

ILLINOIS

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

PRODUCTION NOTE

University of Illinois at Urbana-Champaign Library Large-scale Digitization Project, 2007.

UNIVERSITY OF ILLINOIS ULLETIN

Vol. XXXIX

Tune 23, 1942

No. 44

ENGINEERING EXPERIMENT STATION **BULLETIN SERIES No. 336**

MOMENTS IN L-BEAM BRIDGES

A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION **UNIVERSITY OF ILLINOIS**

IN COOPERATION WITH

THE PUBLIC ROADS ADMINISTRATION FEDERAL WORKS AGENCY

AND

THE DIVISION OF HIGHWAYS **STATE OF ILLINOIS**

> BY NATHAN M. NEWMARK

> > AND

CHESTER P. SIESS

PRICE: ONE DOLLAR PUBLISHED BY THE UNIVERSITY OF ILLINOIS **URBANA**

[Issued weekly. Entered as second-class matter December 11, 1912, at the post office at Urbana, Illinois, under the Act of August 24, 1912. Acceptance for mailing at the special rate of postage provided for in section 1103

THE Engineering Experiment Station was established by act of the Board of Trustees of the University of Illinois on December 8, 1903. It is the purpose of the Station to conduct investigations and make studies of importance to the engineering, manufacturing, railway, mining, and other industrial interests of the State.

The management of the Engineering Experiment Station is vested in an Executive Staff composed of the Director and his Assistant, the Heads of the several Departments in the College of Engineering, and the Professor of Chemical Engineering. This Staff is responsible for the establishment of general policies governing the work of the Station, including the approval of material for publication. All members of the teaching staff of the College are encouraged to engage in scientific research, either directly or in coöperation with the Research Corps. composed of full-time research assistants, research graduate assistants, and special investigators.

To render the results of its scientific investigations available to the public, the Engineering Experiment Station publishes and distributes a series of bulletins. Occasionally it publishes circulars of timely interest, presenting information of importance, compiled from various sources which may not readily be accessible to the clientele of the Station, and reprints of articles appearing in the technical press written by members of the staff and others.

The volume and number at the top of the front cover page are merely arbitrary numbers and refer to the general publications of the University. Above the title on the cover is given the number of the Engineering Experiment Station bulletin, circular, or reprint which should be used in referring to these publications.

For copies of publications or for other information address

THE ENGINEERING EXPERIMENT STATION, **UNIVERSITY OF ILLINOIS,** URBANA, ILLINOIS

UNIVERSITY OF ILLINOIS ENGINEERING EXPERIMENT STATION BULLETIN SERIES NO. 336

MOMENTS IN I-BEAM BRIDGES

A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION UNIVERSITY OF ILLINOIS

IN COOPERATION WITH

THE PUBLIC ROADS ADMINISTRATION FEDERAL WORKS AGENCY

 AND

THE DIVISION OF HIGHWAYS **STATE OF ILLINOIS**

 BY

NATHAN M. NEWMARK RESEARCH ASSISTANT PROFESSOR OF CIVIL ENGINEERING

 AND

CHESTER P. SIESS

SPECIAL RESEARCH ASSOCIATE IN THEORETICAL AND APPLIED MECHANICS

PUBLISHED BY THE UNIVERSITY OF ILLINOIS

PRICE: ONE DOLLAR

UNIVERSITY
OF ILLINOIS
II PRESS II

$\sf CONTENTS$

CONTENTS (CONCLUDED)

 $\bf{4}$

LIST OF FIGURES

LIST OF TABLES

This page is intentionally blank.

MOMENTS IN I-BEAM BRIDGES

I. INTRODUCTION

1. Object and Scope of Investigation.—The studies reported in this bulletin were undertaken in an attempt to obtain a better understanding of the behavior of the type of structure commonly called the I-beam bridge, consisting of a concrete slab continuous over steel beams. This type of structure is so simple in appearance and so conveniently constructed that it has found widespread use for highway bridges. Consequently, an investigation of the action of this type of bridge with a view toward development of better design methods appeared desirable.

The data contained herein are based entirely on analytical considerations, and apply to simple-span right bridges consisting of slabs supported by five identical parallel beams, uniformly spaced. Moments are determined at various points in the slabs and in the beams for different positions of a concentrated load on the structure. The flexibility of the beams is taken into account in the analysis. Influence values for moment are given for a group of structures of various proportions and of different relative stiffnesses of slab and beams. From the influence values, moment coefficients are determined for a number of bridges of different span lengths subjected to standard highway truck loads. General relations bearing on the design of I-beam bridges are derived from the results of the analyses.

The numerical values of moment coefficients are generally applicable also to structures with concrete instead of steel beams, as well as to structures with steel beams anchored to the concrete slab by means of shear connectors.

The analytical work on I-beam bridges has been supplemented by tests of models of a number of bridges. These experimental studies are to be reported in a later publication.

2. Acknowledgment.—The work reported herein was done as a part of an investigation of the effect of concentrated loads on reinforced concrete bridge slabs, which is being conducted by the Engineering Experiment Station of the University of Illinois in cooperation with the Public Roads Administration of the Federal Works Agency and the Illinois Division of Highways. The investigation is under the administrative direction of DEAN M. L. ENGER, Director of the Engineering Experiment Station; PROFESSOR F. B. SEELY, Head of the Department of Theoretical and Applied Mechanics; and PROFESSOR W. C. HUNTINGTON, Head of the Department of Civil Engineering.

An Advisory Committee, with the following personnel, is in general charge of the program of the investigation:

Representing the Public Roads Administration: E. F. KELLEY, Chief, Division of Tests; RAYMOND ARCHIBALD, Senior Structural Engineer. (Until his death in December 1939, A. L. GEMENY, Senior Engineer, was a representative. From January 1940 until June 1941, L. A. PALMER, Associate Research Specialist, served on the Committee.)

Representing the Illinois Division of Highways: M. J. FLEMING, Assistant Chief Highway Engineer; A. BENESCH, Engineer of Grade Separations.

Representing the University of Illinois: F. E. RICHART, Research Professor of Engineering Materials; N. M. NEWMARK, Research Assistant Professor of Civil Engineering.

Consultants to the Advisory Committee, from the University of Illinois: W. M. WILSON, Research Professor of Structural Engineering; T. C. SHEDD, Professor of Structural Engineering.

The whole investigation is under the general guidance of Professor Richart. The work reported herein was done under the direction of the senior author, and was started as a thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering in the Graduate School of the University of Illinois, by the junior author, then Special Research Graduate Assistant in Theoretical and Applied Mechanics.

Detailed calculations have been made by C. P. SIESS, the late E. D. OLSON, W. M. PECKHAM, and L. E. GOODMAN, Research Graduate Assistants; by W. L. COLLINS, Assistant Professor of Theoretical and Applied Mechanics, and by L. N. MUIR, student in Civil Engineering. Some additional calculations have been made by F. L. EHASZ, Instructor in Theoretical and Applied Mechanics, and by W. H. BRETT, G. B. SIMPSON, and W. F. NUMRICH, students in Civil Engineering.

3. Method of Analysis.—The data reported in this bulletin were obtained by computation based on the method of analysis described in Bulletin 304* of the Engineering Experiment Station. Certain assumptions are involved in the analysis. These assumptions are:

(1) The resultant of the normal stresses acting on any crosssection of the slab is a pure couple.

8

^{*&}quot;A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," Univ. of Ill. Eng. Exp. Sta. Bul. 304, 1938.

(2) The material in the slab is homogeneous, elastic, isotropic, and of constant thickness.

(3) Flexural strains vary linearly through the depth of the slab.

(4) The beams exert only vertical forces on the slab; there is no shear between the top flange of the beams and the bottom of the slab. (An approximate study of T-beam action is made, however.)

(5) The reaction of the beam acts on the slab along a line, and is not distributed over a finite width.

(6) A beam and the slab directly over it deflect alike; that is, the slab does not separate from the beams.

(7) The edge beams on either side of the bridge are located at the edge of the slab.

(8) Both the slab and the beams are simply supported at the ends of the span.

In addition, the value of Poisson's ratio for concrete is assumed to be zero for practically all the calculations.

The detailed calculations for the effect of concentrated loads on I-beam bridges are long and tedious, and would serve no useful purpose if given here. The calculations were made by means of infinite trigonometric series, with as many as 16 terms being considered for some of the structures analyzed. With the particular choice of dimensions of the structure and relative stiffness of the beams, it was possible to use different combinations of the same numerical values for a number of structures. This shortened the great amount of computation required for the 20 basically different structures for which influence coefficients are tabulated in Appendix A. In certain cases the slow convergence of the series made it necessary to estimate the effect of the terms in the series that were neglected. It was possible to find reasonably accurate expressions for the sums of the neglected terms in every instance. It is believed that the influence values reported in Appendix A are, with few exceptions, accurate to the number of significant figures reported.

4. *Notation*.—The following notation is used throughout this bulletin. The longitudinal direction is always taken as the direction of the beams.

 $a =$ span of bridge, center to center of supports.

- $b = \text{transverse spacing of beams.}$
- $c =$ diameter of uniformly loaded circular area representing a wheel load.
- $h =$ total depth of slab.

ILLINOIS ENGINEERING EXPERIMENT STATION

- $I =$ moment of inertia per unit of width of the cross-section of the slab: for a homogeneous material, $I = h^3/12$.
- $E =$ modulus of elasticity of the material in the slab.
- μ = Poisson's ratio of lateral contraction for the material in the slab.
- $N = \frac{EI}{1 u^2}$ = measure of stiffness of an element of the slab.
- E_b = modulus of elasticity of the material in a beam.
- I_b = moment of inertia of the cross-section of a beam (or transformed moment of inertia of a T-beam).
- $H = \frac{E_b I_b}{aN} =$ a dimensionless coefficient which is a measure of

the stiffness of the beam relative to that of the slab.

- $P =$ concentrated load applied to the slab.
- $q =$ load per unit of length uniformly distributed along a beam.
- $w =$ load per unit of area uniformly distributed over the slab: also weight of the slab, per unit of area.
- M_t = transverse bending moment per unit of length acting on a longitudinal section of the slab, positive when producing compression at the top of the slab.
- M_l = longitudinal bending moment per unit of width acting on a transverse section of the slab, positive when producing compression at the top of the slab.
- M_b = bending moment in a beam or in a T-beam, positive when producing compression at the top.
- $\begin{bmatrix} M_{ot} \ M_{al} \end{bmatrix}$ = modified bending moments under a unit load uniformly distributed over a circular area at the center of an infinitely long simply supported slab. M_{ot} is in the direction transverse to the supports, M_{ol} in the direction parallel to the supports.
	- $k =$ coefficient in Equations (19) and (20); effective proportion of wheel load to be used in computing maximum moments in a beam due to standard truck loading.
	- $s =$ length defined by Equation (21), in terms of which maximum moments in beams are stated.

10

MOMENTS IN I-BEAM BRIDGES

II. RESULTS OF ANALYSES FOR CONCENTRATED LOADS

5. Tables of Influence Values for Moments in Slab and in Beams.— Tables of numerical values of influence coefficients for moments at various points in slab and beams due to unit loads are given in Appendix A, Tables 1 to 96 inclusive, for bridges having ratios of spacing of beams, b , to span of bridge, a , of 0.1, 0.2, and 0.3. These correspond to ratios of width of roadway to length of span of 0.4. 0.8, and 1.2.

The relative stiffness of the beams, compared to that of the slab. is determined by the dimensionless coefficient H which is defined by the relation

$$
H = \frac{E_b I_b}{aN}
$$

where E_b = modulus of elasticity of the material in a beam,

 I_b = moment of inertia of cross-section of beam,

- $N = \frac{EI}{1 \mu^2},$
- $E =$ modulus of elasticity of the material in the slab,
- $I =$ moment of inertia per unit of width of the cross-section of the slab.

 μ = Poisson's ratio, generally taken as zero in the data given here.

The values of H range between a low value, corresponding to a very flexible beam, and a value of infinity, corresponding to a beam that does not deflect.

Table I gives an outline of the tables of influence coefficients for moment. In general, the coefficients are tabulated in the following order for each structure:

Transverse moment in the slab at mid-span.

Longitudinal moment in the slab at mid-span,

Moment in the beams at mid-span,

Transverse moment in the slab at quarter-point.

Longitudinal moment in the slab at quarter-point,

Moment in the beams at quarter-point.

For some structures moments at mid-span only are tabulated. For all except one structure, all of the beams have the same stiffness. In Table 63 the structure has edge beams having 20 per cent greater

Ratio of Spac- ing of Beams to Span of Bridge, b/a	Relative Stiff- ness of Beams, H	Remarks	Table Numbers, inclusive, (Appendix A)
0.1	0.5 $\frac{1}{2}$ $\frac{1}{5}$ $\frac{5}{10}$ 20 infinity	Moments at mid-span and quarter-point Moments at mid-span and quarter-point Moments at mid-span and quarter-point Moments at mid-span and quarter-point Moments at mid-span only Moments at mid-span only Moments at mid-span and quarter-point	$1 - 6$ $7 - 12$ $13 - 18$ 19-24 25 26 27-32
0.2	$\frac{1}{2}$ 10 infinity 4.8 and 4.0	Moments at mid-span and quarter-point Moments at mid-span only, structure with stiffened edge beams	33-38 $39 - 44$ $45 - 50$ $51 - 56$ 57-62 63
0.3	$\begin{smallmatrix}1.5\3\6\15\end{smallmatrix}$ infinity 0.727 2.96	Moments at mid-span and quarter-point Moments at mid-span only, Poisson's ratio $= 0.2$ Moments at mid-span only, Poisson's ratio $= 0.2$	64-69 70-75 76-81 82-87 88-93 94 95
		Corrective moments under load, M_{ol} and M_{ol}	96

TABLE I

OUTLINE OF TABLES OF INFLUENCE COEFFICIENTS FOR MOMENT IN SLAB AND BEAMS

stiffness than the interior beams. For all except two of the structures the value of Poisson's ratio is zero. In the two structures of Tables 94 and 95, the value of Poisson's ratio is 0.2.

Table 96 gives the corrective moments under the load, M_{ot} and M_{ol} , which are to be added as indicated in the other tables for loads directly over the point considered.

Points on the bridges are located by the coordinate system shown in Fig. 1. The beams or points on the slab directly over beams are denoted by the letters A, B, C, D, and E, as shown. Therefore, beams A and E are always at the edges of the bridge. The longitudinal center line of a panel of the slab is denoted by the two letters corresponding to the beams supporting the panel. Thus, AB denotes the longitudinal center line of the outer panel of the slab. Points on a particular beam or center line of a panel of the slab are denoted by the proportion of the span from the left end of the bridge. It was convenient in the calculations to divide the span into twelve parts. In general, influence ordinates are tabulated for loads at the twelfth points of the span for structures with $b/a = 0.1$. For the other structures influence values are generally given for loads at the center and quarter-points of the span.

It will be noted that influence values for twisting moments in the

FIG. 1. DIAGRAM SHOWING NOTATION USED FOR I-BEAM BRIDGES

slab are not given. For a complete analysis to obtain maximum principal moments and their directions, theoretically it would be necessary to know the twisting moment. However, for the type of loading leading to maximum moments, the directions of principal moment must be very nearly in the transverse and longitudinal directions, and actually the twisting moments must be small. Since the reinforcement in I-beam bridges is invariably placed in the transverse and longitudinal directions, it is felt that the significant design moments are given with sufficient accuracy without taking into account the effect of the twisting moment.

Primarily for convenience in making the calculations the value of Poisson's ratio was generally taken as zero. The uncertainty as to the behavior of reinforcement in concrete slabs makes it questionable whether any other value of Poisson's ratio leads to more accurate determination of stresses in the steel. For a slab reinforced with a steel sheet, in which Poisson's ratio effects could take place in the steel as well as in the concrete, the action might be clearer, and there

might be more justification for considering a value other than zero. However, in any case, the effect of Poisson's ratio, μ , on the maximum moments is not large, and may be estimated approximately from the following relations:

$$
M_t^{\mu} \cong M_t^{\mu=0} + \mu M_t^{\mu=0}
$$

$$
M_t^{\mu} \cong M_t^{\mu=0} + \mu M_t^{\mu=0}
$$

$$
M_b^{\mu} \cong M_b^{\mu=0}.
$$

The superscript indicates the value of μ considered.

The accuracy of these approximations has been investigated in one case, for a structure with $b/a = 0.3$ and $H = 3$. Moments at mid-span for such a structure are given in Tables 70, 71, and 72, for $\mu = 0$, and in Table 95 for $\mu = 0.2$. The influence coefficients for moments given by the approximate relations differ from the correct influence coefficients by less than 0.004 for the slab and by less than 0.005 for the beams. These values correspond to about 2 per cent of the maximum moments. It should be noted that the largest discrepancies occur only for loads over an edge beam. For interior loads, the approximation is even better.

An investigation of the effect of different stiffnesses in the various beams of a structure was also made. Moments at mid-span are given in Tables 45, 46, and 47 for a structure with $b/a = 0.2$ and $H = 4$, and in Table 63 for a corresponding structure with the edge beams stiffened 20 per cent. The change in influence coefficient for moment is less than 0.008 for both slab and beams. This corresponds to less than 4 per cent of the maximum moments.

6. Moment Directly Beneath a Concentrated Load.-In Tables 1 to 95 inclusive, M_{ot} and M_{ol} appear as part of the influence coefficients for transverse and longitudinal moment in the slab. These quantities are modified moments per unit of width in the directions transverse to the supports and parallel to the supports, respectively, under the center of a unit load uniformly distributed over a circular area at the center of an infinitely long slab simply supported on two opposite edges. The modification in the moments is made to take account of the fact that the ordinary theory of flexure of slabs does not apply in the immediate vicinity of a concentrated load. Westergaard* has suggested a method of using the so-called thick slab theory in order

^{*}H. M. Westergaard, "Computation of Stresses in Bridge Slabs Due to Wheel Loads,"
Public Roads, Volume 11, No. 1, March 1930, pages 1-23 (see especially page 8).

to compute stresses under a concentrated load. He has given an expression for the equivalent diameter of a circular loaded area from which moments can be computed by the ordinary theory of flexure so as to obtain the same tensile stress at the bottom of the slab that is given by the thick slab theory. Although there is some question regarding the use of such a procedure for computing stresses in the reinforcement of a concrete slab, it is convenient, and the precedent for its use is well established.

Numerical values of M_{ot} and M_{ol} for a Poisson's ratio of zero, are given in Table 96 of Appendix A. Values of M_{ot} are given in two parts. One part, M' , is a function only of the ratio of thickness of the slab to the span of the panel, and the other part, M'' , is a function only of the ratio of the diameter of the loaded area to the thickness of the slab. The numerical values were computed by means of Westergaard's* equations.

For practical purposes, the following approximate formula for M_{ot} is sufficiently accurate when the diameter of the loaded area is equal to or greater than the depth of the slab, and the depth of the slab is greater than one-fiftieth of the span, for values of c/b (the ratio of the diameter of the loaded area to the span of the slab) between 0.04 and 0.25 :

$$
M_{ot} = \frac{1.16}{3 + 10 \frac{c}{b}}.\t(1)
$$

The limits of applicability of the formula include all ordinary dimensions of I-beam bridges.

The value of M_{ol} is obtained by the relation

$$
M_{ol} = M_{ot} - 0.080. \tag{2}
$$

Equations (1) and (2) differ from corresponding approximations given by Westergaard† which were developed for a Poisson's ratio of 0.15. The formulas given here are for a Poisson's ratio of zero.

To keep in mind the relative magnitude of the moments due to various causes, it is desirable to remember that for ordinary proportions of I-beam bridges M_{ot} may vary between 0.20 and 0.28, while M_{ol} may vary between 0.12 and 0.20.

^{*}See preceding footnote.
†See preceding reference, Equations (66) and (62).

7. Transverse Moments in Slab at Center of Panel.-In studying the moments in I-beam bridges it is helpful to make use of the concept of an influence surface. Such a surface is a generalization of an influence line and is constructed from a base plane or datum plane on the structure by laying off ordinates proportional in value at every point to the particular influence caused by a unit load at the point. The surface can be represented graphically by a contour map, in which each contour line connects all points on the structure at which the influence is the same. A cross-section of an influence surface is an influence line for loads on a particular line on the structure.

Influence surfaces are shown for transverse moments at the centers of panels of several structures in Figs. B-1 to B-5 inclusive, of Appendix B. These moments, in the direction perpendicular to traffic, are the most important moments in the slab as far as design is concerned. The influence surfaces are characterized generally by a high peak value at the point where the influence is considered. The peak value is dependent on the value of M_{ot} . The surfaces drop off to relatively low values a short distance away. To obtain maximum positive moments from truck loads it is always necessary to have a heavy wheel load at the particular point considered.

Common to all the influence surfaces for transverse moment at the center of a panel, whether at mid-span or at the quarter-line, for either interior or exterior panels, is the characteristic of a more or less bowl-shaped depression of the surface in the same panel as the high peak value, and only a short distance away. This characteristic, and others, can be explained by a consideration of the behavior of the slab, which may be thought of as divided into several component actions, as follows:

(1) The behavior of the particular panel of the slab acting as a single simply-supported rectangular slab. The long narrow rectangular panel acts practically in the same way as an infinitely long slab. An influence surface for transverse moment in an infinitely long slab has been given by Westergaard.* There is a maximum negative transverse moment at the center due to a load approximately one panel length away, along the longitudinal center line. The peak influence value has the magnitude M_{ot} . The negative maximum is about -0.02 for a Poisson's ratio of zero.

(2) The effect of continuity of the panel with the adjacent panels, with the beams assumed not to deflect. The effect of the continuity is to reduce the moments in the loaded panel by an amount approxi-

^{*}See preceding reference, upper left hand part of Fig. 14.

FIG. 2. INFLUENCE LINES FOR TRANSVERSE MOMENT IN SLAB AT BC AT MID-SPAN, LOAD ON CENTER LINE

mately half as great as the reduction in the case of a panel with fixed edges. The moments in all panels due to actions (1) and (2) may be obtained from the tabulated influence values for $H =$ infinity, corresponding to a slab continuous over non-deflecting supports.

(3) The effect of the deflections of the beams. In general, this effect produces positive moments in a panel for load in the panel. The secondary depression near the peak value, in the same panel, still remains, as is shown in Figs. B-2, B-3, and B-5, and is caused chiefly by action (1) since actions (2) and (3) are relatively smooth with no major peaks or depressions.

Ordinarily action (3) more or less counterbalances action (2), although the magnitude of (3) may be greater or less than (2) depending on the stiffness of the beams. In almost every case action (1) is predominant.

The effect of the relative stiffness of the beams on the transverse moments at the center of an interior panel, at mid-span of the bridge, is shown in Fig. 2, which gives the influence of load on the center line for a number of values of H for bridges with $b/a = 0.1$ and 0.2. The difference in ordinate between the curve for $H = \text{infinity}$ and the curve for any other value of H shows the magnitude of the effect of deflection of the beams. Naturally the effect is greater for small values of H, or for the less stiff beams. The value of M_{ot} used in Fig. 2 has been arbitrarily taken as 0.232, which is a more or less representative value corresponding to a 15 in. diameter of loaded area and a 6 ft. 3 in. spacing of stringers.

From Fig. 2, there seems to be some correspondence in moments between a structure with $b/a = 0.2$ having some particular value of H and a structure with $b/a = 0.1$ having some larger value of H.

The values $H = 5$ for $b/a = 0.1$ and $H = 2$ for $b/a = 0.2$ correspond to reasonable designs for bridges of about 60 ft. span and 30 ft. span, respectively, for a roadway of about 24 ft. Evidently, for the proportions ordinarily encountered, the effect of deflection of the beams, action (3) , is greater than the effect of continuity, action (2) , although both these effects are small compared with the value of M_{at} . That is, the major part of the maximum moment is practically independent of the relative stiffness of the beams, and to a certain extent, of the proportions of the structure.

From Fig. 2 and from the influence surfaces Figs. B-1 to B-5 inclusive, it appears that negative transverse moments at the center of one panel are generally obtained with loads in other panels. The negative moments are small compared with the maximum positive moment.

8. Transverse Moments in Slab Over a Beam.—Influence surfaces for transverse moments in the slab over intermediate and center beams at both mid-span and at quarter-point for a structure with $H = 5$ and $b/a = 0.1$ are shown in Figs. B-6 to B-9 inclusive. Influence surfaces at mid-span only for a structure with $H = 2$ and $b/a = 0.2$ are shown in Figs. B-10 and B-11. The influence surfaces all have the same general characteristics, with a peculiar singularity at the point where the influence is computed. The nature of the singularity is best ex-

18

FIG. 3. INFLUENCE LINES FOR TRANSVERSE MOMENT IN SLAB AT C AT MID-SPAN, LOAD ON CENTER LINE

plained by reference to Fig. 3, which shows influence lines for transverse moment over a center beam at mid-span, with load on the center line, for a number of values of H for $b/a = 0.1$ and 0.2. Two curves, differing only near C, are shown for each structure: the lower curve of each pair refers to the theoretical effect of a truly concentrated load or point load; the upper curve of each pair has a cusp, and indicates the effect of a load distributed over a particular size of circular area. The point marked with a symbol " x " for each value of H represents the influence of a point load directly over the point C.

As a preliminary to the explanation of these curves for transverse

ILLINOIS ENGINEERING EXPERIMENT STATION

moments in the slab over a beam, it is desirable to consider the moment transverse to the fixed edge of an infinitely large cantilever slab, due to a point load on the slab. This problem has been studied by Westergaard,* and the result is given in Equation (3):

$$
M_t = -\frac{P}{\pi} \cos^2 \theta \tag{3}
$$

where θ is the angle between the normal to the fixed edge and the line from the point where the moment is computed to the position of the load P. The contour lines on the influence surface for M_t are a series of radial lines. The maximum influence is on the normal to the edge and is $-\frac{1}{\pi}$ = -0.318, for a unit load at any distance from the edge. As the load approaches the support the influence

does not change, but when the load is directly over the support the influence is, of course, zero. There is a singularity in the influence surface at the point where the influence is considered.

The moment due to a load uniformly distributed over an area can be obtained by integrating Equation (3). For example, the moment due to a unit load uniformly distributed over a circular area of diameter c tangent to the edge at the point where the influence is computed, is $-\frac{3}{4\pi} = -0.239$. When the center of the circle is over the edge, only half the circle is on the slab, and the moment is $-\frac{1}{4\pi}$ = -0.080. Contour lines on the influence surface are no longer radial when the load is considered distributed over an area.

Now consider the case of an infinitely large slab continuous over a rigid beam. By a consideration of the moment distribution concept, it is evident that the moment in the slab over the beam due to a single load on the slab to one side of the beam, is exactly onehalf of the fixed-edge moment for a load on the slab cantilevered from a fixed support. From these considerations, and the previous integrations, the following results are obtained:

As a point load approaches a line support the maximum moment

^{*}H. M. Westergaard, "Computation of Stresses in Bridge Slabs Due to Wheel Loads," Public Roads, Vol. 11, No. 1, March 1930, p. 1-23, esp. p. 20-21.

over the support is constant for all positions of the load away from the support, namely

$$
-\frac{1}{2} \cdot \frac{P}{\pi} = -0.159P \tag{4}
$$

but changes abruptly to zero when the load passes over the support.

For a load distributed uniformly over a circular area of diameter c, when the circular area is balanced directly over the support, with a semicircle of load on each side, the maximum moment over the support is

$$
-\frac{1}{4} \cdot \frac{P}{\pi} = -0.080P. \tag{5}
$$

With the circle tangent to the support, the maximum moment over the support is

$$
-\frac{3}{8} \cdot \frac{P}{\pi} = -0.119P = -0.159P + 0.040P. \tag{6}
$$

When the center of the circular area is at a distance c from the support, the maximum moment over the support is approximately

$$
-0.46\frac{P}{\pi} = -0.146P = -0.159P + 0.013P.
$$
 (7)

The following procedure was used to obtain the curves in Fig. 3 from the tabulated influence values. The curve for a point load was drawn first, the point close to C being located at a distance corresponding to 0.159 P below the point corresponding to a load directly upon beam C. There is an abrupt change in ordinate at C, of course. Then points were located at distances $c/2$ and c away from the beam. In the figure, c was taken as one-fifth of the spacing of the beams. At the distances $c/2$ and c from the beam, ordinates of 0.040 P, and 0.013 P, respectively, were drawn from the curve corresponding to a concentrated load to get the curve corresponding to a load distributed over a circular area. The influence surfaces in Figs. B-6 to B-11 inclusive were obtained by drawing curves for other points in a manner similar to that described.

In general, maximum negative transverse moments over a beam

are obtained for loads on the same transverse lines as the point where the moment is computed. Maximum positive moments may be obtained either with a load directly over the point where the influence is desired, or for loads at a considerable distance away. The maximum positive value for a load directly over the beam should be used without reduction because the width of the flange of the beam is likely to be as great as the diameter of the loaded area.

The negative transverse moments over a beam increase as the stiffness of the beams increases, or as the deflections decrease. Consequently, the negative moments are greater at the quarter-point of the span than at mid-span.

9. Longitudinal Moments in Slab.—Influence surfaces for longitudinal moment in the slab at the centers of exterior and interior panels are shown in Figs. B-12 to B-16 inclusive. The surfaces are characterized by a high peak at the point where the influence is computed, with the surface having relatively small elevations elsewhere. The major part of the maximum longitudinal moment is the quantity M_{ol} which is the moment in an infinitely long simply-supported panel.

The longitudinal moments are computed on the basis that the slab and the beams act independently in resisting longitudinal flexure. Consequently, if there is any interaction, or so-called T-beam action of the beams with the slab, the moments are not correct. The local effect of a concentrated load, however, should not be greatly reduced by T-beam action.

For small values of H the longitudinal moments in the slab become large, since the slab may account for a large part of the longitudinal flexural resistance of the structure. However, such low values of H are not of practical importance in the I-beam bridge.

Longitudinal moments in the slab over a beam are related to the moments in the beam, since at corresponding points the slab and the beam have the same curvature in the longitudinal direction. For a point over a beam the following relation applies, when Poisson's ratio is zero:

$$
M_l = \frac{N}{EI} M_b = \frac{M_b}{aH}.
$$
\n(8)

Consequently, maximum longitudinal moments in the slab over a beam may be computed from the maximum beam moments, but the slab moments are not likely to be important.

FIG. 4. INFLUENCE LINES FOR MOMENT IN CENTER BEAM AT MID-SPAN, LOAD ON CENTER LINE

10. Moments in Beams.—Influence surfaces for moments at midspan of exterior, intermediate, and center beams are shown in Figs. B-17, B-18, and B-19 for a structure with $H = 5$ and $b/a = 0.1$, and Figs. B-20, B-21, and B-22 for a structure with $H = 2$ and $b/a = 0.2$. These influence surfaces have no unusually sharp peaks or singularities of the type found in the influence surfaces for moments in the slab. Maximum moments at the center line will be obtained for loads on the center line. For groups of concentrated loads a maximum moment in a certain beam may be obtained without placing a load on that beam.

The influence surfaces for edge beams have a much higher and steeper peak than the influence surfaces for other beams. If a concentrated load can come over an edge beam the maximum moment will be larger than the maximum moment in an interior beam. However, for a group of concentrated loads the net effect may be larger for an interior beam than for the edge beam.

The influence surfaces may also be interpreted as moment diagrams due to a concentrated load at the center of the particular beam considered, in view of the reciprocal relation between loads and longitudinal curvatures described in Bulletin 304.* The moments at various points in the beams are given directly; the longitudinal moments in the slab are $1/aH$ times the numerical quantities recorded. It can be noted that the moments on any cross-section of the structure tend to become more nearly uniformly distributed across the section when the section is removed further from the point of application of the load.

Typical influence lines for moment at mid-span of the center beam for different stiffnesses of the beams are shown in Fig. 4 for structures with $b/a = 0.1$ and 0.2. The moment for a single concentrated load varies considerably with H , and, even for the highest values of H considered, the maximum moment differs considerably from the maximum for infinitely stiff beams. These influence lines may also be interpreted as moment diagrams in the manner described previously. It appears that the moments tend to become more uniformly distributed across the structure as the beams become less stiff.

From the interpretation of the influence lines as moment diagrams it appears that the various curves in Fig. 4 are related in the following way. The total moment on the center cross-section is $\frac{1}{4}$ Pa, which is determinable by statics. The sum of the beam moments in the five beams (that is, the ordinates at A, B, C, D, and E) plus the total longitudinal moment in the slab, which is $\frac{1}{aH}$ times the area under a particular curve, must equal the static
moment of $\frac{1}{4}$ Pa. The total longitudinal moment in the slab is relatively small, and can be approximated on the assumption that it is related to the total static moment in the same way that the

^{*&}quot;A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams,"
Univ. of Ill. Eng. Exp. Sta., Bul. 304, 1938; Theorem III, p. 94.

FIG. 5. MOMENT DIAGRAMS FOR BEAMS, LOAD AT MID-SPAN ON CENTER BEAM, $H = 5$, $b/a = 0.1$

total stiffness of the slab is related to the total stiffness of the structure. The relation can be expressed as follows:

total longitudinal moment in slab
\nstatic moment\n
$$
\approx \frac{4bN}{5E_bI_b + 4bN}
$$
\n
$$
\approx \frac{1}{1 + \frac{5}{4} \cdot \frac{aH}{b}}
$$
\n(9)

For example, with $b/a = 0.1$ and $H = 2$, Equation (9) gives the result that the total longitudinal moment carried by the slab is only about 3.8 per cent of the total static moment.

A clearer idea of the way an I-beam bridge acts is obtained by a study of the moment diagrams for all the beams in a bridge with

Ratio of Spacing of Beams to Span of Bridge, b/a	Relative Stiffness of Beams H	Table Numbers (Appendix A)
0.1	0.5 5 10 and 20	$\begin{array}{c} 97 \\ 98 \\ 99 \end{array}$ 100 101
0.2	10	102 103 104 105
0.3	$_{3}^{1\,.\,5}$ 6 15	106 107 108 109 \sim

TABLE II OUTLINE OF TABLES OF INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS

 $b/a = 0.1$ and $H = 5$, for a concentrated load at the center of the center beam. The diagrams are shown in Fig. 5. The moments at mid-span of the various beams are as follows:

The longitudinal moment carried by the slab is negligible.

These moments may be accounted for by the approximation of assigning some proportion of the load to each of the beams: namely,

But this interpretation is valid only for computing the moments at mid-span. Actually the loads on the various beams differ materially.

The moment diagrams for all beams except the center beam are approximately sine curves, and the loads are therefore approximately sine curves, with the load applied downward. However, for the center beam part of the moment diagram is concave upward which means that there is a distributed upward load on the center beam, as well as a concentrated downward load. The moment diagram for beam C is made up, approximately, of the triangular moment diagram for the concentrated load P and a negative sine curve of moment with maximum ordinate of $0.149Pa$. Consequently, the load on beam C consists of the load P downward and a distributed upward

load, more or less of a sine curve, with intensity approximately three times the downward load on beam B. The maximum tension

tending to separate the beam and the slab is roughly $1.5 \frac{P}{\cdots}$.

For a pair of loads, P , at the centers of beams B and C the maximum tension between the slab and beam C is roughly P/a . For H-20 loading on a 60 ft. span, neglecting the effect of the front wheels, the tension between the center beam and the slab is of the order of 270 lb. per ft. due to the live loading. This is less than the pressure between the beam and the slab due to the weight of the slab, but for shorter spans the effect may be greater, and there may be an actual tendency for the slab to separate from the beams.

The explanation of the effect is simple. A concentrated load applied to a beam produces a relatively smooth deflection curve, but the same load applied to a slab produces a larger local deflection with smaller effects away from the load than in the beam. A concentrated load applied to a combined slab and beam tends to pull the slab away from the beam at some distance from the load.

11. Deflections of Beams.---Influence coefficients for deflections of the beams at mid-span and at quarter-point for loads at mid-span and at quarter-points, are given in Tables 97 to 109 inclusive of Appendix A, for 14 structures. An outline of the tables of influence coefficients for deflection is given in Table II. The coefficients given are such that actual deflections for concentrated loads are obtained by multiplying the tabulated coefficients by the quantity Pa^3/E_bI_b .

In order to evaluate the trend in the numerical coefficients, the following values are given for load on a single simply supported beam of span a :

A study of the influence tables for deflection indicates that the deflections of the beams due to a single concentrated load are much more nearly alike than the moments in the beams. The uniformity of the deflection is better for low values of H .

12. Approximate Method for Effect of T-Beam Action.—Although the effect of T-beam action cannot be determined by the method of analysis used herein, the principal effect of such action can be approximated. It is assumed that a full panel width of the slab acts with the beam as a T-beam, either due to the bond between the concrete and the steel beam or due to shear connectors of one kind or another. The value of E_bI_b is computed for the transformed section of the T-beam, and H is determined in the usual way from the formula

$$
H = E_b I_b / aN.
$$

Then the structure is analyzed as if the supporting beams actually had the value of H computed.

The transverse moments in the slab are assumed to be resisted by the slab, as if no T-beam action existed. The moments in the beams are assumed to be resisted by the T-beam section. This assumption leads to longitudinal stresses in the concrete, which are generally fairly small. Local longitudinal flexure of the slab is assumed to take place in the usual way for loads at the centers of panels. Further than this, the writers are not at present prepared to go.

III. MOMENTS IN I-BEAM BRIDGES FOR STANDARD TRUCK LOADS

13. Description of Bridges Analyzed.—Live load moments in slab and beams for standard truck loads were computed for over 50 structures of various dimensions from the influence coefficients given in Appendix A. The structures analyzed have widths of roadway of 20, 24, 28, and 32 ft., corresponding to beam spacings of 5, 6, 7, and 8 ft., respectively. For the narrower roadways the possibility of loads coming over the edge beams was taken into account. For all spacings, values of b/a of 0.1, 0.2, and 0.3 were considered, giving spans of 80, 70, 60, 50, 40, 35, 30, 25, 26%, 23%, 20, and $16\frac{2}{3}$ ft. The values of H considered were 2, 5, 10, 20, and infinity for $b/a = 0.1$; 1, 2, 4, 10, and infinity for $b/a = 0.2$; 1.5, 3, and infinity for $b/a = 0.3$. The low values of H are generally below the values for which practical designs can be made. Some of the structures analyzed represent extreme conditions, but were investigated to determine trends in the moment coefficients beyond the range of practical applications.

All the bridges are simply supported at the ends, the slab as well as the beams being supported at the abutments. The beams in each bridge are assumed to have the same stiffness. The influence of interior diaphragms between beams is neglected. The edge beams are assumed to be at the edge of the slab, and the effects of curbs, sidewalks, and handrail are also neglected. The calculations are applicable to structures with T-beam action, providing a proper interpretation is made of the results.

For each structure, moments were computed at mid-span and at quarter-point in the slab, but only the maximum moments are reported here. Moment coefficients were determined separately for rear wheel loads and for front wheel loads of standard trucks placed in the position producing maximum moments.

14. Standard Truck Loading.—The standard truck loading for which moments are given is that specified as the H truck loading in "Standard Specifications for Highway Bridges," The American Association of State Highway Officials, Third Edition, 1941. The standard H truck has the wheels of each axle spaced 6 ft. apart, with front and rear axles spaced 14 ft. apart. Each of the rear wheels carries a weight which is four-tenths of the total weight of the truck, and each of the front wheels carries a weight of one-tenth the total weight of the truck or one-fourth the rear wheel weight. The weight of the truck in tons is designated by a numeral following H, as H-20. The rear wheel load P , in terms of which the maximum moment coefficients are stated, is the weight on a rear wheel increased by a certain proportion. the impact factor, given by the relation

$$
\frac{50 \text{ ft.}}{L + 125 \text{ ft.}}, \text{ with a maximum of 0.30,}
$$

where L is the length of the beams (for computing maximum moment in the beams) or the spacing of the beams (for computing maximum moment in the slab). The specified maximum value of 0.30 applies for values of L less than 41 ft. 8 in. That is, the impact factor for slab moments is always 0.30. The front wheel load is always taken as $P/4$.

Each truck is considered to occupy the central part of a 10 ft. traffic lane. Therefore, the distance between the center of a wheel and the face of a curb is taken as a minimum of 2 ft.; and the distance between the centers of the nearest wheels of trucks in adjacent lanes is taken as a minimum of 4 ft. For the purposes of spacing of the

ILLINOIS ENGINEERING EXPERIMENT STATION

TABLE III

MAXIMUM TRANSVERSE MOMENTS AT CENTER OF PANEL OF SLAB DUE TO TRUCK LOADING

Moments are given in terms of a coefficient to be multiplied by the rear wheel load, P , for standard truck loadings according to 1941 A. A. S. H. O. specifications. The relative magnitude of the front wheel load, P/A ,

†Coefficients designated thus are for quarter-point of span; coefficients in parentheses are computed for mid-span only.

loads the face of the curb has been considered to be over the edge beam, but supplementary calculations have also been made with the face of the curb 2 ft. outside of the edge beam for beam spacings of 5 and 6 ft.

One or two lanes of loading only, depending on which gives maximum moments, are considered in the calculations. Only one truck in each lane is considered, but each truck is assumed to be traveling in the direction which produces maximum effects. This takes account of the possibility of one truck passing another truck in the adjacent lane. The uniform lane loads described in the specifications have not been considered, nor have truck train loadings been considered.

The specifications consider another type of truck loading, the H-S truck, which is the same as a standard H truck with an extra pair of heavy wheels 14 ft. from the first pair. The maximum moment coefficients given herein can be applied approximately to the H-S loadings by adding to the moment coefficients three to four times the front wheel effects to take account of the added heavy pair of wheels.

The rear wheel load is assumed to be distributed uniformly over a circular area of diameter c where c is taken as 15 in. for H-15 and H-20 loadings.

15. Maximum Transverse Moments in Slab at Center of Panel.-Maximum positive and negative transverse moments at the center of a panel of the slab, due to standard truck loads, are given in Table III. The moments due to front wheel loads are listed separately from those due to rear wheel loads. The numerical values may be in error by several units in the last decimal place recorded.

As an indication of the way in which trucks are placed to produce maximum moments, the following data are given for a structure with $b/a = 0.1$ and $H = 5$. The maximum positive moment is at BC, and for beam spacings of 5 ft. and 6 ft., only one truck is on the bridge, with one rear wheel at mid-span of BC, the other rear wheel on the other side of C. For beam spacings of 7 ft. and 8 ft., two trucks are on the bridge; one truck with one rear wheel at mid-span of BC, the other rear wheel on the other side of B; and the second truck in an adiacent lane with the rear wheels at about the third point of the span. The maximum negative moment is at the quarter-point on line BC. When wheels are not less than 2 ft. from the edge beam, one truck straddles beam B at about the one-sixth point of the span, with a wheel in or fairly close to the bowl-shaped depression in the influence surface. The other truck is close to beam E near the quarter- or

FIG. 6. MAXIMUM POSITIVE TRANSVERSE MOMENTS AT CENTER OF PANEL OF SLAB DUE TO REAR WHEELS OF STANDARD TRUCKS

third-point of the span. For the 5 and 6 ft. beam spacings, somewhat greater negative moments are obtained with wheels closer to the edge beams. The largest moments are reported in Table III for all cases.

The maximum positive moment coefficients for rear wheel loads should depend in some measure on the relative deflections of the beams. Since the deflection of a beam loaded with a given total load depends on the quantity $\frac{a^3}{E_b I_b}$, or, what amounts to the same thing, upon a^2/H for a given slab, the coefficients were plotted against values of a^2/H to determine whether a functional relation existed. A fairly well-defined trend was observed which indicated
that a linear relation with the quantity a/\sqrt{H} could be expected. Therefore, in Fig. 6 are plotted values of the moment coefficient as

ordinates against values of $\frac{a}{-10 \text{ ft} \cdot \sqrt{H}}$ as abscissas. The straight

line shown in Fig. 6 appears to represent the data reasonably well, and is on the safe side for most practical cases. Hence, the following empirical relation is proposed for the maximum positive transverse moment in the slab due to rear wheels of standard trucks:

$$
\frac{M_t}{P} \text{ (for rear wheels only)} = M_{ot} - 0.040 + 0.024 \frac{a}{10 \text{ ft.} \sqrt{H}}. \quad (10)
$$

The value of M_{ot} is obtained from Equation (1), namely,

$$
M_{ot} = \frac{1.16}{3 + 10 c/b}
$$

The additional moment due to the front wheels of the truck loading is given to a good degree of approximation by the relation

$$
\frac{M_t}{P}
$$
 (for front wheels only) = 0.010
$$
\frac{a - 28 \text{ ft.}}{10 \text{ ft.} \sqrt{H}}
$$
. (11)

Equation (11) is applicable only to spans greater than 28 ft. In Equation (11) account is taken of the fact that the front wheel loads are one-fourth of the rear wheel loads.

Then, for standard H truck loads, the maximum positive transverse moment at centers of panels of the slab is as follows:

$$
M_t = P \left[\frac{1.16}{3 + 10 c/b} - 0.040 + 0.024 \frac{a}{10 \text{ ft.} \sqrt{H}} + 0.010 \frac{a - 28 \text{ ft.}}{10 \text{ ft.} \sqrt{H}} \right]
$$
(12)

where the last term is to be dropped for spans less than 28 ft.

The maximum positive moment at the quarter-point of the span differs very little from that at mid-span. A rough measure of the moment at the quarter-point is obtained by use of Equation (12) with a fictitious value of H twice the actual value.

TABLE IV

MAXIMUM TRANSVERSE MOMENTS IN SLAB OVER A BEAM DUE TO TRUCK LOADING

Moments are given in terms of a coefficient to be multiplied by the rear wheel load, P , for standard truck loadings according to 1941 A. A. S. H. O. specifications. The relative magnitude of the front wheel load, P/A , direction in each lane.

†Coefficients designated thus are for mid-span; coefficients in parentheses are computed for mid-span only, for rear wheels only.

MOMENTS IN I-BEAM BRIDGES

The maximum negative moment at the center of a panel of the slab varies rather erratically, but does not become appreciably $larger than -0.065P$ for any case of loading considered. If no wheel is less than 2 ft. from an edge beam, a somewhat better approximation for the effect of all wheels, reasonably on the safe side, is as follows:

$$
M_t = -0.065P + 0.004 \frac{a}{10 \text{ ft.} \sqrt{H}} P.
$$
 (13)

For wheels over edge beams, the last term in Equation (13) should be dropped.

Only maximum moments are considered here, but the moments in the various panels of the bridges analyzed do not differ enough to warrant special treatment for interior and exterior panels.

16. Maximum Transverse Moments in Slab Over a Beam.-Maximum positive and negative transverse moments in the slab over a beam, due to standard truck loads, are given in Table IV. The coefficients are given for the effect of all wheels. The numerical values may be in error by as much as one unit in the second decimal place, because of difficulty in determining the exact positions of the trucks to give maximum moments.

The maximum negative moments are at the quarter-point of the span, but the negative moments at mid-span are not very much less than those at the quarter-point. The moments over the various beams do not differ greatly either. The maximum negative moments are generally obtained with the rear wheels of a truck or of two trucks on the quarter-line. The contributions from the front wheels are generally small, and are usually such as to reduce the negative moment due to the rear wheels.

The following empirical relation summarizes fairly well the maximum negative transverse moments over a beam:

$$
M_t = -P\left(0.240 - \frac{0.110}{H^{1/3}}\right). \tag{14}
$$

The maximum positive transverse moments in the slab over a beam are found at mid-span, but do not differ greatly for the various beams. The moments at the quarter-point may be from 10 to 20 per cent less than those at mid-span. The moments due to the front wheels may be as much as one-fifth the total moment for the longer span struc-

TABLE V

MAXIMUM LONGITUDINAL MOMENTS AT CENTER OF PANEL OF SLAB DUE TO TRUCK LOADING

Moments are given in terms of a coefficient to be multiplied by the rear wheel load, P , for standard truck loadings according to 1941 A. A. S. H. O. specifications. The relative magnitude of the front wheel load, P/A ,

٠

tures. The maximum moments may be obtained with a truck having a rear wheel load at mid-span over the point considered, or alternatively, for trucks at other points on the span in the bulb-shaped region of the influence surface, or with combinations of one truck at mid-span, and the other near the third-point of the span.

The following empirical relation may be used as a reasonable approximation for the maximum positive transverse moments over a heam:

$$
M_t = P\left(\frac{0.150}{H^{1/3}} + \frac{a - 30 \text{ ft.}}{1000 \text{ ft.}}\right).
$$
 (15)

17. Maximum Longitudinal Moments in Slab.-Maximum longitudinal moments at the center of a panel of the slab are given in Table V for front and rear wheels of standard truck loadings. The numerical values may be in error by several units in the last decimal place recorded. It is practically impossible to load the structures so as to obtain negative longitudinal moments; consequently no negative values are reported. The longitudinal moments do not differ materially between inner and outer panels of the structure. However, the maximum moment is at mid-span of the structure, and is generally obtained with two trucks having rear wheels at the center line of the bridge.

The following empirical relation is proposed for the maximum longitudinal moment at the centers of panels of the slab, due to the rear wheels only of standard trucks: \overline{h}

$$
\frac{M_l}{P} \text{ (for rear wheels only)} = M_{ol} - 0.015 + \frac{0.070 + \frac{0}{50 \text{ ft.}}}{H}. \quad (16)
$$

The value of M_{ol} is obtained from Equations (1) and (2), namely,

$$
M_{ol} = \frac{1.16}{3 + 10 c/b} - 0.080.
$$

The additional moment due to the front wheels of the truck loading is given to a good degree of approximation by the relation

$$
\frac{M_l}{P} \text{(for front wheels only)} = 0.3 \left(1 - \frac{28 \text{ ft.}}{a} \right) \frac{0.070 + \frac{b}{50 \text{ ft.}}}{H}. (17)
$$

Equation (17) is applicable only to spans greater than 28 ft.

TABLE VI

MAXIMUM MOMENTS AT MID-SPAN OF BEAMS DUE TO TRUCK LOADING

Moments are given in terms of a coefficient to be multiplied by the product of the rear wheel
load P and the span a , for standard truck loadings according to 1941 A. A. S. H. O. specifications.
The relative magnitude

Then, for standard H truck loads the maximum longitudinal moment at centers of panels of the slab is as follows:

$$
M_{l} = P \left[\frac{1.16}{3 + 10 c/b} - 0.095 + \left(0.070 + \frac{b}{50 \text{ ft.}} \right) \frac{1}{H} + 0.3 \left(1 - \frac{28 \text{ ft.}}{a} \right) \left(0.070 + \frac{b}{50 \text{ ft.}} \right) \frac{1}{H} \right]
$$
(18)

where the last term is to be dropped for spans less than 28 ft.

The maximum longitudinal moment in the slab at the quarter-line is generally from 75 to 80 per cent of the maximum longitudinal moment at mid-span.

Span of Bridge $^a_{\rm ft.}$	Spacing of Beams ь ft.	Relative Stiffness of Beams H	Maximum Moment in Beam	Moment Coefficient in Terms of Pa	
				Rear Wheels	Front Wheels
35	$\overline{7}$	$\frac{1}{2}$ 10 infinity	\bf{B} $\overset{\mathbf{B},\mathbf{C}}{\underset{\mathbf{C}}{\mathbf{C}}}\mathbf{B}$	0.234 0.267 0.306 0.337 0.371	0.011 0.013 0.015 0.015 0.016
30	$\boldsymbol{6}$	1 $\overline{2}$ $\overline{4}$ 10 infinity	C ACBC C BC C	0.214 $0.238*$ 0.243 $0.258*$ 0.275 $0.277*$ 0.303 0.313	0.003 0.004 0.004 0.004 0.005 0.005 0.005 0.006
25	$\rm 5$	1 $\overline{2}$ 4 10 infinity	CACAC C C C C	0.190 $0.214*$ 0.214 $0.229*$ 0.239 0.258 0.280	0.000 0.000 0.000 0.000 0.000 0.000 0.000
2633	8	1.5 $\overline{\mathbf{3}}$ infinity	$_{\rm{B}}^{\rm{B}}$	0.286 0.328 0.438	0.000 0.000 0.000
231/3		1.5 \mathbf{a} infinity	$_{\rm c}^{\rm c}$ B, C	0.255 0.306 0.363	0.000 0.000 0.000
20	6	1.5 3 infinity	$_{\rm C}^{\rm C}$	0.235 $0.242*$ 0.269 0.310	0.000 0.000 0.000 0.000
1635	$\tilde{\text{o}}$	1.5 3 infinity	CACC CC	0.200 $0.215*$ 0.230 $0.233*$ 0.275	0.000 0.000 0.000 0.000 0.000

TABLE VI-(CONCLUDED) MAXIMUM MOMENTS AT MID-SPAN OF BEAMS DUE TO TRUCK LOADING

18. Maximum Moments in Beams.-Maximum moments at midspan of the beams due to standard truck loads are given in Table VI. The moments due to front and rear wheel loads are listed separately. Coefficients are given only for the beam having the greatest moment in a particular structure. Generally, the maximum moments in the interior beams do not differ markedly. In all cases, coefficients are given for load positions with no wheel less than 2 ft. from an edge beam. Where a greater moment is obtained with a wheel less than 2 ft. from an edge beam, the larger moment is also tabulated. The moment coefficients may be in error by several units in the last decimal place recorded.

The greatest beam moments are probably obtained at the section where the total moment in the bridge is greatest, but only moments at

FIG. 7. EFFECTIVE PROPORTION OF WHEEL LOAD TO BE USED IN COMPUTING MAXIMUM MOMENTS AT MID-SPAN OF INTERIOR BEAMS **DUE TO REAR WHEELS OF STANDARD TRUCKS**

mid-span were computed. The difference between the moment at midspan and the greatest moment is negligible, as can be seen by a study of the total moments at the corresponding points, as follows: For spans of less than 26.5 ft. the maximum moment in the bridge occurs at mid-span, for only rear wheels on the bridge. For spans greater than 26.5 ft., the maximum moment in the bridge occurs at a point

1.4 ft. from the center of the bridge. The maximum moment at this point differs from the maximum moment at the center of the bridge by amounts, in terms of the center moment, varying from zero at 26.5 ft. to a maximum of 1.24 per cent at 28 ft., and then decreasing with the span to values of 1.06 per cent at 30 ft., 0.56 per cent at 40 ft., 0.24 per cent at 60 ft., and 0.14 per cent at 80 ft.

The maximum moments in the beams can be most conveniently stated in terms of an effective proportion, k , of a single rear wheel load which, acting alone on a beam, would produce the same moment. That is.

$$
M_b \text{ (for rear wheels only)} = \frac{1}{4} kPa. \tag{19}
$$

Since there are 4 wheel loads on the bridge and 5 beams, the value of k should not be less than 0.8 , which corresponds to an equal division of moment between the beams, neglecting the moment carried by the slab.

However, the moments will generally not be equally apportioned between the beams. It is expedient to express k in the following way:

$$
k = \frac{b}{s} \left(1 - \frac{b}{aH} \right) \tag{20}
$$

where s is some length which depends on the characteristics of the The quantity $\left(1-\frac{b}{aH}\right)$ in Equation (20) takes acstructure. count approximately of the part of the total moment carried by a panel of the slab. If the longitudinal flexural resistance of the slab is to be neglected the term is to be dropped from Equation (20).

In Fig. 7 are plotted values of s, computed from the actual moments given in Table VI for loads not less than 2 ft. from an

edge beam, against values of $\frac{a}{10 \text{ ft} \cdot \sqrt{H}}$. The points lie in a fairly well-defined band. A reasonably safe value of s appears to be given by the following empirical relation:

$$
s = 4.40 \text{ ft.} + 0.42 \frac{a}{10\sqrt{H}}.\tag{21}
$$

When loads may approach an edge beam, a similar relation can be derived, which is valid for values of b of 6 ft. or less:

$$
s = 4.40 \text{ ft.} + 0.21 \frac{a}{10\sqrt{H}}.\tag{22}
$$

The effect of the front wheel loads can be stated in much the same fashion, in terms of the total moment at the center of the bridge due to 4 front wheels at a distance of 14 ft. from the center;

$$
M_b
$$
 (for front wheels only) = $\frac{1}{16}kP(a - 28 \text{ ft.})$ (23)

where the value of k is the same as for the rear wheels, with a minimum value of 0.8. Equation (23) is applicable only to spans greater than 28 ft.

To summarize, for standard truck loading, the maximum moment in the beams is obtained by the following relation:

$$
M_b = k \left[\frac{1}{4} Pa + \frac{1}{16} P (a - 28 \text{ ft.}) \right]
$$
 (24)

where the last term is to be dropped for spans less than 28 ft., and where k is given by the following equation:

$$
k = \frac{b\left(1 - \frac{b}{aH}\right)}{4.40 \text{ ft.} + 0.42 \frac{a}{10\sqrt{H}}},
$$
(25)

for loads not less than 2 ft. from an edge beam; and by Equation (26),

$$
k = \frac{b\left(1 - \frac{b}{aH}\right)}{4.40 \text{ ft.} + 0.21 \frac{a}{10\sqrt{H}}},
$$
(26)

for loads over or approaching an edge beam when b is 6 ft. or less. In either case, k is not to be taken as less than 0.8 for a 5-beam bridge, or in general, not less than the number of rear wheels on the bridge divided by the number of beams.

IV. DEAD LOAD MOMENTS IN I-BEAM BRIDGES

19. Effects to Be Considered.—The dead load moments in I-beam bridges are dependent upon the manner in which the bridge is constructed. For example, consider the most common type of construction, a bridge having steel beams with a concrete slab resting on the beams. The beams are erected first and carry their own weight. Then the form-work for the slab is placed on the beams, which puts an additional temporary load on each beam. The slab is cast, adding to the load carried by the beams, since each beam carries the part of the weight of the slab transferred to it by the forms. The edge beams may have less load than interior beams at this stage of construction since the curb, handrail, and sidewalks may not be constructed until later. All of the longitudinal flexure due to the weight of the concrete must be carried by the beams. When the concrete has hardened and the forms are removed, the structure acts as an I-beam bridge. A load applied to or removed from any one beam affects the whole structure. This means that removal of the form-work may produce stresses somewhat different from those put in the beams when the forms were built. Furthermore, when the forms are removed the concrete slab acts to carry its own weight in the transverse direction, as a beam continuous over several supports. If deflections of the beams are temporarily prevented, the action of the concrete slab is to change the loads on the beams due to the weight of the concrete. The load on the outside and center beams is reduced, that on the intermediate beams is increased. The change in load on the beams causes relative deflections of the beams in the bridge, and changes in moments in the structure, both in the beams and in the slab. The effect may be large. Finally, the construction of any edge details such as a curb or handrail puts a load on the outside beams, and produces stresses throughout the structure.

If T-beam action is considered, it should be remembered that the stresses in the beams have a different distribution for moments applied before the concrete has set and for moments applied afterward when T-beam action can take place. This is of considerable importance when the beams are shored during the construction of the slab.

For a structure with concrete beams and slab the action is different

since the whole weight of the structure is carried by the form-work until the structure is capable of acting as a unit. In such a case, the loads on the structure may be considered to be line loads on the beams due to the weight of the beams, curbs, and other edge details, and a uniform load on the slab due to the weight of the slab.

The influence tables given in Appendix A make it possible to compute moments in I-beam bridges due to line loads and uniform loads. The following procedure is recommended:

For a uniform line load on any beam the influence may be computed from the area under the influence diagram drawn for that beam. However, it is accurate enough for practical purposes to divide the line load up into arbitrary concentrations acting at the twelfth, sixth, or quarter points, along the span, depending on the data given in the influence tables, and to take the sum of the effects of the individual concentrations as the effect of the line load.

For a uniform load on the slab the best procedure is to compute the line reactions on the beams assuming temporarily no deflections of the beams. The slab may be considered as a wide beam spanning in the transverse direction, continuous over several supports. The line reactions on the beams are then applied to the composite structure by the method described in the preceding paragraph. The moments in the beams are obtained from the line loads alone; the moments in the slab are the sums of the moments due to the action of the slab as a wide transverse beam, and the moments due to the effect of the line loads on the beams.

The following approximate relation permits some simplification in dealing with line loads and distributed loads: It is assumed that uniform line loads of equal magnitude on all beams produce only negligible stresses in the structure except in the beams, and the beam stresses are equal and may be computed as if each beam carries its own line load independently.

The moments at the centers of panels and over the beams, in a slab carrying a uniform load per unit of area, w , and continuous over 5 rigid supports spaced at a distance b apart, are as follows:

Support A and E **B** and **D** \mathcal{C} $\frac{1}{2}wb$ **Simple Beam Reaction** w_b w_b $-\frac{3}{28}$ wb $\frac{2}{28}wb$ $rac{4}{28}wb$ Change in Reaction Due to Continuity

The reactions per unit of length on the supports for the loading considered are as follows, where a positive sign indicates a downward force on the support:

The changes in reaction on the supports due to continuity of the slab are the quantities that need to be taken into account in estimating the effect of removing the form-work from the I-beam bridge with steel beams and concrete slab.

20. Moments in Beams.—The dead load moments in the beams may be estimated reasonably well by dividing the total dead load moment at the center of the bridge equally among the beams. In general, this amounts to the same thing as considering each beam to carry its own weight and the weight of one panel of the slab. The added weight of the curbs, sidewalk, and handrails will more or less make up for the fact that only a half panel of the slab is carried by an edge beam.

If the beams are rigid the effect of the changes in the reactions due to the weight of the slab, when the form-work is removed, is to increase the moment in the intermediate beams by an amount of oneseventh the moment due to the weight of one panel of the slab; but for ordinary stiffnesses of the beams the increase is much less. For example, the changes in the moments in the beams for a structure with $b/a = 0.1$ and $H = 5$, in terms of the moment $wba^2/8$ due to the weight of one panel of the slab, are as follows: at A, a decrease of 1.9 per cent; at B, an increase of 1.0 per cent; and at C, an increase of 2.1 per cent.

Corresponding changes for a structure with $b/a = 0.2$ and $H = 2$ are: at A, a decrease of 3.4 per cent; at B, an increase of 1.9 per cent; and at C, an increase of 2.5 per cent.

The following empirical equations represent fairly well the relative

changes in moments in the beams due to the changed reactions caused by removal of the form-work, in terms of the moment $wba^2/8$:

The equations give correct values for $H=0$, and for $H=\text{infinity}$, and agree reasonably well with moments computed for the structures for which influence tables are available. However, for ordinary values of H , it does not appear necessary to consider the changes in moment.

21. Moments in Slab.-The only dead load moments of any importance in the slab are the transverse moments. These are subject to a great deal of uncertainty, but fortunately the moments are small compared with the maximum live load moments. An indication of the possible variations in dead load moments in the slab is shown in Table VII, which gives, for several structures, the transverse moments in the slab, at mid-span and at quarter-point, for the following cases of loading:

 (1) A uniform load per unit of area, w, corresponding to the dead load of the slab, taking into account the effect of the weight of the slab due to removing the form-work.

(2) A uniform load per unit of area, w_1 , corresponding to a load applied over the entire slab to the completed composite structure. Such a load might be produced by additional paving material placed on the completed structure.

 (3) A uniform line load per unit of length, q, applied to the edge beams on both sides of the bridge. Such a load might correspond to the weight of the curbs, handrails, and sidewalk placed on the structure after the concrete slab is hardened.

It is difficult to draw any general conclusions regarding the mag-

TABLE VII

TRANSVERSE MOMENTS IN SLAB DUE TO SEVERAL TYPES OF DEAD LOAD

(1) The load w is a uniform load per unit of area corresponding to the dead load of the slab, taking into account the effect of the weight of the slab due to removing the form-work.
(2) The load w_1 is a uniform load per unit of area applied to the entire slab on the composite structure.

(3) The load q is a line load per unit of length applied to the edge beams, A and E, only.

nitude of the dead load moments in the slab without referring to a particular construction procedure. The dead load moments can vary over a wide range of values depending on when the curb, handrail, and sidewalk are cast; that is, whether they are constructed with the slab or afterward. However, the magnitude of the dead load moments in the slab is not likely to be of great importance in comparison with that of the maximum live load moments. For this reason, it is felt that, in general, provision should be made in the

design for a positive transverse moment of $\frac{1}{8}wb^2$ at the centers of panels of the slab, and for a negative transverse moment of $\frac{1}{10}wb^2$

over the beams, when no more definite provisions for the structure can be made.

V. DISCUSSION OF RESULTS

22. Significance of Results.-The design of an I-beam bridge, after the span and the beam spacing has been chosen, is fundamentally controlled by the maximum moment at the center of the panels of the slab and the maximum moments in the beams. The former influences the depth of the slab; the latter, the cross-section of the supporting beams. Both these moments depend to some extent on the relative stiffness of the beams, that is, upon the quantity H ; consequently, the design cannot be made directly unless H can be estimated in advance.

However, the effect of H upon the moments controlling the design is small. This is fortunate, since the magnitude of H for an I-beam bridge is subject to some uncertainty; its value depends on the quantity N , which in turn depends on the modulus of elasticity and the moment of inertia of the concrete slab. Since the behavior of the structure is influenced by the overall or average moment of inertia, it is reasonable to use for I of the slab the relation

$$
I=\frac{h^3}{12}
$$

which applies to a homogeneous slab of depth h , rather than the moment of inertia of the net section considering cracking to have taken place.

In choosing the modulus of elasticity of the concrete to be used in the calculations, it should be remembered that for live load effects the instantaneous modulus applies, and not the modulus of deformation for long continued loading, which may be considerably less.

Since H varies inversely as the cube of the depth of the slab, it is apparent that the depth of the slab will have a large influence. For example, changing the depth from $6\frac{1}{2}$ in. to 7 in., other things being equal, will reduce H by very nearly 20 per cent. The foregoing information should be kept in mind in interpreting the following results.

A number of designs of structures of different spans, from 25 ft. to 80 ft., were made for both H-15 and H-20 loading. The depth of the slab was determined by use of working stresses of 18 000 lb. per sq. in. in the steel and 1200 lb. per sq. in. in the concrete. A paving allowance of 25 lb. per sq. ft. was considered as an addition to the dead load of the slab. In determining H the modulus of elasticity of the concrete was taken as 3 500 000 lb. per sq. in. Wide-flange beams were used,

with depths up to 36 in., as required by the moments, for stresses of 18 000 lb. per sq. in., with no T-beam action assumed. The values of H appeared to fall in the range given by the following relation:

$$
H \cong (0.5 \text{ to } 0.8) \frac{a}{10 \text{ ft.}} \tag{28}
$$

The slab thicknesses varied from 6 in. to $7\frac{1}{2}$ in.; the differences in thickness required for H-15 and H-20 loads apparently had the effect of making the values of H for the two loadings very nearly the same.

A study of some representative designs of bridges for which no paving allowance was made indicated values of H in the same range, but somewhat less in comparable structures.

That is, for a 30-ft. span with 6-ft. spacing, the value of H is likely to be in the range from 1.5 to 2.4; and for a 60-ft. span with 6-ft. spacing, the value of H is likely to be in the range from 3.0 to 4.8.

With a structure designed neglecting T-beam action, the effect of such action is to increase the value of H to about 2 or 3 times its assumed value, with a consequent slight reduction in stresses in the slab, an increase in beam moment, but a decrease in stresses in the beam due to the increase in section modulus. For a structure designed to take advantage of T-beam action, lighter beams may be used. The value of H for such a design will generally fall in the same range as for a design neglecting interaction.

The general trend of the influence values as shown by the influence surfaces in Appendix B seems to indicate that the maximum moments obtained in this study may be adapted for use in structures with more than five beams. The effect of additional lanes of load coming on a wider bridge should be relatively small, and might be balanced against the improbability of having heavy trucks with full impact simultaneously in all lanes in position to produce maximum moments.

23. Design of Beams.—The total moment in a beam is made up of the dead load moment due to the weight of the beam, the dead load moment due to the weight of the slab, and the live load moment. For a preliminary design the depth of the slab may be estimated as being from 6 to 8 in., depending on the loading and the dimensions of the bridge. The dead load moment may be determined reasonably well for the weight of one panel of the slab and the estimated weight of the beam, plus any paving allowance that may be considered. The live load moment is given by Equations (24) , (25) and (26) . The value of H to be used may be estimated from Equation (28), modified for the design conditions that are contemplated. Alternatively, one may estimate k in Equation (24) from the rather crude approximation.

$$
k \cong \frac{b}{6 \text{ ft.}}
$$

Experience with a given type of design and given working stresses may indicate a better approximation than this.

If provision is made for shear connectors, so that T-beam action may be counted on, it should be noted that unless the beams are supported during construction, the dead load moment is carried by the steel beam alone; the live load moment is carried by the composite section. A more efficient section for T-beam action would have a heavier lower flange than top flange. The same effect may be obtained by using a cover plate on the bottom of the beam.

24. Design of Slab.-As a first approximation to determine the thickness of the slab for ordinary designs the maximum positive live load moment at the center of the panel may be estimated from the following approximate relation:

$$
M_t \cong \left(\frac{1.16}{3 + 10 c/b} + \frac{a}{1000 \text{ ft.}}\right) P
$$

instead of Equation (12). The estimated dead load moment, $\frac{1}{8}wb^2$, may be assumed to add an amount of about ten per cent to the live load moment. With these assumptions the thickness of the slab can be determined, and consequently the value of H , after the beam is designed. For ordinary designs no further revision is necessary, but in any case not more than two trials should be required to determine the depth of the slab and the cross-section of the beams.

For ordinary designs, approximately one-fourth as much steel should be used for negative reinforcement in the transverse direction at the centers of the panels as is used for positive reinforcement. Over the beams, generally, the transverse reinforcement at the bottom should be about one-half, and that at the top about two-thirds to three-quarters, of the positive transverse steel at the center of a panel.

The longitudinal reinforcement required at the center of a panel in the slab may be nearly as much as the maximum transverse steel for short span structures, but is considerably less for long span structures. Whether the structure is seriously impaired if the longitudinal steel furnished is not adequate is a matter open to discussion, and the answer cannot be settled by analysis alone. As a tentative recommendation it is suggested that the ratio of longitudinal to transverse reinforcement in the central part of panels, for ordinary designs, be approximately as follows:

longitudinal reinforcement
transverse reinforcement
 $\approx 1 - \frac{a}{200 \text{ ft}}$.

The longitudinal reinforcement to be provided over the beams is a function of several factors which are difficult to estimate easily. When nothing is done to insure T-beam action it is suggested that in the outer parts of the panel provision be made for a moment of about $P/4H$, which in most cases will require about half the longitudinal reinforcement used in the central part of the panel. With full T-beam action assured, little or no longitudinal steel is required over or near the beams.

25. Moments in Continuous I-Beam Bridges.—No analyses of continuous I-beam bridges are available. It is felt, however, that the magnitudes of the maximum moments in continuous bridges may be approximately determined by suitable modifications of the relations derived herein for simple span bridges. The following principles seem to offer a reasonable basis for an analysis of a continuous bridge:

(1) Maximum moments in the slab in the interior of a span may be computed as for a simple-span bridge with the same beams, but with a span equal to the distance between the "points of inflection" for the continuous bridge. Maximum moments in the slab near a support may possibly be determined on the basis that the beams are rigid. This may not be on the safe side; consequently, it may be desirable to use the same design of the slab that is developed for the central parts of the spans.

(2) The proportion of a single wheel load to be used in determining moments in the beams may be computed also as for a simple-span bridge with the same beams but with a span equal to the distance between "points of inflection" for the continuous bridge. The moments

due to a wheel load should be determined in the usual way for a continuous beam. This procedure should give reasonably good results for positive moments in the beams, and will probably be on the safe side for negative moments in the beams, since the position of loads producing maximum negative moments is a considerable distance away from the supports, and the maximum negative moments are likely to be fairly uniformly distributed across the structure.

Longitudinal reinforcement in the slab for negative moments should be provided in the region where the beam moments are negative.

VI. SUMMARY

26. Summary.—This bullet in contains the results of analyses of the type of bridge commonly called the I-beam bridge, which consists of a slab continuous over supporting beams, with the beams running in the direction of traffic. Influence values are given for moments in the slab and in the beams for twenty simple-span right bridges of basically different proportions, each having five identical parallel beams, uniformly spaced. The flexibility of the beams is taken into account in the analyses. Maximum moments due to standard truck loadings are determined for 52 bridges of various spans and different relative stiffnesses of beams. Moments due to dead load are considered also.

From the results of the analyses, empirical relations are derived upon which the design of the slab and the beams can be based. Formulas for maximum moments in the slab in the direction transverse to the beams, both at the center of a panel and over a beam, for maximum longitudinal moments in the slab, and for maximum moments in the beams, are given in Chapters III and IV.

For the particular type of structure having a concrete slab and steel beams, with usual working stresses and loads, simple approximate rules for design are given in Chapter V. The moments governing the thickness of the slab and the design of the beams are discussed in Sections 23 and 24, and the most desirable arrangement of the reinforcing steel is described in Section 24. Principles are formulated in Section 25 for extending the applicability of the rules for design to continuous I-beam bridges.

It appears from the character of the results obtained in the analyses that the moment coefficients for the structure with five supporting beams may also be used for wider bridges having more than five beams.

The influence values given in this bulletin may also be used to

compute moments in structures in which the traffic runs in the direction transverse to the beams. However, no consideration was given to such structures in the present work.

The results of the analyses are applicable also to structures with concrete instead of steel beams, and to structures in which T-beam action between the steel beams and the concrete slab is assured by means of anchors or shear connectors.

The analyses contained herein have been supplemented by tests of $\frac{1}{4}$ -scale models of a number of I-beam bridges. The experimental studies in general support the validity of the analyses. A report of the experimental work will be made in a later publication.

This page is intentionally blank.

APPENDIX A

TABLES OF INFLUENCE COEFFICIENTS FOR MOMENTS IN SLAB, MOMENTS IN BEAMS, AND DEFLECTIONS OF BEAMS

This page is intentionally blank.

TABLE 1

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Example 19

Relative Streams H = 0.5

Relative Streams H = 0.5

Relative Proportions of Bridge $b/a = 0.1$

due to a unit concentrated load applied along various longitudinal lines A, AB, B, etc., as shown in

Fig. 1. The

TABLE 2

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 0.5$

Relative Proportions of Bridge $b/a = 0.1$

Numerical values of longitudinal moment per unit of width in slab on various longitudinal lines Numerical values of longitudinal moment per unit of what in sino on various longitudinal lines
the same in the calculation of the load applied along various longitudinal lines A, AB, B, etc., as shown in
Fig. 1. The longi

TABLE 3

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 0.5$
Relative Proportions of Bridge $b/a = 0.1$

Numerical values of moment in beams divided by span of bridge due to a unit concentrated load
applied along various longitudinal lines A, AB, B, etc., as shown in Fig. 1. The longitudinal position
of the load is indicated

MOMENTS IN I-BEAM BRIDGES

TABLE 4

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 0.5$
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 1

TABLE 5

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE $\begin{tabular}{p{0.8cm}} \bf Relative~Stiffness~of~Beams~{\it H}=0.5\\ \bf Relative~Proportions~of~Bridge~b/a=0.1\\ \bf See~sub-leading~of~Table~2\\ \end{tabular}$

60

MOMENTS IN I-BEAM BRIDGES

TABLE 6

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF SPAN OF BRIDGE $\begin{array}{c} \mbox{Relative Stiffness of Beams } H = 0.5 \\ \mbox{Relative Proportions of Bridge } b/a = 0.1 \\ \mbox{See sub-leading of Table 3} \end{array}$

TABLE 7

${\bf {\large InFLUENCE\ CoEFFICIENTS\ for\ Transverse\,}\scriptstyle{MonENT\ IN\ SLAB\ AT\atop\ MID-SPAN\ OF\ BRIDGE}}$

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H = 1 \\ \mbox{Relative Proportions of Bridge } b/a = 0.1 \\ \mbox{See sub-leading of Table 1} \end{array}$

 $\frac{1}{2}$

62

MOMENTS IN I-BEAM BRIDGES

TABLE 8

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H=1$ Relative Proportions of Bridge $b/a=\rm 0.1$ See sub-heading of Table 2

TABLE 9

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE Relative Stiffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 3

TABLE 10

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H=1\\ \mbox{Relative Proportions of Bridge } b/a=0.1\\ \mbox{See sub-leading of Table 1} \end{array}$

ī

ï

MOMENTS IN I-BEAM BRIDGES

TABLE 11

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H=1\\ \mbox{Relative Proportions of Bridge } b/a=0.1\\ \mbox{See sub-leading of Table 2} \end{array}$

TABLE 12

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Sitffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.1$ See sub-heading of Table 3

66

TABLE 13

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H=2 \\ \mbox{Relative Proportions of Bridge } b/a=0.1 \\ \mbox{See sub-leading of Table 1} \end{array}$

. . . .

TABLE 14

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.1$ See sub-heading of Table 2

TABLE 15

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 3

TABLE 16

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H = 2 \\ \mbox{Relative Proportions of Bridge } b/a = 0.1 \end{array}$

See sub-heading of Table 1

TABLE 17

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

TABLE 18

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF SPAN OF BRIDGE

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H=2\\ \mbox{Relative Proportions of Bridge } b/a=0.1\\ \mbox{See sub-leading of Table 3} \end{array}$

TABLE 19

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H = 5 \\ \mbox{Relative Proportions of Bridge } b/a = 0.1 \\ \mbox{See sub-leading of Table 1} \end{array}$

TABLE 20

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 5$
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 2

Moment on Line	Longitu- dinal Position of Load	Values of Influence Coefficient for Moment Transverse Location of Load								
		AB	$\frac{1}{12}$ 3/12 quarter $\frac{4}{12}$ 5/12	0.004 0.008 0.012 0.016 0.019	0.003 0.006 0.010 0.020 0.062	0.002 0.005 0.008 0.011 0.015	0.002 0.004 0.006 0.010 0.013	0.002 0.004 0.005 0.006 0.007	0.001 0.002 0.003 0.004 0.004	0.001 0.001 0.002 0.002 0.002
center	0.022		$(M_{ol} +$ 10.011	0.018	0.004	0.007	0.005	0.002	0.000	-0.002
ВC	$\frac{1}{12}$ 312 quarter $\frac{4}{12}$ 5/12 center	0.002 0.004 0.006 0.007 0.008 0.008	0.002 0.004 0.006 0.010 0.013 0.004	0.002 0.004 0.006 0.009 0.013 0.016	0.002 0.004 0.007 0.017 0.060 $\int M_{ol}$ – 10.001	0.002 0.004 0.006 0.009 0.012 0.016	0.002 0.004 0.006 0.009 0.012 0.003	0.002 0.003 0.005 0.006 0.007 0.007	0.001 0.002 0.003 0.004 0.004 0.005	0.001 -0.001 0.001 0.002 0.002 0.002

TABLE 21

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 5$
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 3

TABLE 22

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 5$
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 1

TABLE 23

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 5$
Relative Proportions of Bridge $b/a = 0.1$
See sub-heading of Table 2

Ť

75

-

TABLE 24

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE**

Relative Stiffness of Beams $H = 5$
Relative Proportions of Bridge $b/a = 0.1$ See sub-heading of Table 3

TABLE 25

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB, LONGITUDINAL MOMENT IN SLAB, AND MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.1$
See sub-headings of Tables 1, 2, and 3. Influence coefficients are given only for loads at mid-span.

TABLE 26

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB, LONGITUDINAL MOMENT IN SLAB, AND MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 20$
Relative Proportions of Bridge $b/a = 0.1$
See sub-headings of Tables 1, 2, and 3. Influence coefficients are given only for loads at mid-span.

TABLE 27

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table $1\,$

TABLE 28

${\bf {\large Influence~ Coefficients~for~Longtutdinal~Moment~in~Stab~at}~M1D-SPAN~OF~BRIDGE}$

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.1$ See sub-heading of Table 2

TABLE 29

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 3

TABLE 30

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

 $\begin{tabular}{l} Relative Stiffness of Beams~H = Infinity\\ Relative Proportions of Bridge~b/a = 0.1\\ See sub-leading of Table~1\\ \end{tabular}$

TABLE 31

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

 $\begin{tabular}{l} Relative Stiffness of Beams~H = Infinity\\ Relative Proportions of Bridge~b/a=0.1\\ See sub-leading of Table~2\\ \end{tabular}$

TABLE 32

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.1$

See sub-heading of Table 3

TABLE 33

${\bf {\large\bf {\small\sc InFLUENCE\textcolor{black}{\large\sc CDEFICIENTS\textcolor{black}{\large\sc FOR\textcolor{black}{\large\sc TheANSVERSE\textcolor{black}{\large\sc The MOMENT\textcolor{black}{\large\sc In\textcolor{black}{\sc The MID-SPAN\textcolor{black}{\large\sc CP\textcolor{black}{\sc The DNDSE\textcolor{black}{\large\sc The MID-SPAN\textcolor{black}{\sc The MINDS}}}}}}$

Relative Stiffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 1

TABLE 34 ${\bf Influence~Coefficients~for~Longitudinal~Momentum~in~Stab~at~Mm-span~or~Brlog}$

Relative Stiffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 35

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE Relative Stiffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 3

TABLE 36

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 1

TABLE 37

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 38

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE**

Relative Stiffness of Beams $H = 1$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 3

TABLE 39

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 1

TABLE 40 INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 41

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 3

TABLE 42 INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 1

TABLE 43

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 44

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2$
Relative Proportions of Bridge $b/a = 0.2$

TABLE 45

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 4$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 1

TABLE 46

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 4$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 47

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE Relative Stiffness of Beams $H = 4$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 3

TABLE 48 INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE Relative Stiffness of Beams $H = 4$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 1

TABLE 49

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 4$

TABLE 50

0.006

 $\begin{array}{c} 0.011 \\ 0.005 \end{array}$

 0.010

0.004

 $\begin{array}{c} 0.005 \\ 0.003 \end{array}$

0.002

0.002

 0.000

0.000

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE**

 $\begin{array}{c} 0.004 \\ 0.003 \end{array}$

center

 912

 0.010
 0.004

 $\begin{array}{c} 0.011 \\ 0.005 \end{array}$

Relative Stiffness of Beams $H = 4$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 3

TABLE 51

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 1

TABLE 52 INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 53

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 3

TABLE 54 INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.2$
Son sub-banding of Table 1

TABLE 55

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 2

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 3

TABLE 56

TABLE 57 INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT

MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 1

TABLE 58

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 59

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.2$

See sub-heading of Table 3

TABLE 60

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 1

TABLE 61

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 2

TABLE 62 INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE** Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.2$ See sub-heading of Table 3

Values of Influence Coefficient for Moment Longitu-Moment dinal \sim Transverse Location of Load in Position Beam of Load $\mathbf{A}\mathbf{B}$ \bf{B} BC $_{\rm C}$ CD D DE E \mathbf{A} $\begin{array}{c} 0.069 \\ 0.050 \\ 0.025 \end{array}$ -0.011
 -0.009
 -0.005 $\begin{array}{c} 0.033 \\ 0.003 \\ 0.001 \end{array}$ $\begin{array}{r} -0.001 \\ -0.001 \\ 0.000 \end{array}$ $\begin{array}{c} 0.188 \\ 0.125 \\ 0.063 \end{array}$ $\mathbf A$ quarter θ θ $\bf{0}$ $\bf{0}$ $\check{0}$ $\overline{0}$ $\ddot{\mathbf{0}}$ $\overline{0}$ center 912 $\check{\mathbf{0}}$ $\check{\mathbf{0}}$ $\ddot{\mathbf{0}}$ $\overline{0}$ $\begin{array}{c} 0.121 \\ 0.091 \\ 0.046 \end{array}$ $\begin{array}{c} 0.188 \\ 0.125 \\ 0.063 \end{array}$ $\begin{array}{c} 0.098 \\ 0.071 \\ 0.035 \end{array}$ -0.018
 -0.015
 -0.008 $\begin{array}{c} 0.005 \\ 0.005 \end{array}$ $\mathbf{0}$ $\overline{0}$ $\mathbf{0}$ B quarter $\overline{0}$ $\tilde{0}$ center Ω Ω Ω $\check{\mathbf{0}}$ $\ddot{\mathbf{0}}$ 0.003 $\bar{0}$ 912 $\overline{0}$ -0.023
 -0.020
 -0.010 0.103 $\overline{0}$ $\overline{0}$ $\mathbf C$ quarter $\mathbf{0}$ -0.023 $\boldsymbol{0}$ 0.103 0.188 -0.020
 -0.010 $\bf{0}$ center $\begin{matrix} 0 \\ 0 \end{matrix}$ $\begin{matrix} 0 \\ 0 \end{matrix}$ $\begin{array}{c} 0.076 \\ 0.038 \end{array}$ $\begin{array}{c} 0.125 \\ 0.063 \end{array}$ $\begin{array}{c} 0.076 \\ 0.038 \end{array}$ Ω $\check{\mathbf{0}}$ $\check{\mathbf{0}}$ 912

TABLE 63

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB, LONGITUDINAL MOMENT IN SLAB AND MOMENT IN BEAMS AT MID-SPAN OF BRIDGE, STRUCTURE WITH STIFFENED EDGE BEAMS

 $\begin{tabular}{c} Relative Stiffness of Edge Beams \textit{ $H=4.8$} \\ Relative Stiffness of Interior Beams \textit{ $H=4.0$} \\ Relative Proportions of Bridge \textit{ $b/a=0.2$} \\ See sub-headings of Tables 1, 2, and 3. Influence coefficients are given only for loads at mid-span. \end{tabular}$

TABLE 64

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT

MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 1.5$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

Values of Influence Coefficient for Moment Longitu-
dinal Mo ment Posi-Transverse Location of Load on tion Line of Load \overline{A} AB \mathbf{R} BC \overline{C} CD D DE E $^{-0.021}_{\{M_{ot}\}} \substack{M_0 \ 0.010}$ AB -0.018 0.027 0.003 -0.007 -0.008 -0.005 -0.003 -0.001 quarter center -0.031 0.036 -0.002 -0.010 -0.010 -0.007 -0.004 -0.002 \bf{B} 0.020 0.062 0.027 -0.012 -0.008 -0.003 quarter -0.038 -0.018 -0.013 center -0.059 -0.052 0.129 -0.037 -0.025 -0.023 -0.018 -0.011 -0.004 BC $|quarter| - 0.023$ -0.009 0.020 -0.014 0.021 -0.005 -0.013 -0.012 -0.008 $\int_{0.004}^{M_{ot}}$ center \vert -0.034 -0.019 0.025 0.027 -0.013 -0.019 -0.017 -0.011 \overline{C} quarter $\begin{array}{c} -0.016 \\ -0.023 \end{array}$ -0.015 $\begin{array}{c} 0.024 \\ -0.041 \end{array}$ $\begin{array}{c} 0.061 \\ 0.129 \end{array}$ -0.015 -0.016 -0.021
 -0.027 -0.021 0.024 -0.027 -0.023 -0.030 -0.030 -0.041

TABLE 65 INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE Relative Stiffness of Beams $H = 1.5$
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 2

TABLE 66

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 1.5$
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 3

TABLE 67 INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE Relative Stiffness of Beams $H = 1.5$
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 1

TABLE 68

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

TABLE 69

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE**

Relative Stiffness of Beams $H = 1.5$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 3

TABLE 70

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 3$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

TABLE 71

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 3$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 2

TABLE 72

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 3$
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 3

TABLE 73

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 3$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

TABLE 74

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 3$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 2

TABLE 75

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE**

Relative Stiffness of Beams $H = 3$
Relative Proportions of Bridge $b/a = 0.3$

ILLINOIS ENGINEERING EXPERIMENT STATION

TABLE 76

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 6$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

TABLE 77 INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 6$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 2

TABLE 78

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 6$
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 3

TABLE 79 INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE Relative Stiffness of Beams $H=6$ Relative Proportions of Bridge $b/a=0.3$

See sub-heading of Table 1

 α

ILLINOIS ENGINEERING EXPERIMENT STATION

TABLE 80

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H=6$ Relative Proportions of Bridge $b/a=0.3$ See sub-heading of Table 2

TABLE 81

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE**

Relative Stiffness of Beams $H = 6$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 3

TABLE 82

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 15$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

TABLE 83 INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 15$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 2

ILLINOIS ENGINEERING EXPERIMENT STATION

TABLE 84

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 15$
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 3

TABLE 85

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 15$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

TABLE 86

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H = 15$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 2

TABLE 87

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE**

Relative Stiffness of Beams $H = 15$
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 3

ILLINOIS ENGINEERING EXPERIMENT STATION

TABLE 88

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

TABLE 89 INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 2

TABLE 90

INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 3

TABLE 91 INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 1

ILLINOIS ENGINEERING EXPERIMENT STATION

TABLE 92

INFLUENCE COEFFICIENTS FOR LONGITUDINAL MOMENT IN SLAB AT QUARTER-POINT OF SPAN OF BRIDGE

Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.3$ See sub-heading of Table 2

TABLE 93 INFLUENCE COEFFICIENTS FOR MOMENT IN BEAMS AT QUARTER-POINT OF **SPAN OF BRIDGE** Relative Stiffness of Beams $H =$ Infinity
Relative Proportions of Bridge $b/a = 0.3$

See sub-heading of Table 3

TABLE 94

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB, LONGITUDINAL MOMENT IN SLAB, AND MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

 $\begin{array}{c} \mbox{Relative Stiffness of Beams } H = 0.727 \\ \mbox{Relative Proportions of Bridge } b/a = 0.3 \\ \mbox{Poisson's Ratio} = 0.2 \end{array}$

Numerical values of transverse and longitudinal moments per unit of width in slab, and of moment
in beams divided by span of bridge, due to a unit concentrated load applied at mid-span of bridge on
lines A, AB, etc., as s

 M_{ol} "' = $M_{ol} + 0.2 M_{ol}$.

ILLINOIS ENGINEERING EXPERIMENT STATION

TABLE 95

INFLUENCE COEFFICIENTS FOR TRANSVERSE MOMENT IN SLAB, LONGITUDINAL MOMENT IN SLAB, AND MOMENT IN BEAMS AT MID-SPAN OF BRIDGE

Relative Stiffness of Beams $H = 2.96$
Relative Proportions of Bridge $b/a = 0.3$
Poisson's Ratio = 0.2 \bullet

Numerical values of transverse and longitudinal moments per unit of width in slab, and of moment
in beams divided by span of bridge, due to a unit concentrated load applied at mid-span of bridge
on lines A, AB, B, etc., a

TABLE 96

NUMERICAL VALUES OF M_{ot} and M_{ol} as Functions of Diameter of Loaded Area, Thickness of Slab, and Spacing of Supports

The values of $M_{\theta i}$ are for use in Table 16.05 does not Extractly
per unit of width, in the directions transverse to the supports and parallel to the supports, respectively,
under the center of a unit load uniformly di

$M_{ot} = M' + M''$

$M_{ol} = M_{ol} - 0.080.$

The following approximate formula for M_{of} is accurate enough for practical purposes when c is equal to or greater_sthan h, and h is greater than 0.02b, for values of c/b between 0.04 and 0.25. 1.16

$$
M_{ot} = \frac{1}{3 + 10c/b}
$$

 (1)

TABLE 97

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative Proportions of Bridge $b/a = 0.1$ Relative Stiffness of Beams $H = 0.5$

Numerical values of coefficient for deflection of beams due to a concentrated load applied along various longitudinal lines A, AB, B, etc., as shown in Fig. 1.
The longitudinal position of the load is indicated by the dis

118

TABLE $98\,$

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative of Table 97. To obtain defective Propose of Beams $H = 1.0$
Relative 2008 of Table 97. To obtain defective Propositions of Bridge b/a a set to be multiplied by the quantity $P_{\theta}^{a/RL}$.

MOMENTS IN I-BEAM BRIDGES

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative States of Beams $H = 2.0$
Relative Proportions of Bridge $b/a = 0.1$
See sub-heading of Table 97. To obtain deflections, the tabulated ocefficients are to be multiplied by the quantity Pa^3/B_4I_6 .

120

TABLE $100\,$

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative Stiffness of Beams $H=5.0$ Relative Proportions of Bridge $b/a=0.1$

See sub-heading of Table 97. To obtain deflections, the tabulated coefficients are to be multiplied by the quantity Pa/R_4h ,

MOMENTS IN I-BEAM BRIDGES

122

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN

Relative Stiffness of Beams $H=10$ and 20 Relative Proportions of Bridge $b/a=0.1$

See sub-heading of Table 97. To obtain deflections, the tabulated coefficients are to be multiplied by the quantity Pa/B_4I_b ,

Тавье 102

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTEE-POINT

multiplied by the quentity Pol/E.I. Relative Stiffness of Beams $H = 1.0$
To obtain definitive Propertions of Bridge b/a
To obtain definitive Propertions of Bridge b/a as one. Son out hooding of Table 07

MOMENTS IN I-BEAM BRIDGES

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative Stiffness of Beams $H=2.0$ Relative Proportions of Bridge $b/a=0.2$

To obtain deflections, the tabulated coefficients are to be multiplied by the quantity Pa³/Ed1. of Table 07 $\mathbf{c}_{\alpha\alpha}$

124

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT ТАВLЕ 104

Relative Stiffness of Beams $H=4.0$ Relative Proportions of Bridge $b/a=0.2$

See sub-heading of Table 97. To obtain deflections, the tabulated coefficients are to be multiplied by the quantity Pa³/Ed.

MOMENTS IN I-BEAM BRIDGES

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative Stiffness of Beams $H = 10$
Relative Proportions of Bridge $b/a = 0.2$
See sub-heading of Table 97. To obtain deflections, the tabulated coefficients are to be multiplied by the quantity Pa^2/B_4I_6 ,

126

ТАВLЕ 106

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative Stiffness of Beams $H=1.5$ Relative Proportions of Bridge $b/a=0.3$

See sub-heading of Table 97. To obtain deflections, the tabulated coefficients are to be multiplied by the quantity $PaJB_4L$,

MOMENTS IN I-BEAM BRIDGES

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT

Relative of Table 97. To obtain deflections, the tabulated ocefficients are to be multiplied by the quantity Pa^3B_4L .
See sub-heading of Table 97. To obtain deflections, the tabulated coefficients are to be multiplied

ILLINOIS ENGINEERING EXPERIMENT STATION

INFLUENCE COEFFICIENTS FOR DEFLECTION OF BEAMS AT MID-SPAN AND QUARTER-POINT **ТАВLЕ 108**

Relative Stiffness of Beams $H=6.0$ Relative Proportions of Bridge $b/a=0.3$

See sub-heading of Table 97. To obtain deflections, the tabulated coefficients are to be multiplied by the quantity Pa^3/B_4I_3 .

MOMENTS IN I-BEAM BRIDGES

$130\,$

APPENDIX B

INFLUENCE SURFACES FOR MOMENTS IN I-BEAM BRIDGES

This page is intentionally blank.

138

MOMENTS IN I-BEAM BRIDGES

141

FIG. B-15. INFLUENCE SURFACE FOR LONGITUDINAL MOMENT IN SLAB AT AB AT MID-SPAN. $H=2, \; b/a=0.2$

MOMENTS IN I-BEAM BRIDGES

FIG. B-18. INFLUENCE SURFACE FOR MOMENT IN BEAM B AT MID-SPAN. $H = 5, b/a = 0.1$

ILLINOIS ENGINEERING EXPERIMENT STATION

FIG. B-19. INFLUENCE SURFACE FOR MOMENT IN BEAM C AT MID-SPAN. $H=5,\ b/a=0.1$

FIG. B-20. INFLUENCE SURFACE FOR MOMENT IN BEAM A AT MID-SPAN. $H = 2$, $b/a = 0.2$

144

MOMENTS IN I-BEAM BRIDGES

This page is intentionally blank.

RECENT PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION+

Bulletin No. 304. A Distribution Procedure for the Analysis of Slabs Continuous
Over Flexible Beams, by Nathan M. Newmark. 1938. One dollar.
Circular No. 34. The Chemical Engineering Unit Process—Oxidation, by

Donald B. Keyes. 1938. Fifty cents.

Circular No. 35. Factors Involved in Plate Efficiencies for Fractionating Columns, by Donald B. Keyes. 1938. Twenty cents.

Bulletin No. 305. Summer Cooling in the Warm-Air Heating Research Residence with Cold Water, by Alonzo P. Kratz, Seichi Konzo, Maurice K. Fahnestock and Edwin L. Broderick. 1938. Ninety cents.
Bulletin No. 305. Summer Cool

for Cable Sheathing, by Herbert F. Moore, Bernard B. Betty, and Curtis W. Dollins. One dollar. 1938.

Reprint No. 12. Fourth Progress Report of the Joint Investigation of Fissures

in Railroad Rails, by H. F. Moore. 1938. None available.
Bulletin No. 307. An Investigation of Rigid Frame Bridges: Part I, Tests of
Reinforced Concrete Knee Frames and Bakelite Models, by Frank E. Richart,
Thomas J. Dolan

Bulletin No. 308. An Investigation of Rigid Frame Bridges: Part II, Laboratory Tests of Reinforced Concrete Rigid Frame Bridges, by W. M. Wilson, R. W. Kluge, and J. V. Coombe. 1938. Eighty-five cents. Bulletin No. 309. The Effects of Errors or Variations in the Arbitrary Con-

Batter No. 309. The Enects of Equations by George H. Dell. 1938. *Sixty cents.*
Bulletin No. 310. Fatigue Tests of Butt Welds in Structural Steel Plates, by
Bulletin No. 310. Fatigue Tests of Butt Welds in Structural Steel

Part I, Tests of Strength Properties of Wrought Steel Car Wheels, by Thomas J.

Dolan and Rex L. Brown. 1939. Seventy cents.
Circular No. 36. A Survey of Sulphur Dioxide Pollution in Chicago and
Vicinity, by Alamjit D. Singh. 1939. *Forty cents.*
Circular No. 37. Papers Presented at the Second Confere

ing, Held at the University of Illinois, March 8-9, 1939. 1939. Fifty cents.

Circular No. 38. Papers Presented at the Twenty-sixth Annual Conference on Highway Engineering, Held at the University of Illinois, March 1-3, 1939. 1939. Fifty cents.

Bulletin No. 313. Tests of Plaster-Model Slabs Subjected to Concentrated Loads, by Nathan M. Newmark and Henry A. Lepper, Jr. 1939. Sixty cents.
Bulletin No. 314. Tests of Reinforced Concrete Slabs Subjected to Concen-

trated Loads, by Frank E. Richart and Ralph W. Kluge. 1939. Eighty cents.

Bulletin No. 315. Moments in Simple Span Bridge Slabs with Stiffened Edges,

by Vernon P. Jensen. 1939. One dollar.
Bulletin No. 316. The Effect of Range of Stress on the Torsional Fatigue
Strength of Steel, by James O. Smith. 1939. Forty-five cents.
Bulletin No. 317. Fatigue Tests of Connection An

Reprint No. 13. First Progress Report of the Joint Investigation of Continuous
Welded Rail, by H. F. Moore. 1939. Fifteen cents.

Reprint No. 14. Fifth Progress Report of the Joint Investigation of Fissures in
Railroad Rails, by H. F. Moore. 1939. Fiften cents.
Circular No. 39. Papers Presented at the Fifth Short Course in Coal Utiliza-

tion, Held at the University of Illinois, May 23-25, 1939. 1939. Fifty cents.

Reprint No. 15. Stress, Strain, and Štructural Damage, by H. F. Moore. 1940. None available.

Investigation of Oil-fired Forced-Air Furnace Systems in Bulletin No. 318. the Research Residence, by A. P. Kratz and S. Konzo. 1939. Ninety cents.

Bulletin No. 319. Laminar Flow of Sludges in Pipes with Special Reference to Sewage Sludge, by Harold E. Babbitt and David H. Caldwell. 1939. Sixty-five cents.

†Copies of the complete list of publications can be obtained without charge by addressing the Engineering Experiment Station, Urbana, Ill.

Bulletin No. 320. The Hardenability of Carburizing Steels, by Walter H. Bruckner. 1939. Seventy cents.

Bulletin No. 321. Summer Cooling in the Research Residence with a Condensing Unit Operated at Two Capacities, by A. P. Kratz, S. Konzo, M. K. Fahne-

stock, and E. L. Broderick. 1940. Seventy cents.

Circular No. 40. German-English Glossary for Civil Engineering, by A. A.

Brielmaier. 1940. Fifty cents.

Bulletin No. 322. An Investigation of Rigid Frame Bridges: Part III, Tests of Structural Hinges of Reinforced Concrete, by Ralph W. Kluge. 1940. Forty cents.

Circular No. 41. Papers Presented at the Twenty-seventh Annual Conference on Highway Engineering, Held at the University of Illinois, March 6-8, 1940. 1940. Fifty cents.

Reprint No. 16. Sixth Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1940. Fifteen cents.

Reprint No. 17. Second Progress Report of the Joint Investigation of Continuous Welded Rail, by H. F. Moore, H. R. Thomas, and R. E. Cramer. 1940. Fifteen cents.

Reprint No. 18. English Engineering Units and Their Dimensions, by E. W.
Comings. 1940. Fifteen cents.
Reprint No. 19. Electro-organic Chemical Preparations, Part II, by Sherlock
Swann, Jr. 1940. Thirty cents.

Reprint No. 20. New Trends in Boiler Feed Water Treatment, by F. G. Straub. 1940. Fifteen cents.

Bulletin No. 323. Turbulent Flow of Sludges in Pipes, by H. E. Babbitt and D. H. Caldwell. 1940. Forty-five cents.
Bulletin No. 324. The Recovery of Sulphur Dioxide from Dilute Waste Gases by Chemical Regeneration of the A 1940. One dollar.

Bulletin No. 325. Photoelectric Sensitization of Alkali Surfaces by Means of Electric Discharges in Water Vapor, by J. T. Tykociner, Jacob Kunz, and L. P. Garner. 1940. Forty cents.

Bulletin No. 326. An Analytical and Experimental Study of the Hydraulic Ram, by W. M. Lansford and W. G. Dugan. 1940. Seventy cents.

Railwich No. 327. Fatigue Tests of Welded Joints in Structural Steel Plates, by
W. M. Wilson, W. H. Bruckner, J. V. Coombe, and R. A. Wilde. 1941. One dollar.
Bulletin No. 328. A Study of the Plate Factors in the Fractiona

tion of the Ethyl Alcohol-Water System, by D. B. Keves and L. Byman. 1941. Seventy cents.

*Bulletin No. 329. A Study of the Collapsing Pressure of Thin-Walled Cylinders, by R. G. Sturm. 1941. Eighty cents.

*Bulletin No. 330. Heat Transfer to Clouds of Falling Particles, by H. F. Johnstone, R. L. Pigford, and J. H. Chapin. 1941. Seventy cents.

*Bulletin No. 331. Tests of Cylindrical Shells, by W. M. Wilson and E. D. Olson. 1941. One dollar.

*Reprint No. 21. Seventh Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1941. Fifteen cents.
*Bulletin No. 332. Analyses of Skew Slabs, by Vernon P. Jensen. 1941. One

dollar.

*Bulletin No. 333. The Suitability of Stabilized Soil for Building Construction, by E. L. Hansen. 1941. Fifty cents.

*Circular No. 42. Papers Presented at the Twenty-eighth Annual Conference on Highway Engineering, Held at the University of Illinois, March 5-7, 1941. 1941.

Fifty cents.

*Bulletin No. 334. The Effect of Range of Stress on the Fatigue Strength of

*Bulletin No. 334. The Effect of Range of Stresses in Gear Tooth Fillets, by

*Bulletin No. 354. The Stresses in Gear Tooth Fillets

*Bulletin No. 335. A Photoelastic Study of Stresses in Gear Tooth Fillets, by Thomas J. Dolan and Edward L. Broghamer. 1942. Forty-five cents.

*Circular No. 43. Papers Presented at the Sixth Short Course in Coal Utilization, Held at the University of Illinois, May 21-23, 1941. 1942. Fifty cents.

*Bulletin No. 336. Moments in I-Beam Bridges, by Nathan M. Newmark and

Chester P. Siess. 1942. One dollar.

*A limited number of copies of bulletins starred are available for free distribution.

UNIVERSITY OF ILLINOIS

Colleges and Schools at Urbana

- COLLEGE OF LIBERAL ARTS AND SCIENCES.—General curriculum with majors in the humanities and sciences; a new general curriculum with fields of concentration in mathematics and physical science, biological science, social sci
- COLLEGE OF COMMERCE AND BUSINESS ADMINISTRATION.—Fields of concentration in accountancy, banking and finance, commerce and law, commercial teaching, economics, industrial administration, management, marketing, and public a
- COLLEGE OF ENGINEERING.—Curricula in agricultural engineering, ceramics, ceramic engineering, chemical engineering, civil engineering, electrical engineering, engineering
ing physics, general engineering, mechanical engineering, metallurgical engineering, and mining engineering.
- COLLEGE OF AGRICULTURE.—Curricula in agriculture, dairy technology, floriculture, general home economics, nutrition and dietetics, and vocational agriculture; pre-professional training in forestry.
- COLLEGE OF EDUCATION.—Curricula in education, agricultural education, home economics education, and industrial education. The University High School is the practice school of the College of Education.
- COLLEGE OF FINE AND APPLIED ARTS.-Curricula in architecture, art, landscape architecture, music, and music education.

COLLEGE OF LAW.-Professional curriculum in law.

SCHOOL OF JOURNALISM.-General and special curricula in journalism.

SCHOOL OF PHYSICAL EDUCATION,—Curricula in physical education for men and for women.

LIBRARY SCHOOL.-Curriculum in library science.

GRADUATE SCHOOL.-Advanced study and research.

Summer Session.-Courses for undergraduate and graduate students.

University Extension Division.-Courses taught by correspondence, extramural courses. science aids service, speech aids service, and visual aids service.

Colleges in Chicago

COLLEGE OF DENTISTRY.-Professional curriculum in dentistry. COLLEGE OF MEDICINE.—Professional curriculum in medicine. COLLEGE OF PHARMACY.—Professional curriculum in pharmacy.

University Experiment Stations, and Research and Service Organizations at Urbana

State Scientific Surveys and Other Divisions at Urbana

STATE GEOLOGICAL SURVEY STATE NATURAL HISTORY SURVEY STATE WATER SURVEY

STATE DIAGNOSTIC LABORATORY (for Animal Pathology) **U. S. SOYBEAN PRODUCTS LABORATORY**

For general catalog of the University, special circulars, and other information, address THE REGISTRAR, UNIVERSITY OF ILLINOIS

URBANA, ILLINOIS

