S,t}N^c_{S,t} + W^h_{S,t}N^h_{S,t} + p_{\ell,t}(\ell_t - \ell_{t-1}) +$$
$$+ R^c_{M,t}z^c_{KH,t}K^c_{H,t-1} + R^h_{M,t}z^h_{KH,t}K^h_{H,t-1} + ac_{KE,t} + ac_{EE,t} = \frac{Y_t}{X_t} + q_t IH_t + \frac{1-\delta_{KE,t}}{A_{K,t}}K_{E,t-1} + L_{E,t} \tag{A.51}$$

and

$$L_{E,t} \leq \rho_E L_{E,t-1} + (1-\rho_E) A_{ME,t} E_t \Big[m_K K_{E,t} - m_N (W^c_{H,t} N^c_{H,t} + W^h_{H,t} N^h_{H,t} + W^c_{S,t} N^c_{S,t} + W^h_{S,t} N^h_{S,t}) \Big] \tag{A.52}$$

The Lagrangian:

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta_E^t \Big\{ &(1-\eta) \log(C_{E,t} - \eta C_{E,t-1}) + \lambda^*_{E,t} \Big[\frac{Y_t}{X_t} + q_t IH_t + \frac{1-\delta_{KE,t}}{A_{K,t}} K_{E,t-1} + L_{E,t} + \varepsilon_{E,t} - C_{E,t} - \frac{K_{E,t}}{A_{K,t}} - \\
&- \frac{R_{E,t} L_{E,t-1}}{\pi_t} - W^c_{H,t} N^c_{H,t} - W^h_{H,t} N^h_{H,t} - W^c_{S,t} N^c_{S,t} - W^h_{S,t} N^h_{S,t} - K_{B,t} - p_{\ell,t}(\ell_t - \ell_{t-1}) - R^c_{M,t} z^c_{KH,t} K^c_{H,t-1} - \\
&- R^h_{M,t} z^h_{KH,t} K^h_{H,t-1} - ac_{KE,t} - ac_{EE,t} \Big] + \psi_{E,t} \Big[\rho_E L_{E,t-1} + (1-\rho_E) A_{ME,t} E_t \Big(m_K K_{E,t} - m_N (W^c_{H,t} N^c_{H,t} + \\
&+ W^h_{H,t} N^h_{H,t} + W^c_{S,t} N^c_{S,t} + W^h_{S,t} N^h_{S,t}) \Big) - L_{E,t} \Big] \Big\}
\end{aligned} \tag{A.53}$$

The first order conditions are:

$$C_{E,t}: \qquad \lambda^*_{E,t} = u_{CE,t} = \frac{A_{p,t}(1-\eta)}{C_{E,t} - \eta C_{E,t-1}} \tag{A.54}$$

$$L_{E,t}: \qquad u_{CE,t} \Big(1 - \lambda_{E,t} - \frac{\partial ac_{EE,t}}{\partial L_{E,t}} \Big) = \beta_E E_t \Big[u_{CE,t+1} \Big(\frac{R_{E,t+1}}{\pi_{t+1}} - \rho_E \lambda_{E,t+1} \Big) \Big] \tag{A.55}$$

$$K_{E,t}: \qquad \beta_E \, E_t \big[u_{CE,t+1} (1 - \delta_{KE,t+1} + R_{K,t+1} z_{KE,t+1}) \big] = u_{CE,t} \Big[\frac{1}{A_{K,t}} + \frac{\partial ac_{KE,t}}{\partial K_{E,t}} - \lambda_{E,t} m_k (1-\rho_E) A_{ME,t} \Big] \tag{A.56}$$

$$K^h_{H,t}: \qquad \mu_h q_t IH_t = R^h_{M,t} z^h_{KH,t} K^h_{H,t-1} \tag{A.57}$$

$$K^c_{H,t}: \qquad \alpha(1-\mu_c) \frac{Y_t}{X_t} = R^c_{M,t} z^c_{KH,t} K^c_{H,t-1} \tag{A.58}$$

$$N^c_{H,t}: \qquad \frac{Y_t}{X_t} (1-\alpha)(1-\sigma) = N^c_{H,t} W^c_{H,t} \big[1 + (1-\rho_E) \lambda_{E,t} A_{ME,t} m_N \big] \tag{A.59}$$

$$N^c_{S,t}: \qquad \frac{Y_t}{X_t} (1-\alpha) \, \sigma = N^c_{S,t} W^c_{S,t} \big[1 + (1-\rho_E) \lambda_{E,t} A_{ME,t} m_N \big] \tag{A.60}$$

$$N^h_{H,t}: \qquad q_t \, IH_t \, (1 - \mu_h - \mu_b - \mu_l)(1-\sigma) = N^h_{H,t} W^h_{H,t} \big[1 + (1-\rho_E) \lambda_{E,t} A_{ME,t} m_N \big] \tag{A.61}$$

$$N^h_{S,t}: \qquad q_t\, IH_t\,(1-\mu_h-\mu_b-\mu_l)\sigma = N^h_{S,t}W^h_{S,t}\big[1+(1-\rho_E)\lambda_{E,t}A_{ME,t}m_N\big] \tag{A.62}$$

$$K_{B,t}: \qquad K_{B_t} = \mu_b q_t IH_t \tag{A.63}$$

$$\ell_t: \qquad u_{CE,t}p_{\ell,t} = \beta_E E_t\Big[u_{CE,t+1}\Big(\mu_c\frac{IH_{t+1}}{\ell_t}q_{t+1}+p_{\ell,t+1}\Big)\Big] \tag{A.64}$$

$$\ell_t \text{ normalized to 1:} \qquad u_{CE,t}p_{\ell,t} = \beta_E E_t\Big[u_{CE,t+1}\Big(\mu_c IH_{t+1}q_{t+1}+p_{\ell,t+1}\Big)\Big] \tag{A.65}$$

$$\text{combining it with profit cond.} \qquad u_{CE,t}p_{\ell,t} = \beta_E E_t\Big[u_{CE,t+1}p_{\ell,t+1}\Big(1+R_{\ell,t+1}\Big)\Big] \tag{A.66}$$

$$z_{KE,t}: \qquad R_{K,t} = \frac{\partial\delta_{KE,t}}{\partial z_{KE,t}} = b_{KE}(\zeta'_E z_{KE,t}+1-\zeta'_E) \tag{A.67}$$

Profit maximization yields the following conditions:

$$\begin{aligned}\pi_t = \frac{Y_t}{X_t} &+ q_t IH_t - R_{K,t}z_{KE,t}K_{E,t-1} - R^c_{M,t}z^c_{KH,t}K^c_{H,t-1} - R^h_{M,t}z^h_{KH,t}K^h_{H,t-1} - W^c_{H,t}N^c_{H,t}- \\ &- W^c_{S,t}N^c_{S,t} - W^h_{H,t}N^h_{H,t} - W^h_{S,t}N^h_{S,t} - R_{\ell,t}p_{\ell,t}\ell_{t-1} - K_{B,t}\end{aligned} \tag{A.68}$$

Hence,

$$K_{E,t}: \qquad \frac{Y_{t+1}}{X_{t+1}}\,\frac{\alpha\mu_c}{K_{E,t}} = R_{K,t+1}z_{KE,t+1} \tag{A.69}$$

$$\tfrac{Y_t}{X_t}\,\alpha\,\mu_c = K_{E,t-1}R_{K,t}z_{KE,t}$$

$$K^c_{H,t}: \qquad \frac{Y_{t+1}}{X_{t+1}}\,\frac{\alpha(1-\mu_c)}{K^c_{H,t}} = R^c_{M,t+1}z^c_{KH,t+1} \tag{A.70}$$

$$\tfrac{Y_t}{X_t}\,\alpha(1-\mu_c) = K^c_{H,t-1}R^c_{M,t}z^c_{KH,t}$$

$$N^c_{H,t}: \qquad \frac{Y_t}{X_t}\,\frac{(1-\alpha)(1-\sigma)}{N^c_{H,t}} = W^c_{H,t} \tag{A.71}$$

$$\tfrac{Y_t}{X_t}\,(1-\alpha)(1-\sigma) = N^c_{H,t}W^c_{H,t}$$

$$N^c_{S,t}: \qquad \frac{Y_t}{X_t}\frac{(1-\alpha)\sigma}{N^c_{S,t}} = W^c_{S,t} \tag{A.72}$$

$$\tfrac{Y_t}{X_t}(1-\alpha)\sigma = N^c_{S,t}W^c_{S,t}$$

$$z_{KE,t}: \qquad \frac{Y_t}{X_t}\frac{\alpha\mu}{z_{KE,t}} = R_{K,t}\,K_{E,t-1} \tag{A.73}$$

The respective adjustment costs of capital and loans are:

$$ac_{KE,t} = \frac{\phi_{KE}}{2}\frac{(K_{E,t}-K_{E,t-1})^2}{K_E}, \tag{A.74}$$

$$ac_{EE,t} = \frac{\phi_{EE}}{2}\frac{(L_{E,t}-L_{E,t-1})^2}{L_E}. \tag{A.75}$$

The depreciation functions $\delta^c_{KE,t}$ takes the following form:

$$\delta_{KE,t} = \delta_{KE} + b_{KE}\big[0.5\,\zeta'_E(z_{KE,t})^2 + (1-\zeta'_E)z_{KE,t} + (0.5\,\zeta'_E-1)\big], \tag{A.76}$$

where $\zeta'_E = \frac{\zeta_E}{1-\zeta_E}$ and $b_{KE} = \frac{1}{\beta_E}[1-\lambda_E(1-\rho_E)m_K]-(1-\delta_{KE})$ implies a unitary steady-state utilization rate.

A.2 Steady State Derivations

In this section we derive the steady state of the economy. Due to the complexity of the model we only show the key steps and results of this exercise. Before we can start to derive the central expressions, we first have to compute the steady state equations for the respective interest rates and multipliers. Based on the first order conditions of the agents, we end up with:

$$R_H = \frac{1}{\beta_H} \tag{A.77}$$

$$R_E = \frac{1}{\beta_B} - \frac{\gamma_E(1-\rho_D)+(1-\beta_B)\rho_D}{\beta_B}\frac{1-\beta_B R_H}{1-\beta_B\rho_D}, \tag{A.78}$$

$$R_S = \frac{1}{\beta_B} - \frac{\gamma_S(1-\rho_D)+(1-\beta_B)\rho_D}{\beta_B}\frac{1-\beta_B R_H}{1-\beta_B\rho_D}, \tag{A.79}$$

$$R^c_M = \frac{1}{\beta_H} - (1-\delta^c_{KH}), \tag{A.80}$$

$$R_M^h = \frac{1}{\beta_H} - (1 - \delta_{KH}^h), \tag{A.81}$$

$$R_K = \frac{1}{\beta_E}[1 - \lambda_E(1 - \rho_E)m_K] - (1 - \delta_{KE}), \tag{A.82}$$

$$\lambda_B = \frac{1 - \beta_B R_H}{1 - \beta_B \rho_B}, \tag{A.83}$$

$$\lambda_E = \frac{1 - \beta_E R_E}{1 - \beta_E \rho_E}, \tag{A.84}$$

$$\lambda_S = \frac{1 - \beta_S R_S}{1 - \beta_S \rho_S}. \tag{A.85}$$

The next step involves to derive the housing-consumption, price-rent and housing ratio. From the saver's problem we obtain first consumption ratio and the price-rent ratio:

$$\frac{qH_H}{C_H} = \frac{j}{1 - (1 - \delta_{HH})\beta_H} = oo_3, \tag{A.86}$$

$$\frac{q_r}{q} = \frac{1 - (1 - \delta_{Hr})\beta_H}{\Omega_r} = oo_6. \tag{A.87}$$

From the borrower's problem we can determine the ratio $\frac{H_r}{H_S}$, which later helps us to pin down the housing-consumption ratio of the impatient household. The housing ratio takes the form:

$$\frac{H_S}{H_r} = \Omega_r \left(\frac{\theta_S}{1 - \theta_S} \right) \left[\frac{1 - (1 - \delta_{Hr})\beta_H}{\Omega_r} \right]^{\kappa_S} \cdot \\ \cdot \left\{ 1 - (1 - \delta_{HS})[1 - \mu G(\bar{\omega})]\beta_S - \lambda_S(1 - \rho_S)[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1 - \delta_{HS}) \frac{1}{R_S} \right\}^{-\kappa_S} \tag{A.88}$$

$$\Rightarrow \frac{H_S}{H_r} = \Gamma^*. \tag{A.89}$$

Using this result, we are able to define the housing-consumption ratio of the borrower:

$$\frac{qH_S}{C_S} = \frac{j}{1 + \left(\frac{1-\theta_S}{\theta_S}\right)^{\frac{1}{\kappa_S}} \left(\frac{\Omega_r}{\Gamma^*}\right)^{\frac{\kappa_S - 1}{\kappa_S}}} \left\{1-(1-\delta_{HS})[1-\mu G(\bar{\omega})]\beta_S - \lambda_S(1-\rho_S)[\Gamma(\bar{\omega}) - \mu G(\bar{\omega})](1-\delta_{HS})\frac{1}{R_S}\right\}^{-1}. \tag{A.90}$$

$$\Rightarrow \frac{qH_S}{C_S} = oo_4. \tag{A.91}$$

Similarly we can obtain the output ratios from the entrepreneurs side. We summarize below all important results, which are crucial for the next steps.

$$K_H^c = Y\frac{\alpha(1-\mu_c)}{XR_M^c} = Yoo_1, \tag{A.92}$$

$$K_H^h = qIH\frac{\mu_h}{R_M^h} = qIHoo_2, \tag{A.93}$$

$$\frac{qH_H}{C_H} = oo_3 \Rightarrow qH_H = oo_3 C_H, \tag{A.94}$$

$$\frac{qH_S}{C_S} = oo_4 \Rightarrow qH_S = oo_4 C_S, \tag{A.95}$$

$$\frac{qH_r}{C_S} = \frac{qH_S}{\Gamma^* C_S} = \frac{1}{\Gamma^*}\ \frac{qH_S}{C_S} = \frac{1}{\Gamma^*}oo_4 = oo_5 \Rightarrow qH_r = oo_5 C_S, \tag{A.96}$$

$$\frac{q_r}{q} = oo_6, \tag{A.97}$$

$$K_E = Y\frac{\alpha\mu_c}{XR_K} = Yoo_7, \tag{A.98}$$

$$K_B = \mu_b qIH, \tag{A.99}$$

$$N_H^c W_H^c = Y\frac{(1-\alpha)(1-\sigma)}{X[1+(1-\rho_E)\lambda_E m_N]}, \tag{A.100}$$

$$N_S^c W_S^c = Y\frac{(1-\alpha)\sigma}{X[1+(1-\rho_E)\lambda_E m_N]}, \tag{A.101}$$

$$N_H^h W_H^h = qIH\frac{(1-\mu_h-\mu_b-\mu_\ell)(1-\sigma)}{1+(1-\rho_E)\lambda_E m_N}, \tag{A.102}$$

$$N_S^h W_S^h = qIH\frac{(1-\mu_h-\mu_b-\mu_\ell)\sigma}{1+(1-\rho_E)\lambda_E m_N}. \tag{A.103}$$

The collateral constraints of the borrower and entrepreneur deliver:

$$L_S = m_S(1-\delta_{HS})qH_S\frac{1}{R_S}, \tag{A.104}$$

$$L_E = m_K K_E - m_N(N_H^c W_H^c + N_H^h W_H^h + N_S^c W_S^c N_S^h W_S^h). \tag{A.105}$$

In addition to this, we can rewrite the housing market clearing condition:

$$\begin{aligned} qIH &= \delta_{HH}qH_H + qH_S\Big\{1-(1-\delta_{HS})[1-\mu G(\bar{\omega})]\Big\} + \delta_{Hr}qH_r \\ &= \delta_{HH}oo_3C_H + oo_4C_S\Big\{1-(1-\delta_{HS})[1-\mu G(\bar{\omega})]\Big\} + \delta_{Hr}oo_5C_S \\ &= \delta_{HH}oo_3C_H + C_S\Big\{oo_4 - oo_4(1-\delta_{HS})[1-\mu G(\bar{\omega})]\Big\} + \delta_{Hr}oo_5C_S. \end{aligned} \tag{A.106}$$

And let

$$oo_8 = X[1+(1-\rho_E)\lambda_E m_N], \tag{A.107}$$

$$oo_9 = 1+(1-\rho_E)\lambda_E m_N. \tag{A.108}$$

Since labor enters the utility function via a CES aggregator we have to work with consumption-output ratios. This implies we have to rewrite the budget constraint of both household types and the entrepreneur, using the ratios derived above. Starting with the patient agent, we find that the budget constraint can be written as:

$$
\begin{aligned}
&C_H\Big\{1+\delta_{HH}oo_3-\delta_{HH}oo_3\Big[\frac{(1-\mu_h-\mu_b-\mu_\ell)(1-\sigma)}{oo_9}+(R^h_M-\delta^h_{KH})oo_2+m_N(1-R_H)\gamma_E\frac{(1-\mu_h-\mu_b-\mu_\ell)}{oo_9}\Big]\Big\}+\\
&+C_S\Big\{(1-R_H)\gamma_S[\Gamma(\bar\omega)-\mu G(\bar\omega)](1-\delta_{HH})\frac{oo_4}{R_S}+\delta_{Hr}oo_5-oo_5oo_6\Omega_r-\Big\{oo_4-oo_4(1-\delta_{HS})[1-\mu G(\bar\omega)]+\delta_{Hr}oo_5\Big\}\cdot\\
&\cdot\Big[\frac{(1-\mu_h-\mu_b-\mu_\ell)(1-\sigma)}{oo_9}+(R^h_M-\delta^h_{KH})oo_2+m_N(1-R_H)\gamma_E\frac{(1-\mu_h-\mu_b-\mu_\ell)}{oo_9}\Big]\Big\}=Y\Big\{(R^c_M-\delta^c_{KH})oo_1+\\
&+\frac{(1-\alpha)(1-\sigma)}{oo_8}-(1-R_H)\gamma_E\Big[m_Koo_7-m_N\frac{(1-\alpha)}{oo_8}\Big]+\frac{X-1}{X}\Big\}.
\end{aligned}
\tag{A.109}
$$

The expressions between the curly brackets are simply constants and therefore we can write:

$$
C_HT_1+C_ST_2=YT_3\Rightarrow\frac{C_H}{Y}T_1+\frac{C_S}{Y}T_2=T_3. \tag{A.110}
$$

In the same fashion we can write the borrower's budget constraint as:

$$
\begin{aligned}
&C_S\Big[1+oo_5oo_6\Omega_r+oo_4\{1-(1-\delta_{HS})[1-\mu G(\bar\omega)]\}-(1-R_S)[\Gamma(\bar\omega)-\mu G(\bar\omega)](1-\delta_{HS})\frac{oo_4}{R_S}-\\
&-\Big\{oo_4-oo_4(1-\delta_{HS})[1-\mu G(\bar\omega)]+\delta_{Hr}oo_5\Big\}\frac{(1-\mu_h-\mu_b-\mu_\ell)\sigma}{oo_9}\Big]=Y\frac{1-\alpha)\sigma}{oo_8}+\\
&+C_Hoo_3\delta_{HH}\frac{(1-\mu_h-\mu_b-\mu_\ell)\sigma}{oo_9},
\end{aligned}
\tag{A.111}
$$

$$
\Rightarrow C_ST_4=YT_5+C_HT_6\Rightarrow\frac{C_S}{Y}T_4=T_5+\frac{C_H}{Y}T_6. \tag{A.112}
$$

Similarly the entrepreneur's budget constraint becomes:

$$
\begin{aligned}
&C_E+Y\Big\{\delta_{KE}oo_7+(R_E-1)\Big[m_Koo_7-m_N\frac{(1-\alpha)}{oo_8}\Big]+\frac{(1-\alpha)}{oo_8}+R^c_Moo_1-\frac{1}{X}\Big\}=C_H\delta_{HH}oo_3\Big[1-R^h_Moo_2-\\
&-\frac{(1-\mu_h-\mu_b-\mu_\ell)}{oo_9}-\mu_b+m_N(R_E-1)\frac{(1-\mu_h-\mu_b-\mu_\ell)}{oo_9}\Big]+C_S\Big\{oo_4-oo_4(1-\delta_{HS})[1-\mu G(\bar\omega)]+\delta_{Hr}oo_5\Big\}\cdot\\
&\cdot\Big[1-R^h_Moo_2-\frac{(1-\mu_h-\mu_b-\mu_\ell)}{oo_9}-\mu_b+m_N(R_E-1)\frac{(1-\mu_h-\mu_b-\mu_\ell)}{oo_9}\Big],
\end{aligned}
\tag{A.113}
$$

$$
\Rightarrow C_E+YT_7=C_HT_8+C_ST_9\Rightarrow\frac{C_E}{Y}+T_7=\frac{C_H}{Y}T_8+\frac{C_S}{Y}T_9. \tag{A.114}
$$

What we have just derived is a system of three equations and 3 unknowns. They are summarized below:

$$
\frac{C_H}{Y}T_1+\frac{C_S}{Y}T_2=T_3, \tag{A.115}
$$

$$\frac{C_S}{Y}T_4 = T_5 + \frac{C_H}{Y}T_6, \tag{A.116}$$

$$\frac{C_E}{Y} + T_7 = \frac{C_H}{Y}T_8 + \frac{C_S}{Y}T_9. \tag{A.117}$$

We now solve for $\frac{C_H}{Y}$, $\frac{C_S}{Y}$ and $\frac{C_E}{Y}$. The final result for the three ratios is:

$$\frac{C_S}{Y} = \frac{T_6T_3 + T_1T_5}{T_1T_4 + T_6T_2}, \tag{A.118}$$

$$\frac{C_H}{Y} = \frac{C_S}{Y}\frac{T_4}{T_6} - \frac{T_5}{T_6}, \tag{A.119}$$

$$\frac{C_E}{Y} = \frac{C_H}{Y}T_8\frac{C_S}{Y}T_9 - T_7. \tag{A.120}$$

We can again rewrite the housing market clearing condition and find:

$$\frac{qIH}{Y} = \delta_{HH}oo_3\frac{C_H}{Y} + \frac{C_S}{Y}\Big\{oo_4 - oo_4(1-\delta_{HS})[1-\mu G(\bar{\omega})] + \delta_{Hr}oo_5\Big\}. \tag{A.121}$$

In order to derive the steady state in levels from the ratios, we now have to work out Y. This means we have to pin down N_H^c and N_S^c. Along the way we are also able to derive the steady state N_H^h and N_S^h. Algebraic rearrangement of the four optimality conditions involving saver's and borrower's labor choice yields. Starting with the patient household we find:

$$N_H^c = \left\{\frac{\frac{Y}{C_H}\frac{(1-\alpha)(1-\sigma)}{oo_8}}{\tau\left[1+\frac{qIH}{Y}\frac{(1-\mu_h-\mu_b-\mu_\ell)}{1-\alpha}\right]^{\frac{\chi_S-\kappa_H^N}{1+\kappa_H^N}}}\right\}^{\frac{1}{1+\chi_H}}, \tag{A.122}$$

and

$$N_H^h = N_H^c\frac{N_H^h}{N_H^c}, \tag{A.123}$$

where

$$\frac{N_H^h}{N_H^c} = \left[\frac{qIH}{Y}\frac{X(1-\mu_h-\mu_b-\mu_\ell)}{1-\alpha}\right]^{\frac{1}{1+\kappa_H^N}} \tag{A.124}$$

Symmetrically, we can find the same result for the borrower's labor choice:

$$N_S^c = \left\{ \frac{\frac{Y}{C_S}\frac{(1-\alpha)\sigma}{oo_8}}{\tau\left[1+\frac{qIH}{Y}\frac{(1-\mu_h-\mu_b-\mu_\ell)}{1-\alpha}\right]^{\frac{\chi_S-\kappa_S^N}{1+\kappa_S^N}}} \right\}^{\frac{1}{1+\chi_S}}, \tag{A.125}$$

$$N_S^h = N_S^c \frac{N_S^h}{N_S^c}, \tag{A.126}$$

where

$$\frac{N_S^h}{N_S^c} = \left[\frac{qIH}{Y}\frac{X(1-\mu_h-\mu_b-\mu_\ell)}{1-\alpha}\right]^{\frac{1}{1+\kappa_S^N}}. \tag{A.127}$$

Finally we can pin down the level of output in the steady state:

$$\begin{aligned} Y &= (K_H^c)^{\alpha(1-\mu_c)}(K_E)^{\alpha\mu_c}(N_H^c)^{(1-\alpha)(1-\sigma)}(N_S^c)^{(1-\alpha)\sigma} \\ &= Y^\alpha (oo_1)^{\alpha(1-\mu_c)}(oo_7)^{\alpha\mu_c}(N_H^c)^{(1-\alpha)(1-\sigma)}(N_S^c)^{(1-\alpha)\sigma} \\ Y &= \left[(oo_1)^{\alpha(1-\mu_c)}(oo_7)^{\alpha\mu_c}(N_H^c)^{(1-\alpha)(1-\sigma)}(N_S^c)^{(1-\alpha)\sigma}\right]^{\frac{1}{1-\alpha}}. \end{aligned} \tag{A.128}$$

Having the derived the level of output, we then can then move on to define the consumption, residential investment and the different types of capital stocks:

$$K_H^c = oo_1 Y, \tag{A.129}$$

$$K_H^h = oo_2 Y \frac{qIH}{Y}, \tag{A.130}$$

$$qIH = Y\frac{qIH}{Y}, \tag{A.131}$$

$$K_B = \mu_b qIH, \tag{A.132}$$

$$C_H = Y\frac{C_H}{Y}, \tag{A.133}$$

$$C_S = Y\frac{C_S}{Y}, \tag{A.134}$$

$$C_E = Y\frac{C_E}{Y}, \tag{A.135}$$

$$IH = \left(K_H^h\right)^{\mu_h}\left(N_H^h\right)^{(1-\mu_h-\mu_b-\mu_\ell)(1-\sigma)}\left(N_S^h\right)^{(1-\mu_h-\mu_b-\mu_\ell)\sigma} K_B^{\mu_b}. \tag{A.136}$$

House and rental prices are defined according to the following conditions:

$$q = \frac{qIH}{q}, \tag{A.137}$$

$$qr = qoo_6. \tag{A.138}$$

The individual housing stocks can be computed from the market clearing condition of the housing sector.

$$\begin{aligned} IH &= \delta_{HH}H_H + \Big\{1-(1-\delta_{HS})[1-\mu G(\bar{\omega})]\Big\}H_S + \delta_{Hr}H_r \\ \Rightarrow \frac{IH}{H_H} &= \delta_{HH} + \Big\{1-(1-\delta_{HS})[1-\mu G(\bar{\omega})]\Big\}\frac{H_S}{H_H} + \delta_{Hr}\frac{H_r}{H_H}, \end{aligned} \tag{A.139}$$

where we need the following housing ratios ratios:

$$\frac{H_S}{H_H} = \frac{oo_4 C_S}{oo_3 C_H}, \tag{A.140}$$

$$\frac{H_r}{H_H} = \frac{oo_5 C_S}{oo_3 C_H}, \tag{A.141}$$

in order to work out H_H, H_S and H_r:

$$H_H = \frac{IH}{\delta_{HH} + \left\{1-(1-\delta_{HS})[1-\mu G(\bar{\omega})]\right\}\frac{oo_4 C_S}{oo_3 C_H} + \delta_{Hr}\frac{oo_5 C_S}{oo_3 C_H}}, \tag{A.142}$$

$$H_H = H_H\frac{H_S}{H_H}, \tag{A.143}$$

$$H_r = H_H\frac{H_r}{H_H}. \tag{A.144}$$

Based on these results it is straightforward to solve for the other steady states values.