UNIVERSITY OF ILLINOIS ENGINEERING EXPERIMENT STATION BULLETIN NO. 439

Studies of Slab and Beam Highway Bridges, Part VI.

Moments in Simply Supported Skew I-Beam Bridges

by

T. Y. Chen

C. P. Siess

N. M. Newmark

UNIVERSITY OF ILLINOIS BULLETIN

A REPORT OF AN INVESTIGATION

Conducted by

THE ENGINEEERING EXPERIMENT STATION

UNIVERSITY OF ILLINOIS

In Cooperation with

THE OFFICE OF NAVAL RESEARCH

and

THE ILLINOIS DIVISION OF HIGHWAYS

and

THE DEPARTMENT OF COMMERCE

BUREAU OF PUBLIC ROADS

Price: One dollar

UNIVERSITY OF ILLINOIS BULLETIN

Volume 54, Number 37; January, 1957. Published seven times each month by the University of Illinois. Entered as second-class matter December 11, 1912, at the post office at Urbana, Illinois, under the Act of August 24, 1912. Office of Publication, 207 Administration Building, Urbana, Ill.

Studies of Slab and Beam Highway Bridges, Part VI.

Moments in Simply Supported Skew I-Beam Bridges

by

T. Y. Chen

RESEARCH ASSISTANT PROFESSOR
OF CIVIL ENGINEERING

C. P. Siess

RESEARCH PROFESSOR OF CIVIL ENGINEERING

N. W. Newmark

RESEARCH PROFESSOR OF STRUCTURAL ENGINEERING

ENGINEERING EXPERIMENT STATION BULLETIN NO. 439

4050—1-57—61282

UNIVERSITY
OF ILLINOIS
:: PRESS ::

CONTENTS

CONTENTS (Concluded)

FIGURES

TABLES

I. INTRODUCTION

1. Object and Scope of Investigation

The type of structure considered in this investigation is a simple-span skew bridge, which consists of a concrete slab of uniform thickness supported by five identical steel beams, uniformly spaced and parallel to the direction of traffic. When the angle of skew is zero, the structure reduces to a simple-span right I-beam bridge, for which both analytical and experimental data are available. The analytical results, which were obtained by computations based on a distribution procedure developed by Newmark,(1)* were reported in a paper by Newmark and Siess;(2) and the experimental data were reported by Newmark, Siess, and Penman.(3) While results of an experimental study of the skew I-beam bridge were made available in a paper by Newmark, Siess, and Peckham,(4) no analysis, exact or approximate, of such a bridge, so far as known, has been presented. Consequently, an analytical investigation of the behavior of this type of structure with a view toward development of better design criteria appeared desirable, and would serve, it is hoped, as a supplement to the works cited above.

The data here are based entirely on analytical considerations, and were obtained by means of the method of finite differences. The difference method is by no means new, and has been used by numerous authors in the solutions of various structural problems.** Although it yields results that are only approximate, its use is justifiable, particularly where complicated boundary conditions are encountered which may prevent the expression of exact solutions.

The flexibility of the beams is taken into account in the analysis. Moments are determined at various points in the slab and in the beams for different positions of a concentrated load on the bridge. Influence values for moments and deflections are given for a group of structures of various proportions and relative stiffnesses of the slab and the beams, and of different angles of skew. Influence surfaces for moments and deflections are shown for several of the structures studied. From the influence values, moment coefficients were determined for a number of skew bridges of different span lengths and subjected to standard highway truck loads. Some general relations pertaining to the design of skew I-beam bridges have been derived from the results of analyses.

* Superscripts in parentheses refer to corresponding entries in the References.

** For a listing of some of the works pertaining to the application of difference equations to slab problems, see Ref. 5.

2. Method of Analysis

The data reported here were obtained by means of difference equations. The assumptions used in the analysis, in addition to those usually embodied in the ordinary theory of medium thick plates, are:

(1) The beams exert only vertical forces on the slab; there is no shear between the beams and the slab.

(2) The effect of any diaphragms is neglected.

(3) The reaction of the beam acts on the slab along a line, and is not distributed over a finite width.

(4) A beam and the slab directly over it deflect alike.

(5) The edge beams on each side of the bridge are located at the edge of the slab.

(6) Both the slab and the beams are simply supported at the ends of the span.

The values of Poisson's ratio for concrete is assumed to be zero for all calculations.*

Difference equations are derived for a network of points in skew coordinates, in which the elemental mesh is a parallelogram formed by lines parallel to the sides of the skew panel. A network of points in triangular coordinates (where the elemental mesh is a triangle) might have been used; Jensen(5) used it in his analysis of skew slab

* For a discussion concerning the assumption that Poisson's ratio for concrete is zero, see Ref. 2, pp. 13-14.

bridges with curbs. However, the skew network is better adapted to the solution of the present problem in obtaining the maximum numerical accuracy. Rectangular or square networks could also be conveniently used for certain angles of skew and proportions of the bridge, but the skew network (or the triangular network, for that matter) is more widely applicable.

The skew network of points used is formed by two sets of parallel lines. As shown in Fig. 1, the first set of lines, drawn parallel to the abutments, divides the length of the span into eight equal spaces. The second set of lines, drawn parallel to the beams, divides each slab panel between two consecutive beams into two equal segments. The difference equations are then applied to this network of points to determine the effect at any point caused by a load at any given position on the bridge. It is convenient to break the load into symmetrical and anti-symmetrical components. In this way, the analysis of each particular problem reduces, in general, to the solution of two sets of simultaneous equations: one set having 32 unknowns for the symmetrical loading, and the other having 31 unknowns for the anti-symmetrical loading. The solutions of these two sets of equations give the required results when added. The chosen spacing of the nodal points in the network in Fig. 1 was based primarily on the limiting capacity of the ILLIAC digital computer. The ILLIAC could solve only 39 or fewer simultaneous equations at the time these calculations were made (1953).

The results obtained by means of difference equations are approximate. To estimate the degree of accuracy which may be expected of the difference solutions, comparisons are made between the exact values for right I-beam bridges obtained by Newmark and Siess[2] and the approximate values determined from difference equations for corresponding right bridges. In the absence of exact analyses of the skew bridge, these comparisons for

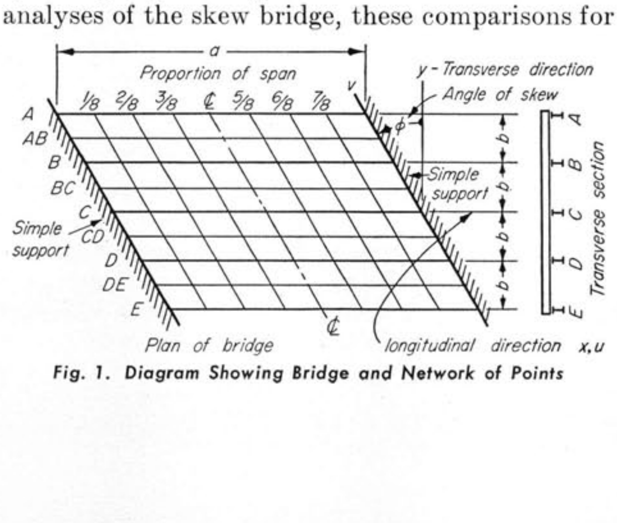


Fig. 1. Diagram Showing Bridge and Network of Points

the right bridge are assumed to give indications regarding the accuracy of the approximate results obtained for the skew bridge.

3. Notation

The following notation is used throughout this work. The longitudinal direction is always taken as the direction of the beams; the transverse direction is that perpendicular to the beams; and the skew direction is parallel to the abutments.

- a = span of bridge, center to center of supports.
- b = transverse spacing of beams.
- c = diameter of uniformly loaded circular area representing a wheel load.
- EI = product of modulus of elasticity of the material in the slab and moment of inertia per unit of width of the cross-section of the slab.
- μ = Poisson's ratio for the material in the slab, taken equal to zero in the numerical data given here.
- $N = \dfrac{EI}{1-\mu^2}$, a measure of stiffness of a slab element.
- E_bI_b = product of modulus of elasticity of beam material and the moment of inertia of the beam cross-section.
- $H = \dfrac{E_bI_b}{aN}$, a dimensionless coefficient which is a measure of the stiffness of the beam relative to that of the slab.
- P = concentrated load.
- p = load per unit of area uniformly distributed over the slab.
- q = load per unit of length uniformly distributed along a beam.
- w = deflection of slab, positive downward; with subscript indicating the deflection at a particular point denoted by the subscript.
- x, y = rectangular coordinates.
- u, v = skew coordinates, as shown in Fig. 2.
- λ_x, λ_y, λ_u, λ_v = distances between points or lines of the network as defined in Fig. 2.
- φ = angle of skew, as shown in Fig. 1.
- $K = \lambda_y/\lambda_x$, an abbreviation.
- $A = (K/\cos\varphi)^2$, an abbreviation.
- $B = K\tan\varphi$, an abbreviation.
- $C = (1-\mu)K^2$, an abbreviation.
- $D = (A+C)B$, an abbreviation.

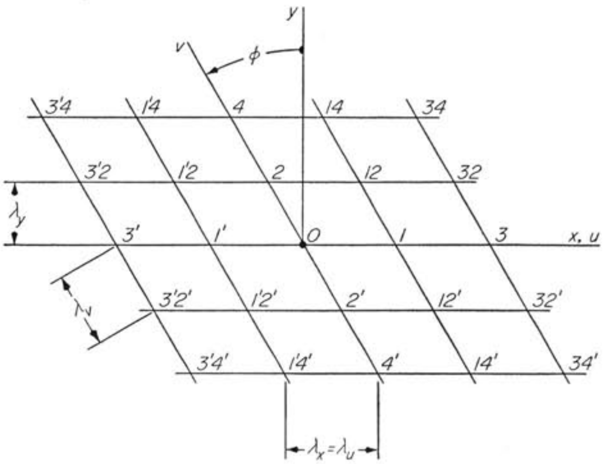


Fig. 2. Network of Points Surrounding a Typical Interior Point

$J = K^4E_bI_b/\lambda_yN = K^4Ha/\lambda_y$, a dimensionless number proportional to the relative stiffness H.

V_x, V_y = vertical shear per unit of length, acting on sections normal to the x and y axes respectively, positive on a rectangular element of a slab when acting upward on the side of the element having the smaller values of x or y respectively.

M_x, M_y = bending moment per unit width of slab in the direction of the x or y axis respectively, positive when producing compression at the top of the slab.

M_{xy} = twisting moment per unit width of slab in the directions of x and y, positive when tending to produce compression at the top of the slab in the direction of the line $x = y$.

M_b = bending moment in a beam, positive when producing compression at the top.

R_x, R_y = reactions per unit of length of slab, defined in the same manner as V_x and V_y.

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, Laplace's operator in two variables.

$U = \nabla^2 w$.

k = coefficient in Eqs. (51) and (52); effective proportion of wheel load used in computing maximum moments in a beam due to standard truck loading.

s = length defined by Eqs. (53) and (54), in terms of which maximum moments in beams are stated.

m = coefficient in Eq. (54), which defines the length s for different angles of skew φ.

4. Acknowledgment

The work reported here was carried out in the Structural Research Laboratory of the Department of Civil Engineering, University of Illinois. First this study was part of a cooperative research project between the Engineering Experiment Station of the University of Illinois and the Office of Naval Research, under Contract NONR-1834(03), Project NR-064-183. Subsequently it became part of an investigation of slab and beam highway bridges, in cooperation with the Illinois Division of Highways and the U. S. Department of Commerce, Bureau of Public Roads.

The subject matter of this bulletin is based on a doctoral thesis by T. Y. Chen submitted to the Graduate College of the University of Illinois in 1954. The thesis was written under the direction of N. M. Newmark.

II. DERIVATION OF EQUATIONS

5. General Concepts

The ordinary theory of flexure of medium thick slabs is well-known, and the derivations of the fundamental equations are available in a number of places in the literature.* A summary of the relations necessary for the present work is given below.

The fundamental differential equation for the flexure of slabs is

$$N\nabla^2(\nabla^2 w) = p, \tag{1}$$

in which w is the deflection of the neutral surface of the slab, p is the intensity of load, $N = \frac{EI}{1-\mu^2}$ is the stiffness of the slab, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace's operator in two variables. The equation for the deflection, w, must also satisfy the boundary conditions of the slab. When such a solution has been found, the moments, shears, and reactions may be obtained by differentiation according to the following formulas:

Moments:

$$\left.\begin{aligned} M_x &= -N\left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -N\left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2}\right) \\ M_{xy} &= -N(1-\mu)\frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{2}$$

Shears:

$$\left.\begin{aligned} V_x &= -N\frac{\partial}{\partial x}(\nabla^2 w) \\ V_y &= -N\frac{\partial}{\partial y}(\nabla^2 w) \end{aligned}\right\} \tag{3}$$

Reactions:

$$\left.\begin{aligned} R_x &= V_x + \frac{\partial M_{xy}}{\partial y} = -N\left[\frac{\partial^3 w}{\partial x^3} + (2-\mu)\frac{\partial^3 w}{\partial x \partial y^2}\right] \\ R_y &= V_y + \frac{\partial M_{xy}}{\partial x} = -N\left[\frac{\partial^3 w}{\partial y^3} + (2-\mu)\frac{\partial^3 w}{\partial x^2 \partial y}\right] \end{aligned}\right\} \tag{4}$$

The ordinary theory of flexure is not valid in the neighborhood of a concentrated load where the assumptions made in the theory are not fulfilled. For the determination of the tensile stresses at the point of application of a concentrated load, resort is usually made to Westergaard's[(8)] suggestion of using, with the ordinary theory, an equivalent circular area of load distribution which is determined by a special theory of Nádai.[(9)]

* For example, Ref. (7).

The differential equation for a beam in the x direction is

$$E_b I_b \frac{\partial^4 w}{\partial x^4} = q, \tag{5a}$$

in which $E_b I_b$ is the flexural rigidity of the beam, and q is the load per unit of length along the beam. The bending moment at any point in the beam is given by

$$M_b = -E_b I_b \frac{\partial^2 w}{\partial x^2} \tag{5b}$$

6. Difference Operators for Skew Network

Consider a simple-span slab bridge supported on five equally spaced beams, having a parallelogram-shaped plan with any angle of skew, and with any given proportions of the span and spacing of the beams. A network of points may be defined by the intersections of two sets of equally spaced lines: one set drawn parallel to the beams, the other parallel to the ends of the span. The x axis will be taken parallel to the beams and, consequently, parallel to the first set of lines. The angle of skew, φ, will be defined as the angle between the y axis and the simply-supported edges of the slab as shown in Fig. 1.

The problem is to find a number of equations for the deflections of the slab at the points of the network, the number of equations to be equal to that of the points in the network. Consider a typical node point O and the points in its neighborhood as shown in Fig. 2. The various points surrounding O have been numbered for identification. These numbers as subscripts on the deflection w will indicate the deflection of the slab at the corresponding points. In addition to the x and y axes, another pair of axes, u and v, are drawn, with the u axis coinciding with the x axis, and the v axis making an angle with the y axis equal to the angle of skew, φ. The dimensions λ_x, λ_u, and λ_v are defined as the distances between successive points of the network in the directions of the x, u, and v axes, respectively, while λ_y is defined as the perpendicular distance between successive lines drawn parallel to the x axis as shown in Fig. 2.

Reference to Fig. 2 will show the following relations:

$$\left.\begin{aligned} \lambda_u &= \lambda_x \\ \lambda_v &= \lambda_y/\cos\varphi \end{aligned}\right\} \qquad \text{(a)}$$

With the u and v directions established, derivatives of any function of x and y may be expressed in terms of corresponding derivatives with respect to u and v.

A point of rectangular coordinates (x, y) as shown in Fig. 3 is located in the plane by skew coordinates (u, v), given by the transformation

$$x = u - v\sin\varphi \qquad y = v\cos\varphi \qquad \text{(b)}$$

The partial derivatives, indicated by subscripts, of x and y with respect to u and v are

$$\left.\begin{aligned} x_u &= 1; & x_v &= -\sin\varphi \\ y_u &= 0; & y_v &= \cos\varphi \end{aligned}\right\} \qquad \text{(c)}$$

Consider a function $f(u, v)$, in which u, v are related to x, y by means of Eqs. (b). The first derivatives of f with respect to u and v are

$$\left.\begin{aligned} f_u &= f_x x_u + f_y y_u = f_x \\ f_v &= f_x x_v + f_y y_v = -f_x\sin\varphi + f_y\cos\varphi \end{aligned}\right\} \qquad \text{(d)}$$

from which one obtains

$$f_y = \frac{f_u\sin\varphi + f_v}{\cos\varphi} \qquad \text{(e)}$$

The second derivatives of f with respect to u and v are obtained by "squaring" the operators f_u and f_v, and by means of their "product."

$$f_{uu} = f_{xx} \qquad \text{(f)}$$

$$f_{vv} = f_{xx}\sin^2\varphi - 2f_{xy}\sin\varphi\cos\varphi + f_{yy}\cos^2\varphi \qquad \text{(g)}$$

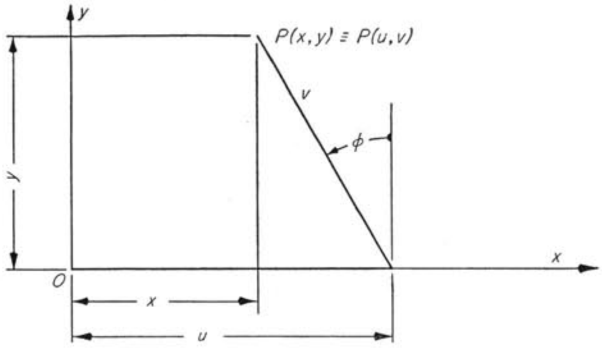


Fig. 3. Relationship Between Rectangular and Skew Coordinates

$$f_{uv} = -f_{xx}\sin\varphi + f_{xy}\cos\varphi \qquad \text{(h)}$$

Substituting Eqs. (f) and (h) into Eq. (g),

$$f_{vv} = f_{uu}\sin^2\varphi - 2\sin\varphi\,(f_{uv} + f_{uu}\sin\varphi) + f_{yy}\cos^2\varphi$$

f_{yy} is obtained in terms of f_{uu}, f_{vv}, and f_{uv} as

$$f_{yy} = \frac{1}{\cos^2\varphi}\,(f_{uu}\sin^2\varphi + 2f_{uv}\sin\varphi + f_{vv}) \qquad \text{(i)}$$

and the Laplacian operator in skew coordinates becomes, by Eqs. (f) and (i),

$$\nabla^2 f = f_{xx} + f_{yy} = \frac{1}{\cos^2\varphi}\,(f_{uu} + 2f_{uv}\sin\varphi + f_{vv}) \quad \text{(j)}$$

Also, from Eqs. (f) and (h),

$$f_{xy} = \frac{1}{\cos\varphi}\,(f_{uu}\sin\varphi + f_{uv}) \qquad \text{(k)}$$

The ∇^2-operator in skew coordinates is transformed into the corresponding difference operator by substituting for the derivatives f_{uu}, f_{uv}, and f_{vv} their expressions in difference form, which may be found in a number of places in the literature.* The difference expressions for f_{uu}, f_{uv}, and f_{vv} for a typical interior point O as shown in Fig. 2 are

$$\left.\begin{aligned} (f_{uu})_o &= \frac{1}{\lambda_u^2}\,(f_{1'} - 2f_o + f_1) \\ (f_{uv})_o &= \frac{1}{4\lambda_u\lambda_v}\,(-f_{1'2} + f_{12} + f_{1'2'} - f_{12'}) \\ (f_{vv})_o &= \frac{1}{\lambda_v^2}\,(f_{2'} - 2f_o + f_2) \end{aligned}\right\} \qquad \text{(m)}$$

Substituting Eqs. (m) in Eqs. (f), (i), (j), and (k), noting Eqs. (a), and using the abbreviations

$$K = \frac{\lambda_y}{\lambda_x}; \quad B = K\tan\varphi; \quad A = B^2 + K^2 = \frac{K^2}{\cos^2\varphi}, \qquad \text{(n)}$$

one obtains the following expressions for f_{xx}, f_{yy}, $\nabla^2 f$, and f_{xy}:

$$(f_{xx})_o = \frac{K^2}{\lambda_y^2}\,(f_{1'} - 2f_o + f_1) \qquad \text{(o)}$$

$$(f_{yy})_o = \frac{1}{\lambda_y^2}\Big[B^2(f_{1'} + f_1) - (2 + 2B^2)f_o + \frac{B}{2}(-f_{1'2} + f_{12} + f_{1'2'} - f_{12'}) + f_{2'} + f_2\Big] \qquad \text{(p)}$$

$$(\nabla^2 f)_o = \frac{1}{\lambda_y^2}\Big[A(f_{1'} + f_1) - (2 + 2A)f_o + \frac{B}{2}(-f_{1'2} + f_{12} + f_{1'2'} - f_{12'}) + f_{2'} + f_2\Big] \qquad \text{(q)}$$

* For example, Reference (6), pp. 69 and 167-9.

$$(f_{xy})_o = \frac{K}{\lambda_y^2}\Big[B\,(f_{1'} + f_1) - 2Bf_o + \frac{1}{4}\,(-f_{1'2} + f_{12} + f_{1'2'} - f_{12'})\Big]. \quad \text{(r)}$$

7. Difference Equations for Skew Network

With the derivatives and the Laplacian operator of any function expressed in finite differences, the difference equations governing the deflection of the neutral surface of the slab at the node points may be derived. It is convenient first to break the differential equation (1) into two parts, as Marcus did[10]:

$$N\nabla^2 U = p, \quad (6)$$

and,

$$\nabla^2 w = U. \quad (7)$$

Equations (6) and (7) may be expressed in terms of finite differences by the use of Eq. (q). For an interior point O as defined by the network of Fig. 2, the results so obtained are

$$N(\nabla^2 U)_o = \frac{N}{\lambda_y^2}\Big[A\,(U_{1'} + U_1) - (2+2A)\,U_o + \frac{B}{2}\,(-U_{1'2} + U_{12} + U_{1'2'} - U_{12'}) + U_{2'} + U_2\Big] = \bar{p}_o, \quad (8)$$

$$(\nabla^2 w)_o = \frac{1}{\lambda_y^2}\Big[A\,(w_{1'} + w_1) - (2 + 2A)\,w_o + \frac{B}{2}\,(-w_{1'2} + w_{12} + w_{1'2'} - w_{12'}) + w_{2'} + w_2\Big] = U_o. \quad (9)$$

In Eq. (8) the quantity $\bar{p}_o$ is the equivalent combined effects in terms of load per unit of area of all the loads that act at point O. Thus, if at point O, there act a uniformly distributed load of p_o per unit of area, a line load of q_o per unit of length in the x direction, and a concentrated load of P_o, $\bar{p}_o$ is given by

$$\bar{p}_o = p_o + \frac{q_o}{\lambda_y} + \frac{P_o}{\lambda_x \lambda_y} = p_o + \frac{q_o}{\lambda_y} + \frac{KP_o}{\lambda_y^2}. \quad \text{(10a)}$$

If point O lies on an exterior edge of the slab, $\bar{p}_o$ is given by

$$\bar{p}_o = p_o + \frac{q_o}{\frac{1}{2}\lambda_y} + \frac{P_o}{\frac{1}{2}\lambda_x \lambda_y} = p_o + \frac{2q_o}{\lambda_y} + \frac{2KP_o}{\lambda_y^2}. \quad \text{(10b)}$$

(a) *General Interior Point*

Consider a general interior point of the slab, such as O shown in Fig. 2. Assume that O is also a point on a beam. If the beam and the slab over it are assumed to deflect alike, the load per unit of length carried by the beam at point O, is, from Eq. (5a), given by the difference expression

$$E_b I_b \left(\frac{\partial^4 w}{\partial x^4}\right)_o = E_b I_b \frac{K^4}{\lambda_y^4}\,(w_{3'} - 4w_{1'} + 6w_o - 4w_1 + w_3).$$

This load carried by the beam, expressed in terms of equivalent uniform load intensity per unit of area, is

$$\frac{E_b I_b}{\lambda_y} \cdot \frac{K^4}{\lambda_y^4}\,(w_{3'} - 4w_{1'} + 6w_o - 4w_1 + w_3)$$

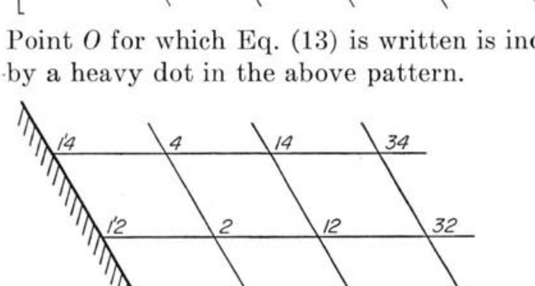
or,

$$\frac{NJ}{\lambda_y^4}\,(w_{3'} - 4w_{1'} + 6w_o - 4w_1 + w_3) \quad (11)$$

in which

$$J = \frac{E_b I_b}{N\lambda_y} \cdot K^4. \quad (12)$$

Expression (11) is to be incorporated into the left-hand side of Eq. (8). With this in mind, and substituting into Eq. (8) expansions of U at the various points similar to Eq. (9), one obtains, after some simplification, the equation for a general interior point O, which may be given more conveniently in pattern form as follows:



$$w = \frac{\bar{p}_o \lambda_y^4}{N} \quad (13)$$

Point O for which Eq. (13) is written is indicated by a heavy dot in the above pattern.

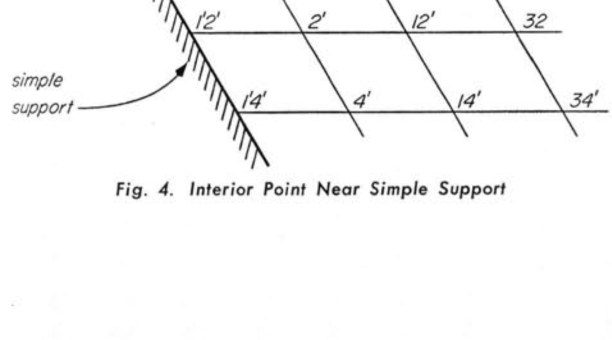


Fig. 4. Interior Point Near Simple Support

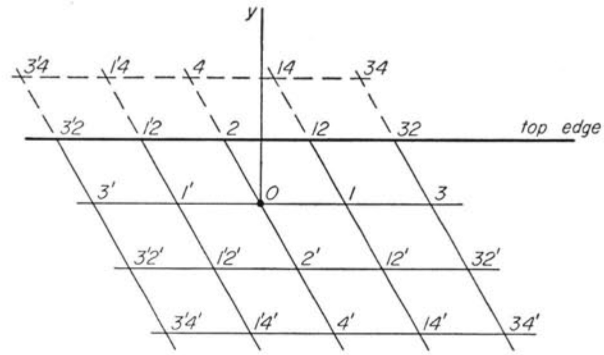


Fig. 5. Interior Point Near Top Edge

If point O is not on a beam, it is necessary merely to drop all the J-terms from Eq. (13).

(b) *Interior Point near Simple Support*

When point O is near the simple supports, the effect of the boundaries must be taken into account. The boundary conditions at the simple supports are: there shall be no deflection at the supports, no moment in the slab in a direction normal to the supports, and no moment in the beams. This requires that

$$\left.\begin{aligned} (w)_{\text{support}} &= 0; \\ (\nabla^2 w)_{\text{support}} &= U_{\text{support}} = 0; \\ (M_b)_{\text{support}} &= -E_b I_b \left(\frac{\partial^2 w}{\partial x^2}\right)_{\text{support}} = 0. \end{aligned}\right\} \quad (14a)$$

For a node point O on a beam near the left simple support (Fig. 4), the first two of these conditions may be written as

$$\left.\begin{aligned} w_{1'4} = w_{1'2} = w_{1'} = w_{1'2'} = w_{1'4'} = 0; \\ U_{1'2} = U_{1'} = U_{1'2'} = 0. \end{aligned}\right\} \quad (14b)$$

The third condition may be stated as

$$(M_b)_{1'} = \frac{-E_b I_b}{\lambda_x^2}(w_{3'} - 2w_{1'} + w_o) = 0$$

from which it follows, noting Eq. (14b), that the deflection of the fictitious point 3′ of the beam is given by

$$w_{3'} = -w_o. \quad (14c)$$

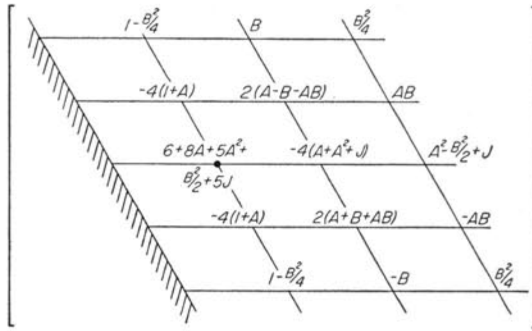
Expression (11), on substitution of Eq. (14c), becomes

$$\frac{NJ}{\lambda_y^4}(5w_o - 4w_1 + w_3). \quad (15)$$

Modified by expression (15), which gives the share of load carried by the beam, and by the second of Eqs. (14b), Eq. (8) becomes

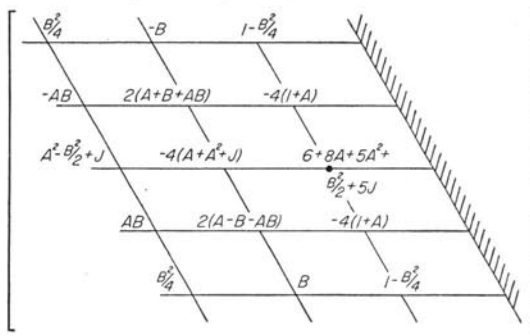
$$\frac{N}{\lambda_y^2}\Big[A U_1 - (2 + 2A) U_o + \frac{B}{2}(U_{12} - U_{12'}) + U_2 + U_{2'}\Big] + \frac{NJ}{\lambda_y^4}(5w_o - 4w_1 + w_3) = \bar{p}_o.$$

Expressing U_1, U_o, U_{12}, . . . , U_2 by equations similar to Eq. (9), and using the first of Eqs. (14b), one obtains



$$w = \frac{\bar{p}_o \lambda_y^4}{N} \quad (16)$$

The equation for a point O on a beam and near the right simple support may be found to be



$$w = \frac{\bar{p}_o \lambda_y^4}{N} \quad (17)$$

If point O is not on a beam, the terms involving J are to be dropped from Eqs. (16) and (17).

(c) *Interior Point near Edge Beam*

Consider an interior point O near the top edge beam of the bridge as shown in Fig. 5. The edge beam is considered to deflect equally with the slab. One of the boundary conditions along the edge is that there shall be no moment normal to the edge of the slab. This condition may be stated as

$$\begin{aligned} (M_y)_{\text{edge}} = 0 &= -N\left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2}\right)_{\text{edge}} \\ &= -N\left[\nabla^2 w - (1 - \mu)\frac{\partial^2 w}{\partial x^2}\right]_{\text{edge}}, \end{aligned}$$

or,

$$U_{\text{edge}} = (1 - \mu)\left(\frac{\partial^2 w}{\partial x^2}\right)_{\text{edge}}. \quad (18)$$

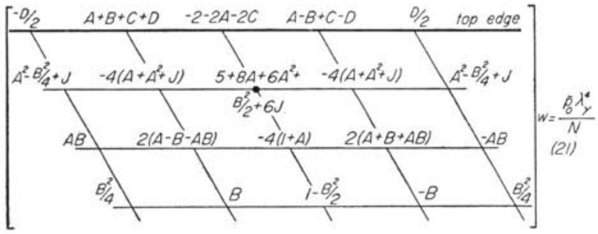
Using the abbreviation

$$C = (1 - \mu) K^2, \quad (19)$$

Eq. (18), when applied to points 1′2, 2, and 12 on the edge beam, gives, in terms of finite differences

$$\left.\begin{aligned} U_{1'2} &= \frac{C}{\lambda_y^2}(w_{3'2} - 2w_{1'2} + w_2); \\ U_2 &= \frac{C}{\lambda_y^2}(w_{1'2} - 2w_2 + w_{12}); \\ U_{12} &= \frac{C}{\lambda_y^2}(w_2 - 2w_{12} + w_{32}). \end{aligned}\right\} \quad (20)$$

Now applying Eq. (8) to point O, treating expression (11) as was done in the derivation of the equation for a general interior point, and expressing $U_{1'2}$, U_2, and U_{12} by means of Eqs. (20), and $U_{1'}$, U_o, U_1, $U_{1'2'}$, $U_{2'}$, and $U_{12'}$ by equations similar to Eq. (9), one obtains



in which D is an abbreviation as given by

$$D = B(A + C). \quad (22)$$

Again, the J-terms are to be dropped from Eq. (21) if point O is not on a beam.

(d) *General Edge Point*

To find the deflection equation which applies

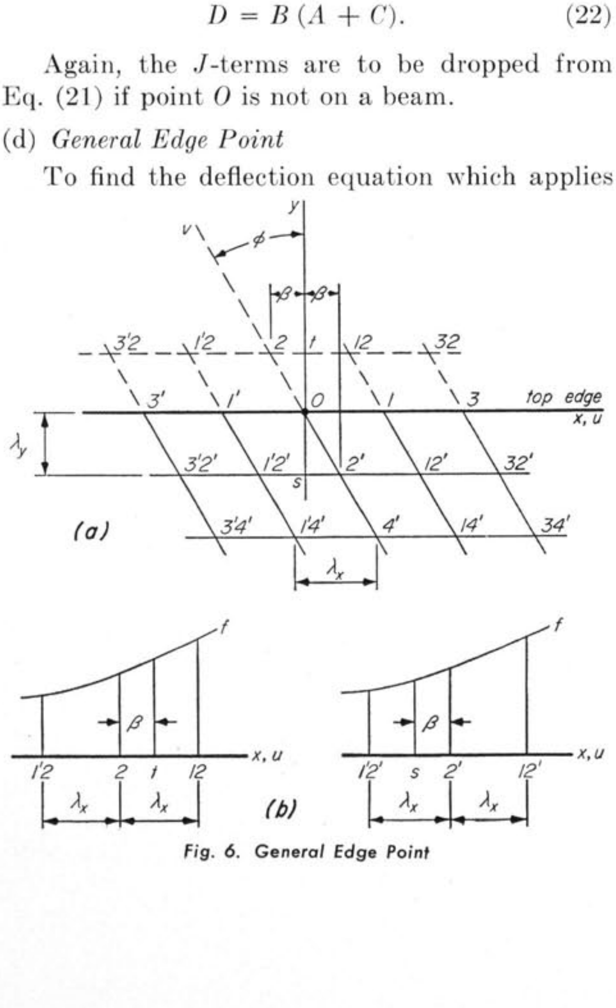


Fig. 6. General Edge Point

when point O is a general point on the top edge beam as shown in Fig. 6(a), it is necessary first to use again the condition (18) with the derivatives expressed by finite differences. Thus,

$$\left.\begin{aligned} U_{1'} &= \frac{C}{\lambda_y^2}(w_{3'} - 2w_{1'} + w_o); \\ U_o &= \frac{C}{\lambda_y^2}(w_{1'} - 2w_o + w_1); \\ U_1 &= \frac{C}{\lambda_y^2}(w_o - 2w_1 + w_3). \end{aligned}\right\} \quad (23)$$

The other boundary condition along the edge beam is that the vertical reaction normal to the edge of the slab shall be transmitted to the beam as a vertical load. This condition may be expressed as

$$(R_y)_{\text{edge}} = -E_b I_b \left(\frac{\partial^4 w}{\partial x^4}\right)_{\text{edge}}. \quad (24a)$$

Now, from the second of Eqs. (4), noting that

$$\begin{aligned} R_y &= -N\left[\frac{\partial^3 w}{\partial y^3} + (2 - \mu)\frac{\partial^3 w}{\partial x^2 \partial y}\right] \\ &= -N\left[\frac{\partial}{\partial y}(\nabla^2 w) + (1 - \mu)\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}\right)\right] \\ &= -N\left[\frac{\partial U}{\partial y} + (1 - \mu)\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}\right)\right], \end{aligned}$$

one finds that Eq. (24a), when applied to point O, Fig. 6(a), may be rewritten as

$$\left[\frac{E_b I_b}{N}\left(\frac{\partial^4 w}{\partial x^4}\right) - \left(\frac{\partial U}{\partial y}\right) - (1-\mu)\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2}\right)\right]_o = 0. \quad (24b)$$

In Eq. (24b), the first term may be expressed in terms of differences without difficulty; the other terms containing $\left(\frac{\partial U}{\partial y}\right)_o$ and $\left(\frac{\partial}{\partial y}\frac{\partial^2 w}{\partial x^2}\right)_o$ require special treatment so that the results, when substituted into Eq. (8), will give an equation which does not involve any of the fictitious node points.

Let the y axis drawn through point O intersect the two grid lines in the x direction adjacent to the edge beam at points s and t as shown in Fig. 6(a). Denote each of the distances $s2'$ and $t2$ by

$$\beta = \lambda_y \tan \varphi.$$

Consider a function, f, for which the slope in the y direction is to be evaluated at point O. By passing a plane tangent to f at point O, the following relations may be found by reference to

Fig. 6(b):

$$\left(\frac{\partial f}{\partial y}\right)_o = \frac{f_t - f_s}{2\lambda_y}, \qquad \text{(a)}$$

$$f_t = f_2 + \frac{f_{12} - f_{1'2}}{2\lambda_x}\beta = f_2 + \frac{1}{2}B\,(f_{12} - f_{1'2}), \qquad \text{(b)} \qquad (25)$$

$$f_s = f_{2'} - \frac{f_{12'} - f_{1'2'}}{2\lambda_x}\beta = f_{2'} - \frac{1}{2}B\,(f_{12'} - f_{1'2'}). \qquad \text{(c)}$$

It should be noted that Eqs. (25a) and (25b) are approximations to the more accurate expressions for f_t and f_s which are

$$f_t = f_2 + \frac{f_{12} - f_2}{\lambda_x}\beta = f_2 + B\,(f_{12} - f_2)$$

$$f_s = f_{2'} - \frac{f_{2'} - f_{1'2'}}{\lambda_x}\beta = f_{2'} - B\,(f_{2'} - f_{1'2'})$$

It will be found, however, that the use of the more accurate expressions would lead to equations similar to Eqs. (27) and (28), from which it appears that the terms involving the fictitious node points could not be so conveniently eliminated as is possible with the use of the approximate expressions. Hence Eqs. (25a) and (25b) are used as an expedient to avoid this difficulty.

From Eq. (25) it follows that the first derivative of any function, f, with respect to y, evaluated at point O, may be expressed in terms of finite differences in the neighborhood of point O by means of the equation

$$\left(\frac{\partial f}{\partial y}\right)_o = \frac{1}{2\lambda_y}\left\{\left[f_2 + \frac{1}{2}B\,(f_{12} - f_{1'2})\right] - \left[f_{2'} - \frac{1}{2}B\,(f_{12'} - f_{1'2'})\right]\right\}.$$

Hence,

$$\left(\frac{\partial U}{\partial y}\right)_o = \frac{1}{2\lambda_y}\left\{\left[U_2 + \frac{1}{2}B\,(U_{12} - U_{1'2})\right] - \left[U_{2'} - \frac{1}{2}B\,(U_{12'} - U_{1'2'})\right]\right\},$$

$$\left(\frac{\partial}{\partial y}\frac{\partial^2 w}{\partial x^2}\right)_o = \frac{\partial}{\partial y}\left[\frac{K^2}{\lambda_y^2}(w_{1'} - 2w_o + w_1)\right]$$

$$= \frac{K^2}{2\lambda_y^3}\left\{\left[w_{1'2} + \frac{1}{2}B\,(w_2 - w_{3'2})\right] - \left[w_{1'2'} - \frac{1}{2}B\,(w_{2'} - w_{3'2'})\right]\right.$$

$$-2\left[w_2 + \frac{1}{2}B\,(w_{12} - w_{1'2})\right] + 2\left[w_{2'} - \frac{1}{2}B(w_{12'} - w_{1'2'})\right]$$

$$\left. + \left[w_{12} + \frac{1}{2}B\,(w_{32} - w_2)\right] - \left[w_{12'} - \frac{1}{2}B(w_{32'} - w_{2'})\right]\right\}. \qquad (26)$$

Substituting Eqs. (26) in Eq. (24b), and expressing the other derivative, $\left(\frac{\partial^4 w}{\partial x^4}\right)_o$, by differences in the usual way, one obtains the condition at point O on the edge beam:

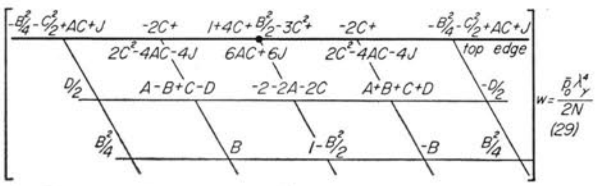
$$\begin{aligned} 0 = {} & J\,(w_{3'} - 4w_{1'} + 6w_o - 4w_1 + w_3) \\ & - \frac{1}{2}\lambda_y^2\left\{\left[U_2 + \frac{1}{2}B\,(U_{12} - U_{1'2})\right] - \left[U_{2'} - \frac{1}{2}B\,(U_{12'} - U_{1'2'})\right]\right\} \\ & - \frac{1}{2}C\left\{\left[w_{1'2} + \frac{1}{2}B\,(w_2 - w_{3'2})\right] - \left[w_{1'2'} - \frac{1}{2}B\,(w_{2'} - w_{3'2'})\right]\right. \\ & - 2\left[w_2 + \frac{1}{2}B\,(w_{12} - w_{1'2})\right] + 2\left[w_{2'} - \frac{1}{2}B\,(w_{12'} - w_{1'2'})\right] \\ & \left. + \left[w_{12} + \frac{1}{2}B\,(w_{32} - w_2)\right] - \left[w_{12'} - \frac{1}{2}B\,(w_{32'} - w_{2'})\right]\right\}. \end{aligned} \qquad (27)$$

Equations (8) and (27) may now be combined so as to eliminate U_2, U_{12}, and $U_{1'2}$. The result is

$$\begin{aligned} \frac{\overline{p}_o\lambda_y^4}{2N} = {} & J\,(w_{3'} - 4w_{1'} + 6w_o - 4w_1 + w_3) \\ & + \lambda_y^2\left[U_{2'} - \frac{1}{2}B\,(U_{12'} - U_{1'2'})\right] \\ & + \frac{1}{2}A\lambda_y^2\,(U_1 + U_{1'}) \\ & - \lambda_y^2\,(1 + A)\,U_o \\ & - \frac{1}{2}C\left\{\left[w_{1'2} + \frac{1}{2}B\,(w_2 - w_{3'2})\right] - \left[w_{1'2'} - \frac{1}{2}B\,(w_{2'} - w_{3'2'})\right]\right. \end{aligned} \qquad (28)$$

$$
\left.\begin{aligned}
&-2\left[w_2+\frac{1}{2}B\,(w_{12}-w_{1'2})\right]\\
&+2\left[w_{2'}-\frac{1}{2}B\,(w_{12'}-w_{1'2'})\right]\\
&+\left[w_{12}+\frac{1}{2}B\,(w_{32}-w_2)\right]\\
&-\left[w_{12'}-\frac{1}{2}B\,(w_{32'}-w_{2'})\right]\Bigg\}.
\end{aligned}\right\} \quad (28)
$$

In this equation, the U's at the interior points, namely: $U_{2'}$, $U_{12'}$, and $U_{1'2'}$, may be replaced by their expansions similar to that given in Eq. (9). The U's at the edge points, namely: $U_{1'}$, U_o, and U_1, are given by Eqs. (23). The deflections of the fictitious points, $w_{3'2}$, $w_{1'2}$, w_2, w_{12}, and w_{32}, which appear in the last bracketed term, may be eliminated by using the three equations found by equating the right sides of Eqs. (23) to the corresponding expansions of $U_{1'}$, U_o, and U_1 similar to Eq. (9). After these operations are performed on Eq. (28), the final result is given by

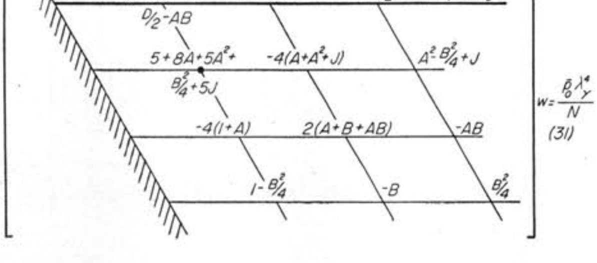


(e) *Interior Point near Corner*

When point O is an interior point on a beam and near a sharp corner of the slab, as shown in Fig. 7, the boundary conditions at the simple support and along the top edge beam are expressed by Eqs. (14b) and (20) as

$$
\left.\begin{aligned}
&w_{1'2} = w_{1'} = w_{1'2'} = w_{1'4'} = 0;\\
&U_{1'2} = U_{1'} = U_{1'2'} = 0;\\
&U_2 = \frac{C}{\lambda_y^2}\,(w_{1'2} - 2w_2 + w_{12});\\
&U_{12} = \frac{C}{\lambda_y^2}\,(w_2 - 2w_{12} + w_{32}).
\end{aligned}\right\} \quad (30)
$$

Then Eq. (8), with U_o, U_1, $U_{2'}$, and $U_{12'}$ given by equations similar to Eq. (9), and modified by Eqs. (30), and the expression (15) which accounts for the load carried by the beam, becomes



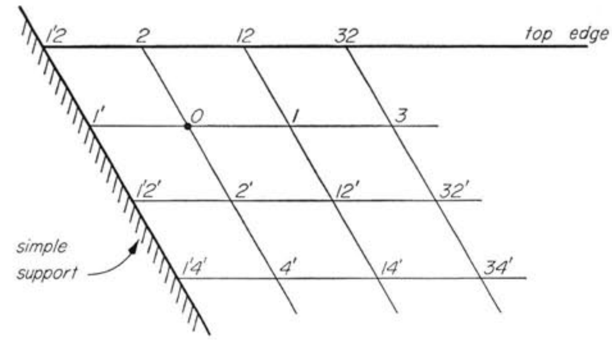
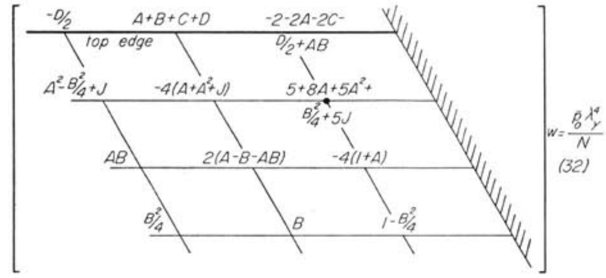


Fig. 7. Interior Point Near Sharp Corner

In an analogous manner, the deflection equation for a point O on a beam near a blunt corner may be found as



In Eqs. (31) and (32) the J-terms are to be omitted if point O does not lie on a beam.

(f) *Edge Point near Corner*

It remains to determine the equations which apply when point O is on the edge beam and near a corner. Consider such a point near a sharp corner as shown in Fig. 8. The derivation proceeds exactly as for a general edge point up to and including the development of Eq. (28).

The boundary conditions at the simple support and the top edge are stated by Eqs. (14b) and the second and third of Eqs. (23). They are

$$w_{1'} = w_{1'2'} = w_{1'4'} = 0; \qquad (33a)$$

$$U_{1'} = U_{1'2'} = 0; \qquad (33b)$$

$$U_o = \frac{C}{\lambda_y^2}\,(w_{1'} - 2w_o + w_1); \qquad (33c)$$

$$U_1 = \frac{C}{\lambda_y^2}\,(w_o - 2w_1 + w_3). \qquad (33d)$$

Two additional conditions will be specified at the corner point, 1′, namely: the slope of the deflected surface in the direction of the simply-supported edge shall be zero, and the moment M_y shall vanish. That is,

$$\left(\frac{\partial w}{\partial v}\right)_{1'} = 0, \qquad (34a)$$

$$(M_y)_{1'} = 0. \qquad (34b)$$

Eq. (34a) may be written as

$$\frac{w_{1'2} - w_{1'2'}}{2\lambda_v} = 0,$$

which, together with Eq. (33a), gives:

$$w_{1'2} = 0. \qquad (35a)$$

The condition specified by Eq. (34b) gives an expression in difference form similar to Eqs. (20), namely:

$$U_{1'} = \frac{C}{\lambda_y^2}(w_{3'} - 2w_{1'} + w_o),$$

which, modified by Eqs. (33a) and (33b), gives

$$w_{3'} = -w_o. \qquad (35b)$$

The following operations are then performed on Eq. (28):

1. Replacing $U_{2'}$ and $U_{12'}$ by their expansions similar to Eq. (9).
2. Eliminating the terms in the brackets, $[w_2 + \frac{1}{2}B\,(w_{12} - w_{1'2})]$, and $[w_{12} + \frac{1}{2}B\,(w_{32} - w_2)]$, in a similar manner as was done in the derivation of the equation for a general edge point, by using the two equations found by equating the right sides of Eqs. (33c) and (33d) to the corresponding expansions of U_o and U_1 similar to Eq. (9).
3. Eliminating the deflections w_2, $w_{3'2}$, $w_{2'}$, and $w_{3'2'}$ in the brackets $[w_{1'2} + \frac{1}{2}B\,(w_2 - w_{3'2})]$, and $[w_{1'2'} - \frac{1}{2}B\,(w_{2'} - w_{3'2'})]$, by using the relation expressing the slope at point 1′ in the x direction of the deflected surface. This slope is taken as the average of the slopes in the x direction at points 1′2 and 1′2′, that is,

$$\left(\frac{\partial w}{\partial x}\right)_{1'} = \frac{1}{2}\left[\left(\frac{\partial w}{\partial x}\right)_{1'2} + \left(\frac{\partial w}{\partial x}\right)_{1'2'}\right].$$

Expressed in terms of differences, this relation is

$$\frac{w_o - w_{3'}}{2\lambda_x} = \frac{1}{2}\left[\frac{(w_2 - w_{3'2}) + (w_{2'} - w_{3'2'})}{2\lambda_x}\right],$$

which, by means of Eq. (35b), gives

$$4w_o = (w_2 - w_{3'2}) + (w_{2'} - w_{3'2'}). \qquad (36)$$

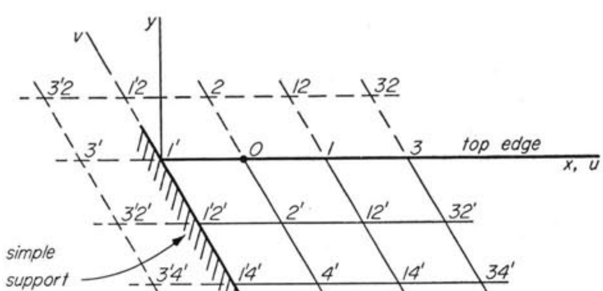
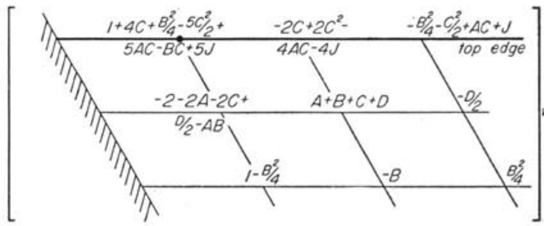
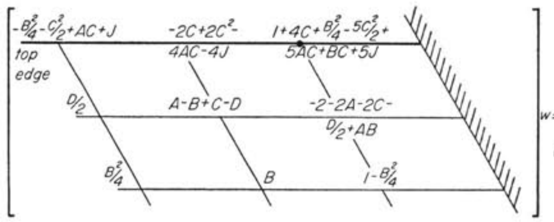


Fig. 8. Edge Point Near Sharp Corner

The final result of all the above operations on Eq. (28) together with the use of Eqs. (33), (35), and (36) is the equation



When point O is on the top edge beam and adjacent to a blunt corner, one finds, in a similar manner, the equation



In the derivation of Eq. (37), in addition to the conditions expressed by Eqs. (33a) and (33b), two conditions expressed by Eqs. (34) have been specified for the corner point 1′. In order to test if the two additional conditions are suitable and justified, two checks are available, and will now be applied to Eq. (37). First, as a check on the coefficients of all the deflections except w_o in Eq. (37), one may use the reciprocal relationship that the coefficient of any w_1 in an equation expressing a unit load at point 2 is equal to that of w_2 in an equation expressing a unit load at point 1. Thus, the coefficient of w_1 in Eq. (37) must be the same as that of $w_{1'}$ in Eq. (29); that of $w_{2'}$ in Eq. (37) the same as that of w_2 in Eq. (31); that of $w_{12'}$ in Eq. (37) the same as that of $w_{1'2}$ in Eq. (21), and so on.

A further observation regarding the sum of all the coefficients affords a check on the coefficient of w_o. The sum of all the coefficients in Eq. (37) may be obtained from Eq. (29) by discarding the coefficients of points on the simply supported edge, ($w_{1'}$, $w_{1'2'}$, and $w_{1'4'}$); by changing the sign of the coefficients of the points outside of the slab boundaries, ($w_{3'}$, $w_{3'2'}$, and $w_{3'4'}$); and then adding algebraically. The coefficients of deflections on the simply supported edge vanish, and those of deflections exterior to the slab must be "reflected" back across the simply supported edge with opposite sign.

Eqs. (37) and (38) have been checked by the procedure discussed above, and the results show reasonably good evidence that suitable corner conditions have been used. For a more complete discussion of the singularity at a corner, reference should be made to Jensen's work.[5] It is to be noted that the use of an equation such as Eq. (9) in the expansion of U at the corner would lead to a final result which violates the reciprocal relationship between the coefficients.

A sufficient number of equations have now been derived to permit the determination of deflections at all points of the network for symmetrical or anti-symmetrical loading. Typical points have been taken from the upper half of the bridge shown in Fig. 1. For unsymmetrical loading, the equations pertaining to the lower half of the bridge may be written readily by analogy with those given here. It may be more convenient, as explained in Art. 11, to treat the unsymmetrical loading as the resultant of its symmetrical and anti-symmetrical component loadings, for each of which equations need be written only for the points in one half of the bridge.

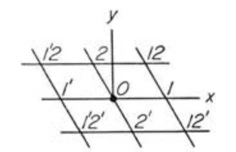
8. Difference Expressions for Moments

After the deflections have been determined for a particular bridge under a given loading from a set of simultaneous equations, it remains to determine the bending and twisting moments. As before, the derivatives required in Eqs. (2) are to be expressed in terms of finite differences. For the network defined in Fig. 2, the derivatives of the deflections for a point O are, from Eqs. (o), (p) and (r),

$$\left.\begin{aligned}
\left(\frac{\partial^2 w}{\partial x^2}\right)_o &= \frac{K^2}{\lambda_y^2}\,(w_{1'}-2w_o+w_1)\\
\left(\frac{\partial^2 w}{\partial y^2}\right)_o &= \frac{1}{\lambda_y^2}\Big[B^2\,(w_{1'}+w_1)-2\,(1+B^2)\,w_o\\
&\quad +\frac{1}{2}\,B\,(-w_{1'2}+w_{12}+w_{1'2'}-w_{12'})\\
&\quad + w_{2'} + w_2\Big]\\
\left(\frac{\partial^2 w}{\partial x \partial y}\right)_o &= \frac{K}{\lambda_y^2}\Big[B\,(w_{1'}+w_1)-2Bw_o\\
&\quad +\frac{1}{4}\,(-w_{1'2}+w_{12}+w_{1'2'}-w_{12'})\Big]
\end{aligned}\right\}\quad (39)$$

(a) *Moments in Slab*

For an interior point O, the moments in the slab are obtained by substituting Eqs. (39) into Eqs. (2). The results are



$$\begin{aligned}
(M_x)_o = -\frac{N}{\lambda_y^2}\Big[&(K^2+\mu B^2)\\
&(w_{1'}-2w_o+w_1)\\
&+\mu\,(w_2-2w_o+w_{2'})\\
&+\frac{1}{2}\,\mu B\,(-w_{1'2}+w_{12}\\
&+w_{1'2'}-w_{12'})\Big] \qquad (40a)
\end{aligned}$$

$$\begin{aligned}
(M_y)_o = -\frac{N}{\lambda_y^2}\Big[&(B^2+\mu K^2)\,(w_{1'}-2w_o+w_1)\\
&+(w_2-2w_o+w_{2'})\\
&+\frac{1}{2}\,B(-w_{1'2}+w_{12}+w_{1'2'}-w_{12'})\Big] \qquad (40b)
\end{aligned}$$

$$\begin{aligned}
(M_{xy})_o = -\frac{NC}{K\lambda_y^2}\Big[&B\,(w_{1'}-2w_o+w_1)\\
&+\frac{1}{4}\,(-w_{1'2}+w_{12}+w_{1'2'}-w_{12'})\Big]. \qquad (40c)
\end{aligned}$$

When point O is on a simply supported edge, such as shown in Fig. 9, Eqs. (40) become modified by the conditions

$$w_2 = w_o = w_{2'} = 0; \qquad (41a)$$

$$U_o = 0; \qquad (41b)$$

$$\left(\frac{\partial w}{\partial v}\right)_o = 0. \qquad (41c)$$

As seen by reference to Eq. (9), the second of these conditions, Eq. (41b), becomes

$$\begin{aligned}
-\frac{1}{2}\,B\,(w_{1'2} - w_{1'2'}) &+ \frac{1}{2}\,B\,(w_{12} - w_{12'})\\
&+ A\,(w_{1'} + w_1) = 0. \qquad (41d)
\end{aligned}$$

The third condition, Eq. (41c), may be written as

$$\begin{aligned}
\left(\frac{\partial w}{\partial v}\right)_o &= \frac{1}{2}\left[\left(\frac{\partial w}{\partial v}\right)_{1'} + \left(\frac{\partial w}{\partial v}\right)_1\right]\\
&= \frac{1}{2}\left(\frac{w_{1'2} - w_{1'2'}}{2\lambda_v} + \frac{w_{12} - w_{12'}}{2\lambda_v}\right) = 0,
\end{aligned}$$

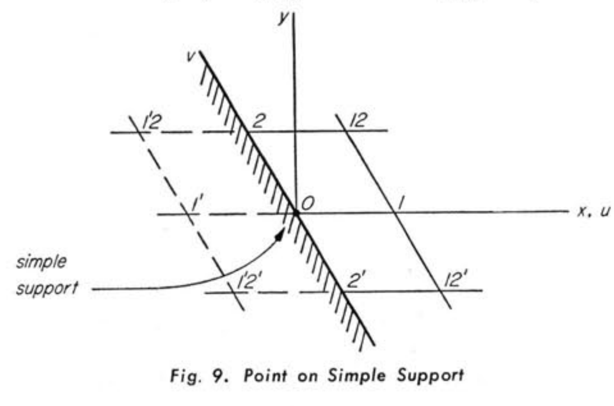


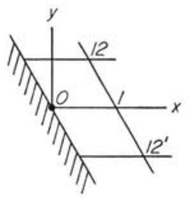
Fig. 9. Point on Simple Support

or, $$(w_{1'2} - w_{1'2'}) + (w_{12} - w_{12'}) = 0. \qquad (41e)$$

From Eqs. (41d) and (41e) one obtains

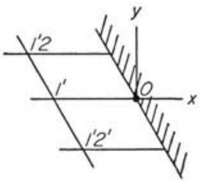
$$w_{1'} = -\frac{B}{A}(w_{12} - w_{12'}) - w_1. \qquad (41f)$$

Substitution of Eqs. (41) into Eqs. (40) gives the moments at a point O on a simply supported edge as



$$\left.\begin{aligned}(M_x)_o &= -(M_y)_o \\ &= -\frac{N}{\lambda_y^2}\frac{C\sin 2\varphi}{2K} \\ &\quad (w_{12'} - w_{12}); \\ (M_{xy})_o &= \frac{N}{\lambda_y^2}\frac{C\cos 2\varphi}{2K} \\ &\quad (w_{12'} - w_{12}).\end{aligned}\right\} \quad (42)$$

Similarly, at a point O on the other simply supported edge, the moments are given by



$$\left.\begin{aligned}(M_x)_o &= -(M_y)_o \\ &= -\frac{N}{\lambda_y^2}\frac{C\sin 2\varphi}{2K} \\ &\quad (w_{1'2} - w_{1'2'}); \\ (M_{xy})_o &= \frac{N}{\lambda_y^2}\frac{C\cos 2\varphi}{2K} \\ &\quad (w_{1'2} - w_{1'2'}).\end{aligned}\right\} \quad (43)$$

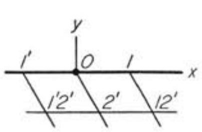
For a point O along the top edge beam (Fig. 6(a)),

$$(M_y)_o = 0, \qquad (44a)$$

and consequently,

$$\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right)_o = 0.$$

Therefore, the moment in the x direction is given by



$$\left.\begin{aligned}(M_x)_o &= -N(1-\mu^2) \\ &\quad \left(\frac{\partial^2 w}{\partial x^2}\right)_o \\ &= -\frac{NK^2}{\lambda_y^2}(1-\mu^2) \\ &\quad (w_{1'} - 2w_o + w_1)\end{aligned}\right\} \quad (44b)$$

The twisting moment at O is given by Eq. (40c), from which however, the deflections of the fictitious points $w_{1'2}$ and w_{12} are to be eliminated by using the approximate relation:

$$\left(\frac{\partial w}{\partial x}\right)_o = \frac{1}{2}\left[\left(\frac{\partial w}{\partial x}\right)_2 + \left(\frac{\partial w}{\partial x}\right)_{2'}\right].$$

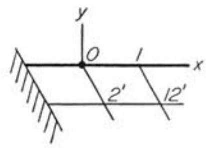
Expressed in terms of differences, this relation is

$$\frac{w_1 - w_{1'}}{2\lambda_x} = \frac{1}{2}\left(\frac{w_{12} - w_{1'2}}{2\lambda_x} + \frac{w_{12'} - w_{1'2'}}{2\lambda_x}\right),$$

which, when substituted into Eq. (40c), gives

$$\left.\begin{aligned}(M_{xy})_o = -\frac{NC}{K\lambda_y^2}\Big[&B(w_{1'} - 2w_o + w_1) \\ &+ \frac{1}{2}(-w_{1'} + w_1 + w_{1'2'} - w_{12'})\Big]\end{aligned}\right\} \quad (44c)$$

When point O is on the top edge beam and near a sharp corner (Fig. 8), Eqs. (44) become further modified to read as follows:

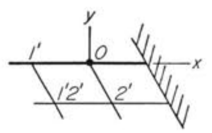


$$(M_x)_o = -\frac{NK^2}{\lambda_y^2}(1 - \mu^2)$$
$$(-2w_o + w_1); \qquad (45a)$$
$$(M_y)_o = 0; \qquad (45b)$$

$$(M_{xy})_o = -\frac{NC}{K\lambda_y^2}\left[B(-2w_o + w_1) + \frac{1}{2}(w_1 - w_{12'})\right]. \qquad (45c)$$

Similarly, when point O is on the top edge beam and near a blunt corner, the moments are given by

$$(M_x)_o = -\frac{NK^2}{\lambda_y^2}(1 - \mu^2)$$
$$(-2w_o + w_{1'}); \qquad (46a)$$



$$(M_y)_o = 0; \qquad (46b)$$
$$(M_{xy})_o = -\frac{NC}{K\lambda_y^2}\left[B(w_{1'} - 2w_o) + \frac{1}{2}(-w_{1'} + w_{1'2'})\right]. (46c)$$

(b) *Moments in Beams*

To determine the bending moment in a beam at an interior point O, the derivative in Eq. (5b) is expressed in terms of differences as

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_o = \frac{K^2}{\lambda_y^2}(w_{1'} - 2w_o + w_1).$$

Hence, the moment at an interior point of a beam is given by

$$(M_b)_o = -\frac{K^2}{\lambda_y^2}E_b I_b(w_{1'} - 2w_o + w_1). \qquad (47$$

When point O is near a simple support, Eq. (47) is modified by the condition that the deflection of a point on the support is zero. When point O is on the simple support, $(M_b)_o$ of course vanishes.

III. APPLICATION OF EQUATIONS

9. Description of Bridge Analyzed

Analysis was made by the difference equations derived in the preceding chapter to determine the numerical values of influence coefficients for effects at various points in the slab and beams due to unit loads for bridges having angles of skew, φ, of 30, 45, and 60 degrees; and ratios of spacing of beams, b, to span of bridge, a, of 0.1 and 0.2. These ratios of b/a correspond to ratios of width of roadway to length of span of 0.4 and 0.8 respectively.

The relative stiffness of the beams, compared to that of the slab, is determined by the dimensionless coefficient, H, which is defined by the relation

$$H = \frac{E_b I_b}{aN}, \tag{48a}$$

and is, therefore, related to the dimensionless coefficient, J, appearing in the difference equations, according to the equation

$$J = \frac{aK^4}{\lambda_y} \cdot H. \tag{48b}$$

The values of H considered in the analysis are 2, 5, and 10 for $b/a = 0.1$; and 1, 2, and 5 for $b/a = 0.2$.

For the sake of convenience in the calculations and because of the uncertainty regarding the behavior of reinforcement in concrete slabs, the value of Poisson's ratio, μ, was taken as zero throughout the analysis.

Points on the bridges are located by the coordinate system shown in Fig. 1. The beams or points on the slab directly over beams are denoted by the letters A, B, C, D, and E, as shown. The longitudinal center line of a panel of the slab is denoted by the two letters corresponding to the beams supporting the panel. Thus, AB denotes the longitudinal center line of the outer panel bounded by the beams A and B. Points on a particular beam or center line of a slab panel are denoted by the proportion of the span from the left end of the bridge. It was convenient in the calculations to divide the span into eight equal parts.

Eighteen structures were considered. For each structure, influence coefficients for loads at the eighth points of the span were determined for the following effects:

- Moment in beams A, B, and C at mid-span,
- Moment in beam C at the quarter and 3/8 points,
- Transverse moment in the slab on center lines of panels AB and BC, at mid-span,
- Transverse moment in the slab over beams B and C at mid-span,
- Deflection of beam C at mid-span.

Numerical values of influence coefficients for these effects due to unit loads are given in Tables A1 through A42 of Appendix A. Table 1 gives an outline of the tables. It will be noted that influence values for twisting moments and longitudinal moments are not given. It is felt that the effect of the twisting moments is small, and that for the relative proportions of the bridges considered here, the longitudinal moments are probably not significant enough to govern the design of the slab. For shorter spans, the longitudinal moments may become, theoretically, as important as the transverse moments. However, both tests and judgment indicate that the bridge cannot be seriously harmed even if the required theoretical amount of longitudinal steel is not provided (See Ref. 11, p. 1009).

10. Method of Determining Influence Surfaces

In determining the ordinates to an influence surface for an effect at a particular point due to

Table 1
Outline of Tables of Influence Coefficients

Description	Ratio of Spacing of Beams to Span of Bridge, b/a	Relative Stiffness of Beams, H	Angle of Skew, $\phi°$	Table Numbers, Inclusive (Appendix A)
Moment in beams A, B, and C at mid-span	0.1	2, 5, 10	30	1–3
			45	4–6
Moment in beam C at quarter and 3/8 points of span			60	7–9
	0.2	1, 2, 5	30	10–12
			45	13–15
			60	16–18
Transverse moment in slab on lines AB, B, BC, and C at mid-span	0.1	2, 5, 10	30	19–21
			45	22–24
			60	25–27
	0.2	1, 2, 5	30	28–30
			45	31–33
			60	34–36
Deflection of beam C at mid-span	0.1	2, 5, 10	30	37
			45	38
			60	39
	0.2	1, 2, 5	30	40
			45	41
			60	42

a unit load at any point on the slab, it is convenient to make use of the fact that this surface may itself be regarded as the deflection of the slab, to some scale, due to an imposed system of fixed loads. Where an effect at any point in a slab is a linear function of the deflection, as in the case of difference equations, this system of fixed loads may be very simply determined by the method described in Ref. (12). This method may be summarized as follows:

Where an effect Q_a at any point a in a slab is a linear function of the deflections, as in the equation

$$Q_a = Aw_a + Bw_b + Cw_c + \ldots + Mw_m,$$

in which $A, B, C, \ldots M$ are constants, and $w_a, w_b, w_c, \ldots w_m$, are the deflections at the points $a, b, c, \ldots m$, an influence surface for Q_a may be obtained as the deflection of the slab due to a system of loads, $A, B, C, \ldots M$, applied at the points $a, b, c, \ldots m$ respectively.

Assume, for example, that the influence ordinates for the moment in beam B at the quarter-point (Fig. 1) are required. Since the moment at this point is, from Eq. (47), given by

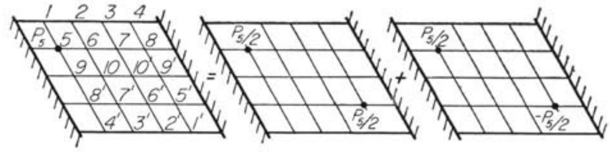
$$(M_b)_{B-1/4} = -\frac{K^2 E_b I_b}{\lambda_y^2}(w_{B-1/8} - 2w_{B-1/4} + w_{B-3/8}),$$

the influence surface for $(M_b)_{B-1/4}$ may be obtained as the deflection of the slab due to a fixed system of loads, $-K^2E_bI_b/\lambda_y^2$, $2K^2E_bI_b/\lambda_y^2$, and $-K^2E_bI_b/\lambda_y^2$, applied at the points B–1/8, B–1/4, and B–3/8, respectively.

11. Treatment of Unsymmetrical Loading

When the loading is unsymmetrical, it is expedient, in applying the difference equations, to break it into symmetrical and anti-symmetrical parts which may be added to give the required result. Consider, for instance, a simply supported skew slab with a network of points as shown in Fig. 10(a), and suppose we wish to obtain the deflected surface of this slab under a load P_5 applied at point 5. A direct application of the difference equations to this system would involve the solution of a set of 20 simultaneous equations with the deflections at the numbered points as unknowns. It is advantageous, however, to resolve the system into its symmetrical and anti-symmetrical components.

As shown in Fig. 10(b), the symmetrical system consists of two loads, each of magnitude $P_5/2$, one applied at point 5, and the other at point 5′. A set of ten simultaneous equations is obtained by ap-



(a) Unsymmetrical *(b) Symmetrical* *(c) Anti-symmetrical*

Fig. 10. Treatment of Unsymmetrical Loading

plying the difference equations to this system, noting that the points identified by the primed numbers have deflections equal to those of the corresponding points having unprimed numbers. The anti-symmetrical system, shown in Fig. 10(c), consists of a load of $P_5/2$ applied at point 5, and an equal load, but of opposite sign, applied at point 5′. Another set of ten equations may be written, noting that the points identified by the primed numbers have deflections equal in magnitude but opposite in sign to those of the corresponding points having unprimed numbers. The sum of the two deflections at any point gives the required result.

In a similar manner, the analysis of the skew I-beam bridge for an unsymmetrical loading with a network of points as shown in Fig. 1 reduces to the solution of two sets of simultaneous equations: the first set corresponding to the symmetrical component system has 32 unknowns, and the second set corresponding to the anti-symmetrical component system has 31 unknowns (noting that in this case the deflection at the mid-point of the beam C must vanish). The two deflections at any point when added give the required ordinate of the deflected surface of the slab due to the given loading.

The advantage of this procedure lies in that otherwise one would have to deal with a set of 63 equations instead of two sets of equations, one having 32 and the other 31 unknowns. The set of 63 equations would require much more time and labor to solve, and call for much greater precision in the calculations in order to obtain a degree of accuracy in the final results comparable to that of the solutions to the set of 32 or 31 equations.

12. Solution of Simultaneous Equations

Using the method of determining influence surfaces by systems of fixed loads, and the technique of treating unsymmetrical loadings described in Arts. 10 and 11, the determination of the influence coefficients for the various effects enumerated previously involves the solution of 17 sets of simultaneous equations for each structure, or a total of 306 sets of equations for all the 18 structures considered.

These equations were solved by means of the ILLIAC electronic digital computer. The details of the coding of the problems and the preparation of the problem tapes suitable for automatic machine solution would serve no useful purpose if given here. However, a brief description of the manner in which the problems are solved, and a word about the speed of the machine in the solution of simultaneous equations and the accuracy of the results obtained may be of some interest.

In the ILLIAC Computer Laboratory a library code is available for use in the solution of simultaneous linear algebraic equations (the method of solution being that of elimination) which contain numerical values of the coefficients of the unknowns and the constant terms. As shown by the difference equations derived in Chapter II, the coefficients of the unknown deflections of a skew I-beam bridge due to a given loading are, in general, functions of the quantities A, B, C, D, and J, which in turn are functions of the independent parameters φ, b/a, and H for a given network of points. A direct application of the library code to the present study would then require that the numerical values of the coefficients be computed in advance on a desk calculator from the quantities A, B, C, D, and J for a given set of φ, b/a, and H values for each set of simultaneous equations to be solved.

As the investigation involves some 300 sets of simultaneous equations, this preliminary work in the evaluation of the coefficients on a desk calculator and the preparation of tapes on which numerical values of about 1000 coefficients were to be punched for each set of 31 or 32 equations, would be most laborious and time-consuming. A special automatic program was therefore prepared for this investigation to include, in addition to the library code, an auxiliary code which automatically calculates on the ILLIAC the numerical values of the coefficients for given values of A, B, C, etc.

The "input" of the special code consists of three tapes: the master tape, parameter tape, and coefficient tape. The coefficient tape represents in code the equations to be solved with the coefficients and the load terms expressed in their original literal form, i.e., as functions of A, B, C, etc. The parameter tape bears the numerical values of A, B, C, and J as well as the load terms, previously computed on a desk calculator for a given structure and loading (the value of D need not be supplied since, D being equal to $[A+C]B$, the code was devised to calculate D when the values of A, B, and C are given). The master tape bears instructions to the ILLIAC in code language and directs it first to calculate the coefficients from the values of A, B, C, and J supplied on the parameter tape, then to substitute these results together with the load terms into the equations indicated on the coefficient tape, and lastly to solve the equations with the use of the library code and to punch the results on an "output" tape.

The master and the coefficient tapes, once made up, may be used for any set of parameter values. Thus, to obtain the input of a different problem, only the parameter tape needs to be changed. As the parameter tape is quite short, bearing only four quantities A, B, C, and J which define a particular bridge and about 30 load terms for a given loading, most of which are zero for the determination of influence ordinates for a particular effect, the preparation of all the necessary parameter tapes was a relatively simple matter.

As an illustration, consider a bridge with $\varphi = 45°$, $b/a = 0.1$, and $H = 2$. With a network of points as shown in Fig. 1, $\lambda_y = b/2$, $\lambda_x = a/8$, and $K = \lambda_y/\lambda_x = 4(b/a) = 0.4$. The quantities A, B, C, and J for the structure are then:

$$A = (K/\cos\varphi)^2 = 0.32$$
$$B = K\tan\varphi = 0.4$$
$$C = (1-\mu^2)K^2$$
$$= K^2 = 0.16 \qquad (\text{for } \mu = 0)$$

and

$$J = aK^4H/\lambda_y = 2K^4H/(b/a) = 1.024$$

For the determination of influence ordinates for the beam moment at point B–1/4 (Fig. 1), the loads to be applied in order that the deflections of the structure give directly the required influence ordinates, as explained in Art. 10, are:

$$P_{B-1/8} = -K^2(E_bI_b/\lambda_y^2)$$
$$P_{B-1/4} = 2K^2(E_bI_b/\lambda_y^2)$$

and

$$P_{B-3/8} = -K^2(E_bI_b/\lambda_y^2)$$

The three loaded points B–1/8, B–1/4, and B–3/8 being interior points, the corresponding load terms given by $(\overline{p}\lambda_y^4/N)$, in which $\overline{p}$ is, by Eq. (10a), (KP/λ_y^2), are then:

$$(\overline{p}\lambda_y^4/N)_{B-1/8} = (K/\lambda_y^2)\,(-K^2E_bI_b/\lambda_y^2)\,(\lambda_y^4/N)$$
$$= -K^3(E_bI_b/N) = -K^3Ha$$
$$= -0.128a$$
$$(\overline{p}\lambda_y^4/N)_{B-1/4} = 0.256a$$
$$(\overline{p}\lambda_y^4/N)_{B-3/8} = -0.128a$$

The load terms for all the other points are zero.

Only the numerical values of the load terms computed above need be used on the parameter tape, with the understanding that the roots to the equations given by the ILLIAC are each to be multiplied by a.

The numerical values of A, B, C, J and the load terms, such as computed above, constitute all the necessary data to be supplied to the ILLIAC via the parameter tape for the solution of a particular problem. It should be noted that the ILLIAC does not handle quantities whose absolute values exceed 1. It is generally necessary, therefore, before the numerical data may be punched on the parameter tape, to "scale" down quantities sufficiently to ensure that at no stage of the computation is this requirement violated. This scaling may be simply done by moving the decimal point in the numerical data towards the left one or more places as needed. A reverse scaling of course should be applied to convert the machine solutions to the actual solutions of the original problem.

To solve a set of equations, the three input tapes are fed in turn into the machine via an electronic "read-in" device; this takes about 3 minutes. Almost immediately, the machine starts to issue the output tape which bears the required answers, and completes it in a matter of seconds. Hence, it takes the machine about 3½ minutes to solve a set of 32 or 31 equations, or about one hour to solve all the 17 sets of equations for each of the 18 structures considered. In comparison, it would take a skilled computer, provided with an ordinary desk calculator, probably at least 50 hours to solve a set of 32 equations, or about 850 hours to solve the 17 sets of equations for each structure. Undoubtedly, without the benefit of this tremendous saving in time and labor obtained with the ILLIAC, it would not have been possible to make available now the data contained here.

Regarding the accuracy of the ILLIAC results, it suffices to state that the answers, being given accurate to the tenth decimal figure, have a degree of accuracy more than adequate for all practical purposes. In several problems, the machine solutions were checked for accuracy by substituting them back into the original equations, and the residuals were found to be, in the worst case, only of the order of 3×10^{-9}.

IV. COMPARISON OF EXACT AND DIFFERENCE SOLUTIONS FOR RIGHT BRIDGES

13. Remarks

The results obtained by the method of finite differences are approximate. The degree of approximation depends upon, among other things, the fineness of the network of points used, the nature of the effect for which the analysis is made, and the order of the difference expressions substituted for the differential derivatives in the derivation of the equations. Where an exact solution is available, the degree of approximation may be determined by a direct comparison of the results obtained.

In order to obtain an idea of the accuracy of the influence values reported, it was necessary, in the absence of any exact analysis of skew I-beam bridges, to make comparisons of the results for right I-beam bridges. In the following discussions the bridges considered are identical to the corresponding skew bridges, and the networks of points used in the difference solutions are the same except that the angle of skew is taken as zero. The exact values (exact in the sense that the solution to the differential equation is exact, and to the extent that the assumptions are justified) are taken from Ref. 2. It is assumed that these comparisons of exact and difference solutions for right bridges would indicate the degree of approximation that might be expected in the results obtained by the difference solution for corresponding skew bridges.

14. Moments in Beams

Computations of influence coefficients for moments in beams were made for two right I-beam bridges: one with $b/a=0.1$ and $H=5$, and the other with $b/a=0.2$ and $H=2$. The numerical values of the influence coefficients for the moments in beams A, B, and C at mid-span, are given in Table 2, which also contains the corresponding exact values obtained by a distribution procedure, and listed in Ref. 2. The coefficients given are such that the actual moments for concentrated loads are obtained by multiplying the tabulated values by the quantity Pa.

Comparisons of the values given in Table 2 shows that the results obtained by the use of difference equations do not differ materially from the exact values. At most points, there is perfect agreement to the unit in the last place in the figures given; at other points the discrepancies range from 0.001 to 0.003; and at one or two points the discrepancies are 0.004 for the bridge with $b/a=0.1$ and $H=5$, and 0.005 for the bridge with $b/a=0.2$ and $H=2$.

Similar comparisons of influence values for moments in the beams at the quarter points for the two bridges* also showed equally good agreement between the difference and the exact solutions.

It may be concluded, therefore, that the difference method and the network chosen are satisfactory for determining the moments in the beams. One may be reasonably sure that the influence values given for moments in the beams of skew I-beam bridges are also accurate enough for design purposes.

15. Transverse Moments in Slab at Center of Panel

(a) *Comparison of Exact and Difference Solutions*

Difference equations were applied to determine the influence coefficients for transverse moments in the slab on lines AB and BC at mid-span of two right bridges, one with $b/a=0.1$ and $H=5$, and the other with $b/a=0.2$ and $H=2$. The numerical values of these coefficients are given in Table 3, which also contain the corresponding exact values as reported in Ref. 2. In the set of approximate values in Table 3, two values are given at each of certain points: the one that appears in parentheses is the value obtained directly from the difference equations, while the other, written above the parentheses, is the "improved" value, obtained in a manner to be described later.

A comparison of the values given in Table 3 shows that while the approximate values at most points agree with the corresponding exact values within 0.002, appreciable discrepancies are found

* Results of these comparisons are not given here.

Table 2
Comparison of Exact and Difference Solutions for Influence Coefficients for Moment in Beams of Right Bridge

Relative Proportions of Bridge $b/a = 0.1$
Relative Stiffness of Beams $H = 5$

Moment in Beam	Transverse Location of Load	Exact Solution 1/8	Exact Solution 2/8	Exact Solution 3/8	Exact Solution Center	Difference Solution 1/8	Difference Solution 2/8	Difference Solution 3/8	Difference Solution Center
A at center	A	0.035	0.072	0.117	0.172	0.034	0.072	0.116	0.170
	A B	0.030	0.059	0.087	0.111	0.029	0.058	0.087	0.115
	B	0.023	0.044	0.061	0.067	0.023	0.044	0.061	0.068
	B C	0.016	0.030	0.038	0.040	0.016	0.030	0.039	0.040
	C	0.009	0.017	0.021	0.022	0.009	0.017	0.021	0.022
	C D	0.004	0.007	0.009	0.009	0.004	0.007	0.009	0.009
	D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	D E	−0.003	−0.005	−0.007	−0.008	−0.003	−0.005	−0.007	−0.007
	E	−0.006	−0.010	−0.013	−0.014	−0.005	−0.010	−0.013	−0.014
B at center	A	0.023	0.044	0.061	0.067	0.023	0.044	0.061	0.068
	A B	0.018	0.037	0.062	0.091	0.018	0.038	0.062	0.093
	B	0.015	0.033	0.059	0.107	0.015	0.032	0.059	0.105
	B C	0.015	0.032	0.054	0.080	0.015	0.032	0.054	0.082
	C	0.015	0.031	0.044	0.050	0.015	0.030	0.044	0.051
	C D	0.013	0.024	0.032	0.033	0.013	0.024	0.032	0.034
	D	0.009	0.016	0.020	0.021	0.009	0.016	0.020	0.021
	D E	0.004	0.008	0.010	0.010	0.004	0.008	0.010	0.010
	E	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C at center	A	0.009	0.017	0.021	0.022	0.009	0.017	0.021	0.022
	A B	0.013	0.025	0.033	0.034	0.013	0.025	0.033	0.035
	B	0.015	0.031	0.044	0.050	0.015	0.030	0.044	0.051
	B C	0.014	0.030	0.052	0.077	0.014	0.030	0.051	0.079
	C	0.013	0.029	0.055	0.101	0.013	0.028	0.053	0.098

Relative Proportions of Bridge $b/a = 0.2$
Relative Stiffness of Beams $H = 2$

Moment in Beam	Transverse Location of Load	Exact Solution 1/8	Exact Solution 2/8	Exact Solution 3/8	Exact Solution Center	Difference Solution 1/8	Difference Solution 2/8	Difference Solution 3/8	Difference Solution Center
A at center	A	0.036	0.075	0.120	0.174	0.036	0.074	0.118	0.172
	A B	0.028	0.055	0.083	0.101	0.027	0.055	0.083	0.106
	B	0.020	0.037	0.050	0.055	0.019	0.037	0.050	0.056
	B C	0.011	0.021	0.026	0.028	0.011	0.021	0.027	0.029
	C	0.006	0.009	0.012	0.013	0.005	0.009	0.012	0.013
	C D	0.002	0.003	0.003	0.004	0.002	0.003	0.004	0.004
	D	−0.001	−0.001	−0.001	−0.001	0.000	0.000	−0.001	−0.001
	D E	−0.002	−0.002	−0.003	−0.003	−0.001	−0.002	−0.003	−0.003
	E	−0.002	−0.003	−0.004	−0.004	−0.002	−0.003	−0.004	−0.004
B at center	A	0.020	0.037	0.050	0.055	0.019	0.037	0.050	0.056
	A B	0.017	0.038	0.062	0.084	0.018	0.038	0.063	0.087
	B	0.016	0.036	0.066	0.112	0.016	0.037	0.066	0.111
	B C	0.017	0.035	0.058	0.078	0.017	0.035	0.058	0.081
	C	0.016	0.031	0.042	0.047	0.016	0.030	0.042	0.047
	C D	0.011	0.021	0.027	0.029	0.011	0.021	0.027	0.029
	D	0.007	0.012	0.015	0.016	0.006	0.012	0.015	0.016
	D E	0.003	0.005	0.006	0.007	0.003	0.005	0.007	0.007
	E	−0.001	−0.001	−0.001	−0.001	0.000	0.000	−0.001	−0.001
C at center	A	0.006	0.009	0.012	0.013	0.005	0.009	0.012	0.013
	A B	0.011	0.021	0.026	0.028	0.011	0.021	0.027	0.028
	B	0.016	0.031	0.042	0.047	0.016	0.030	0.042	0.047
	B C	0.016	0.035	0.056	0.077	0.016	0.035	0.057	0.080
	C	0.015	0.034	0.064	0.109	0.015	0.035	0.063	0.108

in the values at certain points along the center line of the panel considered. Approximate values that differ materially from exact values have been put in parentheses.

The discrepancies may be explained by the following considerations. In the exact solution, a unit point load is considered, except when the load is directly at the point where the influence is desired, in which case the load is considered to be distributed over a small circular area. In the difference solution, however, a concentrated load at any point is, in effect, converted into a load uniformly distributed over an area contained within the elemental mesh of the network used. The degree of approximation obtainable from a difference analysis is directly affected by the mesh size; the smaller the meshes are, the better the results become. For a given network, the accuracy of the difference analysis for an effect at a point caused by a given loading is also dependent on the nature of the true deflected surface of the slab. For instance, the true deflected shape of the slab directly over a beam in the x direction for any position of a concentrated load is a fairly smooth curve with no cusp, irregularity, or steep slope at any point along the beam. The curvature in the x direction (to which the moment in the beam is in direct proportion) at any point along the beam is quite accurately approximated by its equivalent difference expression, even for a fairly coarse network as used here. This has been shown by the good agreement between the difference and exact values of the influence coefficients for moments in the beams. However, in the determination of transverse moments in the slab, difficulties arise in that the true deflected surface of the slab exhibits, for

Table 3
Comparison of Exact and Difference Solutions for Coefficients for Transverse Moment in Slab at Center of Panel at Mid-span of Right Bridge

Relative Proportions of Bridge $b/a = 0.1$
Relative Stiffness of Beams $H = 5$

Moment on Line	Transverse Location of Load	Exact Solution 1/8	Exact 2/8	Exact 3/8	Exact Center	Difference Solution 1/8	Difference 2/8	Difference 3/8	Difference Center
A B	A	−0.007	−0.015	−0.023	−0.028	−0.007	−0.015	−0.023	−0.028
	A B	0.002	0.005	−0.009	M_{oy} −0.011	0.002	0.007	−0.009	M_{oy} −0.012
								(0.026)	(0.144)
	B	0.008	0.018	0.030	0.037	0.008	0.018	0.030	0.037
	B C	0.007	0.015	0.017	0.006	0.007	0.015	0.018	0.006
	C	0.004	0.007	0.005	0.003	0.004	0.006	0.005	0.003
	C D	0.001	0.000	−0.001	0.000	0.001	0.001	−0.001	−0.001
	D	−0.001	−0.003	−0.004	−0.004	−0.001	−0.003	−0.004	−0.005
	D E	−0.003	−0.005	−0.007	−0.007	−0.003	−0.005	−0.007	−0.007
	E	−0.004	−0.007	−0.009	−0.009	−0.004	−0.007	−0.009	−0.009
B C	A	−0.015	−0.030	−0.042	−0.048	−0.015	−0.029	−0.042	−0.048
	A B	−0.001	−0.001	−0.007	−0.021	−0.001	−0.002	−0.005	−0.020
	B	0.010	0.021	0.031	0.035	0.010	0.021	0.030	0.035
	B C	0.016	0.031	0.021	M_{oy} +0.011	0.015	0.032	0.020	M_{oy} +0.012
								(0.057)	(0.166)
	C	0.015	0.029	0.041	0.047	0.015	0.029	0.041	0.047
	C D	0.009	0.016	0.017	0.006	0.009	0.016	0.017	0.006
	D	0.001	0.000	−0.004	−0.006	0.001	0.000	−0.003	−0.007
	D E	−0.006	−0.012	−0.017	−0.018	−0.005	−0.011	−0.017	−0.018
	E	−0.011	−0.021	−0.028	−0.030	−0.011	−0.021	−0.027	−0.030

Relative Proportions of Bridge $b/a = 0.2$
Relative Stiffness of Beams $H = 2$

Moment on Line	Transverse Location of Load	Exact Solution 1/8	Exact 2/8	Exact 3/8	Exact Center	Difference Solution 1/8	Difference 2/8	Difference 3/8	Difference Center
A B	A	−0.010	−0.019	−0.026	−0.032	−0.009	−0.018	−0.026	−0.032
	A B	0.002	−0.004	−0.012	M_{oy} −0.007	0.002	−0.005	−0.014	M_{oy} −0.010
						(0.009)	(0.027)	(0.075)	(0.182)
	B	0.014	0.028	0.039	0.042	0.014	0.027	0.038	0.041
	B C	0.007	0.013	0.009	0.006	0.007	0.012	0.010	0.004
	C	0.000	−0.001	−0.003	−0.004	0.000	−0.001	−0.003	−0.004
	C D	−0.003	−0.006	−0.007	−0.008	−0.003	−0.005	−0.007	−0.008
	D	−0.004	−0.006	−0.008	−0.009	−0.003	−0.006	−0.008	−0.008
	D E	−0.003	−0.005	−0.007	−0.007	−0.003	−0.005	−0.007	−0.007
	E	−0.002	−0.004	−0.005	−0.005	−0.002	−0.004	−0.005	−0.006
B C	A	−0.016	−0.031	−0.040	−0.045	−0.016	−0.030	−0.040	−0.045
	A B	−0.001	−0.004	−0.012	−0.019	−0.001	−0.004	−0.011	−0.020
	B	0.013	0.025	0.033	0.035	0.013	0.024	0.032	0.034
	B C	0.013	0.015	0.005	M_{oy} +0.010	0.013	0.013	0.003	M_{oy} +0.008
						(0.020)	(0.045)	(0.094)	(0.198)
	C	0.015	0.029	0.038	0.041	0.015	0.028	0.037	0.040
	C D	0.004	0.006	0.001	−0.004	0.004	0.006	0.002	−0.005
	D	−0.004	−0.010	−0.014	−0.016	−0.004	−0.009	−0.014	−0.016
	D E	−0.009	−0.015	−0.019	−0.021	−0.008	−0.014	−0.019	−0.021
	E	−0.009	−0.016	−0.020	−0.022	−0.008	−0.016	−0.020	−0.022

certain positions of the load, abrupt changes in slopes in the neighborhood of the point at which the influence is considered. Consequently, a network of points which is not fine enough to account for such sharp changes in slopes tends to give results that differ materially from the true values for those load positions.

The moment for the "exact" solution in Table 3, when the unit load is directly at the point where the transverse moment in the slab is desired, is given by a quantity, M_{oy}, followed by a numerical quantity. An explanation of this may be had by considering the behavior of the slab as divided into three component actions:

(1) The behavior of the particular panel of the slab acting as a single simply-supported slab.

(2) The effect of panel continuity with the adjacent panels for the beams assumed to be nondeflecting.

(3) The effect of flexibility of the beams.

In action (1) the ordinary theory of plate flexure is not suitable for computing the moments in the slab directly beneath a concentrated load because the assumptions serving as a basis for the ordinary theory are not valid in the immediate vicinity of the concentrated load. In order to compute stresses under the concentrated load, resort is made to a method suggested by Westergaard.[8] He has derived expressions for M_{oy} and M_{ox}, which represent, respectively, modified transverse and longitudinal moments under the center of a unit load uniformly distributed over a circular area at the center of an infinitely long slab simply supported on two opposite edges. For the small b/a values of the bridges studied in this investigation, one may, without introducing appreciable error, consider the long narrow panel as practically equivalent to an infinitely long slab for which Westergaard's expressions for M_{oy} and M_{ox} apply.

The modified moments, M_{oy} and M_{ox}, are functions of the ratio of the slab thickness to the slab span, and also functions of the ratio of the diameter of the loaded area to the slab thickness. However, for practical purposes, the following formulas are

Table 4
Corrections for Influence Coefficients for Transverse Moment in Slab at Center of Panel at Mid-span of Right Bridge
Relative Stiffness of Beams H = Infinity

b/a	Moment on Line	Transverse Location of Load	Method of Analysis	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center
0.1	A B	A B	Exact Solution	....	....	−0.019	M_{oy} −0.025
			Difference Solution	....	....	0.016	0.131
			Correction	....	....	−0.035	M_{oy} −0.156
	B C	B C	Exact Solution	....	....	−0.024	M_{oy} −0.039
			Difference Solution	....	....	−0.013	0.115
			Correction	....	....	−0.037	M_{oy} −0.154
0.2	A B	A B	Exact Solution	−0.003	−0.016	−0.029	M_{oy} −0.025
			Difference Solution	0.004	0.016	0.060	0.167
			Correction	−0.007	−0.032	−0.089	M_{oy} −0.192
	B C	B C	Exact Solution	−0.004	−0.019	−0.040	M_{oy} −0.039
			Difference Solution	0.003	0.013	0.051	0.151
			Correction	−0.007	−0.032	−0.091	M_{oy} −0.190

sufficiently accurate for all ordinary dimensions of I-beam bridges:*

$$M_{oy} = \frac{1.16}{3 + 10\,(c/b)} \qquad (49a)$$

$$M_{ox} = M_{oy} - 0.080 \qquad (49b)$$

in which c is the diameter of the loaded area, and b is the span of the slab. Equations (49) differ from corresponding expressions given by Westergaard in that the formulas given are for Poisson's ratio equal to zero.

In Table 3 (and in subsequent tables for transverse moments in the slab at the center of a panel) the moment directly beneath a concentrated load is expressed by M_{oy}, which represents action (1) and may be evaluated by Eq. (49a) when the ratio c/b is known, followed by a numerical quantity which represents the combined effects (2) and (3).

(b) *Improvement of Results*

It was shown in the preceding paragraph that the difference solutions for transverse moment in the slab at the center of a panel of a right I-beam bridge differ appreciably from the corresponding exact solutions for certain unit load positions along the panel's longitudinal center line. A method is suggested here to improve the difference solutions for these load positions:

Apply the difference equations to a right bridge with rigid beams (i.e., H = infinity), and determine the influence values for transverse moment at the panel's center for positions of a load along the center line. Subtract these values from the corresponding exact values which may be obtained from the tables given in Ref. 2. The remainders may be used as corrections, to be added to the corresponding difference results for an I-beam bridge with a value of H other than infinity.

Table 4 gives the corrections computed for right bridges with $b/a = 0.1$ and 0.2. These corrections, though obtained for bridges with H = infinity, may be applied without serious error to the influence values for any of the bridges considered in this investigation. This is true for the transverse moment in the slab at a point due to a load at that point, because the behavior of the panel acting as a single simply-supported slab is much more important than the effect of continuity with the adjacent panels and the effect of beam flexibility. In fact, using a method similar to that suggested by Jensen,[5] and considering a single slab panel, it can be shown that essentially the same corrections are obtained for the transverse moment under a concentrated load as for the corresponding corrections given in Table 4. The following table gives the values of the corrections obtained for a unit load of a single slab panel supported along two parallel edges by beams of relative stiffness H:

b/a	H	Correction for Moment under Load
0.1	2	$M_{oy} - 0.152$
	5	$M_{oy} - 0.152$
	10	$M_{oy} - 0.152$
	∞	$M_{oy} - 0.153$
0.2	1	$M_{oy} - 0.187$
	2	$M_{oy} - 0.188$
	5	$M_{oy} - 0.190$
	∞	$M_{oy} - 0.191$

In the above table, M_{oy} is given by Eq. (49a). It is seen that these corrections are practically independent of the stiffness of the beams within the range of the b/a and H values considered. Also, they do not differ materially from corresponding corrections given in Table 4, in which continuity was considered.

The effect of continuity of a panel with adjacent panels is more important, however, in the determination of the corrections listed in Table 4 for load positions other than at the point considered. This is why the method used was adopted in preference to Jensen's method in which the effect of continuity is not considered. However, the

* See pp. 14-15, Ref. 2.

Table 5
Comparison of Exact and Difference Solutions for Influence Coefficients for Transverse Moment in Slab Over Beam at Mid-span of Right Bridge

Moment on Line	Transverse Location of Load	Values of Influence Coefficient for Moment							
		Exact Solution				Difference Solution			
		Longitudinal Position of Load							
		1/8	2/8	3/8	Center	1/8	2/8	3/8	Center
					$b/a=0.1$ $H=5$				
B	A	−0.014	−0.030	−0.047	−0.057	−0.014	−0.030	−0.046	−0.058
	A B	0.004	0.011	0.020	−0.079	0.003	0.010	0.018 (0.015)	−0.081 (−0.026)
	B	0.015	0.035	0.061	0.083	0.015	0.035	0.060	0.082
	B C	0.015	0.031	0.045	−0.043	0.015	0.030	0.044 (0.041)	−0.039 (0.006)
	C	0.009	0.015	0.011	0.004	0.009	0.014	0.012	0.004
	C D	0.003	0.001	−0.003	0.003	0.003	0.002	−0.002	−0.002
	D	−0.003	−0.006	−0.008	−0.009	−0.003	−0.006	−0.008	−0.009
	D E	−0.006	−0.011	−0.014	−0.015	−0.006	−0.010	−0.013	−0.014
	E	−0.008	−0.014	−0.017	−0.019	−0.007	−0.014	−0.017	−0.019
C	A	−0.016	−0.030	−0.039	−0.042	−0.016	−0.029	−0.039	−0.042
	A B	−0.005	−0.013	−0.023	−0.018	−0.005	−0.012	−0.021	−0.024
	B	0.006	0.008	0.001	−0.006	0.005	0.007	0.002	−0.006
	B C	0.016	0.032	0.046	−0.044	0.015	0.031	0.044 (0.041)	−0.042 (0.006)
	C	0.020	0.044	0.074	0.097	0.020	0.044	0.072	0.095
					$b/a=0.2$ $H=2$				
B	A	−0.019	−0.038	−0.056	−0.065	−0.019	−0.037	−0.054	−0.064
	A B	0.011	0.023	0.011	−0.058	0.010	0.021 (0.017)	0.010	−0.068 (−0.031)
	B	0.027	0.056	0.088	0.118	0.027	0.056	0.085	0.118 (0.105)
	B C	0.018	0.037	0.033	−0.031	0.018	0.036 (0.032)	0.031	−0.034 (−0.007)
	C	0.003	0.001	−0.006	−0.012	0.002	0.001	−0.005	−0.010
	C D	−0.006	−0.013	−0.017	−0.015	−0.005	−0.011	−0.016	−0.017
	D	−0.008	−0.014	−0.017	−0.018	−0.007	−0.013	−0.017	−0.018
	D E	−0.006	−0.012	−0.015	−0.016	−0.006	−0.011	−0.015	−0.016
	E	−0.005	−0.009	−0.011	−0.012	−0.005	−0.009	−0.011	−0.012
C	A	−0.014	−0.026	−0.034	−0.036	−0.014	−0.025	−0.033	−0.036
	A B	−0.011	−0.021	−0.027	−0.027	−0.010	−0.019	−0.027	−0.029
	B	−0.001	−0.005	−0.013	−0.020	−0.001	−0.005	−0.012	−0.018
	B C	0.017	0.034	0.032	−0.035	0.016	0.032 (0.029)	0.032 (0.028)	−0.041 (−0.011)
	C	0.028	0.058	0.090	0.123	0.028	0.058	0.088	0.123 (0.110)

corrections at those points other than where the influence is considered are found also to be practically independent of the value of H for the bridges studied.

The corrections given in Table 4 have been applied to the approximate values in Table 3. The improved values, written above the values in parentheses, are seen to agree reasonably well with the exact values. The applicability of the corrections given in Table 4 to bridges with values of H different from those in Table 3 has also been checked, and the results found to be equally satisfactory.

So far we have considered the corrections for influence values of transverse moment in the slab at the panel's center at mid-span of right bridges. It is proposed that these corrections may also be applied without serious error to corresponding skew bridges. For the values of b/a considered, the narrow panels may each be regarded as an infinitely long slab. As long as the load positions considered are along the center line of the particular panel and not too remote from the point at which the influence is considered, the effect of the skew supports is practically negligible. Furthermore, if the small difference between rectangular meshes (used for a right bridge) and parallelogram-shaped meshes (which should actually be used for skew bridges) is neglected in arriving at the corrections, we may feel reasonably sure that the corrections in Table 4 should also serve to improve influence values for skew bridges. Admittedly, the corrections are not too good for points close to the ends of the span. Generally, though, the influence coefficients and the corrections at those points are small and any errors involved would probably not lead to appreciable errors in the evaluation of significant design moments.

16. Transverse Moments in Slab Over Beams

(a) *Comparison of Exact and Difference Solutions*

Two right bridges, one with $b/a=0.1$ and $H=5$, and the other with $b/a=0.2$ and $H=2$, were analyzed by the difference method to determine the influence coefficients for transverse moment in the slab at the mid-points of beams B and C (Fig. 1). Table 5 gives the numerical values of the coefficients together with the corresponding exact values. It will be noted that two values are given at certain points in the set of difference values in Table 5. The value that appears in parentheses is the value obtained directly from the difference equations, while the one written above the parentheses is the improved value, obtained in a manner to be discussed later.

Comparisons of the approximate and exact values in Table 5 show that, for the bridge with $b/a=0.1$ and $H=5$, the largest discrepancies occur for the load at or near the center of each panel adjacent to the beam considered. There is also a discrepancy, much smaller than the previous ones, but still of the order of 0.005 or 0.006, for a load at the center of the next remote panel. For the bridge with $b/a=0.2$ and $H=2$, we find that in addition to the discrepancies described above, there is another difference of the order of 0.013 in the values for a load at the point at which the influence is considered.

As a preliminary to the explanation of these discrepancies, consider the moment transverse to the fixed edge of an infinitely large cantilever slab, due to a point load $P=1$ on the slab. This problem has been studied by Westergaard,[(8)] and the result is given by Eq. (50):

$$M_y = -\frac{1}{\pi}\cos^2\theta \tag{50}$$

where M_y is the transverse moment at point O along the fixed edge (taken as the x axis) due to a load $P=1$ placed at a point such that the angle between the normal to the fixed edge at point O (taken as the y axis) and the line joining the load point and point O is given by θ, (Fig. 11(a)). The contour lines on the influence surface for M_y are, therefore, a series of radial lines. As the load approaches the support along a radial line the influence does not change, but when the load is directly over the support the influence becomes zero. There is a singularity in the influence surface at point O. However, if the load is considered distributed over an area, the contour lines on the influence surface are no longer radial, and the moment at point O, which will depend on the distance of the load from the support and the shape and the dimensions of the loaded area, is to be obtained by integrating Eq. (50).

Consider a load $P=1$, first at the point (O, λ_y), and then at the point (λ_x, λ_y), as shown in Fig. 11(b) and 11(c), respectively. Moments at the origin due to the load, considered either as concentrated at the load point or distributed over a rectangle of dimensions λ_x, λ_y centered about the load point, may be found for each of the load positions either directly from Eq. (50), or by integrating Eq. (50). The values of the moments for ratios $K=\frac{\lambda_y}{\lambda_x}=0.4$ and $K=0.8$ (corresponding to $b/a=0.1$ and $b/a=0.2$, respectively) are given in the following table:

	Position of Load $P=1$	(O, λ_y)		(λ_x, λ_y)	
	$K=\lambda_y/\lambda_x$	0.4	0.8	0.4	0.8
	b/a	0.1	0.2	0.1	0.2
Moment M_y	P, concentrated	−0.318	−0.318	−0.044	−0.124
at Fixed	P, distributed	−0.222	−0.279	−0.052	−0.130
Edge	Difference	−0.096	−0.039	−0.008	0.006

Consider the case of an infinitely large slab over a rigid beam. By a consideration of Newmark's moment distribution concept,[(1)] it may be shown that the moment in the slab over the beam (due to a single load on the slab to one side of the beam) is exactly one-half of the fixed-edge moment for the load on the slab cantilevered from a fixed support. Hence, one may take one-half of the values shown in the above table as corresponding values for a load on an infinitely large slab continuous over a rigid beam. Take, from the table, one-half of the differences in the moments caused by the difference in assumptions regarding

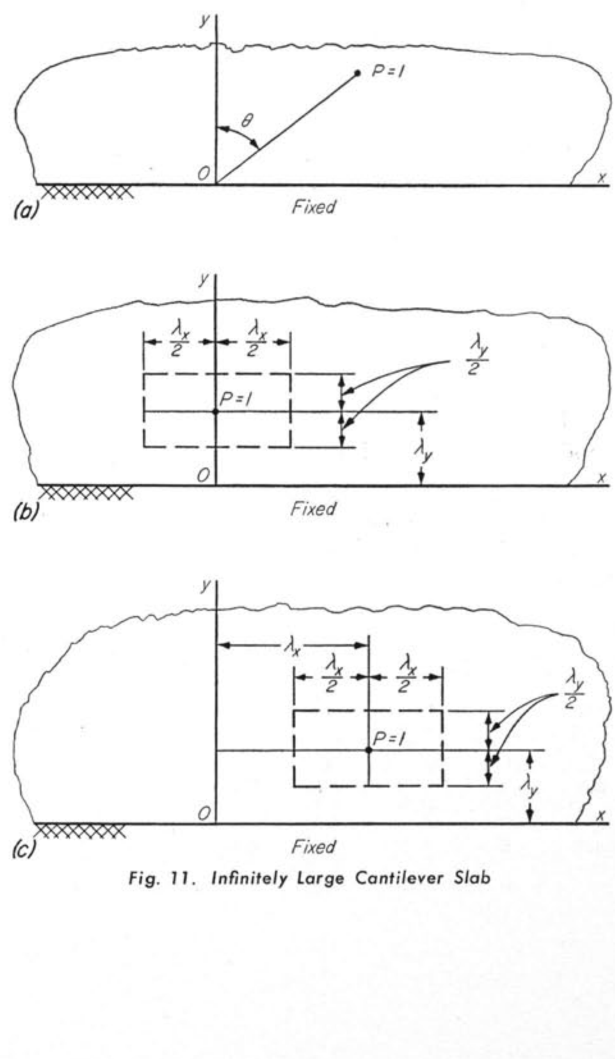


Fig. 11. Infinitely Large Cantilever Slab

the load distribution and compare them with the discrepancies which have been found for corresponding load positions between the exact and approximate influence values in Table 5. It is interesting to note that they are of a comparable order of magnitude.

The coarseness of the network used in the difference analysis of the I-beam bridge is again the primary cause of the discrepancies found in the results.

(b) *Improvement of Results*

A procedure is indicated here for improving some of the influence values that suffer from serious errors, so as to make up in a practical way for the defect of the coarse network:

Consider the corrections to be applied to influence values for load positions on the center line of each panel adjacent to a beam in a right bridge. Two right bridges with rigid and nondeflecting beams (H = infinity), and with $b/a = 0.1$, and $b/a = 0.2$, are analyzed to determine the influence values for moments in the slab over the beams. These values, together with the exact values, are given in Table 6, and the corrections are obtained as the differences between the two sets of values.

The corrections listed in Table 6 have been applied to the values in Table 5, and found to give fairly satisfactory results, as shown by the improved values in Table 5. It is noted that these corrections would be in error as the stiffness of the beams becomes smaller. Within the range of the values of H of the bridges considered, however, it has been found that these corrections may be applied without regard to the effect of beam stiffness.

For the corrections in Table 6 to be applicable to skew bridges, a slight modification is necessary. Suppose we wish to obtain the corrections to be applied to the influence values for transverse moment in the skew bridge slab at the mid-point of beam B. In Fig. 12a is shown the network of the

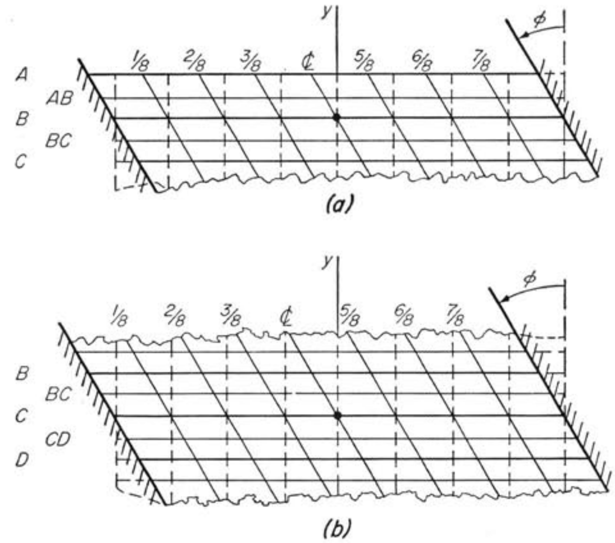


Fig. 12. Relative Positions of Networks of Points of Right and Skew Bridges

skew bridge superimposed on the network of the corresponding right bridge, so that the beams B of both bridges fall along the same line and their mid-points coincide with each other. It is seen that the points on the skew bridge do not coincide with the corresponding points on the right bridge except at points on line B common to both bridges. The corrections for the points on the skew bridge are found by interpolation from the corresponding values for the right bridge. Provided that we limit this procedure to load positions along lines AB and BC not too remote from the y axis (Fig. 12a) the corrections obtained for the skew bridge would not be too much in error. The same method may be used to obtain the corrections for moment in the slab at the center of skew bridge beam C (Fig. 12b) by interpolation from the corrections for the right bridge.

Table 7 gives the corrections for skew bridges based on corrections obtained for corresponding right bridges and listed in Table 6. Corrections are not given for points beyond the first grid lines from the bridge center line because it is felt they would probably be in error.

Table 6

Corrections for Influence Coefficients for Transverse Moment in Slab Over Beam at Mid-span of Right Bridge

Relative Stiffness of Beams H = Infinity

b/a	Moment on Line	Transverse Location of Load	Values of Influence Coefficient for Moment — Exact Solution 1/8	2/8	3/8	Center	Difference Solution (Longitudinal Position of Load) 1/8	2/8	3/8	Center	Corrections 1/8	2/8	3/8	Center
0.1	B	A B	0.000	0.000	−0.003	−0.111	0.000	0.000	−0.006	−0.056	0.000	0.000	0.003	−0.055
		B C	0.000	0.000	−0.001	−0.093	0.000	0.000	−0.004	−0.048	0.000	0.000	0.003	−0.045
	C	B C	0.000	0.000	−0.002	−0.097	0.000	0.000	−0.005	−0.049	0.000	0.000	0.003	−0.048
0.2	B	A B	0.000	−0.002	−0.025	−0.011	−0.001	−0.006	−0.025	−0.074	0.001	0.004	0.000	−0.037
		B C	0.000	0.000	−0.016	−0.093	−0.001	−0.004	−0.019	−0.066	0.001	0.004	0.003	−0.027
	C	B C	0.000	−0.001	−0.016	−0.097	−0.001	−0.004	−0.020	−0.067	0.001	0.003	0.004	−0.030

Table 7
Corrections for Influence Coefficients for Transverse Moment in Slab Over Beam at Mid-span of Skew Bridge
Relative Stiffness of Beams H = Infinity

Angle of Skew, $\phi°$	Moment on Line	Transverse Location of Load	Corrections for Influence Coefficients $b/a=0.1$			$b/a=0.2$		
			Longitudinal Position of Load					
			3/8	Center	5/8	3/8	Center	5/8
30	B	A B	0.002	−0.043	−0.001	0.003	−0.019	−0.016
		B C	0.000	−0.037	0.002	−0.005	−0.008	0.004
	C	B C	0.002	−0.038	0.001	0.004	−0.010	−0.007
45	B	A B	0.001	−0.024	−0.008	0.004	−0.006	−0.033
		B C	−0.006	−0.020	0.001	−0.020	0.001	0.004
	C	B C	0.001	−0.020	−0.006	0.003	0.001	−0.025
60	B	A B	0.000	−0.001	−0.034	0.003	0.004	−0.023
		B C	−0.029	−0.003	0.000	−0.011	0.004	0.003
	C	B C	0.001	−0.001	−0.031	0.002	0.004	−0.013

Previous comparisons of the values in Table 5 indicated another discrepancy exists for a position of the load at the point where the influence is considered. It has been found that this discrepancy may be neglected for the bridges with $b/a=0.1$, $H=2$, 5, and 10, or for the bridge with $b/a=0.2$ and $H=5$. For $b/a=0.2$ and $H=1$, and $b/a=0.2$ and $H=2$, however, corrections are needed, and they are, respectively, 0.021 and 0.013. These corrections are assumed to be the same for skew bridges, and are applied without modification.

17. Deflections of Beams

Influence coefficients for mid-span deflection of the center beam of right bridges with $b/a=0.1$ and $H=5$, and $b/a=0.2$ and $H=2$, have been obtained by the difference equations. The numerical values of the coefficients for loads at the quarter and center points are given in Table 8 together with the corresponding exact values. These values are given so that the actual deflections for concentrated loads are obtained by multiplying the tabulated values by the quantity Pa^3/E_bI_b.

A study of Table 8 shows the approximate values compare quite satisfactorily with the exact values. The maximum discrepancies occur for loads at the point where the deflection is considered, and are about 5 to 6% of the exact values.

Table 8
Comparison of Exact and Difference Solutions for Influence Coefficients for Deflection of Center Beam at Mid-span of Right Bridge

b/a	H	Transverse Location of Load	Values of Influence Coefficient for Deflection: Exact Solution		Difference Solution	
			Longitudinal Position of Load			
			2/8	Center	2/8	Center
0.1	5	A	0.00163	0.00229	0.00168	0.00236
		A B	0.00252	0.00355	0.00257	0.00363
		B	0.00332	0.00477	0.00338	0.00487
		B C	0.00393	0.00585	0.00401	0.00605
		C	0.00417	0.00634	0.00429	0.00665
0.2	2	A	0.00092	0.00130	0.00096	0.00135
		A B	0.00207	0.00291	0.00210	0.00296
		B	0.00323	0.00460	0.00325	0.00464
		B C	0.00429	0.00633	0.00438	0.00653
		C	0.00477	0.00720	0.00496	0.00763

V. INFLUENCE COEFFICIENTS AND INFLUENCE SURFACES FOR MOMENTS AND DEFLECTIONS IN SKEW BRIDGES

18. Moments in Beams

Influence coefficients for moments in beams A, B, and C at mid-span and in beam C at the quarter and ⅜ points of the span are given in Tables 1 through 18 of Appendix A for the 18 bridges studied. Typical influence surfaces for moments in the beams at mid-span for $b/a=0.1$ and $H=5$, and $b/a=0.2$ and $H=2$, with $\varphi=30°$, $45°$, and $60°$, are shown in the form of contour maps in Appendix B.

In general, these surfaces show characteristics similar to those for corresponding right bridges. There are no unusually sharp peaks. The influence surfaces for edge beams have a much higher peak than such surfaces for other beams. However, the influence surfaces for the skew bridges have lower peaks at the point where the influence is considered, and generally have lower relative "altitudes" throughout the regions common to the skew and the right bridges. For a given stiffness H and ratio b/a, the difference in altitude of the surfaces increases with the increase of the angle of skew, φ. As the beams become stiffer, however, the difference in altitude of the surfaces decreases. The effect of skew on the difference in altitude is much less for edge beams than for the other beams.

Table 9 gives the numerical values of the influence surfaces' peak ordinates as well as their relative values expressed as percentages of the corresponding peak ordinates for the right bridges. For a single concentrated load, Table 9 may also be considered to give the relative values of the maximum moments in the beams at the points considered as compared to right bridges. For $H=5$ with $b/a=0.1$, and $H=2$ with $b/a=0.2$, the reduction in moments in interior beams may be as much as 30 to 40% for a 60° skew, and about 15% for a 30° skew. The reductions for edge beams are much smaller, being about 10% for a 60° skew and about 5% for a 30° skew. Of course, these considerations are valid only for a single concentrated load, since the maximum moment at a given point on a beam for groups of loads may be obtained without placing a load at the point or even on the beam.

Maximum moments in beams at mid-span of skew I-beam bridges produced by standard highway trucks have been evaluated by the use of influence surfaces similar to those shown in Appendix B. The results obtained and discussions are presented in detail in Chapter VI.

19. Transverse Moments in Slab at Center of Panel

Influence coefficients for transverse moment in the slab on lines AB and BC at mid-span are

Table 9

Comparison of Maximum Influence Coefficients for Moment in Beams at Mid-span of Bridge of Different Angles of Skew

b/a	0.1						0.2					
H	2		5		10		1		2		5	
$\phi°$	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent
					Moment in Beam A							
0	0.147	100	0.172	100	0.189	100	0.149	100	0.174	100	0.202	100
30	0.131	89	0.164	95	0.185	98	0.138	93	0.167	96	0.198	98
45	0.125	85	0.160	93	0.182	96	0.132	89	0.162	93	0.194	96
60	0.121	82	0.154	90	0.177	94	0.120	81	0.153	88	0.188	93
					Moment in Beam B							
0	0.094	100	0.107	100	0.119	100	0.095	100	0.112	100	0.137	100
30	0.071	76	0.092	86	0.108	91	0.078	82	0.099	88	0.130	95
45	0.064	68	0.085	79	0.102	86	0.067	71	0.089	79	0.121	88
60	0.054	57	0.075	70	0.093	78	0.044	46	0.066	59	0.100	73
					Moment in Beam C							
0	0.084	100	0.101	100	0.116	100	0.090	100	0.109	100	0.135	100
30	0.061	73	0.083	82	0.102	88	0.072	80	0.094	86	0.126	93
45	0.056	67	0.078	77	0.097	84	0.063	70	0.085	78	0.118	87
60	0.047	56	0.070	69	0.089	77	0.041	46	0.063	58	0.097	72

Table 10
Comparison of Maximum Influence Coefficients for Transverse Moment in Slab at Center of Panel at Mid-span of Bridge of Different Angles of Skew
(M_{oy} is assumed to be 0.232)

b/a	0.1						0.2					
H	2		5		10		1		2		5	
$\phi°$	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent
					Moment on Line AB							
0	0.224	100	0.221	100	0.220	100	0.224	100	0.225	100	0.224	100
30	0.214	95	0.215	97	0.215	98	0.218	97	0.219	97	0.218	97
45	0.207	92	0.209	95	0.209	95	0.210	94	0.210	93	0.208	93
60	0.194	87	0.195	88	0.195	89	0.192	86	0.188	84	0.184	82
					Moment on Line BC							
0	0.255	100	0.243	100	0.234	100	0.250	100	0.242	100	0.227	100
30	0.236	93	0.232	96	0.226	97	0.237	95	0.231	96	0.220	97
45	0.231	91	0.226	93	0.220	94	0.240	96	0.228	94	0.213	94
60	0.231	91	0.220	91	0.211	90	0.243	97	0.225	93	0.201	88

given in Tables 19 through 36 of Appendix A. Discrepancies are expected to occur for certain load positions along the center line of the panel considered. At those points, corrections have been applied as explained previously. The values that have been corrected are retained in parentheses for the sake of record, and the improved values are written directly above the parentheses.

Influence surfaces for $b/a = 0.1$ and $H = 5$, and $b/a = 0.2$ and $H = 2$, are shown in Appendix B. These surfaces have an unusually high peak at the point where the influence is considered. They drop off from this high peak value to relatively low values only a short distance away. To obtain maximum positive moments from truck loads, it is always necessary to have a heavy wheel load at the particular point considered. The negative transverse moments at the center of a panel are generally obtained with loads in other panels. The negative moments will generally be smaller than the maximum positive moments.

The peak ordinate for the skew bridge is smaller than that for the right bridge, and the difference increases as the angle of skew increases. A comparison of peak ordinates showing the effect of skew is made in Table 10. The actual values of the peak ordinates given are based on a value of M_{oy} of 0.232 (which is obtained from Eq. (49a) with $c/b = 0.2$). The relative values are expressed as percentages of the corresponding peak values for the right bridge. It seems from Table 10 that the effect of skew on the peak ordinates for transverse moment in the slab is not so pronounced as for moments in the beams.

20. Transverse Moments in Slab Over Beams

Influence coefficients for transverse moment in the slab over beams B and C at mid-span for the 18 structures studied are given in Tables 19 through 36 of Appendix A. Where corrections have been made in the manner described previously, the improved values are given above the values in parentheses. The values after improvement are believed to be free at least of flagrant errors, although errors of the order of 0.005 may exist at some points.

Influence surfaces for a group of structures, with $b/a = 0.1$ and $H = 5$, and with $b/a = 0.2$ and $H = 2$, are shown in Appendix B.

The influence surfaces all have the same general characteristics, with a singularity at the point where the influence is computed. The nature of the singularity may be explained for a typical structure by reference to Fig. 13, which shows the influence lines for transverse moment in the slab at the mid-point of beam C for loads on the skew center line of a bridge with $\varphi = 30°$, $b/a = 0.1$, and $H = 5$. Two curves differing only near C are shown. The lower curve refers to the theoretical effect of a truly concentrated load and the upper curve has a cusp, and indicates the effect of a load distributed over a particular size of circular area. The point

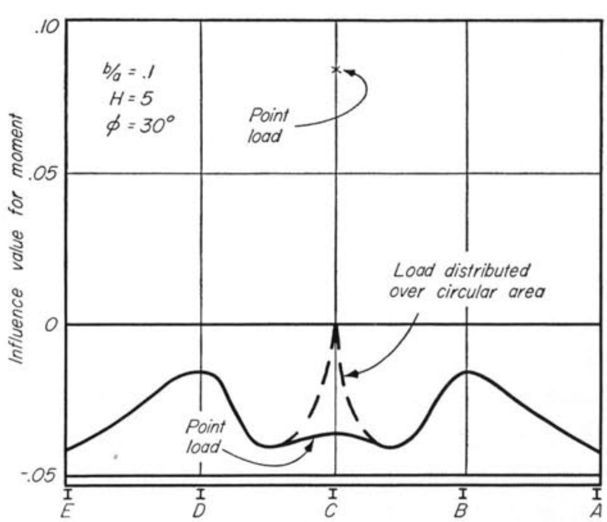


Fig. 13. Influence Line for Transverse Moment in Slab at C at Mid-span, Load on Skew Center Line

marked "x" represents the influence of a point load directly over point C.

An explanation of these curves may be obtained by considering again the moment transverse to the fixed edge of an infinitely large cantilever slab. For a truly unit point load at any distance from the support on a line at 30° to the normal of the support, the moment at the support is, by Eq. (50), −0.239. As the load approaches the support along the 30° line, the influence does not change, but when the load is directly over the support the influence is zero. The moment due to a load uniformly distributed over an area may be found by integrating Eq. (50). Thus, the moments at the support due to a unit load uniformly distributed over a circular area of diameter c, on a 30° line to the normal, and at perpendicular distances from the support of $c/2$, c, $3c/2$ and $2c$, are, respectively, −0.209, −0.231, −0.236, and −0.237. When the center of the circle is over the edge, only one-half of the load on the circle is on the slab, and the moment is −0.080.

Now, in the case of an infinitely large slab over a rigid beam, we have stated previously that the moment in the slab over the beam due to a single load on the slab to one side of the beam is exactly one-half of the fixed-edge moment for a load on the slab cantilevered from a fixed support. From these and the previous integrations, the following results are obtained:

As a point load approaches a line support, on a line at 30° to the normal to the support, the moment is constant for all positions of the load away from the support, and is equal to

$$(\tfrac{1}{2})(-0.239) = -0.119.$$

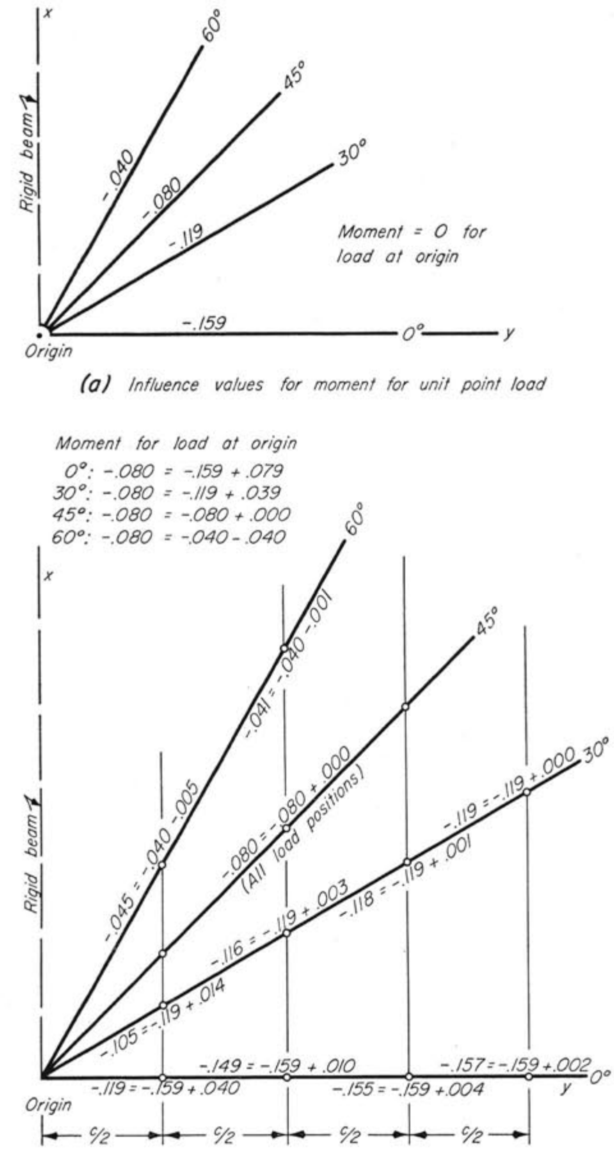
This value changes abruptly to zero when the load is directly over the support.

For a load distributed uniformly over a circle of diameter c, when the circle is balanced over the support with a semi-circle of load on each side, the moment over the support is

$$2(\tfrac{1}{2})(-0.080) = -0.080 = -0.119 + 0.039.$$

When the center of the circle (on a 30° line to the normal) is at distances of $c/2$, c, $3c/2$, and $2c$ perpendicularly from the support, the moments are, respectively:

$$(\tfrac{1}{2})(-0.209) = -0.105 = -0.119 + 0.014;$$
$$(\tfrac{1}{2})(-0.231) = -0.116 = -0.119 + 0.003;$$
$$(\tfrac{1}{2})(-0.236) = -0.118 = -0.119 + 0.001;$$
$$(\tfrac{1}{2})(-0.237) = -0.119 = -0.119 + 0.000.$$



(a) Influence values for moment for unit point load

(b) Influence values for moment for unit load distributed over a circular area of diameter c

Fig. 14. Influence Values for Moment Over Beam in Infinitely Large Slab Continuous Over Rigid Beam

The following procedure was then used to obtain the curves in Fig. 13 from the tabulated values. The curve for a point load was drawn first, the point close to C being located at a distance corresponding to 0.119 below the point corresponding to the load directly upon beam C. From this point close to C, another point corresponding to a circular distributed load directly over the support was located by measuring up a distance of 0.039. Then points were located at perpendicular distances of $c/2$, c, $3c/2$, and $2c$, from the support, by measuring up from the curve corresponding to a concentrated load ordinates of 0.014, 0.003, 0.001, and 0.000 respectively, to obtain the curve corresponding to a load distributed over a circular area. In Fig. 13, c was taken as ⅕ of the spacing of the beams.

The influence surfaces in Appendix B were obtained by drawing curves in a manner similar to that just described, using the results shown in Fig. 14 for the moments in an infinitely large slab over a rigid beam due to various positions of a truly point load and a load distributed over a circle.

21. Deflections of Beams

Influence values for deflection of the beam C at mid-span for the 18 skew bridges are given in Tables 37 through 42 of Appendix A. Influence surfaces are shown for several structures in Appendix B.

A comparison of the peak ordinates of the influence surfaces for deflection is shown in Table 11. The percentage reductions caused by the skew in the peak deflections are almost the same as the corresponding percentage reductions in peak ordinates for moment at the mid-point of beam C. The influence surfaces also indicate a more non-uniform distribution in deflections across a given section than the corresponding surfaces for right bridges.

Table 11
Comparsion of Maximum Influence Coefficients for Deflection of Center Beam at Mid-span of Bridge of Different Angles of Skew

b/a	0.1						0.2					
H	2		5		10		1		2		5	
$\phi°$	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent	Value of Coef.	Per Cent
0	0.00511	100	0.00634	100	0.00762	100	0.00562	100	0.00720	100	0.00961	100
30	0.00325	64	0.00504	80	0.00672	88	0.00445	79	0.00626	87	0.00911	95
45	0.00279	55	0.00452	71	0.00617	81	0.00386	69	0.00558	78	0.00837	87
60	0.00242	47	0.00401	63	0.00554	73	0.00267	47	0.00420	59	0.00684	71

VI. MOMENTS IN BEAMS IN SKEW I-BEAM BRIDGES

22. Description of Bridges Analyzed

With the use of influence surfaces similar to those given in Appendix B, live load beam moments at mid-span for standard trucks were computed for 72 structures of varying dimensions. The structures analyzed had roadway widths of 32, 28, 24, and 20 ft, corresponding to beam spacings of 8, 7, 6, and 5 ft, respectively. For all spacings, values of b/a of 0.1 and 0.2 were considered, giving spans of 80, 70, 60, 50, 40, 35, 30, and 25 ft. The values of H considered were 2, 5, and 10 for $b/a=0.1$; and 1, 2, and 5 for $b/a=0.2$. The angles of skew considered were 30°, 45°, and 60°.

All the bridges were simply-supported at the ends, the slab as well as the beams being supported at the abutments. The beams in each bridge were assumed to have the same stiffness. The influence of interior diaphragms between the beams, and of shear connectors between the slab and beams, was neglected. The edge beams were assumed to be at the edge of the slab, and the effects of curbs, sidewalks, and handrails were also neglected.

Moments were computed at mid-span only for each structure. With standard trucks placed in the position producing maximum moments in the beams at mid-span of bridges, coefficients were determined separately for moments due to rear wheel loads and for corresponding moments due to front wheel loads.

23. Standard Truck Loading

The standard truck loading for which moments are given is that specified as the H-truck loading in "Standard Specifications for Highway Bridges," AASHO, 6th edition, 1953. The standard H-truck has the wheels of each axle spaced 6 ft apart, with front and rear axles spaced 14 ft apart. Each of the rear wheels carries a weight of 0.40 the total weight of the truck, and each of the front wheels carries a weight of 0.10 the total weight of the truck, or one-fourth of the rear wheel weight. The rear wheel load, P, in terms of which the moment coefficients are stated, is the weight on a rear wheel increased by an impact factor given in the specifications. The front wheel load is always taken as $P/4$.

Each truck was assumed to occupy the central part of a 10 ft traffic lane. Therefore the distance between the centers of the nearest wheels of trucks in adjacent lanes was taken as a minimum of 4 ft, and the distance between the center of a wheel and the face of a curb was taken as a minimum of 2 ft. The possibility of wheel loads coming over the edge beams was not taken into account.

Only one or two lanes of loading, depending on which gave larger moments, were considered in the calculations. Only one truck in each lane was considered, but the two trucks in adjacent lanes were assumed to be traveling in the same or opposite directions as might be required to produce maximum effects. Uniform lane loads, or truck train loadings described in the specifications were not considered.

Empirical relations for estimating maximum mid-span moments produced by this H loading are given in Art. 25. In Art. 26 it is indicated how these relations may be extended to obtain corresponding moments produced by the H-S loading which is also given in the AASHO Specifications.

24. Maximum Moments in Beams at Mid-span

Coefficients for maximum moments at mid-span of the beams due to standard truck loads are given in Table 12. To obtain the actual moments, these coefficients are to be multiplied by the product of the rear wheel load P and the span a; the relative magnitude of the front wheel load $P/4$ has already been taken into account. In Table 12 the effects due to rear wheel loads and the corresponding effects of front wheel loads are listed separately. Coefficients for the beam having the greatest moment at mid-span of a particular structure due to rear wheel loads and to combined rear and front wheel loads are italicized. For comparison, the coefficients for corresponding right bridges are also given. These values were taken directly from Reference 2, or obtained by interpolation or otherwise from the data contained

Table 12

Maximum Moments at Mid-span of Beams Due to Truck Loading

Moments are given in terms of a coefficient to be multiplied by the product of the rear wheel load P and the span a, for standard truck loadings according to 1953 A.A.S.H.O. specifications. The relative magnitude of the front wheel load, $P/4$, is taken into account in the coefficients. No wheel is assumed to be less than 2 ft from an edge beam. Two lanes of loading are considered with trucks heading in either direction in each lane.

Span of Bridge a ft	Spacing of Beams b ft	Relative Stiff-ness of Beams H	Maximum Moment in Beam	$\varphi=0°$ Rear Wheels	$\varphi=0°$ Front Wheels	$\varphi=0°$ Total	$\varphi=30°$ Rear Wheels	$\varphi=30°$ Front Wheels	$\varphi=30°$ Total	$\varphi=45°$ Rear Wheels	$\varphi=45°$ Front Wheels	$\varphi=45°$ Total	$\varphi=60°$ Rear Wheels	$\varphi=60°$ Front Wheels	$\varphi=60°$ Total
			A	0.257	0.047	0.304	*0.213*	0.036	*0.249*	*0.193*	0.033	*0.226*	*0.185*	0.030	*0.215*
		2	B	*0.278*	0.040	*0.318*	0.208	0.027	0.235	0.183	0.024	0.207	0.154	0.024	0.178
			C	0.254	0.035	0.289	0.181	0.020	0.201	0.158	0.019	0.177	0.134	0.017	0.151
			A	0.254	0.047	0.301	0.236	0.044	0.280	0.228	0.041	*0.269*	*0.215*	0.038	*0.253*
80	8	5	B	*0.307*	0.045	*0.352*	*0.257*	0.036	*0.293*	*0.234*	0.034	0.268	0.205	0.031	0.236
			C	0.282	0.041	0.323	0.230	0.030	0.260	0.216	0.028	0.244	0.190	0.024	0.214
			A	0.252			0.249	0.044	0.293	0.244	0.044	0.284	0.233	0.040	0.273
		10	B	*0.327*			*0.300*	0.043	*0.343*	*0.274*	0.040	*0.314*	*0.243*	0.038	*0.281*
			C	0.312			0.277	0.038	0.315	0.268	0.035	0.303	0.236	0.034	0.270
			A	0.234	0.040	0.274	*0.188*	0.030	*0.218*	*0.171*	0.026	*0.197*	*0.161*	0.025	*0.186*
		2	B	*0.254*	0.035	*0.289*	0.187	0.023	0.210	0.164	0.010	0.174	0.136	0.021	0.157
			C	0.244	0.031	0.275	0.166	0.019	0.185	0.154	0.015	0.169	0.128	0.014	0.142
			A	0.228	0.040	0.268	0.206	0.035	0.241	0.196	0.033	0.229	*0.191*	0.031	*0.222*
70	7	5	B	*0.277*	0.039	*0.316*	*0.231*	0.031	*0.262*	0.202	0.029	0.231	0.182	0.027	0.209
			C	0.266	0.036	0.302	0.218	0.028	0.246	*0.204*	0.025	*0.229*	0.180	0.023	0.203
			A	0.215			0.215	0.035	0.250	0.210	0.035	0.245	0.208	0.033	0.241
		10	B	*0.297*			*0.264*	0.036	*0.300*	0.241	0.034	0.275	0.215	0.032	0.247
			C	0.288			0.260	0.034	0.294	*0.246*	0.031	*0.277*	*0.222*	0.029	*0.251*
			A	0.203	0.032	0.235	*0.163*	0.022	*0.185*	*0.151*	0.018	*0.169*	*0.142*	0.019	*0.161*
		2	B	*0.228*	0.029	*0.257*	0.163	0.018	0.181	0.141	0.016	0.157	0.116	0.016	0.132
			C	0.226	0.027	0.253	0.162	0.015	0.177	0.146	0.013	0.159	0.120	0.011	0.131
			A	0.197	0.030	0.227	0.178	0.026	0.204	0.173	0.026	0.199	0.169	0.025	*0.194*
60	6	5	B	0.248	0.032	0.280	0.201	0.025	0.226	0.175	0.023	0.198	0.142	0.023	0.165
			C	*0.252*	0.031	*0.283*	*0.206*	0.023	*0.229*	*0.190*	0.020	*0.210*	*0.170*	0.016	0.186
			A	0.183			0.186	0.027	0.213	0.181	0.027	0.208	0.181	0.026	0.207
		10	B	0.256			0.228	0.029	0.257	0.209	0.027	0.236	0.178	0.027	0.205
			C	*0.263*			*0.240*	0.028	*0.268*	*0.226*	0.027	*0.253*	*0.208*	0.023	*0.231*
			A	0.169	0.021	0.190	0.132	0.013	0.145	0.128	0.013	0.141	*0.121*	0.012	*0.133*
		2	B	0.195	0.021	0.216	0.134	0.012	0.146	0.127	0.010	0.137	0.099	0.010	0.109
			C	*0.212*	0.022	*0.234*	*0.152*	0.013	*0.165*	*0.136*	0.011	*0.147*	0.110	0.009	0.119
			A	0.159	0.021	0.180	0.145	0.017	0.162	0.140	0.016	0.156	0.141	0.018	0.159
50	5	5	B	0.204	0.023	0.227	0.161	0.018	0.179	0.142	0.018	0.160	0.124	0.016	0.140
			C	*0.228*	0.024	*0.252*	*0.190*	0.016	*0.206*	*0.174*	0.016	*0.190*	*0.154*	0.014	*0.168*
			A	0.154			0.156	0.019	0.175	0.151	0.019	0.170	0.149	0.019	0.168
		10	B	0.209			0.190	0.020	0.210	0.173	0.019	0.192	0.149	0.018	0.167
			C	*0.230*			*0.218*	0.021	*0.239*	*0.204*	0.020	*0.224*	*0.182*	0.017	*0.199*
			A	0.222	0.020	0.242	0.199	0.018	0.217	*0.179*	0.015	*0.194*	*0.138*	0.011	*0.149*
		1	B	*0.258*	0.017	*0.275*	*0.210*	0.014	*0.224*	0.177	0.013	0.190	0.107	0.011	0.118
			C	0.242	0.017	0.259	0.190	0.011	0.201	0.164	0.010	0.174	0.098	0.007	0.105
			A	0.226	0.021	0.247	0.214	0.020	0.234	0.198	0.019	0.217	*0.172*	0.015	*0.187*
40	8	2	B	*0.296*	0.020	*0.316*	*0.255*	0.019	*0.274*	*0.223*	0.017	*0.240*	0.154	0.015	0.169
			C	0.288	0.019	0.307	0.244	0.016	0.260	0.220	0.014	0.234	0.146	0.011	0.157
			A	0.234	0.019	0.253	0.224	0.019	0.243	0.214	0.020	0.234	0.210	0.018	0.228
		5	B	0.337	0.024	0.361	*0.315*	0.024	*0.339*	0.282	0.023	0.305	*0.222*	0.021	*0.243*
			C	*0.342*	0.024	*0.366*	0.308	0.022	0.330	*0.284*	0.022	*0.306*	0.220	0.018	0.238
			A	0.197	0.012	0.209	0.176	0.012	0.188	*0.159*	0.011	*0.170*	*0.120*	0.008	*0.128*
		1	B	*0.234*	0.011	*0.245*	*0.187*	0.011	*0.198*	0.153	0.010	0.163	0.093	0.009	0.102
			C	0.228	0.010	0.238	0.180	0.009	0.189	0.152	0.007	0.159	0.088	0.006	0.094
			A	0.201	0.012	0.213	0.187	0.013	0.200	0.175	0.012	0.187	0.147	0.011	0.158
35	7	2	B	0.267	0.013	0.280	0.226	0.013	0.239	0.195	0.014	0.209	0.131	0.011	0.142
			C	*0.267*	0.013	*0.280*	*0.230*	0.012	*0.242*	*0.202*	0.011	*0.213*	*0.134*	0.009	*0.143*
			A	0.202	0.011	0.213	0.198	0.013	0.211	0.196	0.015	0.211	0.173	0.012	0.185
		5	B	0.299	0.014	0.313	0.272	0.017	0.289	0.244	0.018	0.262	0.184	0.016	0.200
			C	*0.320*	0.017	*0.337*	*0.288*	0.015	*0.303*	*0.260*	0.016	*0.276*	*0.198*	0.014	*0.212*
			A	0.171	0.004	0.175	0.153	0.005	0.158	0.143	0.006	0.149	*0.101*	0.005	*0.106*
		1	B	0.201	0.003	0.204	0.162	0.006	0.168	0.122	0.006	0.128	0.074	0.006	0.080
			C	*0.214*	0.003	*0.217*	*0.168*	0.006	*0.174*	*0.144*	0.005	*0.149*	0.082	0.002	0.084
			A	0.169	0.003	0.172	0.163	0.007	0.170	0.152	0.006	0.158	*0.125*	0.006	*0.131*
30	6	2	B	0.228	0.003	0.231	0.195	0.008	0.203	0.161	0.009	0.170	0.107	0.007	0.114
			C	*0.246*	0.003	*0.249*	*0.214*	0.006	*0.220*	*0.186*	0.007	*0.193*	0.120	0.006	0.126
			A	0.175	0.004	0.179	0.168	0.006	0.174	0.159	0.007	0.166	0.149	0.007	0.156
		5	B	0.250	0.004	0.254	0.226	0.008	0.234	0.203	0.010	0.213	0.148	0.009	0.157
			C	*0.286*	0.005	*0.291*	*0.257*	0.007	*0.264*	*0.240*	0.009	*0.249*	*0.178*	0.008	*0.186*
			A	0.142	0.000	0.142	0.126	0.001	0.127	0.117	0.002	0.119	*0.084*	0.002	*0.086*
		1	B	0.170	0.000	0.170	0.132	0.001	0.133	0.107	0.002	0.109	0.059	0.004	0.063
			C	*0.190*	0.000	*0.190*	*0.154*	0.002	*0.156*	*0.128*	0.003	*0.131*	0.070	0.002	0.072
			A	0.160	0.000	0.160	0.137	0.001	0.138	0.128	0.003	0.131	*0.105*	0.003	*0.108*
25	5	2	B	0.185	0.000	0.185	0.154	0.002	0.156	0.134	0.002	0.136	0.082	0.006	0.088
			C	*0.214*	0.000	*0.214*	*0.188*	0.002	*0.190*	*0.164*	0.003	*0.167*	0.102	0.003	0.105
			A	0.130	0.000	0.130	0.144	0.001	0.145	0.135	0.002	0.137	0.121	0.003	0.124
		5	B	0.205	0.000	0.205	0.180	0.001	0.181	0.157	0.003	0.160	0.116	0.005	0.121
			C	*0.256*	0.000	*0.256*	*0.230*	0.001	*0.231*	*0.210*	0.004	*0.214*	*0.154*	0.005	*0.159*

Table 13
Effect of Skew on Maximum Moments at Mid-span of Beams
Expressed in terms of percentage reductions in moments in corresponding right bridges

Span of Bridge a ft	Spacing of Beams b ft	Relative Stiffness of Beams H	$\varphi = 30°$ Rear Wheels	$\varphi = 30°$ Combined Rear and Front Wheels	$\varphi = 45°$ Rear Wheels	$\varphi = 45°$ Combined Rear and Front Wheels	$\varphi = 60°$ Rear Wheels	$\varphi = 60°$ Combined Rear and Front Wheels
80	8	2	23.4	21.7	30.6	28.9	33.5	32.4
		5	16.3	16.8	23.8	23.6	30.0	28.1
		10	8.3		16.2		25.7	
70	7	2	26.0	24.6	32.7	31.8	36.6	35.6
		5	16.6	17.1	26.4	27.5	31.0	29.7
		10	11.1		17.2		25.3	
60	6	2	28.6	28.0	33.8	34.3	37.8	37.4
		5	18.3	19.1	24.6	25.8	32.5	31.4
		10	8.7		14.1		20.9	
50	5	2	28.3	29.5	35.8	37.2	42.9	43.2
		5	16.7	18.3	23.7	24.6	32.5	33.3
		10	5.2		11.3		20.9	
40	8	1	18.6	18.5	30.6	29.5	46.5	45.8
		2	13.9	13.3	24.7	24.1	41.9	40.8
		5	7.9	7.4	17.0	16.4	35.1	33.6
35	7	1	20.1	19.2	32.1	30.6	48.7	47.8
		2	13.9	13.6	24.3	23.9	44.9	43.6
		5	10.0	10.1	18.7	18.1	38.1	37.1
30	6	1	21.5	19.8	32.7	31.3	52.8	51.2
		2	13.0	11.6	24.4	22.5	49.3	47.4
		5	10.1	9.4	16.1	14.4	37.8	36.1
25	5	1	18.9	17.9	32.6	31.1	55.8	54.7
		2	12.1	11.2	23.4	22.0	50.9	49.5
		5	10.2	9.8	18.0	16.4	39.8	37.9

therein. In all cases, coefficients are given for load positions with no wheel less than 2 ft from an edge beam which is assumed to be directly under the face of the curb. The moment coefficients given may be in error by several units in the last decimal place recorded.

The critical truck positions corresponding to the maximum moments as listed in Table 12 are not shown. It was found that in most cases the front wheel effects do not exercise appreciable decisive influence on the location of the critical truck positions. In general, a maximum moment in a certain beam may be obtained without placing a load on that beam. The critical truck position is usually such that one of the rear wheels of the truck, or of each of the two trucks, is on or quite close to the skew center line. Where two trucks were used to produce a maximum mid-span moment in the edge beam A or in the first interior beam B, the trucks were generally found to be heading in the same direction for the longer-span bridges ($b/a = 0.1$), and opposite directions for the shorter-span bridges ($b/a = 0.2$). For the maximum mid-span moment in the central beam C, however, the two trucks were almost in every case anti-symmetrically staggered with respect to both the skew and the longitudinal center lines of the bridge, that is, the trucks traveled in opposite directions and were at equal distances from, and on opposite sides of, the two center lines. For some bridges, it was found that there were several critical truck positions giving practically identical moments in a certain beam.

Based on a study of Table 12, the following observations may be made regarding the maximum moment produced at mid-span of beams by standard truck loadings:

1. For a given skew angle and given values of a and b, the maximum moment at the mid-span of a beam due to rear wheel loads always increases with H. This in general is also true for the corresponding moment produced by front wheel loads, but the effect of H is comparatively much smaller than for rear wheel loads.

2. The effects due to front wheel loads vary from zero to about 17% of the corresponding effects of rear wheel loads for the bridges considered. This percentage is smaller for shorter bridges, since the critical load positions are such that the front wheels are generally very close to the abutments, or not on the bridge at all.

3. For all of the right bridges considered, and with no wheels less than 2 ft from an edge beam, the maximum mid-span beam moment always occurs in an interior beam, that is, beam B or C. For skew bridges, while the maximum moment generally occurs in an interior beam, there is a tendency for an edge beam to become the critical beam. This tendency becomes more pronounced for one or a combination of the following characteristics of the bridge: (a) small H; (b) large angle of skew, φ; and (c) large span a (for a given ratio b/a).

4. The effect of skew is always a reduction in beam moments produced by rear wheel loads, and the greater the angle of skew, the larger is this reduction. The same result, but of much smaller magnitude, can be observed in the beam moments

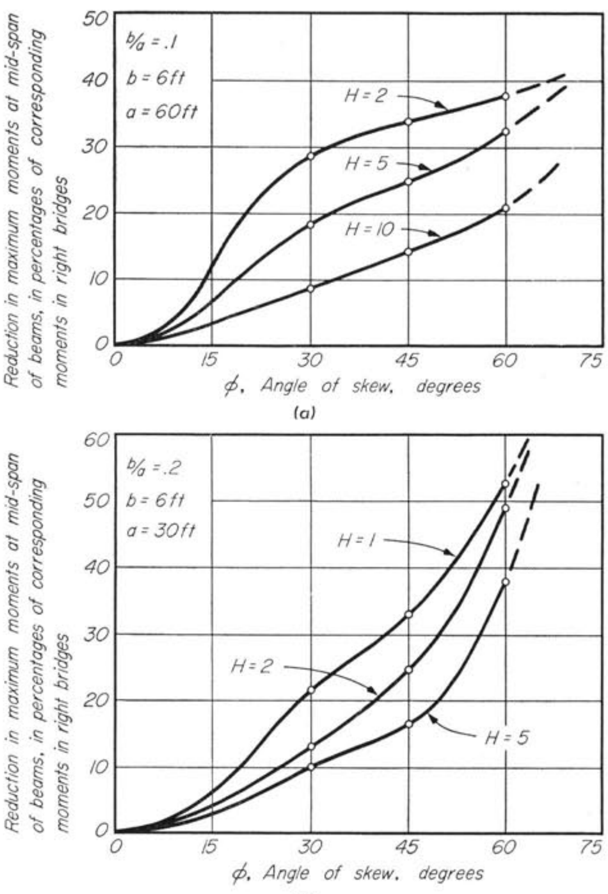


Fig. 15. Reduction in Maximum Moments at Mid-span of Beams Due to Rear Wheels of Standard Trucks

produced by corresponding front wheel loads on bridges 40 ft or more in span. For bridges 35 ft or less in span, mid-span beam moments due to front wheel loads are actually slightly larger for skew bridges than for corresponding right bridges. However, since rear wheel results are always much larger than corresponding front wheel effects, the consequence of skew on the combined rear and front wheel effects is still predominantly a reduction in mid-span beam moments. Percentagewise, this reduction is generally less than the corresponding reduction for the rear wheel effects.

25. Effect of Skew on Maximum Moment at Mid-span of Beams

The effect of skew is to reduce the beam moments in an I-beam bridge. The reductions in maximum mid-span beam moments caused by standard highway trucks may be calculated from the data in Table 12. The results are given in Table 13 where they are expressed in percentages of the corresponding moments in right bridges. Both the effect due to rear wheel loads alone and that due to combined rear and front wheel loads are listed in Table 13 for each structure considered. It appears that for the truck loadings considered, these two effects are nearly the same percentagewise for the majority of structures considered. In general, the percentage reductions are smaller for stiffer beams for a given ratio of b/a, and larger for greater angles of skew. For $b/a = 0.1$, and $H = 5$, the average reductions for spans between 80 and 50 ft range from about 17 to 31% for angles of skew from 30° to 60°. For $b/a = 0.2$, and $H = 2$, the average reductions for spans between 40 and 25 ft range from about 13 to 46% for angles of skew from 30° to 60°.

The percentage reductions in maximum mid-span beam moments caused by rear wheel loads alone are also shown graphically as a function of the angle of skew φ, in Figs. 15a and b for two typical bridges.

The maximum mid-span beam moments produced by rear wheel loads on skew bridges may be conveniently stated (as was done for right I-beam bridges in Ref. 2) in terms of an effective proportion, k, of a single rear wheel load which, when acting alone on a beam, would produce the same moment. That is,

$$M_b \text{ (for rear wheels only)} = (1/4)kPa \qquad (51)$$

This effective proportion, k, for a particular angle of skew, may be expressed as

$$k = \frac{b}{s}\left(1 - \frac{b}{aH}\right) \qquad (52)$$

where s is a length depending on the characteristics of the structure.

Values of s, computed from the actual moments given in Table 12, are plotted against values of $\dfrac{a}{10 \text{ ft} \sqrt{H}}$ in Figs. 16a, b, and c for the three angles of skew considered. The points seem to lie in a fairly well-defined band for $\varphi = 30°$ and 45°; for $\varphi = 60°$, there is a much wider scatter. However, reasonably safe values of s may be obtained by a set of empirical relations for $\varphi = 0°$, 30°, 45°, and 60° as follows:

$$\left.\begin{aligned} s_0 &= 4.40 \text{ ft.} + 0.42\, a/10 \text{ ft.} \sqrt{H}; \quad \text{(a)} \\ s_{30} &= 4.40 \text{ ft.} + 0.81\, a/10 \text{ ft.} \sqrt{H}; \quad \text{(b)} \\ s_{45} &= 4.40 \text{ ft.} + 1.07\, a/10 \text{ ft.} \sqrt{H}; \quad \text{(c)} \\ s_{60} &= 4.40 \text{ ft.} + 1.32\, a/10 \text{ ft.} \sqrt{H}, \quad \text{(d)} \end{aligned}\right\} \qquad (53)$$

in which the numerical subscripts to s denote

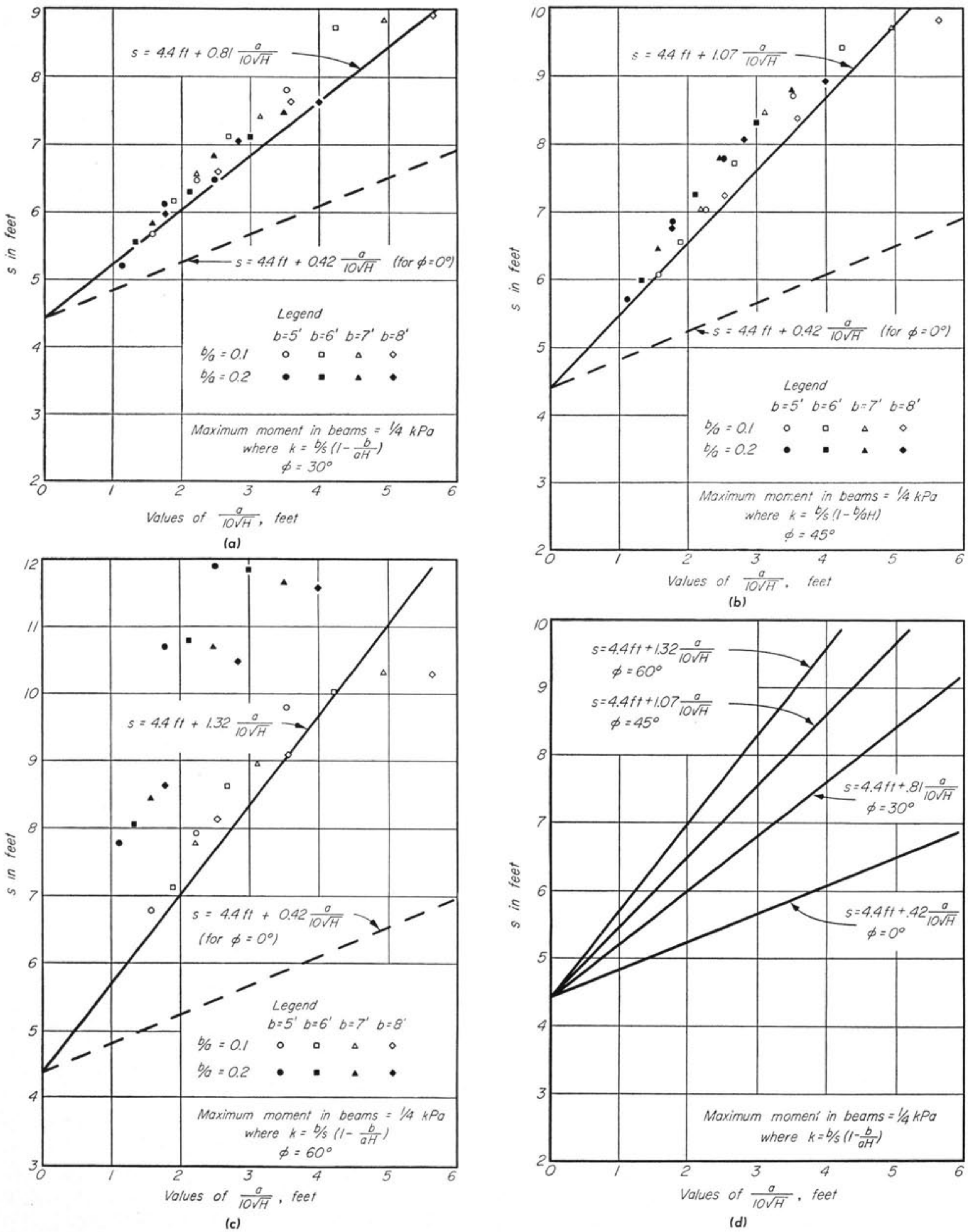


Fig. 16. Effective Proportion of Wheel Load To Be Used in Computing Maximum Moments at Mid-span of Beams Due to Rear Wheels of Standard Trucks

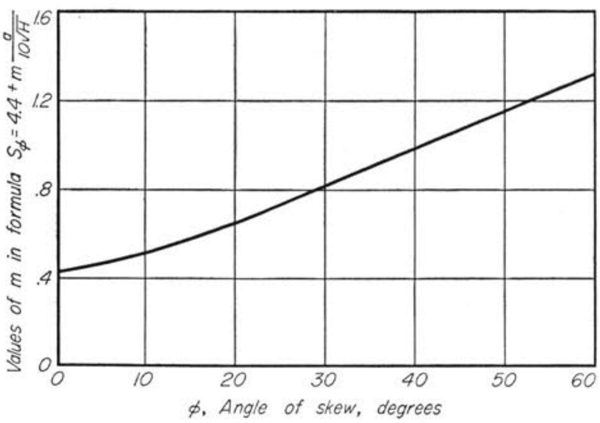


Fig. 17. Value of m as a Function of Skew

the angle of skew in degrees. Fig. 16d gives a graphical summary of Eqs. (53).

For any angle of skew, φ, between 0° and 60°, it appears that the safe value of s may be empirically expressed by

$$s_\varphi = 4.40 \text{ ft.} + m\, a/10 \text{ ft.} \sqrt{H}, \qquad (54)$$

where m is a parameter which depends on the angle of skew, φ, only. The variation of m as a function of φ is given by the curve shown in Fig. 17. This curve makes it convenient to find the "safe" value of s for a bridge having any angle of skew between 0° and 60°. With this value of s found from Fig. 17 and Eq. (54), the maximum mid-span beam moment in the bridge produced by rear wheels only may be calculated by using successively Eqs. (52) and (51).

As an alternative, the maximum mid-span beam moment in a skew bridge produced by rear wheels only may also be found in the following manner. Since this moment is, by Eq. (51), proportional to k, and hence, by Eq. (52), inversely proportional to s, it may be found as the corresponding moment in a right bridge multiplied by the ratio, s_0/s_φ, in which s_φ and s_0 are the values of s for the skew bridge in question and the corresponding right bridge. Figure 18 has been prepared to show a plot of curves giving values of s_0/s_φ for different angles of skew and values of $a/10\sqrt{H}$. These values of s_0/s_φ have been computed using Eqs. (53), and may be treated as reduction factors to be applied to maximum mid-span beam moments in right bridges to obtain the corresponding moments in skew bridges.

To illustrate the use of the empirical relations described above, consider the following example:

Given: A skew I-beam bridge has the following characteristics:
Span $a = 60$ ft.
Spacing of beams $b = 6$ ft.
Relative stiffness of beams $H = 4$
Angle of Skew $\varphi = 40°$

To find: Maximum mid-span beam moment in the bridge produced by rear wheels of standard trucks.

From Fig. 17, we obtain $m = 0.98$ for $\varphi = 40°$. Substituting this into Eq. (54), we find the value of s to be

$$s_{40} = 4.40 + 0.98\,(60/10\sqrt{4}) = 7.34 \text{ ft.}$$

Equation (52) gives the value of the effective proportion k as

$$k = (6/7.34)\,(1 - 6/60 \times 4) = 0.796.$$

Hence, by Eq. (51),

$$M_b \text{ (rear wheels)} = (1/4)\,(0.796)\,Pa$$
$$= 0.199\,Pa = 11.9\,P \text{ ft}$$

Or, for a corresponding right bridge, Eq. (53a) gives

$$s_0 = 4.40 + 0.42\,(60/10\sqrt{4}) = 5.66 \text{ ft.}$$

From Eq. (52), $k = (6/5.66)(1 - 6/60 \times 4) = 1.033$
Therefore for the right bridge,

$$M_b \text{ (rear wheels)} = (1/4)(1.033)Pa = 0.258\,Pa.$$

Now, from Fig. 18, for $a/10\sqrt{H} = 3$, and $\varphi = 40°$, we obtain

$$s_0/s_{40} = 0.77.$$

Therefore for the skew bridge in question,

$$M_b \text{ (rear wheels)} = 0.77\,(0.258\,Pa)$$
$$= 0.199\,Pa = 11.9\,P \text{ ft.}$$

For the effect of front wheel loads, the use of the following formula for right bridges having spans greater than 28 ft has been suggested in Ref. 2:

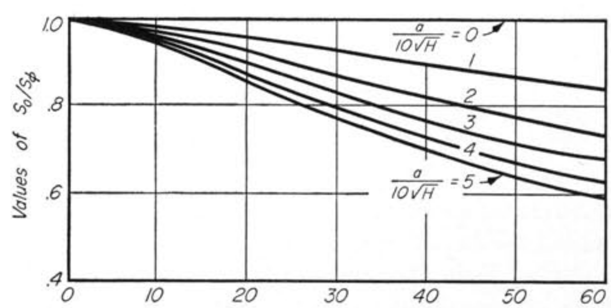


Fig. 18. Value of s_0/s_φ as a Function of Skew

$$M_b \text{ (front wheels only)} = (1/16)kP\,(a - 28 \text{ ft}), \quad (55)$$

in which the value of k is the same as for the rear wheels. For right bridges having spans equal to or less than 28 ft, the front wheel effects are to be taken as zero. A similar empirical relation could not be conveniently formulated for skew bridges. Table 13 shows that as a result of skew the percentage reductions in maximum mid-span beam moments for rear wheel loads are very nearly the same as those for combined rear and front wheel loads. This suggests a practical and simple procedure for estimating roughly the front wheel effects for skew bridges. The procedure consists of first finding the maximum mid-span beam moment in the corresponding right bridge by using Eq. (55), and then multiplying this by the same reduction factor used for rear wheel effects, s_0/s_φ, to obtain the front wheel effects. Thus, for the illustrative example given, the maximum mid-span beam moment produced by front wheels in the right bridge is, by Eq. (55),

$$M_b \text{ (front wheels, right bridge)} = (1/16)(1.033)\,P\,(60-28) = 2.066\,P \text{ ft}$$

Hence, the front wheel effect for the skew bridge is (s_0/s_φ) times the above value, that is,

$$M_b \text{ (front wheels, skew bridge)} = 0.77 \times 2.066\,P = 1.59\,P \text{ ft (approximately)}$$

26. Moments Produced by H-S Loading

In addition to the H loading, another system of loading, the H-S loading, is also given in the AASHO Specifications. The H-S loading consists of a tractor truck followed by a single-axle semi-trailer. The axle loads and spacing for the tractor truck are exactly the same as for the H loading. The weight on the trailer axle is the same as that on the rear axle of the H-truck. The distance between the trailer wheels is also 6 ft. The trailer axle is located at a variable distance v from the rear axle of the tractor truck. This distance v may vary from 14 to 30 ft, whichever value governs the design.

Using the influence surfaces for beam moments in the same manner as was done for the H loading, maximum mid-span beam moments were determined for the H-S loading for 30 structures. Because of lack of space the results are not given here. It was found, however, that the conclusions regarding the effect of skew on beam moments for the H loading are also generally valid for the H-S loading, so far as the moments produced by the tractor truck are concerned. The critical positions for maximum mid-span beam moments produced by the H-S loading are slightly shifted from the corresponding critical positions for the H loading. This shift in critical position generally causes a slight decrease in the moments produced by the tractor truck as compared to the corresponding moments produced by the H-truck. The combined maximum beam moments due to all three axles of the H-S loading are, of course, always larger than those due to the H loading because of the additional effects of the trailer axle.

No attempt has been made to estimate empirically the moment produced separately by each of the three axles of the H-S loading, as was done for the H loading in the preceding section. However, it was found that the theoretical value of a combined maximum mid-span beam moment produced by all three axles of the H-S loading is very closely given by the sum of the corresponding rear axle moment of the H-truck and five times the front axle moment of the H-truck. That is,

$$M_b \text{ (all 3 axles, } H\text{-}S \text{ loading)} \approx M_b \text{ (rear axle, } H \text{ loading)} + 5M_b \text{ (front axle, } H \text{ loading)} \quad (56)$$

This equation has been found to give values with an accuracy of within 1 or 2% for practically all of the structures considered and within 4 or 5% for one or two bridges outside the practical range.

The results of this study suggest a convenient way of estimating the total live load design moments for the H-S loading, once the corresponding moments produced separately by the rear and front axles of the H-truck have been found as described in Art. 25. For example, continuing with the illustrative example in Art. 25 and assuming that the maximum mid-span beam moment due to the H-S loading is desired, we have, from previous calculations,

$$M_b \text{ (rear axle, } H \text{ loading)} = 11.9\ P \text{ ft}$$
$$M_b \text{ (front axle, } H \text{ loading)} = 1.59\,P \text{ ft}$$

Hence, by Eq. (56), the combined mid-span beam moment due to the three axles of the H-S loading is

$$M_b \text{ (all 3 axles, } H\text{-}S \text{ loading)} = 11.9\,P \text{ ft} + 5\,(1.59\,P \text{ ft}) = 18.3\,P \text{ ft}$$

27. Moments Due to Line Loads and Uniform Loads

The dead load moments in an I-beam bridge depend on the manner in which the bridge is constructed. For a discussion of the various effects

Table 14
Moments at Mid-span of Beams Due to Uniformly Distributed Load Over Entire Bridge

Moments are given by a coefficient to be multiplied by wa^2b, in which w is the uniform load per unit of area, a the span, and b the spacing of the beams.

$b/a = 0.1$

Moment in Beam	$\varphi=0°$			$\varphi=30°$			$\varphi=45°$			$\varphi=60°$		
	$H=2$	$H=5$	$H=10$	$H=2$	$H=5$	$H=10$	$H=2$	$H=5$	$H=10$	$H=2$	$H=5$	$H=10$
A	0.091	0.088		0.061	0.075	0.077	0.056	0.073	0.077	0.058	0.074	0.079
B	0.098	0.104		0.057	0.077	0.090	0.048	0.069	0.083	0.043	0.063	0.077
C	0.101	0.110		0.060	0.081	0.097	0.053	0.074	0.090	0.044	0.066	0.082
*Sum	0.479	0.494	0.499	0.296	0.385	0.431	0.261	0.358	0.410	0.246	0.340	0.394

$b/a = 0.2$

Moment in Beam	$\varphi=0°$			$\varphi=30°$			$\varphi=45°$			$\varphi=60°$		
	$H=1$	$H=2$	$H=5$	$H=1$	$H=2$	$H=5$	$H=1$	$H=2$	$H=5$	$H=1$	$H=2$	$H=5$
A	0.077	0.077	0.071	0.065	0.072	0.071	0.060	0.069	0.071	0.047	0.059	0.068
B	0.090	0.100	0.111	0.066	0.081	0.099	0.056	0.073	0.092	0.038	0.054	0.076
C	0.095	0.108	0.122	0.069	0.087	0.107	0.058	0.076	0.099	0.034	0.053	0.079
*Sum	0.429	0.462	0.486	0.331	0.393	0.447	0.290	0.360	0.425	0.204	0.279	0.367

* Sum of the moments at mid-span of all the beams.

to be considered in calculating the dead load moments, reference is made to University of Illinois Engineering Experiment Station Bulletin 336[2] by Newmark and Siess.

For any type of dead load, the moments may be found by using influence surfaces such as those given in Appendix B. For example, the moment at mid-span of a beam produced by a uniformly distributed load over the entire bridge may be found as the volume under the influence surface plotted for that beam multiplied by the load intensity. The moment at mid-span of a beam produced by a uniform line load may be found as the area under the influence curve for the beam multiplied by the load intensity.

Tables 14 and 15 give coefficients for moments at mid-span of the beams due to a uniform load of w per unit of area over the entire bridge, and a line load of q per unit length of the beams. Values for right bridges based on data contained in Reference 2 are also included. In the preparation of the two tables, the areas under influence curves have been calculated by Simpson's ⅓-rule, and the volumes under influence surfaces have been calculated by the method of average end-areas. Tables 14 and 15 also give the sum of moments at mid-span of all five beams in each structure concerned.

It appears from Tables 14 and 15 that for both right and skew bridges, the moments at mid-span of the beams due to either uniform load over the entire bridge or line loads on all five beams are practically the same for all beams. That is, if the moment at mid-span of any beam is expressed as a percentage of the sum of moments in the beams across the bridge center line, this value is roughly 20%, with an average error of 2 to 3% in most structures considered, and a maximum error of 7% in one or two structures.

Table 15
Moments at Mid-span of Beams Due to Uniform Line Loads on Beams

Moments are given in terms of a coefficient to be multiplied by qa^2, in which q is the line load per unit of length of a beam, and a the span.

$b/a = 0.1$

Moment in Beam	Load on Beam	$\varphi=0°$			$\varphi=30°$			$\varphi=45°$			$\varphi=60°$		
		$H=2$	$H=5$	$H=10$	$H=2$	$H=5$	$H=10$	$H=2$	$H=5$	$H=10$	$H=2$	$H=5$	$H=10$
A	A, E	0.056	0.068		0.045	0.064	0.078	0.044	0.064	0.078	0.049	0.067	0.079
	B, D	0.044	0.041		0.029	0.035	0.033	0.026	0.033	0.033	0.026	0.033	0.034
	C	0.020	0.015		0.011	0.011	0.008	0.009	0.010	0.008	0.009	0.010	0.009
	All Beams	0.120	0.124		0.085	0.110	0.119	0.079	0.107	0.119	0.084	0.110	0.122
B	A, E	0.044	0.041		0.029	0.034	0.034	0.026	0.033	0.033	0.025	0.032	0.033
	B, D	0.050	0.053		0.028	0.039	0.047	0.023	0.034	0.043	0.021	0.031	0.040
	C	0.026	0.029		0.014	0.021	0.025	0.012	0.018	0.023	0.010	0.016	0.020
	All Beams	0.120	0.123		0.071	0.094	0.106	0.061	0.085	0.099	0.056	0.079	0.093
C	A, E	0.039	0.030		0.023	0.022	0.015	0.020	0.020	0.015	0.016	0.019	0.017
	B, D	0.052	0.058		0.031	0.043	0.052	0.027	0.039	0.048	0.023	0.034	0.042
	C	0.029	0.036		0.017	0.026	0.036	0.015	0.024	0.033	0.014	0.022	0.030
	All Beams	0.120	0.124		0.071	0.091	0.103	0.062	0.083	0.096	0.053	0.075	0.089
*Sum	All Beams	0.600	0.618	0.624	0.383	0.499	0.553	0.342	0.467	0.532	0.333	0.453	0.519

$b/a = 0.2$

Moment in Beam	Load on Beam	$\varphi=0°$			$\varphi=30°$			$\varphi=45°$			$\varphi=60°$		
		$H=1$	$H=2$	$H=5$	$H=1$	$H=2$	$H=5$	$H=1$	$H=2$	$H=5$	$H=1$	$H=2$	$H=5$
A	A, E	0.062	0.076	0.094	0.058	0.074	0.092	0.058	0.073	0.091	0.054	0.071	0.089
	B, D	0.036	0.033	0.027	0.029	0.030	0.027	0.026	0.029	0.026	0.018	0.022	0.023
	C	0.013	0.009	0.001	0.009	0.007	0.003	0.008	0.007	0.004	0.004	0.006	0.005
	All Beams	0.111	0.118	0.122	0.096	0.111	0.122	0.092	0.109	0.121	0.076	0.099	0.117
B	A, E	0.036	0.033	0.027	0.029	0.030	0.027	0.026	0.028	0.026	0.017	0.021	0.022
	B, D	0.047	0.053	0.063	0.034	0.044	0.057	0.029	0.039	0.054	0.020	0.030	0.046
	C	0.025	0.028	0.031	0.017	0.022	0.026	0.014	0.019	0.023	0.009	0.013	0.018
	All Beams	0.108	0.114	0.121	0.080	0.096	0.110	0.069	0.086	0.103	0.046	0.064	0.086
C	A, E	0.025	0.017	0.003	0.018	0.014	0.005	0.014	0.014	0.009	0.006	0.009	0.010
	B, D	0.050	0.057	0.062	0.036	0.045	0.053	0.029	0.038	0.047	0.016	0.025	0.035
	C	0.031	0.041	0.055	0.024	0.033	0.050	0.021	0.030	0.046	0.015	0.023	0.038
	All Beams	0.106	0.115	0.120	0.078	0.092	0.108	0.064	0.082	0.102	0.037	0.057	0.083
*Sum	All Beams	0.544	0.579	0.606	0.430	0.506	0.572	0.386	0.472	0.550	0.281	0.383	0.489

* Sum of the moments at mid-span of all 5 beams loaded with q.

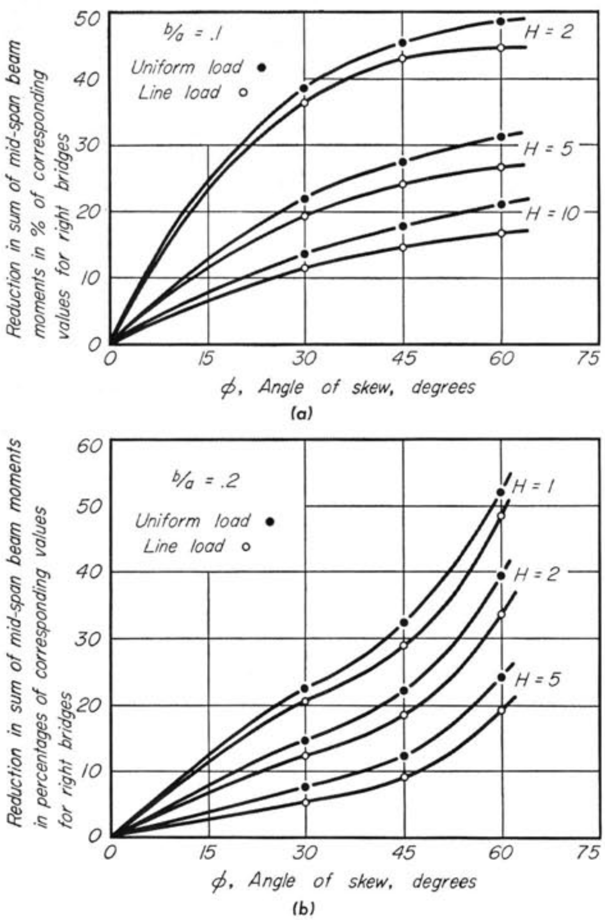


Fig. 19. Reduction in Sum of Moments in Beams at Mid-span Due to Line Loads on All Beams and Uniform Load Over Entire Bridge

A further observation may be made regarding the sum of beam moments across the center line of the right bridge produced by uniform surface load and uniform line loads on all five beams. This sum is very approximately given by

$$\Sigma M = (1/8)\, Wa\,(1 - b/aH)^* \qquad (55a)$$

in which W is the total load on the bridge, and is therefore given by

$$\left.\begin{array}{ll} W = 4wab & \text{for uniform load on entire bridge,} \\ \text{and } W = 5qa & \text{for uniform line loads on all five beams} \end{array}\right\} \qquad (55b)$$

For skew bridges, the sum of beam moments across the center line of the bridge is less than that in the corresponding right bridge. The percentage reductions in this sum of beam moments as a result of skew for the various skew bridges considered are shown by the curves in Fig. 19.

* As explained in Ref. (2), p. 41, the quantity $(1\text{-}b/aH)$ takes account approximately of the part of the total moment carried by the slab.

28. Maximum Mid-span Beam Moments and Absolute Maximum Beam Moments

A distinction should be made between the maximum mid-span beam moments as reported and the absolute maximum beam moments which should be used in design. Except for right bridges, the two are different. To locate the section of absolute maximum moment due to a given loading is a relatively simple matter for a single isolated beam, but is much more involved for an I-beam bridge. For a skew bridge the distance from the mid-point of a beam to the section of absolute maximum moment is probably larger for edge beams than for interior beams. It is also larger for larger angles of skew. The same trend holds more or less proportionately for the difference between the maximum mid-span and absolute maximum moments. For the central beam, the critical section is not likely to be too remote from mid-span, irrespective of the angle of skew. Hence, the maximum mid-span moment should not differ much from the absolute maximum moment.

In Art. 24 it was found that the maximum mid-span beam moment in an I-beam bridge generally occurs in an interior beam and there is a marked tendency for the edge beam to become the critical beam for large φ, small H, and large a (for a given ratio b/a). If it may be assumed that this tendency also holds for the absolute maximum beam moments, then the following conclusions seem to be reasonable:

(1) Since the central or the first interior beams are critical for most of the bridges considered with φ less than 60°, the absolute maximum beam moments may be taken as equal to the maximum mid-span beam moments given in Table 12.

(2) For bridges with φ equal to 60° or more, the difference between the absolute maximum and maximum mid-span beam moments becomes larger. Also the effect of H and a on this difference is more pronounced than for bridges having φ less than 60°. This explains in part the much wider scatter of points shown in Fig. 16c. For $\varphi = 60°$, the maximum mid-span beam moments listed in Table 12 are expected to be somewhat less than the absolute maximum beam moments by varying amounts, depending on H, b/a and a. However, as indicated in Fig. 16c, in the derivation of the empirical relationship for s, the straight line represented by Eq. (53a) was made to give the lowest value of s for all the bridges considered except for two or three structures whose proportions do not lie within the practical ranges of I-beam

bridges. That is, the use of the empirical formula, Eq. (53a), and the formulas derived gives a little larger maximum mid-span beam moments than those given in Table 12. This excess in moments will serve to compensate more or less for the difference between the maximum mid-span and absolute maximum beam moments.

For practical design purposes the empirical relations given for estimating maximum beam moments in skew I-beam bridges may be used without involving appreciable error.

29. Application to Composite Bridges

In I-beam bridges, composite action of the slab with the beams may exist due either to bond between the concrete and the beams or to shear connectors of some kind. This composite action may be complete or limited, depending on the degree of the bond and the effectiveness of the shear connectors. When composite action is present, a panel of the slab, or part of it, acts as the upper flange of the composite beam. The composite bridge therefore has a larger value of H, and its beams have relatively larger moments compared with the non-composite bridge. As the distance from the neutral axis of the composite section to the bottom flange is considerably greater than in a non-composite design, the stress controlling the design of a composite beam is decreased much less proportionately than the stiffness is increased. Therefore if composite action is taken advantage of in the design of an I-beam bridge, lighter steel beams may generally be used.

While the analyses reported do not consider composite action, the results may still be applied with reasonable accuracy to composite structures, provided that proper values of H are used, based on known or assumed amounts of interaction of the slab with the beams. For bridges with properly designed shear connectors, full composite action may be expected. If it is further assumed that for the proportions of the bridges considered, a full panel width of the slab acts with the beam as a composite section, the value of E_bI_b may be computed for the transformed section of the T-beam, and H determined in the usual way from the formula

$$H = E_bI_b/aN.$$

The structure may then be regarded as if the beams actually had the value of H computed. It is true that for edge beams, only half panel widths are effective, but curbs and sidewalks which the present study did not consider will more or less make up for this discrepancy.

VII. SUMMARY AND CONCLUSIONS

30. General Summary

An analytical study has been made of the behavior of the simply-supported skew I-beam bridge, which consists of a slab continuous over five equally spaced beams parallel to the direction of traffic. Because of the complicated nature of the boundary conditions in the problem, no exact solution has been available. In this study, the numerical method of finite differences was used.

Difference equations have been developed for a general system of skew coordinates to permit the analyses of the structures for any angle of skew, φ, ratio of beam spacing to span, b/a, and relative stiffness of the beams and slab, H.

A total of 18 skew bridges were considered, having physical characteristics defined by combinations of the following parameter values: $\varphi = 30°$, $45°$, and $60°$; $H = 2$, 5, and 10 for $b/a = 0.1$; and $H = 1$, 2, and 5 for $b/a = 0.2$. With the aid of the ILLIAC, the University of Illinois electronic digital computer, influence coefficients for each of the 18 structures were computed for moments in beams at five locations at or near mid-span, for transverse moments in the slab at four locations at mid-span, and for deflections of the center beam at mid-span.

The results obtained were approximate. In order to test the degree of approximation, difference analyses were made for a group of right I-beam bridges for which exact solutions are available. The influence values obtained were compared with the corresponding exact values. Based on these comparisons, conclusions were drawn regarding the accuracy of the difference solutions and the nature and amount of the corrections, where needed, to be applied to the approximate results. These corrections were then applied, with some modification, to the influence values determined for the skew bridges wherever discrepancies were believed to exist. It is believed that after these improvements the tabulated values of the influence coefficients given are free of serious error, and are in general sufficiently accurate for practical purposes. From the tabulated values, influence surfaces in the form of contour maps have been prepared for several structures.

Maximum live load moments at mid-span of beams for standard trucks were computed for 72 structures, having beam spacings of 5, 6, 7, and 8 ft for different values of the variables, b/a, H, and φ. One or two lanes of loading were used in the calculations, depending on which gave larger moments. Only one truck in each lane was considered, but the two trucks in adjacent lanes were assumed to be traveling in the same or opposite directions as required to produce maximum effects. The distance between the center of a wheel and the edge of the bridge was taken as a minimum of 2 ft, and the distance between the centers of the nearest wheels of trucks in adjacent lanes was taken as a minimum of 4 ft. Based on a study of the computed coefficients for maximum beam moments at mid-span, empirical relationships were developed for use in estimating the maximum live load beam moment at mid-span of any skew bridge with physical characteristics within the ranges considered in this investigation.

Tables and graphs have also been prepared to facilitate the calculation of dead load moments in beams at mid-span of skew bridges. The two types of dead load considered were: (1) uniformly distributed load over the entire bridge; and (2) uniform line loads on all five beams.

31. Conclusions

(a) *Difference Equations as a Method of Analysis of I-beam Bridges*

(1) The method of finite differences for analysis is satisfactory for the determination of beam moments in I-beam bridges. Even for a coarse network of points as used in this study, the results obtained are quite reliable.

(2) Serious errors are to be expected, however, in the difference solutions for slab moments in the I-beam bridge for certain load positions, unless a much finer network of points is chosen. Although, as in this investigation, corrections may be derived to improve the results as a remedial measure; the

difference method is satisfactory for determining slab moments in I-beam bridges only if a relatively fine network can be used.

(b) *Moments in Beams at Mid-span*

(3) The effect of skew is always a reduction in beam moments for rear wheel loads of standard H-trucks, and the greater the angle of skew, the larger is this reduction. For $b/a = 0.1$, and $H = 5$, the average reductions for spans between 80 and 50 ft range from about 17 to 31% for angles of skew from 30° to 60°; and for $b/a = 0.2$, and $H = 2$, the average reductions for spans between 40 and 25 ft range from about 13 to 46% for angles of skew from 30° to 60°. These percentages are in terms of the corresponding moments in right bridges.

(4) The effects of front wheel loads vary from zero to about 17% of the corresponding effects of rear wheel loads. This percentage is smaller for shorter bridges. As a result of skew, the beam moments for front wheel loads are in general also reduced, but the amount of reduction is much smaller than for rear wheel loads. Thus the total reductions in beam moments for combined rear and front wheels are, percentagewise, practically the same as the reductions in beam moments for rear wheels alone.

(5) In general, front wheel results do not affect the critical truck loading positions which produce the maximum moment in a beam. For the truck loads considered, the critical load position is generally one where the rear wheels are quite close to the skew center line of the bridge.

(6) The maximum beam moment at mid-span of a skew bridge for the truck loading considered may be estimated by using the empirical relationships given in Eqs. (51) to (54), together with the use of the curves in Figs. 17 and 18.

(7) For a uniformly distributed load over the entire bridge, or for uniform line loads on all 5 beams, the mid-span moment in a beam may be assumed to be roughly ⅕ of the sum of beam moments across the skew center line of the bridge.

VIII. REFERENCES

1. Newmark, N. M.: "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Supports," University of Illinois Engineering Experiment Station Bulletin 304, 1938.
2. Newmark, N. M., and Siess, C. P.: "Moments in I-Beam Bridges," University of Illinois Engineering Experiment Station Bulletin 336, 1942.
3. Newmark, N. M., Siess, C. P., and Penman, R. R.: "Studies of Slab and Beam Highway Bridges, Part I — Tests of Simple-Span Right I-Beam Bridges," University of Illinois Engineering Experiment Station Bulletin 363, 1946.
4. Newmark, N. M., Siess, C. P., and Peckham, W. M.: "Studies of Slab and Beam Highway Bridges, Part II — Tests of Simple-Span Skew I-Beam Bridges," University of Illinois Engineering Experiment Station Bulletin 375, 1948.
5. Jensen, V. P.: "Analysis of Skew Slabs," University of Illinois Engineering Experiment Station Bulletin 332, 1941.
6. Salvadori, M. G., and Baron, M. L.: "Numerical Methods in Engineering," New York: Prentice-Hall, 1952, pp. 233-4.
7. Timoshenko, S.: "Theory of Plates and Shells," New York: McGraw-Hill, 1940.
8. Westergaard, H. M.: "Computation of Stresses in Bridge Slabs due to Wheel Loads," Public Roads, Vol. 11, No. 1, 1930, pp. 1-23.
9. Nádai, A.: "Die elastischen Platten," Berlin: Julius Springer, 1925.
10. Marcus, H.: "Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten," Berlin: Julius Springer, 1932.
11. Richart, F. E., Newmark, N. M., and Siess, C. P.: "Highway Bridge Floors," University of Illinois Engineering Experiment Station Reprint No. 45, 1949.
12. Newmark, N. M.: "Note on Calculation of Influence Surfaces in Plates by Use of Difference Equations," Journal of Applied Mechanics, Vol. 8, No. 2, June 1941, p. A-92.

APPENDIX A: TABLES OF INFLUENCE COEFFICIENTS FOR MOMENTS IN BEAMS, MOMENTS IN SLABS, AND DEFLECTIONS OF BEAMS

The tables contained herein give numerical values of influence coefficients due to a unit concentrated load applied along various longitudinal lines A, AB, B, etc., as shown in Fig. 1. The longitudinal position of the load is indicated by the distance from the left end of the bridge, shown as a proportion of the span. The quantities φ, H, b, and a are defined in Chapter I. Poisson's ratio is zero. The quantity M_{oy} is given by Eq. (49a).

The numerical values given are such that:

(1) Actual moments in beams for concentrated loads are obtained by multiplying the tabulated values for moments in beams by the quantity Pa.

(2) Actual transverse moments per unit of length in slab for concentrated loads are obtained by multiplying the tabulated values for transverse moments in slab by the quantity P.

(3) Actual deflections of beams for concentrated loads are obtained by multiplying the tabulated values for deflections of beams by the quantity Pa^3/E_bI_b.

In the tables for slab moments, where two values are given at any point, the one in the parentheses is believed to be in error, and is replaced by the improved value which appears above the value in parentheses. The need for this correction and the method of estimating its value are discussed in Chapter IV.

Table A-1

Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=30°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment, Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.022	0.048	0.082	0.131	0.083	0.049	0.023
	A B	0.021	0.043	0.069	0.089	0.059	0.037	0.017
	B	0.019	0.036	0.049	0.053	0.042	0.027	0.013
	B C	0.014	0.025	0.031	0.031	0.027	0.018	0.009
	C	0.009	0.014	0.017	0.017	0.014	0.010	0.005
	C D	0.004	0.006	0.006	0.006	0.004	0.003	0.002
	D	0.000	−0.001	−0.002	−0.002	−0.002	−0.002	−0.001
	D E	−0.003	−0.006	−0.008	−0.009	−0.008	−0.006	−0.003
	E	−0.006	−0.011	−0.014	−0.014	−0.012	−0.009	−0.005
B at center	A	0.014	0.028	0.043	0.054	0.050	0.035	0.018
	A B	0.011	0.022	0.037	0.067	0.044	0.025	0.011
	B	0.008	0.017	0.034	0.071	0.034	0.016	0.006
	B C	0.007	0.017	0.033	0.052	0.025	0.012	0.005
	C	0.008	0.017	0.027	0.030	0.019	0.010	0.004
	C D	0.007	0.014	0.018	0.017	0.012	0.007	0.003
	D	0.005	0.008	0.010	0.009	0.007	0.004	0.002
	D E	0.002	0.003	0.003	0.002	0.002	0.001	0.001
	E	−0.001	−0.002	−0.003	−0.003	−0.002	−0.001	0.000
C at 2/8-point	A	0.009	0.016	0.020	0.021	0.018	0.013	0.007
	A B	0.013	0.022	0.026	0.022	0.016	0.010	0.004
	B	0.017	0.032	0.030	0.021	0.012	0.006	0.002
	B C	0.019	0.048	0.030	0.015	0.007	0.002	0.001
	C	0.021	0.059	0.024	0.009	0.003	0.000	0.000
	C D	0.022	0.044	0.019	0.008	0.001	0.000	−0.001
	D	0.017	0.024	0.016	0.007	0.002	0.000	0.000
	D E	0.010	0.013	0.011	0.006	0.003	0.001	0.000
	E	0.003	0.006	0.007	0.006	0.004	0.002	0.001
C at 3/8-point	A	0.008	0.014	0.017	0.018	0.017	0.013	0.007
	A B	0.009	0.017	0.022	0.023	0.018	0.011	0.005
	B	0.009	0.020	0.030	0.027	0.017	0.009	0.003
	B C	0.008	0.020	0.045	0.027	0.012	0.004	0.001
	C	0.008	0.022	0.057	0.021	0.006	0.001	0.000
	C D	0.010	0.026	0.044	0.017	0.005	0.001	0.000
	D	0.012	0.023	0.026	0.015	0.006	0.002	0.001
	D E	0.010	0.016	0.016	0.012	0.007	0.004	0.002
	E	0.005	0.008	0.010	0.010	0.008	0.005	0.002
C at center	A	0.006	0.012	0.016	0.018	0.017	0.014	0.008
	A B	0.006	0.012	0.019	0.023	0.023	0.017	0.009
	B	0.005	0.012	0.021	0.032	0.028	0.018	0.008
	B C	0.004	0.011	0.021	0.049	0.030	0.015	0.006
	C	0.004	0.011	0.025	0.061	0.025	0.011	0.004

Table A-2

Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=30°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment, Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.032	0.067	0.110	0.164	0.111	0.069	0.033
	A B	0.028	0.057	0.086	0.107	0.076	0.050	0.024
	B	0.023	0.043	0.057	0.061	0.051	0.035	0.018
	B C	0.015	0.027	0.032	0.033	0.031	0.022	0.011
	C	0.007	0.013	0.015	0.016	0.015	0.011	0.006
	C D	0.002	0.003	0.004	0.004	0.004	0.004	0.002
	D	−0.001	−0.002	−0.003	−0.003	−0.003	−0.002	−0.001
	D E	−0.004	−0.007	−0.008	−0.009	−0.008	−0.006	−0.003
	E	−0.006	−0.010	−0.013	−0.013	−0.012	−0.009	−0.005
B at center	A	0.019	0.036	0.051	0.061	0.056	0.042	0.022
	A B	0.014	0.029	0.048	0.082	0.057	0.033	0.015
	B	0.011	0.024	0.048	0.092	0.048	0.024	0.010
	B C	0.011	0.025	0.045	0.068	0.037	0.020	0.009
	C	0.012	0.024	0.035	0.038	0.029	0.018	0.008
	C D	0.010	0.018	0.022	0.022	0.019	0.013	0.006
	D	0.006	0.010	0.012	0.012	0.010	0.008	0.004
	D E	0.002	0.003	0.004	0.004	0.003	0.003	0.001
	E	−0.002	−0.003	−0.004	−0.003	−0.003	−0.002	−0.001
C at 2/8-point	A	0.006	0.010	0.014	0.016	0.015	0.012	0.007
	A B	0.012	0.020	0.025	0.024	0.019	0.012	0.006
	B	0.019	0.034	0.035	0.028	0.018	0.010	0.005
	B C	0.025	0.061	0.042	0.024	0.013	0.007	0.003
	C	0.031	0.078	0.038	0.018	0.008	0.004	0.001
	C D	0.030	0.057	0.030	0.016	0.007	0.004	0.001
	D	0.019	0.029	0.024	0.016	0.009	0.005	0.002
	D E	0.010	0.015	0.016	0.013	0.009	0.005	0.003
	E	0.003	0.006	0.008	0.009	0.008	0.006	0.003
C at 3/8-point	A	0.007	0.012	0.015	0.016	0.016	0.012	0.007
	A B	0.010	0.020	0.024	0.026	0.023	0.016	0.008
	B	0.014	0.027	0.037	0.036	0.027	0.016	0.007
	B C	0.014	0.031	0.062	0.042	0.023	0.011	0.004
	C	0.014	0.037	0.080	0.038	0.017	0.007	0.003
	C D	0.017	0.039	0.061	0.031	0.015	0.008	0.003
	D	0.017	0.030	0.034	0.026	0.016	0.009	0.004
	D E	0.012	0.019	0.020	0.020	0.014	0.009	0.004
	E	0.005	0.008	0.011	0.013	0.012	0.009	0.005
C at center	A	0.007	0.012	0.016	0.016	0.015	0.012	0.007
	A B	0.008	0.016	0.023	0.026	0.026	0.021	0.011
	B	0.009	0.019	0.030	0.039	0.036	0.026	0.013
	B C	0.008	0.018	0.033	0.065	0.044	0.024	0.010
	C	0.007	0.019	0.040	0.083	0.040	0.019	0.007

Table A-3
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=10$
Angle of Skew $\phi=30°$

Moment in Beam	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A at center	A	0.039	0.080	0.128	0.185	0.129	0.081	0.039
	A B	0.032	0.064	0.094	0.116	0.084	0.056	0.028
	B	0.023	0.043	0.056	0.060	0.052	0.037	0.019
	B C	0.013	0.023	0.028	0.029	0.028	0.021	0.011
	C	0.005	0.008	0.010	0.011	0.011	0.009	0.005
	C D	0.000	0.000	0.000	0.001	0.001	0.001	0.001
	D	−0.002	−0.004	−0.005	−0.005	−0.005	−0.003	−0.002
	D E	−0.004	−0.006	−0.008	−0.008	−0.007	−0.006	−0.003
	E	−0.004	−0.007	−0.010	−0.010	−0.009	−0.007	−0.004
B at center	A	0.020	0.038	0.052	0.060	0.056	0.042	0.022
	A B	0.016	0.033	0.055	0.091	0.065	0.038	0.017
	B	0.013	0.031	0.060	0.108	0.060	0.031	0.013
	B C	0.014	0.032	0.056	0.080	0.047	0.028	0.013
	C	0.015	0.030	0.040	0.043	0.036	0.024	0.012
	C D	0.012	0.020	0.024	0.024	0.022	0.017	0.009
	D	0.006	0.010	0.012	0.012	0.011	0.009	0.005
	D E	0.002	0.002	0.003	0.003	0.003	0.002	0.001
	E	−0.002	−0.004	−0.005	−0.005	−0.004	−0.003	−0.002
C at 2/8-point	A	0.003	0.005	0.007	0.009	0.010	0.008	0.005
	A B	0.010	0.016	0.022	0.023	0.019	0.014	0.007
	B	0.020	0.035	0.038	0.033	0.024	0.015	0.007
	B C	0.030	0.070	0.052	0.033	0.020	0.011	0.005
	C	0.040	0.094	0.051	0.027	0.014	0.007	0.003
	C D	0.036	0.066	0.039	0.024	0.013	0.008	0.004
	D	0.019	0.030	0.028	0.022	0.015	0.009	0.004
	D E	0.009	0.014	0.018	0.016	0.012	0.008	0.004
	E	0.002	0.004	0.007	0.008	0.008	0.006	0.003
C at 3/8-point	A	0.004	0.008	0.010	0.011	0.010	0.008	0.005
	A B	0.010	0.020	0.023	0.025	0.024	0.018	0.010
	B	0.016	0.031	0.042	0.041	0.033	0.022	0.011
	B C	0.019	0.041	0.075	0.055	0.034	0.019	0.008
	C	0.021	0.050	0.099	0.053	0.027	0.014	0.006
	C D	0.022	0.049	0.074	0.043	0.024	0.014	0.006
	D	0.019	0.034	0.039	0.034	0.024	0.015	0.007
	D E	0.012	0.019	0.021	0.022	0.018	0.012	0.006
	E	0.003	0.006	0.008	0.010	0.010	0.008	0.004
C at center	A	0.005	0.009	0.011	0.011	0.010	0.008	0.004
	A B	0.009	0.017	0.024	0.025	0.025	0.021	0.012
	B	0.012	0.024	0.036	0.044	0.042	0.031	0.016
	B C	0.012	0.026	0.044	0.079	0.056	0.032	0.014
	C	0.012	0.028	0.055	0.102	0.055	0.028	0.012

Table A-4
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=45°$

Moment in Beam	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A at center	A	0.020	0.044	0.077	0.125	0.077	0.045	0.021
	A B	0.020	0.042	0.067	0.081	0.051	0.030	0.013
	B	0.019	0.035	0.047	0.047	0.034	0.021	0.009
	B C	0.014	0.024	0.028	0.027	0.021	0.013	0.006
	C	0.009	0.014	0.015	0.014	0.011	0.007	0.003
	C D	0.004	0.006	0.006	0.005	0.003	0.002	0.001
	D	0.001	0.000	0.000	−0.001	−0.001	−0.001	−0.001
	D E	−0.002	−0.004	−0.005	−0.005	−0.005	−0.004	−0.002
	E	−0.005	−0.008	−0.009	−0.009	−0.008	−0.005	−0.003
B at center	A	0.011	0.023	0.036	0.048	0.046	0.034	0.017
	A B	0.009	0.018	0.031	0.058	0.042	0.023	0.009
	B	0.007	0.015	0.030	0.064	0.029	0.012	0.004
	B C	0.007	0.016	0.031	0.045	0.018	0.008	0.003
	C	0.008	0.017	0.025	0.025	0.012	0.005	0.002
	C D	0.007	0.013	0.015	0.013	0.007	0.003	0.001
	D	0.005	0.008	0.008	0.006	0.003	0.001	0.000
	D E	0.002	0.003	0.002	0.001	0.001	0.000	0.000
	E	−0.001	−0.002	−0.003	−0.002	−0.001	−0.001	0.000
C at 2/8-point	A	0.010	0.017	0.022	0.023	0.021	0.015	0.008
	A B	0.012	0.022	0.027	0.024	0.018	0.010	0.004
	B	0.014	0.030	0.031	0.022	0.012	0.005	0.002
	B C	0.016	0.044	0.030	0.014	0.005	0.001	−0.001
	C	0.019	0.053	0.020	0.006	−0.001	−0.002	−0.002
	C D	0.021	0.037	0.012	0.002	−0.003	−0.003	−0.002
	D	0.014	0.018	0.008	0.001	−0.003	−0.003	−0.002
	D E	0.008	0.088	0.004	0.000	−0.002	−0.002	−0.001
	E	0.001	0.001	0.001	0.000	−0.001	−0.001	−0.001
C at 3/8-point	A	0.007	0.013	0.016	0.018	0.017	0.013	0.007
	A B	0.007	0.014	0.020	0.022	0.018	0.012	0.005
	A	0.007	0.016	0.027	0.026	0.017	0.008	0.003
	B C	0.007	0.016	0.040	0.027	0.011	0.003	0.000
	C	0.006	0.020	0.052	0.018	0.004	−0.001	−0.001
	C D	0.009	0.025	0.038	0.011	0.001	−0.002	−0.001
	D	0.011	0.020	0.020	0.008	0.001	−0.001	−0.001
	D E	0.008	0.012	0.011	0.006	0.002	0.000	0.000
	E	0.003	0.005	0.005	0.004	0.003	0.001	0.001
C at center	A	0.005	0.010	0.013	0.015	0.015	0.012	0.007
	A B	0.004	0.009	0.014	0.020	0.021	0.016	0.009
	B	0.004	0.009	0.016	0.027	0.027	0.018	0.008
	B C	0.003	0.008	0.019	0.043	0.030	0.014	0.005
	C	0.003	0.009	0.023	0.056	0.023	0.009	0.003

Table A-5
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=45°$

Moment in Beam	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A at center	A	0.030	0.064	0.106	0.160	0.107	0.066	0.031
	A B	0.028	0.057	0.085	0.101	0.069	0.044	0.021
	B	0.023	0.043	0.055	0.056	0.045	0.030	0.015
	B C	0.016	0.026	0.030	0.030	0.027	0.018	0.009
	C	0.007	0.012	0.015	0.015	0.013	0.010	0.005
	C D	0.002	0.004	0.005	0.005	0.004	0.004	0.002
	D	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	0.000
	D E	−0.002	−0.004	−0.005	−0.005	−0.005	−0.003	−0.002
	E	−0.004	−0.007	−0.009	−0.009	−0.008	−0.005	−0.003
B at center	A	0.016	0.032	0.046	0.056	0.054	0.041	0.021
	A B	0.013	0.025	0.041	0.074	0.055	0.031	0.013
	B	0.009	0.022	0.044	0.085	0.043	0.020	0.008
	B C	0.010	0.024	0.044	0.061	0.030	0.015	0.007
	C	0.012	0.023	0.032	0.033	0.022	0.013	0.006
	C D	0.010	0.017	0.020	0.018	0.014	0.009	0.004
	D	0.006	0.009	0.010	0.010	0.008	0.005	0.003
	D E	0.002	0.003	0.004	0.004	0.003	0.002	0.001
	E	−0.001	−0.002	−0.002	−0.001	−0.001	0.000	0.000
C at 2/8-point	A	0.007	0.012	0.016	0.018	0.017	0.013	0.007
	A B	0.011	0.020	0.025	0.025	0.020	0.013	0.006
	B	0.017	0.033	0.035	0.029	0.019	0.010	0.004
	B C	0.021	0.057	0.042	0.023	0.012	0.005	0.001
	C	0.028	0.073	0.033	0.014	0.005	0.001	0.000
	C D	0.029	0.050	0.021	0.010	0.003	0.001	0.000
	D	0.017	0.023	0.016	0.009	0.004	0.001	0.000
	D E	0.008	0.011	0.011	0.007	0.004	0.002	0.001
	E	0.002	0.004	0.006	0.006	0.005	0.003	0.002
C at 3/8-point	A	0.007	0.012	0.015	0.016	0.015	0.012	0.007
	A B	0.009	0.017	0.023	0.025	0.022	0.016	0.008
	B	0.011	0.023	0.034	0.034	0.026	0.016	0.007
	B C	0.012	0.026	0.056	0.041	0.022	0.010	0.003
	C	0.012	0.033	0.074	0.034	0.013	0.005	0.001
	C D	0.016	0.037	0.054	0.023	0.010	0.004	0.002
	D	0.016	0.027	0.029	0.019	0.011	0.005	0.002
	D E	0.011	0.016	0.016	0.014	0.009	0.006	0.003
	E	0.004	0.007	0.009	0.010	0.009	0.006	0.003
C at center	A	0.006	0.011	0.015	0.015	0.014	0.011	0.006
	A B	0.006	0.013	0.020	0.023	0.024	0.020	0.012
	B	0.007	0.015	0.025	0.035	0.035	0.025	0.013
	B C	0.007	0.015	0.028	0.059	0.043	0.023	0.009
	C	0.006	0.017	0.037	0.078	0.037	0.017	0.006

Table A-6
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=10$
Angle of Skew $\phi=45°$

Moment in Beam	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A at center	A	0.037	0.078	0.125	0.182	0.126	0.080	0.038
	A B	0.033	0.064	0.095	0.111	0.078	0.051	0.024
	B	0.024	0.043	0.055	0.057	0.047	0.033	0.017
	B C	0.014	0.023	0.027	0.028	0.026	0.019	0.009
	C	0.005	0.009	0.011	0.012	0.011	0.009	0.005
	C D	0.000	0.001	0.002	0.002	0.002	0.002	0.001
	D	−0.002	−0.003	−0.003	−0.003	−0.002	−0.001	−0.001
	D E	−0.003	−0.004	−0.005	−0.006	−0.005	−0.003	−0.002
	E	−0.003	−0.006	−0.007	−0.007	−0.007	−0.005	−0.003
B at center	A	0.018	0.035	0.048	0.057	0.054	0.041	0.022
	A B	0.015	0.029	0.049	0.085	0.065	0.037	0.016
	B	0.012	0.028	0.056	0.102	0.056	0.028	0.012
	B C	0.013	0.031	0.054	0.073	0.040	0.023	0.011
	C	0.015	0.028	0.038	0.039	0.030	0.019	0.009
	C D	0.011	0.019	0.022	0.021	0.019	0.013	0.006
	D	0.005	0.009	0.011	0.011	0.010	0.007	0.004
	D E	0.002	0.003	0.003	0.004	0.003	0.002	0.001
	E	−0.002	−0.003	−0.003	−0.003	−0.002	−0.001	−0.001
C at 2/8-point	A	0.004	0.007	0.009	0.010	0.011	0.009	0.005
	A B	0.010	0.017	0.022	0.024	0.021	0.015	0.008
	B	0.018	0.034	0.038	0.034	0.025	0.015	0.007
	B C	0.026	0.067	0.052	0.033	0.019	0.010	0.004
	C	0.038	0.089	0.047	0.023	0.011	0.005	0.002
	C D	0.035	0.060	0.030	0.017	0.008	0.004	0.002
	D	0.017	0.025	0.022	0.016	0.010	0.005	0.002
	D E	0.007	0.012	0.014	0.011	0.008	0.005	0.003
	E	0.002	0.004	0.007	0.007	0.007	0.005	0.003
C at 3/8-point	A	0.005	0.009	0.011	0.011	0.011	0.009	0.005
	A B	0.009	0.018	0.022	0.025	0.024	0.018	0.010
	B	0.014	0.027	0.039	0.040	0.033	0.022	0.011
	B C	0.016	0.035	0.069	0.054	0.033	0.017	0.007
	C	0.019	0.047	0.093	0.049	0.024	0.011	0.004
	C D	0.021	0.048	0.067	0.035	0.018	0.010	0.005
	D	0.018	0.031	0.034	0.027	0.018	0.011	0.005
	D E	0.011	0.017	0.019	0.019	0.014	0.009	0.004
	E	0.003	0.006	0.008	0.010	0.009	0.007	0.004
C at center	A	0.005	0.009	0.012	0.012	0.010	0.008	0.004
	A B	0.007	0.015	0.022	0.024	0.024	0.021	0.013
	B	0.010	0.020	0.031	0.040	0.040	0.031	0.016
	B C	0.010	0.022	0.038	0.073	0.055	0.031	0.013
	C	0.010	0.025	0.051	0.097	0.051	0.025	0.010

Table A-7
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=60°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.019	0.043	0.075	0.121	0.075	0.043	0.019
	A B	0.021	0.045	0.068	0.074	0.044	0.024	0.009
	B	0.021	0.037	0.046	0.041	0.026	0.014	0.005
	B C	0.016	0.025	0.027	0.023	0.015	0.008	0.003
	C	0.010	0.015	0.015	0.013	0.008	0.005	0.002
	C D	0.006	0.008	0.008	0.007	0.004	0.002	0.001
	D	0.003	0.004	0.004	0.003	0.002	0.001	0.001
	D E	0.001	0.002	0.002	0.001	0.001	0.001	0.000
	E	0.000	0.000	0.000	0.000	0.001	0.001	0.000
B at center	A	0.009	0.018	0.029	0.041	0.044	0.034	0.018
	A B	0.008	0.016	0.027	0.048	0.041	0.022	0.008
	B	0.007	0.015	0.029	0.054	0.027	0.011	0.003
	B C	0.007	0.018	0.031	0.036	0.014	0.005	0.001
	C	0.009	0.018	0.023	0.019	0.008	0.003	0.001
	C D	0.008	0.013	0.014	0.010	0.004	0.001	0.000
	D	0.006	0.008	0.007	0.005	0.002	0.001	0.000
	D E	0.003	0.004	0.003	0.002	0.001	0.001	0.000
	E	0.000	0.001	0.001	0.001	0.001	0.001	0.000
C at 2/8-point	A	0.010	0.019	0.025	0.027	0.025	0.018	0.010
	A B	0.010	0.021	0.028	0.027	0.021	0.013	0.005
	B	0.012	0.026	0.031	0.024	0.014	0.006	0.001
	B C	0.013	0.036	0.030	0.016	0.005	0.000	−0.001
	C	0.017	0.043	0.017	0.004	−0.002	−0.004	−0.002
	C D	0.018	0.026	0.005	−0.002	−0.005	−0.004	−0.002
	D	0.011	0.010	0.000	−0.004	−0.005	−0.004	−0.002
	D E	0.004	0.002	−0.003	−0.005	−0.005	−0.004	−0.002
	E	−0.001	−0.003	−0.004	−0.005	−0.005	−0.004	−0.002
C at 3/8-point	A	0.006	0.012	0.016	0.019	0.018	0.014	0.008
	A B	0.006	0.012	0.018	0.022	0.020	0.014	0.006
	B	0.006	0.012	0.023	0.026	0.020	0.011	0.004
	B C	0.006	0.014	0.034	0.028	0.014	0.005	0.000
	C	0.006	0.019	0.044	0.019	0.005	0.000	−0.001
	C D	0.010	0.024	0.030	0.008	0.000	−0.002	−0.001
	D	0.011	0.017	0.014	0.004	−0.001	−0.002	−0.001
	D E	0.007	0.009	0.006	0.001	−0.001	−0.001	0.000
	E	0.002	0.003	0.002	0.001	0.000	0.000	0.000
C at center	A	0.003	0.007	0.010	0.012	0.013	0.011	0.006
	A B	0.003	0.006	0.009	0.014	0.017	0.015	0.009
	B	0.003	0.006	0.011	0.021	0.024	0.018	0.008
	B C	0.002	0.006	0.014	0.035	0.029	0.015	0.005
	C	0.003	0.008	0.022	0.047	0.022	0.008	0.003

Table A-8
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=60°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.029	0.061	0.102	0.154	0.103	0.062	0.029
	A B	0.029	0.058	0.085	0.092	0.060	0.036	0.015
	B	0.025	0.043	0.053	0.049	0.036	0.022	0.010
	B C	0.017	0.026	0.029	0.027	0.022	0.014	0.006
	C	0.009	0.014	0.016	0.015	0.013	0.008	0.004
	C D	0.004	0.007	0.009	0.009	0.007	0.005	0.002
	D	0.002	0.004	0.005	0.005	0.004	0.003	0.001
	D E	0.001	0.002	0.002	0.002	0.002	0.001	0.001
	E	0.000	0.000	0.000	0.000	0.001	0.001	0.000
B at center	A	0.013	0.026	0.038	0.049	0.050	0.039	0.021
	A B	0.011	0.021	0.035	0.063	0.053	0.030	0.011
	B	0.009	0.020	0.040	0.075	0.038	0.017	0.006
	B C	0.010	0.025	0.044	0.051	0.023	0.011	0.004
	C	0.012	0.024	0.031	0.027	0.016	0.009	0.004
	C D	0.010	0.017	0.018	0.016	0.010	0.006	0.003
	D	0.006	0.010	0.011	0.009	0.007	0.005	0.002
	D E	0.004	0.005	0.006	0.006	0.005	0.003	0.002
	E	0.001	0.002	0.003	0.004	0.004	0.003	0.002
C at 2/8-point	A	0.009	0.016	0.021	0.022	0.021	0.016	0.009
	A B	0.010	0.021	0.027	0.028	0.023	0.015	0.007
	B	0.014	0.030	0.036	0.031	0.021	0.011	0.004
	B C	0.017	0.049	0.042	0.025	0.012	0.004	0.001
	C	0.025	0.063	0.029	0.011	0.002	−0.001	−0.001
	C D	0.026	0.038	0.012	0.003	−0.001	−0.002	−0.001
	D	0.013	0.015	0.007	0.002	0.000	−0.001	0.000
	D E	0.006	0.006	0.004	0.001	0.000	0.000	0.000
	E	0.001	0.002	0.003	0.002	0.001	0.001	0.000
C at 3/8-point	A	0.007	0.013	0.017	0.018	0.017	0.013	0.008
	A B	0.007	0.015	0.021	0.025	0.024	0.018	0.009
	B	0.009	0.018	0.030	0.034	0.028	0.018	0.008
	B C	0.009	0.021	0.049	0.043	0.024	0.011	0.003
	C	0.011	0.031	0.067	0.032	0.013	0.004	0.001
	C D	0.016	0.036	0.045	0.017	0.007	0.003	0.001
	D	0.015	0.024	0.022	0.012	0.006	0.003	0.001
	D E	0.010	0.014	0.012	0.009	0.006	0.004	0.002
	E	0.003	0.006	0.008	0.008	0.007	0.005	0.002
C at center	A	0.005	0.010	0.014	0.015	0.014	0.011	0.006
	A B	0.005	0.010	0.015	0.020	0.022	0.019	0.012
	B	0.005	0.011	0.018	0.030	0.033	0.025	0.013
	B C	0.005	0.011	0.022	0.051	0.043	0.024	0.009
	C	0.005	0.015	0.035	0.070	0.035	0.015	0.005

Table A-9
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=10$
Angle of Skew $\phi=60°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.036	0.075	0.121	0.177	0.122	0.076	0.037
	A B	0.034	0.066	0.094	0.102	0.069	0.043	0.019
	B	0.025	0.044	0.053	0.051	0.040	0.026	0.013
	B C	0.015	0.024	0.027	0.027	0.023	0.016	0.007
	C	0.006	0.011	0.013	0.014	0.013	0.009	0.004
	C D	0.003	0.005	0.007	0.007	0.006	0.005	0.002
	D	0.001	0.002	0.003	0.003	0.003	0.002	0.001
	D E	0.000	0.000	0.000	0.001	0.001	0.000	0.000
	E	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	0.000
B at center	A	0.015	0.029	0.042	0.051	0.051	0.040	0.022
	A B	0.013	0.025	0.040	0.074	0.062	0.036	0.015
	B	0.011	0.026	0.051	0.093	0.050	0.024	0.009
	B C	0.013	0.032	0.054	0.064	0.031	0.017	0.008
	C	0.015	0.028	0.036	0.033	0.023	0.014	0.007
	C D	0.012	0.018	0.021	0.019	0.015	0.010	0.005
	D	0.006	0.010	0.012	0.012	0.010	0.007	0.003
	D E	0.003	0.005	0.007	0.007	0.006	0.004	0.002
	E	0.001	0.001	0.002	0.003	0.003	0.002	0.001
C at 2/8-point	A	0.006	0.011	0.014	0.015	0.014	0.011	0.006
	A B	0.009	0.018	0.024	0.026	0.023	0.017	0.009
	B	0.016	0.032	0.039	0.036	0.027	0.017	0.008
	B C	0.021	0.059	0.053	0.034	0.019	0.009	0.003
	C	0.034	0.081	0.041	0.019	0.007	0.002	0.000
	C D	0.034	0.048	0.020	0.009	0.003	0.001	0.001
	D	0.014	0.018	0.012	0.008	0.004	0.002	0.001
	D E	0.006	0.009	0.009	0.006	0.004	0.003	0.001
	E	0.002	0.004	0.006	0.006	0.005	0.004	0.002
C at 3/8-point	A	0.006	0.011	0.014	0.015	0.013	0.010	0.006
	A B	0.008	0.016	0.022	0.025	0.025	0.020	0.012
	B	0.011	0.023	0.035	0.040	0.035	0.024	0.012
	B C	0.013	0.029	0.062	0.055	0.034	0.018	0.006
	C	0.017	0.043	0.086	0.045	0.021	0.009	0.003
	C D	0.021	0.046	0.057	0.026	0.013	0.007	0.003
	D	0.017	0.028	0.028	0.020	0.013	0.007	0.003
	D E	0.011	0.015	0.016	0.014	0.010	0.007	0.003
	E	0.004	0.007	0.009	0.010	0.009	0.007	0.004
C at center	A	0.005	0.010	0.013	0.014	0.012	0.009	0.005
	A B	0.006	0.012	0.018	0.022	0.023	0.020	0.013
	B	0.007	0.016	0.025	0.035	0.038	0.030	0.016
	B C	0.008	0.017	0.031	0.064	0.055	0.032	0.013
	C	0.009	0.023	0.047	0.089	0.047	0.023	0.009

Table A-10
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=1$
Angle of Skew $\phi=30°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.026	0.055	0.091	0.138	0.091	0.055	0.026
	A B	0.024	0.049	0.073	0.084	0.057	0.036	0.017
	B	0.020	0.036	0.046	0.046	0.036	0.024	0.011
	B C	0.013	0.022	0.026	0.026	0.021	0.014	0.007
	C	0.007	0.012	0.014	0.014	0.011	0.008	0.004
	C D	0.003	0.005	0.006	0.006	0.005	0.004	0.002
	D	0.001	0.002	0.002	0.002	0.002	0.001	0.001
	D E	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	E	−0.001	−0.002	−0.002	−0.002	−0.002	−0.001	−0.001
B at center	A	0.013	0.026	0.037	0.046	0.045	0.034	0.018
	A B	0.011	0.023	0.038	0.062	0.051	0.030	0.013
	B	0.010	0.022	0.042	0.078	0.042	0.021	0.009
	B C	0.011	0.025	0.042	0.053	0.030	0.016	0.007
	C	0.012	0.023	0.030	0.030	0.021	0.013	0.006
	C D	0.009	0.016	0.019	0.018	0.013	0.009	0.004
	D	0.006	0.009	0.011	0.010	0.008	0.006	0.003
	D E	0.003	0.005	0.006	0.006	0.005	0.003	0.002
	E	0.001	0.001	0.002	0.002	0.002	0.001	0.001
C at 2/8-point	A	0.006	0.011	0.015	0.016	0.015	0.012	0.006
	A B	0.009	0.018	0.023	0.023	0.019	0.013	0.007
	B	0.014	0.028	0.032	0.028	0.020	0.012	0.006
	B C	0.020	0.047	0.040	0.025	0.015	0.007	0.003
	C	0.026	0.065	0.033	0.016	0.007	0.003	0.001
	C D	0.025	0.040	0.021	0.011	0.005	0.002	0.001
	D	0.013	0.018	0.013	0.009	0.005	0.002	0.001
	D E	0.007	0.009	0.008	0.006	0.004	0.002	0.001
	E	0.002	0.004	0.004	0.004	0.003	0.002	0.001
C at 3/8-point	A	0.006	0.011	0.014	0.016	0.015	0.012	0.006
	A B	0.008	0.015	0.021	0.023	0.022	0.016	0.009
	B	0.010	0.021	0.031	0.032	0.027	0.018	0.008
	B C	0.012	0.026	0.050	0.041	0.025	0.013	0.005
	C	0.013	0.033	0.069	0.035	0.017	0.008	0.003
	C D	0.016	0.035	0.047	0.025	0.013	0.007	0.003
	D	0.014	0.024	0.025	0.019	0.012	0.007	0.003
	D E	0.009	0.015	0.015	0.013	0.010	0.006	0.003
	E	0.004	0.007	0.009	0.009	0.007	0.005	0.003
C at center	A	0.005	0.009	0.012	0.014	0.013	0.011	0.006
	A B	0.006	0.011	0.017	0.021	0.021	0.018	0.010
	B	0.007	0.014	0.023	0.031	0.031	0.023	0.012
	B C	0.007	0.015	0.028	0.051	0.041	0.024	0.010
	C	0.007	0.018	0.037	0.072	0.037	0.018	0.007

Table A-11
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=30°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.034	0.071	0.114	0.167	0.115	0.071	0.034
	A B	0.030	0.059	0.086	0.098	0.069	0.044	0.021
	B	0.021	0.039	0.049	0.049	0.040	0.028	0.014
	B C	0.012	0.021	0.024	0.025	0.022	0.015	0.008
	C	0.005	0.009	0.011	0.011	0.010	0.007	0.004
	C D	0.002	0.003	0.004	0.004	0.003	0.003	0.001
	D	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	D E	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001
	E	−0.001	−0.002	−0.002	−0.003	−0.002	−0.002	−0.001
B at center	A	0.015	0.029	0.041	0.049	0.047	0.036	0.019
	A B	0.014	0.029	0.048	0.076	0.063	0.038	0.017
	B	0.013	0.030	0.057	0.099	0.057	0.030	0.013
	B C	0.015	0.033	0.055	0.068	0.041	0.024	0.011
	C	0.015	0.028	0.036	0.036	0.028	0.019	0.009
	C D	0.010	0.017	0.021	0.020	0.017	0.012	0.006
	D	0.005	0.009	0.011	0.011	0.009	0.007	0.003
	D E	0.002	0.004	0.005	0.005	0.004	0.003	0.001
	E	0.000	0.000	0.000	0.000	0.000	0.000	0.000
C at 2/8-point	A	0.004	0.008	0.010	0.011	0.011	0.009	0.005
	A B	0.009	0.016	0.021	0.023	0.020	0.015	0.008
	B	0.016	0.030	0.036	0.033	0.026	0.017	0.008
	B C	0.026	0.060	0.052	0.035	0.022	0.012	0.005
	C	0.037	0.085	0.047	0.025	0.013	0.006	0.002
	C D	0.033	0.052	0.030	0.018	0.010	0.005	0.003
	D	0.014	0.021	0.018	0.014	0.009	0.006	0.003
	D E	0.006	0.010	0.011	0.009	0.007	0.005	0.002
	E	0.002	0.004	0.005	0.005	0.005	0.003	0.002
C at 3/8-point	A	0.004	0.008	0.011	0.012	0.011	0.009	0.005
	A B	0.008	0.016	0.022	0.024	0.023	0.018	0.010
	B	0.013	0.026	0.036	0.039	0.033	0.023	0.012
	B C	0.016	0.036	0.065	0.055	0.036	0.020	0.008
	C	0.020	0.048	0.091	0.051	0.027	0.013	0.005
	C D	0.022	0.047	0.061	0.036	0.021	0.012	0.006
	D	0.016	0.028	0.031	0.026	0.018	0.012	0.006
	D E	0.010	0.016	0.018	0.017	0.013	0.009	0.004
	E	0.003	0.006	0.008	0.009	0.008	0.006	0.003
C at center	A	0.004	0.008	0.010	0.011	0.010	0.008	0.005
	A B	0.006	0.013	0.019	0.022	0.023	0.019	0.011
	B	0.010	0.019	0.029	0.037	0.037	0.029	0.015
	B C	0.011	0.022	0.039	0.066	0.054	0.033	0.014
	C	0.011	0.027	0.052	0.094	0.052	0.027	0.011

Table A-12
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=30°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.044	0.091	0.141	0.198	0.141	0.091	0.044
	A B	0.035	0.066	0.096	0.108	0.078	0.050	0.024
	B	0.019	0.035	0.044	0.045	0.038	0.027	0.014
	B C	0.008	0.013	0.016	0.017	0.016	0.012	0.006
	C	0.001	0.003	0.004	0.004	0.004	0.003	0.002
	C D	−0.001	−0.001	−0.001	−0.001	−0.001	0.000	0.000
	D	−0.001	−0.002	−0.002	−0.002	−0.002	−0.001	−0.001
	D E	−0.001	−0.002	−0.002	−0.002	−0.002	−0.001	−0.001
	E	−0.001	−0.001	−0.002	−0.002	−0.001	−0.001	−0.001
B at center	A	0.015	0.028	0.039	0.045	0.043	0.033	0.018
	A B	0.017	0.036	0.060	0.093	0.078	0.049	0.023
	B	0.020	0.046	0.081	0.130	0.081	0.046	0.021
	B C	0.022	0.047	0.073	0.088	0.057	0.035	0.017
	C	0.017	0.032	0.041	0.042	0.035	0.025	0.013
	C D	0.010	0.016	0.020	0.020	0.018	0.013	0.007
	D	0.003	0.006	0.008	0.008	0.007	0.005	0.003
	D E	0.001	0.001	0.001	0.002	0.002	0.001	0.001
	E	−0.001	−0.002	−0.002	−0.002	−0.002	−0.002	−0.001
C at 2/8-point	A	0.001	0.001	0.002	0.002	0.003	0.002	0.001
	A B	0.006	0.012	0.016	0.019	0.019	0.015	0.009
	B	0.016	0.031	0.038	0.037	0.031	0.022	0.011
	B C	0.032	0.074	0.068	0.050	0.033	0.019	0.009
	C	0.051	0.112	0.070	0.042	0.024	0.013	0.006
	C D	0.043	0.067	0.043	0.029	0.018	0.011	0.006
	D	0.014	0.022	0.023	0.020	0.015	0.010	0.005
	D E	0.004	0.009	0.012	0.012	0.009	0.006	0.003
	E	0.001	0.002	0.003	0.003	0.003	0.002	0.001
C at 3/8-point	A	0.001	0.003	0.003	0.003	0.003	0.003	0.002
	A B	0.007	0.014	0.019	0.021	0.021	0.018	0.011
	B	0.016	0.030	0.041	0.044	0.039	0.029	0.015
	B C	0.023	0.049	0.084	0.074	0.051	0.031	0.014
	C	0.032	0.070	0.123	0.076	0.045	0.025	0.011
	C D	0.031	0.062	0.080	0.052	0.033	0.021	0.010
	D	0.017	0.030	0.035	0.032	0.026	0.018	0.009
	D E	0.008	0.013	0.017	0.018	0.015	0.010	0.005
	E	0.001	0.002	0.004	0.004	0.004	0.003	0.002
C at center	A	0.002	0.003	0.004	0.004	0.004	0.003	0.001
	A B	0.006	0.013	0.019	0.021	0.020	0.017	0.011
	B	0.013	0.025	0.036	0.042	0.041	0.032	0.018
	B C	0.016	0.034	0.054	0.086	0.073	0.046	0.021
	C	0.019	0.043	0.077	0.126	0.077	0.043	0.019

Table A-13
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=1$
Angle of Skew $\phi=45°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.025	0.053	0.087	0.132	0.087	0.052	0.024
	A B	0.025	0.050	0.071	0.075	0.048	0.028	0.012
	B	0.020	0.035	0.042	0.038	0.027	0.017	0.007
	B C	0.013	0.021	0.023	0.021	0.016	0.010	0.004
	C	0.007	0.011	0.012	0.011	0.009	0.006	0.003
	C D	0.004	0.006	0.007	0.006	0.005	0.003	0.002
	D	0.002	0.003	0.004	0.004	0.003	0.002	0.001
	D E	0.001	0.002	0.002	0.002	0.002	0.001	0.001
	E	0.000	0.001	0.001	0.001	0.001	0.001	0.000
B at center	A	0.010	0.020	0.029	0.038	0.040	0.032	0.017
	A B	0.009	0.018	0.030	0.051	0.046	0.028	0.012
	B	0.008	0.020	0.038	0.067	0.037	0.018	0.007
	B C	0.011	0.025	0.040	0.044	0.023	0.012	0.005
	C	0.012	0.022	0.027	0.024	0.015	0.009	0.004
	C D	0.009	0.015	0.016	0.014	0.010	0.006	0.003
	D	0.005	0.008	0.009	0.008	0.006	0.004	0.002
	D E	0.003	0.005	0.006	0.005	0.004	0.003	0.001
	E	0.001	0.003	0.003	0.003	0.003	0.002	0.001
C at 2/8-point	A	0.006	0.012	0.016	0.018	0.017	0.013	0.007
	A B	0.008	0.016	0.022	0.024	0.021	0.015	0.007
	B	0.011	0.024	0.031	0.029	0.021	0.013	0.006
	B C	0.016	0.040	0.039	0.026	0.015	0.007	0.002
	C	0.023	0.056	0.028	0.013	0.005	0.001	0.000
	C D	0.022	0.030	0.012	0.005	0.001	0.000	0.000
	D	0.009	0.010	0.005	0.002	0.000	0.000	0.000
	D E	0.004	0.004	0.002	0.001	0.000	0.000	0.000
	E	0.001	0.001	0.001	0.001	0.000	0.000	0.000
C at 3/8-point	A	0.005	0.010	0.013	0.015	0.015	0.012	0.007
	A B	0.006	0.012	0.018	0.021	0.021	0.016	0.009
	B	0.008	0.016	0.026	0.031	0.027	0.018	0.009
	B C	0.009	0.021	0.043	0.040	0.026	0.014	0.005
	C	0.012	0.030	0.060	0.032	0.015	0.007	0.002
	C D	0.016	0.033	0.038	0.018	0.009	0.004	0.002
	D	0.013	0.020	0.018	0.012	0.007	0.004	0.002
	D E	0.008	0.011	0.010	0.008	0.005	0.003	0.002
	E	0.003	0.005	0.006	0.005	0.004	0.003	0.002
C at center	A	0.003	0.007	0.009	0.011	0.011	0.009	0.005
	A B	0.004	0.008	0.012	0.016	0.018	0.016	0.010
	B	0.005	0.010	0.016	0.024	0.027	0.022	0.012
	B C	0.005	0.012	0.022	0.043	0.039	0.023	0.010
	C	0.006	0.016	0.033	0.063	0.033	0.016	0.006

Table A-14
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=45°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.033	0.069	0.111	0.162	0.110	0.068	0.033
	A B	0.031	0.060	0.085	0.089	0.059	0.036	0.016
	B	0.022	0.038	0.046	0.043	0.032	0.021	0.010
	B C	0.013	0.020	0.023	0.022	0.018	0.012	0.006
	C	0.006	0.010	0.011	0.011	0.010	0.007	0.003
	C D	0.003	0.005	0.006	0.006	0.005	0.003	0.002
	D	0.001	0.002	0.003	0.003	0.003	0.002	0.001
	D E	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	E	0.000	0.000	0.000	0.000	0.000	0.000	0.000
B at center	A	0.012	0.024	0.034	0.042	0.043	0.034	0.019
	A B	0.011	0.023	0.038	0.065	0.059	0.037	0.016
	B	0.012	0.027	0.051	0.089	0.051	0.026	0.011
	B C	0.015	0.033	0.053	0.058	0.032	0.018	0.008
	C	0.015	0.027	0.033	0.030	0.022	0.014	0.007
	C D	0.010	0.017	0.019	0.017	0.013	0.009	0.004
	D	0.005	0.009	0.010	0.010	0.008	0.006	0.003
	D E	0.003	0.005	0.006	0.006	0.005	0.003	0.001
	E	0.001	0.002	0.003	0.003	0.003	0.002	0.001
C at 2/8-point	A	0.005	0.010	0.013	0.014	0.013	0.011	0.006
	A B	0.008	0.016	0.022	0.024	0.023	0.017	0.009
	B	0.014	0.028	0.036	0.035	0.028	0.019	0.009
	B C	0.021	0.053	0.052	0.037	0.022	0.011	0.004
	C	0.033	0.076	0.042	0.021	0.010	0.004	0.001
	C D	0.030	0.041	0.020	0.010	0.005	0.002	0.001
	D	0.011	0.014	0.010	0.006	0.004	0.002	0.001
	D E	0.004	0.006	0.006	0.004	0.003	0.002	0.001
	E	0.001	0.003	0.003	0.003	0.003	0.002	0.001
C at 3/8-point	A	0.005	0.009	0.012	0.013	0.012	0.010	0.006
	A B	0.006	0.014	0.020	0.023	0.023	0.019	0.011
	B	0.010	0.021	0.032	0.037	0.034	0.024	0.012
	B C	0.013	0.030	0.057	0.054	0.037	0.020	0.008
	C	0.018	0.044	0.083	0.046	0.024	0.012	0.004
	C D	0.022	0.044	0.051	0.027	0.015	0.008	0.004
	D	0.015	0.024	0.024	0.018	0.012	0.008	0.004
	D E	0.009	0.013	0.013	0.012	0.009	0.006	0.003
	E	0.003	0.006	0.007	0.007	0.006	0.005	0.002
C at center	A	0.004	0.007	0.010	0.011	0.011	0.008	0.005
	A B	0.005	0.010	0.015	0.019	0.020	0.018	0.011
	B	0.007	0.014	0.022	0.031	0.034	0.028	0.015
	B C	0.008	0.017	0.031	0.057	0.052	0.032	0.014
	C	0.010	0.024	0.047	0.085	0.047	0.024	0.010

Table A-15
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=45°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.043	0.089	0.138	0.194	0.138	0.088	0.043
	A B	0.037	0.069	0.095	0.100	0.068	0.042	0.019
	B	0.020	0.035	0.042	0.041	0.033	0.022	0.011
	B C	0.009	0.014	0.017	0.018	0.016	0.011	0.005
	C	0.003	0.005	0.006	0.007	0.006	0.005	0.002
	C D	0.001	0.001	0.002	0.002	0.002	0.002	0.001
	D	0.000	0.000	0.000	0.000	0.001	0.000	0.000
	D E	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	E	0.000	0.000	−0.001	−0.001	0.000	0.000	0.000
B at center	A	0.013	0.024	0.034	0.040	0.040	0.032	0.017
	A B	0.014	0.029	0.049	0.082	0.076	0.050	0.024
	B	0.018	0.042	0.074	0.121	0.075	0.042	0.019
	B C	0.022	0.047	0.072	0.078	0.047	0.028	0.013
	C	0.017	0.031	0.038	0.037	0.029	0.020	0.010
	C D	0.010	0.016	0.019	0.019	0.016	0.011	0.005
	D	0.004	0.007	0.009	0.009	0.008	0.006	0.003
	D E	0.002	0.003	0.004	0.004	0.004	0.003	0.001
	E	0.000	0.000	0.000	0.000	0.001	0.000	0.000
C at 2/8-point	A	0.002	0.004	0.005	0.005	0.005	0.004	0.002
	A B	0.006	0.013	0.018	0.021	0.021	0.018	0.011
	B	0.015	0.030	0.039	0.040	0.034	0.024	0.013
	B C	0.027	0.068	0.070	0.052	0.034	0.020	0.008
	C	0.048	0.104	0.064	0.038	0.021	0.010	0.004
	C D	0.042	0.056	0.032	0.019	0.012	0.007	0.003
	D	0.011	0.016	0.015	0.013	0.009	0.006	0.003
	D E	0.004	0.007	0.009	0.008	0.006	0.004	0.002
	E	0.001	0.003	0.004	0.004	0.003	0.003	0.001
C at 3/8-point	A	0.003	0.005	0.006	0.006	0.006	0.005	0.003
	A B	0.006	0.014	0.019	0.022	0.022	0.019	0.012
	B	0.013	0.026	0.038	0.043	0.040	0.030	0.016
	B C	0.019	0.042	0.077	0.074	0.052	0.031	0.014
	C	0.029	0.065	0.115	0.070	0.041	0.022	0.009
	C D	0.032	0.060	0.069	0.041	0.026	0.016	0.008
	D	0.016	0.027	0.029	0.025	0.019	0.013	0.006
	D E	0.008	0.013	0.015	0.015	0.012	0.008	0.004
	E	0.002	0.004	0.006	0.006	0.006	0.004	0.002
C at center	A	0.003	0.005	0.007	0.007	0.006	0.005	0.003
	A B	0.005	0.011	0.017	0.020	0.020	0.017	0.011
	B	0.010	0.020	0.030	0.037	0.039	0.032	0.018
	B C	0.013	0.027	0.045	0.077	0.071	0.046	0.022
	C	0.017	0.039	0.071	0.118	0.071	0.039	0.017

Table A-16
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=1$
Angle of Skew $\phi=60°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.024	0.050	0.082	0.120	0.080	0.048	0.022
	A B	0.025	0.047	0.061	0.057	0.035	0.018	0.006
	B	0.018	0.029	0.031	0.024	0.015	0.008	0.003
	B C	0.011	0.015	0.015	0.011	0.007	0.004	0.002
	C	0.005	0.007	0.007	0.006	0.005	0.003	0.001
	C D	0.003	0.004	0.005	0.004	0.003	0.002	0.001
	D	0.002	0.003	0.003	0.003	0.002	0.002	0.001
	D E	0.001	0.002	0.002	0.002	0.002	0.001	0.001
	E	0.001	0.002	0.002	0.002	0.002	0.001	0.001
B at center	A	0.005	0.010	0.016	0.023	0.027	0.023	0.013
	A B	0.005	0.010	0.019	0.031	0.032	0.022	0.009
	B	0.006	0.015	0.027	0.044	0.027	0.013	0.005
	B C	0.009	0.020	0.030	0.028	0.015	0.007	0.003
	C	0.009	0.016	0.018	0.014	0.008	0.005	0.002
	C D	0.007	0.010	0.010	0.008	0.006	0.003	0.002
	D	0.004	0.005	0.006	0.005	0.004	0.002	0.001
	D E	0.002	0.003	0.004	0.003	0.003	0.002	0.001
	E	0.002	0.003	0.003	0.003	0.002	0.002	0.001
C at 2/8-point	A	0.004	0.009	0.012	0.014	0.014	0.012	0.006
	A B	0.004	0.010	0.015	0.017	0.017	0.013	0.006
	B	0.006	0.015	0.022	0.023	0.018	0.012	0.005
	B C	0.010	0.025	0.029	0.022	0.013	0.006	0.002
	C	0.016	0.035	0.020	0.009	0.003	0.000	−0.001
	C D	0.013	0.015	0.005	0.001	−0.001	−0.001	−0.001
	D	0.004	0.003	0.000	−0.001	−0.001	−0.001	0.000
	D E	0.001	0.000	−0.001	−0.001	−0.001	0.000	0.000
	E	0.000	0.000	−0.001	−0.001	0.000	0.000	0.000
C at 3/8-point	A	0.003	0.005	0.007	0.009	0.010	0.008	0.005
	A B	0.003	0.006	0.009	0.013	0.014	0.012	0.007
	B	0.004	0.008	0.015	0.021	0.021	0.015	0.008
	B C	0.005	0.014	0.028	0.030	0.022	0.013	0.005
	C	0.009	0.023	0.041	0.024	0.013	0.006	0.002
	C D	0.012	0.022	0.022	0.011	0.005	0.002	0.001
	D	0.008	0.011	0.008	0.004	0.002	0.001	0.000
	D E	0.004	0.004	0.003	0.002	0.001	0.001	0.000
	E	0.001	0.002	0.002	0.001	0.001	0.001	0.000
C at center	A	0.001	0.002	0.003	0.004	0.005	0.004	0.003
	A B	0.001	0.002	0.004	0.006	0.008	0.009	0.006
	B	0.001	0.004	0.007	0.012	0.016	0.015	0.009
	B C	0.002	0.006	0.014	0.026	0.027	0.019	0.008
	C	0.005	0.012	0.024	0.041	0.024	0.012	0.005

Table A-17
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=60°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.032	0.066	0.106	0.153	0.105	0.065	0.030
	A B	0.031	0.058	0.076	0.072	0.044	0.024	0.008
	B	0.020	0.033	0.036	0.030	0.020	0.011	0.005
	B C	0.012	0.017	0.017	0.014	0.010	0.006	0.003
	C	0.006	0.008	0.009	0.008	0.007	0.004	0.002
	C D	0.003	0.005	0.006	0.006	0.005	0.003	0.001
	D	0.002	0.003	0.004	0.004	0.003	0.002	0.001
	D E	0.001	0.002	0.003	0.002	0.002	0.001	0.001
	E	0.001	0.002	0.002	0.002	0.002	0.001	0.001
B at center	A	0.007	0.014	0.021	0.028	0.031	0.027	0.015
	A B	0.006	0.014	0.025	0.043	0.044	0.030	0.014
	B	0.009	0.021	0.040	0.066	0.039	0.021	0.008
	B C	0.014	0.030	0.043	0.041	0.022	0.011	0.005
	C	0.013	0.022	0.025	0.020	0.013	0.008	0.004
	C D	0.009	0.013	0.014	0.012	0.009	0.006	0.003
	D	0.005	0.007	0.008	0.007	0.006	0.004	0.002
	D E	0.003	0.005	0.005	0.005	0.004	0.003	0.001
	E	0.002	0.003	0.004	0.004	0.003	0.002	0.001
C at 2/8-point	A	0.005	0.009	0.013	0.014	0.014	0.011	0.006
	A B	0.005	0.011	0.017	0.021	0.021	0.016	0.009
	B	0.009	0.020	0.029	0.031	0.026	0.018	0.008
	B C	0.014	0.037	0.043	0.034	0.021	0.011	0.004
	C	0.024	0.054	0.032	0.016	0.007	0.002	0.000
	C D	0.020	0.023	0.009	0.003	0.000	0.000	0.000
	D	0.006	0.004	0.001	0.000	0.000	0.000	0.000
	D E	0.001	0.001	0.000	0.000	0.000	0.000	0.000
	E	0.000	0.001	0.001	0.001	0.001	0.000	0.000
C at 3/8-point	A	0.004	0.007	0.009	0.011	0.011	0.009	0.005
	A B	0.004	0.008	0.012	0.016	0.018	0.016	0.010
	B	0.005	0.012	0.022	0.029	0.029	0.022	0.011
	B C	0.008	0.021	0.042	0.045	0.033	0.020	0.008
	C	0.015	0.035	0.062	0.037	0.020	0.009	0.003
	C D	0.018	0.033	0.033	0.017	0.009	0.005	0.002
	D	0.011	0.015	0.013	0.008	0.005	0.003	0.001
	D E	0.005	0.007	0.006	0.005	0.003	0.002	0.001
	E	0.002	0.003	0.004	0.004	0.003	0.002	0.001
C at center	A	0.002	0.004	0.006	0.007	0.007	0.006	0.004
	A B	0.002	0.004	0.007	0.010	0.012	0.012	0.008
	B	0.003	0.007	0.012	0.019	0.023	0.021	0.012
	B C	0.004	0.011	0.021	0.039	0.041	0.028	0.013
	C	0.008	0.019	0.037	0.063	0.037	0.019	0.008

Table A-18
Influence Coefficients for Moment in Beams

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=60°$

Moment in Beam	Transverse Location of Load	Values of Influence Coefficient for Moment — Longitudinal Position of Load 1/8	2/8	3/8	Center	5/8	6/8	7/8
A at center	A	0.042	0.086	0.134	0.188	0.133	0.085	0.041
	A B	0.038	0.069	0.089	0.084	0.054	0.030	0.011
	B	0.020	0.032	0.036	0.032	0.023	0.014	0.006
	B C	0.010	0.015	0.017	0.016	0.012	0.008	0.004
	C	0.004	0.007	0.009	0.009	0.007	0.005	0.002
	C D	0.003	0.005	0.006	0.006	0.005	0.003	0.001
	D	0.002	0.003	0.003	0.003	0.003	0.002	0.001
	D E	0.001	0.002	0.002	0.002	0.002	0.001	0.000
	E	0.000	0.001	0.001	0.001	0.001	0.001	0.000
B at center	A	0.009	0.017	0.024	0.030	0.032	0.027	0.015
	A B	0.008	0.019	0.034	0.060	0.063	0.044	0.021
	B	0.014	0.034	0.062	0.100	0.062	0.034	0.015
	B C	0.021	0.044	0.063	0.060	0.034	0.019	0.008
	C	0.016	0.028	0.032	0.028	0.020	0.013	0.006
	C D	0.010	0.016	0.018	0.016	0.014	0.009	0.004
	D	0.005	0.008	0.010	0.010	0.008	0.005	0.003
	D E	0.003	0.005	0.006	0.006	0.005	0.003	0.001
	E	0.001	0.003	0.003	0.003	0.003	0.002	0.001
C at 2/8-point	A	0.004	0.007	0.009	0.010	0.010	0.008	0.004
	A B	0.005	0.012	0.018	0.022	0.022	0.019	0.011
	B	0.011	0.025	0.036	0.039	0.035	0.025	0.013
	B C	0.020	0.054	0.064	0.051	0.034	0.019	0.008
	C	0.039	0.085	0.053	0.030	0.015	0.007	0.002
	C D	0.032	0.037	0.017	0.009	0.005	0.003	0.001
	D	0.007	0.008	0.005	0.004	0.003	0.002	0.001
	D E	0.002	0.003	0.003	0.003	0.002	0.002	0.001
	E	0.001	0.002	0.003	0.003	0.002	0.002	0.001
C at 3/8-point	A	0.003	0.007	0.009	0.010	0.009	0.007	0.004
	A B	0.004	0.010	0.015	0.019	0.021	0.018	0.012
	B	0.009	0.018	0.030	0.037	0.037	0.029	0.016
	B C	0.013	0.032	0.061	0.066	0.050	0.031	0.013
	C	0.024	0.055	0.094	0.059	0.034	0.018	0.007
	C D	0.028	0.050	0.050	0.028	0.017	0.009	0.004
	D	0.013	0.020	0.019	0.014	0.010	0.007	0.003
	D E	0.007	0.010	0.010	0.009	0.007	0.004	0.002
	E	0.003	0.005	0.006	0.006	0.005	0.004	0.002
C at center	A	0.003	0.005	0.007	0.008	0.008	0.006	0.003
	A B	0.003	0.007	0.011	0.014	0.016	0.015	0.010
	B	0.006	0.012	0.019	0.027	0.031	0.027	0.016
	B C	0.008	0.018	0.033	0.058	0.060	0.042	0.020
	C	0.014	0.033	0.059	0.097	0.059	0.033	0.014

Table A-19
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=30°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.008	−0.017	−0.028	−0.039	−0.032	−0.020	−0.009
	A B	−0.002	0.001	−0.014	M_{oy}−0.018	−0.012	0.004	0.001
				(0.021)	(0.138)	(0.023)		
	B	0.005	0.014	0.029	0.040	0.024	0.011	0.004
	B C	0.006	0.012	0.014	0.008	0.017	0.010	0.004
	C	0.004	0.005	0.002	0.001	0.006	0.006	0.003
	C D	0.000	−0.002	−0.004	−0.004	−0.001	0.001	0.001
	D	−0.003	−0.006	−0.008	−0.007	−0.005	−0.003	−0.001
	D E	−0.004	−0.008	−0.010	−0.010	−0.008	−0.006	−0.003
	E	−0.006	−0.010	−0.013	−0.013	−0.011	−0.008	−0.004
B	A	−0.014	−0.030	−0.051	−0.077	−0.067	−0.042	−0.019
	A B	−0.005	−0.004	0.007	−0.067	0.002	0.003	0.001
				(0.005)	(−0.024)	(0.003)		
	B	0.006	0.021	0.052	0.093	0.055	0.026	0.010
	B C	0.011	0.024	0.033	−0.025	0.040	0.022	0.009
					(0.012)	(0.038)		
	C	0.008	0.011	0.005	−0.002	0.012	0.012	0.006
	C D	0.002	−0.002	−0.010	−0.011	−0.003	0.002	0.001
	D	−0.005	−0.012	−0.017	−0.017	−0.013	−0.007	−0.003
	D E	−0.009	−0.018	−0.023	−0.023	−0.020	−0.014	−0.007
	E	−0.013	−0.023	−0.028	−0.029	−0.026	−0.019	−0.010
B C	A	−0.017	−0.035	−0.053	−0.070	−0.066	−0.048	−0.024
	A B	−0.010	−0.016	−0.019	−0.034	−0.023	−0.012	−0.004
	B	0.000	0.004	0.014	0.026	0.022	0.014	0.007
	B C	0.007	0.019	0.008	M_{oy}+0.004	0.011	0.023	0.010
				(0.045)	(0.158)	(0.048)		
	C	0.010	0.021	0.033	0.040	0.028	0.016	0.007
	C D	0.006	0.009	0.005	−0.004	0.006	0.004	0.001
	D	−0.002	−0.009	−0.018	−0.021	−0.014	−0.008	−0.004
	D E	−0.011	−0.023	−0.033	−0.035	−0.028	−0.019	−0.010
	E	−0.020	−0.037	−0.046	−0.047	−0.040	−0.028	−0.015
C	A	−0.018	−0.037	−0.053	−0.064	−0.062	−0.049	−0.026
	A B	−0.011	−0.021	−0.033	−0.045	−0.040	−0.025	−0.010
	B	−0.002	−0.004	−0.008	−0.023	−0.012	0.000	0.003
	B C	0.005	0.014	0.030	−0.034	0.029	0.023	0.011
				(0.028)	(0.004)	(0.028)		
	C	0.011	0.029	0.060	0.102	0.060	0.029	0.011

Table A-20
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=30°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.007	−0.015	−0.023	−0.030	−0.025	−0.017	−0.008
	A B	0.000	0.005	−0.011	M_{oy}−0.017	−0.010	0.007	0.003
				(0.024)	(0.139)	(0.025)		
	B	0.007	0.017	0.029	0.035	0.026	0.015	0.007
	B C	0.007	0.013	0.013	0.006	0.017	0.012	0.006
	C	0.003	0.004	0.002	0.002	0.006	0.006	0.003
	C D	0.000	−0.002	−0.003	−0.002	−0.001	0.001	0.001
	D	−0.002	−0.004	−0.005	−0.005	−0.004	−0.002	−0.001
	D E	−0.003	−0.005	−0.007	−0.007	−0.006	−0.004	−0.002
	E	−0.004	−0.006	−0.008	−0.009	−0.008	−0.006	−0.003
B	A	−0.013	−0.027	−0.043	−0.059	−0.053	−0.036	−0.017
	A B	−0.001	0.003	0.014	−0.069	0.006	0.010	0.006
				(0.012)	(−0.026)	(0.007)		
	B	0.012	0.029	0.054	0.079	0.056	0.033	0.015
	B C	0.014	0.027	0.031	−0.033	0.040	0.025	0.012
					(0.004)	(0.038)		
	C	0.007	0.009	0.004	0.001	0.010	0.011	0.007
	C D	0.000	−0.003	−0.007	−0.006	−0.003	0.001	0.001
	D	−0.005	−0.009	−0.011	−0.012	−0.010	−0.006	−0.003
	D E	−0.007	−0.012	−0.015	−0.015	−0.014	−0.010	−0.005
	E	−0.008	−0.014	−0.018	−0.019	−0.017	−0.013	−0.007
B C	A	−0.014	−0.028	−0.040	−0.050	−0.048	−0.036	−0.019
	A B	−0.005	−0.008	−0.009	−0.024	−0.014	−0.005	−0.001
	B	0.004	0.011	0.019	0.027	0.025	0.018	0.009
	B C	0.011	0.023	0.009	M_{oy}+0.000	0.011	0.025	0.012
				(0.046)	(0.154)	(0.048)		
	C	0.012	0.023	0.032	0.037	0.029	0.019	0.009
	C D	0.006	0.009	0.006	−0.002	0.009	0.007	0.003
	D	−0.002	−0.006	−0.011	−0.013	−0.008	−0.005	−0.002
	D E	−0.008	−0.016	−0.022	−0.023	−0.019	−0.013	−0.007
	E	−0.013	−0.024	−0.030	−0.031	−0.027	−0.020	−0.011
C	A	−0.014	−0.024	−0.038	−0.043	−0.042	−0.033	−0.018
	A B	−0.008	−0.015	−0.024	−0.029	−0.028	−0.018	−0.008
	B	0.001	0.000	−0.004	−0.014	−0.008	0.000	0.003
	B C	0.009	0.019	0.033	−0.040	0.028	0.025	0.013
				(0.031)	(−0.002)	(0.027)		
	C	0.015	0.035	0.060	0.084	0.060	0.035	0.015

Table A-21
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=10$
Angle of Skew $\phi=30°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.007	−0.013	−0.019	−0.023	−0.021	−0.014	−0.007
	A B	0.001	0.006	−0.010	M_{oy}−0.017	−0.009	0.008	0.004
				(0.025)	(0.139)	(0.026)		
	B	0.008	0.018	0.027	0.031	0.024	0.016	0.007
	B C	0.007	0.012	0.010	0.003	0.016	0.011	0.005
	C	0.002	0.002	0.001	0.001	0.003	0.004	0.002
	C D	−0.001	−0.002	−0.003	−0.002	−0.002	0.000	0.000
	D	−0.002	−0.004	−0.004	−0.004	−0.004	−0.003	−0.001
	D E	−0.002	−0.004	−0.005	−0.005	−0.005	−0.003	−0.002
	E	−0.002	−0.004	−0.005	−0.005	−0.005	−0.004	−0.002
B	A	−0.012	−0.024	−0.036	−0.047	−0.043	−0.030	−0.015
	A B	0.002	0.007	0.017	−0.070	0.007	0.013	0.008
				(0.015)	(−0.027)	(0.008)		
	B	0.015	0.032	0.052	0.067	0.053	0.034	0.016
	B C	0.014	0.025	0.026	−0.041	0.036	0.023	0.011
					(−0.004)	(0.034)		
	C	0.004	0.005	0.002	−0.001	0.006	0.007	0.005
	C D	−0.002	−0.005	−0.007	−0.006	−0.006	−0.001	0.000
	D	−0.004	−0.008	−0.010	−0.010	−0.009	−0.006	−0.003
	D E	−0.005	−0.009	−0.011	−0.011	−0.010	−0.008	−0.004
	E	−0.005	−0.008	−0.011	−0.012	−0.011	−0.008	−0.004
B C	A	−0.011	−0.022	−0.031	−0.037	−0.036	−0.027	−0.014
	A B	−0.003	−0.004	−0.004	−0.019	−0.010	−0.002	0.001
	B	0.006	0.013	0.020	0.025	0.023	0.017	0.009
	B C	0.011	0.023	0.006	M_{oy}−0.006	0.007	0.024	0.011
				(0.043)	(0.148)	(0.044)		
	C	0.011	0.020	0.028	0.031	0.025	0.017	0.009
	C D	0.005	0.007	0.003	−0.005	0.008	0.006	0.003
	D	−0.003	−0.006	−0.010	−0.011	−0.007	−0.004	−0.002
	D E	−0.007	−0.012	−0.016	−0.016	−0.014	−0.010	−0.005
	E	−0.009	−0.015	−0.019	−0.020	−0.019	−0.014	−0.007
C	A	−0.011	−0.020	−0.027	−0.030	−0.029	−0.023	−0.013
	A B	−0.006	−0.012	−0.019	−0.020	−0.020	−0.015	−0.007
	B	0.001	0.000	−0.004	−0.011	−0.007	−0.002	0.001
	B C	0.009	0.019	0.030	−0.047	0.023	0.022	0.012
				(0.028)	(−0.009)	(0.022)		
	C	0.016	0.034	0.054	0.070	0.054	0.034	0.016

Table A-22
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=45°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.009	−0.018	−0.029	−0.040	−0.035	−0.023	−0.011
	A B	−0.003	0.000	−0.015	M_{oy}−0.025	−0.013	0.004	0.001
				(0.020)	(0.131)	(0.022)		
	B	0.004	0.014	0.029	0.038	0.020	0.009	0.003
	B C	0.006	0.011	0.012	0.010	0.014	0.007	0.003
	C	0.003	0.003	0.001	0.002	0.006	0.004	0.002
	C D	0.000	−0.002	−0.003	−0.002	0.001	0.001	0.001
	D	−0.002	−0.005	−0.005	−0.004	−0.002	−0.001	−0.001
	D E	−0.003	−0.006	−0.007	−0.006	−0.005	−0.003	−0.002
	E	−0.004	−0.007	−0.009	−0.008	−0.007	−0.005	−0.002
B	A	−0.013	−0.028	−0.047	−0.075	−0.072	−0.049	−0.023
	A B	−0.007	−0.008	0.000	−0.043	−0.010	0.002	0.003
				(−0.001)	(−0.019)	(−0.002)		
	B	0.003	0.017	0.047	0.096	0.051	0.024	0.010
	B C	0.010	0.022	0.022	−0.003	0.034	0.018	0.008
				(0.028)	(0.017)	(0.033)		
	C	0.007	0.009	0.003	0.001	0.013	0.010	0.004
	C D	0.001	−0.003	−0.009	−0.008	0.000	0.002	0.001
	D	−0.005	−0.011	−0.014	−0.013	−0.008	−0.005	−0.002
	D E	−0.008	−0.015	−0.018	−0.017	−0.014	−0.009	−0.005
	E	−0.011	−0.019	−0.022	−0.022	−0.018	−0.013	−0.006
B C	A	−0.015	−0.031	−0.048	−0.065	−0.067	−0.051	−0.027
	A B	−0.011	−0.018	−0.021	−0.031	−0.024	−0.012	−0.003
	B	−0.003	0.000	0.009	0.026	0.023	0.015	0.008
	B C	0.005	0.017	0.007	M_{oy}−0.001	0.011	0.022	0.009
				(0.044)	(0.153)	(0.048)		
	C	0.010	0.021	0.034	0.041	0.024	0.012	0.004
	C D	0.007	0.008	0.004	−0.001	0.005	0.001	−0.001
	D	−0.002	−0.008	−0.016	−0.018	−0.011	−0.008	−0.005
	D E	−0.010	−0.022	−0.030	−0.029	−0.022	−0.016	−0.009
	E	−0.019	−0.034	−0.040	−0.039	−0.032	−0.022	−0.011
C	A	−0.014	−0.029	−0.043	−0.055	−0.057	−0.047	−0.026
	A B	−0.010	−0.018	−0.026	−0.038	−0.037	−0.024	−0.008
	B	−0.004	−0.005	−0.006	−0.018	−0.011	0.000	0.004
	B C	0.003	0.011	0.026	−0.008	0.021	0.023	0.012
				(0.025)	(0.012)	(0.027)		
	C	0.010	0.028	0.058	0.106	0.058	0.028	0.010

Table A-23
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=45°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.008	−0.015	−0.023	−0.030	−0.027	−0.018	−0.009
	A B	−0.001	0.004	−0.011	M_{oy} −0.023	−0.009	0.007	0.003
				(0.024)	(0.1333)	(0.026)		
	B	0.007	0.017	0.029	0.035	0.023	0.013	0.006
	B C	0.007	0.012	0.011	0.008	0.016	0.010	0.005
	C	0.003	0.003	0.002	0.003	0.006	0.005	0.003
	C D	0.000	−0.002	−0.002	−0.001	0.001	0.002	0.001
	D	−0.002	−0.003	−0.003	−0.003	−0.002	−0.001	0.000
	D E	−0.002	−0.004	−0.005	−0.005	−0.004	−0.003	−0.001
	E	−0.003	−0.005	−0.006	−0.006	−0.005	−0.004	−0.002
B	A	−0.011	−0.024	−0.039	−0.056	−0.055	−0.039	−0.019
	A B	−0.003	0.001	0.010	−0.044	−0.004	0.010	0.007
				(0.009)	(−0.020)	(0.004)		
	B	0.010	0.027	0.051	0.080	0.055	0.032	0.014
	B C	0.013	0.025	0.020	−0.011	0.037	0.023	0.010
				(0.026)	(0.009)	(0.036)		
	C	0.006	0.007	0.003	0.002	0.011	0.011	0.006
	C D	0.000	−0.003	−0.006	−0.005	−0.001	0.002	0.001
	D	−0.004	−0.008	−0.010	−0.009	−0.007	−0.004	−0.002
	D E	−0.006	−0.010	−0.012	−0.012	−0.010	−0.007	−0.004
	E	−0.006	−0.012	−0.014	−0.015	−0.013	−0.010	−0.005
B C	A	−0.012	−0.024	−0.035	−0.046	−0.047	−0.036	−0.019
	A B	−0.006	−0.009	−0.009	−0.021	−0.015	−0.005	0.001
	B	0.002	0.008	0.016	0.027	0.025	0.019	0.010
	B C	0.009	0.022	0.008	M_{oy} −0.006	0.010	0.024	0.011
				(0.045)	(0.148)	(0.047)		
	C	0.011	0.022	0.031	0.035	0.025	0.015	0.007
	C D	0.006	0.008	0.003	−0.001	0.008	0.005	0.002
	D	−0.002	−0.007	−0.011	−0.011	−0.007	−0.004	−0.002
	D E	−0.008	−0.015	−0.019	−0.019	−0.015	−0.010	−0.006
	E	−0.012	−0.021	−0.026	−0.026	−0.022	−0.016	−0.008
C	A	−0.011	−0.022	−0.032	−0.038	−0.038	−0.031	−0.017
	A B	−0.007	−0.012	−0.019	−0.025	−0.025	−0.018	−0.007
	B	0.000	0.000	−0.002	−0.012	−0.009	−0.001	0.003
	B C	0.007	0.016	0.029	−0.018	0.017	0.023	0.013
				(0.028)	(0.002)	(0.023)		
	C	0.014	0.032	0.057	0.085	0.057	0.032	0.014

Table A-24
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=10$
Angle of Skew $\phi=45°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.007	−0.013	−0.019	−0.024	−0.021	−0.015	−0.008
	A B	0.000	0.006	−0.009	M_{oy} −0.023	−0.007	0.009	0.004
				(0.026)	(0.133)	(0.028)		
	B	0.008	0.018	0.027	0.031	0.023	0.014	0.007
	B C	0.007	0.011	0.008	0.005	0.015	0.010	0.005
	C	0.002	0.001	0.001	0.002	0.005	0.004	0.002
	C D	−0.001	−0.002	−0.002	−0.001	0.000	0.001	0.001
	D	−0.002	−0.003	−0.003	−0.003	−0.002	−0.001	−0.001
	D E	−0.002	−0.003	−0.004	−0.004	−0.003	−0.002	−0.001
	E	−0.002	−0.003	−0.004	−0.004	−0.004	−0.003	−0.002
B	A	−0.010	−0.021	−0.033	−0.044	−0.043	−0.031	−0.016
	A B	0.000	0.005	0.015	−0.045	−0.003	0.013	0.010
				(0.014)	(−0.021)	(0.005)		
	B	0.014	0.030	0.050	0.069	0.052	0.034	0.016
	B C	0.014	0.024	0.016	−0.019	0.034	0.022	0.010
				(0.022)	(0.001)	(0.033)		
	C	0.004	0.004	0.001	0.001	0.007	0.008	0.004
	C D	−0.002	−0.005	−0.006	−0.005	−0.003	0.000	0.000
	D	−0.004	−0.007	−0.008	−0.008	−0.007	−0.004	−0.002
	D E	−0.004	−0.007	−0.009	−0.009	−0.008	−0.006	−0.003
	E	−0.004	−0.007	−0.009	−0.010	−0.009	−0.007	−0.004
B C	A	−0.009	−0.018	−0.027	−0.034	−0.034	−0.026	−0.014
	A B	−0.004	−0.004	−0.004	−0.016	−0.010	−0.002	0.002
	B	0.005	0.011	0.017	0.024	0.023	0.018	0.010
	B C	0.010	0.022	0.005	M_{oy} −0.012	0.007	0.023	0.010
				(0.042)	(0.142)	(0.044)		
	C	0.010	0.020	0.027	0.029	0.022	0.015	0.007
	C D	0.005	0.006	0.001	−0.003	0.007	0.005	0.002
	D	−0.003	−0.007	−0.010	−0.009	−0.006	−0.003	−0.002
	D E	−0.006	−0.012	−0.014	−0.014	−0.012	−0.008	−0.004
	E	−0.008	−0.013	−0.017	−0.017	−0.015	−0.011	−0.006
C	A	−0.009	−0.017	−0.023	−0.026	−0.026	−0.021	−0.012
	A B	−0.005	−0.009	−0.015	−0.018	−0.019	−0.014	−0.007
	B	0.001	0.000	−0.002	−0.010	−0.008	−0.003	0.000
	B C	0.007	0.016	0.027	−0.026	0.011	0.021	0.012
				(0.026)	(−0.006)	(0.017)		
	C	0.015	0.032	0.052	0.069	0.052	0.032	0.015

Table A-25
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=60°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.009	−0.018	−0.028	−0.038	−0.035	−0.024	−0.012
	A B	−0.004	0.000	−0.012	M_{oy} −0.038	−0.008	0.007	0.003
				(0.023)	(0.118)	(0.027)		
	B	0.004	0.015	0.031	0.038	0.016	0.007	0.003
	B C	0.007	0.012	0.015	0.014	0.011	0.006	0.002
	C	0.004	0.005	0.005	0.006	0.007	0.004	0.002
	C D	0.001	0.001	0.002	0.004	0.004	0.002	0.001
	D	0.000	0.000	0.001	0.002	0.002	0.001	0.001
	D E	0.001	0.000	0.000	0.001	0.001	0.001	0.000
	E	−0.001	−0.001	−0.001	0.000	0.000	0.000	0.000
B	A	−0.010	−0.020	−0.035	−0.060	−0.066	−0.049	−0.024
	A B	−0.007	−0.009	−0.005	−0.004	−0.031	0.006	0.007
					(−0.003)	(0.003)		
	B	0.003	0.017	0.046	0.107	0.053	0.027	0.011
	B C	0.010	0.023	0.004	0.030	0.029	0.016	0.006
				(0.033)	(0.033)			
	C	0.008	0.012	0.011	0.012	0.015	0.009	0.003
	C D	0.003	0.001	0.000	0.004	0.006	0.004	0.001
	D	−0.001	−0.003	−0.003	0.000	0.001	0.001	0.000
	D E	−0.003	−0.005	−0.005	−0.003	−0.002	−0.001	−0.001
	E	−0.004	−0.007	−0.007	−0.005	−0.004	−0.002	−0.001
B C	A	−0.009	−0.018	−0.028	−0.043	−0.050	−0.043	−0.023
	A B	−0.007	−0.012	−0.014	−0.015	−0.011	−0.003	−0.004
	B	−0.003	0.001	0.012	0.037	0.036	0.025	0.013
	B C	0.006	0.021	0.017	M_{oy} −0.001	0.022	0.027	0.010
				(0.054)	(0.153)	(0.059)		
	C	0.013	0.028	0.045	0.050	0.026	0.013	0.004
	C D	0.010	0.015	0.015	0.012	0.008	0.003	0.000
	D	0.003	0.001	−0.002	−0.002	−0.001	−0.002	−0.002
	D E	−0.003	−0.009	−0.012	−0.010	−0.007	−0.005	−0.003
	E	−0.011	−0.017	−0.018	−0.015	−0.011	−0.007	−0.003
C	A	−0.004	−0.009	−0.016	−0.024	−0.031	−0.029	−0.017
	A B	−0.004	−0.006	−0.008	−0.013	−0.016	−0.010	−0.001
	B	−0.001	0.002	0.006	0.003	0.005	0.011	0.010
	B C	0.004	0.014	0.029	0.035	0.008	0.031	0.016
				(0.028)	(0.036)	(0.039)		
	C	0.013	0.033	0.066	0.125	0.066	0.033	0.013

Table A-26
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=60°$

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
		Values of Influence Coefficient for Moment — Longitudinal Position of Load						
A B	A	−0.007	−0.015	−0.023	−0.029	−0.028	−0.020	−0.010
	A B	−0.002	0.004	−0.008	M_{oy} −0.037	−0.005	0.010	0.004
				(0.027)	(0.119)	(0.030)		
	B	0.007	0.018	0.030	0.034	0.019	0.011	0.005
	B C	0.007	0.011	0.011	0.012	0.013	0.008	0.003
	C	0.003	0.003	0.004	0.006	0.007	0.005	0.002
	C D	0.000	0.001	0.002	0.003	0.004	0.003	0.001
	D	0.000	0.000	0.001	0.002	0.002	0.001	0.001
	D E	0.000	0.000	0.000	0.000	0.001	0.000	0.000
	E	−0.001	−0.001	−0.001	−0.001	0.000	0.000	0.000
B	A	−0.008	−0.017	−0.029	−0.046	−0.051	−0.039	−0.020
	A B	−0.004	−0.001	0.005	−0.006	−0.029	0.011	0.010
					(−0.005)	(0.005)		
	B	0.009	0.025	0.050	0.088	0.054	0.032	0.015
	B C	0.013	0.024	−0.003	0.019	0.031	0.020	0.008
				(0.026)	(0.022)			
	C	0.007	0.008	0.006	0.008	0.014	0.010	0.005
	C D	0.001	−0.001	−0.001	0.002	0.005	0.004	0.002
	D	−0.002	−0.003	−0.003	−0.001	0.001	0.001	0.000
	D E	−0.003	−0.004	−0.004	−0.003	−0.002	−0.001	−0.001
	E	−0.003	−0.005	−0.006	−0.005	−0.004	−0.003	−0.001
B C	A	−0.006	−0.013	−0.021	−0.030	−0.035	−0.030	−0.017
	A B	−0.004	−0.006	−0.005	−0.009	−0.007	0.000	0.004
	B	0.002	0.007	0.016	0.032	0.032	0.024	0.013
	B C	0.009	0.024	0.014	M_{oy} −0.012	0.016	0.027	0.011
				(0.051)	(0.142)	(0.053)		
	C	0.013	0.025	0.036	0.039	0.024	0.014	0.006
	C D	0.008	0.010	0.008	0.007	0.009	0.005	0.002
	D	0.000	−0.002	−0.004	−0.002	0.001	0.000	0.000
	D E	−0.004	−0.007	−0.008	−0.007	−0.004	−0.003	−0.002
	E	−0.007	−0.011	−0.012	−0.011	−0.008	−0.005	−0.003
C	A	−0.004	−0.009	−0.014	−0.019	−0.021	−0.019	−0.011
	A B	−0.002	−0.004	−0.005	−0.010	−0.013	−0.009	−0.003
	B	0.001	0.004	0.006	0.000	0.000	0.004	0.005
	B C	0.006	0.016	0.028	0.018	−0.005	0.025	0.015
				(0.027)	(0.019)	(0.026)		
	C	0.014	0.033	0.058	0.095	0.058	0.033	0.014

Table A-27
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Relative Stiffness of Beams $H=10$
Angle of Skew $\phi=60°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.007	−0.013	−0.019	−0.024	−0.022	−0.016	−0.008
	A B	0.000	0.006	−0.007	M_{oy} −0.037	−0.004	0.012	0.006
				(0.028)	(0.119)	(0.031)		
	B	0.008	0.018	0.028	0.030	0.020	0.012	0.006
	B C	0.007	0.010	0.009	0.009	0.014	0.008	0.004
	C	0.002	0.002	0.003	0.005	0.006	0.005	0.002
	C D	0.000	0.000	0.001	0.002	0.003	0.002	0.001
	D	0.000	0.000	0.000	0.000	0.001	0.001	0.000
	D E	−0.001	−0.001	−0.001	−0.001	0.000	0.000	0.000
	E	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	0.000
B	A	−0.008	−0.016	−0.025	−0.037	−0.040	−0.031	−0.016
	A B	−0.001	0.003	0.010	−0.009	−0.030	0.013	0.012
					(−0.008)	(0.004)		
	B	0.012	0.028	0.049	0.074	0.052	0.033	0.016
	B C	0.014	0.023	−0.009	0.009	0.029	0.019	0.008
				(0.020)	(0.012)			
	C	0.004	0.004	0.003	0.005	0.010	0.008	0.004
	C D	−0.001	−0.003	−0.003	0.000	0.002	0.003	0.001
	D	−0.002	−0.004	−0.004	−0.003	−0.001	−0.001	0.000
	D E	−0.002	−0.004	−0.004	−0.004	−0.003	−0.002	−0.001
	E	−0.002	−0.004	−0.005	−0.005	−0.004	−0.003	−0.002
B C	A	−0.005	−0.011	−0.016	−0.022	−0.025	−0.022	−0.012
	A B	−0.002	−0.002	−0.001	−0.006	−0.006	0.001	0.004
	B	0.004	0.009	0.017	0.027	0.027	0.020	0.011
	B C	0.010	0.023	0.009	M_{oy} −0.021	0.010	0.024	0.010
				(0.046)	(0.133)	(0.047)		
	C	0.011	0.021	0.029	0.030	0.021	0.013	0.006
	C D	0.006	0.006	0.003	0.003	0.008	0.005	0.002
	D	−0.002	−0.004	−0.005	−0.003	0.000	0.000	0.000
	D E	−0.004	−0.007	−0.007	−0.006	−0.004	−0.003	−0.002
	E	−0.004	−0.008	−0.009	−0.008	−0.007	−0.005	−0.002
C	A	−0.004	−0.008	−0.012	−0.015	−0.016	−0.013	−0.008
	A B	−0.002	−0.003	−0.006	−0.009	−0.011	−0.009	−0.004
	B	0.002	0.003	0.004	−0.002	−0.004	−0.001	0.002
	B C	0.006	0.015	0.025	0.006	−0.014	0.020	0.013
				(0.024)	(0.007)	(0.017)		
	C	0.015	0.032	0.052	0.075	0.052	0.032	0.015

Table A-28
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=1$
Angle of Skew $\phi=30°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.011	−0.020	−0.030	−0.040	−0.034	−0.022	−0.010
	A B	−0.001	−0.008	−0.020	M_{oy} −0.014	−0.016	−0.001	0.006
		(0.006)	(0.024)	(0.069)	(0.178)	(0.073)	(0.031)	(0.013)
	B	0.015	0.030	0.042	0.045	0.037	0.024	0.012
	B C	0.009	0.013	0.011	0.011	0.017	0.013	0.006
	C	0.001	0.001	0.000	0.001	0.004	0.004	0.002
	C D	−0.002	−0.004	−0.004	−0.003	−0.001	0.000	0.000
	D	−0.003	−0.005	−0.005	−0.005	−0.004	−0.002	−0.001
	D E	−0.003	−0.005	−0.005	−0.005	−0.005	−0.003	−0.002
	E	−0.002	−0.004	−0.005	−0.006	−0.005	−0.004	−0.002
B	A	−0.015	−0.030	−0.047	−0.066	−0.065	−0.046	−0.022
	A B	0.001	0.009	0.017	−0.033	−0.009	0.020	0.016
				(0.014)	(−0.014)	(0.007)		
	B	0.023	0.052	0.089	0.150	0.092	0.057	0.027
					(0.129)			
	B C	0.021	0.035	0.026	0.007	0.046	0.031	0.015
				(0.031)	(0.015)	(0.042)		
	C	0.005	0.003	−0.002	−0.002	0.008	0.009	0.005
	C D	−0.004	−0.009	−0.013	−0.011	−0.006	−0.002	−0.001
	D	−0.007	−0.012	−0.015	−0.014	−0.011	−0.007	−0.004
	D E	−0.007	−0.012	−0.015	−0.015	−0.012	−0.009	−0.005
	E	−0.006	−0.011	−0.014	−0.015	−0.013	−0.009	−0.005
B C	A	−0.014	−0.027	−0.039	−0.048	−0.049	−0.038	−0.020
	A B	−0.006	−0.009	−0.011	−0.020	−0.016	−0.006	0.001
	B	0.006	0.016	0.026	0.034	0.035	0.028	0.016
	B C	0.011	0.011	−0.001	M_{oy} +0.005	0.001	0.013	0.012
		(0.018)	(0.043)	(0.090)	(0.195)	(0.092)	(0.045)	(0.019)
	C	0.017	0.030	0.039	0.040	0.032	0.021	0.009
	C D	0.005	0.003	−0.002	−0.005	0.002	0.002	0.001
	D	−0.006	−0.013	−0.018	−0.017	−0.013	−0.008	−0.004
	D E	−0.010	−0.018	−0.023	−0.023	−0.019	−0.013	−0.007
	E	−0.012	−0.021	−0.026	−0.026	−0.022	−0.016	−0.009
C	A	−0.012	−0.023	−0.032	−0.038	−0.038	−0.031	−0.017
	A B	−0.008	−0.014	−0.021	−0.028	−0.029	−0.021	−0.010
	B	0.000	0.000	−0.004	−0.014	−0.013	−0.004	0.001
	B C	0.011	0.024	0.038	−0.002	0.019	0.031	0.020
				(0.034)	(0.008)	(0.026)		
	C	0.026	0.056	0.092	0.153	0.092	0.056	0.026
					(0.132)			

Table A-29
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=30°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.009	−0.018	−0.026	−0.032	−0.028	−0.019	−0.010
	A B	−0.001	−0.007	−0.019	M_{oy} −0.013	−0.015	−0.001	0.006
		(0.006)	(0.025)	(0.070)	(0.179)	(0.074)	(0.031)	(0.013)
	B	0.015	0.028	0.039	0.041	0.035	0.024	0.012
	B C	0.008	0.011	0.008	0.008	0.015	0.012	0.006
	C	0.000	−0.001	−0.002	0.000	0.002	0.002	0.001
	C D	−0.003	−0.004	−0.005	−0.004	−0.003	−0.001	−0.001
	D	−0.002	−0.004	−0.005	−0.005	−0.004	−0.003	−0.001
	D E	−0.002	−0.004	−0.004	−0.004	−0.004	−0.003	−0.001
	E	−0.002	−0.003	−0.004	−0.004	−0.003	−0.002	−0.001
B	A	−0.014	−0.028	−0.043	−0.056	−0.055	−0.040	−0.020
	A B	0.003	0.011	0.018	−0.038	−0.012	0.019	0.016
				(0.015)	(−0.019)	(0.004)		
	B	0.025	0.053	0.083	0.123	0.085	0.056	0.027
					(0.110)			
	B C	0.020	0.030	0.017	−0.005	0.040	0.028	0.013
				(0.022)	(0.003)	(0.036)		
	C	0.001	−0.001	−0.006	−0.006	0.002	0.004	0.003
	C D	−0.006	−0.011	−0.013	−0.012	−0.009	−0.005	−0.002
	D	−0.006	−0.011	−0.013	−0.013	−0.011	−0.007	−0.004
	D E	−0.006	−0.009	−0.011	−0.012	−0.010	−0.007	−0.004
	E	−0.004	−0.007	−0.009	−0.009	−0.008	−0.006	−0.003
B C	A	−0.011	−0.022	−0.031	−0.037	−0.037	−0.030	−0.016
	A B	−0.004	−0.005	−0.007	−0.016	−0.013	−0.005	0.001
	B	0.008	0.016	0.025	0.031	0.031	0.025	0.014
	B C	0.010	0.009	−0.006	M_{oy} −0.011	−0.005	0.009	0.011
		(0.017)	(0.041)	(0.085)	(0.189)	(0.086)	(0.041)	(0.018)
	C	0.014	0.026	0.033	0.034	0.028	0.019	0.009
	C D	0.003	0.001	−0.005	−0.007	0.001	0.002	0.001
	D	−0.006	−0.012	−0.016	−0.015	−0.011	−0.007	−0.004
	D E	−0.008	−0.015	−0.018	−0.018	−0.015	−0.010	−0.005
	E	−0.008	−0.014	−0.017	−0.018	−0.015	−0.011	−0.006
C	A	−0.010	−0.018	−0.025	−0.028	−0.028	−0.022	−0.012
	A B	−0.006	−0.012	−0.019	−0.024	−0.024	−0.019	−0.010
	B	−0.001	−0.002	−0.006	−0.015	−0.015	−0.008	−0.002
	B C	0.012	0.023	0.033	−0.015	0.009	0.026	0.018
				(0.029)	(−0.005)	(0.016)		
	C	0.026	0.054	0.083	0.122	0.083	0.054	0.026
					(0.109)			

Table A-30
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=30°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.007	−0.014	−0.019	−0.023	−0.020	−0.015	−0.008
	A B	0.000	−0.007	−0.019	M_{oy} −0.014	−0.017	−0.002	0.006
		(0.007)	(0.025)	(0.070)	(0.178)	(0.072)	(0.030)	(0.013)
	B	0.013	0.024	0.032	0.034	0.030	0.021	0.011
	B C	0.006	0.006	0.002	0.002	0.010	0.008	0.004
	C	−0.002	−0.003	−0.004	−0.003	−0.002	−0.001	0.000
	C D	−0.003	−0.005	−0.005	−0.005	−0.004	−0.003	−0.001
	D	−0.002	−0.003	−0.004	−0.004	−0.004	−0.003	−0.001
	D E	−0.001	−0.002	−0.003	−0.003	−0.003	−0.002	−0.001
	E	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001
B	A	−0.012	−0.024	−0.034	−0.042	−0.041	−0.031	−0.016
	A B	0.004	0.012	0.016	−0.047	−0.018	0.016	0.015
				(0.013)	(−0.028)	(−0.002)		
	B	0.024	0.047	0.069	0.083	0.069	0.048	0.025
	B C	0.016	0.021	0.002	−0.023	0.028	0.020	0.009
				(0.007)	(−0.015)	(0.024)		
	C	−0.003	−0.008	−0.012	−0.012	−0.006	−0.003	−0.001
	C D	−0.007	−0.012	−0.014	−0.014	−0.012	−0.008	−0.004
	D	−0.005	−0.009	−0.011	−0.011	−0.010	−0.007	−0.004
	D E	−0.003	−0.006	−0.007	−0.007	−0.006	−0.005	−0.002
	E	−0.001	−0.002	−0.003	−0.003	−0.003	−0.002	−0.001
B C	A	−0.008	−0.015	−0.021	−0.024	−0.024	−0.019	−0.010
	A B	−0.002	−0.002	−0.004	−0.014	−0.012	−0.005	0.000
	B	0.007	0.014	0.020	0.024	0.023	0.018	0.010
	B C	0.007	0.002	−0.016	M_{oy} −0.012	−0.015	0.002	0.007
		(0.014)	(0.034)	(0.075)	(0.178)	(0.076)	(0.034)	(0.014)
	C	0.010	0.018	0.023	0.024	0.020	0.014	0.007
	C D	0.000	−0.004	−0.010	−0.011	−0.002	0.000	0.000
	D	−0.006	−0.011	−0.014	−0.013	−0.011	−0.007	−0.004
	D E	−0.006	−0.010	−0.012	−0.012	−0.010	−0.007	−0.004
	E	−0.003	−0.006	−0.008	−0.008	−0.007	−0.006	−0.003
C	A	−0.006	−0.011	−0.014	−0.016	−0.015	−0.012	−0.007
	A B	−0.005	−0.011	−0.016	−0.018	−0.018	−0.016	−0.010
	B	−0.003	−0.006	−0.011	−0.017	−0.017	−0.012	−0.005
	B C	0.007	0.016	0.022	−0.031	−0.005	0.017	0.014
				(0.018)	(−0.021)	(0.002)		
	C	0.023	0.045	0.066	0.080	0.066	0.045	0.023

Table A-31
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=1$
Angle of Skew $\phi=45°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.011	−0.020	−0.029	−0.037	−0.033	−0.022	−0.010
	A B	−0.002	−0.008	−0.022	M_{oy} −0.022	−0.015	0.002	0.009
		(0.005)	(0.024)	(0.067)	(0.170)	(0.074)	(0.034)	(0.016)
	B	0.015	0.031	0.045	0.047	0.034	0.022	0.010
	B C	0.010	0.014	0.015	0.016	0.017	0.012	0.005
	C	0.003	0.004	0.005	0.007	0.008	0.006	0.003
	C D	0.000	0.001	0.002	0.003	0.004	0.003	0.001
	D	0.000	0.000	0.001	0.001	0.001	0.001	0.000
	D E	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	E	0.000	−0.001	−0.001	0.000	0.000	0.000	0.000
B	A	−0.010	−0.020	−0.031	−0.048	−0.054	−0.041	−0.020
	A B	0.000	0.007	0.019	0.000	−0.017	0.024	0.019
				(0.015)	(0.006)	(0.016)		
	B	0.022	0.051	0.090	0.163	0.094	0.058	0.027
					(0.142)			
	B C	0.022	0.036	0.016	0.033	0.045	0.028	0.012
				(0.036)	(0.032)	(0.041)		
	C	0.007	0.007	0.006	0.009	0.015	0.011	0.005
	C D	−0.001	−0.003	−0.003	0.001	0.004	0.003	0.001
	D	−0.003	−0.004	−0.004	−0.002	0.000	0.000	0.000
	D E	−0.003	−0.004	−0.004	−0.003	−0.002	−0.001	−0.001
	E	−0.002	−0.004	−0.004	−0.004	−0.003	−0.002	−0.001
B C	A	−0.006	−0.012	−0.019	−0.026	−0.029	−0.025	−0.014
	A B	−0.002	−0.001	0.000	−0.002	−0.001	0.005	0.007
	B	0.007	0.018	0.031	0.046	0.047	0.037	0.021
	B C	0.013	0.017	0.005	M_{oy} +0.008	0.007	0.019	0.014
		(0.020)	(0.049)	(0.096)	(0.198)	(0.098)	(0.051)	(0.021)
	C	0.021	0.038	0.050	0.050	0.036	0.022	0.009
	C D	0.009	0.011	0.009	0.009	0.011	0.007	0.003
	D	−0.001	−0.003	−0.004	−0.002	0.000	0.000	0.000
	D E	−0.004	−0.007	−0.007	−0.006	−0.004	−0.003	−0.002
	E	−0.005	−0.008	−0.009	−0.008	−0.007	−0.004	−0.002
C	A	−0.004	−0.007	−0.011	−0.015	−0.017	−0.016	−0.009
	A B	−0.002	−0.002	−0.004	−0.008	−0.011	−0.009	−0.004
	B	0.003	0.007	0.009	0.003	0.000	0.004	0.005
	B C	0.010	0.025	0.040	0.030	0.010	0.036	0.023
				(0.037)	(0.029)	(0.035)		
	C	0.027	0.058	0.097	0.168	0.097	0.058	0.027
					(0.147)			

Table A-32
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=45°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.009	−0.018	−0.025	−0.031	−0.028	−0.020	−0.010
	A B	−0.001	−0.007	−0.021	M_{oy} −0.022	−0.016	0.002	0.008
		(0.006)	(0.025)	(0.068)	(0.170)	(0.073)	(0.034)	(0.015)
	B	0.015	0.030	0.041	0.043	0.033	0.021	0.010
	B C	0.009	0.011	0.011	0.013	0.016	0.011	0.005
	C	0.001	0.002	0.003	0.005	0.006	0.004	0.002
	C D	−0.001	−0.001	0.000	0.001	0.002	0.002	0.001
	D	−0.001	−0.001	0.000	0.000	0.000	0.000	0.000
	D E	−0.001	−0.001	−0.001	−0.001	0.000	0.000	0.000
	E	0.000	−0.001	−0.001	−0.001	−0.001	−0.001	0.000
B	A	−0.010	−0.020	−0.030	−0.043	−0.047	−0.037	−0.019
	A B	0.001	0.009	0.020	−0.008	−0.024	0.021	0.018
				(0.016)	(−0.002)	(0.009)		
	B	0.023	0.051	0.083	0.131	0.086	0.056	0.027
					(0.118)			
	B C	0.020	0.030	0.004	0.018	0.039	0.025	0.011
				(0.024)	(0.017)	(0.035)		
	C	0.003	0.001	−0.001	0.002	0.008	0.007	0.004
	C D	−0.003	−0.006	−0.006	−0.003	0.000	0.001	0.000
	D	−0.003	−0.005	−0.006	−0.005	−0.003	−0.002	−0.001
	D E	−0.003	−0.004	−0.005	−0.004	−0.003	−0.002	−0.001
	E	−0.002	−0.003	−0.004	−0.004	−0.004	−0.003	−0.001
B C	A	−0.006	−0.011	−0.016	−0.022	−0.024	−0.021	−0.012
	A B	−0.002	0.000	0.001	−0.003	−0.004	0.001	0.005
	B	0.007	0.017	0.027	0.038	0.038	0.030	0.017
	B C	0.012	0.012	−0.003	M_{oy} −0.004	−0.003	0.013	0.012
		(0.019)	(0.044)	(0.088)	(0.186)	(0.088)	(0.045)	(0.019)
	C	0.017	0.031	0.039	0.039	0.029	0.019	0.008
	C D	0.006	0.005	0.002	0.004	0.008	0.005	0.002
	D	−0.003	−0.006	−0.006	−0.004	−0.002	−0.001	0.000
	D E	−0.004	−0.007	−0.007	−0.006	−0.004	−0.003	−0.002
	E	−0.004	−0.006	−0.007	−0.007	−0.006	−0.004	−0.002
C	A	−0.004	−0.008	−0.012	−0.014	−0.015	−0.013	−0.008
	A B	−0.002	−0.004	−0.006	−0.010	−0.013	−0.011	−0.006
	B	0.002	0.004	0.003	−0.003	−0.006	−0.003	0.000
	B C	0.009	0.022	0.034	0.014	−0.005	0.027	0.019
				(0.031)	(0.013)	(0.020)		
	C	0.026	0.054	0.084	0.132	0.084	0.054	0.026
					(0.119)			

Table A-33
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=45°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.007	−0.014	−0.019	−0.022	−0.021	−0.015	−0.008
	A B	0.000	−0.007	−0.022	M_{oy} −0.024	−0.018	0.001	0.008
		(0.007)	(0.025)	(0.067)	(0.168)	(0.071)	(0.033)	(0.015)
	B	0.013	0.025	0.033	0.035	0.028	0.019	0.010
	B C	0.006	0.006	0.004	0.007	0.012	0.008	0.004
	C	−0.001	−0.001	−0.001	0.001	0.002	0.002	0.001
	C D	−0.002	−0.002	−0.002	−0.001	−0.001	0.000	0.000
	D	−0.001	−0.001	−0.002	−0.002	−0.001	−0.001	0.000
	D E	−0.001	−0.001	−0.001	−0.001	−0.001	−0.001	0.000
	E	0.000	−0.001	−0.001	−0.001	−0.001	−0.001	0.000
B	A	−0.009	−0.018	−0.026	−0.034	−0.036	−0.028	−0.015
	A B	0.002	0.010	0.018	−0.019	−0.034	0.015	0.016
				(0.014)	(−0.013)	(−0.001)		
	B	0.023	0.046	0.069	0.088	0.070	0.048	0.024
	B C	0.016	0.019	−0.013	−0.001	0.028	0.018	0.007
				(0.007)	(−0.002)	(0.024)		
	C	−0.002	−0.006	−0.009	−0.006	−0.001	0.000	0.000
	C D	−0.006	−0.009	−0.009	−0.008	−0.006	−0.003	−0.001
	D	−0.003	−0.006	−0.007	−0.006	−0.005	−0.004	−0.002
	D E	−0.002	−0.004	−0.005	−0.004	−0.004	−0.003	−0.001
	E	−0.001	−0.002	−0.002	−0.002	−0.002	−0.002	−0.001
B C	A	−0.004	−0.009	−0.012	−0.016	−0.017	−0.014	−0.008
	A B	−0.001	0.000	0.001	−0.005	−0.007	−0.003	0.001
	B	0.006	0.014	0.020	0.026	0.026	0.020	0.011
	B C	0.008	0.004	−0.016	M_{oy} −0.019	−0.016	0.004	0.008
		(0.015)	(0.036)	(0.075)	(0.171)	(0.075)	(0.036)	(0.015)
	C	0.011	0.020	0.026	0.026	0.020	0.014	0.007
	C D	0.001	−0.002	−0.005	−0.003	0.003	0.003	0.001
	D	−0.004	−0.007	−0.008	−0.006	−0.004	−0.003	−0.001
	D E	−0.004	−0.006	−0.007	−0.006	−0.005	−0.003	−0.001
	E	−0.002	−0.004	−0.005	−0.005	−0.004	−0.003	−0.002
C	A	−0.003	−0.006	−0.009	−0.010	−0.010	−0.008	−0.005
	A B	−0.003	−0.005	−0.009	−0.011	−0.013	−0.012	−0.007
	B	−0.001	−0.002	−0.004	−0.010	−0.012	−0.009	−0.004
	B C	0.006	0.015	0.023	−0.006	−0.022	0.016	0.014
				(0.020)	(−0.007)	(0.003)		
	C	0.022	0.045	0.067	0.085	0.067	0.045	0.022

Table A-34
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=1$
Angle of Skew $\phi=60°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.009	−0.016	−0.021	−0.025	−0.023	−0.015	−0.007
	A B	0.000	−0.004	−0.019	M_{oy} −0.040	−0.010	0.010	0.013
		(0.017)	(0.028)	(0.070)	(0.152)	(0.079)	(0.042)	(0.020)
	B	0.017	0.036	0.051	0.050	0.031	0.018	0.008
	B C	0.013	0.020	0.023	0.021	0.016	0.010	0.004
	C	0.007	0.010	0.012	0.012	0.010	0.006	0.003
	C D	0.004	0.007	0.008	0.008	0.007	0.005	0.002
	D	0.003	0.005	0.006	0.006	0.005	0.003	0.001
	D E	0.002	0.004	0.004	0.004	0.003	0.002	0.001
	E	0.002	0.003	0.004	0.004	0.003	0.002	0.001
B	A	−0.001	−0.002	−0.004	−0.012	−0.019	−0.016	−0.008
	A B	0.002	0.010	0.026	0.040	0.019	0.038	0.025
				(0.023)	(0.036)	(0.042)		
	B	0.023	0.056	0.100	0.183	0.104	0.062	0.028
					(0.162)			
	B C	0.026	0.046	0.047	0.060	0.045	0.026	0.010
				(0.058)	(0.056)	(0.042)		
	C	0.015	0.023	0.026	0.026	0.022	0.014	0.006
	C D	0.008	0.013	0.016	0.017	0.015	0.010	0.005
	D	0.005	0.009	0.012	0.012	0.010	0.007	0.003
	D E	0.004	0.008	0.009	0.009	0.008	0.005	0.003
	E	0.004	0.007	0.008	0.008	0.007	0.005	0.002
B C	A	0.005	0.010	0.013	0.013	0.010	0.006	0.003
	A B	0.005	0.012	0.020	0.026	0.029	0.028	0.019
	B	0.010	0.025	0.045	0.069	0.073	0.057	0.031
	B C	0.021	0.034	0.028	M_{oy} +0.011	0.027	0.034	0.022
		(0.028)	(0.066)	(0.119)	(0.201)	(0.118)	(0.066)	(0.029)
	C	0.031	0.058	0.075	0.072	0.049	0.029	0.013
	C D	0.020	0.032	0.036	0.035	0.028	0.018	0.008
	D	0.011	0.018	0.022	0.022	0.019	0.012	0.006
	D E	0.008	0.013	0.017	0.017	0.014	0.010	0.005
	E	0.007	0.012	0.015	0.015	0.013	0.009	0.005
C	A	0.005	0.010	0.014	0.015	0.013	0.010	0.005
	A B	0.005	0.010	0.015	0.017	0.016	0.012	0.008
	B	0.006	0.014	0.022	0.026	0.026	0.023	0.015
	B C	0.010	0.027	0.046	0.062	0.047	0.049	0.028
				(0.044)	(0.058)	(0.060)		
	C	0.029	0.065	0.109	0.190	0.109	0.065	0.029
					(0.169)			

Table A-35
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=2$
Angle of Skew $\phi=60°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.008	−0.015	−0.020	−0.024	−0.022	−0.016	−0.008
	A B	0.000	−0.004	−0.021	M_{oy} −0.044	−0.014	0.008	0.012
		(0.007)	(0.028)	(0.068)	(0.148)	(0.075)	(0.040)	(0.019)
	B	0.016	0.033	0.045	0.044	0.029	0.017	0.008
	B C	0.010	0.016	0.018	0.017	0.014	0.009	0.004
	C	0.005	0.008	0.009	0.010	0.009	0.006	0.003
	C D	0.003	0.005	0.007	0.007	0.006	0.004	0.002
	D	0.002	0.004	0.005	0.005	0.004	0.003	0.001
	D E	0.002	0.003	0.003	0.003	0.002	0.002	0.001
	E	0.001	0.002	0.002	0.002	0.002	0.001	0.001
B	A	−0.002	−0.004	−0.008	−0.016	−0.022	−0.019	−0.010
	A B	0.001	0.009	0.022	0.028	0.006	0.017	0.021
				(0.019)	(0.024)	(0.029)	(0.030)	
	B	0.023	0.052	0.088	0.149	0.091	0.056	0.027
					(0.136)			
	B C	0.022	0.036	0.030	0.044	0.037	0.022	0.009
				(0.041)	(0.040)	(0.034)		
	C	0.009	0.013	0.015	0.017	0.016	0.011	0.005
	C D	0.004	0.006	0.009	0.011	0.011	0.007	0.003
	D	0.003	0.005	0.007	0.008	0.007	0.005	0.002
	D E	0.002	0.004	0.006	0.006	0.005	0.003	0.001
	E	0.002	0.003	0.004	0.004	0.004	0.003	0.001
B C	A	0.004	0.007	0.008	0.007	0.004	0.002	0.000
	A B	0.004	0.009	0.015	0.018	0.020	0.019	0.013
	B	0.009	0.021	0.036	0.054	0.057	0.045	0.025
	B C	0.017	0.025	0.013	M_{oy} −0.007	0.012	0.025	0.018
		(0.024)	(0.057)	(0.104)	(0.183)	(0.103)	(0.057)	(0.025)
	C	0.025	0.045	0.058	0.056	0.039	0.024	0.011
	C D	0.014	0.022	0.025	0.025	0.022	0.015	0.007
	D	0.007	0.011	0.015	0.016	0.014	0.009	0.005
	D E	0.005	0.008	0.011	0.011	0.010	0.007	0.003
	E	0.004	0.007	0.009	0.009	0.008	0.006	0.003
C	A	0.003	0.006	0.008	0.009	0.007	0.005	0.002
	A B	0.003	0.007	0.010	0.011	0.009	0.006	0.003
	B	0.005	0.011	0.016	0.017	0.014	0.012	0.008
	B C	0.009	0.022	0.037	0.044	0.028	0.036	0.022
				(0.035)	(0.040)	(0.041)		
	C	0.027	0.057	0.093	0.153	0.093	0.057	0.027
					(0.140)			

Table A-36
Influence Coefficients for Transverse Moment in Slab at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Relative Stiffness of Beams $H=5$
Angle of Skew $\phi=60°$

Values of Influence Coefficient for Moment — Longitudinal Position of Load

Moment on Line	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
A B	A	−0.007	−0.012	−0.017	−0.020	−0.019	−0.014	−0.007
	A B	0.001	−0.005	−0.023	M_{oy} −0.048	−0.018	0.005	0.011
		(0.008)	(0.027)	(0.066)	(0.144)	(0.071)	(0.037)	(0.018)
	B	0.014	0.027	0.036	0.035	0.025	0.016	0.007
	B C	0.007	0.010	0.011	0.012	0.012	0.008	0.003
	C	0.002	0.004	0.005	0.006	0.006	0.004	0.002
	C D	0.001	0.003	0.004	0.004	0.004	0.003	0.001
	D	0.001	0.002	0.002	0.002	0.002	0.001	0.001
	D E	0.001	0.001	0.001	0.001	0.001	0.001	0.000
	E	0.000	0.001	0.001	0.001	0.001	0.000	0.000
B	A	−0.004	−0.007	−0.011	−0.017	−0.021	−0.018	−0.010
	A B	0.001	0.008	0.018	0.014	−0.010	0.018	0.015
				(0.015)	(0.010)	(0.013)		
	B	0.022	0.045	0.071	0.100	0.072	0.047	0.023
	B C	0.015	0.021	0.007	0.022	0.026	0.015	0.006
				(0.018)	(0.018)	(0.023)		
	C	0.002	0.001	0.002	0.005	0.007	0.005	0.003
	C D	−0.001	−0.001	0.001	0.003	0.004	0.003	0.001
	D	0.000	0.001	0.002	0.002	0.002	0.001	0.001
	D E	0.000	0.001	0.001	0.001	0.001	0.001	0.000
	E	0.000	0.000	0.000	0.001	0.001	0.000	0.000
B C	A	0.001	0.002	0.003	0.002	0.000	−0.002	−0.001
	A B	0.003	0.006	0.009	0.009	0.008	0.008	0.006
	B	0.007	0.015	0.025	0.035	0.037	0.029	0.016
	B C	0.012	0.012	−0.007	M_{oy} −0.031	−0.008	0.012	0.012
		(0.019)	(0.044)	(0.084)	(0.159)	(0.083)	(0.044)	(0.019)
	C	0.016	0.029	0.037	0.036	0.026	0.016	0.008
	C D	0.006	0.009	0.011	0.013	0.014	0.010	0.004
	D	0.002	0.004	0.006	0.008	0.007	0.005	0.003
	D E	0.001	0.003	0.004	0.005	0.004	0.003	0.001
	E	0.001	0.002	0.003	0.003	0.003	0.002	0.001
C	A	0.001	0.001	0.001	0.001	0.001	0.000	0.000
	A B	0.001	0.002	0.003	0.002	0.001	−0.001	−0.001
	B	0.002	0.005	0.007	0.005	0.001	0.001	0.001
	B C	0.006	0.015	0.025	0.021	0.004	0.020	0.015
				(0.023)	(0.017)	(0.017)		
	C	0.023	0.046	0.072	0.100	0.072	0.046	0.023

Table A-37
Influence Coefficients for Deflection in Center Beam at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Angle of Skew $\phi=30°$

Values of Influence Coefficient for Deflection — Longitudinal Position of Load

Relative Stiffness of Beams H	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
2	A	0.00064	0.00120	0.00159	0.00176	0.00167	0.00132	0.00073
	A B	0.00072	0.00137	0.00187	0.00213	0.00206	0.00163	0.00091
	B	0.00082	0.00157	0.00217	0.00254	0.00243	0.00188	0.00102
	B C	0.00092	0.00178	0.00246	0.00299	0.00273	0.00200	0.00105
	C	0.00101	0.00196	0.00277	0.00325	0.00277	0.00196	0.00101
5	A	0.00065	0.00120	0.00157	0.00170	0.00158	0.00122	0.00067
	A B	0.00088	0.00170	0.00231	0.00260	0.00251	0.00201	0.00114
	B	0.00117	0.00225	0.00308	0.00353	0.00339	0.00265	0.00145
	B C	0.00143	0.00276	0.00379	0.00450	0.00415	0.00309	0.00164
	C	0.00163	0.00315	0.00439	0.00504	0.00439	0.00315	0.00163
10	A	0.00047	0.00086	0.00111	0.00119	0.00109	0.00083	0.00045
	A B	0.00091	0.00177	0.00241	0.00269	0.00259	0.00208	0.00120
	B	0.00144	0.00274	0.00373	0.00422	0.00404	0.00317	0.00174
	B C	0.00190	0.00366	0.00497	0.00584	0.00540	0.00405	0.00216
	C	0.00223	0.00429	0.00592	0.00672	0.00592	0.00429	0.00223

Table A-38
Influence Coefficients for Deflection in Center Beam at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Angle of Skew $\phi=45°$

Values of Influence Coefficient for Deflection — Longitudinal Position of Load

Relative Stiffness of Beams H	Transverse Location of Load	1/8	2/8	3/8	Center	5/8	6/8	7/8
2	A	0.00052	0.00099	0.00133	0.00149	0.00143	0.00114	0.00064
	A B	0.00056	0.00108	0.00150	0.00176	0.00175	0.00142	0.00081
	B	0.00063	0.00123	0.00174	0.00210	0.00208	0.00164	0.00090
	B C	0.00071	0.00141	0.00201	0.00253	0.00236	0.00173	0.00090
	C	0.00082	0.00163	0.00236	0.00279	0.00236	0.00163	0.00082
5	A	0.00059	0.00110	0.00145	0.00158	0.00147	0.00114	0.00063
	A B	0.00073	0.00143	0.00198	0.00228	0.00226	0.00186	0.00109
	B	0.00095	0.00187	0.00261	0.00308	0.00305	0.00244	0.00136
	B C	0.00118	0.00233	0.00326	0.00399	0.00375	0.00281	0.00149
	C	0.00142	0.00277	0.00390	0.00452	0.00390	0.00277	0.00142
10	A	0.00048	0.00089	0.00115	0.00122	0.00111	0.00085	0.00046
	A B	0.00078	0.00155	0.00216	0.00247	0.00243	0.00201	0.00120
	B	0.00122	0.00237	0.00328	0.00381	0.00374	0.00299	0.00167
	B C	0.00163	0.00319	0.00441	0.00530	0.00500	0.00378	0.00203
	C	0.00201	0.00389	0.00541	0.00617	0.00541	0.00389	0.00201

Table A-39
Influence Coefficients for Deflection in Center Beam at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.1$
Angle of Skew $\phi=60°$

Relative Stiffness of Beams H	Transverse Location of Load	Values of Influence Coefficient for Deflection — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
2	A	0.00046	0.00088	0.00120	0.00138	0.00135	0.00110	0.00063
	A B	0.00046	0.00089	0.00125	0.00149	0.00153	0.00128	0.00074
	B	0.00049	0.00097	0.00142	0.00177	0.00181	0.00146	0.00081
	B C	0.00055	0.00114	0.00170	0.00216	0.00207	0.00153	0.00078
	C	0.00068	0.00140	0.00205	0.00242	0.00205	0.00140	0.00068
5	A	0.00059	0.00110	0.00146	0.00161	0.00153	0.00120	0.00067
	A B	0.00062	0.00123	0.00172	0.00203	0.00207	0.00175	0.00106
	B	0.00075	0.00151	0.00219	0.00267	0.00273	0.00224	0.00127
	B C	0.00093	0.00192	0.00280	0.00348	0.00336	0.00255	0.00135
	C	0.00121	0.00242	0.00346	0.00401	0.00346	0.00242	0.00121
10	A	0.00055	0.00102	0.00133	0.00144	0.00133	0.00102	0.00055
	A B	0.00067	0.00137	0.00194	0.00228	0.00232	0.00198	0.00122
	B	0.00098	0.00196	0.00281	0.00337	0.00341	0.00280	0.00160
	B C	0.00131	0.00267	0.00384	0.00469	0.00453	0.00348	0.00188
	C	0.00176	0.00345	0.00484	0.00554	0.00484	0.00345	0.00176

Table A-40
Influence Coefficients for Deflection in Center Beam at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Angle of Skew $\phi=30°$

Relative Stiffness of Beams H	Transverse Location of Load	Values of Influence Coefficient for Deflection — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
1	A	0.00050	0.00094	0.00127	0.00142	0.00136	0.00108	0.00060
	A B	0.00065	0.00129	0.00180	0.00209	0.00207	0.00170	0.00099
	B	0.00089	0.00176	0.00246	0.00288	0.00283	0.00227	0.00127
	B C	0.00115	0.00228	0.00321	0.00382	0.00362	0.00275	0.00147
	C	0.00143	0.00279	0.00389	0.00445	0.00389	0.00279	0.00143
2	A	0.00042	0.00080	0.00106	0.00116	0.00109	0.00085	0.00047
	A B	0.00071	0.00143	0.00200	0.00231	0.00228	0.00188	0.00111
	B	0.00116	0.00225	0.00312	0.00361	0.00353	0.00283	0.00158
	B C	0.00161	0.00316	0.00441	0.00521	0.00494	0.00378	0.00205
	C	0.00207	0.00400	0.00552	0.00626	0.00552	0.00400	0.00207
5	A	0.00017	0.00031	0.00040	0.00042	0.00038	0.00028	0.00015
	A B	0.00066	0.00136	0.00190	0.00217	0.00213	0.00175	0.00104
	B	0.00143	0.00274	0.00374	0.00425	0.00411	0.00326	0.00182
	B C	0.00225	0.00443	0.00613	0.00716	0.00680	0.00525	0.00290
	C	0.00312	0.00595	0.00812	0.00911	0.00812	0.00595	0.00312

Table A-41
Influence Coefficients for Deflection in Center Beam at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Angle of Skew $\phi=45°$

Relative Stiffness of Beams H	Transverse Location of Load	Values of Influence Coefficient for Deflection — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
1	A	0.00041	0.00079	0.00108	0.00123	0.00121	0.00098	0.00056
	A B	0.00047	0.00097	0.00140	0.00169	0.00174	0.00149	0.00090
	B	0.00065	0.00132	0.00194	0.00236	0.00241	0.00199	0.00114
	B C	0.00087	0.00181	0.00266	0.00324	0.00315	0.00243	0.00130
	C	0.00120	0.00238	0.00337	0.00386	0.00337	0.00238	0.00120
2	A	0.00041	0.00079	0.00106	0.00119	0.00113	0.00090	0.00051
	A B	0.00055	0.00115	0.00167	0.00200	0.00206	0.00176	0.00108
	B	0.00088	0.00179	0.00258	0.00310	0.00314	0.00259	0.00148
	B C	0.00126	0.00260	0.00377	0.00455	0.00443	0.00344	0.00189
	C	0.00180	0.00352	0.00491	0.00558	0.00491	0.00352	0.00180
5	A	0.00026	0.00049	0.00064	0.00069	0.00064	0.00049	0.00026
	A B	0.00055	0.00121	0.00175	0.00208	0.00211	0.00179	0.00110
	B	0.00117	0.00232	0.00327	0.00383	0.00381	0.00311	0.00177
	B C	0.00184	0.00378	0.00542	0.00647	0.00630	0.00496	0.00279
	C	0.00283	0.00543	0.00745	0.00837	0.00745	0.00543	0.00283

Table A-42
Influence Coefficients for Deflection in Center Beam at Mid-span of Bridge

Relative Proportions of Bridge $b/a=0.2$
Angle of Skew $\phi=60°$

Relative Stiffness of Beams H	Transverse Location of Load	Values of Influence Coefficient for Deflection — Longitudinal Position of Load: 1/8	2/8	3/8	Center	5/8	6/8	7/8
1	A	0.00022	0.00042	0.00060	0.00072	0.00074	0.00063	0.00038
	A B	0.00022	0.00046	0.00070	0.00091	0.00100	0.00091	0.00057
	B	0.00030	0.00067	0.00108	0.00141	0.00152	0.00132	0.00078
	B C	0.00048	0.00111	0.00174	0.00215	0.00216	0.00170	0.00092
	C	0.00080	0.00165	0.00235	0.00267	0.00235	0.00165	0.00080
2	A	0.00029	0.00057	0.00079	0.00091	0.00091	0.00075	0.00043
	A B	0.00031	0.00066	0.00100	0.00128	0.00139	0.00125	0.00080
	B	0.00047	0.00104	0.00162	0.00208	0.00222	0.00191	0.00114
	B C	0.00077	0.00174	0.00268	0.00331	0.00331	0.00263	0.00145
	C	0.00131	0.00263	0.00370	0.00420	0.00370	0.00263	0.00131
5	A	0.00030	0.00058	0.00079	0.00088	0.00084	0.00067	0.00037
	A B	0.00039	0.00086	0.00131	0.00164	0.00176	0.00157	0.00101
	B	0.00074	0.00157	0.00236	0.00293	0.00306	0.00260	0.00154
	B C	0.00124	0.00277	0.00421	0.00514	0.00515	0.00415	0.00235
	C	0.00225	0.00440	0.00608	0.00684	0.00608	0.00440	0.00225

APPENDIX B: INFLUENCE SURFACES FOR MOMENTS IN BEAMS, MOMENTS IN SLAB, AND DEFLECTIONS OF BEAMS

The diagrams contained herein show influence surfaces, in the form of contour maps, for a unit concentrated load on the bridge as shown in Fig. 1. The quantities ϕ, H, b, and a are defined in Chapter I. Poisson's ratio is zero. The quantity M_{oy} is given by Eq. (49a).

The numerical values of ordinates of the influence surfaces shown are such that:

(1) Actual moments in beams for concentrated loads are obtained by multiplying the values of the ordinates by the quantity Pa.

(2) Actual transverse moments per unit of length in the slab for concentrated loads are obtained by multiplying the values of ordinates by the quantity P.

(3) Actual deflections of beams for concentrated loads are obtained by multiplying the values of the ordinates by the quantity Pa^3/E_bI_b.

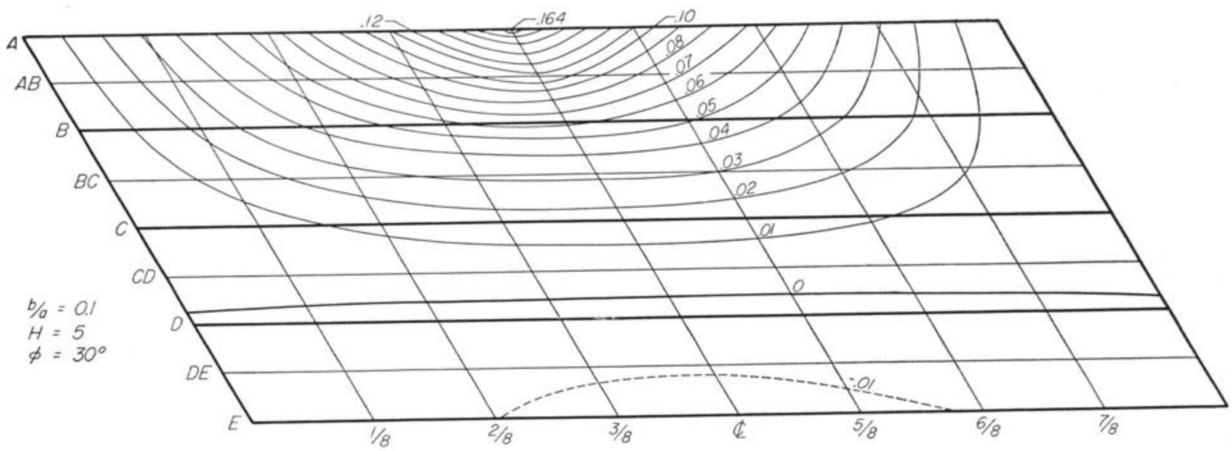


Fig. B-1. Influence surface for moment in beam "A" at mid-span

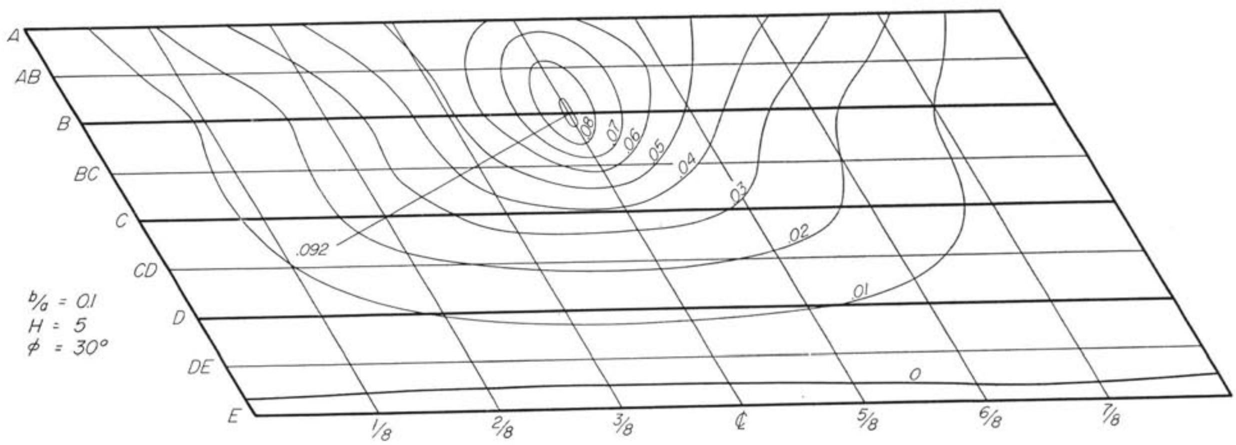


Fig. B-2. Influence surface for moment in beam "B" at mid-span

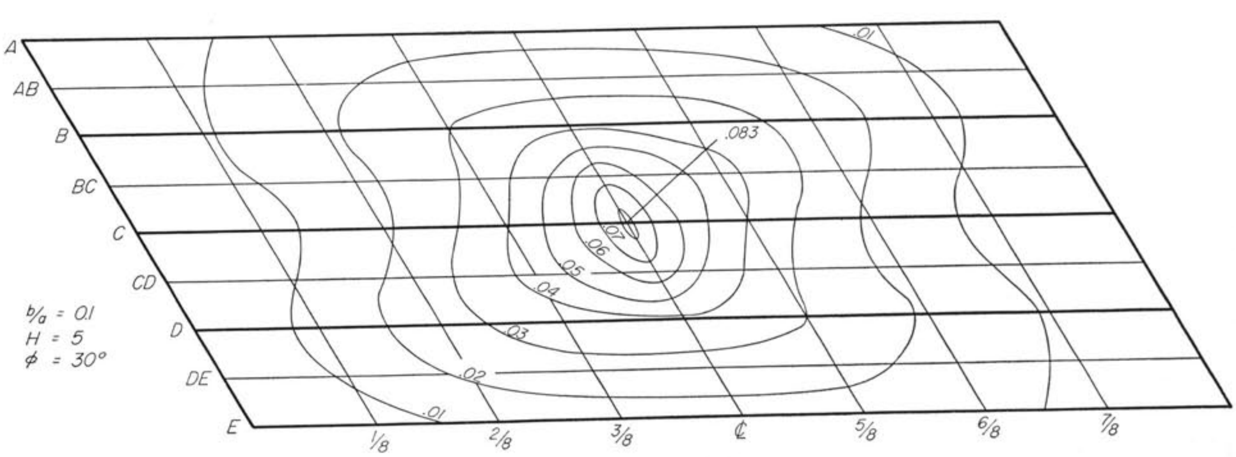


Fig. B-3. Influence surface for moment in beam "C" at mid-span

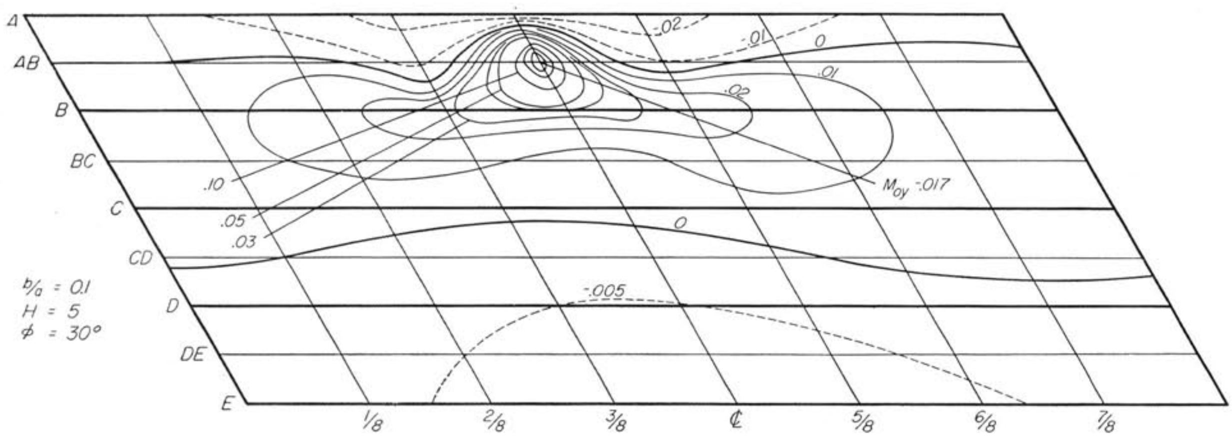


Fig. B-4. Influence surface for transverse moment in slab at AB at mid-span

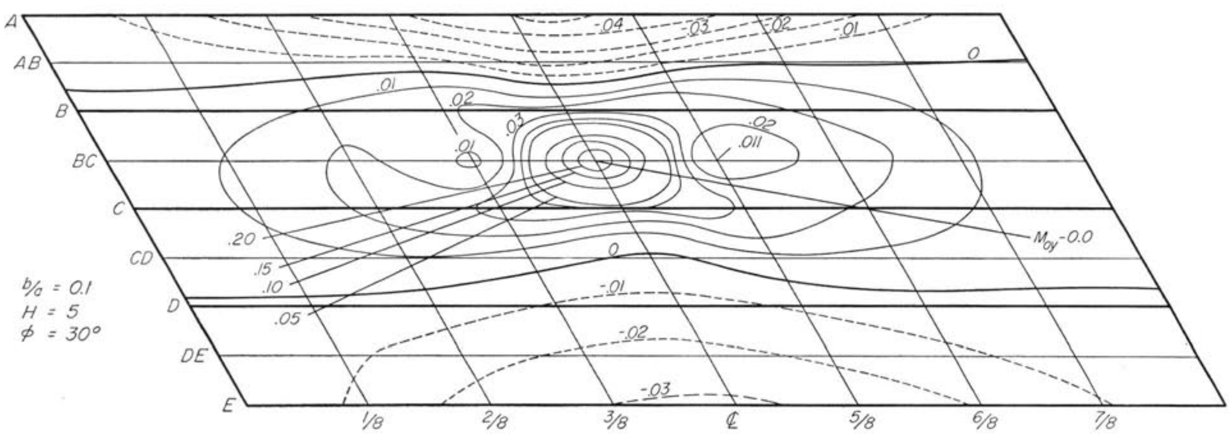


Fig. B-5. Influence surface for transverse moment in slab at BC at mid-span

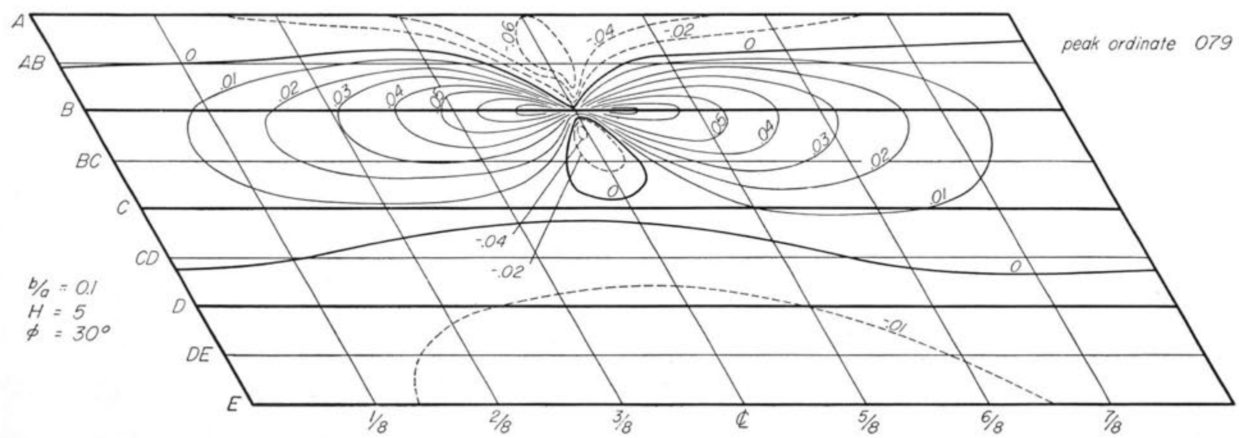


Fig. B-6. Influence surface for transverse moment in slab at B at mid-span

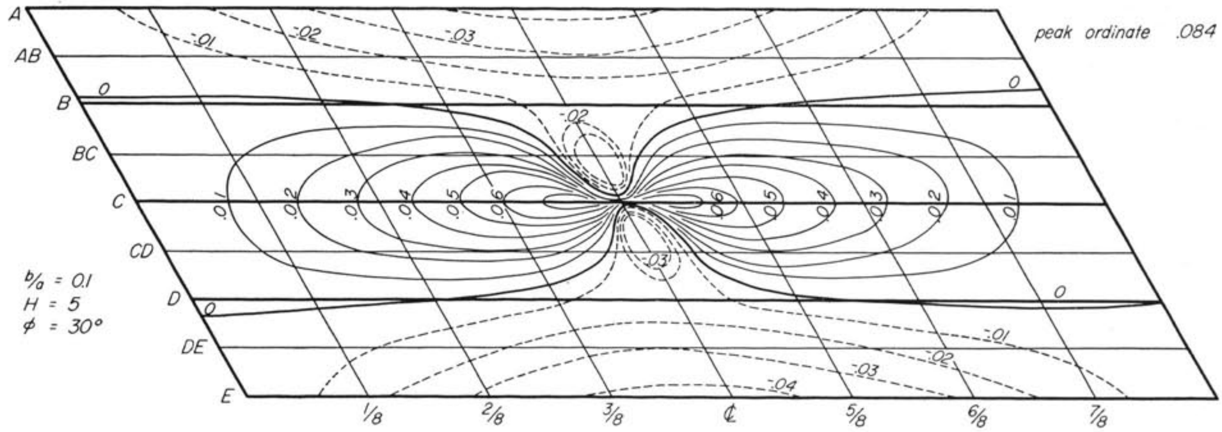


Fig. B-7. Influence surface for transverse moment in slab at C at mid-span

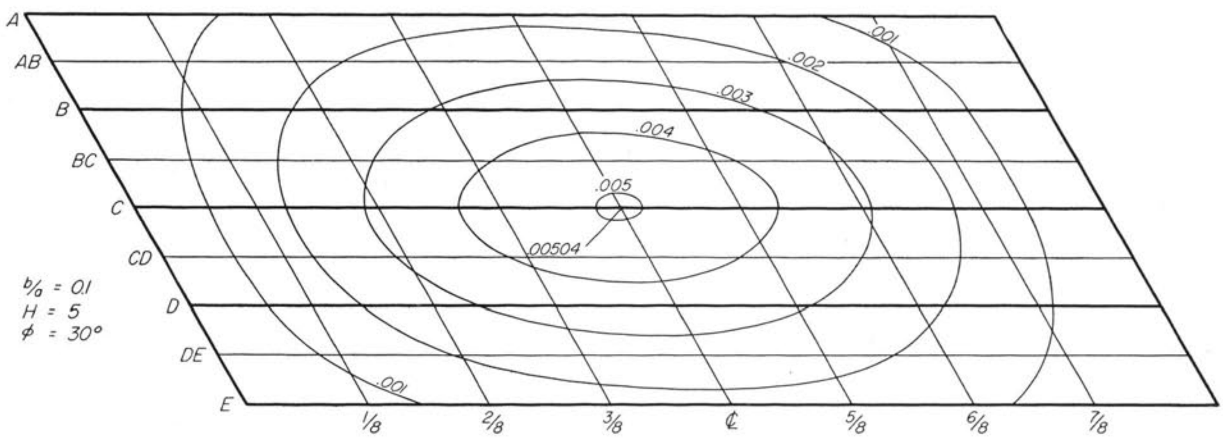


Fig. B-8. Influence surface for deflection of beam C at mid-span

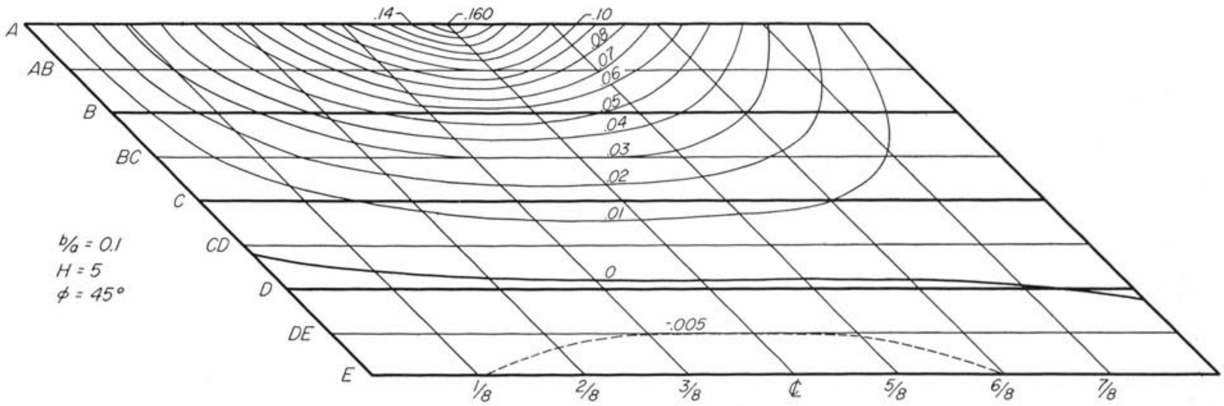


Fig. B-9. Influence surface for moment in beam "A" at mid-span

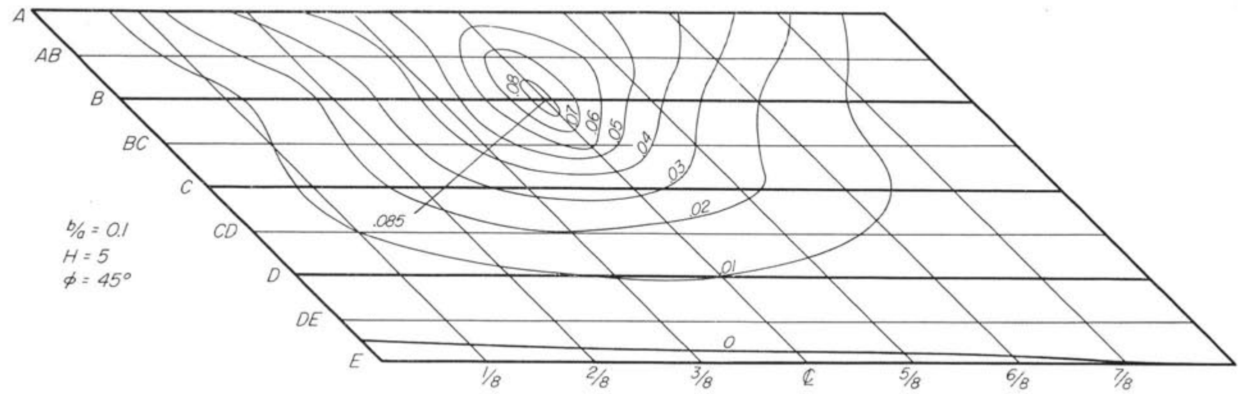


Fig. B-10. Influence surface for moment in beam "B" at mid-span

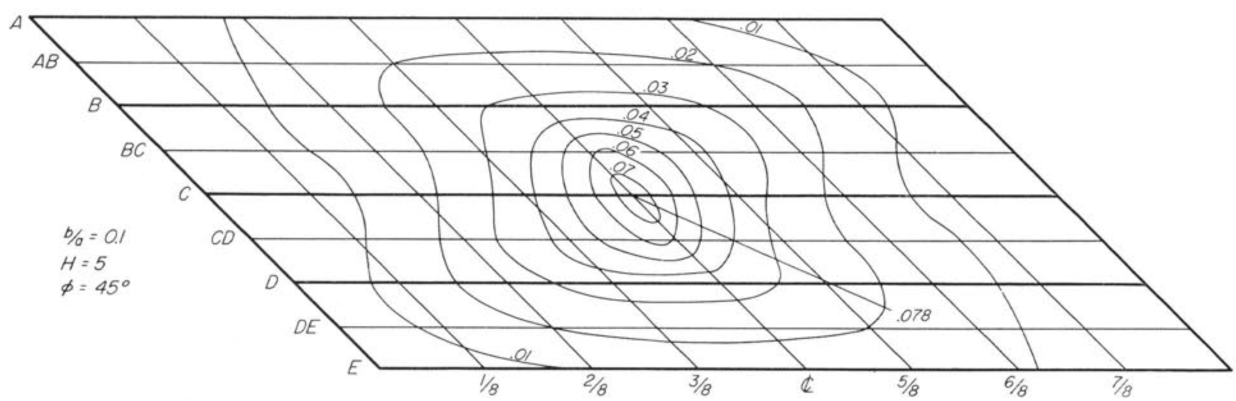


Fig. B-11. Influence surface for moment in beam "C" at mid-span

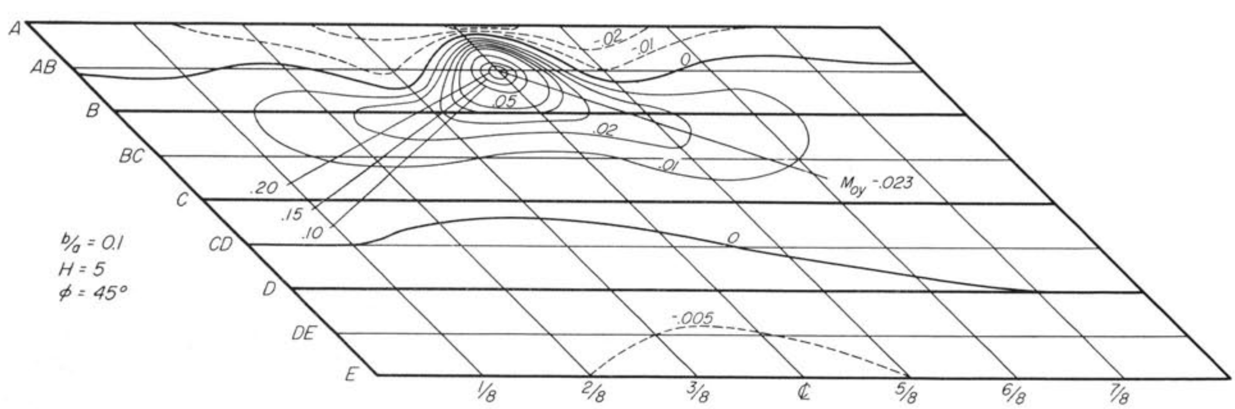


Fig. B-12. Influence surface for transverse moment in slab at AB at mid-span

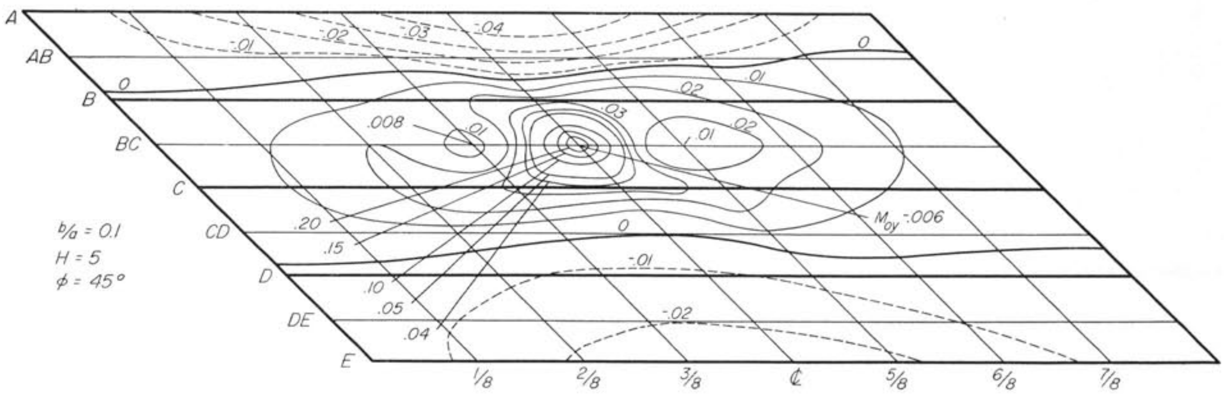


Fig. B-13. Influence surface for transverse moment in slab at BC at mid-span

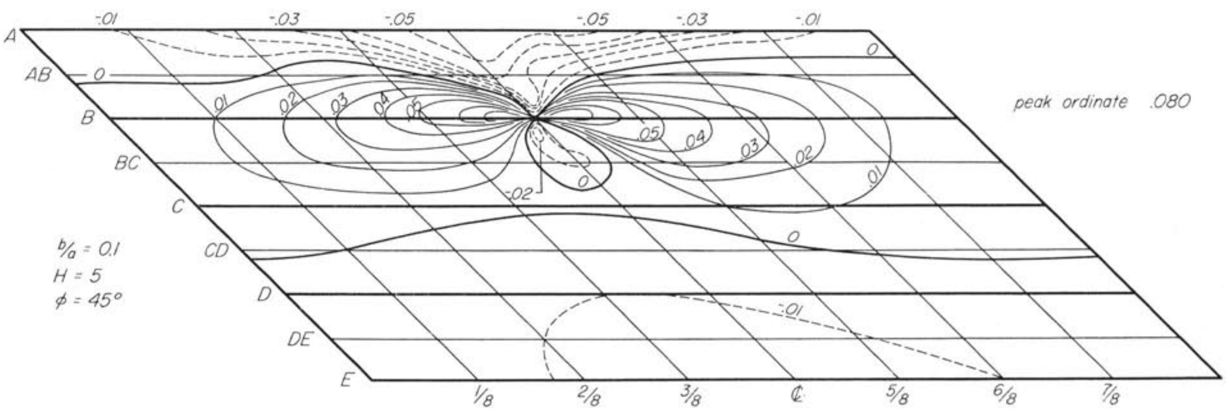


Fig. B-14. Influence surface for transverse moment in slab at B at mid-span

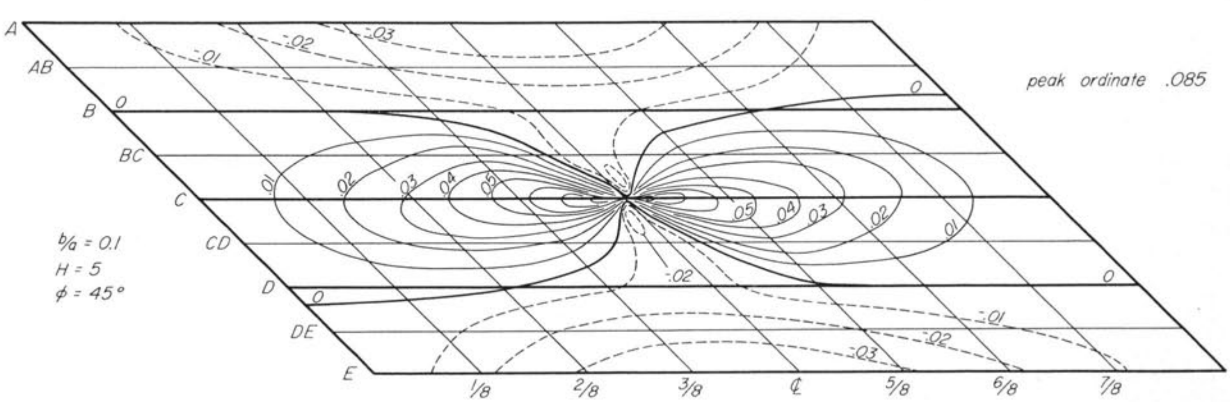


Fig. B-15. Influence surface for transverse moment in slab at C at mid-span

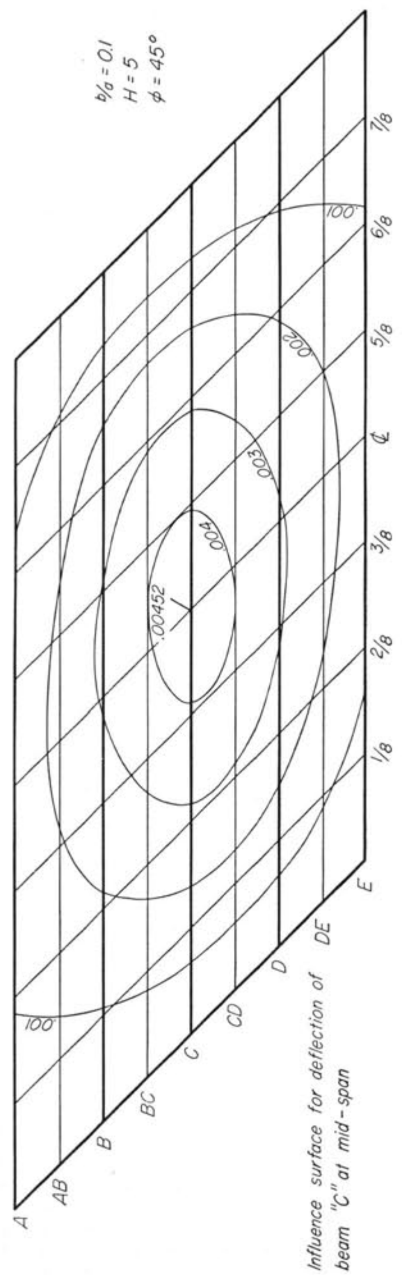


Fig. B-16. Influence surface for deflection of beam "C" at mid-span

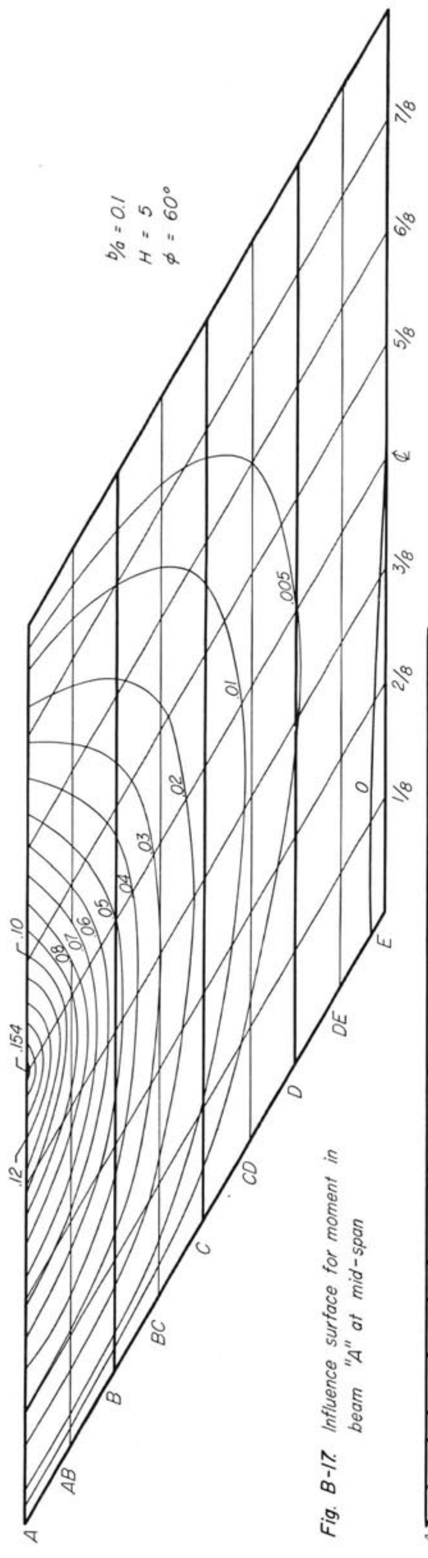


Fig. B-17. Influence surface for moment in beam "A" at mid-span

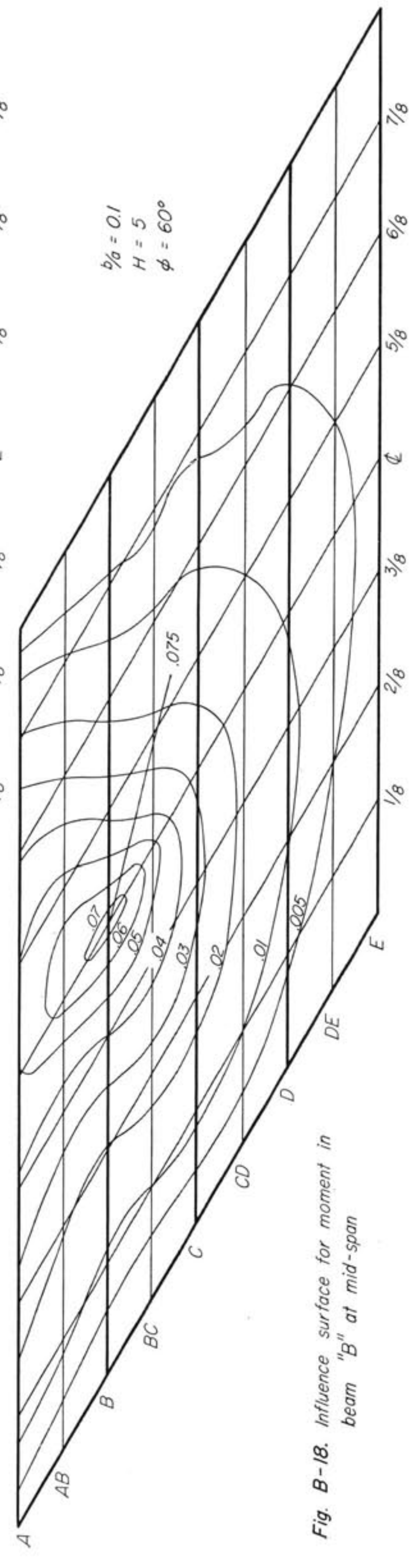


Fig. B-18. Influence surface for moment in beam "B" at mid-span

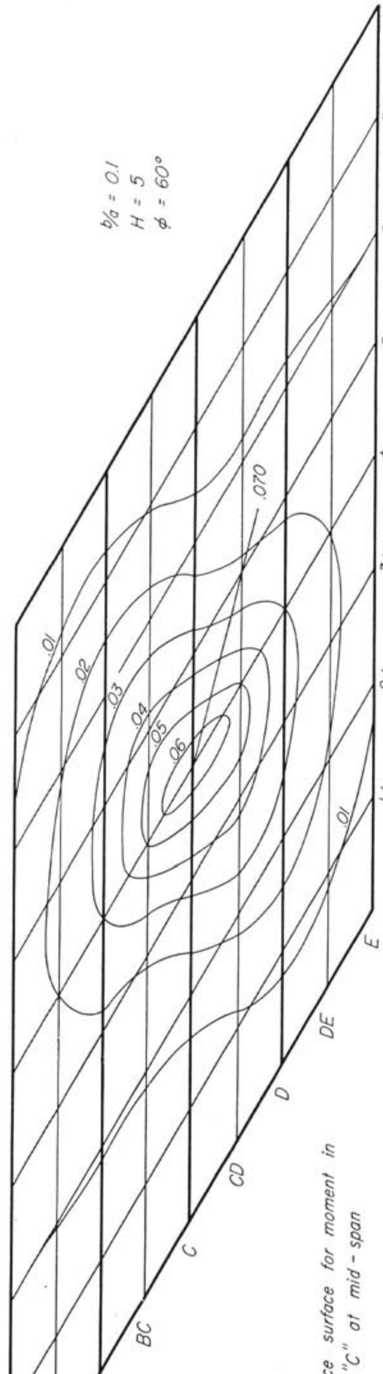


Fig. B-19. Influence surface for moment in beam "C" at mid-span

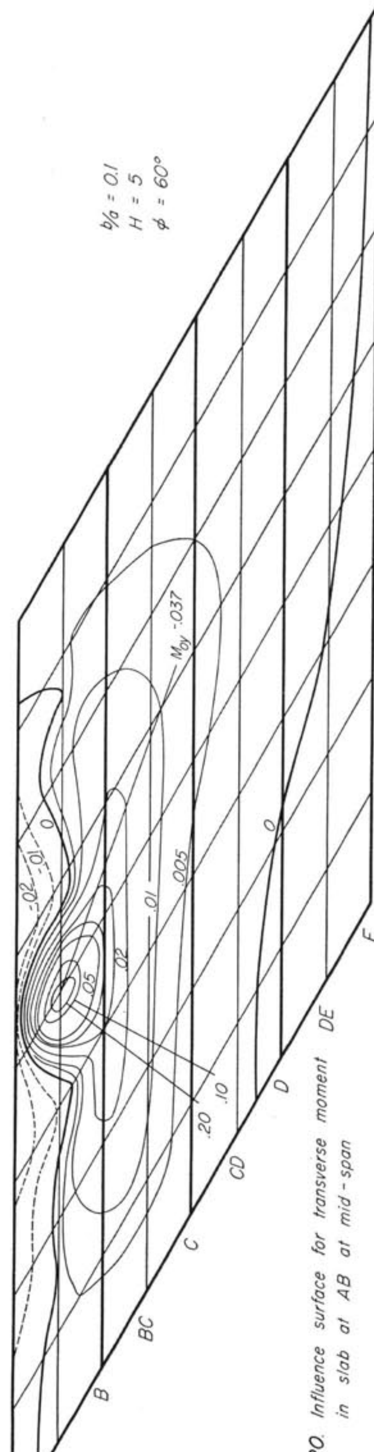


Fig. B-20. Influence surface for transverse moment in slab at AB at mid-span

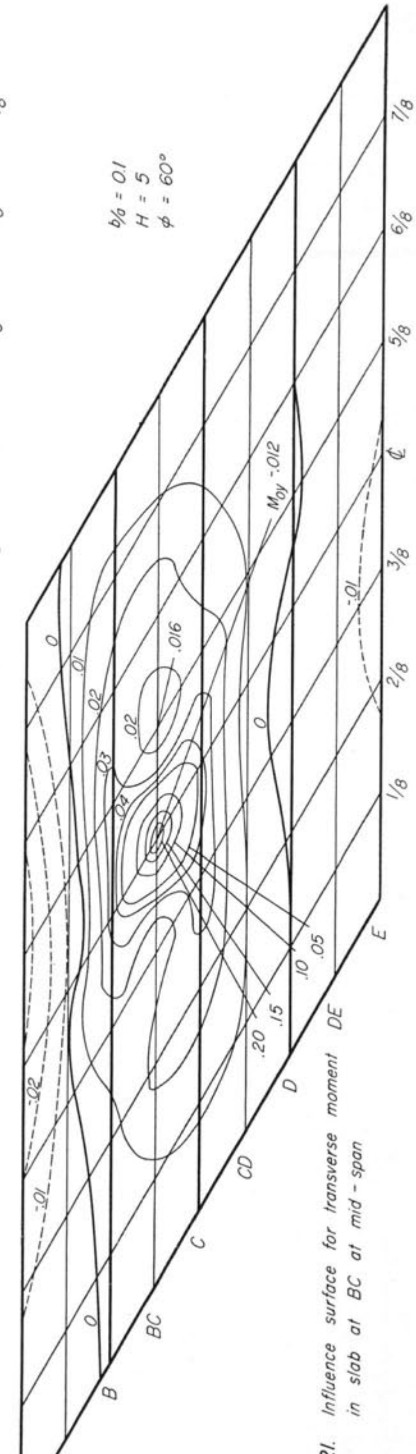


Fig. B-21. Influence surface for transverse moment in slab at BC at mid-span

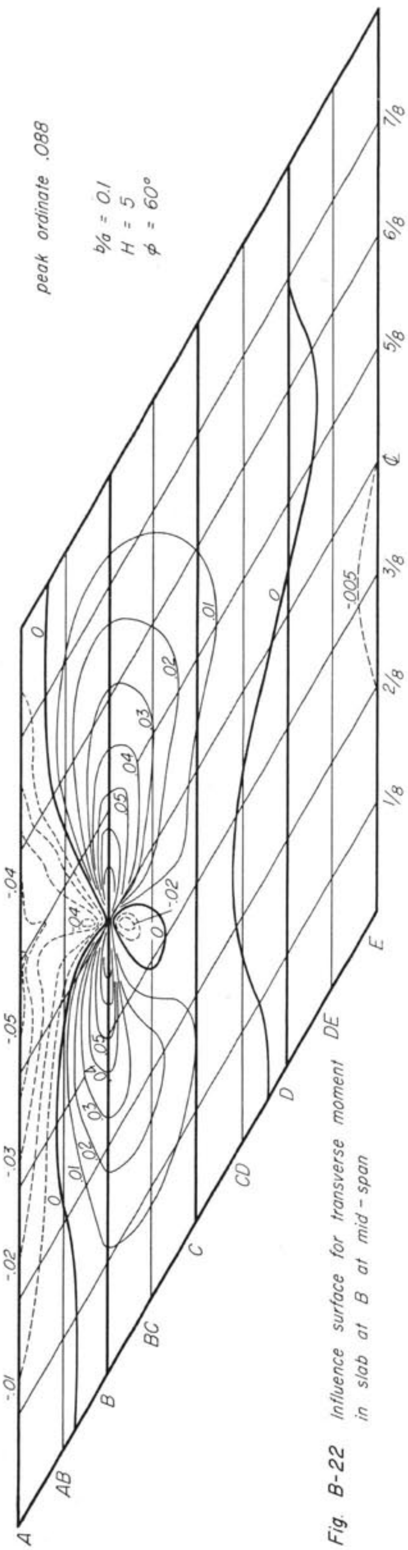


Fig. B-22 Influence surface for transverse moment in slab at B at mid-span

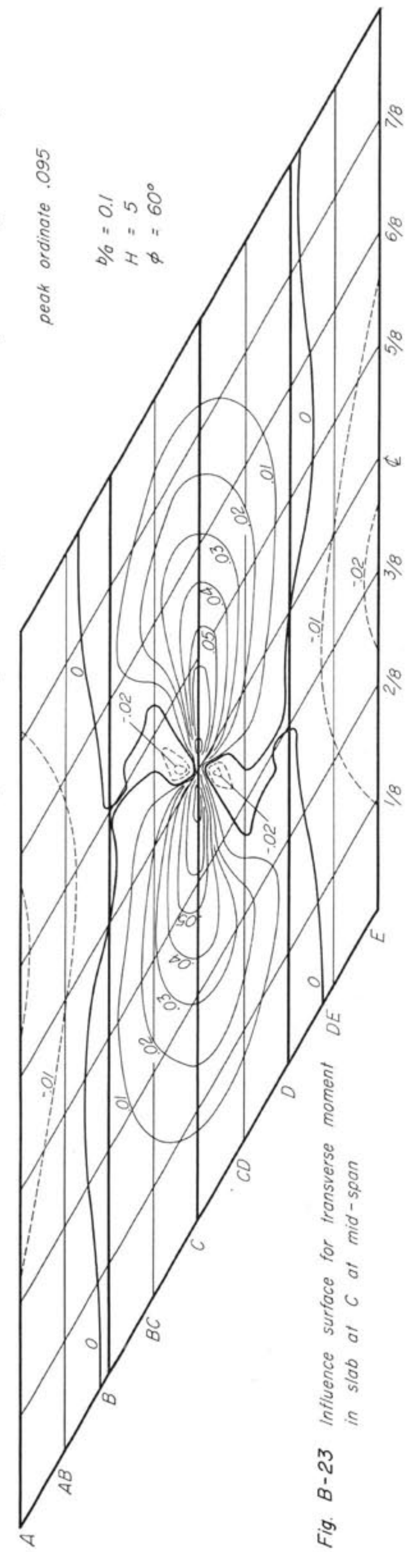


Fig. B-23 Influence surface for transverse moment in slab at C at mid-span

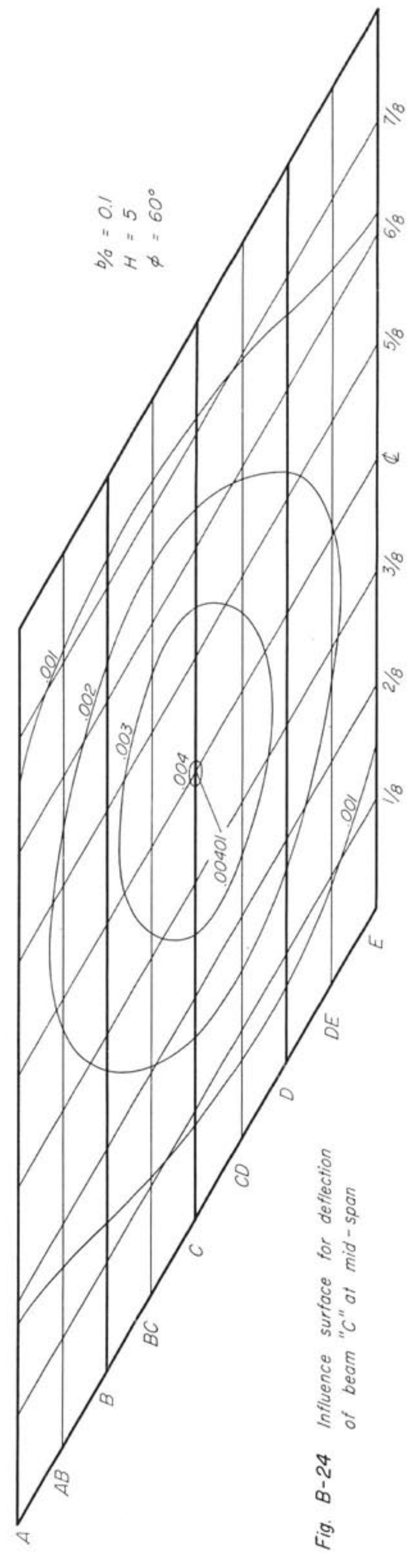


Fig. B-24 Influence surface for deflection of beam "C" at mid-span

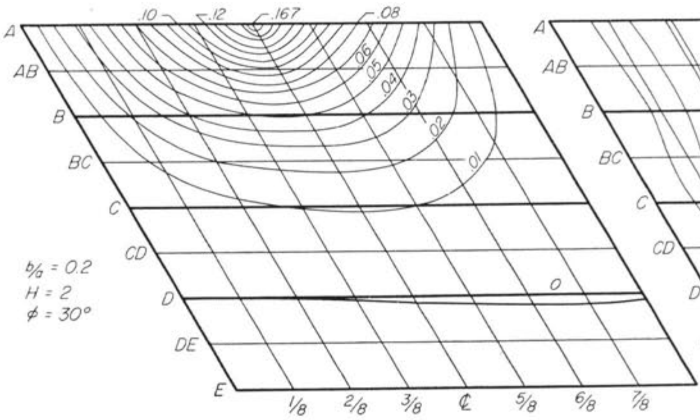


Fig. B-25. Influence surface for moment in beam "A" at mid-span

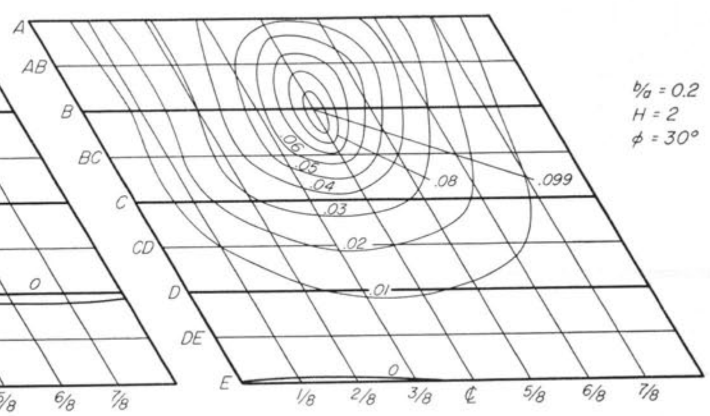


Fig. B-26. Influence surface for moment in beam "B" at mid-span

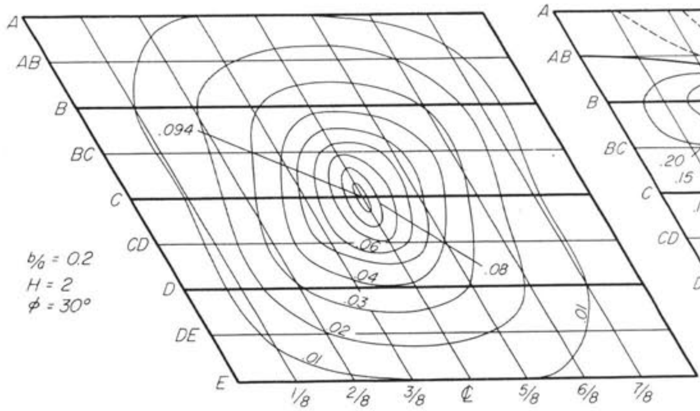


Fig. B-27. Influence surface for moment in beam "C" at mid-span

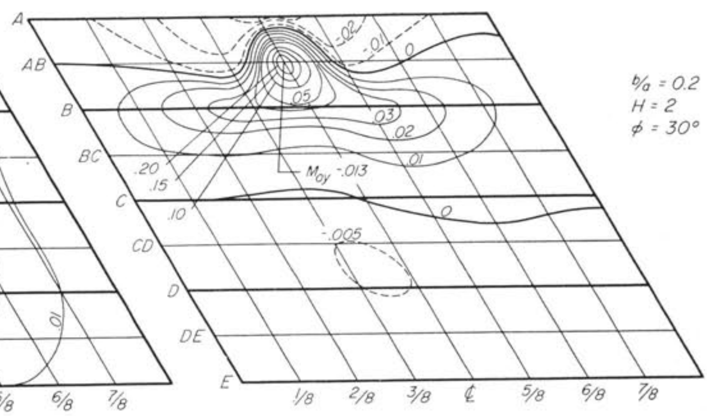


Fig. B-28. Influence surface for transverse moment in slab at AB at mid-span

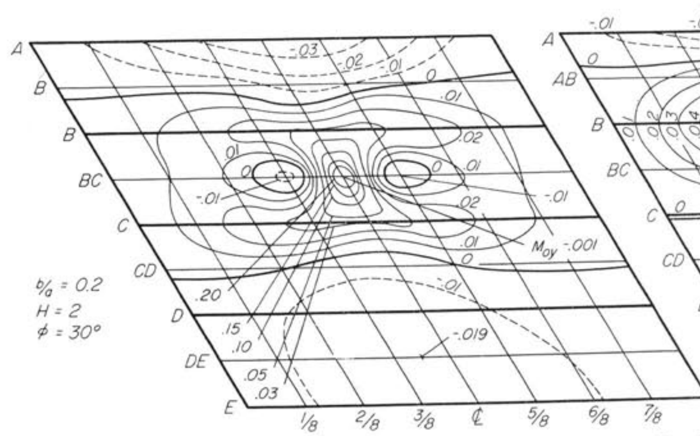


Fig. B-29. Influence surface for transverse moment in slab at BC at mid-span

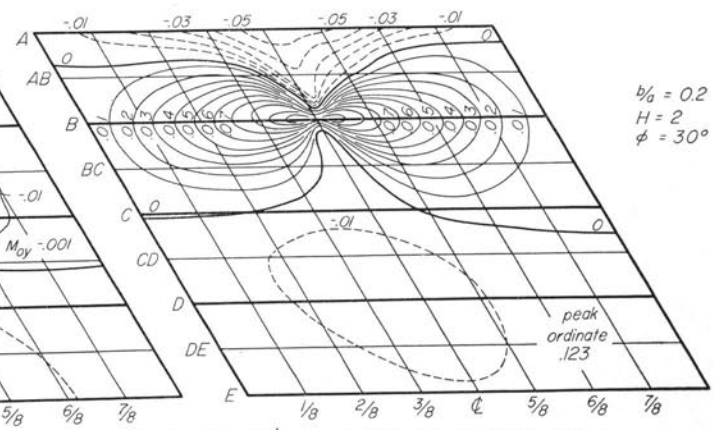


Fig. B-30. Influence surface for transverse moment in slab at B at mid-span

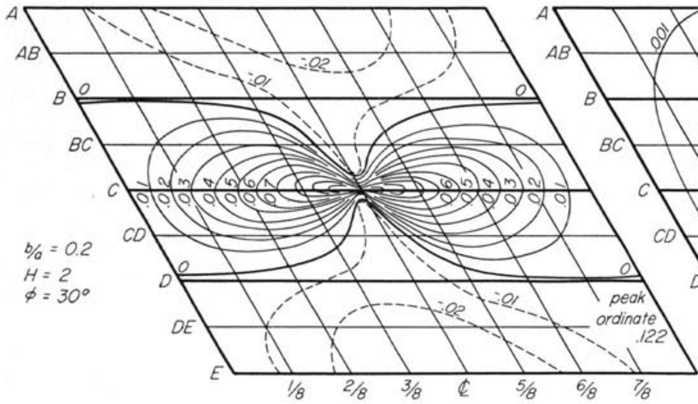


Fig. B-31. Influence surface for transverse moment in slab at C at mid-span

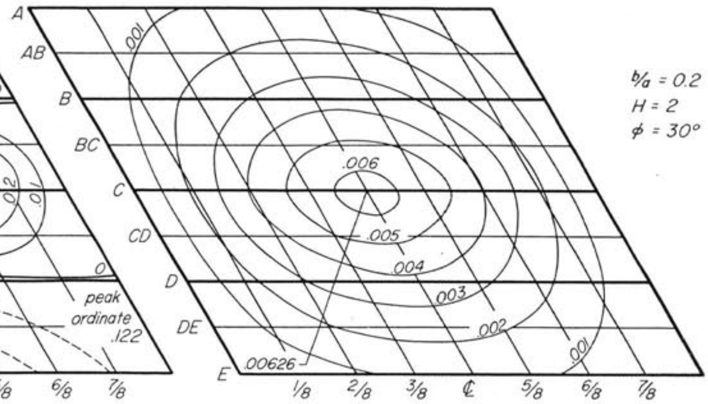


Fig. B-32. Influence surface for deflection of beam "C" at mid-span

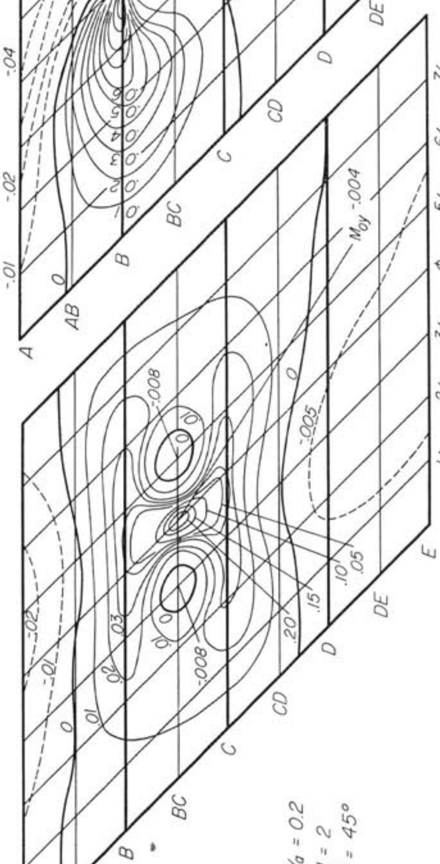


Fig. B-37. Influence surface for transverse moment in slab at BC at mid-span

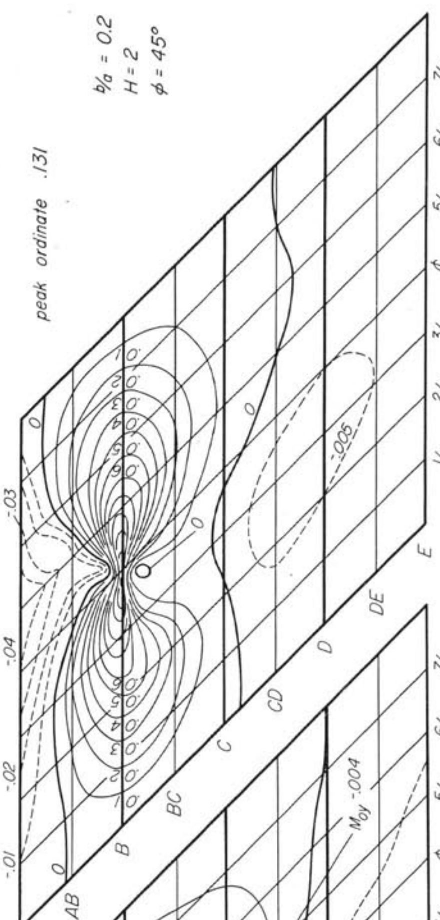


Fig. B-38. Influence surface for transverse moment in slab at B at mid-span

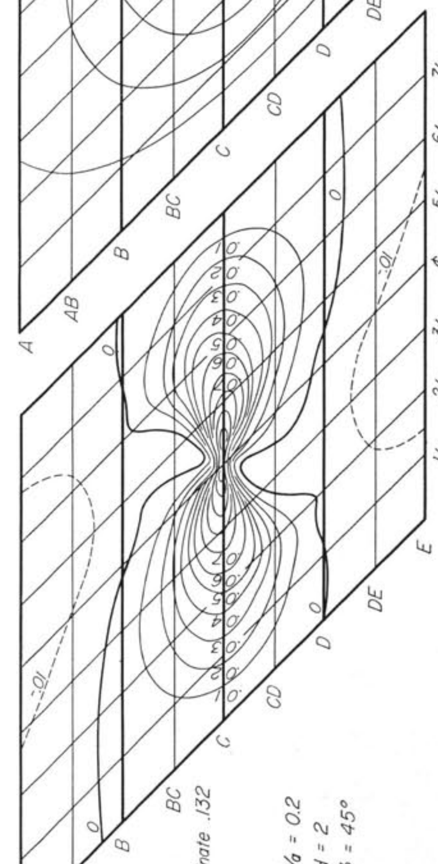


Fig. B-39. Influence surface for transverse moment in slab at C at mid-span

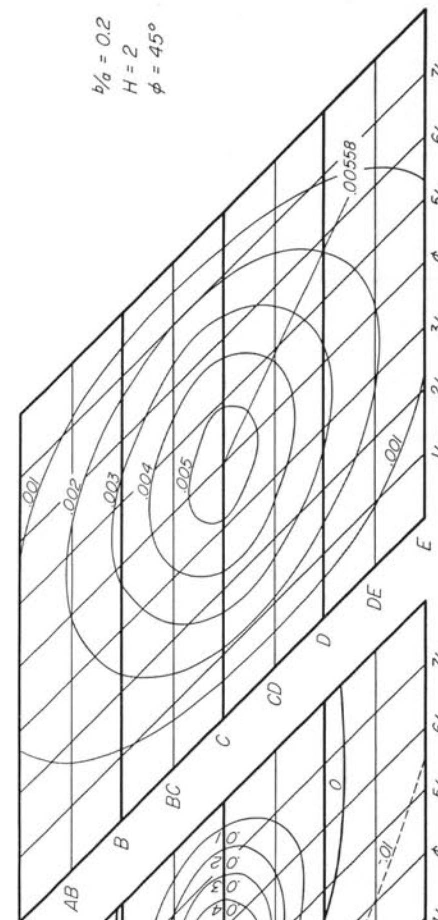


Fig. B-40. Influence surface for deflection of beam "C" at mid-span

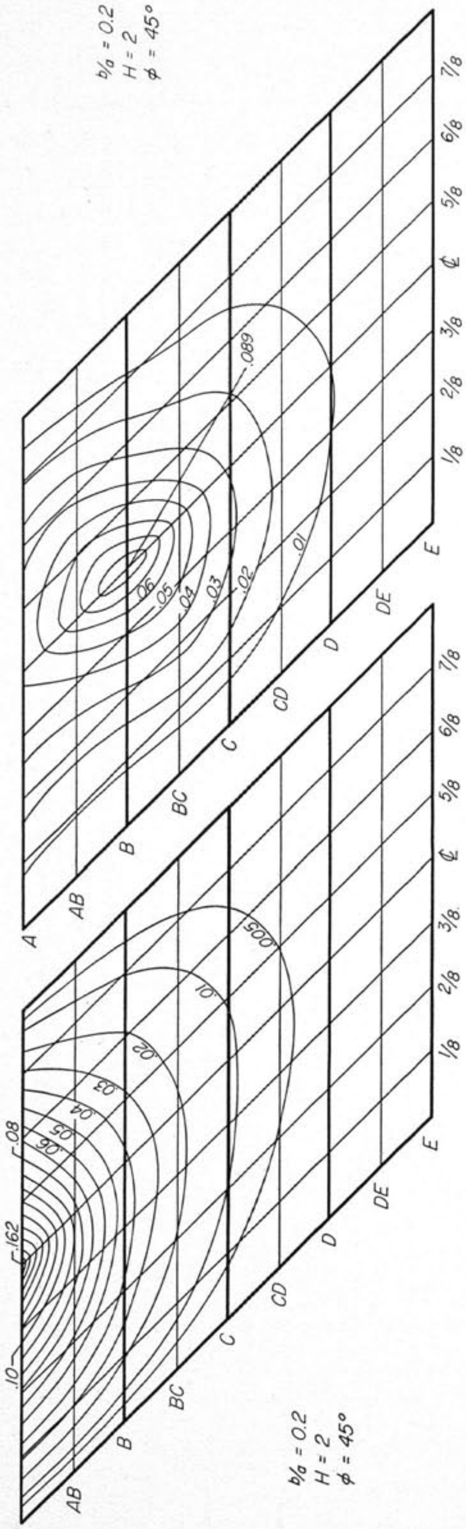


Fig. B-34. Influence surface for moment in beam "B" at mid-span

Fig. B-33. Influence surface for moment in beam "A" at mid-span

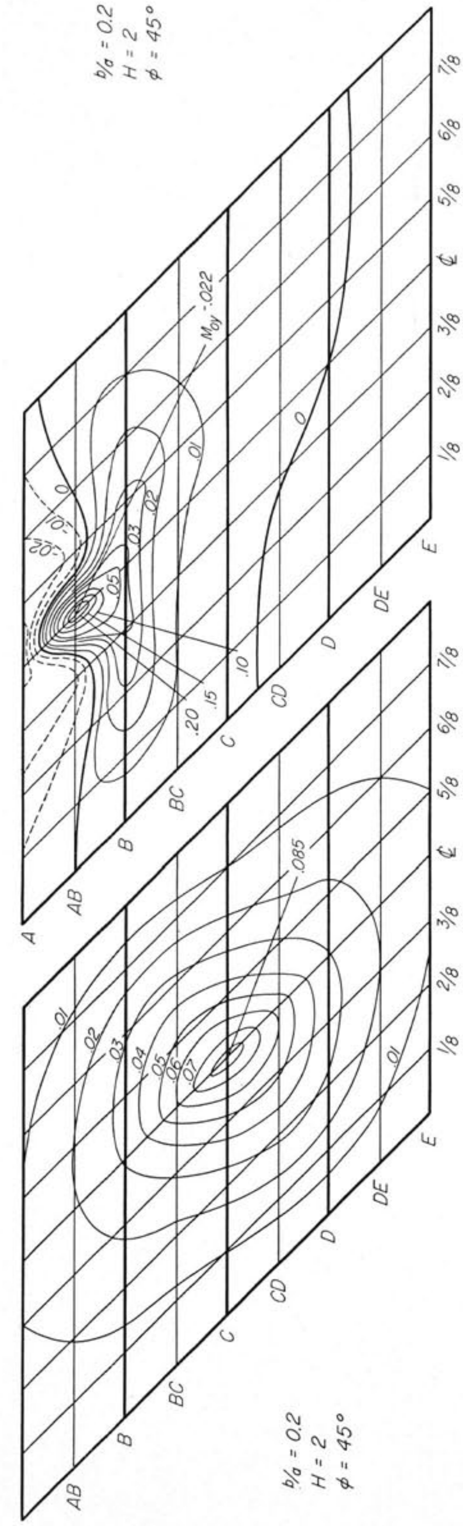


Fig. B-36. Influence surface for transverse moment in slab at AB at mid-span

Fig. B-35. Influence surface for moment in beam "C" at mid-span

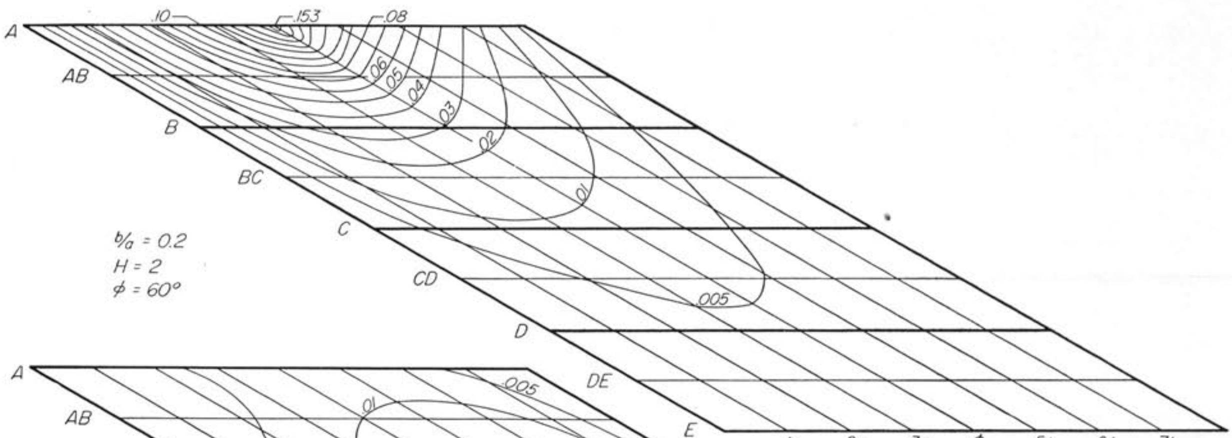


Fig. B-41. Influence surface for moment in beam "A" at mid-span

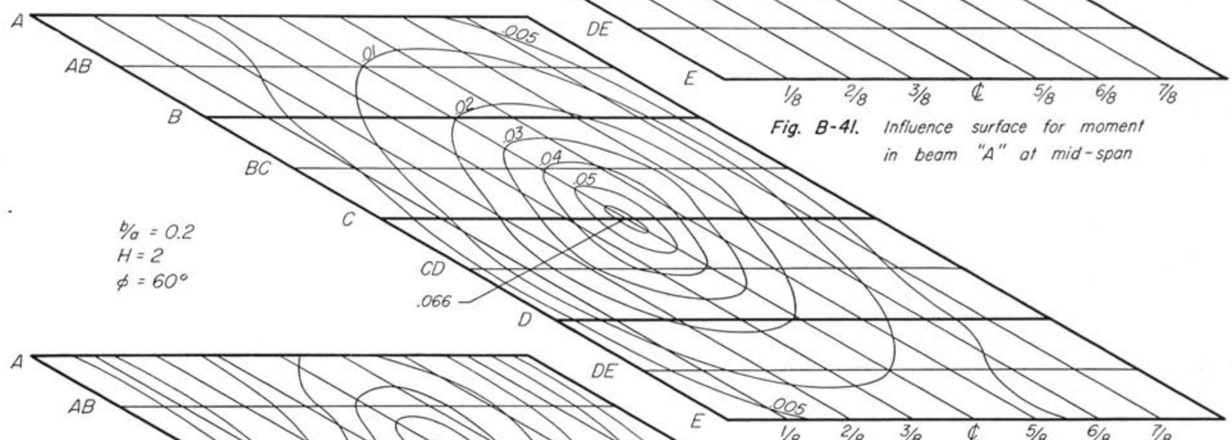


Fig. B-42. Influence surface for moment in beam "B" at mid-span

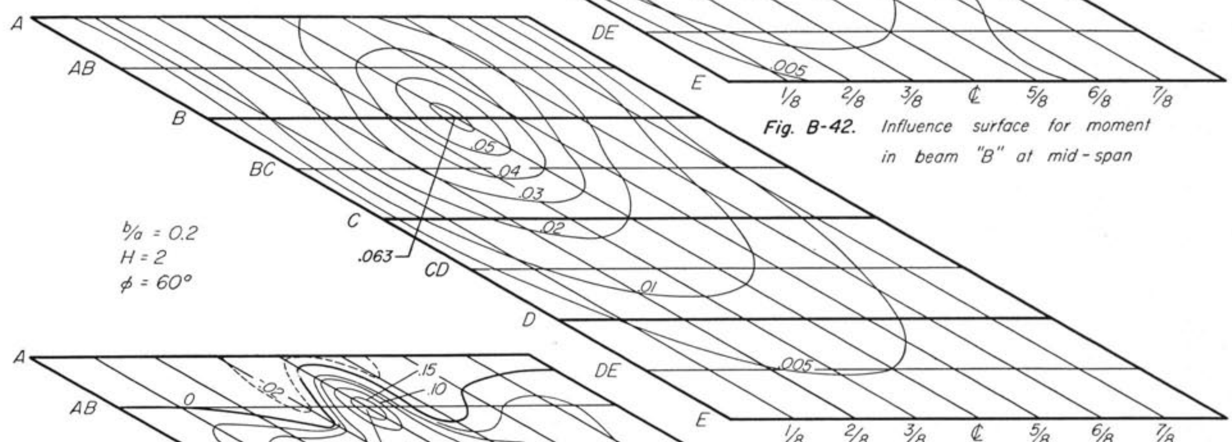


Fig. B-43. Influence surface for moment in beam "C" at mid-span

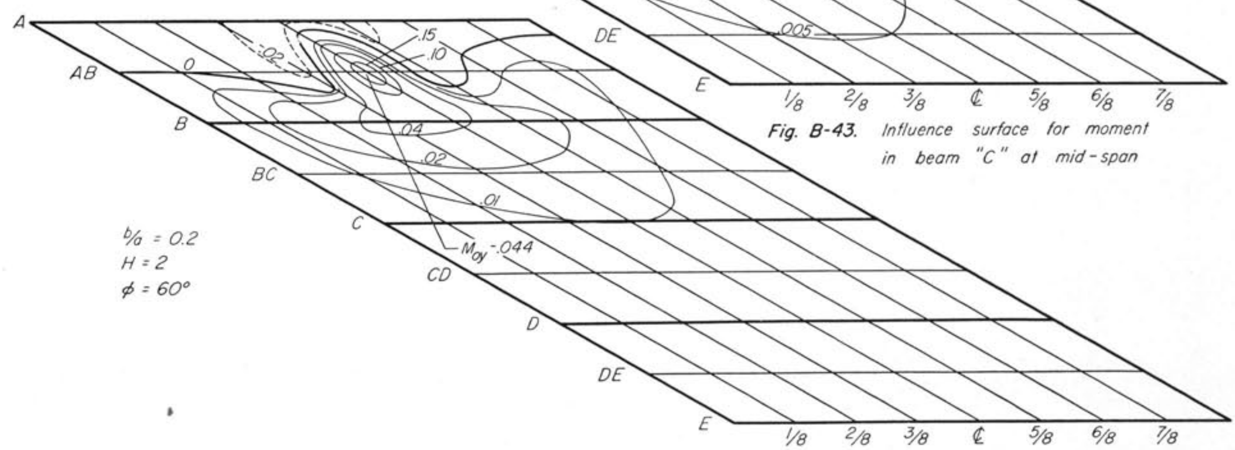


Fig. B-44. Influence surface for transverse moment in slab at AB at mid-span

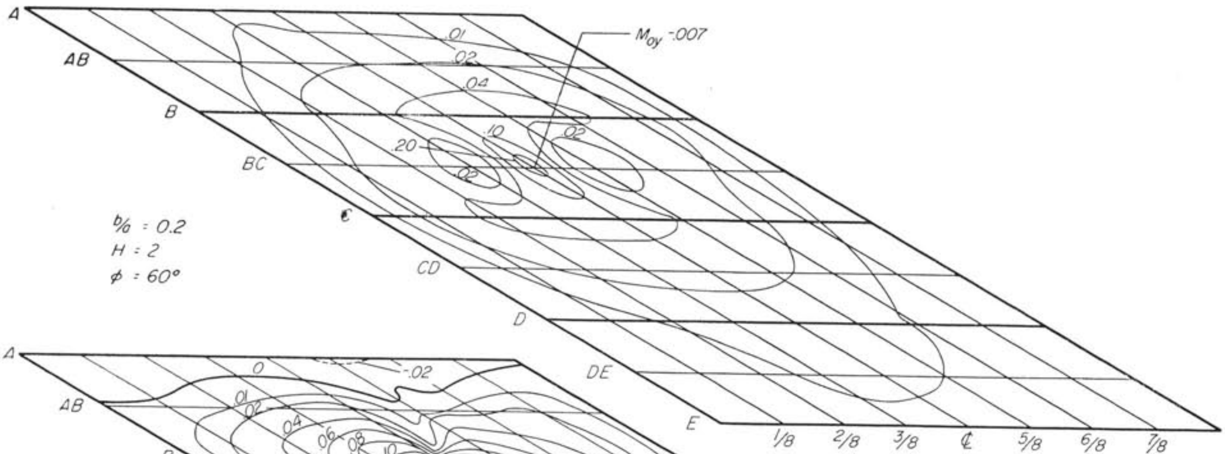


Fig. B-45. Influence surface for transverse moment in slab at BC at mid-span

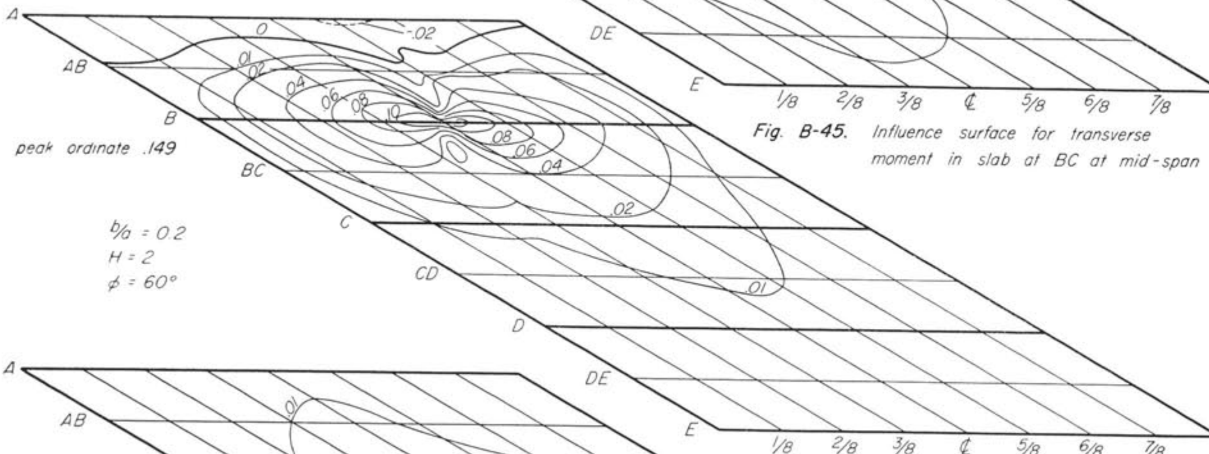


Fig. B-46. Influence surface for transverse moment in slab at B at mid-span

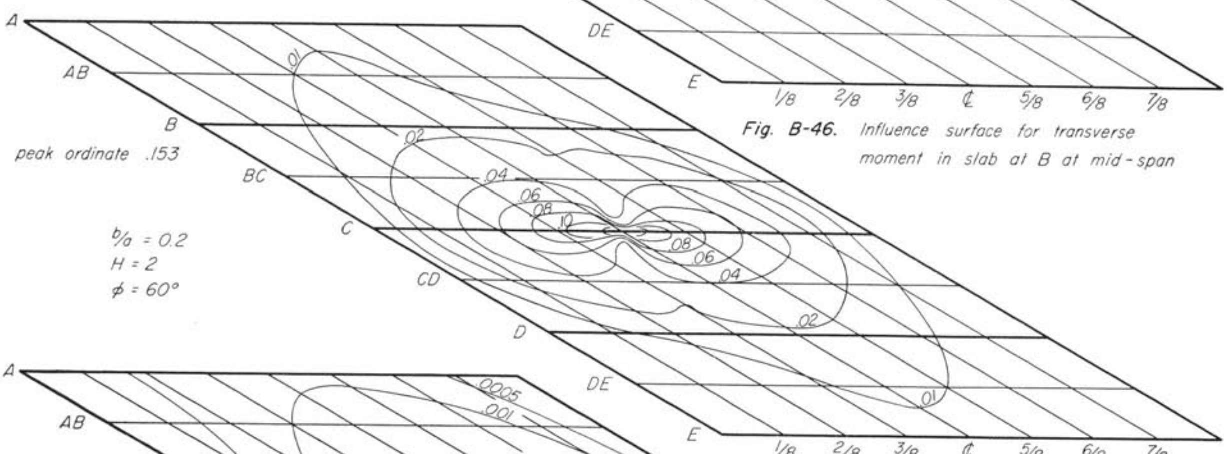


Fig. B-47. Influence surface for transverse moment in slab at C at mid-span

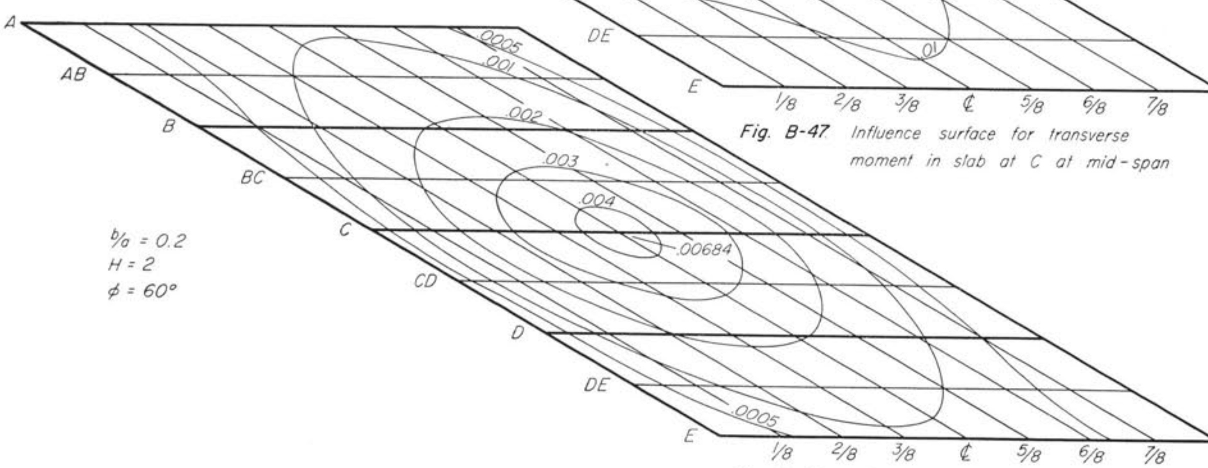


Fig. B-48. Influence surface for deflection of beam C at mid-span

The Engineering Experiment Station was established by act of the University of Illinois Board of Trustees on December 8, 1903. Its purpose is to conduct engineering investigations that are important to the industrial interests of the state.

The management of the Station is vested in an Executive Staff composed of the Director, the Associate Director, the heads of the departments in the College of Engineering, the professor in charge of Chemical Engineering, and the director of Engineering Information and Publications. This staff is responsible for establishing the general policies governing the work of the Station. All members of the College of Engineering teaching staff are encouraged to engage in the scientific research of the Station.

To make the results of its investigations available to the public, the Station publishes a series of bulletins. Occasionally it publishes circulars which may contain timely information compiled from various sources not readily accessible to the Station clientele or may contain important information obtained during the investigation of a particular research project but not having a direct bearing on it. A few reprints of articles appearing in the technical press and written by members of the staff are also published.

In ordering copies of these publications reference should be made to the Engineering Experiment Station Bulletin, Circular, or Reprint Series number which is at the upper left hand corner on the cover. Address

ENGINEERING PUBLICATIONS OFFICE
114 CIVIL ENGINEERING HALL
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS