UNIVERSITY OF ILLINOIS
BULLETIN

Vol. XXXVIII **November 19, 1940** **No. 13**

ENGINEERING EXPERIMENT STATION
BULLETIN SERIES No. 323

TURBULENT FLOW OF SLUDGES IN PIPES

BY

HAROLD E. BABBITT

AND

DAVID H. CALDWELL

PRICE: FORTY-FIVE CENTS

PUBLISHED BY THE UNIVERSITY OF ILLINOIS
URBANA

[Issued weekly. Entered as second-class matter December 11, 1912, at the post office at Urbana, Illinois, under the Act of August 24, 1912. Acceptance for mailing at the special rate of postage provided for in section 1103, Act of October 3, 1917, authorized July 31, 1918]

THE Engineering Experiment Station was established by act of the Board of Trustees of the University of Illinois on December 8, 1903. It is the purpose of the Station to conduct investigations and make studies of importance to the engineering, manufacturing, railway, mining, and other industrial interests of the State.

The management of the Engineering Experiment Station is vested in an Executive Staff composed of the Director and his Assistant, the Heads of the several Departments in the College of Engineering, and the Professor of Chemical Engineering. This Staff is responsible for the establishment of general policies governing the work of the Station, including the approval of material for publication. All members of the teaching staff of the College are encouraged to engage in scientific research, either directly or in coöperation with the Research Corps, composed of full-time research assistants, research graduate assistants, and special investigators.

To render the results of its scientific investigations available to the public, the Engineering Experiment Station publishes and distributes a series of bulletins. Occasionally it publishes circulars of timely interest, presenting information of importance, compiled from various sources which may not readily be accessible to the clientele of the Station, and reprints of articles appearing in the technical press written by members of the staff and others.

The volume and number at the top of the front cover page are merely arbitrary numbers and refer to the general publications of the University. *Above the title on the cover* is given the number of the Engineering Experiment Station bulletin, circular, or reprint which should be used in referring to these publications.

For copies of publications or for other information address

THE ENGINEERING EXPERIMENT STATION,
UNIVERSITY OF ILLINOIS,
URBANA, ILLINOIS

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION
BULLETIN SERIES No. 323

TURBULENT FLOW OF SLUDGES IN PIPES

BY

HAROLD E. BABBITT
Professor of Sanitary Engineering

AND

DAVID H. CALDWELL
Research Assistant in Civil Engineering

PUBLISHED BY THE UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

Price: Forty-five Cents

4000—11-40—19813

UNIVERSITY
OF ILLINOIS
:: PRESS ::

CONTENTS

LIST OF FIGURES

LIST OF TABLES

This page is intentionally blank.

TURBULENT FLOW OF SLUDGES IN PIPES

I. INTRODUCTION

1. *Statement of Problem.*—It is generally known and has been shown by numerous authors[1, 2, 3, 4]* that two distinct types of flow occur when a sludge is caused to flow in a circular pipe. These two types of flow have been termed laminar flow and turbulent flow. The flow is said to be laminar when the particles of flowing sludge move in straight lines parallel to the sides of the pipe. The flow is said to be turbulent when the direction and magnitude of the velocity vectors of the flowing particles vary with time.[5]

Laminar flow relationships for various sludges have been studied at the University of Illinois Sewage Research Laboratory and the results and conclusions, together with equations for predicting friction loss due to sludge flowing in a circular pipe, have been published in Bulletin 319 of the Engineering Experiment Station.[1] It has been pointed out[1, 2, 3] that turbulent flow of sludge in the higher velocity ranges resembles the turbulent flow of other liquids. In Fig. 1 the friction head loss is plotted logarithmically against the mean velocity of flow for various sludges and for water, flowing in a $\frac{3}{8}$-in. pipe. The figure shows that above the critical velocity at which laminar flow changes to turbulent flow, the lines representing the relation of head loss to velocity are approximately straight and approximately parallel. The fact that the lines are almost straight above the critical velocity suggests the possibility of an exponential relation between head loss and velocity. The possibility of plotting the sludge flow data on a diagram similar to the friction-factor chart or Reynolds-Stanton diagram is also suggested.

The relation between head loss and velocity for the turbulent flow of sludge through pipes is not known. The relation has been formulated empirically by some industries using certain types of sludge. For example, in oil well drilling, mud is used as a circulating medium to hold down gas pressure and bring up the cuttings of the drill bit. Various methods and formulas are in use for estimating the friction loss as the drilling mud flows through the drilling machinery, but the formulas are empirical, derived for a certain mud, and good only for that mud. Practice in the design of machinery and pipes for conveying sewage sludge now utilizes empirical

---

*Index numbers refer to items in bibliography.

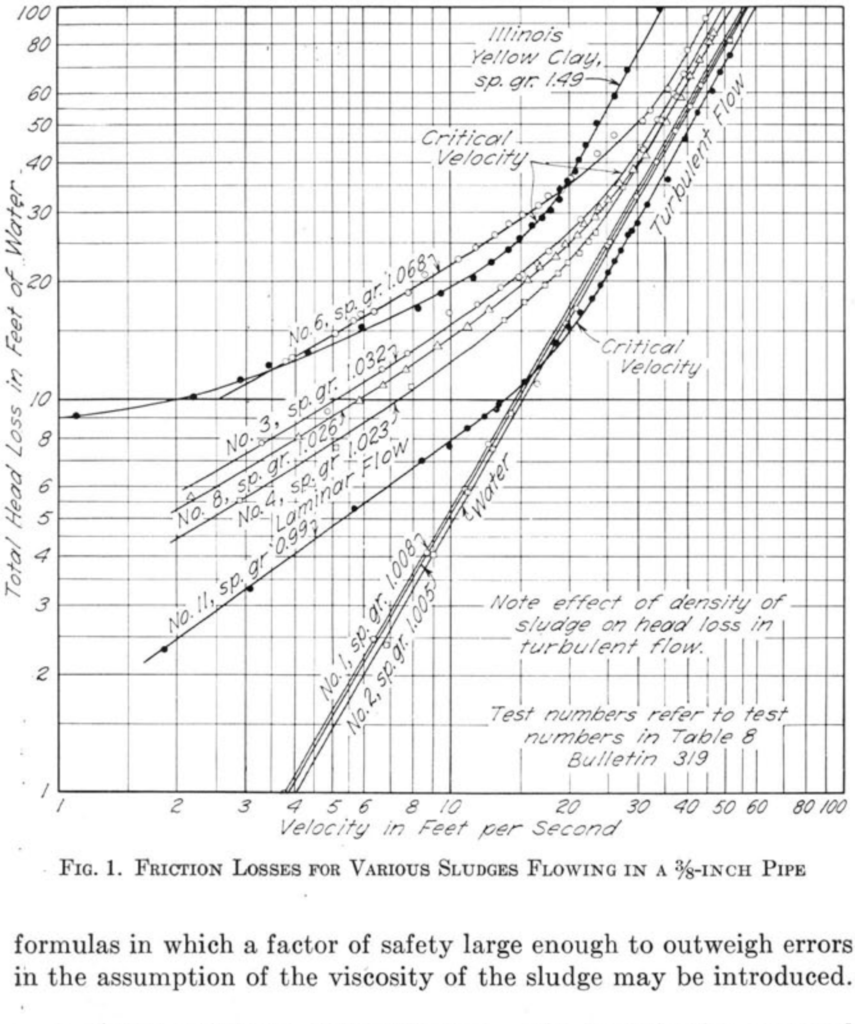


Fig. 1. Friction Losses for Various Sludges Flowing in a ⅜-inch Pipe

formulas in which a factor of safety large enough to outweigh errors in the assumption of the viscosity of the sludge may be introduced.

2. *Object and Scope of Investigation.*—The investigation reported in this bulletin carries on the work reported in Bulletin 319. The latter deals with the laminar flow of sludge in circular pipes and the upper limiting velocity at which laminar flow changes to turbulent flow.

The object of this investigation was to formulate the factors affecting the turbulent flow of sludge and, if possible, to present, for various types of sludge, data which could be used in the design of pipe lines and pumping machinery for conveying various types of sludge. In addition, it was expected that tests to supplement those reported in Bulletin 319 covering wider ranges in the variables would provide additional information of value.

In order to test the validity of equations developed for turbulent flow it was necessary to carry the tests to velocities of flow far above those normally encountered in practice. The apparatus to do this required the use of pumps and power of other types than those usually used for sludge pumping.

3. *Acknowledgments.*—The tests herein described were made as part of the work of the Engineering Experiment Station of the University of Illinois, of which MELVIN L. ENGER, Dean of the College of Engineering, is director, and of the Department of Civil Engineering, of which PROF. W. C. HUNTINGTON is the head.

Some of the labor involved in building the apparatus and in performing the routine duties connected with the tests was done by students employed through the National Youth Administration.

4. *Nomenclature.*—The nomenclature used throughout this report is as follows:

C = coefficient of roughness in Hazen and Williams formula

D = diameter of pipe, feet

f = friction factor in the expression $H = f\frac{L}{D}\cdot\frac{V^2}{2g}$

g = acceleration due to gravity, feet per second per second

H = difference in static head between two points in a pipe, feet of flowing substance

H_w = difference in static head between two points in a pipe, feet of water

L = length of pipe, feet

Q = rate of flow, cubic feet per second

R = radius of pipe, feet

Re = Reynolds number $DV\rho/\mu$, dimensionless

S_p = shearing stress in a flowing material at the boundary or pipe wall, pounds per square foot

S_y = shearing stress at the yield point of a plastic material, called yield value, pounds per square foot

V = mean velocity of flow in a pipe, feet per second

V_c = critical velocity, feet per second

V_{lc} = lower critical velocity ($Re = 2000$), feet per second, or minimum probable value of V_c

V_{uc} = upper critical velocity ($Re = 3000$), feet per second, or maximum probable value of V_c

μ = coefficient of viscosity, pounds per foot second
η = coefficient of rigidity, pounds per foot second
ρ = density of flowing substance, pounds per cubic foot
W = weight of a cubic foot of water, pounds

The system of units employed herein assumes four primary units and dimensions. These are: for mass, the mass pound; for force, the force pound; for length, the foot; and for time, the second. The mass pound is equal to the pound avoirdupois. The force pound is the force which will give one mass pound an acceleration of 32.1740 ft. per sec. per sec. regardless of the value of gravity in the locality. A mass pound is equivalent to a slug in the gravitational system, divided by 32.1740. A force pound is equivalent to 32.1740 poundals in the absolute system of units.

5. *Laminar Flow and Critical Velocity.*—As a result of the investigations reported in Bulletin 319 it was found that the characteristics of a sludge could be completely defined, as far as laminar flow frictional losses were concerned, by two constants. These constants are the *yield value* S_y and the *coefficient of rigidity* η. It was found that the laminar flow factors could be formulated as

$$V = \frac{gD}{8\eta}\left[S_p - \frac{4}{3}S_y + \frac{1}{3}\frac{S_y^4}{S_p^3}\right]^* \qquad (1)$$

which could be further reduced in some cases to

$$\frac{H}{L} = 32\left[\frac{S_y}{6\rho D} + \frac{\eta V}{g\rho D^2}\right] \qquad (2)$$

The critical velocity, or the velocity at which laminar flow changes to turbulent flow, was found to occur within a certain range of velocity. The lower critical velocity, below which the flow will always be laminar, was formulated as

$$V_{lc} = \frac{1000\eta + 1000\sqrt{\eta^2 + \dfrac{D^2 S_y \rho g}{3000}}}{D\rho} \qquad (3)$$

---

*In Bulletin 319 this equation was written as Equation (11) on page 15 in the approximate form $V = \frac{4D}{\eta}\left[S_p - \frac{4}{3}S_y + \frac{1}{3}\frac{S_y^4}{S_p^3}\right]$ in which the coefficient of g was taken as 32. This approximate form is numerically correct. It would be dimensionally correct if the terms on the right hand side were multiplied by one ft. per sec. per sec. because the value of g, which is approximately equal to 32 ft. per sec. per sec., has been replaced by the dimensionless coefficient 32.

TABLE 1
CHARACTERISTICS OF SLUDGES TESTED

Sludge No.	S_y lb. per sq. ft.	η lb. per ft. sec.	Specific Gravity	ρ lb. per cu. ft.	Percentage Moisture by wt.	Percentage Solids by wt.	Type of Sludge
1	0.90	0.015	1.21	75.6	70.2	29.5	Clay suspension
2	0.60	0.015	1.20	75.0	73.5	26.5	Clay suspension
3	0.44	0.011	1.18	73.8	75.0	25.0	Clay suspension
4	0.29	0.010	1.16	72.5	76.7	23.3	Clay suspension
5	0.19	0.008	1.15	72.0	79.7	20.3	Clay suspension
6	0.082	0.006	1.12	70.0	83.9	16.1	Clay suspension
7	0.13	0.013	1.03	65.0	92.2	7.8	Sewage sludge (digested)

The upper critical velocity, above which the flow will usually be turbulent in industrial piping, was formulated as

$$V_{uc} = \frac{1500\eta + 1500\sqrt{\eta^2 + \dfrac{D^2 S_y \rho g}{4500}}}{D\rho} \tag{4}$$

In a given installation it has been noted that the value of the actual critical velocity might occur within the range between the upper and lower critical velocities. The velocity at which the change from laminar to turbulent flow takes place is controlled by various factors, the principal one being the roughness of the pipe.

II. SLUDGES TESTED

6. *Types of Sludges.*—At the beginning of the investigation it was deemed necessary to obtain a sludge whose characteristics would remain constant throughout all the tests so that comparable results might be obtained in different size pipes. The desirable characteristics of the sludge for these tests were decided upon as follows: (a) the sludge particles must remain in suspension so that appreciable settling is avoided; (b) the sludge must be a true plastic, as otherwise the yield point is difficult to detect; (c) sludge particles must be of a character which will not cause wear in the gears of a rotary pump; (d) sludge particles must not disintegrate on prolonged agitation; and (e) the sludge must not exhibit thixotropic properties. Requirements (a) and (b) may be conflicting, because particles which stay in suspension are likely to be colloidal, and cause the sludge to show pseudoplastic properties.

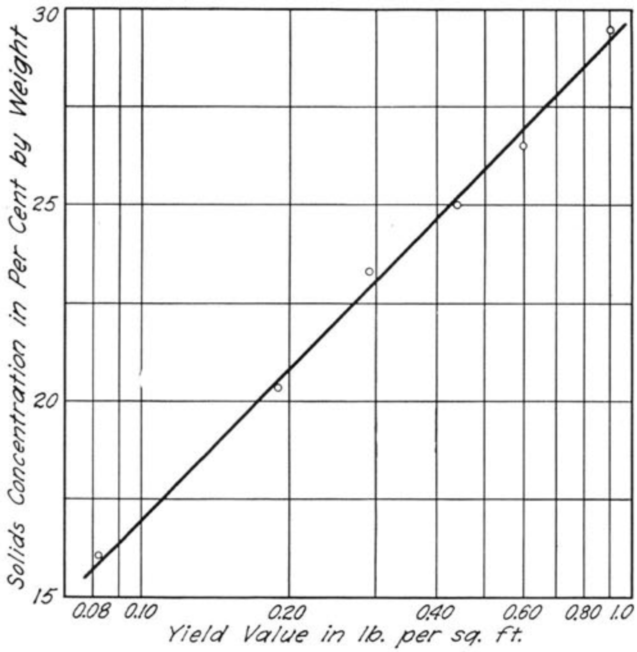


FIG. 2. VARIATION OF YIELD VALUE WITH CONCENTRATION OF SUSPENDED MATTER FOR TENNESSEE BALL CLAY

It was found that a ball clay, mined in Tennessee, most nearly satisfied these requirements. The clay remained in suspension for several hours, and yet did not exhibit colloidal properties to such an extent that the yield value and coefficient of rigidity of the sludge were obscured.

After the preliminary relationships had been established using the ball clay, a sewage sludge was obtained and tests were made on it to show the applicability of the relationships to different types of sludge.

7. *Variation of Yield Value and Coefficient of Rigidity.*—The ball clay was mixed with water to a consistency as thick as the pumps could handle. This material was then run through the test pipes, and measurements of its density, yield value, coefficient of rigidity, per cent solids, and temperatures were observed. The mixture was then diluted with water and the tests were repeated. Eight different mixtures, or sludges, with different yield values, densities, etc., were tested. Table 1 lists the characteristics of the various sludges tested. Figure 2 shows the variation of yield value with the con-

centration of suspended matter for the Tennessee ball clay used in the tests. The line in Fig. 2 shows that, for this ball clay, the yield value may be obtained from the percentage concentration of clay by the following formula

$$S_y = 10^{0.078C-2.32} \tag{5}$$

where C = per cent, by weight, of the concentration of solids in the sludge.

8. *Evidence of Thixotropy During Tests.*—Because the clay was usually circulated for from three to four hours during a test, it was necessary to have a material which did not exhibit thixotropy. Thixotropy is the property, or phenomenon, exhibited by some gels of becoming fluid when shaken. The change is also reversible. Tests were made on the clay mixture to determine if the material had thixotropic properties. Several measurements of friction loss at various velocities of flow were made as soon as possible after the pumps were started. Measurements were again made after the pumps had been running for four hours. This procedure was repeated several times. During each repetition the friction losses, for a constant velocity, were constant, regardless of the amount of agitation of the sludge. It was concluded, therefore, that the clay mixtures used were not thixotropic.

Because of the nature of sewage sludge it might be expected that it would exhibit thixotropy. This was shown by Hatfield[6] and others.[7] In order to stabilize the characteristics of the sludge and to reduce the effect of thixotropy during a test the sludge was circulated for two hours before measurements were taken. It is to be expected that similar results will be obtained by repetitions of the test on sludges with similar characteristics.

III. APPARATUS

9. *Essentials of Apparatus.*—In order to find the relation between the factors involved in turbulent flow it was necessary to construct an apparatus to fulfill the following requirements: (a) to deliver sludge to the upstream end of the test pipe at any desired velocity ranging from velocities less than critical to velocities well above those normally encountered in practice; (b) to maintain an even rate of flow of sludge in order to avoid fluctuations in head loss;

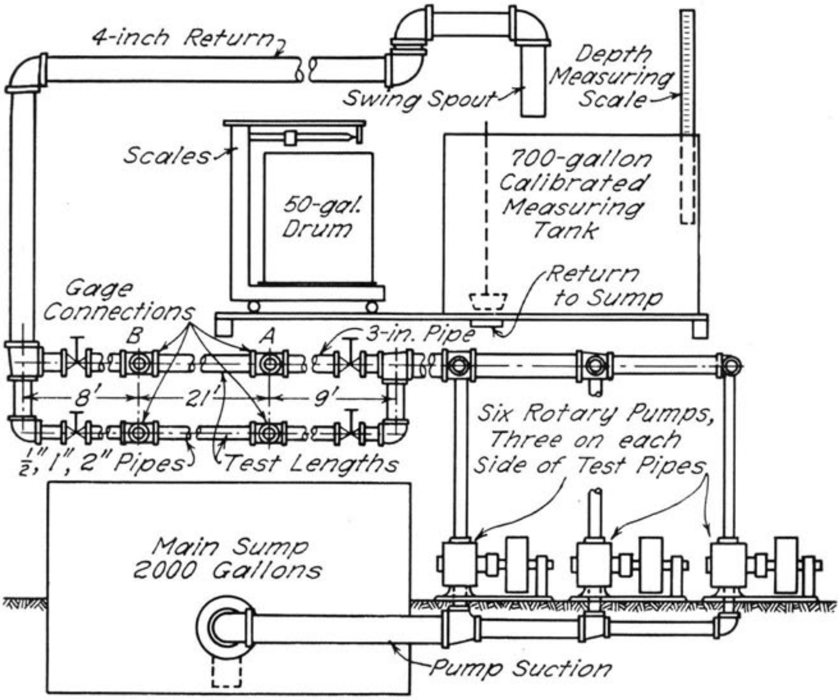


FIG. 3. DIAGRAM OF SLUDGE FLOW APPARATUS

and (c) to develop sufficient pressure to force the thickest sludge through the test pipe at velocities of 35 feet per second, or less. The limiting velocities chosen were approximately 0.5 feet per second to 35 feet per second; to obtain such velocities in the 1-in., 2-in., and 3-in. pipes tested, rates of flow between one gallon per minute and 500 gallons per minute are required.

The apparatus illustrated diagrammatically in Fig. 3 met these requirements with the exception that the maximum velocity obtainable in the 3-in. pipe was 30 feet per second. This apparatus built at the Sewage Testing Laboratory at the University of Illinois consisted of the following essential elements: (a) a variable speed motor; (b) constant displacement pumps of the rotary type to reduce pulsations in the gage; (c) a series of several quickly interchangeable test pipes; (d) a mercury gage of sufficient capacity to measure a loss of head of approximately 100 feet of water; and (e) a weighing scale and a calibrated measuring tank for discharge determinations.

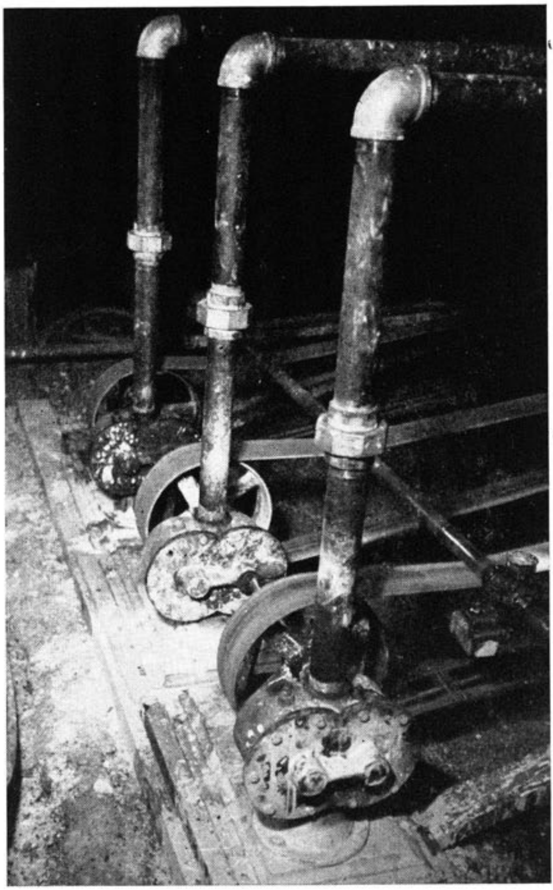


FIG. 4. ROTARY PUMP

10. *Variable Speed Motor.*—From a preliminary study of the pressure demanded it was found that approximately 30 horsepower would be required to drive the pumps at their maximum discharge and maximum pressure. Since a variable-speed electric motor of 30 horsepower was not available, a six-cylinder gasoline motor was obtained. The motor with radiator, transmission, and clutch were taken from an automobile. The rated brake horsepower of the motor was 27.9. Since a gasoline motor develops power approximately proportional to its speed it was assumed that 30 horsepower could be obtained from this motor by allowing it to run at a higher speed than its rated speed of 2500 revolutions per minute.

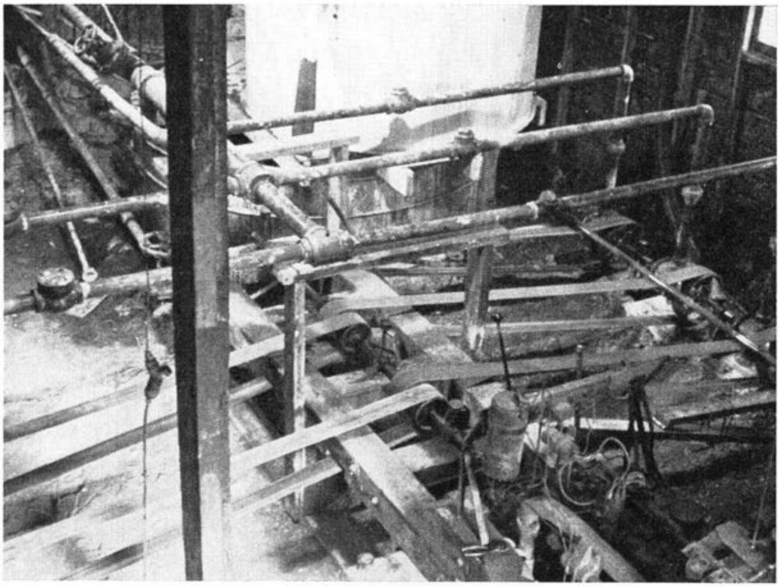


FIG. 5. PUMP ARRANGEMENT

The motor transmission was connected to a central drive shaft from which the pumps were driven by belts, three pumps being on one side of the shaft and two on the other, as shown in Figs. 4 and 5.

11. *Pumps.*—The rotary type pump, Fig. 4, was found to be satisfactory for pumping sludge at constant velocity through a test pipe and was used on the apparatus. Since it was desirable to have a flexible pumping unit for velocity regulation, five rotary pump units were used. The capacity of each pump was 100 gallons per minute at 300 revolutions per minute. The gear ratio between the motor and the pumps was such that the motor ran at approximately 2500 revolutions per minute when the pumps were running at 300 revolutions per minute.

12. *Test Pipes.*—Pipes through which the sludge was pumped ranged in size from ½ inch to 3 inches in diameter. All pipes were new standard black steel taken from stock. Test lengths were 21 feet for the 1-in., 2-in., and 3-in. pipes, and 10 feet for the ½-in. pipe. As shown diagrammatically in Fig. 3, the lengths of pipe upstream and downstream from the test length were at least 40 pipe diameters, assuring steady flow conditions through the test length.

TABLE 2
DIMENSIONS OF TEST PIPES

Nominal Pipe Diameter in.	Actual Internal Pipe Diameter in.	Actual Internal Pipe Diameter ft.	Interior Area sq. in.	Interior Area sq. ft.	Test Length ft.
½	0.622	0.0518	0.304	0.00211	10.00
1	1.05	0.0875	0.864	0.0060	20.88
2	2.07	0.173	3.355	0.0233	20.88
3	3.07	0.256	7.393	0.0512	20.88

The interiors of the 1-in., 2-in., and 3-in. pipes all appeared smooth and without appreciable mill scale. The interior of the ½-in. pipe was the roughest of those tested. It was later found that the friction factors were higher for the ½-in. pipe than for the other three sizes. As a protection against corrosion throughout the duration of the tests, disconnected test pipes were kept full of sludge and the ends securely closed.

13. *Gage Connections.*—Since the gages used were of the two-liquid, differential type, it was necessary to take precautions to keep the sludge flowing in the test pipe from entering the gage via the gage connections. The combination gage connection and mud trap used in previous experiments described in Bulletin 319 was used in these tests. The gage connection proved itself most satisfactory during this investigation, and at no time was trouble encountered from sludge entering the gage.

14. *Gages.*—Two gages were used during the tests. The smaller gage, shown in Fig. 6(a), is an air-water differential gage. Air is introduced over the two legs so that the meniscus in each leg appears on the measuring face. Then, since the pressure is the same over each leg, the difference between the elevations of the two menisci is a direct measure of the friction loss in terms of head of water for the test length. The larger gage, shown in Fig. 6(b), is a differential gage with mercury and water in each gage column. A constriction was provided at the bottom of the gage to reduce pulsations in the gage due to surges from the pumps. The frictional loss is obtained from the differential pressure between a column of mercury and a column of water equal in length to the gage reading.

The smaller gage was used for low rates of flow with thin sludge or water as the flowing material. The mercury-water gage was

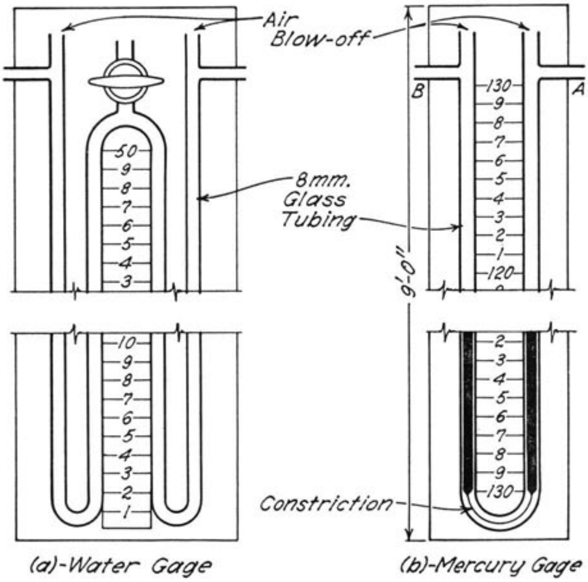


FIG. 6. DIAGRAM OF GAGES

used for thick sludges and high rates of flow. The latter gage was constructed to measure a pressure drop of approximately 50 pounds per square inch, which is equivalent to a hydraulic gradient of approximately 10:1 for a ten-foot test length.

A difficulty encountered at the beginning of the tests resulted from the pulsation of the pumps causing the gage to indicate friction losses greater than actually occurred. This was particularly noticeable at low velocities of flow in the larger pipes. The mercury-water gage as constructed had a constriction at the bottom of the U, to reduce oscillations in the mercury, as shown in Fig. 6(b). The gage, being located at the upstream end of the test pipe, received the surge from the pump on the high pressure leg first, as shown at *A* in Fig. 6. The mercury in this leg was, therefore, forced down by the surge, the constriction in the U tube preventing the mercury from returning before another surge came from the pump. The result was that the indicated mercury head was continually built up in one leg by the surging, causing the gage reading to be always greater than it should be. This trouble was eliminated by making the gage line from the upstream gage connection, *A*, Fig. 3, equal in length to the gage line from the downstream gage connection, *B*. With gage lines of equal length, the surges from the pump were transmitted through the sludge in the test pipe and the water

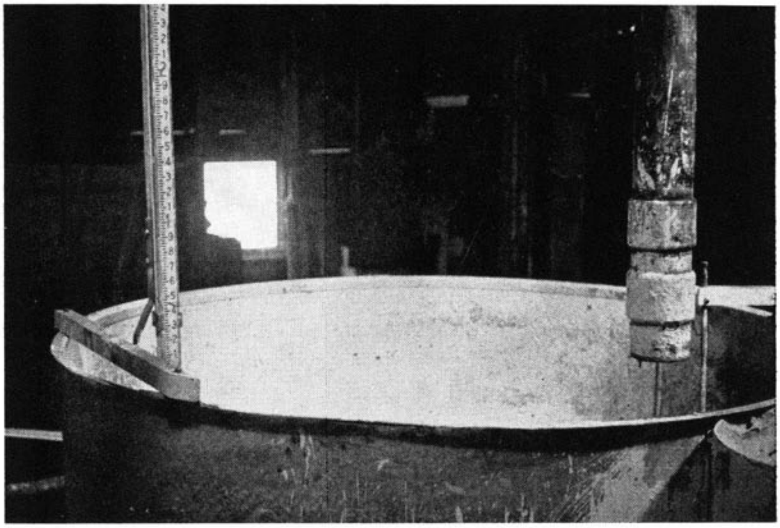


FIG. 7. CALIBRATED TANK USED TO MEASURE VELOCITY OF FLOW IN TEST PIPES

in the gage lines to arrive in each leg of the gage at the same instant; thus the surge effect in one leg was cancelled by the surge effect in the other leg.

15. *Sludge Return Pipe Submergence.*—When the contents of the measuring tank were dropped through the air into the main sump, air bubbles were entrained in the sludge. These entrained bubbles were then sucked up by the pumps and delivered to the test pipes; the result was a sludge of variable and uncertain characteristics. This difficulty was overcome by discharging the returned sludge through a pipe opening below the surface of the sludge in the main sump. The introduction of bubbles into the sludge was carefully guarded against by avoiding surface agitation. Bubbles, once introduced into a thick sludge, do not rise until they become large enough for their buoyant force to overcome the yield value of the sludge.

16. *Rate of Flow Determinations.*—Facilities were provided for measuring rates of flow between 10 pounds of sludge per minute and 7000 pounds per minute. For rates of flow between 10 and 1000 pounds per minute the discharge was weighed. The velocity of flow could be calculated from the weight of material collected, its known density, and the pipe diameter. For rates from 1000 to 7000 pounds per minute the discharge was measured volumetrically

in a container 6 feet in diameter and 4 feet deep, as shown in Figs. 3 and 7. Care was taken during a test to collect sludge to a depth of at least 3 feet in order to reduce the percentage error in the measurement of depth. Depth of sludge was read to the nearest one-hundredth of a foot on the scale shown in Figs. 3 and 7.

IV. Procedure and Tests

17. *Procedure.*—A complete test in each pipe required the determination of the friction loss at various velocities of flow ranging from 0.5 foot per second to the maximum limit for the apparatus, which was about 35 feet per second. The velocities of flow were controlled by regulating the speed of the motor and the number of pumps in operation simultaneously. The time required to collect a known volume of sludge was measured by means of a stop watch reading to 0.1 second. The procedure in the tests on sludges and on water was the same.

Observations were made with each of the eight sludges tested in each of the four test pipes. The time required to make a run in one test pipe and to change the apparatus for the next test pipe was approximately five hours. In addition to making the runs in the test pipes, characteristics of each sludge, such as density, percentage moisture, etc., were measured.

18. *Calibration Tests.*—In order to determine the roughness of the test pipes, runs were made with water, in which the following factors were observed: velocity of flow, head loss, density, and temperature. Results of these runs are compared on the friction factor-Reynolds number chart shown in Fig. 8. The friction factors for the 1-in. and 2-in. pipes are on the line predicted for the new commercial black steel pipe. The 3-in. pipe has slightly smaller friction factors than the 1-in. and 2-in. pipes, indicating a smoother interior. The ½-in. pipe was noticeably rougher, and the friction factors were found to be correspondingly higher.

19. *Observations Made.*—A typical sheet from a test run is shown in Table 3. The data taken for a complete run in one pipe included the temperature of the sludge at various intervals of time, the gage readings, the velocity-measuring apparatus readings, the time to collect the indicated quantity of sludge, the number of pumps operating, and any remarks such as the appearance of the critical velocity noticed by erratic readings of the gage.

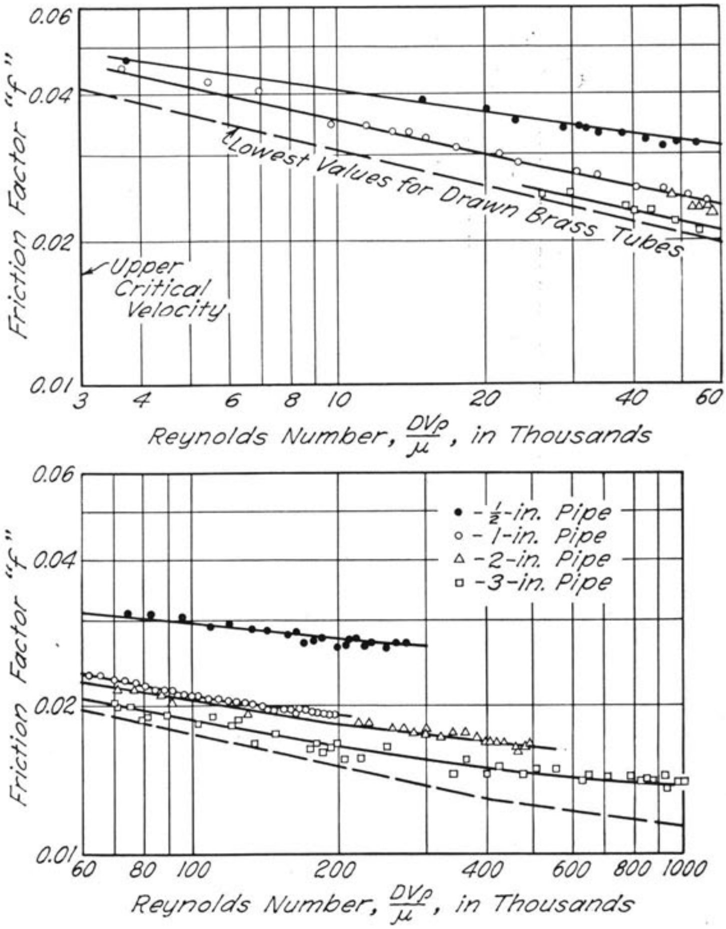


FIG. 8. FRICTION FACTORS FOR TEST PIPES

V. TEST DATA

20. *Method of Plotting Test Data.*—The method adopted for plotting the test data to study the relationship of the various factors affecting the friction loss was to construct a friction factor chart, as shown in Fig. 9. It was found that, when evaluating Reynolds number, $DV\rho/\mu$, if the viscosity of the dispersion medium were used as μ the friction factor chart, obtained by plotting friction factor against Reynolds number, correlated data for all the materials tested ranging from pure water to the thickest sludge. For most industrial sludges the viscosity of the dispersion medium

TABLE 3

TYPICAL DATA FROM TEST RUN

Sludge No. 4; yield value, 0.29; specific gravity, 1.16; diameter of pipe, 1 inch; coefficient of rigidity, 0.010; percentage solids, 23.3; temperature 18 deg. C.

No.	Gage Readings			Weight lb.	Time sec.	$\frac{100H}{L}$	Velocity ft. per sec.	$\frac{V}{4D}$	S_p lb. per sq. ft.	R_e	f
	Up-stream	Down-stream	Sum								
1	9.9	3.3	13.2	20	55.2	22.6	0.84	2.4	0.36	Laminar Flow	
2	12.0	4.4	16.4	50	64.4	28.1	1.80	5.2	0.44		
3	12.7	5.1	17.8	50	39.3	30.5	2.94	8.4	0.48		
4	13.3	5.8	19.1	50	27.4	32.6	4.22	12.1	0.52		
5	13.8	6.3	20.1	50	21.5	34.4	5.38	15.4	0.54		
6	14.4	6.9	21.3	100	34.7	36.5	6.66	19.1	0.58		
7	14.8	7.3	22.1	100	30.7	37.8	7.54	21.6	0.60		
8	15.1	7.7	22.8	100	28.4	39.1	8.14	24.3	0.62		
9	15.9	8.5	24.4	100	24.6	41.8	9.40	26.9	0.66	84 000	0.0268
10	16.8	9.4	26.2	100	22.7	44.9	10.2	29.2	0.71	91 000	0.0244
11	17.7	10.3	28.0	150	32.5	48.0	10.7	30.6	0.76	96 000	0.0239
12	19.3	12.1	31.4	150	30.8	53.8	11.3	32.3	0.85	101 000	0.0239
13	21.3	14.0	35.3	150	28.2	60.5	12.3	35.2	0.96	110 000	0.0224
14	23.2	16.0	39.2	150	26.6	67.2	13.1	37.5	1.06	111 000	0.0220
15	25.2	18.0	43.2	150	25.2	74.1	13.8	39.5	1.17	123 000	0.0220
16	27.2	20.0	47.2	200	31.8	81.0	14.6	41.8	1.28	130 000	0.0216
17	31.8	25.0	56.8	200	28.7	97.4	16.1	46.0	1.54	144 000	0.0212
18	36.5	30.0	66.5	200	26.3	114.0	17.6	50.4	1.80	157 000	0.0210
19	41.0	35.0	76.0	300	36.6	130.0	19.0	54.4	2.06	170 000	0.0204
20	45.5	40.0	85.5	300	34.9	147.0	19.9	57.0	2.32	178 000	0.0208
21	50.0	45.0	95.0	300	32.7	163.0	21.2	60.6	2.58	190 000	0.0204
22	54.5	50.0	104.5	300	31.3	179.0	22.2			198 000	0.0204
23	59.3	55.0	114.3	350	34.8	196.0	23.2			208 000	0.0204
24	64.2	60.0	124.2	350	33.2	213.0	24.4			218 000	0.0200
25	73.5	70.0	143.5	350	30.5	246.0	26.6			238 000	0.0196
26	83.5	80.0	163.5	350	28.4	280.0	28.5			254 000	0.0196
27	101.0	100.0	201.0	350	25.4	346.0	31.8			284 000	0.0192
28	119.0	118.0	237.0	350	23.5	406.0	34.4			308 000	0.0188
29	127.0	126.0	253.0	350	22.6	433.0	35.9			321 000	0.0188

was found to be practically the same as that of water. In determining the values of Reynolds number it was necessary, therefore, to substitute only the known quantities D, V, and ρ for sludge, and μ for water, in the expression $Re = DV\rho/\mu$.

Figure 9 is the friction factor chart correlating all data taken in this investigation. On comparing this figure with Fig. 8, which was obtained from the same pipes using a true fluid (water), it is seen that the agreement between the two figures is good. Friction factors obtained from the sludges are, as a whole, slightly higher than those obtained for water. This fact might have been caused by some corrosion occurring in the pipes during the tests which extended over a six-month period. This agreement, however, is within 5 per cent, which is within the degree of precision of the determination of the roughness of a pipe.

21. *Method of Determining Critical Velocity.*—The method of plotting data to determine the observed critical velocity is shown in Fig. 10. It was found that when the friction loss, expressed in

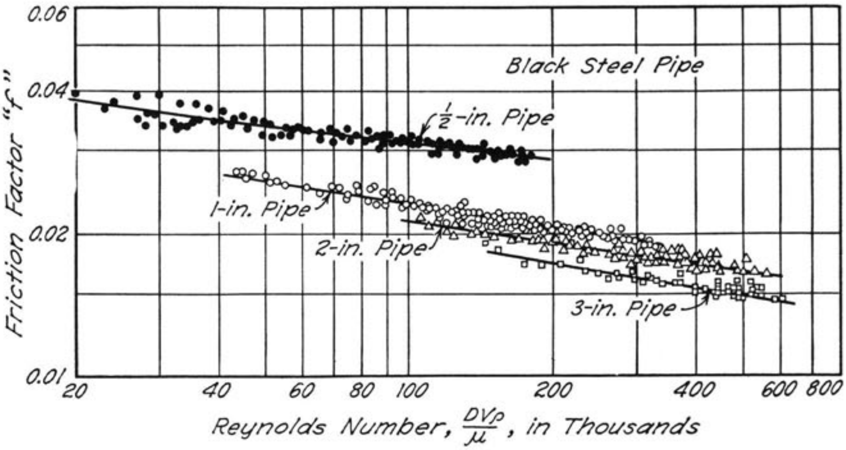


FIG. 9. FRICTION FACTORS FOR SLUDGES IN TEST PIPES

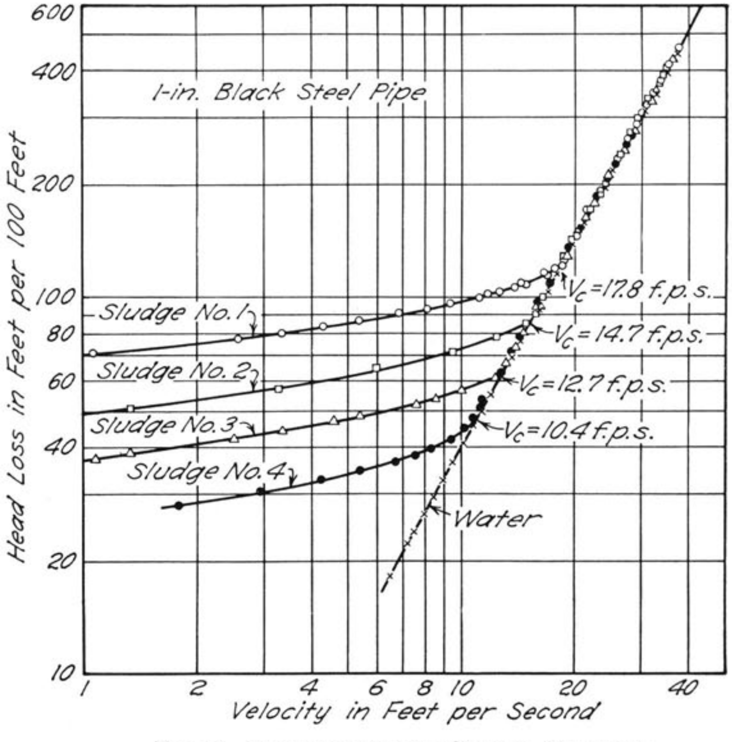


FIG. 10. DETERMINATION OF CRITICAL VELOCITIES

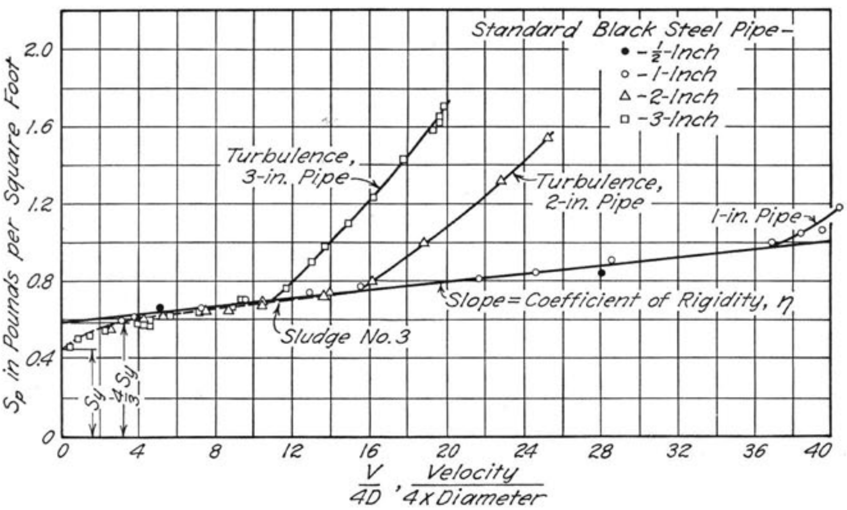


FIG. 11. DETERMINATION OF COEFFICIENTS OF RIGIDITY AND YIELD VALUES

feet of flowing material, was plotted on logarithmic paper against the corresponding mean velocity of flow, the best indication of the critical velocity was obtained. Figure 10 is a graph representing the flow data for the 1-in. pipe. The critical velocities are indicated on the figure. Figures for the other three pipes are similar. On Fig. 10 the line representing the flow of water in the same pipe is also shown, and it can be seen that in the turbulent region, on the figure, the head losses expressed in feet of flowing material are nearly the same for all sludges tested and for water.

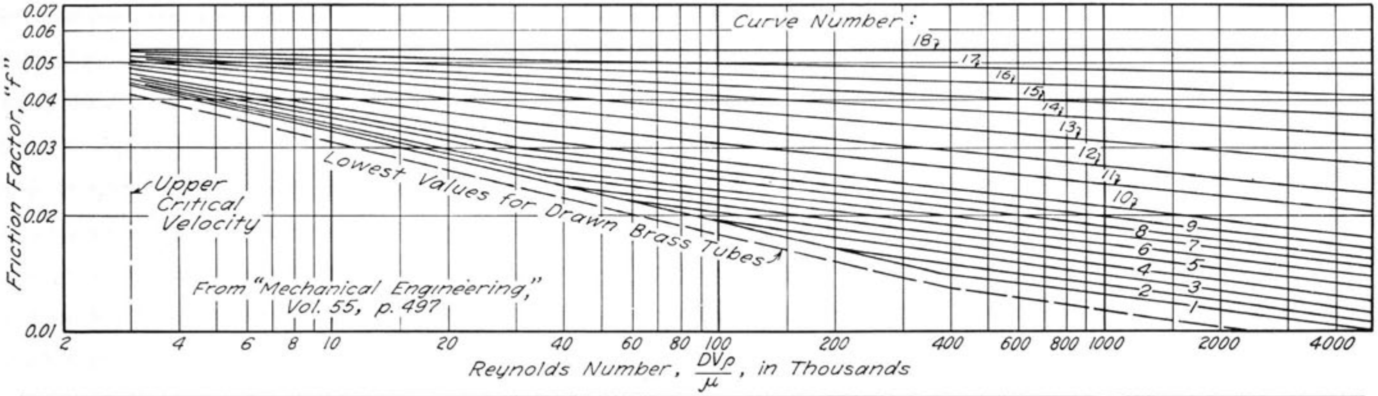
22. *Method of Determining Coefficient of Rigidity and Yield Value.*—The coefficients of rigidity and the yield values of the sludges tested were determined graphically as described in Chapter IV, page 28 of Bulletin 319.[1] Figure 11 is a diagram of this type in which the shear, S_p, at the pipe wall due to the flowing material is plotted against $V/4D$. The slope of the line resulting from connecting the experimental points, for laminar flow only, is the coefficient of rigidity and the intercept of the same line on the S_p axis is 4/3 the yield value. The coefficients of rigidity and the yield values obtained in this way, for the various sludges tested, are given in Table 1.

It can be seen from Fig. 11 that at low velocities of flow, that is, low values of $V/4D$, the experimental points tend to approach the yield value. This tendency is shown by the exact formula for laminar flow, Equation (1). For very low velocities of laminar flow it is probably better to use Equation (1) to compute the friction loss, because for low velocities of flow the last term of Equation (1) becomes appreciable with respect to the first two terms, and cannot be neglected as in the approximate Equation (2). For higher velocities the approximate Equation (2) may be used because at higher velocities the last term in Equation (1) becomes negligible.

VI. Theory of Turbulent Flow

23. *Factors Affecting Turbulent Flow.*—The following factors may be assumed to affect the friction loss of sludge in straight, circular pipes flowing full with a constant average velocity: (a) diameter of pipe; (b) velocity of flow; (c) length of pipe; (d) roughness of pipe; (e) density of the flowing sludge; and (f) yield value and coefficient of rigidity of the sludge. Characteristics of the sludge and its environment, such as particle size, concentration of suspended matter, temperature, etc., will probably affect the friction losses, but will be considered only in their effect on the yield value and the coefficient of rigidity of the sludge. All the factors listed above, except the yield value and rigidity, are included in the coordinates of a friction-factor chart, showing conditions in turbulent flow of sludge, with the provision that the viscosity of the dispersion medium of the sludge must be substituted for the viscosity of the fluid as used in Reynolds number. It is to be emphasized that this statement is true only where the velocity is greater than critical, i.e., where the flow is turbulent. In non-turbulent or laminar flow the conditions of Equation (1) hold.

24. *Formulation of Factors Affecting Turbulent Flow.*—It has been found mathematically[9] and experimentally,[10] that for any pipe a plot of the friction factor, f, in Equation (6) on page 28, against the Reynolds number, as shown in Fig. 12, will show friction losses resulting from the flow of any liquid through any pipe or pipes, provided the roughnesses of the various pipes are dynamically similar. To determine the friction factor for any pipe by the use of such a chart the line to be used should represent the same relative roughness as that of the pipe to be used. Since the chart correlates all factors affecting flow it would be desirable to be able to plot the flow of all types of sludge on a similar diagram. This can be done provided a constant can be found for the sludge similar to the viscosity of a liquid. It has been found, in this investigation,



Type of Conduit	Curve Number in Diagram Above																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Drawn Tubing-Brass, Tin, Lead, Glass, Diameter in Inches ⟶	0.35 up	—	—	—	—	—	—	—	—	—	—	—	—	0.125	—	—	—	0.0625
Clean Steel, Wrought Iron Diameter in Inches ⟶	72	48-66	14-42	6-12	4-5	2-3	$1\frac{1}{2}$	$1-1\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	—	—	—	—	—
Clean Galvanized Iron Diameter in Inches ⟶	—	—	30	10-24	6-8	3-5	$2\frac{1}{2}$	$1\frac{1}{2}-2$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	—	$\frac{3}{8}$	—	$\frac{1}{4}$	—	$\frac{1}{8}$
Best Cast Iron, Cement, Light Riveted Sheet Ducts Diameter in Inches ⟶	—	—	48-96	20-48	12-16	5-10	3-4	$2-2\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	—	—	—	—	—	—	—
Average Cast Iron, Rough Formed Concrete Diameter in Inches ⟶	—	—	96	42-96	24-36	10-20	6-8	4-5	3	—	—	—	—	—	—	—	—	—
First-class Brick, Heavy Riveted Steel Diameter in Inches ⟶	—	—	220	84-204	48-72	20-42	16-18	10-14	8	5	4	3	—	—	—	—	—	—

Note—In drawn tube actual diameter is given. In pipe nominal size of standard weight is given

FIG. 12. FRICTION FACTOR CHART

from the results of about 900 tests using 8 different sludges, that the viscosity of the dispersion medium of the sludges tested can be substituted for the viscosity of the liquid in Reynolds number, and a friction-factor chart constructed which will be approximately the same as the chart obtained for a liquid. Although the viscosities of the dispersion media in the sludges tested are closely equal to that of water it is possible that similar results might be secured with dispersion media of higher viscosities. Tests from which this conclusion is drawn are described in Chapter IV.

For sludges composed essentially of water with suspended material the viscosity of the dispersion medium is nearly the same as that of water so that any of the common hydraulic formulas can be used in finding the turbulent flow friction losses of a sludge.

The use of the viscosity of the dispersion medium in place of the viscosity of a true liquid in Reynolds number is reasonable when it is recalled that in turbulent flow the frictional losses are due essentially to impact losses which depend only on density and velocity. The presence of suspended particles increases the density of the material, but in no way affects the basic viscosity of the dispersion medium. It is the viscosity of the dispersion medium which causes a relatively thin layer of material to move in laminar flow along the walls of the pipe while the material in the center of the pipe is flowing turbulently. In the flow of true liquids the thin layer at the pipe wall is called the "boundary layer." It is due to the boundary layer that the exponent of the velocity factor in the common hydraulic formulas is not 2, as would be expected if the flow were wholly turbulent. It is found, however, that for very rough pipes or for very high velocities in smooth pipes the exponent does become 2, indicating that flow is wholly turbulent and that frictional losses are due only to kinetic energy impact losses, the boundary layer having been reduced to zero thickness.

25. *Friction Factor Chart.*—The most complete summary of available pipe-flow data, prepared by Emory Kemler,[11] and correlated on a friction factor diagram by R. J. S. Pigott, can be found in Mechanical Engineering, Vol. 55, page 497, 1933. A reproduction of this friction factor chart, suitable for use in determining friction factors for most types of pipes, is given in Fig. 12. It has been found in this investigation, as shown in Fig. 9 and explained in Section 24, that such a friction factor chart can be used for solving sludge-flow, friction-loss problems, provided the viscosity of the dispersion medium of the sludge is used in computing the Reynolds number.

TABLE 4

VISCOSITY OF WATER AT VARIOUS TEMPERATURES

32 to 125 deg. F.

Bingham and Jackson, U. S. Bur. Stds. Bul. 14, 75, 1918.

Temperature deg. F.	Viscosity lb. per ft. sec.	Temperature deg. F.	Viscosity lb. per ft. sec.
32	12.1×10^{-4}	80	5.8×10^{-4}
35	11.4×10^{-4}	85	5.4×10^{-4}
40	10.4×10^{-4}	90	5.1×10^{-4}
45	9.5×10^{-4}	95	4.9×10^{-4}
50	8.8×10^{-4}	100	4.6×10^{-4}
55	8.1×10^{-4}	105	4.4×10^{-4}
60	7.6×10^{-4}	110	4.2×10^{-4}
65	7.0×10^{-4}	115	4.0×10^{-4}
70	6.6×10^{-4}	120	3.8×10^{-4}
75	6.2×10^{-4}	125	3.6×10^{-4}

In the use of the chart the Reynolds number is first computed, using the viscosity of water as that of the sludge dispersion medium. Values of the viscosities of water at various temperatures are given in Table 4. Using this Reynolds number and the curve representing the type of conduit to be used, the corresponding friction factor, f, is found. The total head loss is then computed by means of the expression

$$H = f \frac{L}{D} \frac{V^2}{2g} \tag{6}$$

It is to be noted that for high friction factors a relatively large error in determining the viscosity of the dispersion medium will have but slight effect on the value of the friction factor, and that the magnitude of this effect is less than that resulting from a slight error in the assumption of the pipe wall roughness.

Let it be required to determine the pressure required to pump a sludge through 4000 feet of 6-in. cast-iron pipe at the rate of 600 gallons per minute. The characteristics of the sludge will be assumed to be as follows:

$$S_y = 0.020 \text{ lb. per sq. ft.}$$
$$\eta = 0.020 \text{ lb. per ft. sec.}$$
$$\rho = 65 \text{ lb. per cu. ft.}$$

The viscosity of the dispersion medium is assumed to be the same as that of water.

The temperature will be assumed to be 60 deg. F.

The steps in the solution are as follows:

(1) Determine the critical velocity by Equation (4);

$$V_{uc} = \frac{1500\eta + 1500\sqrt{\eta^2 + \dfrac{D^2 S_y \rho g}{4500}}}{D\rho}$$

$$= \frac{1500 \times 0.020 + 1500\sqrt{\overline{0.020^2} + \dfrac{0.020 \times 65 \times \overline{0.5^2} \times 32.2}{4500}}}{65 \times 0.5}$$

$$= 3.4 \text{ ft. per sec.}$$

(2) Determine velocity at which sludge will flow in pipe at specified rate;

$$V = \frac{Q}{A} = \frac{600}{\overline{0.5^2} \times 0.785 \times 60 \times 7.5} = 6.8 \text{ ft. per sec.}$$

(3) If the velocity found in step (2) is less than the critical velocity in step (1), the laminar flow Equation (2) is used. If the actual velocity is greater than the critical velocity, then Reynolds number is computed for the actual velocity as follows:

Find the viscosity, μ, for water from Table 4 at 60 deg. F. then

$$Re = \frac{DV\rho}{\mu} = \frac{0.5 \times 6.8 \times 65}{7.6 \times 10^{-4}} = 290\ 000$$

(4) Using the Reynolds number found in step (3), find the friction factor from Fig. 12 corresponding to a new 6-in. cast iron pipe. The friction factor is

$$f = 0.020$$

(5) Substitute this value of f in Equation (6);

$$H = \frac{0.02 \times 4000 \times \overline{6.8^2}}{0.5 \times 2 \times 32.2} = 115 \text{ feet of sludge}$$

(6) The pressure loss is then

$$\Delta P = H\rho = 115 \times 65 = 7500 \text{ lb. per sq. ft.} = 52 \text{ lb. per sq. in.}$$

TABLE 5

COMPARISON OF HEAD LOSSES COMPUTED BY KESSLER'S FORMULA WITH OBSERVED HEAD LOSSES FOR SLUDGE FLOWING IN A 1-IN. BLACK STEEL PIPE

$S_y = 0.60$ lb. per sq. ft.; $\eta = 0.015$ lb. per ft. sec.; $\rho = 75.0$ lb. per cu. ft.; $V_c = 14.9$ ft. per sec.

Velocity ft. per sec.	Head Loss ft. per 100 ft. (computed by Equation (7))	Head Loss ft. per 100 ft. (observed)	Per Cent Variation
15	86	87	+1.32
18	117	120	+2.65
20	145	145	0.00
22	175	175	0.00
24	200	200	0.00
26	232	232	0.00
28	265	270	+1.88
30	300	310	+3.33
32	339	349	+2.95
34	380	390	+2.65
36	420	430	+2.38
38	465	475	+2.15
40	510	520	+1.96

It must be remembered that below the critical velocity the turbulent flow relationships for a sludge do not hold, and the friction factor chart cannot be used in determining the friction loss. The critical velocity should always be computed by means of Equation (4) or Equation (9), on pages 11 and 32, for the particular sludge in question.

26. *Exponential Type Formulas.*—When sludges in which water is the dispersion medium flow in the turbulent state, the head loss, expressed in feet of the flowing material, will be the same as the head loss for water. This is because the viscosity of the dispersion medium of these sludges is essentially the same as that of water. It was observed, from data similar to those shown in Fig. 10, that the head loss due to the turbulent flow of sludge is independent of the rigidity and the yield value of the sludge. Since water is the dispersion medium in the sludges shown in Fig. 10 it follows that the head loss due to the turbulent flow of sludge in which water is the dispersion medium is dependent on the characteristics of water. This conclusion is corroborated by the data in Fig. 9 which show that sludges with the viscosity of the dispersion medium the same as that of water, and with a density closely that of water, will have a friction factor for the sludge closely equal to the friction factor for water at equal velocities. It is to be noted also, from Fig. 9, that a relatively large difference in Reynolds numbers will cause a relatively small variation in the friction factor. Hence it can be

TABLE 6

COMPARISON OF HEAD LOSSES OBSERVED IN CALUMET TESTS WITH HEAD LOSSES AT HIGHER THAN CRITICAL VELOCITY COMPUTED BY HAZEN AND WILLIAMS FORMULA

Velocity ft. per sec.	Head Loss in Feet of Sludge per 100 Feet of Pipe			Per Cent Variation from Observed Values	
	Computed		Observed (Sp. Gr. = 1.05)	$C = 124$	$C = 121$
	$C = 124$	$C = 121$			
(1)	(2)*	(3)†	(4)	(5)	(6)
1.5	0.14	0.14	0.14	0.0	0.0
2.0	0.23	0.24	0.24	−4.2	0.0
3.0	0.49	0.51	0.51	−3.9	0.0
4.0	0.83	0.87	0.87	−4.6	0.0
5.0	1.3	1.3	1.3	0.0	0.0
6.0	1.8	1.9	1.9	−5.3	0.0
7.0	2.4	2.5	2.5	−4.0	0.0

*The figures in this column were computed by substituting the corresponding velocity from column (1) of this table in Hazen and Williams formula, using C as 124. This value of C was assumed as a median value for the 8-in. pipe shown in Fig. 11, page 1789 of Reference 3.

†The figures in this column were computed in a similar manner to those shown in column (2) except that a value of $C = 121$ was taken in order to make the computed values check with the observed values. The purpose of this procedure was to emphasize the fact that the computed head losses vary inversely as $C^{1.85}$ and that it is possible to select a value of C between the limits of 122 and 136 reported on page 1791 of Reference 3 that will make the computed and observed values of head loss agree.

The observation of head losses in Reference 3 are reported in feet of water. The figures in this column were converted to feet of sludge by dividing the observed values by 1.05, the assumed specific gravity of the sludge.

assumed that the friction factors for water and for sludges with water as a dispersion medium are closely equal at equal velocities. Since all factors in commonly-used hydraulic formulas are, therefore, the same for water and for sludges with water as the dispersion medium, the head loss resulting from the turbulent flow of sludge with water as the dispersion medium is the same as that for the flow of water.

Table 5 shows a comparison of observed velocities for the flow of sludge in black steel pipe with corresponding velocities computed by Kessler's[12] formula applicable to the flow of water in black steel pipe;

$$V = 80.2D^{0.678}\left(\frac{H}{L}\right)^{0.543} \tag{7}$$

The agreement is well within the usual limits of error experienced in the use of the exponential type hydraulic formulas.

In computing head losses due to the turbulent flow of sludge in cast iron pipe, Hazen and Williams formula

$$V = CR^{0.63}\left(\frac{H}{L}\right)^{0.54} 0.001^{-0.04} \tag{8}$$

with the proper value of C, may be used. Comparisons of observed velocities in the Calumet tests[3] with velocities computed by the Hazen and Williams formula are given in Table 6. The comparison shows satisfactory agreement between the observed and the computed velocities.

27. *The Critical Velocity.*—The critical velocity may be defined as the velocity at which laminar flow changes to turbulent flow. The equation for head loss in turbulent flow is Equation (6) which is

$$H = f\frac{L}{D}\frac{V^2}{2g}$$

Equating expressions (2) and (6) and solving for V, we obtain

$$V_c = \frac{32\eta + 32\sqrt{\eta^2 + \dfrac{fS_y\rho D^2 g}{96}}}{f\rho D} \tag{9}$$

which is an expression for the critical velocity.

Methods for the measurement of S_y and η, the yield value and the coefficient of rigidity, respectively, are given in Chapter IV of Bulletin 319. An additional method for determining the yield value is given on page 37 of this Bulletin. The friction factor, f, in Equation (9) varies with the velocity, so that the expression must be solved by trial and error, first assuming a value of f, then obtaining from the friction factor chart the value of f for the velocity found in the first approximation. This second value of f is then substituted in Equation (9) and the critical velocity found. Usually one approximation is all that is necessary.

Equations (3) and (4) are also expressions for critical velocity in terms of controlling factors exclusive of the roughness of the pipe wall, whereas allowance is made for the roughness in Equa-

TABLE 7

OBSERVED AND COMPUTED VALUES OF CRITICAL VELOCITY FOR TESTS MADE IN THIS INVESTIGATION

Sludge No.	S_y lb. per sq. ft.	η lb. per ft. sec.	ρ lb. per cu. ft.	Diameter of Pipe in.	Friction Factor f	Critical Velocity, V_c ft. per sec.		Error Per Cent
						(computed)	(observed)	
1	0.90	0.015	75.7	1	0.0216	17.4	17.5	−0.6
				2	0.0196	16.4	16.0	2.5
				3	0.0172	16.6	15.5	7.0
2	0.60	0.015	75.0	½	0.0316	14.0	14.2	−1.4
				1	0.0220	14.9	14.7	1.4
				2	0.0204	13.5	13.5	0.0
				3	0.0180	13.8	13.8	0.0
3	0.44	0.011	73.8	½	0.0324	11.3	12.0	−5.8
				1	0.0224	12.2	12.9	−5.4
				2	0.0208	11.2	11.0	1.8
				3	0.0180	11.6	11.5	0.9
4	0.29	0.010	72.5	1	0.0236	10.1	10.1	0.0
				2	0.0216	9.2	9.0	2.2
				3	0.0188	9.5	9.5	0.0
5	0.19	0.008	72.0	½	0.0340	7.5	7.7	−2.6
				1	0.0240	8.0	7.9	1.3
				2	0.0208	7.6	7.6	0.0
				3	0.0176	8.0	7.7	3.9
6	0.082	0.006	70.0	½	0.0352	5.2	5.4	−3.7
				1	0.0264	5.2	5.3	−1.9
				2	0.0216	5.1	5.1	0.0
				3	0.0192	5.1	5.1	0.0

tion (9). These equations should not be confused. Where pipe roughness is not considered, the critical velocity, V_c, may be found in a range between V_{lc} and V_{uc}. Where pipe roughness is considered, the critical velocity, V_c, is dependent on the pipe roughness, as shown in Equation (9). Equation (3) shows the minimum value at which turbulent velocity can occur, i.e., the minimum possible value of V_c; and Equation (4) shows the maximum velocity at which laminar flow can exist, i.e., the maximum possible value of V_c.

Table 7 is a summary of critical velocities computed by Equation (9) and critical velocities for corresponding conditions observed in this investigation. Under field conditions the agreement between computed and observed values of the critical velocities cannot be expected to be so precise as in these tests, because, in general, the friction factors for pipes in the field are not so precisely known.

28. *Effect of Pipe Diameter on Critical Velocity.*—If the critical velocities for a given sludge are computed by means of Equation (9) for different sizes of pipes, a curve similar to Fig. 13 is obtained when the critical velocity is plotted against the pipe diameter. This

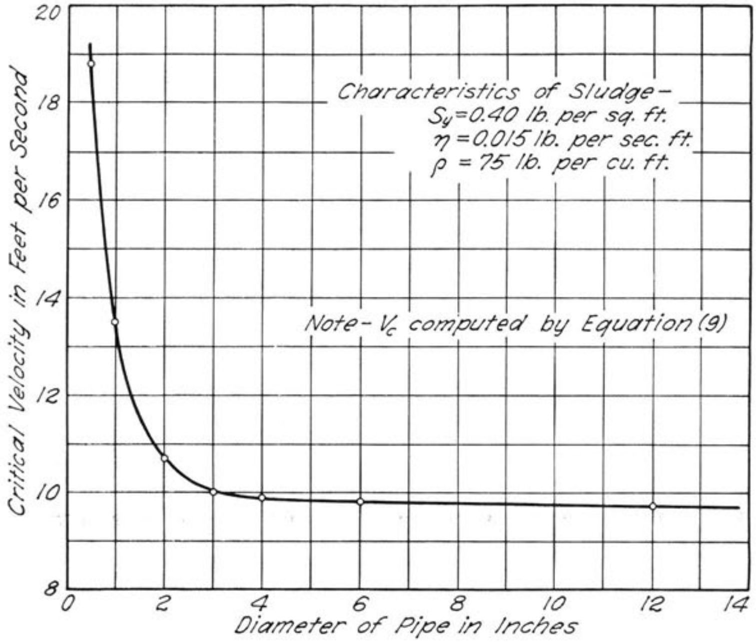


FIG. 13. RELATION OF CRITICAL VELOCITY TO PIPE DIAMETER

figure shows that for ordinary sludges (η less than 0.030) the diameter of the pipe has a negligible effect on the critical velocity for pipes larger than 2 to 3 inches diameter. Above a diameter of 3 inches the critical velocity remains nearly constant for a given sludge. This conclusion is in contrast with what occurs in the case of the critical velocity of a true liquid, which varies inversely as the pipe diameter, as shown by the Reynolds criterion.

In order to show diagrammatically why the critical velocity in the case of sludge is not affected by pipe diameter, computations of velocities and corresponding head losses were made by Equations (2) and (6) and were plotted in Fig. 14. This figure represents the flow of a typical sludge in various diameters of new black steel pipe. The effect of pipe diameter is clearly shown.

29. *Effect of Roughness on Critical Velocity.*—The qualitative effect of roughness on critical velocity is shown by Equation (9) to be of some magnitude. In order to show this effect graphically computations were made, by Equations (2) and (6), of velocities and corresponding head losses for one sludge flowing in two pipes

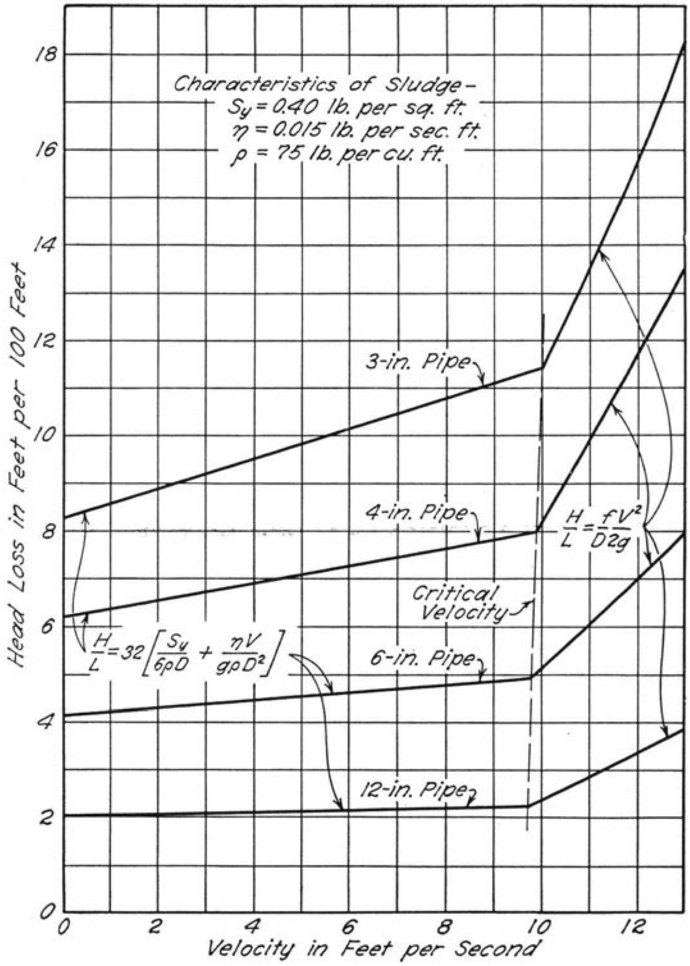


FIG. 14. EFFECT OF PIPE DIAMETER ON CRITICAL VELOCITY OF SLUDGE FLOWING IN NEW BLACK STEEL PIPE

of the same diameter but of different roughnesses. The results were then plotted, as shown in Fig. 15.

30. *Effect of Yield Value on Critical Velocity.*—A simple approximate expression for determining the magnitude of critical velocity may be found useful in design. Such an expression can be based on the fact that the critical velocity for ordinary sludges, i.e., where η is less than 0.030, varies roughly as the yield value, S_y, for pipes

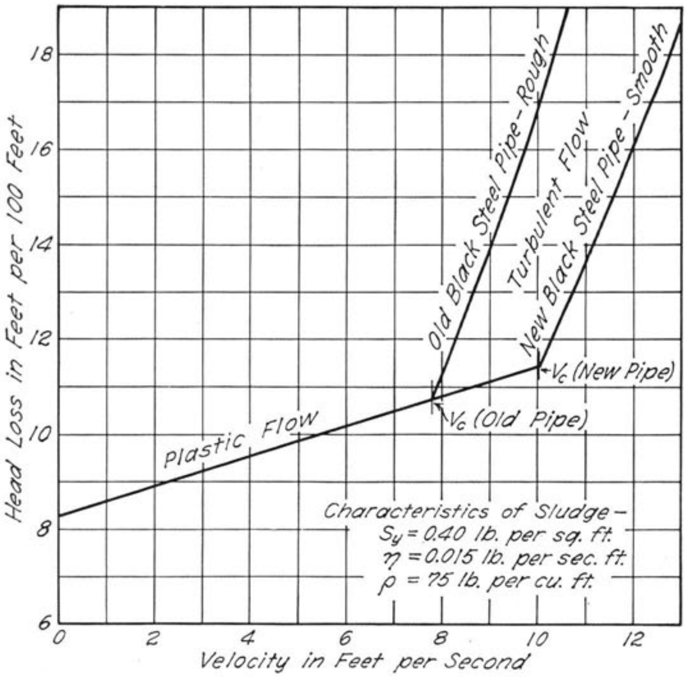


FIG. 15. EFFECT OF PIPE ROUGHNESS ON MAGNITUDE OF CRITICAL VELOCITY IN A 3-INCH PIPE

above 3 to 6 inches in diameter. This fact becomes evident when appropriate numerical values are substituted in Equation (4) as follows:

$$V_{uc} = \frac{1500\eta + 1500\sqrt{\eta^2 + \dfrac{S_y \rho D^2 g}{4500}}}{\rho D}$$

Then if $\eta = 0.03$

$$V_{uc} = \frac{45 + 1500\sqrt{0.0009 + \dfrac{S_y \rho D^2 g}{4500}}}{\rho D}$$

$$= \frac{45}{D\rho} + 22.4\left(\frac{S_y g}{\rho}\right)^{1/2} \text{(approx.)}$$

For values of D above 3 to 6 inches the first term can be neglected, leaving the expression

$$V_{uc} = 22.4\left(\frac{S_y g}{\rho}\right)^{1/2} \qquad (10)$$

The precision of this expression increases as η decreases, as D increases, and as S_y increases. The expression is presented for its general usefulness, but does not give precise results. In extreme cases it may give values of V_c about 40 per cent in error, but in most cases the precision will be much greater.

The yield value of sludge may be quickly and accurately measured by observing the depth to which a thin metallic strip will sink, due to its own weight, into the sludge. The yield value is obtained by equating the downward and upward forces acting on the metal strip. The downward force is the weight of the metal minus the buoyant force of the sludge on the submerged portion, and the upward force is the friction on the sides of the strip due to the yield value of the sludge. At equilibrium these forces are equal and can be equated, and the yield value can be computed. Equation (10) can then be used in computing the approximate critical velocity.

VII. SUMMARY

31. *Practical Applications.*—The formulas studied herein for conditions of flow can be used for two principal purposes: the measurement of the yield value, S_y, and the rigidity, η, of a sludge; and in the design of pipe sizes and in computing the capacity of equipment for conveying and pumping sludge. The factors involved in the use of the formulas for either of these purposes are: D, L, Q, H, S_y, η, ρ, and C. The nomenclature used is explained on page 9.

In the design of pipe sizes and pumping equipment for any particular sludge it is necessary to know its yield value and coefficient of rigidity. Where these values cannot be measured they must be estimated from the assumed characteristics of the sludge and such data as are given in Table 8 of Bulletin 319. Because of the comparatively wide variations in values of S_y and η between sludges with apparently similar characteristics, and because of the limited data available, the values of S_y and η found in practice may vary considerably from those assumed in design. The designer should, therefore, assume values which will assure results within safe limits. The field is open for further research in determining values of these characteristics of sludges and to correlate them with other characteristics.

When S_y, η, and C are known and it is desired to determine H, the remaining factors Q, L, and D must be known or assumed for the particular conditions. The first step is to determine the value of the average velocity of flow, V, from the expression $V = 4Q/\pi D^2$. The values of V_{lc}, V_c, and V_{uc} must then be determined by substituting in Equations (3), (9), and (4), respectively. If V is greater than V_c or V_{uc}, turbulent flow exists, and H is determined as explained on page 29. If the value of V is less than V_c or V_{lc}, laminar flow exists, and H is determined by substitution in Equation (14) from Bulletin 319, as follows:

$$\frac{H_w}{L} = 32\left(\frac{S_y}{6WD} + \frac{\eta V}{gWD^2}\right)$$

32. *Conclusions.*—The work reported herein is a continuation into the field of turbulent flow of the work on the flow of sludges reported in Bulletin No. 319 of the Engineering Experiment Station at the University of Illinois, under the title "Laminar Flow of Sludges in Pipes with Special Reference to Sewage Sludge." It is shown that most sludges, such as drilling muds, clay slurries, sewage sludges, and other aqueous suspensions of fine particles, when flowing in a pipe at a velocity greater than the critical, follow the fundamental laws for the flow of true liquids.

Problems involving the head loss due to friction resulting from the flow of sludges at greater than critical velocity can be solved by means of a chart, such as is shown in Fig. 12, in which the friction factor, f, in Equation (6) is plotted as ordinate against Reynolds number as abscissa. In using the chart for this purpose Reynolds number is computed from the expression

$$Re = DV\rho/\mu$$

by using the viscosity, μ, of the dispersion medium of the sludge. With the known Reynolds number the corresponding friction factor, f, is read from the chart and is substituted in Equation (6) to find the value of H/L.

Problems involving the head loss due to friction resulting from the flow of sludges at greater than critical velocity can be solved, for certain types of sludges, also by means of the familiar exponential formulas, such as those of Hazen and Williams, Saph and Schroeder, Kessler, etc. In order that such formulas may be applied, the sludge must be made up of finely-divided substance dispersed in water, and the velocity of flow must be above the critical.

Effects of such factors as pipe roughness, pipe diameter, and the yield value of the sludge on the critical velocity are shown, and these factors have been formulated into the expression

$$V_c = \frac{32\eta + 32\sqrt{\eta^2 + \dfrac{fS_y\rho D^2 g}{96}}}{f\rho D} \qquad (9)$$

This page is intentionally blank.

APPENDIX

BIBLIOGRAPHY

REFERENCE No.	YEAR	AUTHOR	TITLE
1	1939	Babbitt, H. E. and Caldwell, D. H.	"Laminar Flow of Sludges in Pipes with Special Reference to Sewage Sludge," Univ. of Ill. Eng. Exp. Sta. Bul. 319.
2	1927	Gregory, W. B.	"Pumping Clay Slurry Through a Four-inch Pipe," Mech. Eng., Vol. 49, p. 609.
3	1929	Committee Report, A.S.C.E.	"Friction of Sewage Sludge in Pipes." Progress Report of Committee of the Sanitary Engineering Division, Proc. A.S.C.E., Vol. 55, pt. 2, p. 1773.
4	1939	Wilhelm, R. H. Wroughton, D. M. Loeffel, W. L.	"Flow of Suspensions Through Pipes," Ind. and Eng. Chem., Ind. Ed., Vol. 31, p. 622.
5	1939	Kalinske, A. A.	"Relation of the Statistical Theory of Turbulence to Hydraulics," Proc. A.S.C.E., p. 1387.
6	1938	Hatfield, W. D.	"Viscosity or Pseudo-Plastic Properties of Sewage Sludges," Sewage Works Jour., Vol. 10, pp. 3 and 272.
7	1934	Merkel, Wilhelm	"Die Fliesseigenschaften von Abwasserschlamm," Beiheft zum Gesundheits-Ingenieur, Reihe II, Heft 14.
8	1926	Fraser, E. S. Jones, R. B.	"Motor Vehicles and Their Engines, D. Van Nostrand and Co., New York.
9	1883	Reynolds, Osborn	"An Experimental Investigation of the Circumstances Which Determine Whether Motion of Water Shall Be Direct or Sinuous and of the Law of Resistance in Parallel Channels," Phil. Trans. Royal Soc., London, Vol. 174, pt. 3, p. 935.
10	1914	Stanton and Pannell	"Similarity of Motion in Relation to the Surface Friction of Fluids," Proc. Royal Soc., London, Vol. A214, pp. 199-224.
11	1933	Kemler, Emory	"A Study of the Data on the Flow of Fluids in Pipes," Trans. A.S.M.E., Vol. 55, Hyd. 55-2.
12	1935	Kessler, L. H.	"Experimental Investigation of Friction Losses in Wrought Iron Pipe When Installed with Couplings," Univ. of Wisconsin, Bul. 82.

RECENT PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION†

Bulletin No. 295. Tests of Thin Hemispherical Shells Subjected to Internal Hydrostatic Pressure, by Wilbur M. Wilson and Joseph Marin. 1937. *Thirty cents.*

Circular No. 30. Papers Presented at the Twenty-fourth Annual Conference on Highway Engineering, Held at the University of Illinois, March 3-5, 1937. 1937. *None available.*

Reprint No. 11. Third Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1937. *Fifteen cents.*

Bulletin No. 296. Magnitude and Frequency of Floods on Illinois Streams, by George W. Pickels. 1937. *Seventy cents.*

Bulletin No. 297. Ventilation Characteristics of Some Illinois Mines, by Cloyde M. Smith. 1937. *Seventy cents.*

Bulletin No. 298. Resistance to Heat Checking of Chilled Iron Car Wheels, and Strains Developed Under Long-Continued Application of Brake Shoes, by Edward C. Schmidt and Herman J. Schrader. 1937. *Fifty-five cents.*

Bulletin No. 299. Solution of Electrical Networks by Successive Approximations, by Laurence L. Smith. 1937. *Forty-five cents.*

Circular No. 31. Papers Presented at the Short Course in Coal Utilization, Held at the University of Illinois, May 25-27, 1937. 1938. *None available.*

Bulletin No. 300. Pressure Losses Resulting from Changes in Cross-Sectional Area in Air Ducts, by Alonzo P. Kratz and Julian R. Fellows. 1938. *Sixty-five cents.*

Bulletin No. 301. The Friction of Railway Brake Shoes at High Speed and High Pressure, by Herman J. Schrader. 1938. *Sixty cents.*

Bulletin No. 302. Fatigue Tests of Riveted Joints, by Wilbur M. Wilson and Frank P. Thomas. 1938. *One dollar.*

Circular No. 32. Two Investigations on Transit Instruments, by William H. Rayner. 1938. *Twenty-five cents.*

Circular No. 33. Papers Presented at the Twenty-fifth Annual Conference on Highway Engineering, Held at the University of Illinois, March 2-4, 1938. 1938. *None available.*

Bulletin No. 303. Solutions for Certain Rectangular Slabs Continuous Over Flexible Supports, by Vernon P. Jensen. 1938. *One dollar.*

Bulletin No. 304. A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams, by Nathan M. Newmark. 1938. *One dollar.*

Circular No. 34. The Chemical Engineering Unit Process—Oxidation, by Donald B. Keyes. 1938. *Fifty cents.*

Circular No. 35. Factors Involved in Plate Efficiencies for Fractionating Columns, by Donald B. Keyes. 1938. *Twenty cents.*

Bulletin No. 305. Summer Cooling in the Warm-Air Heating Research Residence with Cold Water, by Alonzo P. Kratz, Seichi Konzo, Maurice K. Fahnestock and Edwin L. Broderick. 1938. *Ninety cents.*

Bulletin No. 306. Investigation of Creep and Fracture of Lead and Lead Alloys for Cable Sheathing, by Herbert F. Moore, Bernard B. Betty, and Curtis W. Dollins. 1938. *One dollar.*

Reprint No. 12. Fourth Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1938. *None available.*

Bulletin No. 307. An Investigation of Rigid Frame Bridges: Part I, Tests of Reinforced Concrete Knee Frames and Bakelite Models, by Frank E. Richart, Thomas J. Dolan, and Tilford A. Olson. 1938. *Fifty cents.*

Bulletin No. 308. An Investigation of Rigid Frame Bridges: Part II, Laboratory Tests of Reinforced Concrete Rigid Frame Bridges, by W. M. Wilson, R. W. Kluge, and J. V. Coombe. 1938. *Eighty-five cents.*

Bulletin No. 309. The Effects of Errors or Variations in the Arbitrary Constants of Simultaneous Equations, by George H. Dell. 1938. *Sixty cents.*

Bulletin No. 310. Fatigue Tests of Butt Welds in Structural Steel Plates, by W. M. Wilson and A. B. Wilder. 1939. *Sixty-five cents.*

---

†Copies of the complete list of publications can be obtained without charge by addressing the Engineering Experiment Station, Urbana, Ill.

Bulletin No. 311. The Surface Tensions of Molten Glass, by Cullen W. Parmelee, Kenneth C. Lyon, and Cameron G. Harman. 1939. *Fifty-five cents.*

Bulletin No. 312. An Investigation of Wrought Steel Railway Car Wheels: Part I, Tests of Strength Properties of Wrought Steel Car Wheels, by Thomas J. Dolan and Rex L. Brown. 1939. *Seventy cents.*

Circular No. 36. A Survey of Sulphur Dioxide Pollution in Chicago and Vicinity, by Alamjit D. Singh. 1939. *Forty cents.*

Circular No. 37. Papers Presented at the Second Conference on Air Conditioning, Held at the University of Illinois, March 8-9, 1939. 1939. *Fifty cents.*

Circular No. 38. Papers Presented at the Twenty-sixth Annual Conference on Highway Engineering, Held at the University of Illinois, March 1-3, 1939. 1939. *Fifty cents.*

Bulletin No. 313. Tests of Plaster-Model Slabs Subjected to Concentrated Loads, by Nathan M. Newmark and Henry A. Lepper, Jr. 1939. *Sixty cents.*

Bulletin No. 314. Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads, by Frank E. Richart and Ralph W. Kluge. 1939. *Eighty cents.*

Bulletin No. 315. Moments in Simple Span Bridge Slabs with Stiffened Edges, by Vernon P. Jensen. 1939. *One dollar.*

Bulletin No. 316. The Effect of Range of Stress on the Torsional Fatigue Strength of Steel, by James O. Smith. 1939. *Forty-five cents.*

Bulletin No. 317. Fatigue Tests of Connection Angles, by Wilbur M. Wilson and John V. Coombe. 1939. *Thirty-five cents.*

Reprint No. 13. First Progress Report of the Joint Investigation of Continuous Welded Rail, by H. F. Moore. 1939. *Fifteen cents.*

Reprint No. 14. Fifth Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1939. *Fifteen cents.*

Circular No. 39. Papers Presented at the Fifth Short Course in Coal Utilization, Held at the University of Illinois, May 23-25, 1939. 1939. *Fifty cents.*

Reprint No. 15. Stress, Strain, and Structural Damage, by H. F. Moore. 1939. *None available.*

Bulletin No. 318. Investigation of Oil-fired Forced-Air Furnace Systems in the Research Residence, by A. P. Kratz and S. Konzo. 1939. *Ninety cents.*

Bulletin No. 319. Laminar Flow of Sludges in Pipes with Special Reference to Sewage Sludge, by Harold E. Babbitt and David H. Caldwell. 1939. *Sixty-five cents.*

Bulletin No. 320. The Hardenability of Carburizing Steels, by Walter H. Bruckner. 1939. *Eighty cents.*

**Bulletin No. 321.* Summer Cooling in the Research Residence with a Condensing Unit Operated at Two Capacities, by A. P. Kratz, S. Konzo, M. K. Fahnestock, and E. L. Broderick. 1940. *Seventy cents.*

**Circular No. 40.* German-English Glossary for Civil Engineering, by A. A. Brielmaier. 1940. *Fifty cents.*

**Bulletin No. 322.* An Investigation of Rigid Frame Bridges: Part III, Tests of Structural Hinges of Reinforced Concrete, by Ralph W. Kluge. 1940. *Forty cents.*

**Circular No. 41.* Papers Presented at the Twenty-seventh Annual Conference on Highway Engineering, Held at the University of Illinois, March 6-8, 1940. 1940. *Fifty cents.*

**Reprint No. 16.* Sixth Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1940. *Fifteen cents.*

**Reprint No. 17.* Second Progress Report of the Joint Investigation of Continuous Welded Rail, by H. F. Moore, H. R. Thomas, and R. E. Cramer. 1940. *Fifteen cents.*

**Reprint No. 18.* English Engineering Units and Their Dimensions, by E. W. Comings. 1940. *Fifteen cents.*

**Reprint No. 19.* Electro-organic Chemical Preparations, Part II, by Sherlock Swann, Jr. 1940. *Thirty cents.*

**Reprint No. 20.* New Trends in Boiler Feed Water Treatment, by F. G. Straub. 1940. *Fifteen cents.*

**Bulletin No. 323.* Turbulent Flow of Sludges in Pipes, by H. E. Babbitt and D. H. Caldwell. 1940. *Forty-five cents.*

---

*A limited number of copies of bulletins starred are available for free distribution.

UNIVERSITY OF ILLINOIS

Colleges and Schools at Urbana

COLLEGE OF LIBERAL ARTS AND SCIENCES.—General curriculum with majors in the humanities and sciences; a new general curriculum with fields of concentration in mathematics and physical science, biological science, social science, and the humanities; specialized curricula in chemistry and chemical engineering; general courses preparatory to the study of law and journalism; pre-professional training in medicine, dentistry, and pharmacy; curriculum in social administration.

COLLEGE OF COMMERCE AND BUSINESS ADMINISTRATION.—Fields of concentration in accountancy, banking and finance, commerce and law, commercial teaching, economics, industrial administration, management, marketing, and public affairs.

COLLEGE OF ENGINEERING.—Curricula in agricultural engineering, ceramics, ceramic engineering, chemical engineering, civil engineering, electrical engineering, engineering physics, general engineering, mechanical engineering, metallurgical engineering, mining engineering, and railway engineering.

COLLEGE OF AGRICULTURE.—Curricula in agriculture, dairy technology, floriculture, general home economics, and nutrition and dietetics; pre-professional training in forestry.

COLLEGE OF EDUCATION.—Curricula in education, agricultural education, home economics education, and industrial education. The University High School is the practice school of the College of Education.

COLLEGE OF FINE AND APPLIED ARTS.—Curricula in architecture, art, landscape architecture, music, and music education.

COLLEGE OF LAW.—Professional curriculum in law.

SCHOOL OF JOURNALISM.—General and special curricula in journalism.

SCHOOL OF PHYSICAL EDUCATION.—Curricula in physical education for men and for women.

LIBRARY SCHOOL.—Curriculum in library science.

GRADUATE SCHOOL.—Advanced study and research.

Summer Session.—Courses for undergraduate and graduate students.

University Extension Division.—Courses taught by correspondence, extramural courses, science aids service, speech aids service, and visual aids service.

Colleges in Chicago

COLLEGE OF DENTISTRY.—Professional curriculum in dentistry.

COLLEGE OF MEDICINE.—Professional curriculum in medicine.

COLLEGE OF PHARMACY.—Professional curriculum in pharmacy.

University Experiment Stations, and Research and Service Organizations at Urbana

AGRICULTURAL EXPERIMENT STATION
ENGINEERING EXPERIMENT STATION
EXTENSION SERVICE IN AGRICULTURE AND HOME ECONOMICS
RADIO STATION (WILL)
BUREAU OF BUSINESS RESEARCH
BUREAU OF COMMUNITY PLANNING
BUREAU OF EDUCATIONAL RESEARCH
BUREAU OF INSTITUTIONAL RESEARCH
UNIVERSITY OF ILLINOIS PRESS

State Scientific Surveys and Other Divisions at Urbana

STATE GEOLOGICAL SURVEY
STATE NATURAL HISTORY SURVEY
STATE WATER SURVEY
STATE DIAGNOSTIC LABORATORY (for Animal Pathology)
U. S. SOYBEAN PRODUCTS LABORATORY

For general catalog of the University, special circulars, and other information, address
THE REGISTRAR, UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS