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LAMINAR FLOW OF SLUDGES IN PIPES WITH SPECIAL REFERENCE TO SEWAGE SLUDGE

BY

HAROLD E. BABBITT
AND
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LAMINAR FLOW OF SLUDGES IN PIPES

I. INTRODUCTION

1. *The Problem.*—In the design of pipe lines through which sludge will flow, or must be pumped, insufficient information is available to make possible precise estimates of the head losses due to friction. In the pumping of sewage sludges the problem has usually been solved by assuming that the common hydraulic formulas applicable to the flow of water may be used in the flow of a sludge, provided the velocity of flow of the sludge is great enough to create turbulence. Measurements of head losses resulting from the flow of sludge in a pipe, as shown in Fig. 1, indicate that such an assumption is only an approximation of the true conditions.

Problems involving the laminar flow of fluids in all sizes of pipes can be solved by the use of Poiseuille's equation. Turbulent flow frictional losses are evaluated by means of the Reynolds-Stanton diagram, Fig. 6. At present the only methods available for determining laminar flow frictional losses for plastic and pseudo-plastic materials flowing in circular pipes have been the use of empirical formulas and information from reports of a few special instances of the flow of sewage sludges, clay slurries, etc., in the literature. Such materials as sewage sludge, aqueous suspensions of clay and sand in dredging, wood pulp suspensions in the paper making industry, and drilling mud in well drilling operations may be required to be pumped through pipes, and for design purposes the frictional losses must be evaluated.

2. *Purposes of Investigation.*—The purposes of this investigation were to formulate the various factors that influence the frictional losses when a sludge is pumped through a circular pipe, to formulate the variable factors that affect the critical velocity, to verify the formulas experimentally, and to present a simple method for measuring the characteristics of individual sludges. For a clear understanding of the characteristics of flow it is necessary to distinguish between viscosity and plasticity.

3. *Viscosity and Viscous Flow.*—Viscosity is the measure of the resistance to flow or deformation of a fluid. The rate of deformation is a linear function of the deforming force. The coefficient of viscosity of a fluid is equal to the tangential force on a unit area of either of two horizontal planes at a unit distance apart required to move one

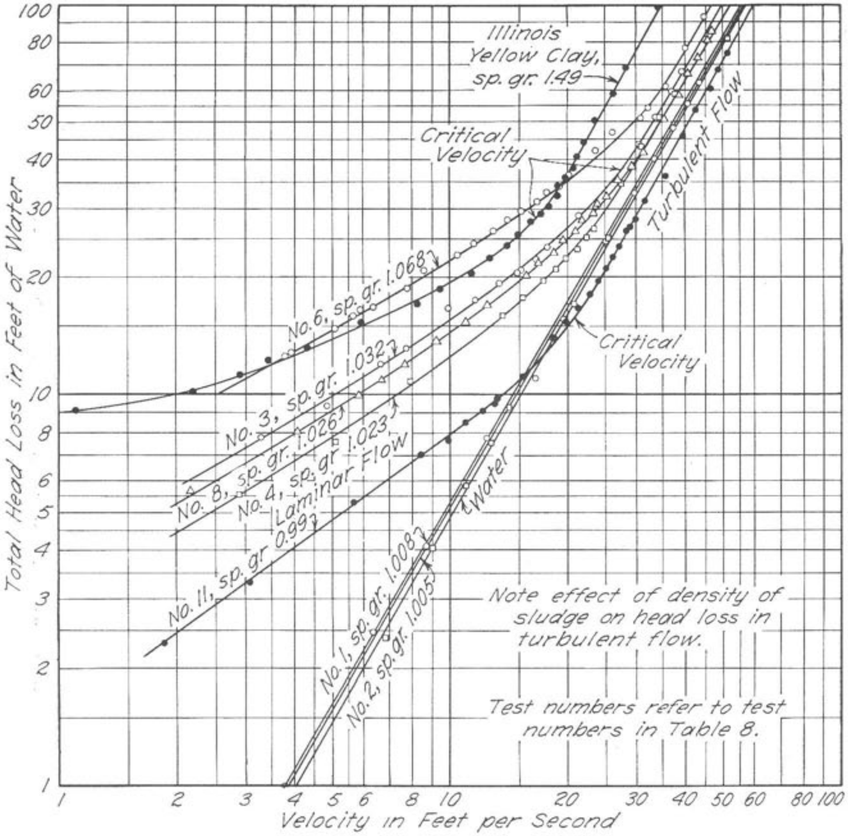

FIG. 1. FRICTION LOSSES OF VARIOUS SLUDGES FLOWING IN A ⅜-IN. PIPE

plane with a unit velocity with reference to the other plane, the space between being filled with the viscous substance. From which it follows that

$$\mu' = \frac{Sx}{v} \tag{1}$$

Where

μ' = coefficient of viscosity
S = tangential unit shearing force
x = distance between planes
v = velocity of one plane with respect to the other

When the C.G.S. system is used, the name given to the coefficient

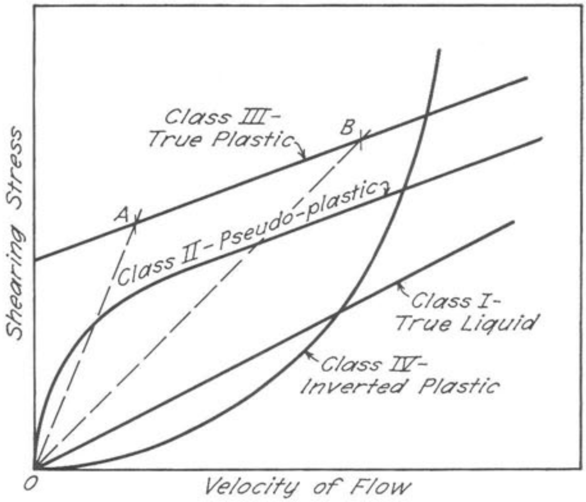

FIG. 2. FLOW CHARACTERISTICS OF DIFFERENT CLASSES OF MATERIALS

of viscosity is the poise. The centipoise is one one-hundredth of a poise. In the F.P.S. system no name is given to the coefficient.

4. *Plasticity and Plastic Flow.*—Plasticity is the property of a substance which enables it to be continuously and permanently deformed in any direction without rupture under a stress exceeding the yield value. After deformation has started, equal increments of stress will produce equal increments in velocity. Reverting to the fundamental conception of flow between two parallel planes, since a part of the applied force S is used up in overcoming the yield value S_y, the equation for plastic flow becomes

$$\eta' = \frac{(S - S_y)\,x}{v} \tag{2}$$

where η' is the coefficient of rigidity of the material, analogous to the coefficient of viscosity of a true fluid.

Figure 2 represents all the recognized types of flow. Curve I represents the flow of a *true liquid*, the slope of the line is proportional to the coefficient of viscosity. Curve II represents the flow of a *pseudoplastic* material. It can be seen that this curve does not obey the fundamental equation of plastic flow, because the line bends towards the origin at low rates of flow. Curve III represents a *true plastic,* and is a graphical representation of Equation (2). The apparent viscosity of the plastic at any point A on curve III, if measured in the usual way for liquids, is proportional to the slope of the line OA. It is evident, therefore, that the apparent viscosity is not constant for different velocities and stresses. It is seen that two different velocities such as A and B in the figure correspond to entirely different viscosity lines OA and OB, the slopes of which are proportional to the apparent viscosity. Curve IV represents the flow of an *inverted plastic* substance. This material is thin at low rates of flow but becomes increasingly thicker as the force increases. It has been found in this investigation that the flow of sludge follows the type of flow illustrated by curve III. It is concluded, therefore, that sewage sludge and clay slurries are true plastics.

5. *Procedure and Nomenclature.*—The experimental procedure has been to compare the theoretically developed formulas with measured friction losses in various sizes of pipes resulting from the flow of sludges from different sources.

The nomenclature used is as follows:

D = Diameter of pipe, feet

g = Acceleration due to gravity, feet per second per second

H = Difference in static head between two points in a pipe, feet of flowing substance

H_w = Difference in static head between two points in a pipe, feet of water

L = Length of pipe, feet

n = Speed of revolution of cylinder in modified Stormer viscometer, revolutions per second

Q = Rate of flow, cubic feet per second

r = Distance from any point within a pipe to the center of the pipe, feet

R = Radius of pipe, feet

Re = Reynolds number $DV\rho/\mu$, dimensionless

r_o = Radius within a pipe at which the shearing stress equals the yield value, feet (see Fig. 4)

S = Shearing stress, pounds per square foot

S_p = Shearing stress in a flowing material at the boundary or pipe wall, pounds per square foot

S_r = Shearing stress in a circular pipe at distance r from the center, pounds per square foot

S_y = Shearing stress at the yield point of a plastic material, called yield value, pounds per square foot

u_y = Intercept on the u axis of line connecting points representing driving force, u, and corresponding speed of revolution, n, in the Stormer viscometer (see Fig. 10)

V = Mean velocity of flow in a pipe, feet per second

v_o = Velocity of plug of radius r_o, feet per second

v_r = Velocity at any distance r, from the center of the pipe, feet per second

V_{lc} = Lower critical velocity ($Re = 2000$), feet per second

V_{uc} = Upper critical velocity ($Re = 3000$), feet per second

μ = Coefficient of viscosity, pounds per foot per second

μ' = Coefficient of viscosity, slugs per foot per second

η' = Coefficient of rigidity, slugs per foot per second

η = Coefficient of rigidity, pounds per foot per second

λ = Slope of line connecting points representing driving force, u, and corresponding speed of revolution, n, in the modified Stormer viscometer (see Fig. 10)

ρ = Density of flowing substance, pounds per cubic foot

6. *Acknowledgments.*—The tests herein described were made as part of the work of the Engineering Experiment Station of the University of Illinois, of which DEAN M. L. ENGER is the Director, and of the Department of Civil Engineering, of which PROFESSOR W. C. HUNTINGTON is the head. MR. LUDWIG STOYKE, a graduate student in the College of Engineering, devoted part of his time to the performance of the preliminary tests and to a search of the literature. Some of the routine work, the setting up of apparatus and similar work, was done by undergraduate students employed through the National Youth Administration. The authors are grateful to DEAN ENGER for his careful analysis of the manuscript and suggestions for improvement. A debt of gratitude is due to DR. W. D. HATFIELD for his advice, assistance, and unfailing interests in the progress of the investigation, and for the use of the facilities of the sewage treatment plant at Decatur, Illinois, in the measurement of sludge. The cooperation and interest of sewage treatment plant operators at Indianapolis and at the larger plants in central Illinois was encouraging and is appreciated.

II. PRINCIPLES OF FLOW

7. *Factors Affecting Friction.*—In an attempt to formulate the factors affecting the friction resulting from the steady uniform flow

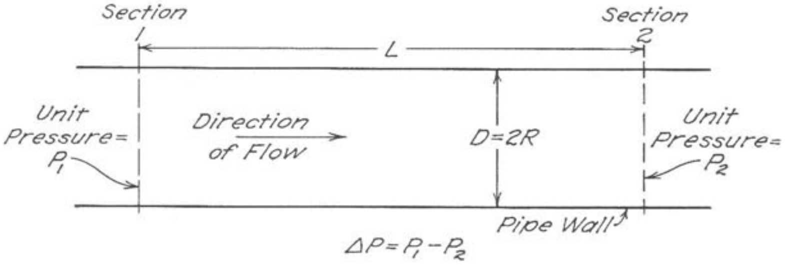

FIG. 3. DIAGRAM OF FLOW BETWEEN TWO SECTIONS OF PIPE

of sludge in a circular pipe certain assumptions will be made and the resulting formulation checked by tests. It will be assumed that the conditions affecting the friction resulting from the laminar flow of sludge in a circular pipe are the velocity of the sludge, the diameter of the pipe, the length of the pipe, and the characteristics of the sludge such as density, rigidity, and yield value. The pressure and temperature will be assumed to affect the friction only through their effect on the characteristics of the sludge. Another factor that might be assumed to affect the friction is the roughness of the pipe walls. However, it is known that, in the laminar flow of fluids, pipe wall roughness does not affect the friction loss. It has been assumed, therefore, that pipe wall roughness will not affect friction loss in the laminar flow of sludge. The friction loss will be assumed to result only from the rubbing of the sludge layers past each other and not from kinetic energy losses.

8. *Theoretical Formulation of Factors Affecting Flow.*—The following mathematical analysis was first presented by Bingham.[1]* The total force producing flow in a pipe between sections 1 and 2, Fig. 3, is $\pi R^2(\Delta P)$ where ΔP is the difference in unit pressure between sections 1 and 2. Since there is no acceleration in steady, uniform flow this force is opposed by an equal force $2\pi RLS_p$

Hence

$$S_p = \frac{R(\Delta P)}{2L} \tag{3}$$

and

$$S_r = \frac{r(\Delta P)}{2L} \tag{4}$$

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*Index numbers refer to items in bibliography.

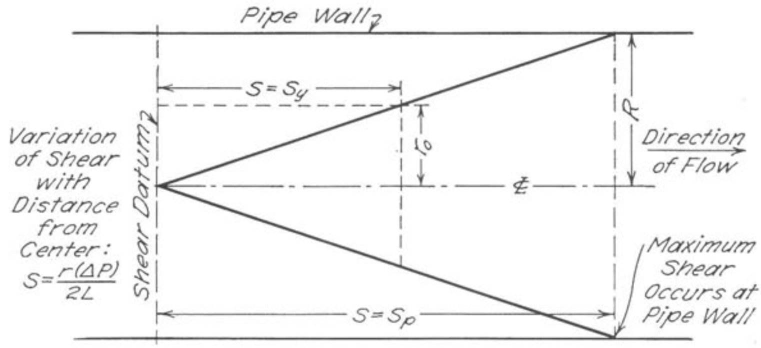


FIG. 4. DISTRIBUTION OF SHEARING FORCES IN A CIRCULAR PIPE

The sludge flowing in the center of the pipe moves as a solid plug, with a radius of r_o. This phenomenon results from the fact that the shear between the moving layers increases from zero at the center of the pipe to a maximum at the pipe wall, as shown by Equation (4), and in Fig. 4, and at some distance between the center of the pipe and the wall the shear will be equal to the yield value of the plastic sludge. Where the shear is less than the yield value there will be no relative motion between adjacent particles which will, therefore, flow together as a solid plug.

If Δp is the unit pressure used in overcoming the friction due to the yield value, the yield value S_y is

$$S_y = \frac{R(\Delta p)}{2L} \tag{5}$$

It also follows that

$$S_y = \frac{r_o \Delta P}{2L} \tag{6}$$

For a circular pipe Equation (2) becomes

$$dv = -\frac{1}{\eta'}(S_r - S_y)dr$$

or

$$dv = -\frac{1}{\eta'}\left[\frac{r(\Delta P)}{2L} - S_y\right]dr \tag{7}$$

The velocity, at any distance r from the center of the pipe, in the region between the plug of radius r_o and the pipe wall, is obtained by integrating Equation (7) from $r = R$ to $r = r$

$$v_r = -\frac{1}{\eta'}\int_R^r\left[\frac{(\Delta P)r}{2L} - S_y\right]dr = \frac{1}{\eta'}\left[\frac{(\Delta P)r^2}{4L} - S_y r\right]_r^R$$

$$v_r = \frac{1}{\eta'}\left[\frac{\Delta P}{4L}(R^2 - r^2) - S_y(R - r)\right] \tag{8}$$

The velocity of the solid plug is obtained by making $r = r_o$ in Equation (8)

$$v_o = \frac{1}{\eta'}\left[\frac{(\Delta P)R^2}{4L} + \frac{LS_y^2}{\Delta P} - S_yR\right] \tag{9}$$

Hence the flow Q is

$$Q = \pi r_o^2 v_o + 2\pi\int_{r_o}^R r v_r dr \tag{10}$$

but

$$\pi r_o^2 v_o = \frac{\pi 4S_y^2L^2}{(\Delta P)^2\eta'}\left[\frac{(\Delta P)R^2}{4L} + \frac{LS_y^2}{\Delta P} - S_yR\right]$$

and

$$2\pi\int_{r_o}^R r v_r dr = \frac{2\pi}{\eta'}\int_{r_o}^R\left[\frac{(\Delta P)}{4L}(R^2r - r^3) - S_y(Rr - r^2)\right]dr$$

$$= \frac{2\pi}{\eta'}\left[\frac{R^4(\Delta P)}{16L} - \frac{R^3S_y}{6} - \frac{\Delta P}{4L}\left(\frac{R^2r_o^2}{2} - \frac{r_o^4}{4}\right) + S_y\left(\frac{Rr_o^2}{2} - \frac{r_o^3}{3}\right)\right]$$

Substituting the value of r_o from Equation (6)

$$2\pi\int_{r_o}^R r v_r dr = \frac{2\pi}{\eta'}\left(\frac{R^4(\Delta P)}{16L} - \frac{R^3S_y}{6} - \frac{R^2LS_y^2}{2(\Delta P)} + \frac{2RL^2S_y^3}{(\Delta P)^2} - \frac{5}{3}\frac{L^3S_y^4}{(\Delta P)^3}\right)$$

Substituting these values in Equation (10)

$$Q = \frac{\pi}{\eta'}\left[\frac{R^4(\Delta P)}{8L} - \frac{R^3S_y}{3} + \frac{2}{3}\frac{L^3S_y^4}{(\Delta P)^3}\right]$$

$$= \frac{\pi R^4}{8L\eta'}\left[(\Delta P) - \frac{4}{3}\left(\frac{2LS_y}{R}\right) - \frac{1}{3(\Delta P)^3}\left(\frac{2LS_y}{R}\right)^4\right]$$

Substituting from Equation (5)

$$Q = \frac{\pi R^4}{8L\eta'}\left[(\Delta P) - \frac{4}{3}(\Delta p) + \frac{(\Delta p)^4}{3(\Delta P)^3}\right]$$

The mean velocity of flow is

$$V = \frac{Q}{\pi R^2} = \frac{R^2}{8L\eta'}\left[\Delta P - \frac{4}{3}\Delta p + \frac{(\Delta p)^4}{3(\Delta P)^3}\right]$$

or, changing ΔP and Δp to terms of shear by Equations (3) and (5),

$$V = \frac{R}{4\eta'}\left[S_p - \frac{4}{3}S_y + \frac{S_y^4}{3S_p^3}\right]$$

In the foregoing equation the units of η' are slugs per second foot. If it is desired to express the coefficient of rigidity in pounds per second foot, η, the left hand member of the equation must be divided by g. Using η instead of η', and taking g as 32 feet per second per second, the equation reduces to

$$V = \frac{4D}{\eta}\left[S_p - \frac{4}{3}S_y + \frac{1}{3}\frac{S_y^4}{S_p^3}\right] \tag{11}$$

Equation (11) may be written

$$V = \frac{4DS_p}{\eta}\left[1 - \frac{4}{3}\left(\frac{S_y}{S_p}\right) + \frac{1}{3}\left(\frac{S_y}{S_p}\right)^4\right]$$

from which it can be shown that the last term may be omitted with little error when $\frac{S_y}{S_p} < 0.5$. The error will be 5.9 per cent when $\frac{S_y}{S_p} = 0.5$ and 1.8 per cent when $\frac{S_y}{S_p} = 0.4$. Omitting the last term, Equation (11) reduces to

$$V = \frac{4D}{\eta}\left(S_p - \frac{4}{3}S_y\right) \tag{12}$$

Bingham[1] has shown that the coefficient of rigidity η and the yield value S_y, are independent of the characteristics of the measuring

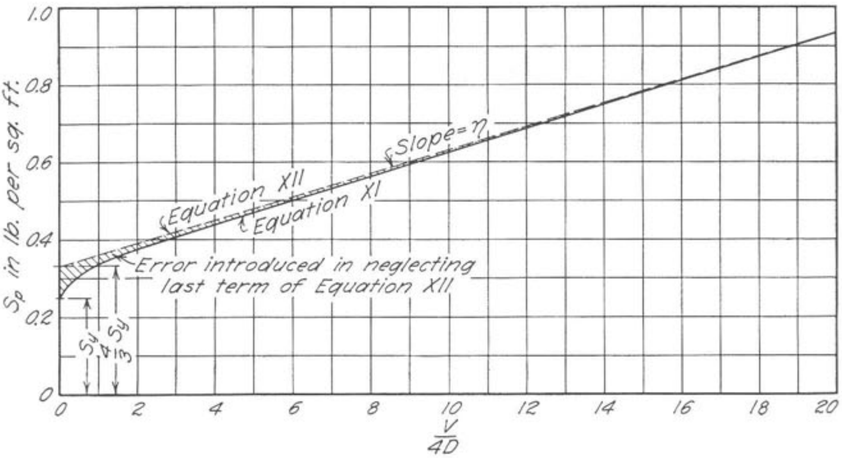


FIG. 5. GRAPHICAL REPRESENTATION OF EQUATIONS 11 AND 12

apparatus and depend only on the nature of the sludge. Both these facts have been corroborated in this investigation. On this basis, if a graph is plotted with S_p as ordinate and $V/4D$ as abscissa, the slope of the resulting line will represent the coefficient of rigidity, η; and, for a given sludge, the same line will represent the flow of that sludge in a pipe of any diameter. Graphs of this type, illustrating Equations (11) and (12), are shown in Fig. 5. The error in neglecting the last term of Equation (11) is thus shown graphically.

For industrial piping, and with sewage sludges, clay slurries, and drilling muds as the flowing material, Equation (12) will yield results within the limits of experimental error in determining the constants S_y and η of the sludge.

Since
$$H\rho\pi R^2 = 2S_p\pi RL$$

then
$$S_p = \frac{H\rho R}{2L}$$

and Equation (12) can be written in another form which may be more convenient in certain cases, as follows:

$$\frac{H}{L} = \frac{16S_y}{3\rho D} + \frac{\eta V}{\rho D^2} \tag{13}$$

TABLE 1
COMPARISON OF COMPUTED AND OBSERVED VALUES OF HEAD LOSS IN PIPES OF VARIOUS SIZES

Type of Sludge and Source	Test* Reference No.	Per Cent Moisture (by weight)	Diameter of Pipe in.	Velocity ft. per sec.	Observed Head Loss ft. per 100 ft.	Computed Head Loss ft. per 100 ft.	Per Cent Variation
Sewage—Imhoff Tank, Calumet Treatment Plant, Chicago, Ill. $S_y = 0.060$, $\eta = 0.021$‡	90	88	5	0.5	1.32	1.34	+1.5
				1.0	1.42	1.43	+0.7
				1.5	1.52	1.53	+0.7
				2.0	1.62	1.63	+0.6
				2.5	1.72	1.73	+0.6
				3.5	1.92	1.92	0.0
				4.5†	2.18	2.17	−0.5
Sewage—Imhoff Tank, Calumet Treatment Plant, Chicago, Ill. $S_y = 0.014$, $\eta = 0.035$‡	36	90	12	0.5	0.144	0.150	+4.2
				0.7	0.152	0.161	+5.9
				0.9	0.162	0.176	+8.6
				1.0	0.169	0.178	+5.3
				1.5	0.200	0.214	+7.0
				2.0	0.242	0.234	−3.3
				2.6†	0.300	0.268	−10.6
Sewage—Imhoff Tank, Calumet Treatment Plant, Chicago, Ill. $S_y = 0.017$, $\eta = 0.026$‡	30	90	8	0.5	0.260	0.265	+1.9
				0.7	0.275	0.287	+4.4
				1.0	0.300	0.315	+5.0
				1.5	0.372	0.361	−3.0
				2.0†	0.480	0.408	−15.0
Clay Slurry from Water Purification Plant $S_y = 0.011$, $\eta = 0.005$‡	50	86.4	4	0.5	0.30	0.32	+6.7
				0.7	0.31	0.34	+9.7
				0.9	0.33	0.35	+6.1
				1.5	0.37	0.40	+8.0
				1.8†	0.41	0.41	0.0
Sewage—Digested, Sewage Disposal Plant, Stuttgart, Germany. $S_y = 0.10$, $\eta = 0.077$‡	73	90	7.9	1.0	1.50	1.57	+4.7
				1.5	1.62	1.72	+6.2
				2.0	1.82	1.86	+2.2
				2.5	2.00	2.00	0.0
				3.0	2.20	2.14	−2.7
				4.0	2.47	2.33	−5.7
				5.0†	2.75	2.71	−1.5
Illinois Yellow Clay suspension—Tests made in this investigation. $S_y = 0.72$, $\eta = 0.028$‡		52	3	2.0	26.1	26.3	+0.8
				3.0	27.2	27.1	−0.4
				4.0	27.9	28.0	+0.4
				5.0	28.9	28.8	−0.3
				6.0	29.7	29.7	0.0
				7.0†	30.5	30.5	0.0

*Refers to numbers in Table 8.
†Observed critical velocity.
‡These values of S_y and η have been determined by plotting appropriate data similarly to Fig. 5, and reading intercepts and slopes as described on page 16.

or

$$\frac{H_w}{L} = \frac{16S_y}{3WD} + \frac{\eta V}{WD^2} \tag{14}$$

Equation (14) has been checked by experiments in this investigation, and by using tests reported in the literature.[2, 3, 4] Table 1 shows a few comparisons of the observed and the computed values of head loss, together with the percentage variation, using Equation (14). It is evident that the agreement between observed and

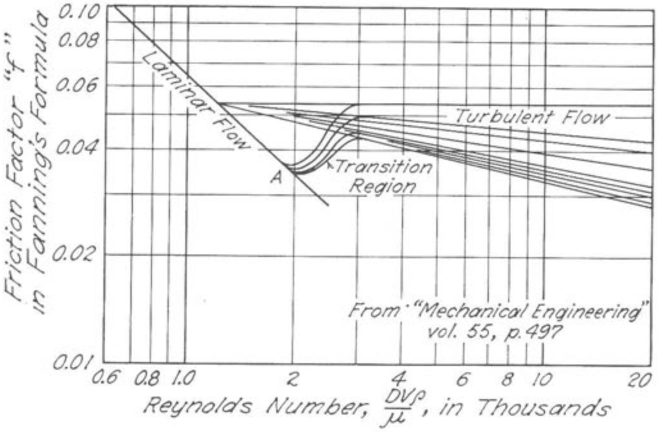


FIG. 6. THE REYNOLDS-STANTON DIAGRAM

computed values of friction head loss is sufficiently precise for practical purposes.

9. *The Critical Velocity.*—The critical velocity will be considered as that velocity below which the friction loss is directly proportional to the velocity and above which the friction loss is directly proportional to some power of the velocity between 1.7 and 2.0. The friction loss resulting from the flow of a liquid through a pipe may be determined by means of Reynolds-Stanton diagram,[5, 6, 7, 8] Fig. 6. Such a diagram is constructed by plotting as ordinates values of the friction factor f in Fanning's formula $H = f\frac{L}{D}\frac{V^2}{2g}$ and as abscissas the corresponding Reynolds number, $DV\rho/\mu$.

Reynolds[9] showed that the critical velocity occurred at a definite value of the Reynolds number. Recent work[10] has shown that in industrial piping the value of the Reynolds number at the critical velocity is approximately 2300.

For circular pipes the flow will be laminar when the Reynolds number is less than 2000.[11] Sometimes laminar flow persists to higher Reynolds numbers, but in industrial piping installations the flow will usually be turbulent above a Reynolds number of 3000.[12] Between these values of the Reynolds number there is uncertainty as to the type of flow which may occur. In order to distinguish between these two values the velocity corresponding to a Reynolds number of 2000 will be designated as V_{lc}, the lower critical velocity, and the velocity corresponding to a Reynolds number of 3000 will be designated as V_{uc}, the upper critical velocity.

In order to determine Reynolds number it is necessary to know the viscosity of the flowing material. Since sludge possesses no definite viscosity, but a varying apparent viscosity, as described on page 10, Reynolds criterion for critical velocity cannot be directly evaluated for sludge. However, if reference is made to Fig. 5, it can be seen from Poiseuille's expression of viscosity

$$\mu = \frac{4DS_p}{V} \tag{15}$$

that the apparent viscosity for any particular value of $V/4D$ is the ratio of the ordinate to the abscissa or $4DS_p/V$.

Poiseuille's equation can be used in this case in determining the apparent viscosity of a sludge at any particular rate of flow by merely substituting the known values of D, S_p, and V. The equation is not good for solving for the friction loss at any other velocity when a sludge is the flowing material, because the apparent viscosity will have changed. The term S_p in Equation (15) can be expressed in terms of $V/4D$ and S_y so that the only variable is V.

Referring to Fig. 5,

$$S_p = \frac{4}{3}S_y + \eta\frac{V}{4D}$$

and from Equation (15),

$$\mu = \frac{16DS_y}{3V} + \eta \tag{16}$$

This is an expression for the viscosity at any value of $V/4D$ in Fig. 5. Substituting this value for μ into Reynolds' criterion for critical velocity, and solving for V it is found that for the lower critical velocity

$$V_{lc} = \frac{1000\eta + 103\sqrt{94\eta^2 + D^2S_y\rho}}{D\rho} \tag{17}$$

and for the upper critical velocity

$$V_{uc} = \frac{1500\eta + 127\sqrt{140\eta^2 + D^2S_y\rho}}{D\rho} \tag{18}$$

To check the validity of Equations (17) and (18), tests were made in various sizes of pipes and with various types of sludges to observe the critical velocities together with the values of S_y and η.

The results of these tests are shown in Table 2, together with observed and computed values of critical velocity taken from various tests reported in the literature.[2, 3] These results show a high degree of correlation between observed and computed values.

III. FACTORS AFFECTING YIELD VALUE AND COEFFICIENT OF RIGIDITY

10. *Yield Value, S_y, and Coefficient of Rigidity, η.*—The yield value and the coefficient of rigidity of a sludge are independent of the dimensions of the pipe through which flow is taking place and are independent of the velocity of flow. Hence, from Equations (13) and (14) it is evident that it is necessary to determine only the values of S_y and η for a given sludge in order to predict friction losses in the laminar flow of the sludge through a pipe. Among the important factors affecting the yield value and the coefficient of rigidity may be included (a) concentration of suspended matter, (b) size and character of particles of suspended matter, (c) nature of the continuous phase, (d) temperature, (e) thixotropy, (f) slippage and seepage, (g) agitation, and (h) gas content.

11. *Concentration of Suspended Matter.*—In this investigation the concentration of suspended matter is taken as the ratio of the weight of dry solids to the weight of the mixture of dry solids and liquid. In the mixture the fine particles suspended in the liquid are termed the dispersed phase and the liquid in which the particles are suspended is termed the continuous phase. The tests made in this investigation show, as in Fig. 11, that the concentration of suspended matter greatly affects the yield value, and affects the coefficient of rigidity to a lesser degree. Bingham[1] (p. 220) has shown that when the concentration of suspended matter is low the material may exhibit no measurable yield value, but as the concentration of suspended matter is increased a measurable yield value will appear and will increase almost in direct proportion to the increase in concentration of suspended matter. The absence of a yield point at low concentrations of suspended matter may be explained on the supposition that the suspended particles are not in contact. When the concentration becomes sufficiently great to force the particles into contact with each other a measurable force

TABLE 2
COMPARISON OF COMPUTED AND OBSERVED VALUES OF CRITICAL VELOCITY

Test* Reference No.	Source and Description of Sludge	Diameter of Pipe in.	S_y	η	Critical Velocity			Source of Information
					Computed		Observed	
					V_{lc}	V_{uc}		
26	Calumet, Ill., Sewage, Imhoff	8	0.057	0.044	4.2	5.7	4.0	Reference† No. 2
27	Calumet, Ill., Sewage, Imhoff	8	0.043	0.040	3.8	5.1	3.7	
28	Calumet, Ill., Sewage, Imhoff	8	0.033	0.036	3.3	4.5	3.5	
29	Calumet, Ill., Sewage, Imhoff	8	0.024	0.034	2.9	3.8	3.0	
30	Calumet, Ill., Sewage, Imhoff	8	0.017	0.026	2.4	3.2	2.7	
31	Calumet, Ill., Sewage, Imhoff	8	0.013	0.018	1.9	2.6	2.2	
32	Calumet, Ill., Sewage, Imhoff	8	0.006	0.016	1.5	2.0	1.8	
33	Calumet, Ill., Sewage, Imhoff	8	0.003	0.016	1.1	1.6	1.3	
34	Calumet, Ill., Sewage, Imhoff	8	0.0015	0.014	0.9	1.3	1.0	
35	Calumet, Ill., Sewage, Imhoff	12	0.020	0.0514	2.8	3.8	3.0	
36	Calumet, Ill., Sewage, Imhoff	12	0.014	0.035	2.1	3.0	2.6	
37	Calumet, Ill., Sewage, Imhoff	12	0.009	0.032	1.8	2.5	2.3	
38	Calumet, Ill., Sewage, Imhoff	12	0.005	0.036	1.6	2.2	2.1	
39	Calumet, Ill., Sewage, Imhoff	12	0.0026	0.025	1.1	1.6	1.7	
40	Calumet, Ill., Sewage, Imhoff	12	0.0015	0.024	1.0	1.4	1.4	
45	Clay Slurry	4	0.020	0.005	2.0	2.7	2.2	Reference† No. 3
46	Clay Slurry	4	0.056	0.005	3.3	4.2	3.6	
47	Clay Slurry	4	0.117	0.005	4.6	5.8	6.0	
48	Clay Slurry	4	0.158	0.005	5.3	6.8	7.0	
49	Clay Slurry	4	0.214	0.005	6.1	7.6	8.5	
50	Clay Slurry	4	0.012	0.005	1.5	2.1	1.8	
20	Decatur, Ill., Sewage, Imhoff	3	0.115	0.0163	5.4	7.3	6.5	This investigation
	Decatur, Ill., Sewage, Imhoff	2	0.091	0.0165	5.6	7.7	7.0	
	Decatur, Ill., Sewage, Imhoff	1	0.075	0.0165	7.6	11.1	7.0	
	Decatur, Ill., Sewage, Imhoff	1	0.065	0.0165	7.4	10.8	7.0	

*Refers to numbers in Table 8.

†In Bibliography.

TABLE 3
EFFECT OF PERCENTAGE OF SOLIDS ON FLOW OF SLUDGE

Character of Sludge	Per Cent Solids (by weight)	S_y	η	H_w* $V = 2$ ft. per sec.	H_w* $V = 4$ ft. per sec.
Digested Sewage Sludge	8	0.236	0.032	4.44	4.85
	14	0.339	0.040	6.31	6.82
	8	0.184	0.030	3.52	3.91
	7	0.136	0.029	2.72	3.11
Tennessee Ball Clay	15.1	0.059	0.011	1.15	1.29
	20.3	0.201	0.014	3.62	3.80
	22.8	0.340	0.015	6.00	6.19
	23.2	0.431	0.017	7.58	7.80
	24.8	0.532	0.019	9.34	9.59
Illinois Yellow Clay	34.0	0.030	0.011	0.65	0.79
	45.0	0.133	0.017	2.49	2.71
	48.5	0.208	0.024	3.86	4.17
	49.0	0.402	0.023	7.15	7.45
	52.0	0.709	0.028	12.46	12.82

*Head loss in feet of water per 100 feet of 6-in. pipe, computed by Equation (14).

is required to cause them to slide over one another, thus exhibiting a yield value. When the concentration of suspended matter is less than the concentration at which a yield value exists the material will, in most cases, exhibit the properties of the continuous phase, the coefficient of rigidity becoming equal to the coefficient of viscosity of the continuous phase. As the concentration of suspended matter is increased the yield value appears, and the coefficient of rigidity is increased over the corresponding viscosity coefficient of the continuous phase. The net effect of an increase in solids concentration is to increase the resistance to flow of the material. Results of tests that show this effect are recorded in Table 3, and are shown in Fig. 7.

12. *Size and Character of Suspended Particles.*—That the size of particles in suspension will affect the flow characteristics of a sludge has been found by other investigators. Bingham[1] conveys the impression that resistance to flow increases as the size of the particle is decreased when he states: "There is abundant evidence that as the diameter of the particles is decreased, the opportunity for the particles touching is increased, which enhances the friction (yield value), but this effect reaches a limit eventually when the particles are so small that their Brownian movement becomes appreciable and strains in the material are not permanent."

Tests made in this investigation corroborate this statement by Bingham. In a mixture of coarse yellow clay with water the yield value found for a concentration of 50 per cent by weight was the same

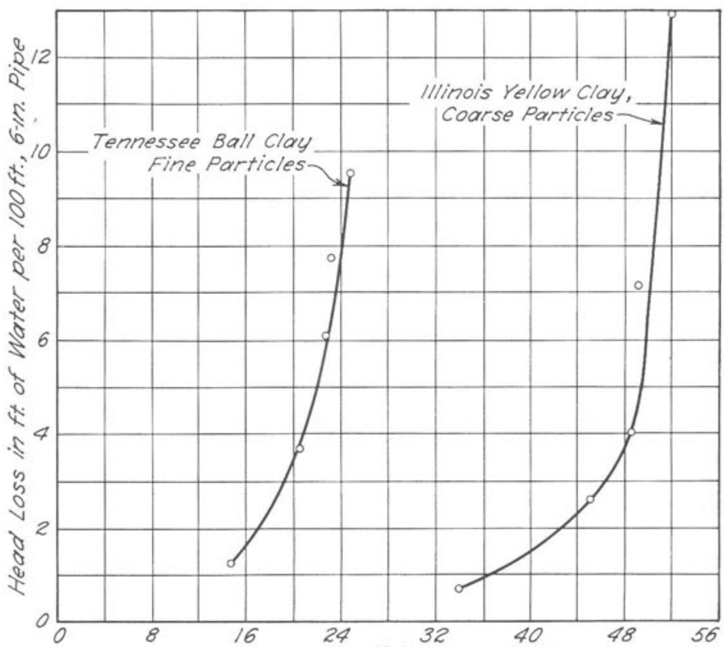


FIG. 7. EFFECT OF PARTICLE SIZE AND CONCENTRATION ON THE FLOW CHARACTERISTICS OF SLUDGES

as the yield value found for a fine Tennessee ball clay at a concentration of 25 per cent. Figure 7 shows the effect of particle size and solids concentration on the frictional resistance to flow of sludge in a six-inch pipe. A sewage sludge of 15 to 20 per cent solids is almost thick enough to handle with a pitch fork and thus would be almost impossible to pump. This characteristic may be due to the gelatinous, water-binding properties of the solids present.

In general, the sizes of particles in the sludges studied in this investigation were small, or even colloidal, although they sometimes gathered together in flocculent masses, so that they either could not be measured by ordinary means, or their measurement would give a dimension of no practical value. Although agitation, temperature, chemical reactions, and other factors may affect the size of the particles in a sludge so as to cause changes in the constants S_y and η there is insufficient information available at the present time to predict the effect of such changes.

TABLE 4
EFFECT OF CONTINUOUS PHASE ON FLOW OF SLUDGE

Characteristics of Sludge	Per Cent Solids (by weight)	Value of S_y	Value of η	H_w* $V = 3$ ft per sec.	H_w* $V = 5$ ft. per sec.
Tennessee Ball Clay and Water	15	0.087	0.0104	1.69	1.83
	18	0.129	0.0124	2.44	2.60
	20	0.210	0.0140	3.86	4.04
Tennessee Ball Clay and Glycerine Diluted with Water	15	0.087	0.0300	2.07	2.45
	18	0.129	0.0322	2.82	3.23
	20	0.210	0.0340	4.24	4.68

*Head loss in feet of water per 100 feet of 6-in. pipe, computed by Equation (14).

13. *Nature of the Continuous Phase.*—Most of the sludges considered in this investigation were mixtures of solids and water, the particles of solids being suspended in the water. Sludges may, however be composed of a mixture of solids suspended in some fluid other than water. Since the viscosity of all fluids cannot be expected to be the same it leads to the conclusion that the coefficient of rigidity of a sludge, and hence its resistance to flow, depends on the viscosity of the continuous phase. In order to demonstrate that the continuous phase is of importance in affecting the coefficient of rigidity, measurements have been made in this investigation of the coefficient of rigidity and the yield value of sludges composed of water and of sludges composed of glycerine as the continuous phase. Some results are reported in Table 4. It will be noted from the table that the continuous phase has no effect on the yield value provided the percentage of suspended matter remains constant. Because of the fact that the specific gravities of the continuous phases in the two sludges shown in Table 4 are almost the same, the percentages by volume of solids in the two sludges were closely alike. The equality of the yield value of these two sludges bears out Bingham's hypothesis to the effect that as long as the percentage by volume of the suspended matter in different mixtures is the same, the yield value is the same, and is independent of the character of the continuous phase, as long as the continuous phase is inert with respect to the suspended particles. This is an interesting fact and should prove useful in evaluating the effect of particle size on yield value since the effect of a change in the continuous phase does not have to be considered. In this investigation no attempt has been made to correlate particle size and yield value.

14. *Temperature.*—Temperature has a marked effect on the viscosity of fluids. In the case of liquids a rise in temperature lowers the

viscosity while in the case of gases the reverse is true. The relations between temperature and viscosity have been determined for water and many other liquids, and since the coefficient of rigidity of sludge depends to a great extent on the viscosity of the continuous phase, an estimate of the effect of temperature on the coefficient of rigidity may be made. No attempt was made to formulate the effect of temperature on the yield value other than to show that an increase in temperature lowered the yield value in the case of sewage sludge. Hatfield[13] also found a decrease in resistance to flow of sewage sludge with increase in temperature.

The effect on a sewage sludge of a rise in temperature is to reduce the yield value and the coefficient of rigidity, as is shown in Table 5. According to Bingham the temperature and viscosity of the continuous phase is without effect on the yield value. This apparent disagreement may be explained by the fact that Bingham's hypothesis refers only to a mixture in which the character of the dispersed phase is not affected by the temperature, whereas, in sewage sludge, the character of the dispersed phase may be affected by temperature.

15. *Thixotropy.*—Thixotropy is the property, or phenomenon, exhibited by some gels of becoming fluid when shaken. The change is also reversible. Such properties have been found in sewage sludges by other investigators and also in this investigation. In measuring the coefficient of rigidity and the yield value of a sludge it is essential that their changes due to agitation be controlled, otherwise a true measurement cannot be made. Hatfield[13] states: "This thixotropic property, first mentioned by Merkel[14] is very important and must be considered both in the determination of the viscosity of the sludges, and in the application of viscosity data to engineering problems, because (1) it is impossible to obtain a sample of quiescent sludge and get it into the viscometer beaker without some agitation or stirring, and (2) during each 100 revolutions of the viscometer cylinder the velocity of revolution has accelerated with each revolution. This acceleration is particularly noticeable at very low velocities on thick sludges, in which case only 10 to 20 revolutions are timed." It was realized that measurements of head loss made by recirculating sewage sludge through any form of apparatus led to erroneous observations due to thixotropy. It is believed the errors due to thixotropic properties are avoided where desired in this investigation through the use of the long tube viscometer, through the absolute measurements made in the apparatus described on pages 47-49, or through direct measurements of flow made with the apparatus at Decatur, described on pages 45-47.

TABLE 5
EFFECT OF TEMPERATURE ON FLOW OF SEWAGE SLUDGE

Type of Sludge and Source	Per Cent Solids (by weight)	Temperature deg. F.	Specific Gravity	S_y	η	H_w* $V = 3$ ft. per sec.
Danville—Digested	7.0	40	1.026	0.121	0.026	2.56
Danville—Digested	7.0	46	1.030	0.136	0.032	2.94
Danville—Digested	7.0	71	1.023	0.105	0.019	2.15
Danville—Digested	7.0	122	1.008	0.042	0.016	1.03
Indianapolis—Digested	7.8	41	1.032	0.236	0.032	4.64
Indianapolis—Digested	7.8	55	1.023	0.142	0.029	2.98

*Head loss in feet of water per 100 feet of 6-in. pipe, computed by Equation (14).

TABLE 6
EFFECT OF AGITATION ON FLOW OF SLUDGE

Character of Sludge	Per Cent Solids (by weight)	Specific Gravity	Before Agitation				After Agitation†			
			H_w* $V = 1$ ft. per sec.	H_w* $V = 3$ ft. per sec.	S_y	η	H_w* $V = 1$ ft. per sec.	H_w* $V = 3$ ft. per sec.	S_y	η
Yellow Clay	52	1.49	9.41	9.79	0.54	0.030	12.33	12.69	0.71	0.028
Decatur, Ill., Sewage sludge	86	1.062	2.16	2.38	0.12	0.017	1.26	1.48	0.07	0.017

*Head loss in feet of water per 100 feet of 6-in. pipe, computed by Equation (14).
†Circulated through a rotary pump for 3 hours.

It is possible that the acceleration of the cylinder in the Stormer viscometer, observed by Hatfield and the present authors, is due partly to a combination of "slippage" and "seepage," as a result of which a layer of water has formed between the revolving cylinder and adjacent sludge, causing slippage. It was found that by smartly pulling the string attached to the drum, thereby causing the cylinder to revolve at a high rate, much of the original resistance could be restored. It is probable, therefore, that some of the thixotropy of sewage sludge is only apparent, and is actually due to seepage and slippage. A similar condition was observed in the swinging pendulum viscometer described on pages 34 and 35. It was found that by stopping the motion of the viscometer and tapping it or gently stirring the sludge in it, much of the original resistance to the swinging of the pendulum was restored.

16. *Agitation.*—Agitation may change the resistance to flow of a sludge in a given pipe line by changing both the yield value and the coefficient of rigidity. Agitation may change the size of particles in the mixture, rearrange or redistribute the particles, or produce manifestations of thixotropy. The pumping of sludge through reciprocating, centrifugal, or rotary pumps or by other mechanical means, is a common cause of agitation, which affects its flow characteristics. It is also possible that merely flowing through a long pipe line may so change the values of η and S_y of a sludge that the resistance to flow near the end of the line is less than at the beginning. It is essential to make note of the effects of this phenomenon in measurements of the factors affecting the flow of sludge, and, where the effect of agitation is of importance, to report it in connection with the measured factors.

Results of tests of the flow of sludges which show the effect of agitation are reported in Table 6. It is to be noted that the resistance to the flow of yellow clay was increased by agitation, probably by the breaking up of the particles, and the resistance to the flow of sewage sludge was decreased by agitation, probably as a result of thixotropy.

17. *Slippage and Seepage.*—One of the assumptions made in the development of the laminar flow equation for sludges was that no slippage occurred at the pipe walls. That slippage is possible, especially when the continuous phase seeps out toward the pipe wall, thereby causing the solids concentration to be less in this region, is apparent, and has been the subject of investigation by others.[1, 14] Slippage is a rare phenomenon in the laminar flow of sludges in pipes, but may occasionally be noticed where the wall with which the sludge is in contact is glass smooth and the yield value of the sludge is high.

An interesting use of slippage is made in a few long pipe lines designed to carry crude petroleum which exhibits a yield point. The pipe line is designed with spiral ridges on the inside of the pipe. About 10 per cent of water is mixed with the crude oil, and the mixture pumped into the pipe. The ridges cause the mixture to revolve like a bullet fired from a rifle. The water, being heavier than the oil, is thrown to the outside of the pipe, where it lubricates the flow of the more viscous oil in the center of the pipe. The phenomenon of slippage is commonly easy to recognize as its occurrence will cause a drop in the friction loss as the velocity of flow is increased. In measuring the resistance offered by a sludge to flow through a pipe, or in a viscometer, errors due to slippage may be overcome by avoiding the use of too smooth a surface in contact with the sludge.

Seepage is the flow of the continuous phase through the dispersed phase, and occurs only when the shearing forces tending to produce flow are less than the yield value. The medium in effect filters through the mixture. Seepage is relatively unimportant in pipes larger than capillary tubes.

18. *Gas Content.*—Bubbles of gas so finely divided as to be unable to rise and escape may occur in a sludge as a result of bacterial fermentation or as a result of mechanical stirring. Because of this the density of the mixture is lowered. Sewage sludges have been observed in this investigation with 15 per cent solids whose density is materially less than that of water. Density theoretically has no effect on the laminar flow of sludges in pipes, but it has been observed that, when the velocity of flow is large enough to cause turbulence, the head loss due to friction is proportional to the density, as is shown in Fig. 1, where the lines representing the flow of sewage sludges and one test on Illinois yellow clay are displaced from the line representing the flow of water by amounts approximately proportional to their densities. Investigations into turbulent flow relationships of sludges should take account of the density factor.

IV. Measurement of Yield Value and Coefficient of Rigidity

19. *Measurements in Pipe Lines.*—Values of S_y and η may be measured in a variety of ways. For sewage sludges and for sludges commonly encountered in industry the simplest means is probably the best. Any existing pipe line through which the sludge may be pumped

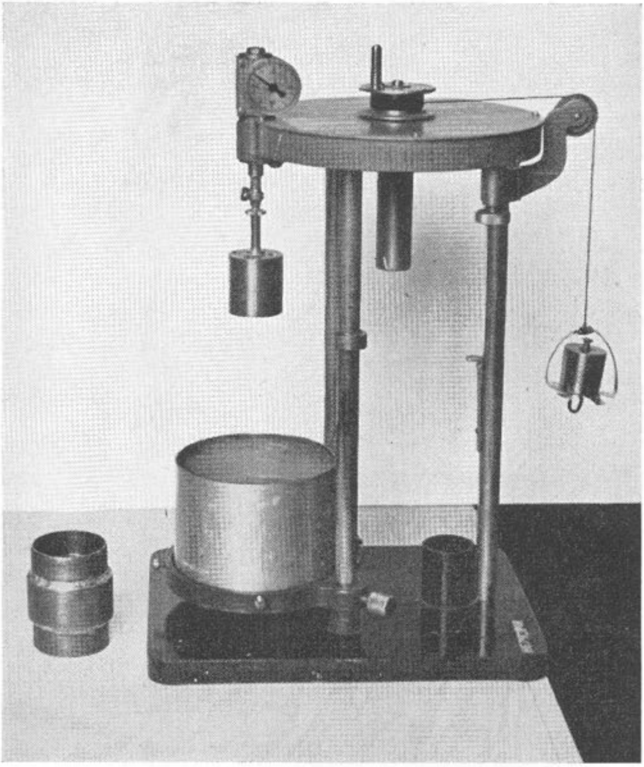
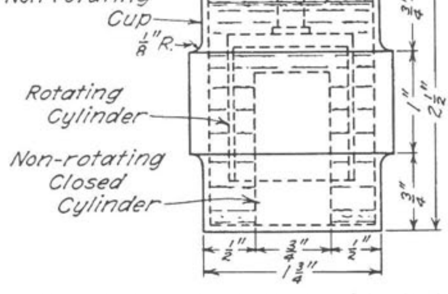


FIG. 8. THE STORMER VISCOMETER



Note: Surfaces in contact with sludge must not be highly polished.

FIG. 9. MODIFIED STORMER VISCOMETER CUP

can be used in the measurement of the values of S_y and η. The observations to be made under such conditions are the friction losses between two points on the pipe line, at two or more velocities below critical. These observations are plotted on a diagram with S_p as ordinate and $V/4D$ as abscissa. The slope of the line formed by connecting the plotted points is the coefficient of rigidity, and the intercept of the line on the S_p axis is 4/3 the yield value, as is seen from Equation (13). The values of S_p are obtained from the head losses, expressed in feet of water, by means of the following equation:

$$S_p = \frac{15.5 H_w D}{L} \tag{19}$$

20. *Measurements with a Modified Stormer Viscometer.*—The Stormer viscometer, illustrated in Fig. 8, can be adapted to the measurement of the yield value and the coefficient of rigidity of a sludge by slight modifications. Sewage sludges in particular have numerous large particles that render the standard Stormer cup useless by binding the rotating cylinder. To overcome this difficulty, and yet to maintain the clearances at a minimum, the cup illustrated in Fig. 9 was designed. Although turbulence occurs sooner with the modified cup than with the standard cup, interference of the particles with the rotating cylinder is believed to have been eliminated.

The procedure when using the modified Stormer viscometer for the measurement of S_y and η is as follows: The material to be measured is poured into the cup to exactly ¼ inch from the top, and the rotating cylinder is inserted. A known weight to act as a driving force is attached to the string wound on the drum; the brake is released and the time for the cylinder to make 100 revolutions is noted, and recorded as revolutions per second, together with the corresponding driving force in grams. After several readings have been taken at various values of driving force a new cupful of material is taken in order to eliminate, as much as possible, errors due to thixotropy.

Plots of observations made with a modified Stormer viscometer for four clay sludges are shown in Fig. 10. The driving force in grams is plotted as ordinate using the symbol u. The speed of revolution is plotted as abscissa using n as the symbol. Figure 10 is a plot of the flow characteristics of an Illinois yellow clay of four different concentrations of suspended matter. In Fig. 11 the pipe flow characteristics of the same four sludges are plotted with S_p as ordinate and $V/4D$ as abscissa. The similarity of the graphs is apparent, and is the basis for

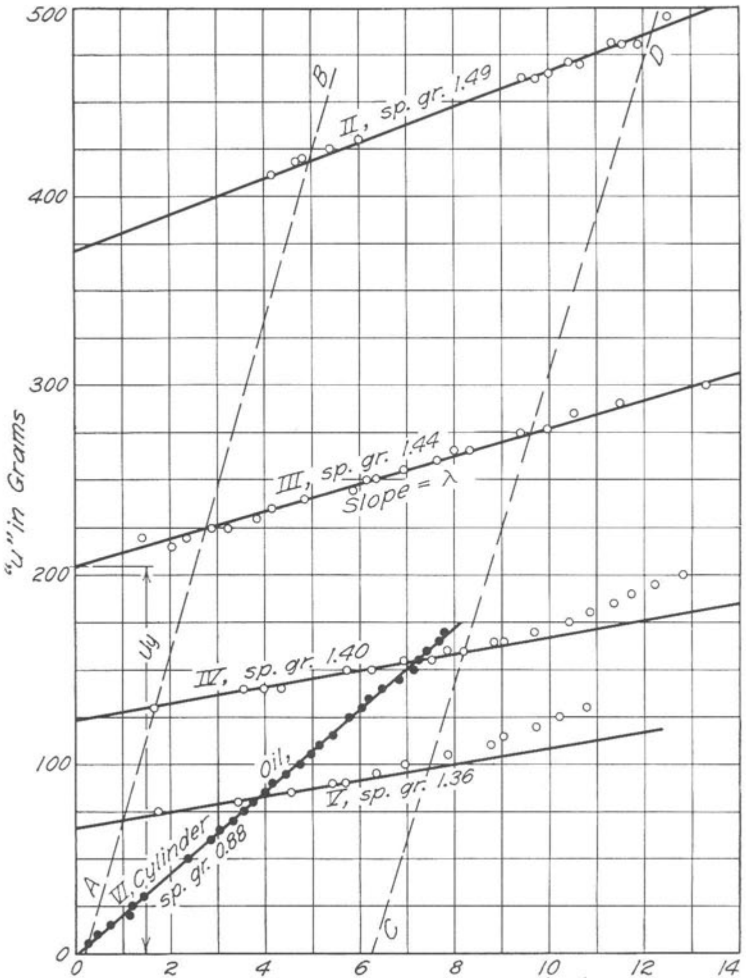


FIG. 10. PLOT OF u AGAINST n FOR CLAY SLURRIES USED FOR CALIBRATION OF THE STORMER VISCOMETER

converting the Stormer data to the pipe flow constants S_y and η. It has been found that in Fig. 10, the intercept u_y on the u axis of a line connecting the points representing driving force, u, and speed of revolution, n, is proportional to the yield value S_y, and that the slope, λ, of the same line, is proportional to the coefficient of rigidity, η. Table 7 presents a comparison of values of u_y and S_y and λ and η, and demon-

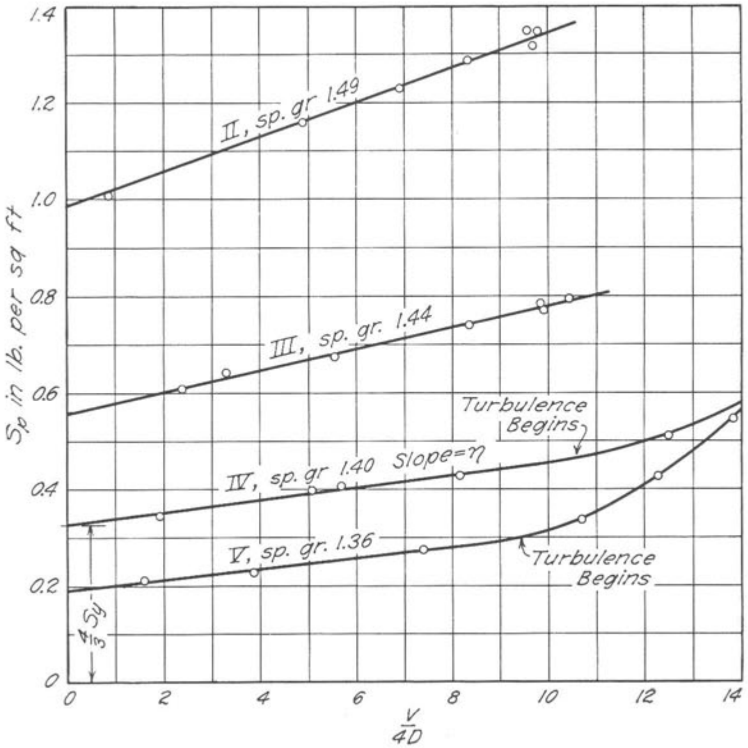


FIG. 11. PLOT OF S_p AGAINST $V/4D$ FOR CLAY SLURRIES USED FOR CALIBRATION OF THE STORMER VISCOMETER

strates that the modified Stormer viscometer may be used in measuring the constants S_y and η.

Any Stormer viscometer can be adapted to the measurement of S_y and η without calibration if the modified cup described in the foregoing is used. The following equations can be used for converting Stormer data to pipe flow constants:

$$S_y = 0.0020u_y \tag{20}$$

$$\eta = 0.0035\lambda \tag{21}$$

At low rates of shear in the modified Stormer viscometer corresponding to small speeds of revolution the relationship between the driving force and the speed of revolution is not linear, which results in the line, connecting the points representing driving force and

TABLE 7

RELATION OF S_y TO u_y AND OF η TO λ

Test No.	Substance	Specific Gravity	Per Cent Solids (by weight)	S_y	u_y	Ratio S_y/u_y	η	λ	Ratio η/λ
I	Yellow clay (plastic solid)............	1.49	52.0	0.72	355	0.0020	0.030	8.0	0.0037
II*	Yellow clay (plastic solid)............	1.49	52.3	0.74	370	0.0020	0.036	9.5	0.0038
III*	Yellow clay (plastic solid)............	1.44	48.9	0.418	205	0.0020	0.022	7.0	0.0032
IV*	Yellow clay (plastic solid)............	1.40	45.3	0.245	124	0.0019	0.014	4.4	0.0032
V*	Yellow clay (plastic solid)............	1.36	42.2	0.139	66	0.0021	0.013	4.0	0.0033
VI*	Cylinder oil........................	0.88	0.0	0.0	0	......	0.074	21.2	0.0035
	Average..........					0.0020			0.0035

*See Fig. 10.

corresponding speed of revolution, bending toward the origin. The explanation of this is that, at low rates of shear, the material shears first at the point of greatest stress, which is in the layers of material next to the rotating cylinder. As the rate of shear is increased, shear takes place progressively further from the rotating cylinder until, at some point, shear is taking place from the rotating cylinder to the cup sides. At this point the relation between the increase in driving force and the increase in speed of revolution becomes linear, and continues in this manner until turbulence is introduced by the high rates of shear. It is essential that the driving force and speed of revolution be measured between the areas bounded by lines AB and CD in Fig. 10, as only between these lines is the relationship between the increase in driving force and the increase in speed of revolution linear.

21. *Other Viscometers Useful in Measuring* S_y *and* η.—Viscometers suitable for measuring the values of S_y and η of a sludge include (1) rotating cylinder viscometers, of which the Stormer and Kampf are examples, (2) falling ball viscometers, (3) swinging pendulum viscometers, and (4) capillary tube viscometers. Of these four only the last one will measure the absolute values of the constants S_y and η. The underlying principles of some of these viscometers are explained in detail in the First and Second Reports on Viscosity and Plasticity of the Academy of Sciences of Amsterdam.[15]

22. *Rotating-cylinder Viscometers.*—Most standard rotating-cylinder viscometers are suitable for measuring the significant constants of a sludge. The Stormer viscometer was used in this investigation because it is a commonly-used type in industry. The method of using the Stormer viscometer is outlined in Section 20.

23. *Falling-ball Viscometers.*—Viscosity can be measured by applying Stokes Law to the observations made on a sphere dropping through the viscous substance. A falling-ball viscometer was tried in this investigation and discarded as impracticable because the falling sphere tended to travel in a helical path instead of in a straight line when the sphere fell sufficiently rapidly to cause turbulence.

24. *Swinging-pendulum Viscometers.*—The swinging-pendulum viscometer operates on the principle that the rate of damping of a swinging arm attached to a heavy pendulum is proportional to the viscosity of the material in which the swinging arm is immersed. A viscometer of this type was constructed and used on some measure-

ments in this investigation. The instrument was not used extensively, however, because of the greater availability and greater simplicity of operation of the Stormer viscometer. It was concluded that measurements of the constants in plastic flow are possible with swinging-pendulum viscometers after calibration.

25. *Capillary-tube Viscometers.*—Capillary-tube viscometers are not satisfactory for measuring the pipe flow constants of the type of sludges studied in this investigation, because of trouble from clogging of the capillary. An outstanding advantage of the capillary-tube viscometer, where the material to be studied will flow through it without clogging the tube, is the possibility of computing S_y and η directly by applying the proper observations to Poiseuille's equation. This type of viscometer is described by Herrick[16] who used it to measure the flow characteristics of rotary drilling mud. Bingham[1] advocates this type of viscometer, which he calls a "plastometer." It was not used in this investigation because of difficulty in attempting to cause sewage sludge to flow through a capillary tube.

V. APPARATUS AND TESTS

26. *Preliminary.*—Due to the fact that sufficient sludge was not available from the Sewage Testing Plant operated by the Engineering Experiment Station to permit tests on a large scale, the preliminary apparatus was set up in the Sanitary Engineering Laboratory to permit recirculation of sludge through glass and rubber tubes about $\frac{1}{4}$ in. in diameter. The sludge in this apparatus was circulated by gravity from a reservoir near the ceiling of the room. This reservoir was intermittently refilled by catching the discharge from the apparatus in containers which were lifted and emptied into the overhead tank. The apparatus was later supplemented by similar equipment, set up in the Sewage Testing Plant, using a section of smooth brass pipe one inch in diameter, in which friction losses were measured, and through which the sludge was recirculated by means of a rotary pump that returned the used sludge to a constant-level tank at a fixed elevation, from which the sludge flowed through the apparatus, the rate of flow being controlled by adjustments of a gate valve.

Observations of friction losses as sludge flowed through such an apparatus proved of little value because of various difficulties. The apparatus served, however, to indicate methods of measuring heads or pressures, and how to overcome clogging by avoiding the slightest

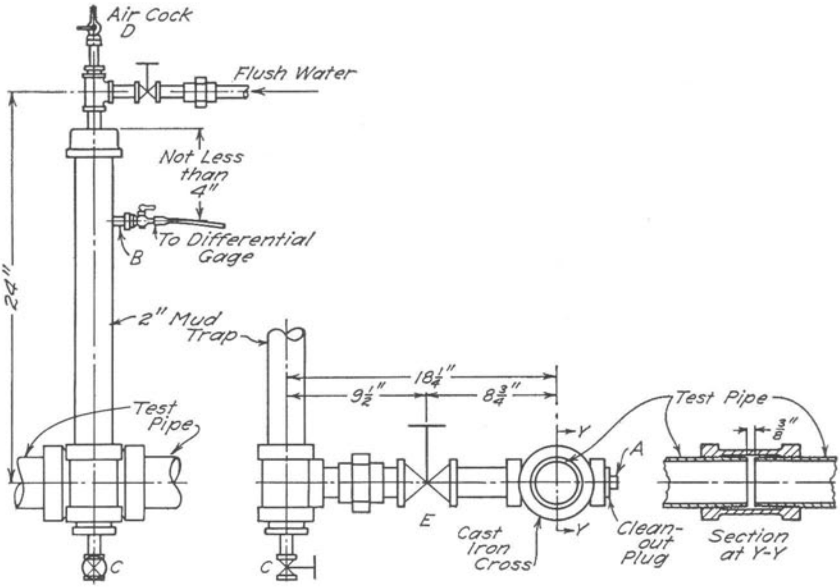


FIG. 12. GAGE CONNECTION AND MUD TRAP

constriction in the sludge pipe, and to draw attention to the impossibility of using partly-open valves to control the flow of sludge, and to other difficulties.

27. *Difficulties in Measurements.*—Tests with the first constant-level apparatus showed that difficulties were due, in part, to the following causes: (1) unsuitable gage connections to the test pipe, (2) entrance of sludge into the gages, (3) attempts to regulate the rate of flow by means of a valve and (4) the recirculation of sludge. All of these difficulties were satisfactorily overcome in subsequent arrangements of the apparatus.

28. *Gage Connections.*—Sludges have been found to give trouble by clogging gage connections unless precautions are taken to avoid the difficulty. An easily-made and satisfactory connection is shown in Fig. 12. This is the type of connection used in the final form of apparatus. The plug, shown at *A* in the figure, was inserted in one of the openings of a cross and the opposite opening was connected, by means of standard pipe-fittings, to the gage. In case of clogging the plug can be removed and the connection cleaned. For pipes 3 inches or larger it is desirable to use a cross with 1½-in. openings on one run.

29. *Clogging of Gages.*—To keep sludge from entering the gages through the gage lines a trap was provided at the gage connection, as shown in Fig. 12. The top of the trap was capped and a $\frac{3}{8}$-in. air cock was tapped into the center of the cap. It has been found that the gage take-off *B* should be located at least 4 inches below the top of the trap in order that entrapped air may not enter the gage lines. Frequent flushings through valves *C* and *D* are necessary during operation in order to expel accumulated sludge and air. Putting a valve between the bottom of the trap and the gage connection on the test pipe, as shown at *E* in the figure, makes it possible to flush the gage lines without shutting down the flow in the test pipe.

30. *Velocity Control.*—The initial set-up contemplated the control of the velocity in the test pipe by means of a valve. It was soon found that this means of velocity control was impossible in the case of several sludges. Such articles as wads of chewing gum, match sticks, etc. found in sewage sludge would clog the valve at low flows so that constant velocity for even a short time could not be maintained. It was found possible to control velocity of flow by either one of two methods: (1) by the use of a constant-level tank in which the level could be changed manually as desired, and (2) by means of a rotary pump with controlled speed. The latter method proved the more successful, particularly at low speeds.

A constant-level tank will not assure constant velocity in the neighborhood of the critical velocity because of the unbalanced conditions in the transition region. On the other hand, a rotary pump driven at constant speed may not maintain a constant head near the transition region. As a result of experience with the preliminary tests, methods of measuring pressures and of catching and weighing sludge passing through the apparatus were so improved as to facilitate the work done later with the long tube viscometer, and with the bank of three pipes set up to measure viscosity directly. The only satisfactory method of forcing sludge through such pipes was found to be with a rotary pump with a closely controllable speed. Any other method of pumping was found to give sudden and frequently apparently inconsistent variations in head loss and velocity of flow.

31. *Recirculation.*—Due to the uncertainty of maintaining constant physical characteristics of a plastic with thixotropic properties on prolonged agitation, recirculation of thixotropic sludges is undesirable. In tests of friction losses in such sludges it is necessary to provide a reservoir of sludge of sufficient capacity to make a number of tests

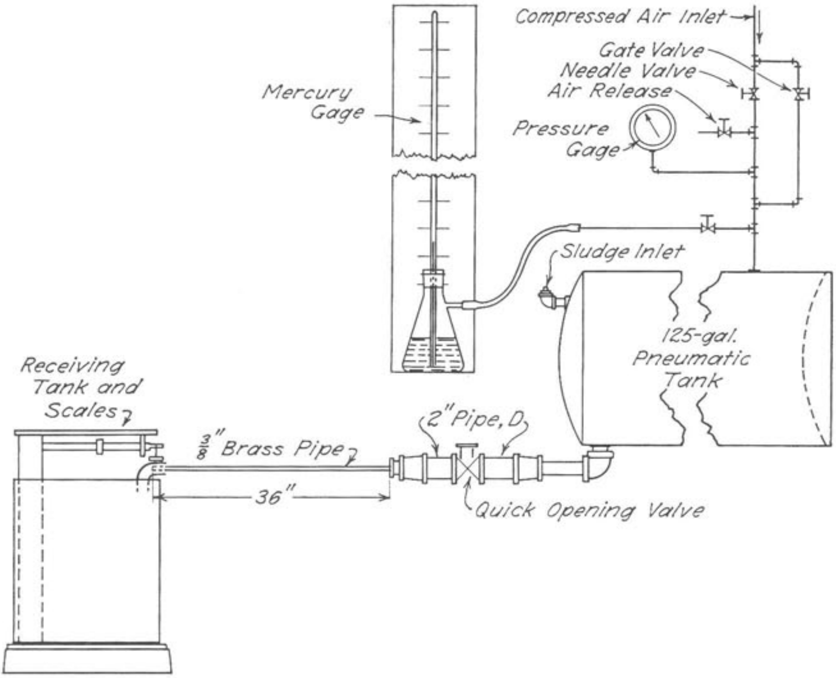


FIG. 13. DIAGRAM OF LONG TUBE VISCOMETER

without reusing the sludge. Where the sludge does not possess thixotropic properties there is no objection to its recirculation through the apparatus when friction losses are to be measured.

32. *Sludge Analyses.*—Analyses of the sludges were made throughout the entire investigation, the analyses including determinations of the total solids, ash, specific gravity, and in some cases the pH value. In all tests, except that for specific gravity, the recommendations of "Standard Methods for Water and Sewage Analyses" of the American Public Health Association were used.

33. *Specific Gravity Measurements.*—In the preliminary tests specific gravities were determined by placing a known volume of sludge in a weighed and calibrated, 500-ml. volumetric flask, and filling up the flask with distilled water, taking care not to entrain air. The flask containing the sludge and the water was then weighed, and the volume of added distilled water was computed. The density of the sludge was then found as the ratio of the weight of the sludge to the

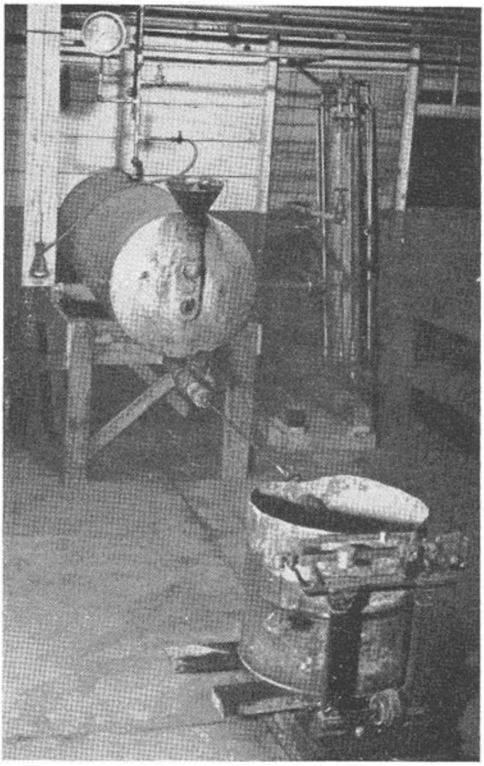


FIG. 14. LONG TUBE VISCOMETER

weight of an equivalent volume of distilled water. A more satisfactory and accurate method was by means of a pycnometer.

In certain regions of flow the specific gravity of the flowing sludge plays an important part (see pages 8 and 28). It is necessary, therefore, to measure the specific gravity with precision. The use of a hydrometer is not feasible for this purpose in relatively thick sludges, because the hydrometer is not free to come to equilibrium. The most satisfactory method of measurement of specific gravity is by means of the pycnometer. In all of these tests specific gravity was measured by means of a calibrated, small-mouth gallon jug which served as the pycnometer. The jug was weighed to the nearest one hundredth of a pound and carefully filled with the sludge to be measured, in such a manner as to avoid the entraining of air, since bubbles entrained in the sludge when pouring into the measuring bottle cannot rise in the sludge. Specific gravity tests should be made immediately after the test runs so that finely divided gas bubbles which affect certain regions of flow do not escape.

34. *Long Tube Viscometer.*—The apparatus illustrated in Figs. 13 and 14 has been designated as a long tube viscometer. In its operation about fifty gallons of the sludge to be tested was poured into the tank through a ⅛-inch mesh wire strainer at *A*, to remove large particles, such as chewing gum, leaves, etc. After quiescent conditions had existed long enough to avoid errors through thixotropic properties of the sludge, air, under pressure, was admitted into the tank. Pressures less than about 25 lb. per sq. in. were observed by means of a mercury gage. Higher pressures, up to a maximum of 60 lb. per sq. in., were measured on a calibrated Bourdon gage. The pressure datum was taken at the elevation of the horizontal discharge tube, corrections being applied to the gage readings, making allowance for the depth of sludge in the tank during a run. The amount of sludge removed during any run was relatively so small that no allowance was necessary for the small amount of change of depth of sludge in the tank during a run.

When the desired pressure was attained, the quick-opening valve was snapped open and sludge was allowed to flow into the container shown on the scales in Fig. 13. During a run the air pressure was maintained constant by the manual adjustment of a needle valve on the air pipe. As the scale beam rose the time was observed, a known weight was added to the scale arm, and the time taken when the beam arose again. The observations thus gave information from which friction loss and the velocity of flow could be computed. A summary of all results is given in Table 8.

Advantages found with this type of viscometer were quick measurements and avoidance of interference by thixotropic properties of the sludge. Occasionally runs on the same sludge were repeated to check this fact. Unfortunately, as designed for these tests, the short section of 2-in. pipe shown at *D* in Fig. 13 precludes the possibility of determining the yield value. Although the friction loss in the 2-in. pipe is negligible when compared with the friction loss in the 3-ft. length of ⅜-in. drawn brass tube, the larger pipe greatly changes the apparent value of the yield value as calculated from the area of the ⅜-in. test pipe. It was necessary, therefore, to calibrate the apparatus with a material of known rigidity and yield value in order to determine the absolute rigidity and yield point of an unknown material. This was done with material measured in the circulating apparatus described on pages 47-49.

TABLE 8
CONSTANTS FOR LAMINAR FLOW OF SLUDGES

Test Reference No.	Source and Description of Sludge	Temperature deg. F.	Per Cent Moisture (by weight)	Volatile Solids Per Cent of Total Solids (by weight)	Specific Gravity	Diameter of Pipe in.	Yield Value S_y	Coefficient of Rigidity η	Source of Information
	Sewage Sludge								
1	Indianapolis, Ind., Activated	36	98.6	66.6	1.008	⅜	0.0	0.001	Long Tube Viscometer
2	Indianapolis, Ind., Primary	40	98.2	68.4	1.005	⅜	0.02	.....	
3	Indianapolis, Ind., Digested	42	92	53.4	1.032	⅜	0.405	0.032	
4	Indianapolis, Ind., Digested	55	92	53.4	1.023	⅜	0.243	0.029	
5	Indianapolis, Ind., Digested	58	96	53.5	1.019	⅜	0.0	.....	
6	Decatur, Ill., Digested	60	86	33.2	1.068	⅜	0.581	0.040	
7	Decatur, Ill., Digested	60	90	34.8	1.050	⅜	0.098	0.015	
8	Springfield, Ill., Digested	58	92.5	44.8	1.026	⅜	0.315	0.030	
9	Springfield, Ill., Digested	60	93.7	44.6	1.021	⅜	0.098	0.018	
10	Decatur, Ill., Imhoff	61	88.3	40.0	1.049	⅜	0.405	0.035	
11	Exp. Sta. Plant, Primary	53	93.3	78.1	0.991	⅜	0.090	0.019	
12	Danville, Ill., Primary	44	97.3	75.3	1.003	⅜	0.0	.....	
13	Kankakee, Ill., Digested	46	92.7	50.8	1.034	⅜	0.026	0.017	
14	Danville, Ill., Digested	46	93	48.5	1.030	⅜	0.234	0.029	
15	Danville, Ill., Digested	122	93	48.5	1.008	⅜	0.072	0.014	
16	Danville, Ill., Digested	71	93	48.5	1.023	⅜	0.180	0.019	
17	Danville, Ill., Digested	40	93	48.5	1.026	⅜	0.205	0.026	
18	Decatur, Ill., Digested	70	87	33	1.06	2	0.141	.....	Decatur, Ill., tests
19	Decatur, Ill., Digested	70	91	33	1.05	2	0.034	.....	
20	Decatur, Ill., Imhoff	60	86	40	1.06	1	0.065	0.0165	Calibration tests with circulating apparatus
21	Decatur, Ill., Imhoff	60	86	40	1.06	2	0.065	0.0165	
22	Decatur, Ill., Imhoff	60	86	40	1.06	3	0.065	0.0165	
26	Calumet, Ill., Imhoff	36-74	90	....	....	8	0.017	0.026	Reference No. 2 (bibliography)
27	Calumet, Ill., Imhoff	36-74	87	....	....	8	0.044	0.040	
28	Calumet, Ill., Imhoff	36-74	88	....	....	8	0.033	0.036	
29	Calumet, Ill., Imhoff	36-74	89	....	....	8	0.024	0.034	
30	Calumet, Ill., Imhoff	36-74	90	....	....	8	0.017	0.026	

TABLE 8.—(CONTINUED)
CONSTANTS FOR LAMINAR FLOW OF SLUDGES

Test Reference No.	Source and Description of Sludge	Temperature deg. F.	Per Cent Moisture (by weight)	Volatile Solids Per Cent of Total Solids (by weight)	Specific Gravity	Diameter of Pipe in.	Yield Value S_y	Coefficient of Rigidity η	Source of Information
	Sewage Sludge								
31	Calumet, Ill., Imhoff	36-74	91	....	....	8	0.013	0.018	Reference No. 2 (bibliography)
32	Calumet, Ill., Imhoff	36-74	92	....	....	8	0.006	0.018	
33	Calumet, Ill., Imhoff	36-74	93	....	....	8	0.003	0.016	
34	Calumet, Ill., Imhoff	36-74	94	....	....	8	0.0015	0.014	
35	Calumet, Ill., Imhoff	54-67	89	....	....	12	0.020	0.0514	
36	Calumet, Ill., Imhoff	54-67	90	....	....	12	0.014	0.035	
37	Calumet, Ill., Imhoff	54-67	91	....	....	12	0.009	0.032	
38	Calumet, Ill., Imhoff	54-67	92	....	....	12	0.0045	0.036	
39	Calumet, Ill., Imhoff	54-67	93	....	....	12	0.0026	0.025	
40	Calumet, Ill., Imhoff	54-67	94	....	....	12	0.0015	0.024	
41	Birmingham, England	....	90	....	....	12	0.054	0.03*	
42	Manchester, England	....	89	....	....	10	0.155	0.03*	
43	Glasgow, Scotland	....	88	....	....	9	0.036	0.03*	
44	Wolverhampton, England	....	90	....	....	4	0.070	0.03*	
68	Stuttgart, Germany, Digested	57-68	85	....	....	7.9	0.412	0.089	Reference No. 4 (bibliography)
69	Stuttgart, Germany, Digested	57-68	86	....	....	7.9	0.276	0.096	
70	Stuttgart, Germany, Digested	57-68	87	....	....	7.9	0.199	0.093	
71	Stuttgart, Germany, Digested	57-68	88	....	....	7.9	0.152	0.089	
72	Stuttgart, Germany, Digested	57-68	89	....	....	7.9	0.118	0.080	
73	Stuttgart, Germany, Digested	57-68	90	....	....	7.9	0.100	0.077	
74	Stuttgart, Germany, Digested	57-68	91	....	....	7.9	0.077	0.079	
75	Stuttgart, Germany, Digested	57-68	82	....	....	7.9	0.128	0.078	
76	Stuttgart, Germany, Digested	57-68	84	....	....	7.9	0.089	0.076	
77	Stuttgart, Germany, Digested	57-68	86	....	....	7.9	0.061	0.068	
78	Stuttgart, Germany, Digested	57-68	88	....	....	7.9	0.046	0.066	
79	Stuttgart, Germany, Digested	57-68	90	....	....	7.9	0.031	0.062	

*η assumed.

TABLE 8.—(CONCLUDED)
CONSTANTS FOR LAMINAR FLOW OF SLUDGES

Test Reference No.	Source and Description of Sludge	Temperature deg. F.	Per Cent Moisture (by weight)	Volatile Solids Per Cent of Total Solids (by weight)	Specific Gravity	Diameter of Pipe in.	Yield Value S_y	Coefficient of Rigidity η	Source of Information
	Sewage Sludge								
90	Calumet, Ill., Imhoff	....	88	....	....	5	0.060	0.021	Unpublished data of Chicago Sanitary District
91	Calumet, Ill., Imhoff	....	90	....	....	5	0.020	0.019	
92	Calumet, Ill., Imhoff	....	92	....	....	5	0.008	0.013	
	Clay								
23	Illinois yellow clay	76	48	0	1.49	1	0.720	0.028	Calibration tests with circulating apparatus
24	Illinois yellow clay	76	48	0	1.49	2	0.720	0.028	
25	Illinois yellow clay	76	48	0	1.49	3	0.720	0.028	
45	Slurry from water purification plant	79	81.4	....	1.13	4	0.020	0.005	Reference No. 3 (bibliography)
46	Slurry from water purification plant	86	76.6	....	1.175	4	0.056	0.005	
47	Slurry from water purification plant	89	70.9	....	1.225	4	0.117	0.005	
48	Slurry from water purification plant	91	67.5	....	1.225	4	0.158	0.005	
49	Slurry from water purification plant	87	64.8	....	1.285	4	0.215	0.005	
50	Slurry from water purification plant	65	86.4	....	1.17	4	0.011	0.005	
55	Tennessee ball clay with water as dispersion medium	82	84.9	0	1.110	...	0.059	0.011	Modified Stormer Viscometer
56	Tennessee ball clay with water as dispersion medium	84	79.3	0	1.140	...	0.201	0.014	
57	Tennessee ball clay with water as dispersion medium	82	77.2	0	1.162	...	0.340	0.015	
58	Tennessee ball clay with water as dispersion medium	82	76.8	0	1.162	...	0.431	0.017	
59	Tennessee ball clay with water as dispersion medium	82	75.2	0	1.145	...	0.532	0.019	
60	Tennessee ball clay with glycerine as dispersion medium	82	85.2	0	1.092	...	0.081	0.0119	
61	Tennessee ball clay with glycerine as dispersion medium	82	83.5	0	1.071	...	0.123	0.0144	
62	Tennessee ball clay with glycerine as dispersion medium	82	81.3	0	1.105	...	0.178	0.0166	
63	Illinois yellow clay with water as dispersion medium	80	66.0	0	1.32	...	0.030	0.011	
64	Illinois yellow clay with water as dispersion medium	78	55.0	0	1.42	...	0.132	0.017	
65	Illinois yellow clay with water as dispersion medium	80	51.5	0	1.44	...	0.208	0.024	
66	Illinois yellow clay with water as dispersion medium	80	51	0	1.46	...	0.401	0.023	
67	Illinois yellow clay with water as dispersion medium	79	48	0	1.49	...	0.709	0.028	
51	Bentonite Paste	Room	55.4	....	....	...	1.73	0.022	Reference No. 15 (bibliography) (Second report)
52	Kaolin paste	Room	45.0	....	....	...	2.35	1.72	
53	Paint paste	Room	68.0	....	....	...	1.80	0.364	
54	Paint paste	Room	....	....	....	...	1.50	0.270	

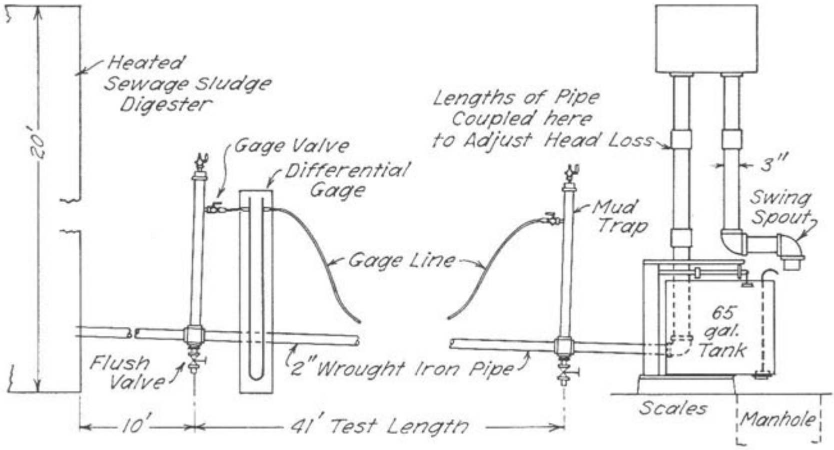


FIG. 15. DIAGRAM OF EQUIPMENT AND APPARATUS USED AT DECATUR, ILLINOIS

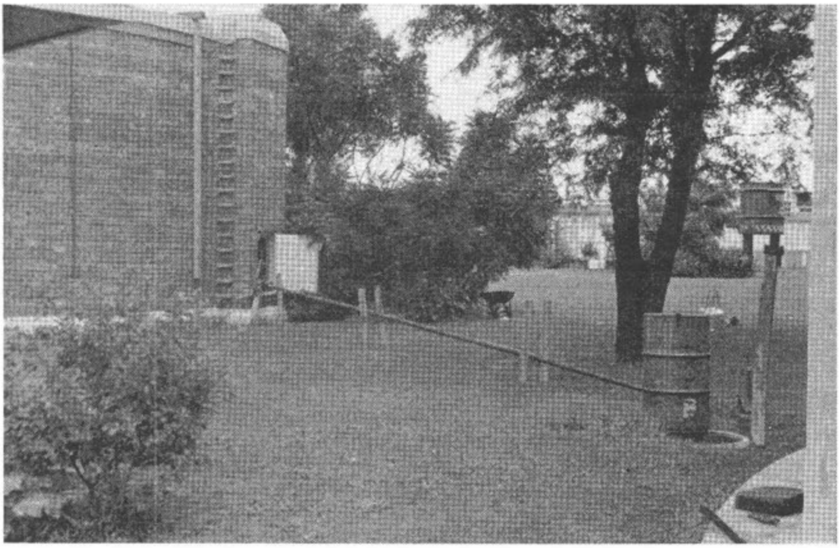


FIG. 16. EQUIPMENT AND APPARATUS USED AT DECATUR, ILLINOIS

Among the important findings resulting from the tests with this viscometer were (a) the true plastic flow properties of sludges, particularly sewage sludges, and (b), by computation after the calibration of the viscometer, the values of the yield value and of the coefficient of rigidity of the sludges tested.

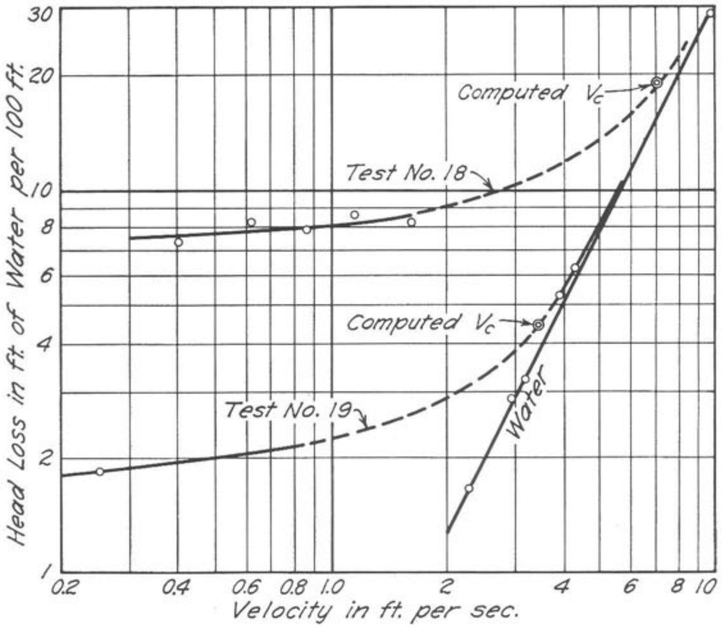


FIG. 17. FRICTION LOSS OF DIGESTED SEWAGE SLUDGE FLOWING IN 2-IN. IRON PIPE AT DECATUR, ILLINOIS

35. *Tests and Apparatus at Decatur, Illinois.*—Conditions at the sewage treatment plant of the Sanitary District at Decatur, Illinois, were unusually favorable for conducting tests on the friction losses for the flow of sewage sludge in pipes without the necessity for recirculating the sludge. The test installation is illustrated in Figs. 15 and 16. The separate sludge digestion and storage tank from which the sludge was drawn is shown at the left of the illustration. Sludge was allowed to run from the tank through a 41-ft. length of 2-in. pipe, discharging, by gravity, into a container resting on a beam scales. The pipe arrangement was such that sludge could be drawn from two different levels in the storage tank. Gage connections and gage-line equipment similar to that finally adopted on the preliminary apparatus were used on this equipment, the upper gage connection being placed on the pipe following a straight run of 10 ft. The rate of flow through the pipe was controlled by raising or lowering the outlet by adding upright sections of straight pipe. Such a method of control was not entirely satisfactory, as clogging of the entrance to the test pipe interfered with the control of the rate of flow in the pipe and with the head-loss

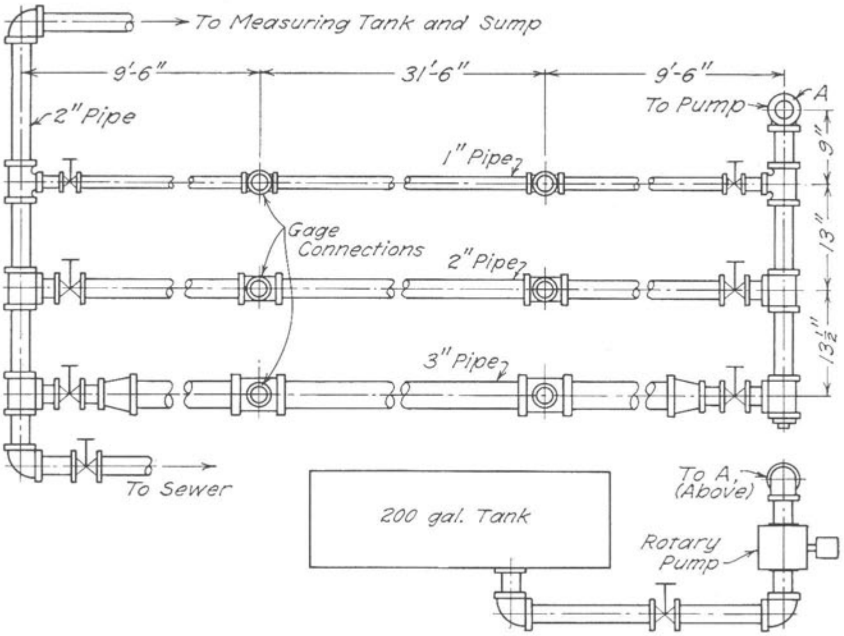


FIG. 18. DIAGRAM OF CIRCULATING APPARATUS USED FOR MEASURING S_y AND η

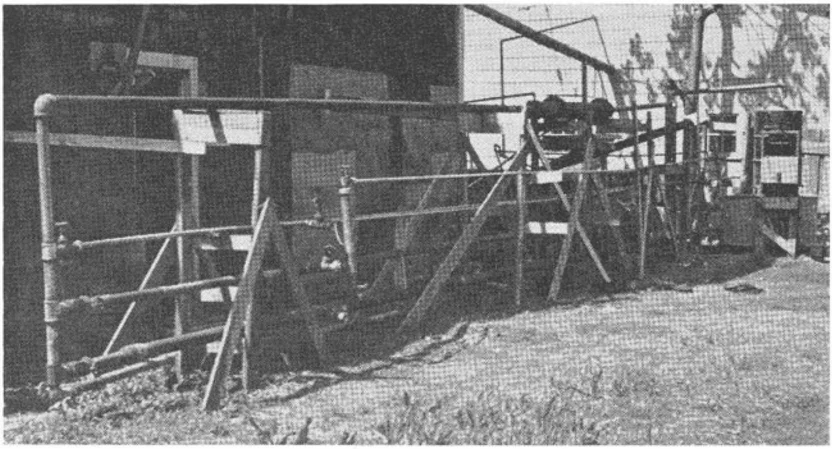


FIG. 19. APPARATUS USED FOR MEASURING S_y AND η

observations in the gage. The maximum head available was about 20 ft. between the top of the sludge in the tank and the lowest point to which the end of the test pipe could be lowered.

Some of the observations made during these tests are shown graphically in Fig. 17. Using the values of the yield value S_y, obtained from the tests and assuming values of the coefficient of rigidity, η, of 0.02 and 0.01 for sludge drawn from the lower and higher levels in the sludge tank, respectively, the computed critical velocities are 7 ft. per sec. and 3.5 ft. per sec., respectively.

36. *Final Recirculating Equipment.*—A special circulating apparatus, consisting of lengths of three different diameters of pipe for measurement of values of S_y and of η was constructed at the University Sewage Treating Plant. The purposes of the apparatus were (1) to demonstrate that the values of S_y and of η are independent of the diameter of the pipe, (2) to correlate the data from the ⅜-in. pipe viscometer with known absolute values of S_y and of η, and (3) to correlate the data obtained by the Stormer viscometer with absolute values of S_y and of η.

The apparatus, illustrated in Figs. 18 and 19, consisted of a tank for holding the sludge, a rotary pump for circulating it, a variable speed motor for driving the pump which permitted quick adjustment to any speed between about 30 r.p.m. and 400 r.p.m., and lengths of 1-in., 2-in. and 3-in. pipe. The sludge could be diverted into any one of the pipes by means of gate valves. Gage connections and gage-line equipment were similar to those used in the Decatur tests.

Loss of head was measured in a mercury U tube. Unsteadiness of level of mercury in the U-tube gage, due to pulsations caused by the rotary pump, were overcome by inserting a small bore tip in each gage line where it entered the gage. This expedient reduced the pulsations of the mercury levels to a negligible amount and gave apparently very precise readings. Velocity of flow was measured by weighing the quantity of sludge discharged through the pipe in a known time.

Figure 20 shows, for comparative purposes, a plot of head loss against velocity in all three pipes in the special circulating apparatus for a well-digested sludge taken from an Imhoff tank at Decatur, Illinois. Before running these tests the sludge was circulated through the pipes at a high velocity for two hours, and the observations were then made as rapidly as possible.

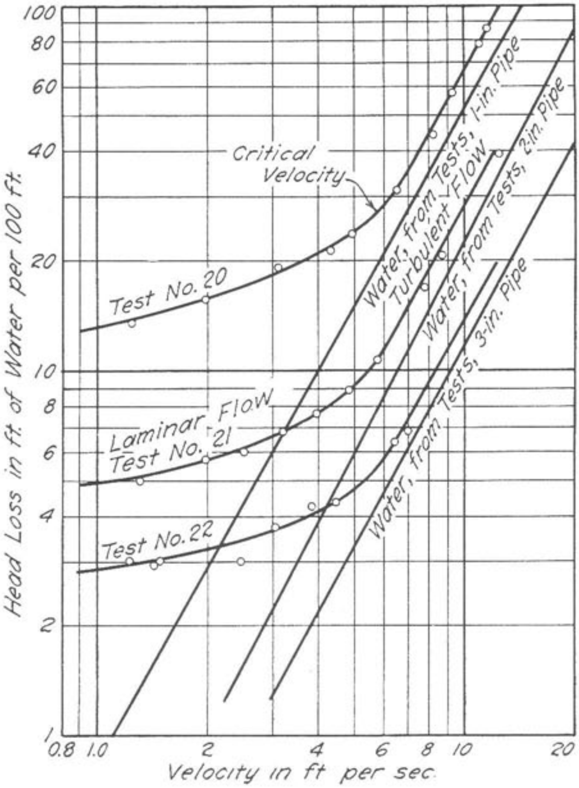


FIG. 20. FRICTION LOSSES OF DECATUR IMHOFF SLUDGE FLOWING IN 1-IN., 2-IN., AND 3-IN. PIPES

The observations in these tests are plotted in Fig. 21 with S_p as ordinate and $V/4D$ as abscissa for the purpose of studying the relation between S_y and η and the pipe diameter. Since the points all fall on the same straight line when the flow is laminar, within the limits of experimental error, it is concluded that S_y and η are independent of the diameter of the pipe in which they are measured.

Tests on a thick yellow clay were also run in the circulating apparatus. The clay, obtained locally, had about 52 per cent solids by weight. Results of the tests are shown in Fig. 21. It is apparent that these results further corroborate the theory that S_y and η are independent of pipe diameter.

In order to correlate the measurements made in the long tube viscometer and the Stormer viscometer with the absolute values of S_y

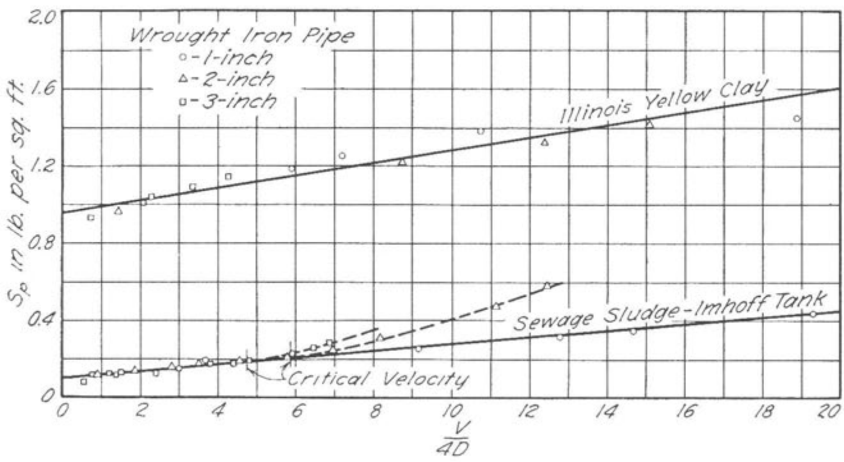


FIG. 21. EFFECT OF PIPE DIAMETER ON FLOW CHARACTERISTICS OF SLUDGE

and η determined in the circulating apparatus, the sludge used in the preceding two tests was run also in the long tube and the Stormer viscometers immediately after the tests in the circulating apparatus had been completed. From the observations with the same sludge in the various viscometers conversion factors were computed by which the relative values of S_y and η found in the viscometers could be converted to absolute values of these factors.

VI. RESULTS AND CONCLUSIONS

37. *Summary.*—It has been shown that sludges such as mixtures of clay and water used in deep-well boring, sewage sludges, sludges from water softening plants, and other similar aqueous suspensions of fine particles, obey the fundamental formula for the flow of a true plastic.

The theoretically developed formula for friction loss when a sludge flows through a straight circular pipe has been experimentally verified for 1-inch, 2-inch, and 3-inch pipes using several different sludges as the flowing material. The theoretical formula applying only to laminar flow is:

$$\frac{H_w}{L} = \frac{16S_y}{3WD} + \frac{\eta V}{WD^2} \qquad (14)^*$$

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*Nomenclature not given here will be found on pages 10 and 11.

It has been shown that for a sludge flowing in a circular pipe a critical velocity is encountered as the velocity of flow is increased. Below the critical velocity the flow is laminar, while above this velocity the flow is turbulent. The critical velocity does not always occur at the same velocity in a given pipe with a given sludge. Based on the known behavior of fluids in this region two formulas for critical velocity have been developed and verified experimentally. One formula gives a velocity that has been called the lower critical velocity, below which the flow will be laminar, and the other formula gives a velocity that has been called the upper critical velocity, above which the flow will be turbulent. Between these two velocities the flow may be either laminar or turbulent or a combination of the two, depending on various factors. The formula for the lower critical velocity is

$$V_{lc} = \frac{1000\eta + 103\sqrt{94\eta^2 + D^2S_y\rho}}{D\rho} \tag{17}$$

The formula for the upper critical velocity

$$V_{uc} = \frac{1500\eta + 127\sqrt{140\eta^2 + D^2S_y\rho}}{D\rho} \tag{18}$$

The yield value, S_y, and the coefficient of rigidity, η, have been shown to be independent of the diameter and roughness of the pipe in which they are measured.

A simple method for the measurement of S_y and η by means of a modified Stormer viscometer has been given, and it has been shown that

$$S_y = 0.0020\mu \tag{20}$$

and

$$\eta = 0.0035\lambda \tag{21}$$

Table 8 lists the yield value and the coefficient of rigidity for sludges measured in this investigation, together with similar values computed from tests reported in the literature. The table also gives important data in regard to the other characteristics of the sludges.

Heretofore no formulation of the factors affecting the friction loss in the flow of sludge through straight circular pipes has been available. Further research as to the factors affecting the turbulent-flow frictional losses of sludges flowing in circular pipes is desirable, as indications are that the common hydraulic formulas for the flow of water are not applicable to the flow of sludges.

APPENDIX

OTHER INVESTIGATIONS

Investigations into the flow characteristics of fluids have been extensive and have resulted in the development of the familiar hydraulic formulas, such as that of Hazen and Williams, and in the universal flow formula of Weisbach which, by means of Stanton's[6] expansion of Reynold's[9] studies, makes possible the solution of problems involving head-loss–velocity relations for any fluid, provided its specific gravity and viscosity are known and constant. Many investigations have been made of the flow characteristics of plastic solids in capillary tubes, an exhaustive treatise on this subject being presented by Bingham[1] in 1922. However, in the words of Bingham, in referring to "hydraulic flow in the plastic state," "So far as known to the author no one has yet used rates of flow high enough to bring about eddy currents which are so troublesome in the case of fluids."

None of the reports of investigations mentioned by Bingham nor found in the literature subsequent to 1922 has attempted to present a formula applicable to the flow of thin sludges in either the laminar or the turbulent region.

A brief summary of references to some pertinent investigations into the flow of fluids and plastics is contained in the bibliography on pages 57 and 58. The following extracts have been taken from reports of investigations into the flow of thin sludges published subsequent to 1919:

1. Nevitt, T. H.[17] The tests were made on a 12-in. cast-iron pipe 240.5 feet long. They gave a variation for Kutter's n from 0.0168 to 0.0181 after five years of service. The sludge was obtained from sedimentation tanks, the age being not more than ten days. Its density was 1.01, moisture content 95.9 per cent, ash, 49.5 per cent, and temperature, 54 deg. F.

2. Clifford, W. E.[18] The author attempted to base the flow of sewage sludge on the laws dealing with viscous fluids. He determined the kinematic viscosity by two methods:

(a) By comparing the flow of sludge with the flow of a homogeneous fluid of known viscosity, measuring the time of efflux through a tube $\frac{5}{16}$ in. in internal diameter and $5\frac{1}{4}$ in. long.

(b) By determining the viscosity directly by measuring the friction loss in a pipe, converting to kinematic viscosity, and applying the general equation of the flow of liquid through a pipe. From a comparison of the flow of a sludge containing 90 per cent moisture with the flow of glycerine with known kinematic viscosity, Clifford found the kinematic viscosity of his sludge to be 0.001055 lb. per ft. per sec. By applying the assumed viscosity to the data for sludges containing 90 per cent moisture flowing in an 8-in. pipe at Calumet he calculated theoretical friction losses which checked closely with observed friction losses.

3. Gregory, W. B.[3] studied the flow of a clay slurry, containing about 85 per cent moisture, through a pipe of 4-in. diameter. The viscosity of the clay slurry was computed by Poiseuille's law, with the conclusion that the substance has no viscosity comparable to that of a homogeneous fluid similar to oil or water. Gregory found that in the viscous or stream-line zone the head loss was almost constant, and explained this by stating that, since the viscous or stream-line resistance in a true liquid varies as the velocity, then the presence of solids suspended in the liquid will cause a variation in the opposite direction, so that the sum of the two is a constant below the critical velocity. "It may be expected that with increase in concentration of solid particles the starting resistance will increase. Also that with equal concentration, a suspension of a finer material will have a smaller starting resistance and, hence, a lower critical velocity." He concluded that

(a) The most economical velocity for pumping is the critical velocity, and

(b) The apparent viscosity of slurry at the critical velocity varied from 24 to 85 times that of water, depending on the concentration of solids.

4. A Committee of the American Society of Civil Engineers[2] reported the results of extensive research and study of the flow of sewage sludge, including valuable data on flow tests in an 8-in. pipe at the Calumet Sewage Disposal Plant in Chicago. The sludges tested varied in moisture content from 89 to 97 per cent, and head loss measurements were made at various velocities in both laminar and turbulent flow. The conclusions were summarized as follows:

(a) Sludge is neither a viscous nor a homogeneous material but is variable in character.

(b) The usual analytical tests do not define its physical qualities, but it seems to behave more like suspended material.

(c) Below the critical velocity sludges have a different friction factor from that found above the critical velocity. As yet the coefficient of flow below the critical velocity cannot be concisely stated; above the critical velocity it can only be expressed in ranges.

(d) Sludge friction losses increase with decrease of moisture content.

(e) Sludge friction losses tend to increase with lower temperature.

(f) Sludge friction losses for high velocities, from about 5 to 6 feet per second or more, tend to follow more closely the characteristic law for the flow of water.

(g) Friction losses for fresh or undigested sludge and for sludge from combined sewage are more erratic and the determination of the friction factor is correspondingly more difficult.

(h) Within the limits of the investigation no law of flow was found.

5. Hubell, G. E.[19] Sewage sludge obtained by sedimentation of sewage from a combined sewer was pumped through 20 466 feet of 8-in., cast-iron pipe. The sludge, which contained 99.1 per cent moisture, gave Hazen and Williams coefficients of 140 and 137 at velocities of 3.15 and 4.06 feet per second, respectively.

6. Woolgar, C. A.[20] The article deals with the measurement of viscosity of drilling fluids by means of a Stormer type of rotating viscometer. It is recommended that such an instrument be not used for liquids with viscosities below 6 centipoises. The flow characteristics of a clay suspension and the relation between the yield point, the viscosity, and the percentage clay content of a well-drilling fluid are discussed.

7. Evans, P. and Reid, A.[21] The article deals with the meaning of the viscosity of a suspension and its measurement. The authors state that unless the conditions of measurement are specifically defined the viscosity is meaningless, since the apparent viscosity decreases with increasing flow. For many drilling fluids (mud) the graph connecting the points representing the applied pressure and the discharge from a long narrow pipe approximates a straight line which, if produced, cuts the pressure axis at a definite point. At low velocities the graph is not straight but bends to the origin, intersecting the pressure axis at a point called the yield point. The slope of the straight line gives the mobility of the mud. The authors give also a method of measurement of the mobility and the yield point of the drilling

mud. They discuss the phenomena of a mud being forced through a small pipe, changes in viscosity, and the effect of thixotropy on the flow of mud.

8. G. D. Hobson.[22] This article discusses the value of various types of viscometers in the measurement of the viscosity of mud or clay-water suspensions. The author points out that the viscosity of such mixtures can be altered by the addition of certain chemicals, and that the effect of entrained air is to give a viscosity value far below that necessary to account for the volume of air present.

9. Andrews, C. A.[23] In discussing the viscosity characteristics of a chemically-doctored drilling fluid the author defines the viscosity of a colloidal suspension and points out the factors which tend to change the viscosity. He discusses also the measurements of the viscosities of drilling fluids under turbulent conditions by means of the Dallwitz and Wegner viscometer. Precise results will not be given by this instrument, if used as described, but reliable and reproducible results can be obtained quickly for comparative purposes.

10. Merkel, W.[4] This article is a report on an experimental and theoretical study of the flow of sewage and plastic materials, largely based on the more general work on plastic materials reported by Bingham,[1] Oswald,[24] and Reiner.[25] The author discusses the various factors affecting the flow of sludge. He made tests of the viscosity of sewage sludge flowing through 20-cm. pipes at the Stuttgart Sewage Disposal Plant. As a result he suggests using a modification of Poiseuille's formula developed by Reiner to estimate the flow of sludge containing 80 to 85 per cent moisture. His tests were made on raw, separate digested, and activated sludge containing from 80 to 94 per cent of moisture.

11. Hatfield, W. D.[13] studied the pseudo-plastic properties and thixotropic properties of sewage sludges. The author measured the viscosities of sludges containing from 90 to 99 per cent moisture by means of the Stormer viscometer, the instrument having been calibrated for absolute viscosities by measuring in it the viscosities of fluids with known viscosities. By plotting shearing stresses against rate of shear he obtained flow curves similar to those obtained for a semi-plastic material. In his first article[13] the author concludes:

(a) A rotational viscometric method of studying the viscous properties of sewage sludge has been described.

(b) The viscous properties of sewage sludge have been shown to be pseudo-plastic, that is, the apparent viscosity decreases as the rate of shear and the shearing stress increase.

(c) The sludge is thixotropic, the pseudo-plastic resistance and, therefore, the apparent viscosity being greatly reduced by stirring or shaking.

(d) The apparent viscosity when plotted against the rate of flow or the percentage of solids produces a straight line on logarithmic coordinates.

(e) Application of the laws of fluid flow to sewage sludge gives valuable information, even though these laws are only approximate when applied to pseudo-plastic flow. Further studies should be made to correct the equations of flow for the pseudo-plastic properties of sludge flow.

In his second article[13] the author compares his results with those of Merkel,[4] and shows that, in general, the viscosities of sewage sludges range from 1000 to 5000 centipoises at low velocities, down to 25 to 100 centipoises at turbulent velocities. He shows that his shearing curves for partly-digested sludges containing 84 to 91 per cent moisture are similar to those of Merkel's in 20-cm. diameter pipes. By computing the viscosities in this study and plotting these values against their corresponding velocities the author shows that these values will plot in a straight line on logarithmic coordinates, giving a slope of 1.4 to 1.6, which checked those given by sludges used in Springfield, Illinois.

12. Herrick, H. N.[16] This is a report of a series of experiments indicating the relationship between various characteristics of plastic solids used in the oil fields. The experiments were made on California muds. The author used a capillary-tube viscometer and gives mathematical formulas for yield point and viscosity values of sludges, together with methods for measuring the constants and variables in the formulas.

13. Traxler, R. N.[26] It is pointed out that the flow properties of dilute suspensions of clay and other minerals are of interest to the pulp and paper technologist. The various concentrations of solids are discussed. The instruments for evaluating the flow properties in such systems are mentioned. It is pointed out that particle size, size distribution, and shape are primary properties of the particles of a mineral

filler and that, dependent on these, are the secondary properties of the compacted powder, such as per cent voids and average void size. The methods for determining both primary and secondary properties are briefly discussed. A simple relationship has been established between concentration of the solids present and the viscosity of the suspension. This relationship makes possible an accurate evaluation of the influence of a dispersed solid on the flow properties of slurries of which they are a part.

14. Report on Viscosity and Plasticity prepared by the Committee for the Study of Viscosity of the Academy of Sciences at Amsterdam. First Report in 1935,[15] Second Report in 1938.[15] This is an exhaustive treatise of the subject, printed in English, in two volumes. The objects of the committee were: (1) "To gather information regarding the phenomena of viscous and plastic deformations as they present themselves in various domains of physics, chemistry, technology, and biology; . . . . (4) To study the methods used for the measurement of viscosity and of related properties of matter, to interpret the meaning of the results given by various technical instruments, and, where possible, to indicate instruments which allow an unambiguous measurement of scientifically well defined quantities."

15. Wilhelm, R. H., Wroughton, D. M. and Loeffel, W. L.[27] This article is a report of experiments on pumping filter-cell and cement rock suspensions through pipes. Head losses were measured in three sizes of pipes at velocities ranging from 0.3 to 14 feet per second. Apparent viscosity characteristics of all suspensions were determined by means of a rotating viscometer.

Flow was found to exist in two states which were termed "plug" and "turbulent" flow. Separate correlations, involving pipe and viscometer data, are presented for each type of flow.

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