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STRENGTH OF WEBS OF I-BEAMS AND GIRDERS

BY
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 86 MAY, 1916

THE STRENGTH OF WEBS OF I-BEAMS AND GIRDERS

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THE STRENGTH OF THE WEBS OF I-BEAMS AND GIRDERS

I. Introduction.

1. *Preliminary.*—In designing beams and girders it is usual to consider that the bending action is resisted by the flanges, and the shearing stress by the web. There is a tendency, however, for webs of deep girders to fail by buckling; and there are complex stresses set up at the junction of the web and flange. There are also crushing stresses in the web over the supports of the girder or under the points of application of concentrated loads.

When a girder is subjected to flexure the material on one side of the neutral axis is subjected to longitudinal tensile stresses and the material on the other side is subjected to longitudinal compressive stresses. The material at any point in the girder is also subjected to longitudinal and to transverse shearing stresses of equal intensity. The longitudinal tensile and compressive stresses are equal to zero at the neutral axis and increase to a maximum at the outer edges of the flanges. The shearing stresses are equal to zero at the outer edges of the flanges and increase to a maximum at the neutral axis. Since the shearing stress is equal to zero at the outer edge of the girder the flange carries little and the web practically all of the shear on a transverse section.

In the design of a girder the longitudinal stress in the outer edge of the flange is limited to a safe value for the material in tension or compression, and the average shearing stress in the web (obtained by dividing the maximum total shear upon a transverse section by the cross-sectional area of the web) is kept within safe limits for the material in shear. Although, according to the elastic theory of beams, points intermediate between the neutral axis and the outer edge of the flange are subjected to both longitudinal tension or compression and to transverse and longitudinal shear, and although it is known that these combined stresses result in diagonal tensile (or compressive) and shearing stresses which are greater than the component stresses producing them, these diagonal stresses are not considered in the design of the girder. The view has been held that the diagonal tensile or compressive stresses do not materially exceed the simple longitudinal stress at the outer edge of the flange, and that the diagonal shearing stress does not materially exceed the shearing stress at the neutral axis.

It has been shown by tests that a tensile (or compressive) stress produces a strain in a direction at right angles to the the line of action of the stress, the term strain being here used to designate deformation and not stress which is used to designate an internal resisting force. Recent tests, notably those of Dr. Becker,* indicate that the tendency of a material to fail depends, within certain limits, upon the *strain,* and that a stress in one direction, while it does not set up lateral *stress,* does set up lateral *strain,* and affects the strength of the material. An analysis of diagonal stresses in a girder shows that, in general, a load which sets up a stress in a diagonal direction sets up a second stress at right angles to that direction. Hence, in considering the strength of a girder to resist a diagonal stress any stress at right angles to that diagonal stress must be taken into account.

The tests reported in this bulletin were made to study the web strains in I-beams and girders so designed that the primary failure would be a web failure. The test data obtained were used in conjunction with a mathematical analysis made to determine the importance of the diagonal strains and the methods of failure of girders.

2. *Previous Tests of the Web Strength of I-Beams and Girders.*—Not many tests of I-beams and girders in which the primary failure was web failure have been made. Table 1 (reprinted from bulletin 68 of the Engineering Experiment Station of the University of Illinois) gives the results of a few such tests.

3. *Acknowledgment.*—This Investigation was a part of the research work of the Department of Theoretical and Applied Mechanics and of the Department of Civil Engineering, and was conducted under the general supervision of Professor A. N. Talbot, of the Department of Theoretical and Applied Mechanics, and Professor I. O. Baker, of the Department of Civil Engineering. Wherever reference has been made to the work of other investigators, credit has been given in the text.

II. MATHEMATICAL ANALYSIS OF STRESS AND STRAINS IN WEB MEMBERS.

4. *Notation and Units.*—The following notations are used:

S=tensile or compressive stress (lb. per sq. in.); (various subscripts are used to denote stresses in various parts of the web).

S_s=shearing stress (lb. per sq. in.); (shearing stresses in various parts of the web are denoted by S'_s, S''_s, etc.).

*Bulletin No. 85, Engineering Experiment Station, University of Illinois.

P = force acting upon a girder or any small element (pounds).

ϕ = angle between the diagonal plane and the plane of one side of a element of a girder.

E = modulus of elasticity in tension or compression (lb. per sq. in.).

F = modulus of elasticity in shear (lb. per sq. in.).

ϵ = unit-strain in inches per inch (an abstract number).

M = bending moment at any section of a girder (pound-inches).

I = moment of inertia of the cross-section of a girder (inches)^4.

c = distance from the neutral axis of a girder to the outer fibers (inches).

V = vertical shear at any section of a girder (pounds).

t = thickness of the web of a girder (inches).

$(a_1\,c_1)$ = "static moment" of any portion of the cross-sectional area of a girder about the neutral axis (inches)^3.

$\left(\frac{l}{r}\right)$ = slenderness ratio for the web of a girder acting as a long column under compression (an abstract number).

h = clear depth of web of girder between flanges (inches).

b = length of a bearing block over a support or under a concentrated load (inches).

Δ_f = deflection of a girder due to direct flexure (inches).

Δ_s = deflection of a girder due to shear (inches).

Δ = deflection of a girder due to both flexure and shear (inches).

a = area of the cross-section of a girder (square inches).

l = length of span of a girder (inches).

l_1 = distance from support of a girder to nearest concentrated load (inches).

λ = Poisson's ratio, the factor of lateral strain (an abstract number).

Throughout this bulletin the term *strain* is used to denote the deformation caused by stress; it is not used as a synonym for stress.

5. *Common Formulas of Girder Design.*—The tensile or com-

TABLE 1.
WEB FAILURE OF I-BEAMS
Reprint from Bulletin 68 of the Engineering Experiment Station of the University of Illinois

1. Beam	12-in., 31.5-lb. I-Beam	12-in., 31.5-lb. I-Beam. Web Planed Thin	12-in., 31.5-lb. I-Beam. Web Planed Thin	12-in., 31.5-lb. I-Beam. Web Planed Thin	Beth'm Girder Beam 30-in., 175-lb.	20-in. Special* Built-up Girder
2. Span, ft.	2.92	3.00	3.00	3.00	20	15
3. Loading	Two Points 4⅞ in. Each Side of Center	Two Points 4⅞ Each Side of Center	Two Points 4⅞ Each Side of Center	Two Points 4⅞ Each Side of Center	Quarter Points	One Load at Quarter Point
4. Material	Steel	Steel	Steel	Steel	Steel	Steel
5. Number tested	2	1	1	1	3	1
6. Tested by	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Marburg (Univ. of Pa.)	Turneaure (Univ. of Wis.)
7. Fiber stress due to direct flexure, lb. per sq. in.	33,500	28,200	19,300	12,700	31,000	Sidewise buckling prevented by bracing
8. Vertical distance between flanges, inches (h)	10.52	10.52	10.52	10.52	26,8	14
9. Thickness of web, inches (t)	0.35	0.28	0.19	0.16	0.69	0.14
10. Slenderness ratio for web $\frac{l}{r} = \sqrt{12}\,\frac{h \sec. 45^\circ}{t}$	148	184	272	324	190	490
11. Load at failure, pounds	190,100	160,500	109,600	72,100	538,400	106,000 (Approx.)
12. Computed fiber stress (shear and also compression) at middle of web, lb. per sq. in.	25,800	27,200	27,400	21,400	14,800	26,500
13. $\frac{4\pi^2 E}{(\frac{l}{r})^2}$	53,900	34,900	16,000	11,300	32,700	4,900
14. Length of block under support, inches	6	6	6	6	12	....
15. Computed fiber stress (compression) in web adjacent to support, lb. per sq. in.	45,300	47,800	48,200	37,600	32,500	Stiffeners used at ends and under load
16. Yield-point strength of material at root of flange, lb. per sq. in.	31,700	33,100	34,000	32,200	28,200	37,700

***Test reported in full in the Journal of the Western Society of Engineers for 1907, p. 788. Failure by sidewise buckling was prevented by bracing. The load at failure is given by Dean Turneaure as that at which very great distortion had taken place and noticeable buckling in the web occurred. Excessive compressive stress in the web adjacent to reactions and concentrated loads was prevented by using stiffeners, well fitted to the flanges.**

pressive stress in the outer fibers of the flanges of a girder is denoted by the equation

$$S = \frac{M\,c}{I} \qquad (1)$$

in which S is the stress in the outer fiber, M the bending moment, I the moment of inertia of the cross sectional area of the girder and c the distance from the neutral axis to the outer fiber. The longitudinal or the transverse shearing stress at any point in a girder is given by the formula

$$S_s = \frac{V}{I\,t}(a_1\ c_1) \qquad (2)$$

in which S_s is the shearing stress, V the total shear on a vertical section through the point, I the moment of inertia of the cross-sectional area of the girder, t the thickness of the beam at the point considered, a_1 the area of that part of the cross-section between the point and the extreme fiber, and c_1 the distance from the center of gravity of the area a_1 to the neutral axis of the beam.

For I-beams and built-up girders it is customary to use an approximate method for obtaining the shearing stress at any transverse section of the web; the total shear V is divided by the cross-sectional area of the web, and the quotient is taken as the shearing stress.

For the derivation of the above formulas the reader is referred to any standard author on the mechanics of materials, such as Merriman, Boyd, Murdock, Slocum, or Morley.

6. *Discussion of the Theory of Web Stress.*—The stresses on a transverse section of a beam in flexure consist of shearing stresses parallel to the section and tensile and compressive stresses normal to the section. The stress on a longitudinal section normal to the force plane consists of a shearing stress parallel to the section. Fig. 1 (a) represents a small element of a beam in flexure. Before the beam was subjected to flexure the element was rectangular. The plane ab, normal to the paper, is a transverse plane, and cb is a longitudinal plane normal to the force plane. The forces acting upon the element will produce a stress upon the oblique plane ac which makes an angle ϕ with bc. This stress may be resolved into two components, one parallel and the other normal to ac. The relation between the stresses on the diagonal plane and the stresses on the faces of the element depends upon the value of ϕ. Let Fig. 1 (a) represent any element having a length dx, a width dy, and a thickness normal to the paper equal to

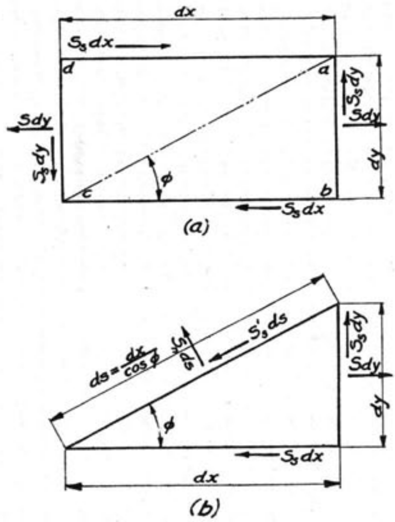

FIG. 1. STRESSES ACTING ON AN ELEMENT OF A GIRDER

unity. The normal stress per unit area on the ends of the element is represented by S and the shearing stress per unit area on the sides of the element, which are normal to the paper, is represented by S_s. The normal stress per unit area on the oblique plane (see Fig. 1 (b)) is represented by S_n and the tangential stress is represented by S'_s. It can be proven that*

$$S_n = \tfrac{1}{2}S\ (1 - \cos 2\phi) + S_s \sin 2\phi \quad \text{.................. (3)}$$
$$S'_s = \tfrac{1}{2}S \sin 2\phi + S_s \cos 2\phi \quad \text{........................ (4)}$$

The stresses S_n and S'_s are functions of ϕ. Their maximum values are given by the equations*

$$\text{Max } S_n = \tfrac{1}{2}S \pm \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \quad \text{.................. (5)}$$
$$\text{Max } S'_s = \sqrt{S_s^2 + (\tfrac{1}{2}S)^2} \quad \text{........................ (6)}$$

It should be noted that Max S_n and Max S'_s are not in the same direction.

The stress S may be either tension or compression. When S is tension the plus sign before the radical is used to find the maximum tensile unit stress S_n, and the minus sign before the radical, to find the maximum compressive unit-stress S_n. The latter is always normal to the former.

If stresses S_1 and S_2 at right angles to each other are acting on small particles, S_1 causes strain at right angles to its direction and

*Merriman, "Mechanics of Materials," 10th ed., p. 264.

thus influences the total strain of the particle in the direction of S_2. If S_1 and S_2 are both tensions or both compressions, the strain in the direction of either stress is *diminished* by the lateral strain due to the other stress; if S_1 is a tension and S_2 a compression (or vice versa) the strain in the direction of either stress is *increased* by the lateral strain due to the other stress. Following what they believe to be the best practice, the writers use the term *stress* to mean an internal resisting force set up by the action of external forces. A stress in any direction causes a lateral *strain* at right angles to that direction, but, accepting the above definition of stress, does not cause a lateral stress.

Whether or not the strain at right angles to a stress influences the *strength* of a material has for a long time been a subject of discussion among students of the mechanics of materials. Three theories of the strength of materials under combined stresses have been advanced:

(a) The strength of a material depends only on the normal *stress.* If this theory is accepted, the transverse strain produced by a stress does not affect the strength, and a particle under the action of two stresses at right angles to each other is just as near failure (and no nearer) as it would be under the action of the greater of the two stresses acting alone. This is called the "maximum stress theory."

(b) The strength of the material is dependent upon the *strain.* If this theory is accepted, a particle under the action of stress in one direction is in greater danger of failure than it would be under the action of two stresses of no greater magnitude, alike in sign, and acting in two directions at right angles to each other. This is known as the "maximum strain theory."

(c) The strength of the material depends on the maximum *shearing* stress set up by the action of the various stresses. This is known as the "maximum shear theory."

The latest, and in some respects the most conclusive, tests which bear on this subject are those of Becker (University of Illinois Engineering Experiment Station, Bulletin No. 85). Becker studied the behavior of thin tubes in which axial tension or compression was produced by means of a testing machine and transverse tension was set up by means of internal water pressure. He found that for steel the second theory, the maximum strain theory, held unless the maximum shearing stress developed was greater than about 0.60 of the tensile or compressive stress. Within certain limits the strength of the material seems to depend on the strain, and beyond those limits it seems to depend on the shearing stress.

For expressing the value of a strain in this bulletin, the numerical value of $E\epsilon$ (the product of the unit strain and the modulus of elasticity, 30,000,000 lb. per sq. in. for steel) is given instead of the numerical value of the strain. Thus, in speaking of the safe working strain for a bar in tension, instead of saying that the safe unit strain is 0.000533 inches per inch length, the writers have used the expression, the strain corresponding to a value of $E\epsilon$ of 16,000 lb. per sq. in. or, more briefly, the $E\epsilon$ value of 16,000 lb. per sq. in. It is thought that this method of expressing the strain enables the engineer, who is not in the habit of thinking in terms of strain, to compare more easily the combined strains in girders with the strains resulting from the simple stresses with which he is familiar. $E\epsilon$ is not a stress, but a simple stress in one direction only with a magnitude equal to $E\epsilon$ would produce the same structural damage to the material as is produced when the strain ϵ occurs.

In the following equation S_1 and S_2 are two stresses at right angles to each other* and λ is Poisson's ratio

$$E\epsilon = S_1 - \lambda S_2 \qquad (7)$$

$$E\epsilon_2 = S_2 - \lambda S_1 \qquad (8)$$

If a stress is compression it is to be taken as negative; if the strain is positive it is an elongation; if negative, a contraction. The stresses S_1 and S_2 produce shearing stresses along planes oblique to their lines of action, and it can be proven that the maximum shearing stress S'_s set up by the two stresses is given by the equation.†

$$S''_s = \tfrac{1}{2}(S_1 - S_2) \qquad (9)$$

III. Tests

7. *Specimens.*—The specimens of the series of tests reported in this bulletin (1914 series) comprised six 12-inch I-beams and two 24-inch built-up girders. The webs of the I-beams were planed thin and the webs of the girders were made of thin plates. The webs in all of these test specimens were thinner than those used in standard practice. Had standard practice been followed, the webs would have been so strong in comparison with the flanges that the primary failure would probably not have been a web failure, but a failure due to some other cause, such as direct flexure or sidewise buckling of the compression flange. To investigate the web strength of beams it was

*Lanza, "Applied Mechanics," pp. 868-869.
†Merriman, "Mechanics of Materials," 10th ed., p. 363.

necessary to use specimens in which web failure would be the primary failure. Hence, thin-webbed specimens were used.

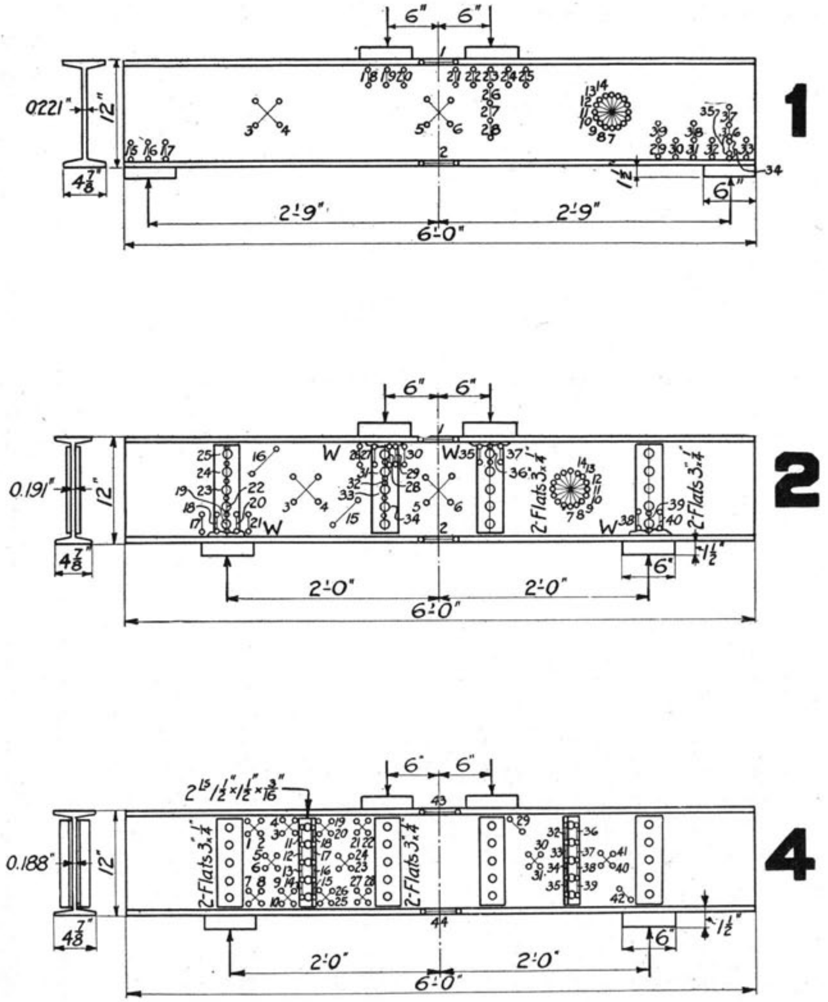

FIG. 2. I-BEAMS 1, 2, AND 4 SHOWING LOCATION OF GAGE LINES (W DENOTES AN OXY-ACETYLENE WELD)

Fig. 2 and 3 show the shape and size of the I-beam specimens. Beams 1 and 5 had no stiffeners; beams 2 and 6 had flat stiffeners on the web adjacent to the bearing blocks. Flats were used instead of angles so that there would be very little resistance to the buckling of

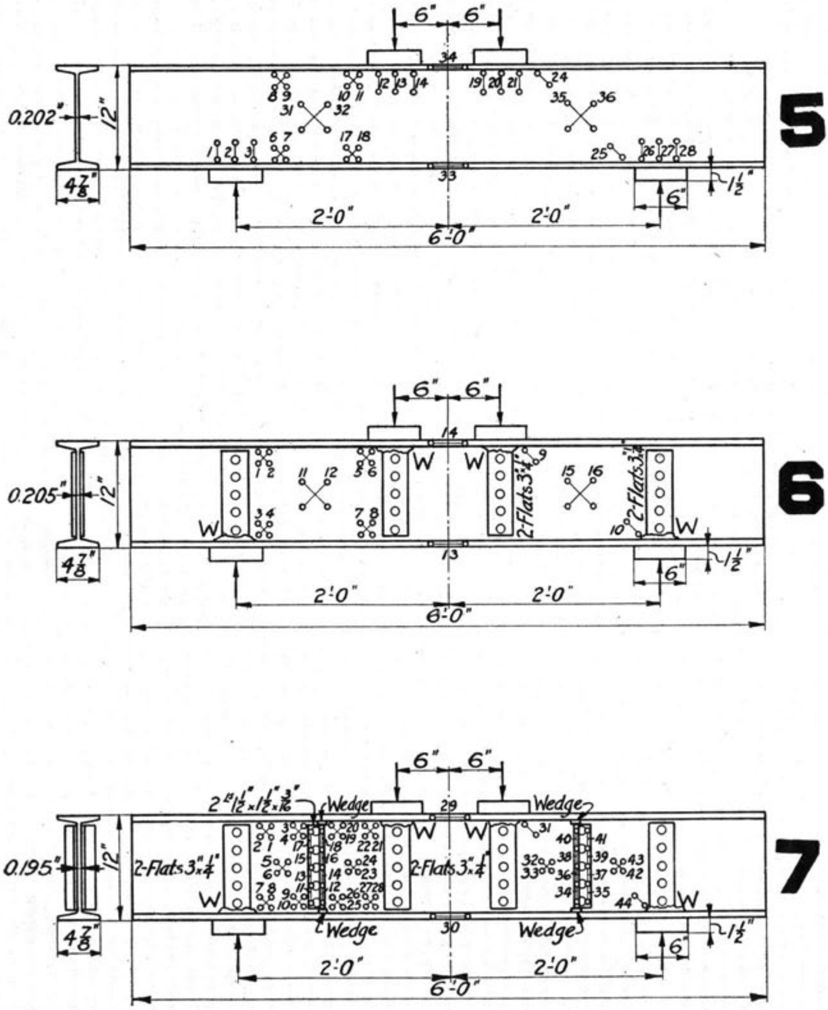

FIG. 3. I-BEAMS 5, 6, AND 7 SHOWING LOCATION OF GAGE LINES (W DENOTES AN OXY-ACETYLENE WELD)

the web. The flats were welded to the flanges to insure a good bearing. For beams 4 and 7 there were flat-stiffeners at the bearing blocks and angle stiffeners between bearing blocks. Fig. 4 and 5 show the built-up girders. Girder 8 had angle-stiffeners at the bearing blocks and at intermediate points; girder 3 had angle-stiffeners at the bearing blocks only.

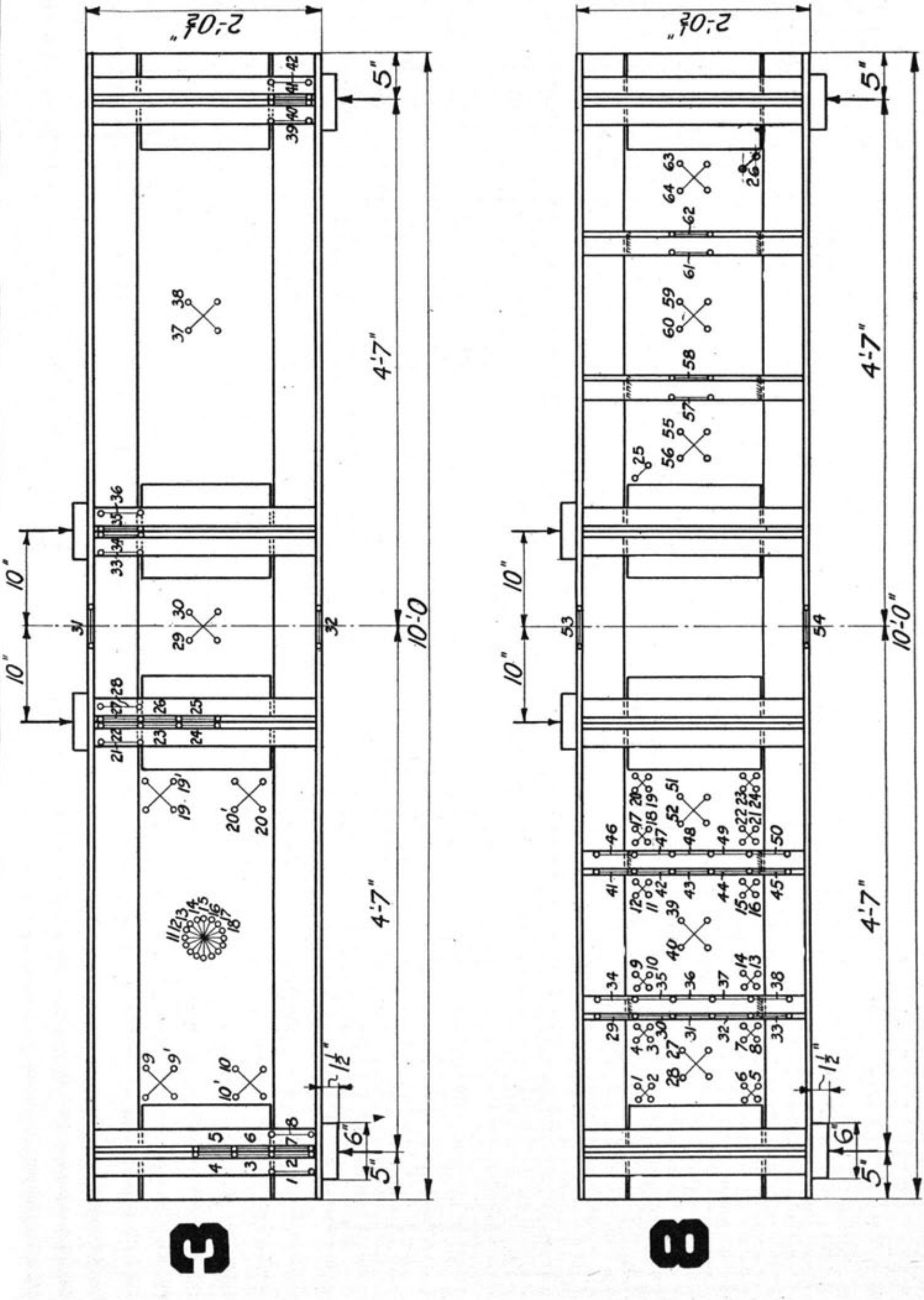

FIG. 4. BUILT-UP GIRDERS 3 AND 8 SHOWING LOCATION OF GAGE LINES

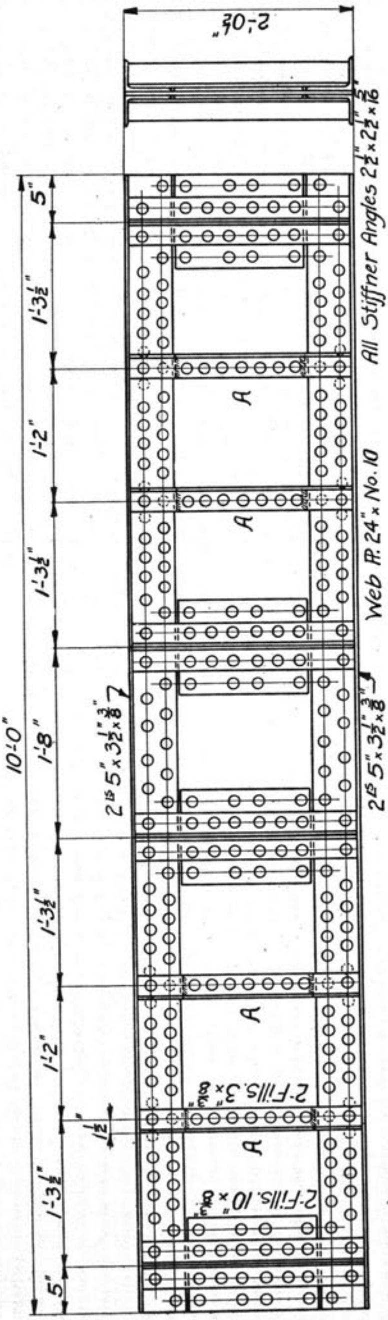

Fig. 5. Working Drawing of Built-up Girders 3 and 8 (Stiffeners marked A omitted for Girder 3)

For the purpose of determining the quality of the material in the beams, test pieces were cut from some portion of each beam which had not been under heavy stress during the test. For the I-beams the specimens were cut from the portion overhanging the end bearing. Tension specimens were cut from the webs, from the flanges, and from the points at which the webs join the flanges. Shear specimens were cut from the webs. For the girders these specimens were cut from the central portion of the webs where the shear was zero. A typical tension specimen is shown in Fig. 6 and a typical shear specimen in Fig. 7.

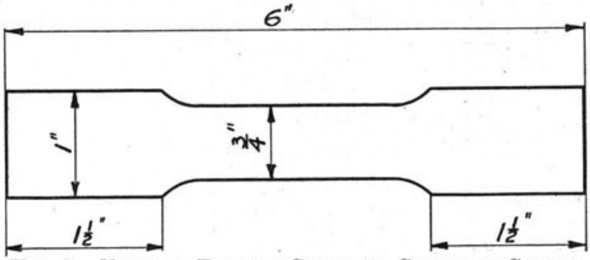

FIG. 6. FORM OF TENSION SPECIMEN CUT FROM GIRDER

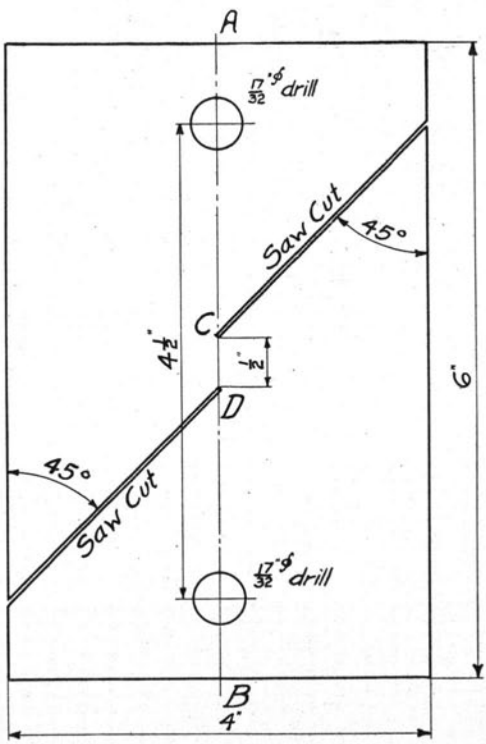

FIG. 7. FORM OF SHEAR SPECIMEN CUT FROM WEB OF GIRDER

The form of the shear specimen was suggested by Mr. Malcolm Westergaard. When axial pull is applied to the shear specimen along the line *AB* (Fig. 7) shear occurs along the line *CD*.

8. *Apparatus.*—Beam 1 was tested in a 200,000-lb. Olsen vertical-screw testing machine. The other tests were made in a 600,000-

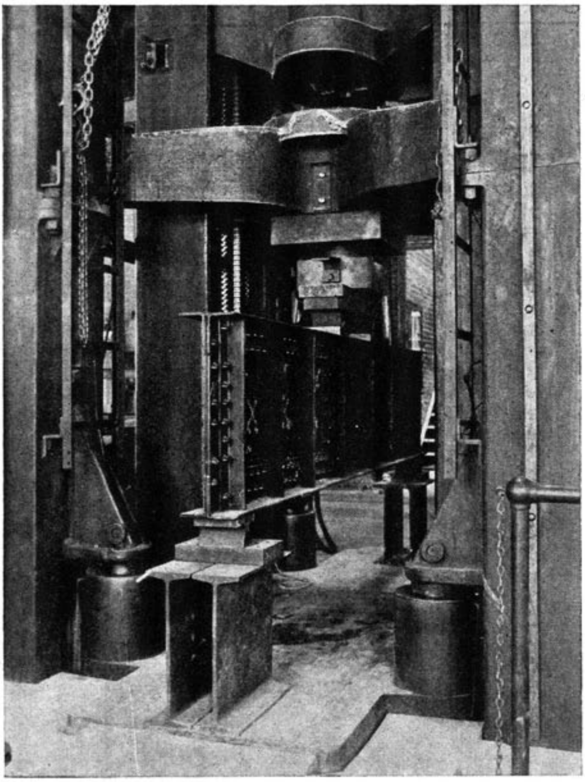

FIG. 8. GIRDER IN TESTING MACHINE IN POSITION FOR TESTING

lb. Riehle vertical-screw testing machine. In all tests, loads were applied at two points equidistant from the center of the span. Fig. 8 shows a specimen in the testing machine.

Strains were measured by means of Berry strain gages. The locations of the gage lines for the different strain measurements are shown in Fig. 2, 3, and 4. Each gage line is denoted by two small circles, one for each end of the line, joined by a straight line. The gage lengths used were two inches and four inches, and, as Fig. 2, 3, and 4, are reproduced to scale, the length of any gage line is apparent. Each gage line was given an identifying number as shown in the figures. For each gage line on one side of the specimen there was a mating line directly opposite on the specimen. The lines for the opposite sides of the specimen were distinguished by the letters *E* or *W*, *N* or *S*, following the line number.

The deflections of beams 3, 4, 5, 6, 7, and 8, were measured at the middle of the span. Several forms of deflectometer were used, the most convenient being the level bar shown in Fig. 9. The point *A* of

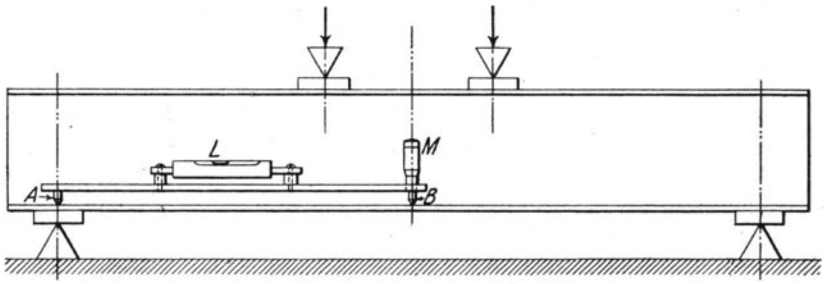

FIG. 9. LEVEL BAR FOR MEASURING DEFLECTION OF GIRDER. (THE POINT *A* CAN BE SCREWED INTO VARIOUS HOLES ALONG THE BAR FOR DIFFERENT LENGTHS OF SPAN)

the instrument is set on the beam over the end bearing, and the point *B*, which is at the end of a micrometer screw *M*, is set at mid-span. With zero load on the beam the micrometer screw is adjusted, raising or lowering the point *B*, until the level bubble *L* is in mid-position. With any load on the beam the same process is repeated, and the difference between this micrometer reading and the reading for zero load gives the deflection at mid-span. The point *A* is placed in a prick-punch hole, and the point *B* in a cold-chisel mark made along the flange of the beam. The sensitiveness of the level bubble was such that twenty seconds change of angle from the horizontal caused the bubble to move one division of its scale.*

9. *Data and Results.*—Readings of the strain gages and readings of the deflectometer were taken for various loads up to the ultimate. The strain gage readings were corrected for variation of temperature by means of an unstressed standard bar.† The value of $E\epsilon$ along

*Since these tests were made a later form of level bar has been constructed in the shops of the Laboratory of Applied Mechanics. In this new level bar, designed by Mr. H. R. Thomas, the screw micrometer shown in Fig. 9 has been replaced by a leveling screw which actuates the plunger of a direct reading dial gage micrometer. This later form of level bar is much quicker in operation than the earlier form.

†For detailed discussion of the use of the strain gage see: Proceedings of the American Society for Testing Materials, 1913, Slater and Moore on "The Use of the Strain Gage in Testing Materials"; also, Bulletin No. 64 of the Engineering Experiment Station of the University of Illinois, "Tests of Reinforced Concrete Buildings under Load," by Talbot and Slater.

each gage line was determined from the strain gage readings assuming a modulus of elasticity of 30,000,000 lb. per sq. in. Curves were plotted for each beam with values of applied loads as ordinates and values of $E\epsilon$ along the gage lines, determined from the strain page readings, as abscissas. These curves are given in Fig. 14-22. The load at failure for each beam is given in Table 2. Fig. 10, 11, and 12 are reproduced from photographs of the beams after failure.

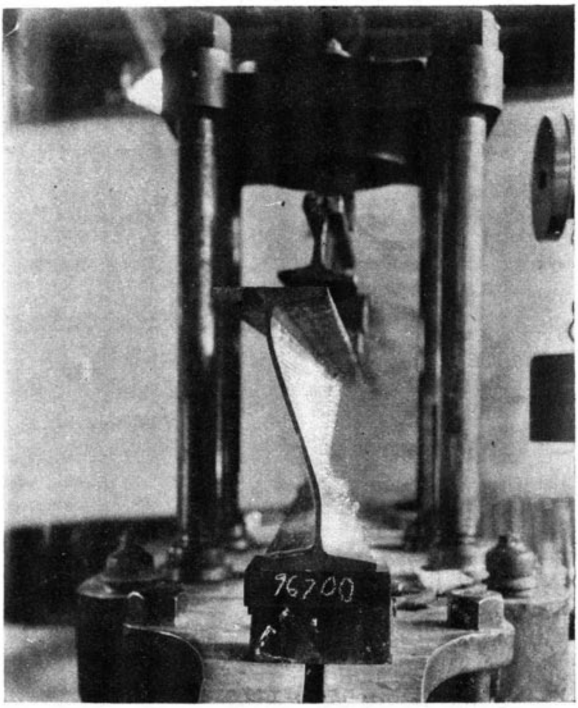

FIG. 10. I-BEAM 1 AFTER FAILURE

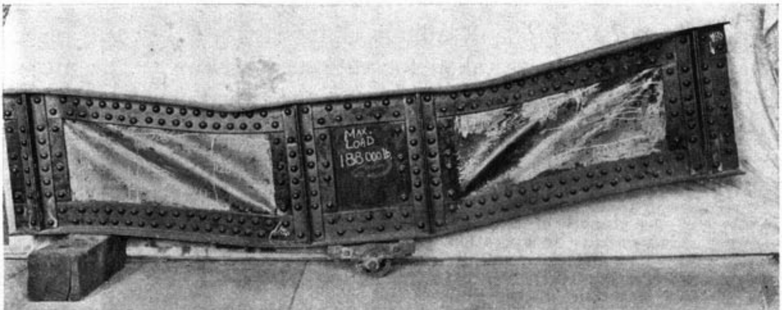

FIG. 11. GIRDER 3 AFTER FAILURE

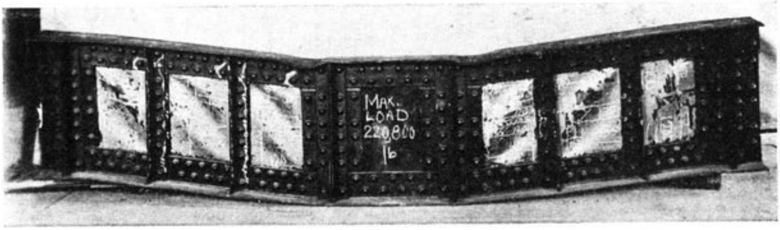

FIG. 12. GIRDER 8 AFTER FAILURE

The local yielding of the built-up girders is shown in Fig. 11 and 12. During the progress of the tests local yielding was made evident by the flaking of the mill scale and the paint on the specimen.

During a test three stages of structural failure were noted: (1) Local overstress, shown by flaking of paint and high local strain gage readings. (2) Signs of general yielding of the girder as a whole. (3) Final collapse under the ultimate load.

10. *Determination of the Yield Point of Girders.*—The writers believe that the second stage of failure, as given in the preceding paragraph, gives the best indication of the limit of load-carrying capacity of girders for static loads. The term "yield point of the girder" is used to designate this stage of general yielding. The general yielding of an I-beam or girder is shown by the "knee" of the load-deflection curve, and an examination of the load-deflection curves shown in Fig. 16-22 shows that the departure of a curve from a straight line is fairly well marked. In determining the yield point of a girder from the load-deflection curve the method of J. B. Johnson was used.* The yield

TABLE 2.

FAILURE OF GIRDERS (1915 TESTS)

Girder Number	Load at Yield Point of Girder, pounds.	Load at Ultimate, pounds.	Manner of Failure.
1	89,500	96,700	Local compression over bearing block followed by twist of web.
2	85,500	138,000	Wrinkling of web due to diagonal compression.
3	101,500	188,000	Wrinkling of web due to diagonal compression.
4	73,500	141,500	Wrinkling of web due to diagonal compression.
5	82,000	104,500	Local compression over bearing block followed by twist of web.
6	86,500	120,500	Wrinkling of web due to diagonal compression.
7	82,000	144,000	Wrinkling of web due to diagonal compression.
8	112,500	220,800	Wrinkling of web due to diagonal compression.

*J. B. Johnson's method consists in finding, by means of drawing a tangent line, the point on any stress-strain curve (or load-deflection curve) at which the strain is increasing 50 per cent more rapidly than its initial rate of increase. See J. B. Johnson, "The Materials of Construction," pp. 18-20.

point obtained from the load-deflection curve does not differ greatly from a yield point obtained from a curve showing the average web-strains at mid-web. The yield point for those girders for which no deflection was measured was determined from the average web-strain (along a 45-degree gage line) at mid-web. The yield point of the girder occurs under loads greater than those causing the first evidence of local structural distress (shown by scaling of paint or excessive strain along individual gage lines) and at loads less than the ultimate. The yield point seems to be a fairly reliable criterion of the beginning of serious structural damage to the girder as a whole. The loads at the yield points, the ultimate loads, and the manner of failure are given in Table 2.

IV. Discussion of Results.

11. *Relation between the Actual and the Theoretical Strains.*—Equation (7) gives the theoretical value of $E\epsilon$ for any gage line in a girder. In this equation S_1 is the computed unit stress along a gage line, and S_2 the computed unit stress normal to the gage line and in the plane of the web. The values of S_1 and S_2 can be determined from equation (3).

To illustrate the use of these equations consider gage line 19 of girder 3 (see Fig. 4 and 16). A total load of 1,000 lb. on the girder will produce on the transverse section at the middle of gage line 19, a compressive stress S of 70.7 lb. per sq. in. as computed by equation (1), and a shearing stress S_s of 153.4 lb. per sq. in. as computed by equation (2). $\phi = -45$ degrees. Substituting these qualities in equation (3) gives

$$S_1 = \tfrac{1}{2}\ (-70.7)\ [1 - \cos(-90^\circ)] + 153.4 \sin\ (-90^\circ) =$$
-188.75 lb. per sq. in.

This is compression along gage line 19. S_2, the stress at right angles to S_1, is also given by equation (3). S and S_s are the same as for S_1, and $\phi = +45^\circ$. Substituting these values in equation (3) gives $S_2 = \tfrac{1}{2}(-70.7)\ (1 - \cos 90^\circ) + 153.4 \sin 90^\circ = 118.05$ lb. per sq. in. This is tension normal to gage line 19.

λ (Poisson's ratio) for steel is about $\tfrac{1}{3}$. Substituting values of S_1, S_2, and λ in equation (7) gives

$E\epsilon = -188.75 - \tfrac{1}{3}(118.05) = -228.10$ lb. per sq, in. for 1,000 lb. load on the girder.

This is slightly less than the value given by the average of the strain gage readings on the two sides of the web at gage line 19, as shown in Fig. 16.

A comparison of the strains as computed by the above method with the strains measured by the use of the strain gage may be made by means of the diagrams shown in Fig. 14-22. In these diagrams the full lines represent values of $E\epsilon$ corresponding to the strain as determined from the readings of the strain gage on the two sides of the girder and the dot and dash lines give the values of $E\epsilon$ as calculated by the formulas given in the preceding pages.

Comparison of measured strain with computed strain was made on 163 gage lines for the eight test pieces. Of these 163 gage lines, 40 were near bearing blocks where local compression materially modified the strains. For 107 of the remaining 123 gage lines the measured strain and the computed strain agreed closely, and for most of the other gage lines there were evidences of local bending action which might explain the discrepancy. The good general agreement between measured strain and computed strain furnishes an experimental confirmation of the theory on which the computation is based. This theory is therefore used as the basis of the computation of the maximum strains, and the shearing stresses in the girder webs.

12. *Maximum Shearing Stress in the Web.*—The longitudinal and transverse shearing stresses in the web of a girder are a maximum at the neutral axis at which the tensile and compressive stresses are zero. Equation (6) shows that it may be possible to have a diagonal shearing stress greater than the longitudinal and transverse shearing stress at the neutral axis. The longitudinal and transverse shearing stresses vary inversely as the thickness of the girder; hence they are much smaller in the flange than in the web adjacent to the flange. The tensile and compressive stresses vary directly as the distance from the neutral axis. Therefore, in the case of the girders under consideration, the maximum diagonal shearing stress is either at the neutral axis or at the inner edge of the flange and under a load. The maximum diagonal shearing stress can be determined by the use of equation (6). The shearing stresses in girders 1 to 8, calculated by the above methods, are given in Table 3.

In all I-beams, except No. 1 and No. 5, there are stiffeners under the loads. The stresses given in Table 3 are at the outer edges of these stiffeners. The moment is slightly greater immediately under the load, but the stiffener helps the web to resist the shear at this point.

The quantities used in the calculation of the maximum shearing stresses are also given in Table 3; the longitudinal tensile or compressive stress in the web at the inner edge of the flange is given in Column 2; the longitudinal and transverse shearing stresses at the same

TABLE 3.

SHEARING STRESSES IN WEBS OF GIRDERS

All stresses are given in pounds per square inch

No. of Test	Stress for 1000 Pounds Load on the Girder				Maximum Shearing Stress in Web at Yield Point of Girder.	Maximum Shearing Stress in Web at Ultimate of Girder.	Yield Point in Shear of Material in Web (from Tests of Specimens).
	Longitudinal Stress at Inner Edge of Flange (Tension or Compression)	Longitudinal and Transverse Shearing Stress at Edge of Flange.	Maximum Diagonal Shearing Stress in Web.	Longitudinal and Transverse Shearing Stress at Neutral Axis‡			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	353	172	246*	206	22,000	23,800	24,400†
2	219	203	230	236*	20,200	32,600	26,500†
3	108¶	151	161*	158	16,300	30,300	25,700
4	220	206	234	240*	17,600	34,000	25,400†
5	236	190	224*	224*	18,300	23,400	25,100†
6	217	190	219	224*	19,400	27,000	26,100†
7	219	198	226	232*	19,000	33,400	23,700†
8	108¶	151	161*	158	18,100	35,500	26,100

*Maximum shearing stress in the web of the girder.
†Taken as 0.60 of the yield point in tension. The yield point in shear was not well defined for these particular specimens.
‡Computed by Equation (2).
¶Stress on center line of inner row of rivets.

point are given in Column 3; the maximum shearing stresses in the web at the inner edge of the flange in Column 4; and the shearing stresses at the neutral axis are given in Column 5. The maximum shearing stress in the web, the greater of the quantities in Columns 4 and 5, is indicated by an asterisk. All of the above stresses are given in lb. per sq. in. per 1000-lb. load on the girders. The maximum shearing stresses in the web when the beam is loaded to the yield point are given in Column 6.

A comparison of Columns 3 and 4 of Table 3 shows that, except for I-beam 1, the diagonal shearing stress in the web at its junction with the flange is not materially greater than the longitudinal and transverse shearing stress at the neutral axis. In the case of I-beam 1 the high diagonal shearing stress is due to the fact that the longitudinal tensile (or compressive) stress at the point at which the flange joins the web is much greater than the transverse and longitudinal shearing stress at the same point.

To illustrate the possible importance of diagonal shearing stresses consider the following numerical example. A girder is supported upon end supports 70 ft. apart. It carries two concentrated loads of 450,000 lb. each, one located 9 ft. 8 in. from each end support. The girder is made up of one web plate 90 in. x $\frac{1}{2}$ in., and (for each flange) two 6 in. x 6 in. x $\frac{3}{4}$ in. angles, and two cover plates 14 in. x $\frac{3}{4}$ in.

Neglecting the weight of the girder, the shear is constant between a load and the adjacent support, and is equal to 450,000 lb. The maximum bending moment occurs under a load and is equal to 450,000 x 116=52,200,000 pound-inches.

The web area equals 90×0.5 or 45 sq. in. and the average shearing stress as usually figured is 10,000 lb. per sq. in. The moment of inertia of the net section is 153,100 (in.)^4 and the distance from the neutral axis to the outer edge of the flange is 46.75 in. The longitudinal stress in the outer fiber, as given by equation (1), is, therefore, 15,930 lb. sq. in.

The maximum diagonal stress occurs at the point at which the flange is connected to the web. For shear consider this point to be on the center line of the inner row of rivets, a distance of $45.25-4.75=40.50$ in. from the neutral axis. The longitudinal stress varies directly as the distance from the neutral axis and at the point in question is equal to 13,800 lb. per sq. in. Consider the shear at the neutral axis on a transverse section under a load. The unit shearing stress at any point is given by equation (2). For the girder consider, $V=450{,}000$ lb., $I=183{,}200$ (in.)^4, $t=\frac{1}{2}$ in., and $a_1c_1=2{,}204$ (in.)^3 for the gross area. Then from equation (2) S_s equals 10,800 lb. per sq. in. Consider next the shear on the same transverse section on the gage line of the angles, a distance of 40.50 in. from the neutral axis. For this point the value of a_1c_1 is 1,794 (in.)^3, and V, I, and t have the same values as in the previous case. S_s equals 8,800 lb. per sq. in.

The maximum diagonal shearing stress at the edge of the angles, S_s^1, given by equation (6), is 11,200 lb. per sq. in. This value is 400 lb. per sq. in. or 3.7 per cent greater than the value of the shearing stress at the neutral axis (10,800 lb. per sq. in.).

In this girder the longitudinal tensile and compressive stresses are as high at a section at which maximum shear occurs as good practice will permit, and therefore the excess of the diagonal shearing stress at the inner edge of the flange over the transverse and longitudinal shearing stress at the neutral axis should, if ever, be of importance. Since this difference is only 3.7 per cent it would seem that the excess of diagonal shearing stress over shearing stress at the neutral axis is not, in general, of much importance. It is true that if the shearing stresses had been low at the inner edge of the flange while the longitudinal tensile and compressive stresses had been high, the diagonal shearing stress might have been materially greater than the shearing stress at the neutral axis. But the very supposition on which this condition is based; namely, that the shearing stresses should be low, makes it certain that they will not be the criterion of strength of the girder, and that the difference between the two low shearing stresses will be of no importance.

The shearing stress at the neutral axis, as computed by equation (2) is 10,800 lb. per sq. in.; whereas the value obtained by dividing the total shear by the gross area of the web (the method usually used in girder design) is 10,000 lb. per sq. in. The excess of the former value over the latter is about 8 per cent. For the I-beams and built-up girders tested the difference between the values obtained by the two methods of calculation is also about 8 per cent. The relation between the transverse and longitudinal shearing stress at the neutral axis, as obtained by equation (2), and the approximate value obtained by dividing the total shear upon a section by the area of the cross-section of the web depends upon the relation between the area of the cross-section of the web and the area of the cross-section of the flange. In the example given above, in which the area of cross-section of the web is not large compared with the area of cross-section of the flange, equation (2) gives values for the shearing stress about 8 per cent higher than those obtained by approximate method. Calculations for girders, in which the area of the web is large compared with the flange area, show that the difference between the sharing stresses as computed by the two methods may be as great as 20 per cent. Therefore, in the design of a girder, if the value obtained by dividing the total shear by the area of cross-section of web is more than 80 per cent of the allowable stress for the material in shear, a check computation of the shearing stress should be made, using the more exact formula, equation (2). The computing of the diagonal shearing stresses at the inner edge of the flange seems unnecessary.

13. *Maximum Tensile and Compressive Strains in the Web.*—The longitudinal tensile and compressive stresses increase with the longitudinal distance of a point from the support and vary as the transverse distance from the neutral axis of the girder. The longitudinal and transverse shearing stresses decrease as the distance from the neutral axis increases, but they are nearly as great at the point at which the flange joins the web as at the neutral axis. However, after this point is passed the decrease is very rapid. The maximum diagonal tensile and compressive stresses depend upon the shearing stress and upon the longitudinal tensile (or compressive) stress. In the case of the beams under consideration the maximum tensile and compressive diagonal stresses occur at the inner edges of the flanges, under the load point for beams which have no stiffeners, and at the outer edge of the middle stiffeners for beams which have stiffeners.

The maximum diagonal tensile or compressive stress is given by equation (5). The values of S and S_s to be used in this equation for a load of 1,000 lb. on the girder are given in Columns 2 and 3 of Table 4. A diagonal tensile or compressive stress at a given point in the web in one direction is accompanied by a second diagonal stress at the same point normal to the first. If the first is the maximum diagonal stress, the second is the minimum. If the longitudinal stress is tension, the maximum diagonal stress is given by equation (5) if the upper or plus sign is used, and the minimum diagonal stress is given when the lower or minus sign is used. In considering the strength of the web it is necessary to get the value of $E\epsilon$ corresponding to the maximum strain. The value of $E\epsilon$ is given in equation (7). Substituting in equation (7) the values of the maximum and the minimum diagonal stress for S_1 and S_2, as given by equation (5) there is obtained

$$E\epsilon=\frac{S}{2}\,(1-\lambda)+(1+\lambda)\,\sqrt{S_s^2+\left(\frac{S}{2}\right)^2}\,\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

Table 4.

Diagonal Tensile and Compressive Stresses in Webs of Girders

All stresses are given in pounds per square inch

No. of Test (1)	Longitudinal Stress in Web Adjacent to Flange for 1000 lb. Load on Girder. (2)	Shearing Stress In Web Adjacent to Flange for 1000 lb. Load on Girder. (3)	Maximum Value of E_ϵ along a Diagonal for 1000 lb. Load on Girder. (4)	Maximum Value of E_ϵ along a Diagonal at Yield Point of Girder. (5)	Compressive Stress along 45-degree Line at Neutral Axis at Yield Point of Girder. (S_y) (6)	Ratio of Clear Vertical Depth to Thickness of Web. (7)	Ultimate Buckling Strength of 45-Degree Strip by Euler's Formula for Fixed-Ended Columns (S_b) (8)	$S_y:S_b$ (9)
1	353	172	447	40,000	18,700	48	21,500	0.87
2	219	203	381	32,500	19,500	55	16,200	1.20
3	104*	151	250	25,400	20,000	101	4,800	4.16
4	220	206	385	28,300	17,300	56	15,800	1.09
5	236	190	377	30,800	17,100	53	17,400	0.98
6	217	199	364	31,400	18,300	52	18,200	1.00
7	219	198	375	30,700	18,800	54	16,900	1.11
8	104*	151	250	27,900	22,200	101	4,800	4.63

*Stress on center line of inner row of rivets.

The values of $E\epsilon$ corresponding to a load of 1,000 lb. on the girder are given in Column 4 of Table 4. The values of $E\epsilon$ at the yield point of the girders are given in Column 5 of Table 4, and are repeated in Column 3 of Table 5. The longitudinal stresses at the

TABLE 5.

COMPARISON OF LONGITUDINAL STRESSES IN EXTREME FIBER WITH MAXIMUM VALUES OF $E\epsilon$ FOR GIRDERS

No. of Test	Longitudinal Stress in outer Fiber of Girder at Yield Point of Girder* lb. per sq. in.	Maximum Value of $E\epsilon$ along a Diagonal at Yield Point of Girder lb. per sq. in.	Ratio of $E\epsilon$ along a Diagonal to Longitudinal Stress
1	36,100	40,000	1.11
2	21,400	32,500	1.52
3	15,800	24,600	1.56
4	18,400	28,300	1.53
5	22,200	30,800	1.39
6	21,400	31,400	1.46
7	20,600	30,700	1.49
8	17,500	27,000	1.54

*Computed by the commonly used flexure formula, Equation (1).

outer edges of the flanges are given in Column 2 of Table 5. Since the transverse stress at the outer edge of the flange is zero, the values in Column 2 of Table 5 represent also values of $E\epsilon$ for the outer fibers of the flanges, and since the values in Column 2 are for the same transverse section of the girder as those in Column 3, a direct comparison of the values of the two columns can be made. The ratios of the values in Column 3 to those in Column 2 are given in Column 4 of Table 5. An examination of these ratios shows that the maximum value of $E\epsilon$ along a diagonal in girder 1 is only slightly greater than the maximum longitudinal stress at the outer edge of the flange, but that for the other girders the values of $E\epsilon$ are much greater than the longitudinal stress at the outer edge of the flange. An examination of equation (5) shows that the relation between the two quantities depends on the relation between the transverse shearing stress and the longitudinal stress. This fact is apparent from Table 5 also, which shows that the values of $E\epsilon$ along a diagonal are not much greater numerically than the longitudinal stresses for cases in which the shearing stress is low compared to the longitudinal stress, but that for cases in which the shearing stress is high compared with the longitudinal stress the values of $E\epsilon$ along a diagonal are decidedly higher than the longitudinal stress at the outer edge of the flange. However, for the girders tested the ratio of the shearing stress to the longitudinal stress in the outer fibers is higher than ordinarily occurs in practice, and this fact tends to exaggerate the excess of the values of $E\epsilon$ over those of the longitudinal stress.

A consideration of the girder described in section 12 will illustrate the possible importance of the diagonal strain in a girder designed in accordance with current practice. The maximum diagonal strain occurs at a point at which the flange is connected to the web. For tensile and compressive strains this point is taken on the center line of the inner row of rivets, a distance of 4.75 in. from the backs of the angles, and 45.25–4.75=40.50 in. from the neutral axis. The longitudinal stress varies directly as the distance from the neutral axis, and at the point in question is equal to $15{,}930\times\frac{40.50}{46.75}=13{,}800$ lb. per sq. in. In computing the shearing stress at this point the numerical values to be used in equation (2) are the same as those used on p. 26, except the value of a_1c_1 which in this case is 1,794 (in.)^3. Using equation (2), $S_s=8{,}800$ lb. per sq. in.

The maximum value of $E\epsilon$ obtained from equation (10), is 19,530 lb. per sq. in. This is 3,600 lb. per sq. in. or 22.6 per cent greater than 15,930 lb. per sq. in., the longitudinal stress at the outer edge of the flange.

In this example the excess of the value of $E\epsilon$ along a diagonal over the longitudinal stress at the outer edge of the flange is so great (22.6 per cent) that it should have been considered in the design of the girder. This indicates that in the design of structural steel girders in which maximum shear and maximum moment occur *at the same transverse section of the web,* it is not safe to allow an average transverse shearing stress of 10,000 lb. per sq. in. and at the same time to allow a longitudinal stress of 16,000 lb. per sq. in. in the outer fibers of the flanges. It is evident that the diagonal strains in such girders should be given special attention.

14. *Buckling of Web.*—The tendency of webs of girders to fail by buckling is well illustrated by the action of the webs of girders 3 and 8 as shown in Fig. 11 and 12. The exact analysis of buckling action in the web of a girder would be extremely complicated, and the following approximate analysis is in common use: A narrow strip of web making an angle of 45 degrees with the longitudinal axis of the girder is regarded as a column carrying an average stress over its cross-section equal to the shearing stress at the neutral axis (which is equal to the compressive stress on a 45 degree line at the neutral axis—the maximum compressive stress at that point). The length of this column is taken as $h\sqrt{2}$, in which h is the clear depth of web. The column is regarded as fixed-ended. Since the web is thin

the slenderness ratio l/r is large for the strip, and Euler's column formula, which gives good results for very slender columns, may be used. Applying Euler's formula for fixed-ended columns, the computed web stress at failure by buckling S_c, becomes

$$S_c=\frac{1.64\ E}{\left(\frac{h}{t}\right)^2} \qquad (11)$$

The compressive stress at the neutral axis at the yield point of the girders tested is given in Column 6 of Table 4; the ultimate strength of a 45-degree strip as obtained by Euler's formula in Column 8; and the relation between these two quantities is given in Column 9.

It will be seen for all girders except 3 and 8 that the load at the yield point of the beam corresponds quite closely to the load for the failure of the web-column as given by Euler's formula. In the case of girder 3 the stiffeners at the ends apparently helped stiffen the whole web, though the unsupported length of web was much greater than is allowable under standard specifications. In the case of girder 8 the web was supported against buckling by stiffeners at intermediate points. It would seem that the webs of girders are capable of developing a shearing stress at the neutral axis equal to the ultimate stress on a 45-degree strip considered as a column, figured by Euler's formula for columns with fixed ends, unless the stress given by that formula is higher than the yield-point strength of the material in shear. An examination of Tables 1 and 3 shows that before the *ultimate* of any test girder had been reached the yield-point strength in shear of the web material was developed (unless failure was due to local compression of the web over bearing blocks), but that the *yield point* of the girder was reached before the web material was stressed to its yield point in shear. It should be kept in mind that the yield point of a girder is the practical limit of its load-carrying capacity.

15. *Local Web Compression Adjacent to Bearing Blocks.*—The exact determination of the stresses in the web of an I-beam or girder adjacent to a bearing block would involve a consideration of the combined effect of the shearing stresses and the longitudinal stresses in the web adjacent to a bearing block. It would also involve a knowledge of the law of variation of the transverse stress from the flange towards the neutral axis and of the distribution of the pressure along

the bearing block. Since exact knowledge of these conditions is lacking, it is thought that a simple, approximate treatment of the local stress adjacent to bearing blocks will serve the structural engineer as well as a more elaborate analysis, especially since the simple analysis gives results in fair agreement with tests.

The determination of the local compressive stress in the web of an I-beam or girder, adjacent to a bearing block is discussed in Bulletin No. 68 of the Engineering Experiment Station of the University of

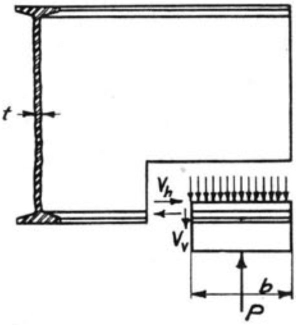

FIG. 13. DIAGRAM OF COMPRESSIVE STRESS IN WEB OF GIRDER OVER A BEARING BLOCK

Illinois. The following is quoted from that bulletin, with a few changes in notation:

"There may be an excessive compressive stress near the junction of web and flange and adjacent to a concentrated load or reaction. What has been referred to previously as failure by twisting of ends of I-beams is in most cases primarily caused by excessive local compression at the root of the flange.

"An approximate method of computing the compressive stress at the root of the flange adjacent to a concentrated load or an end reaction, has been given by C. W. Hudson as follows:* Imagine a small piece cut from the flange and web of an I-beam immediately over a bearing block (Fig. 13), and imagine this piece to be held in equilibrium by the elastic forces which act on it while it is in its place in the beam. The forces are (1) the pressure of the reaction at the bearing block P; (2) the compression in the web which equals $S_w tb$, when S_w=the average intensity of compressive stress, t= the thickness of web, and b=the length of bearing block; (3) a horizontal shearing force V_h; (4) a vertical shearing force V_v and (5) horizontal

*Engineering News, Dec. 9, 1909.

tensile or compressive stresses. Very little of the total shear would be balanced by the small internal shearing stress in the flange of an I-beam, and if the section considered be taken at the root of the flange, an equation may be written without serious error as follows:

$$V_v = V_h = 0$$

Then, the compressive stress on the web is balanced by the reaction on the bearing block. The compressive stress may be regarded as uniformly distributed, and an equation may be written as follows:

$$S_w = \frac{P}{bt} \qquad (12).$$

"In the above discussion the case considered is for the compressive stress adjacent to an end reaction. The reasoning for the compressive stress in the web adjacent to a concentrated load would be similar.

"It is unwise to regard the ultimate compressive fiber stress in the web adjacent to a bearing block as higher than the yield-point strength of the material at the root of the flange. Moreover, the fact should be borne in mind that the material at the root of the flange of an I-beam usually has a yield-point strength somewhat lower than that of the material in the flange or in the web. In the absence of special tests the yield-point strength of the structural steel at the root of the flange of an I-beam may be taken as about 30,000 lb. sq. in."

In comparing Hudson's analysis with the results of tests either of two methods of procedure may be used: (1) The fiber stress at failure, computed by Hudson's formula for beams which failed by local web compression, may be compared with the yield-point strength of the material at the root of the flange. (2) Strain gage measurements directly over bearing blocks may be compared with values given by Hudson's formula. Table 6 gives a statement of the results following the first method of procedure. From this table it is evident that for the beam tested by Marburg and for the 1913 series of beams tested at the University of Illinois, the fiber stress, computed by Hudson's formula, was slightly greater than the yield-point strength of the material at the root of the flange of the beams; and for the 1914 tests at the University of Illinois the stress, computed by Hudson's formula was slightly less than the yield-point strength of the material at the root of the flange.

Tests 1 and 5 of the 1914 series furnish the available data for a comparison of the results of strain gage measurements with the fiber stress as computed by Hudson's formula. It is necessary to make al-

TABLE 6.

FAILURE OF I-BEAMS BY LOCAL COMPRESSION IN THE WEB ADJACENT TO A BEARING BLOCK

I-beam	Tested by	Thickness of Web in.	Length of Bearing Block in.	Load at Failure lb.	Local Compressive Stress Adjacent to Bearing Block by Hudson's Formula lb. per sq. in.	Yield Point of Material at Root of Flange lb. per sq. in.	Remarks
30 in. 175 lb. Beth. Girder	Marburg	0.69	12	538,400	32,200	28,200	
12 in. 31.5 lb. with planed web	U. of Ill.	0.35	6	190,100	45,300	31,700	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.28	6	160,500	47,800	33,100	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.19	6	109,600	48,200	34,000	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.16	6	72,100	37,600	32,300	1913 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.22	6	96,700	36,500	38,000	Test 1 1914 series
12 in. 31.5 lb. with planed web	U. of Ill.	0.20	6	104,500	41,600	44,000	Test 5 1914 series

lowance for the fact that the strain gage readings are taken over a gage length of which the center is necessarily some distance from the root of the flange. The approximate relation between the strain and the distance from the root of the flange can be obtained by comparing the strains indicated by the readings on gage lines 34, 35, 36, 37, and 23, 26, 27, and 28 of Test 1 shown in Fig. 14.

Assuming that the strain varied directly as the distance from the root of the flange, the stress at the root of the flange would be about 1.25 times as great as the stress indicated by the strain gage readings nearest the flange. Table 7 gives the compressive stress in the web

TABLE 7.

COMPRESSIVE STRESS IN WEBS OF I-BEAMS ADJACENT TO A BEARING BLOCK

I-beam	Thickness of Web in.	Length of Bearing Block in.	Gage Line (See Figs. 2 and 3)	Stress for a Load of 1000 lb. lb. per sq. in.	
				From Strain Gage Measurements	Computed by Hudson's Formula
1–1914 series	0.221	6	16 at end	426	376
			35 at end	400	376
5–1914 series	0.202	6	2 at end	500	412
			27 at end	472	412

directly over the center of a bearing block as determined from the strain gage readings (corrected for distance from root of flange), and as computed by Hudson's formula. The results show an actual fiber stress over the center of the bearing blocks slightly greater than that given by Hudson's formula.

Considering both the results of tests to failure and results of strain gage tests, Hudson's formula seems fairly reliable.

The writers wish to call especial attention to the importance of considering the local compression over a bearing block for an I-beam or a girder. If there is not sufficient bearing area in the web, stiffeners must be provided to prevent the flange from folding over towards the web (Fig. 10). Tendencies toward such folding action were observed in several tests, and the inequality of the strains on opposite sides of the web (test 1, gage lines 18, 19, 20, 21, 22, 23, 24, test 5, gage lines 12, 13, 14, 19, and 20) indicates bending action as well as compression. Angle stiffeners placed over the supports as shown in Fig. 8 check this tendency to bend. It is of the highest importance that such stiffeners fit closely against the flange. An excellent illustration of this point is furnished by the behavior of the built-up girders during the tests. The stiffeners at points at which concentrated forces are applied, for girders 3 and 8 are angles having their outstanding legs ground to fit the outstanding legs of the flange angles, whereas the corresponding stiffeners for the I-beams are flats. Girders 3 and 8 maintained a vertical position much better than did the I-beams, indicating that it is an essential feature at points at which concentrated forces are applied, to have the stiffeners support the outstanding legs of the flange as much as possible.

16. *Functions of Stiffeners.*—Stiffeners, as commonly used on plate girders, perform two functions. Those placed at frequent intervals along the girders prevent the web from buckling as a column because of diagonal compression. Stiffeners placed under concentrated loads or over supports distribute the concentrated force, which would otherwise be delivered directly to the flange, over a considerable portion of the depth of the web. These latter stiffeners assist also in preventing the buckling of the web.

The spacing of stiffeners is usually determined more or less arbitrarily. The General Specifications of the American Railway Engineering Association for Steel Railway Bridges, 1910 edition, state, "There shall be web stiffeners generally in pairs, over bearings, at points of concentrated loading, and at other points where the thickness of the web is less than 1/60 of the unsupported distance between flange angles. The distance between stiffeners shall not exceed that given by the following formula, with a limit of six feet (and not greater than the clear depth of the web):

$$d = \frac{t}{40}\,(12{,}000 - s).$$

in which d=clear distance between stiffeners of flange angles, t=thickness of web, s=shear per sq. in."

According to these specifications the spacing of the intermediate stiffeners is in a general way a function of the shearing stress; but stiffeners are required, no matter how low the stress may be, if the ratio of the unsupported width to the thickness of the web exceeds 60.

The ratio of the unsupported width to the thickness of the web for the I-beams tested was less than 60 in all cases. For girders 3 and 8 this ratio was 100, and therefore, according to the specifications quoted, intermediate stiffeners were required. Girder 8 was equipped with intermediate stiffeners in accordance with the specifications; whereas girder 3 was provided with stiffeners only at points at which concentrated forces were applied. The load at the yield point for girder 3

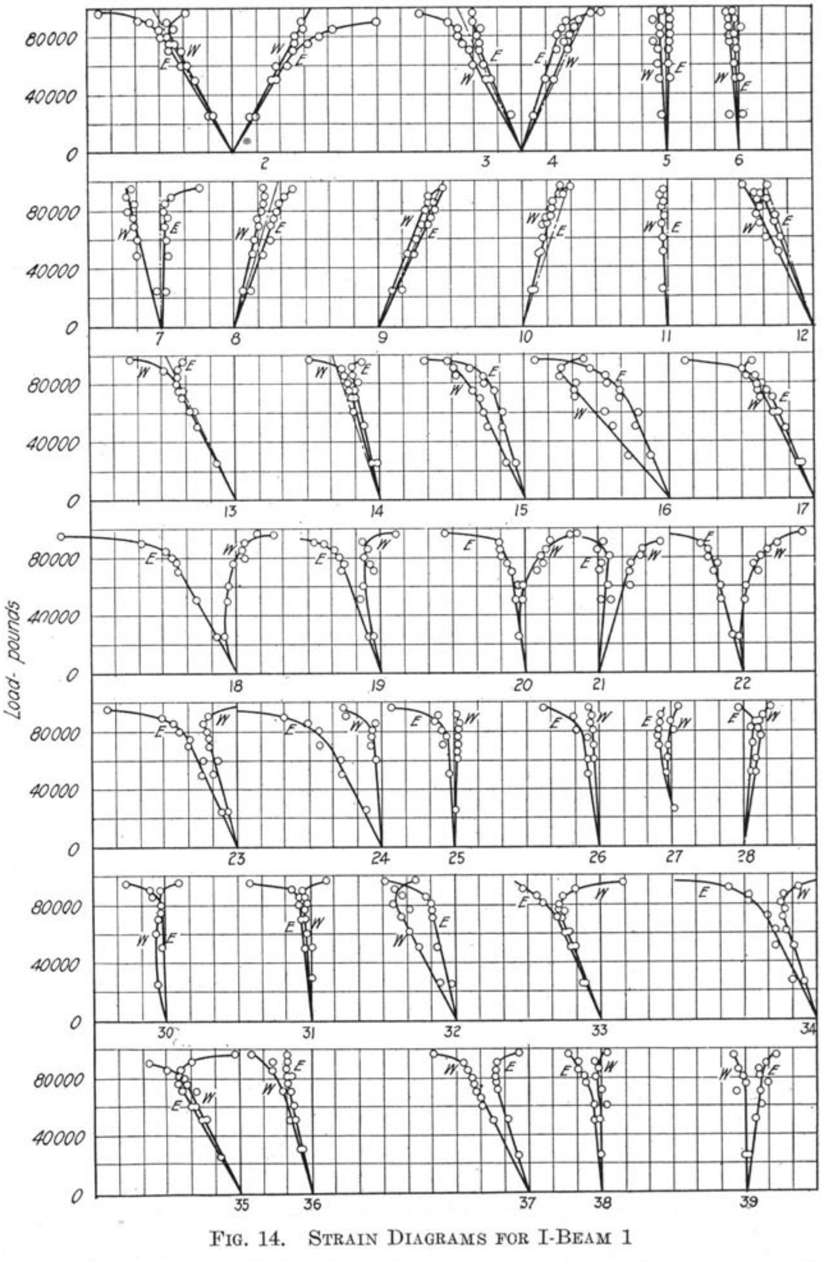

FIG. 14. STRAIN DIAGRAMS FOR I-BEAM 1

Note: 1 horizontal division=10,000 lb. per sq. in. as indicated by the strain gage. Numbers refer to gage lines as given in Fig. 2. Tension plotted to right; compression to left. Measured quantities ————; computed quantities —— — —— — ——.

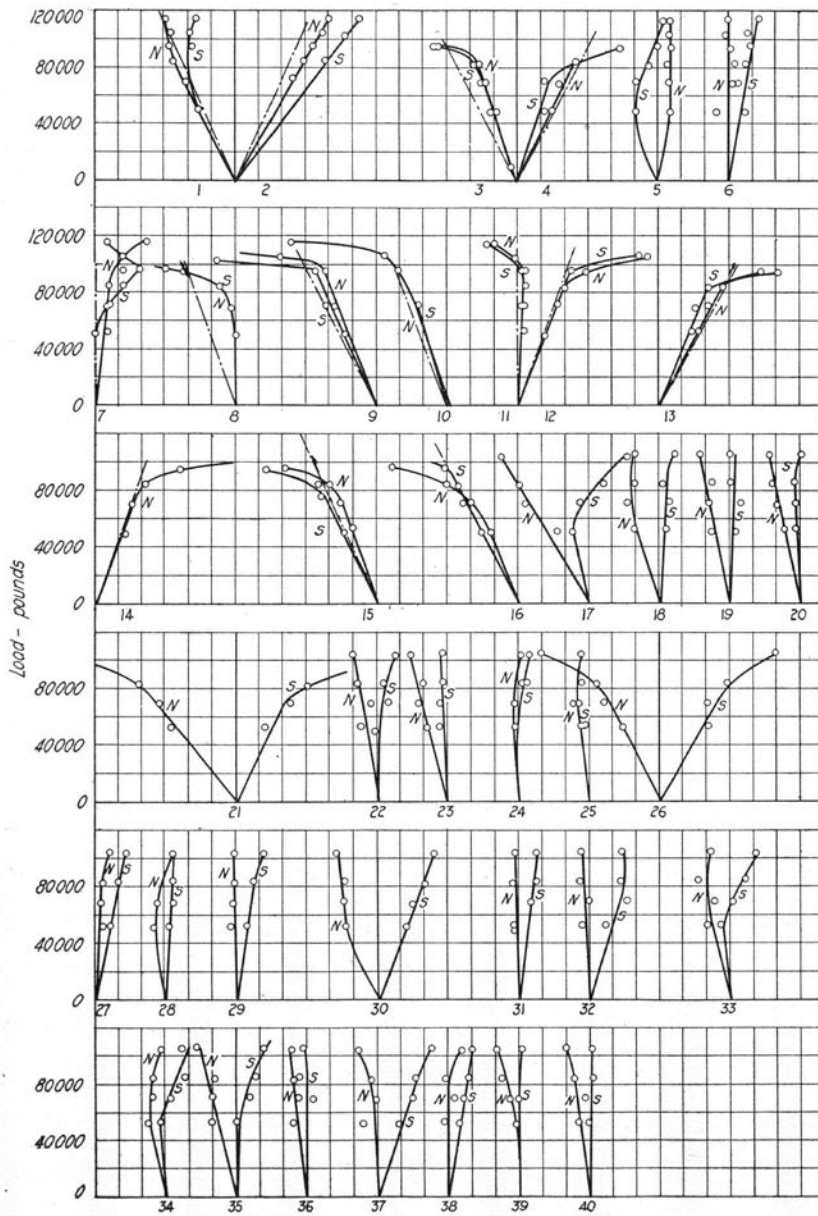

Fig. 15. Strain Diagrams for I-Beam 2

Note: See Fig. 14.

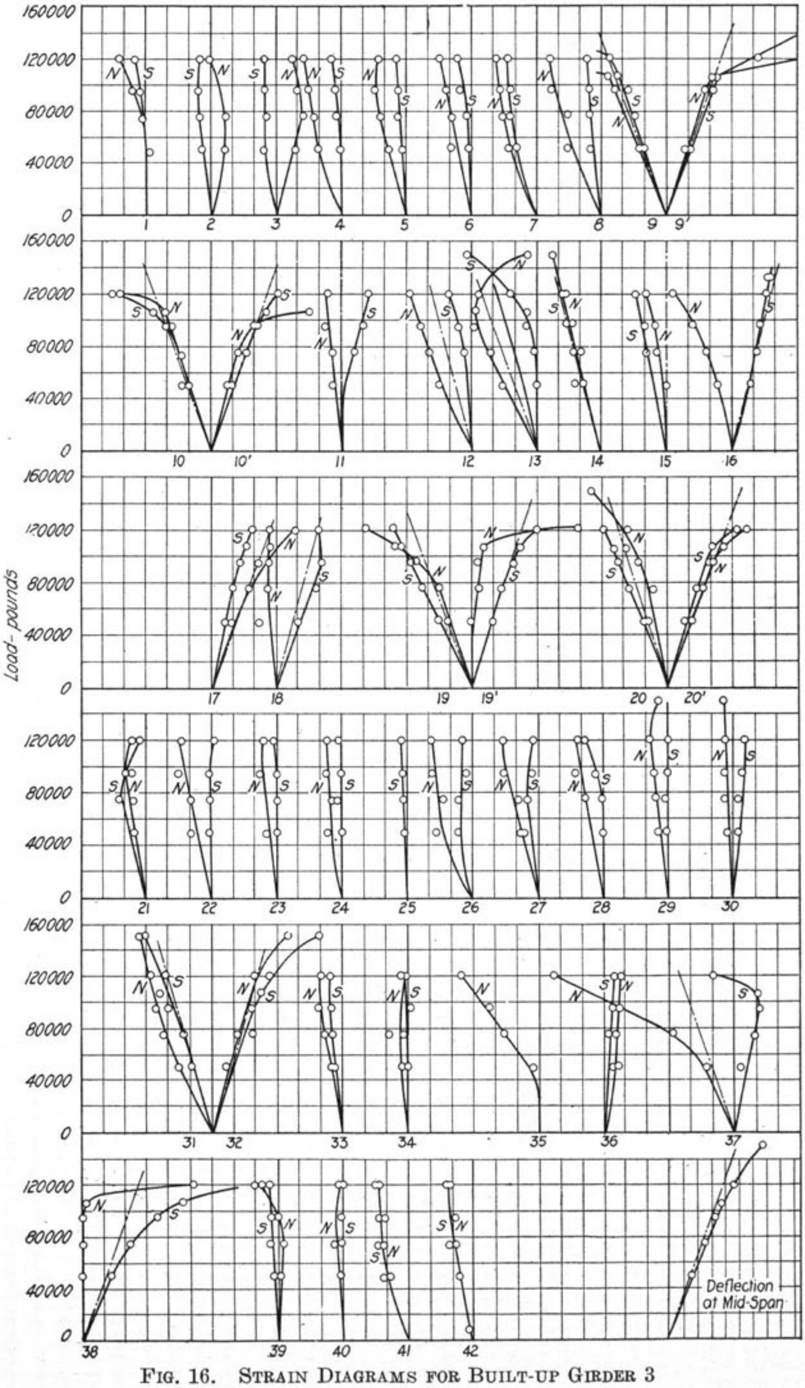

FIG. 16. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 3

Note: See Fig. 22.

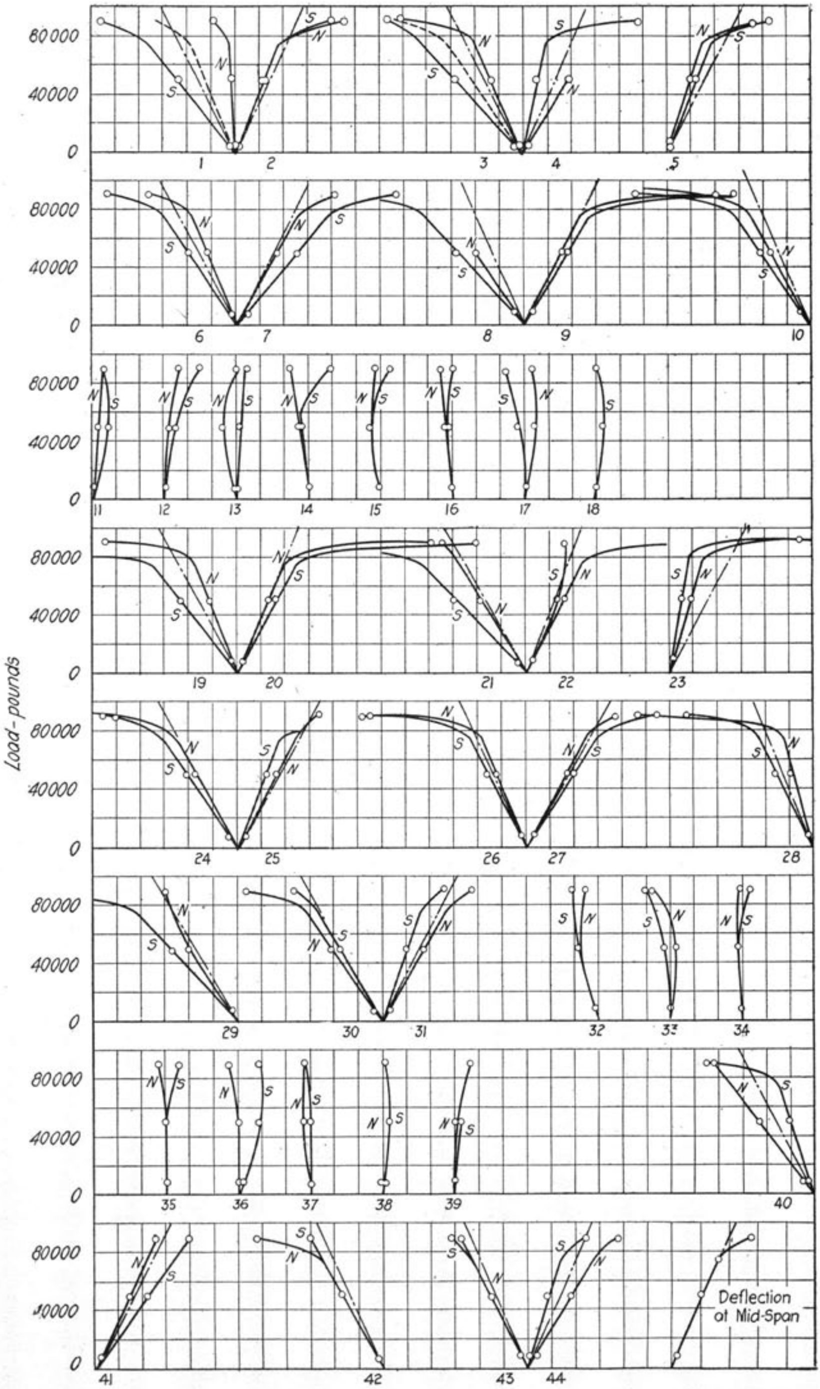

FIG. 17. STRAIN DIAGRAMS FOR I-BEAM 4

Note: See Fig. 14.
1 horizontal division = 0.02 inch (for deflection curve only)

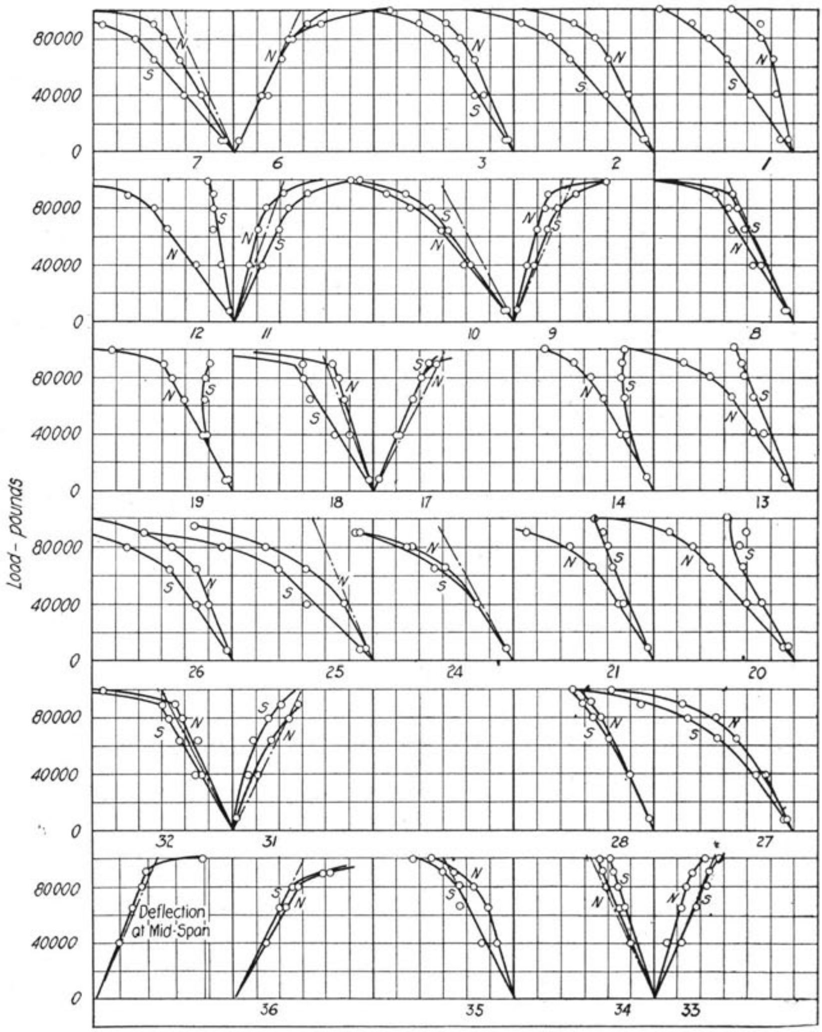

FIG. 18. STRAIN DIAGRAMS FOR I-BEAM 5

Note: 1 horizontal division = 10,000 lb. per sq. in, as indicated by the strain gage.
1 horizontal division = 0.02 inch (for deflection curve only)
Numbers refer to gage lines as given in Fig. 3.
Tension plotted to right; compression to left.
Measured quantities ———; computed quantities —— — —·— — ———.

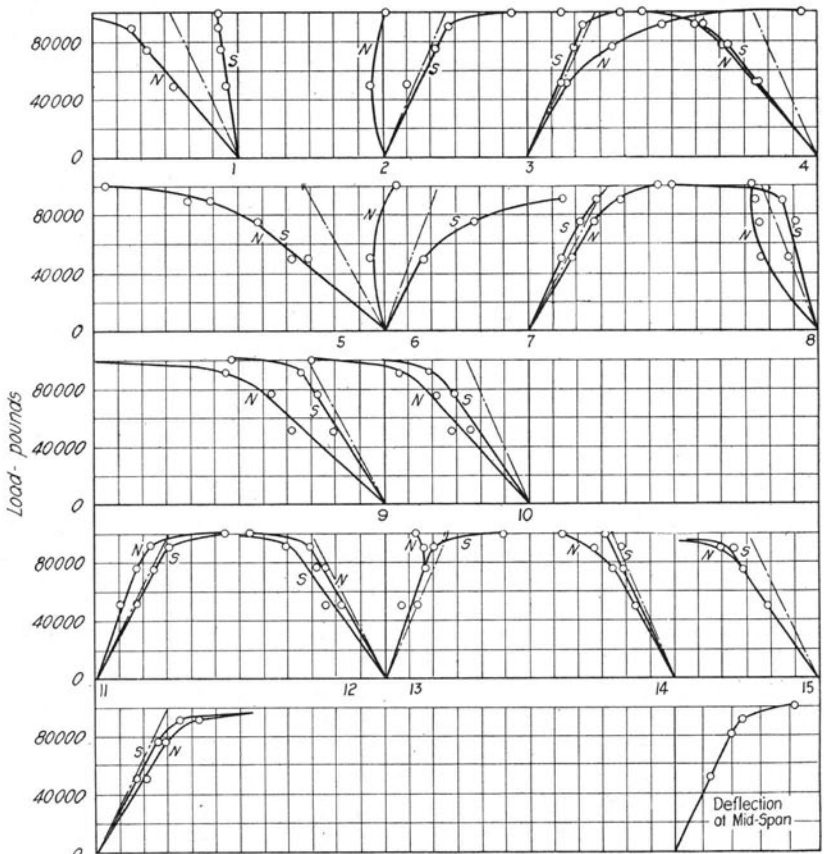

FIG. 19. STRAIN DIAGRAMS FOR I-BEAM 6

Note: See Fig. 18.

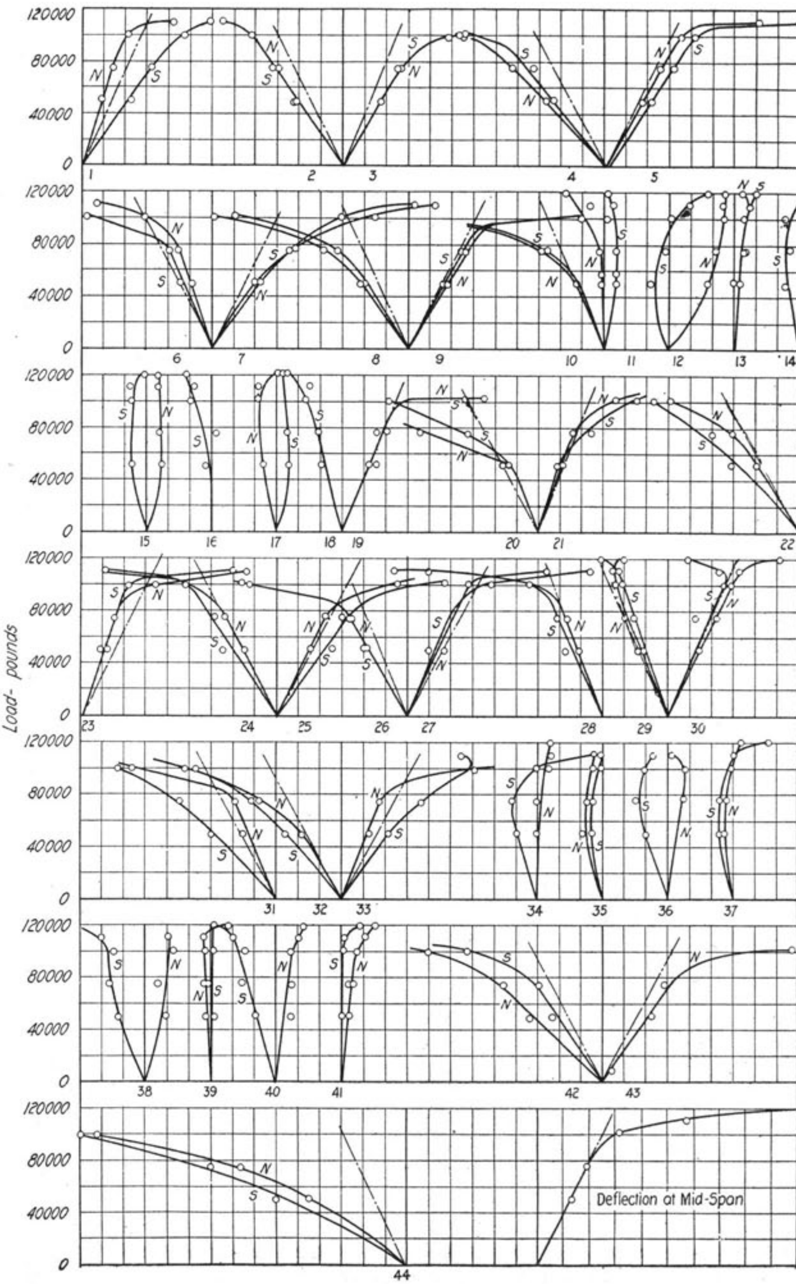

FIG. 20. STRAIN DIAGRAMS FOR I-BEAM 7

Note: See Fig. 18.

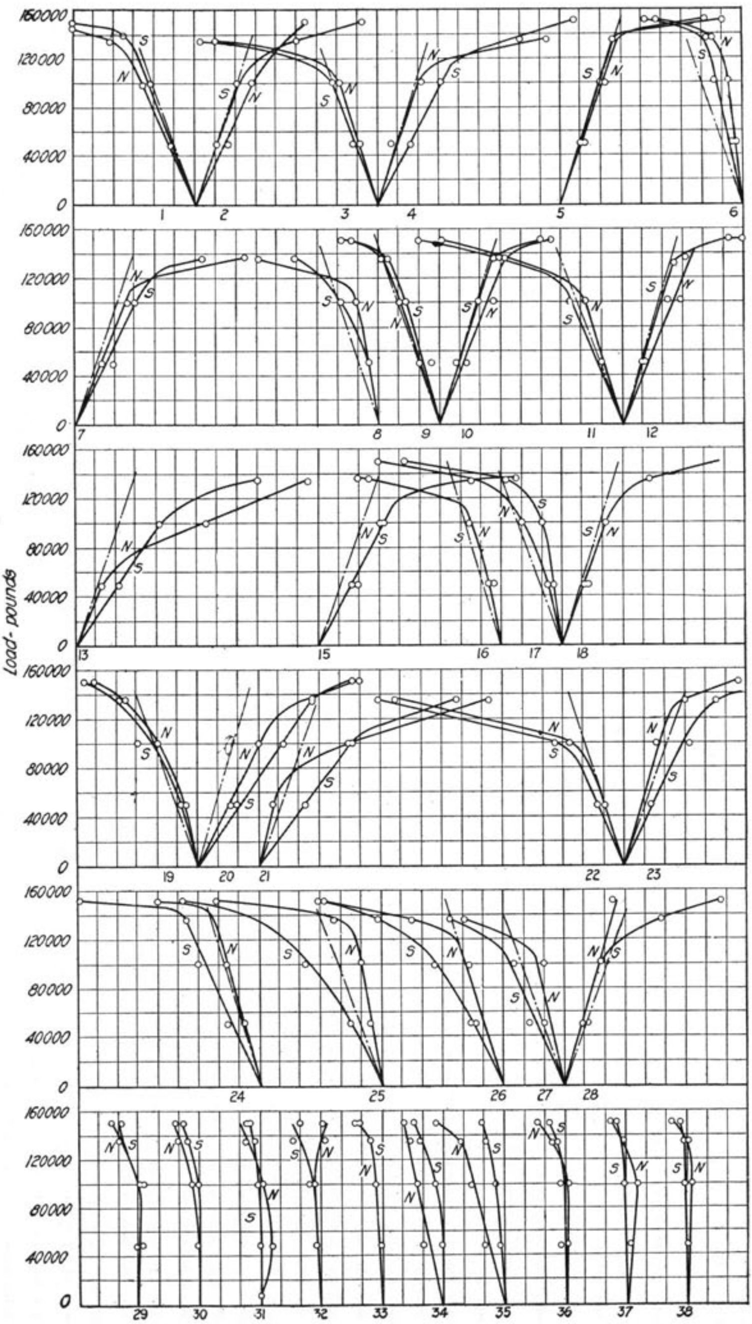

FIG. 21. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 8,—I

Note: See Fig. 22.

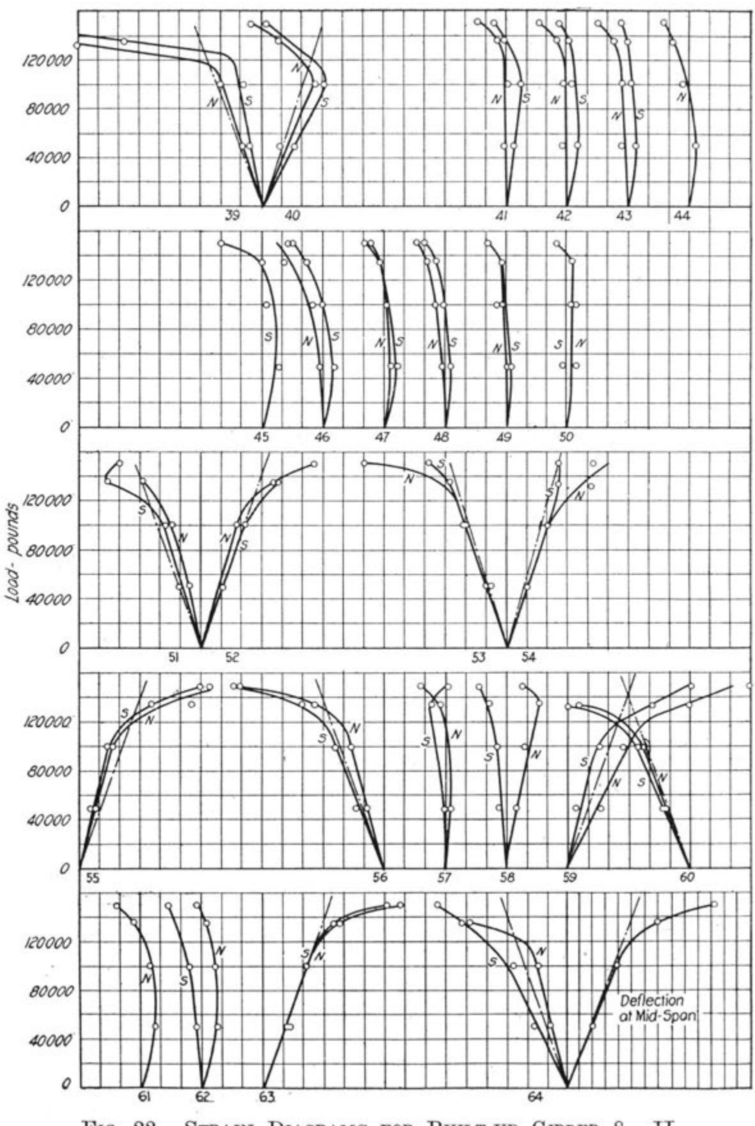

FIG. 22. STRAIN DIAGRAMS FOR BUILT-UP GIRDER 8,—II

Note: 1 horizontal division=10,000 lb. per sq. in. as indicated by the strain gage.
1 horizontal division=0.02 in. (for deflection curve only)
Numbers refer to gage lines as given in Fig. 4.
Tension plotted to right; compression to left.
Measured quantities ———; computed quantities — — —·— —

was 101,500 pounds, and for girder 8, 112,500 pounds. That is, the addition of intermediate stiffeners to a girder for which the ratio of the unsupported width to the thickness of the web was 100, increased the yield point of the girder only about 10 per cent. This is remarkable in view of the fact that the proportions of the girders were such that the web received a relatively higher stress than the flanges, a condition which is usually reversed in practice. While the girder without stiffeners does not have a yield point quite as high as the girder with stiffeners, the fact that the deflection curve is practically straight up to the yield point indicates that it may be safe to build girders without intermediate stiffeners if the ratio of the unsupported width to the thickness of the web exceeds 60. However, it is necessary to decrease the working stress allowable in the web as this ratio becomes greater.

For the determination of the proper working stress for webs without intermediate stiffeners three calculations should be made: (1) The maximum shearing stress (which may be taken equal to the shearing stress at the neutral axis) should not be greater than the safe working stress in shear for the web material. (The strength in shear of web material is discussed in the next section "Strength of Materials in I-beams and Girders".) (2) The value of $E\epsilon$ corresponding to the maximum diagonal strain should not exceed the value of the safe tensile or compressive working stress for the material. (3) The maximum diagonal compressive stress at the neutral axis should not exceed the safe stress as given by Euler's formula for fixed-ended columns, which when applied to webs of I-beams and girders becomes

$$S_w = \frac{1.64\ E}{f\left(\frac{h}{t}\right)^2} \qquad \text{(11a)}$$

in which S_w equals the average working stress in the web, f=the factor of safety, E=the modulus of elasticity (30,000,000 lb. per sq. in. for steel), h=the unsupported depth of the web between flanges, and t=the thickness of the web.

It may be necessary to provide stiffeners for girders having relatively thin webs, irrespective of the stress, in order that the girder may not be injured in handling.

17. *Strength of Material in the I-beams and Girders.*—Specimens for tension tests and shear tests were cut from the beams and girders as described in section 7 "Specimens." The results of the tension tests are given in Table 8, and the results of the shear tests are given in Table 9. In all tension tests the yield point was determined by the

TABLE 8.

TENSION TESTS OF MATERIAL IN I-BEAMS AND GIRDERS

Each value is the average of two or more tests

Girder (or I-beam)	Specimens from	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.	Elongation in 2 inches per cent
1	Flange	40,200	66,800	35.0
	Web	40,600	67,500	29.5
	Root of flange	38,000	65,700	27.7
2	Flange	42,600	67,600	27.5
	Web	44,200	68,200	27.7
	Root of flange	39,200	67,700	31.5
3	Web	41,800	54,100	35.7
4	Flange	37,500	66,900	36.1
	Web	42,300	68,900	29.7
	Root of flange	45,800	70,100	27.0
5	Flange	36,200	66,900	34.2
	Web	41,800	68,900	28.2
	Root of flange	46,000	69,200	27.0
6	Flange	39,300	67,400	33.5
	Web	43,500	66,500	27.0
	Root of flange	49,200	69,800	28.7
7	Flange	41,400	67,700	35.5
	Web	39,500	65,300	28.7
	Root of flange	43,800	68,200	30.2
8	Web	36,900	53,700	36.0

TABLE 9.

SHEARING TESTS OF MATERIAL FROM WEBS OF I-BEAMS AND GIRDERS

Each result is the average of two or more tests

Girder (or I-beam)	Yield Point lb. per sq. in.	Ultimate lb. per sq. in.
1	Not well defined	53,300
2	do.	53,300
3	25,700	49,000
4	Not well defined	52,300
5	do.	54,400
6	do.	50,500
7	do.	53,100
8	26,100	47,900

drop of the beam of the testing machine, and in shear tests by the first noticeable stretch as shown by a pair of dividers. The results of the tension tests indicate that the material at the root of the flanges of the I-beams was well rolled, since the specimens cut from this part of the I-beams showed fully as high strength as did the specimens from the webs and from the flanges. The yield-point strength of the shear specimens cut from the webs of the I-beams was not clearly defined, but definite yield points in shear were obtained for the specimens cut from the webs of the two built-up girders. Several specimens from each built-up girder were tested. These shear tests indicate a ratio of yield-point strength in shear to yield-point strength in tension of about 0.65, a ratio not widely different from the value 0.60 which is commonly used. If a fiber stress of 16,000 lb. sq. in. is allowable for structural steel in tension, a stress of about 10,000 lb. per sq. in. would be allowable for steel in shear. This agrees with the usual practice.

18. *Deflection of Test Girders.*—As a matter of interest, though secondary in importance to the determination of strength properties, the observed deflection at mid span of the girders under load has been compared with the deflection computed by means of the formulas commonly given in texts on the mechanics of materials. The girders tested had such short spans that the deflection due to shear, about 20 per cent of the total, is important. This deflection is calculated from the formula

$$\Delta_s = \frac{P}{2} \; \frac{l_1}{aF} \qquad \text{.................................... (13)}$$

in which Δ_s is the deflection at mid span due to shear, P is the total applied load, l_1 is the distance from support to the near of the two symmetrically spaced loads, F the modulus of elasticity in shear (taken as 12,000,000 lb. per sq. in. for steel) and a is the total area of cross-section of the beam. This formula is readily derived from the discussion given in Merriman's "Mechanics of Materials," 10th ed. p. 320.

The deflection due directly to flexure is *

$$\Delta_s = \frac{Pl_1}{EI}\left(\frac{l^2}{16} - \frac{l_1^2}{12}\right) \qquad \text{............................ (14)..}$$

in which l is the total length of span of the beam, E is the modulus of elasticity in tension and compression (taken as 30,000,000 lb. per sq. in. for steel), I the moment of inertia of the cross-section of the beam, and other symbols are the same as given for equation 14.

The total deflection Δ is then $\Delta_s + \Delta_f$. In the deflection curves of Fig. 16-22, the deflection computed by the preceding formulas is shown by the dot and dash line, and the observed deflection by solid lines. The computed and observed values agree very closely.

The theoretical curve of deflection due to shear alone for a girder loaded at two symmetrical points is made up of two inclined straight lines for the end portions, joined by a horizontal straight line for the

**Boyd, "Strength of Materials," p. 115.

middle portion. The general tendency of the girders to assume such a shape under excessive shearing strain is well shown in Fig. 10 and 11.

19. *Summary.*—The following summary is given:

(1) The measured strains in various parts of the six I-beams and two built-up girders agree closely with the strains as computed by the ordinary elastic theory if due allowance is made for the lateral strain (Poisson's ratio effect).

(2) The maximum shearing stress in an I-beam or a built-up girder is in some cases the shearing stress at the neutral axis, and is in other cases the diagonal shearing stress caused by the combined stresses in the web at its junction with the flange. However, the two are usually nearly equal, and, in general the shearing stress at the neutral axis may be used in designing girders.

(3) A common approximate method of computing the shearing stress in the web of a girder is to divide the total shear upon a transverse section by the area of the cross-section of the web. If the value given by this method is more than 80 per cent of the allowable stress in shear for the material, a check computation for shearing stress should be made, using the more precise formula, equation (2).

(4) The yield point (not the ultimate strength) of the material in shear should be regarded as the ultimate shearing stress which can be developed in the webs of girders. The ratio of the yield point in shear to the yield point in tension for structural steel is about 0.6, and the ratio of the allowable shearing stress to the allowable tensile or compressive stress may be taken at the same value.

(5) The maximum tensile or compressive strain is an I-beam or built-up girder is in some cases the longitudinal strain in the extreme fibers of the flange, and is in other cases the diagonal strain in the web adjacent to the flange. The diagonal strain may be enough greater than the longitudinal strain to make it desirable to consider the former in the design of a girder. The value of $E\epsilon$ corresponding to the maximum strain should not in any case exceed in magnitude the safe working stress of the material in tension. (The safe stress in tension for structural steel is usually taken at 16,000 lb. per sq. in.)

(6) In the case of girders having no stiffeners except at points at which concentrated forces are applied, the web is capable of developing the lowest of the following critical values: (1) the yield-point strength of the material of the web in shear; or (2) the compressive strength of the web as computed by Euler's formula, considering a 45-degree strip as a fixed-ended column subjected to a compressive stress equal to the transverse shearing stress at the neutral axis; or

(3) a diagonal strain equal to the strain at the yield point of the material in tension.

(7) It would seem that the ability to resist buckling of thin webs without intermediate stiffeners had been underestimated.

(8) Stiffeners at supports and under concentrated loads are very necessary. These should be well fitted to the flanges.

(9) The local compressive stress in the web of a girder when no stiffeners are used at points at which concentrated forces are applied may be computed with a fair degree of accuracy by the use of Hudson's formula (see p. 32 for detailed discussion). Even if this stress is low, the use of stiffeners at points at which concentrated forces are applied diminishes the danger of lateral bending of the beam at the junction of the web and the flange.

(10) The deflection of the girders as measured and as computed by the ordinary elastic theory agrees closely when the deflection due to shear is considered (see p. 48 for discussion of formulas). For short-span beams the deflection due to shear may be as much as 20 per cent of the total.

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