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A THERMODYNAMIC ANALYSIS OF
INTERNAL-COMBUSTION ENGINE
CYCLES

BY

GEORGE A. GOODENOUGH

AND

JOHN B. BAKER



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ENGINEERING EXPERIMENT STATION

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JANUARY, 1927

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COMBUSTION ENGINE CYCLES

BY

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ENGINEERING EXPERIMENT STATION

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A THERMODYNAMIC ANALYSIS OF INTERNAL-COMBUSTION ENGINE CYCLES

I. INTRODUCTION

1. *Objects of Investigation.*—In the theoretical analysis of the cycle of an internal combustion engine three degrees of approximation may be observed. The simplest and crudest system of analysis gives the so-called "air standard," which is still used in estimating the efficiencies of engines. In this analysis it is assumed that the medium throughout the cycle is air, or, at least, a gas having the same properties as air. During the combustion phase the air is supposed to receive an amount of heat equal to the heat of combustion of the fuel. Usually the specific heat of the air is taken as a constant. The air standard efficiency deduced from this analysis can hardly be regarded as an approximation; it is always from 10 to 25 per cent higher than the efficiency obtained from more accurate analyses.

In the second system of analysis the properties of the actual gas mixtures are used. It is recognized that the medium compressed is a mixture of fuel and air, and the medium undergoing adiabatic expansion after combustion is an entirely different mixture, having different properties. In this analysis, however, it is assumed that combustion is complete before adiabatic expansion begins.

It is now well known that at the maximum pressure and temperature attained in the cycle the combustion is not complete, and that through the dissociation of the products CO_2 and H_2O as the temperature rises above 2500 deg. F. there will be unburned CO and H_2 in the mixture at the beginning of adiabatic expansion. As the temperature falls during expansion the combustion continues until at the end of the expansion it is practically complete. The third system of analysis takes account of these phenomena. By the method outlined in Bulletin No. 139* the maximum temperature, taking account of dissociation and chemical equilibrium, may be calculated; then, as shown in a later section of the present bulletin, the conditions of adiabatic expansion accompanied by combustion of the unburned CO and H_2 are established, and the temperature at the end of expansion is determined.

It is the principal object of this investigation to apply this accurate system of analysis to the two leading cycles of the internal combustion engine, and to obtain thereby accurate values for ideal efficiencies under

*"An Investigation of the Maximum Temperatures and Pressures Attainable in the Combustion of Gaseous and Liquid Fuels," Univ. of Ill. Eng. Exp. Sta. Bul. 139, 1924.

various conditions. A secondary object is the comparison of the efficiencies obtainable for various liquid fuels.

2. *Acknowledgments.*—Credit is due MR. GEORGE T. FELBECK for a considerable amount of the preliminary work in the preparation of this bulletin. MR. FELBECK outlined the methods to be pursued in the analysis of the various cycles and developed the analysis of the adiabatic expansion with combustion still proceeding. His assistance is gratefully acknowledged. The assistance of MR. ALBERT E. HERSHEY, in offering various suggestions, is also acknowledged.

II. DEFINITIONS AND THERMODYNAMIC LAWS

3. *Gas Mixtures.*—For convenience of reference a condensed statement of the principal laws of gases is here given. For a more complete exposition the reader is referred to Bulletin No. 139.

The unit of weight is taken as the mol, which is the weight in pounds indicated by the molecular weight. Thus 1 mol of oxygen (O_2) = 32 lb., 1 mol of CO_2 = 44 lb., etc. With a pressure of 14.7 lb. per sq. in. and a temperature of 32 deg. F., 1 mol of any gas has the volume 358.7 cu. ft.; for 62 deg. F. the volume is 380.6 cu. ft.

Denoting by v the volume of 1 mol in cubic feet, the equation of a perfect gas is

$$pv = RT$$

where $R = 1544$, with p in lb. per sq. ft., and T absolute temperature Fahrenheit. The product $AR = \frac{1}{778} \times 1544 = 1.985$ is of frequent occurrence.

The volume composition of a gas mixture is also a mol composition. Thus the composition

$$\begin{array}{r} CO_2 = 0.30 \\ H_2O = 0.20 \\ N_2 = \underline{0.50} \\ \hline 1.00 \end{array}$$

signifies that if the CO_2 is separated from the mixture its volume at the same pressure will be 30 per cent of the volume of the original mixture. It also signifies that 1 mol of the mixture contains 0.3 mol of CO_2 , 0.2 mol of H_2O , etc.

The total pressure of the mixture is the sum of the partial pressures of the constituents,

$$p = p_1 + p_2 + p_3 + \dots$$

and the partial pressures are respectively proportional to the number of mols of the constituents; thus

$$p_1 : p_2 : p_3 : \dots = n_1 : n_2 : n_3 : \dots$$

4. *Specific Heats.*—The specific heat of a gas is a function of the temperature, usually a second degree function. Thus for constant pressure, and constant volume, respectively,

$$\begin{aligned}\gamma_p &= a + bT + cT^2 \\ \gamma_v &= a' + bT + cT^2 \\ \text{and } \gamma_p - \gamma_v &= a - a' = AR = 1.985\end{aligned}$$

Expressions for the specific heats of a number of gases are given in Bulletin No. 139, p. 106.

5. *Energy, Thermal Potential, and Entropy.*—The energy of 1 mol of a gas is given by the expression

$$u = \int \gamma_v dT + u_0$$

The energy of a mixture of gases is the sum of the energies of the individual constituents:

$$U = n_1 \int \gamma_{v_1} dT + n_2 \int \gamma_{v_2} dT + \dots + n_1 u_{0_1} + n_2 u_{0_2} + \dots$$

From the definition of thermal potential

$$\begin{aligned}i &= u + Apv = u + ART \\ i &= \int \gamma_p dT + u_0\end{aligned}$$

and for a mixture

$$I = n_1 \int \gamma_{p_1} dT + n_2 \int \gamma_{p_2} dT + \dots + n_1 u_{0_1} + n_2 u_{0_2} + \dots$$

The entropy of 1 mol of gas is

$$s = \int \gamma_p \frac{dT}{T} - AR \log_e p + s_0$$

The symbol h is used to denote the sum

$$\int \gamma_p \frac{dT}{T} + s_0$$

then

$$s = h - AR \log_e p$$

For a gas mixture

$$S = n_1 s_1 + n_2 s_2 + \dots$$

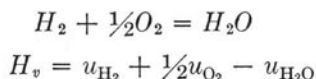
6. *Heats of Combustion.*—For combustion at constant volume

$$H_v = U_1 - U_2$$

and for combustion at constant pressure

$$H_p = I_1 - I_2$$

The subscript 1 refers to the initial mixture of fuel and air, the subscript 2 to the mixture of products. Thus for the reaction

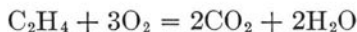


The initial and final temperatures must be the same.

The following notation will be employed frequently:

$$\begin{array}{lll} \Sigma n = z & \Sigma nu_0 = H_0 & \Sigma nc = \sigma''' \\ \Sigma nu = H_v & \Sigma na = \sigma' & \Sigma ns_0 = k \\ \Sigma ni = H_p & \Sigma nb = \sigma'' & \Sigma nh = \lambda \end{array}$$

In the formation of these sums the number of mols n is taken from the reaction equation; the products belonging to the constituents in the fuel mixture are given the positive sign, those belonging to the constituents in the mixture of products the negative sign. For example, consider the reaction



$$\sigma' = \Sigma na = a_{C_2H_4} + 3a_{O_2} - 2a_{CO_2} - 2a_{H_2O}; z = 1 + 3 - 2 - 2 = 0$$

The difference between the heats of combustion H_p and H_v is

$$H_p - H_v = zART$$

Also

$$\begin{array}{l} H_p = H_0 + T(\sigma' + \frac{1}{2}\sigma''T + \frac{1}{3}\sigma'''T^2) \\ H_v = H_0 + T(\sigma' - zAR + \frac{1}{2}\sigma''T + \frac{1}{3}\sigma'''T^2) \end{array}$$

7. *Energy Equation Applied to a Chemical Reaction.*—When an amount dx of a constituent is transformed the work obtained is the product Xdx , and X is regarded as the driving force of the reaction. With several reactions proceeding simultaneously the work is

$$dW = X_1dx_1 + X_2dx_2 + X_3dx_3 + \dots$$

If there is a change of volume during the process the element pdV must be included.

The energy equation for the process is, therefore,

$$dQ = dU + X_1dx_1 + X_2dx_2 + \dots + ApdV$$

8. *Thermodynamic Potentials.*—By the introduction of the potential functions

$$F_v = U - TS$$

$$F_p = U - TS + ApV = I - TS$$

along with the relation $dQ = TdS$, the following equations are obtained from the energy equation:*

$$-dF_v = X_1dx_1 + X_2dx_2 + \dots + ApdV$$

$$-dF_p = X_1dx_1 + X_2dx_2 + \dots - AVdp$$

These equations are valid for isothermal processes only.

9. *Chemical Equilibrium.*—The conditions of equilibrium for any thermodynamic system are the following:

(1) With T and V constant

$$F_v \text{ is a minimum, } dF_v = 0$$

(2) With T and p constant

$$F_p \text{ is a minimum, } dF_p = 0$$

The application of these conditions to a gas reaction gives an equilibrium equation of the general form

$$AR \log_e K_p = \frac{H_0}{T} - \sigma' \log_e T - \frac{1}{2} \sigma'' T - \frac{1}{6} \sigma''' T^2 - (k - \sigma')$$

In this equation the symbol K_p applies to a function of the partial pressures of the constituents when the gas mixture is in equilibrium. The function is given by the equation

$$\log_e K_p = -\sum n \log_e p$$

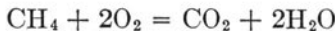
Thus for the reaction $\text{H}_2 + \frac{1}{2} \text{O}_2 = \text{H}_2\text{O}$

$$\log_e K_p = -\log_e p_{\text{H}_2} - \frac{1}{2} \log_e p_{\text{O}_2} + \log_e p_{\text{H}_2\text{O}}$$

whence

$$K_p = \frac{p_{\text{H}_2\text{O}}}{p_{\text{H}_2} p_{\text{O}_2}^{1/2}}$$

for the reaction



$$K_p = \frac{p_{\text{CO}_2} p_{\text{H}_2\text{O}}^2}{p_{\text{CH}_4} p_{\text{O}_2}^2}$$

*"An Investigation of the Maximum Temperatures and Pressures Attainable in the Combustion of Gaseous and Liquid Fuels." Univ. of Ill. Eng. Exp. Sta. Bul. 139, p. 23.

Three equilibrium equations are required for the analysis of the cycles of internal combustion engines:

- (1) For the reaction $\text{H}_2 + \frac{1}{2} \text{O}_2 = \text{H}_2\text{O}$

$$AR \log_e K_p = \frac{102820}{T} - 1.135 \log_e T - 0.4713 \cdot 10^{-3} T \\ + 0.0605 \cdot 10^{-6} T^2 - 2.3$$

- (2) For the reaction $\text{CO} + \frac{1}{2} \text{O}_2 = \text{CO}_2$

$$AR \log_e K_p = \frac{120930}{T} - 3.245 \log_e T + 1.95 \cdot 10^{-3} T - 0.13 \cdot 10^{-6} T^2 + 0.6$$

- (3) For the water-gas reaction $\text{H}_2 + \text{CO}_2 = \text{H}_2\text{O} + \text{CO}$

$$AR \log_e K_p = -\frac{18110}{T} + 2.11 \log_e T - 2.4213 \cdot 10^{-3} T \\ + 0.1905 \cdot 10^{-6} T^2 - 2.9$$

Values of $\log_{10} K_p$ for the first two of these three reactions are given in Tables 30 and 31, and values of K_p for the third in Table 32, Bulletin No. 139 (pp. 144-149).

III. OTTO AND DIESEL CYCLES

10. *Description of Otto Cycle: Assumptions.*—The indicator diagram of the ideal Otto engine is shown in Fig. 1. The line 0-1 indicates the entrance of the charge of fuel and air during the first stroke, the curve 1-2 the compression of this charge on the return stroke. The line 2-3 represents the rise of pressure at constant volume when the charge is ignited and the fuel is burned, and the curve 3-4 the expansion of the products of combustion. At point 4 the exhaust valve opens and the products pass out of the cylinder into the exhaust. The

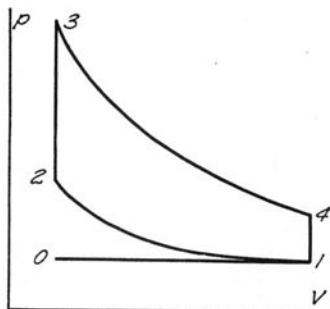


FIG. 1. INDICATOR DIAGRAM OF IDEAL OTTO CYCLE

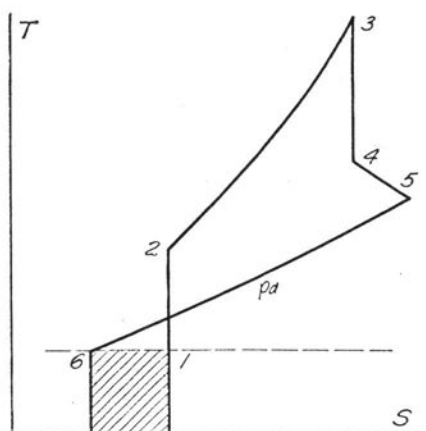


FIG. 2. T-S DIAGRAM OF IDEAL OTTO CYCLE

line 4-1 does not represent a change of state; what occurs at release will be discussed in the next section. On the fourth stroke part of the gas mixture remaining in the cylinder is pushed through the exhaust valve; some of the mixture, however, remains in the clearance space and is mixed with the incoming charge.

To simplify the analysis certain conditions are assumed for this ideal cycle:

- (1) that the pressure of the mixture during the suction stroke and also during the exhaust stroke is the pressure of the atmosphere, and hence the ideal cycle shows no "pumping loss;"
- (2) that all the operations of the cycle are adiabatic, and that there is no loss of heat to the external surroundings;
- (3) that the combustion is at constant volume, that is, line 2-3 is vertical;
- (4) that in the case of a liquid fuel, the fuel is completely vaporized before entering the cylinder.

11. *Analysis of Otto Cycle.*—An accurate analysis of the Otto cycle requires a careful study of all the changes of state of the medium; and for this purpose a representation of the cycle on the T - S plane is useful. In Fig. 2, point 1 represents the state of the fuel mixture at temperature T_1 just at the beginning of adiabatic compression. The vertical line 1-2 represents the adiabatic compression of this mixture, and curve 2-3 represents the change of state during combustion. The vertical line 3-4 represents the adiabatic expansion of the mixture of products. At point 4 the exhaust valve opens and the products of combustion sweep

out into the atmosphere. This irreversible process is accompanied by an increase of entropy and point 5 represents the state of the products mixture at the end of the process. Finally the constant-pressure curve 5-6 represents the cooling of the products from temperature T_5 to the initial temperature T_1 .

It will be observed that the cycle is not closed; the initial point 1 represents the state of the initial mixture at temperature T_1 and atmospheric pressure, while point 6 represents the mixture of products at the same pressure and temperature. The following equalities may be noted:

$$\begin{aligned} T_6 &= T_1 \\ p_6 &= p_5 = p_1 = p_a, \text{ atmospheric pressure} \\ V_4 &= V_1 \\ V_2 &= V_3 = \text{clearance volume} \end{aligned}$$

Referring to Fig. 1, it is seen that the process represented by 4-1 does not involve any work so far as the engine is concerned. Hence (W) the work of the cycle is that obtained in the three changes of state 1-2, 2-3, 3-4. We have then

$$(W) = {}_1W_2 + {}_2W_3 + {}_3W_4$$

and since these changes are all adiabatic

$$A(W) = U_1 - U_4 \quad (1)$$

The process 4-5 may also be regarded as adiabatic, but in the constant pressure change 5-6 heat is rejected to the atmosphere. Thus

$${}_5Q_6 = I_6 - I_5 \quad (2)$$

or

$$-{}_5Q_6 = I_5 - I_6$$

To determine state 5 the energy equation is applied to the change of state 4-5. The work done when the volume is increased from V_4 to V_5 against the pressure p_a is $Ap_a(V_5 - V_4)$ and the change of energy is $U_5 - U_4$. Assuming that the process is adiabatic, the heat equivalent of the work done is equal to the decrease of energy, or

$$Ap_a(V_5 - V_4) = U_4 - U_5$$

whence

$$U_4 + Ap_aV_4 = U_5 + Ap_aV_5 = I_5 \quad (3)$$

Substituting this expression in (2)

$$-{}_5Q_6 = U_4 + Ap_aV_4 - I_6 \quad (4)$$

Equation (1) gives the work of the cycle $A(W)$ and equation (4) gives the heat rejected. The sum of these, as in all heat engine cycles, is the energy supplied. This sum is

$$\begin{aligned} A(W) - {}_5Q_6 &= (U_1 - U_4) + (U_4 + Ap_aV_4 - I_6) \\ &= U_1 + Ap_aV_4 - I_6 \end{aligned} \quad (5)$$

Since

$$V_4 = V_1, \quad Ap_aV_4 = Ap_aV_1$$

and

$$U_1 + Ap_aV_1 = I_1$$

Hence

$$A(W) - {}_5Q_6 = I_1 - I_6 \quad (6)$$

At point 6 the temperature of the products mixture is the initial temperature T_1 , and the temperature is so low that the combustion is complete. The difference $I_1 - I_6$ is, therefore, the heat of combustion H_p of the fuel at constant pressure and at temperature T_1 . It is interesting to note that while the combustion process 2-3 is at constant volume, nevertheless, the energy supplied must be taken as H_p , the heat of combustion at constant pressure. The truth of this statement is evident from the following considerations: the initial state of the mixture is represented by point 1, the final state of the products by point 6, and during the passage from 1 to 6 directly along the isothermal 1-6 the heat H_p is developed. No matter what series of processes intervenes between states 1 and 6, H_p is the energy supplied.

If the chemical energy of the fuel could be transformed directly into electrical energy at constant temperature T_1 , then the heat rejected would be $T_1(S_6 - S_1)$ represented by the area under 1-6. This heat is from 1 to 10 per cent of H_p , depending on the properties of the fuel. With the Otto cycle the heat that must be rejected is represented by the much larger area under 5-6.

The ideal efficiency of the cycle is the ratio of $A(W)$, the heat equivalent of the work, and H_p the energy supplied.

That is

$$\eta = \frac{A(W)}{H_p} \quad (7)$$

12. *Diesel Cycle: Assumptions.*—The indicator diagram of the ideal Diesel engine is shown in Fig. 3. The line 0-1 indicates the entrance of the air during the first stroke, the curve 1-2 the compression of this air on the return stroke. At 2 the fuel is forced into the cylinder and due to the high temperature of the compressed air it ignites. The supply of

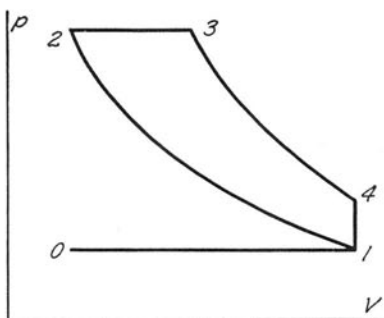


FIG. 3. INDICATOR DIAGRAM OF IDEAL DIESEL CYCLE

fuel continues as the piston moves back, causing a combustion at constant pressure. The curve 3-4 represents the adiabatic expansion of the products of combustion. At point 4 the exhaust valve opens and the products pass out of the cylinder. On the fourth stroke part of the gas mixture is pushed through the exhaust valve; some of the mixture, however, remains in the clearance space and is mixed with the incoming air.

To simplify the analysis certain conditions are assumed for the ideal cycle:

- (1) that the pressure of the air during the suction stroke and also that of the products during the exhaust stroke is the pressure of the atmosphere, or in other words there is no "pumping loss;"
- (2) that all the operations of the cycle are adiabatic;
- (3) that the combustion of the fuel is at constant pressure, that is, the line 2-3 is horizontal;
- (4) that in the case of liquid fuel, the fuel is completely vaporized before entering the cylinder;
- (5) that the fuel is injected into the cylinder without admixture of compressed air, and that it enters the cylinder at the temperature T_1 .

13. *Analysis of Diesel Cycle.*—The analysis follows very closely the analysis of the Otto cycle. The temperature-entropy diagram has the same general appearance as that for the Otto cycle, Fig. 2.

The energy U_1 of the air in the initial state 1 is supplemented by the energy brought in by the oil during the operation 2-3. The energy of 1 mol of oil at temperature T_1 may be denoted by u_1' . To this must be added the equivalent of the work of forcing the oil into the cylinder, and the sum is simply the thermal potential i_1' . For the adiabatic cycle, the work is given by the equation

$$A(W) = U_1 + i_1' - U_4 \quad (8)$$

As in the Otto cycle, the heat rejected is

$$- {}_5Q_6 = U_4 + Ap_a V_1 - I_6 \quad (9)$$

and the sum of the two is

$$\begin{aligned} A(W) - {}_5Q_6 &= U_1 + i_1' + Ap_a V_1 - I_6 \\ &= I_1 + i_1' - I_6 \end{aligned} \quad (10)$$

Again, this expression is that for H_p , the heat of combustion at constant pressure of 1 mol of the oil.

The expression for efficiency is the same as for the Otto cycle; that is,

$$\eta = \frac{A(W)}{H_p} \quad (11)$$

14. *System of Calculation.*—In the expression for the work of the cycle, namely, $A(W) = U_1 - U_4$, the value of the energy U_1 is known from the initial conditions. (In the case of the Diesel cycle the sum $U_1 + i_1'$ is known.) The evaluation of U_4 requires a knowledge of the temperature T_4 and the composition of the gas in state 4. The calculation of T_4 and the composition at state 4 involves the following steps:

(1) The initial mixture is determined, taking account of the effect of the exhaust gas remaining in the clearance space.

(2) Temperature T_2 and pressure p_2 at the end of adiabatic compression are determined.

(3) Taking into consideration the combustion process 2-3, the maximum temperature T_3 and the corresponding pressure p_3 are determined. The effect of dissociation must be taken into account. In the case of the Diesel cycle, the relation between the volumes V_2 and V_3 must receive attention.

(4) Temperature T_4 is determined for the adiabatic expansion 3-4; at the same time the composition at the state 4 is found.

(5) Having the conditions at point 4, the energy U_4 is found, and from this, the work $A(W)$ and the efficiency of the cycle.

These five steps will be considered in detail in the following sections.

15. *Initial Mixture.*—In the operation of the actual engine some of the exhaust gas remains in the clearance space and mixes with the incoming fresh charge. The temperature of the exhaust gas that passes from the cylinder is T_5 (Fig. 2) and is lower than the temperature T_4 at release. However, this drop of temperature does not reach to the gas remaining in the clearance; while the temperature of the trapped gas may be a little lower than T_4 it is probably nearer T_4 than T_5 .

In the estimation of the initial mixture it is assumed, therefore, that the temperature of the gas in the clearance space is T_4 and that the pressure is atmospheric. This gas mixes with the charge which is at temperature T_a and atmospheric pressure; and the resulting mixture is required.

Let V_c = clearance volume

V = total cylinder volume including clearance

$\frac{V}{V_c} = r$, the ratio of the volumes

n_1 = number of mols of entering charge of fuel and air

n_c = number of mols of gas in clearance space

T_1 = final temperature after mixing

The gas equation applied to the gas in the clearance space is

$$p_a V_c = n_c R T_4$$

Applied to the final mixture, it is

$$p_a V = (n_1 + n_c) R T_1$$

Therefore,

$$\frac{V}{V_c} = r = \frac{n_1 + n_c}{n_c} \frac{T_1}{T_4}$$

or

$$\frac{n_1}{n_c} = r \frac{T_4}{T_1} - 1 \quad (12)$$

Since the mixing process is at constant pressure, the thermal potential i is constant, and the equation that represents the interchange of heat is

$$n_1 (T_1 - T_a) \gamma_p' = n_c (T_4 - T_1) \gamma_p'' \quad (13)$$

in which γ_p' denotes the mean specific heat of the fresh charge, and γ_p'' the mean specific heat of the exhaust gas. Taking $\gamma_p''/\gamma_p' = \beta$, the equation becomes

$$\frac{n_1}{n_c} = \beta \frac{T_4 - T_1}{T_1 - T_a} \quad (14)$$

From the two equations (12) and (14) the unknowns n_c and T_1 may be determined.

At the beginning of the calculation the temperature T_4 is unknown. A probable value is assumed and values of n_c and T_1 are obtained. If the value of T_4 ultimately found differs considerably from the assumed value, the calculation may be repeated.

16. *Adiabatic Compression.*—The initial mixture consists of $n_i = n_1 + n_c$ mols at temperature T_1 and volume V . This is compressed adiabatically, as shown by curve 1-2, Fig. 1, to a volume $V_2 (= V_c)$. The temperature T_2 and pressure p_2 at the end of compression are required.

The basic equations for the process are the following:

$$\begin{aligned} 0 &= dQ = dU + A p dV \\ dU &= \gamma_v dT = (a' + bT + cT^2) dT \\ pV &= n_i R T \end{aligned}$$

The elimination of p between these equations gives at once the equation

$$\left(\frac{a'}{T} + b + cT\right) dT + n_i AR \frac{dV}{V} = 0 \tag{15}$$

and integration between limits T_1, V_1 and T_2, V_2 gives the desired relation

$$\begin{aligned} a' \log_e \frac{T_2}{T_1} + b(T_2 - T_1) + \frac{1}{2}c(T_2^2 - T_1^2) &= n_i AR \log_e \frac{V_1}{V_2} \\ &= n_i AR \log_e r \end{aligned} \tag{16}$$

Let

$$a' \log_e T + bT + \frac{1}{2}cT^2 = \phi$$

then equation (16) may be written

$$\Phi_2 - \Phi_1 = 4.571 n_i \log_{10} r \tag{17}$$

(Since $2.3026 AR = 2.3026 \times 1.985 = 4.571$)

Tables of the values of the function ϕ for various temperatures have been calculated. Each constituent, as H_2, CO, H_2O , etc., has its series of values. Hence, at the known temperature T_1 the value of Φ of the initial mixture is

$$\Phi_1 = n_1 \phi_1 + n_2 \phi_2 + \dots \tag{18}$$

Several probable values of the temperature T_2 are assumed and the corresponding values of Φ_2 are calculated. The value of T_2 for which equation (17) is satisfied is the temperature at the end of compression.

The pressure p_2 at the end of compression is deduced from the two equations

$$\begin{aligned} p_1 V_1 &= n_i R T_1 \\ p_2 V_2 &= n_i R T_2 \end{aligned}$$

whence

$$p_2 = p_1 \frac{T_2 V_1}{T_1 V_2} = p_1 r \frac{T_2}{T_1} \tag{19}$$

17. *Combustion Process (Otto Cycle).*—The line 2-3, Fig. 1, represents the combustion of the fuel at constant volume. In the ideal case the process is also adiabatic, that is, no heat is transmitted from the medium during the process of combustion. It is assumed that when state 3 is reached the gas mixture is in equilibrium. The subscript i was used to indicate the initial condition, that is, at point 2; similarly, the subscript e will indicate the equilibrium state, and the subscript p a hypothetical state of complete combustion.

The initial mixture being known, the mixture of the products of complete combustion is readily found. The composition of this mixture may be as follows:

$$\left. \begin{array}{l} \text{CO}_2 = n_1 \text{ mols} \\ \text{H}_2\text{O} = n_2 \text{ mols} \\ \text{O}_2 = n_p' \text{ mols} \\ \text{N}_2 = n'' \text{ mols} \\ \hline \text{Total} \qquad \qquad n_p \text{ mols} \end{array} \right\} \quad (\text{a})$$

It is assumed that at least sufficient oxygen for complete combustion is supplied. Then in the preceding composition n_p' denotes the excess oxygen. The mixture at equilibrium will contain unburned CO and H₂. Denoting by x and y , respectively, the parts of CO and H₂ consumed, the equilibrium mixture has the composition

$$\left. \begin{array}{l} \text{CO}_2 = n_1x \\ \text{CO} = n_1(1-x) \\ \text{H}_2\text{O} = n_2y \\ \text{H}_2 = n_2(1-y) \\ \text{O}_2 = n_e' \\ \text{N}_2 = n'' \\ \hline \text{Total} \qquad \qquad n_e \end{array} \right\} \quad (\text{b})$$

It is convenient to assume the following hypothetical processes:

(1) The fuel mixture is completely burned, giving the final composition (a).

(2) Then the part $1-x$ of the CO₂ and the part $1-y$ of the H₂O are dissociated. The result is the equilibrium mixture (b).

The following relations are readily found:

$$\left. \begin{array}{l} n_e = n_p + \frac{1}{2}n_1(1-x) + \frac{1}{2}n_2(1-y) = n_s - \frac{1}{2}n_1x - \frac{1}{2}n_2y \\ n_e' = n_p' + \frac{1}{2}n_1(1-x) + \frac{1}{2}n_2(1-y) = n_s' - \frac{1}{2}n_1x - \frac{1}{2}n_2y \end{array} \right\} \quad (20)$$

where

$$\left. \begin{array}{l} n_s = n_p + \frac{1}{2}n_1 + \frac{1}{2}n_2 \\ n_s' = n_p' + \frac{1}{2}n_1 + \frac{1}{2}n_2 \end{array} \right\}$$

The state of the mixture in equilibrium at the maximum temperature (the state represented by point 3, Fig. 1) is specified by three variables: x , y , and T_e , the temperature at equilibrium. To determine these three unknowns, three equations are required. A full discussion of these equations and of the methods of using them is given in Bulletin No. 139, Chapter IV. The following is a brief summary:

The first of the required equations is deduced from the energy relations involved. Conceive that the initial mixture is completely burned at the initial temperature (T_2 in this case) and that the heat thus developed is used (1) to dissociate the part $(1-x)$ of CO_2 and the part $(1-y)$ of H_2O , and (2) to raise the temperature of the resulting mixture having the composition (b) from the temperature T_2 to the equilibrium temperature T_e .

Let H_m denote the heat of combustion of the fuel mixture, H_{CO} and H_{H_2} the heats of combustion of CO and H_2 , respectively, all three at constant volume and at temperature T_2 . Also let U' denote the energy of the equilibrium mixture (b) at temperature T_2 and U'' the energy of the same mixture at the equilibrium temperature T_e . Then the energy equation is

$$H_m = n_1(1-x)H_{\text{CO}} + n_2(1-y)H_{\text{H}_2} + U'' - U' \quad (21)$$

The energy difference $U'' - U'$ may be expressed as follows: Let Δu denote the increase of energy of 1 mol of a constituent when the temperature increases from T_2 to T_e . Then taking the constituents from composition (b)

$$U'' - U' = n_1x\Delta u_{\text{CO}_2} + n_1(1-x)\Delta u_{\text{CO}} + n_2y\Delta u_{\text{H}_2\text{O}} + n_2(1-y)\Delta u_{\text{H}_2} + n'_e\Delta u_{\text{O}_2} + n''\Delta u_{\text{N}_2} \quad (22)$$

The three diatomic gases O_2 , N_2 , and CO , having the same energy per mol, may be included in one term; thus

$$[n_1(1-x) + n'_e + n'']\Delta u_D$$

It is evident that x and y appear in the first degree only, and the equation (21) may ultimately be reduced to the form

$$y = b - ax \quad (23)$$

The expressions for the constants a and b are the following:

$$a = \frac{\frac{3}{2}n_1\Delta u_D - n_1\Delta u_{\text{CO}_2} + n_1H_{\text{CO}}}{n_2H_{\text{H}_2} + n_2\Delta u_{\text{H}_2} - n_2\Delta u_{\text{H}_2\text{O}} + \frac{1}{2}n_2\Delta u_D} \quad (24)$$

$$b = \frac{(n'_s + n_1 + n'') \Delta u_D + n_2 \Delta u_{H_2} + n_1 H_{CO} + n_2 H_{H_2} - H_m}{n_2 H_{H_2} + n_2 \Delta u_{H_2} - n_2 \Delta u_{H_2O} + \frac{1}{2} n_2 \Delta u_D} \quad (25)$$

The values of the constants a and b will, of course, vary with the assumed value of the temperature T_e .

The condition of equilibrium at temperature T_e gives the two equations

$$K_{p(\text{CO})} = \frac{p_{\text{CO}_2}}{p_{\text{CO}} p_{\text{O}_2}^{1/2}} = \frac{x}{1-x} \sqrt{\frac{n_e}{n'_e P}} \quad (26)$$

$$K_{p(\text{H}_2)} = \frac{p_{\text{H}_2\text{O}}}{p_{\text{H}_2} p_{\text{O}_2}^{1/2}} = \frac{y}{1-y} \sqrt{\frac{n_e}{n'_e P}} \quad (27)$$

in which P denotes the pressure of the mixture. The unknown pressure is eliminated through the equations

$$\begin{aligned} PV &= n_e RT_e \\ P_2 V &= n_i RT_2 \end{aligned}$$

from which

$$\frac{n_e}{P} = \frac{n_i T_2}{P_2 T_e} \quad (28)$$

Equation (26) thus becomes

$$K_{p(\text{CO})} = \frac{x}{1-x} \sqrt{\frac{n_i T_2}{n'_e P_2}} \frac{1}{\sqrt{T_e}}$$

or more conveniently

$$\log K_{p(\text{CO})} + \frac{1}{2} \log T_e = \log x - \log (1-x) + \frac{1}{2} \log \frac{n_i T_2}{P_2} - \frac{1}{2} \log n'_e \quad (29)$$

A combination of equations (26) and (27) gives the equilibrium constant for the water-gas reaction; thus

$$K_{p(\text{w.g.})} = \frac{y(1-x)}{x(1-y)} = c \quad (30)$$

The elimination of y between equations (23) and (30) gives the quadratic

$$a(c-1)x^2 + [a+b-c(b-1)]x - b = 0$$

and the solution of this is

$$x = \frac{c(b-1) - (a+b) \pm \sqrt{[c(b-1) - (a+b)]^2 + 4ab(c-1)}}{2a(c-1)} \quad (31)$$

The calculation proceeds along the following lines: Several probable values of $T_e (= T_3)$ are chosen, as 4500 deg., 4600 deg., 4700 deg. For each of these temperatures the constants a and b are calculated from equations (24) and (25), and for each the constant c is known. Then from equations (31) and (23) values of x and y for these assumed temperatures are calculated. In this way simultaneous values of T_e , x , y that will satisfy equations (23) and (30) are obtained. There remains the satisfaction of equation (29). The simultaneous values of x and y are substituted in the second member of (29) and thus give the values of a function $R(x, y)$. The corresponding values of T_e are substituted in the first member of (29) thus giving the values of a function $L(T)$. The curves representing these two functions are plotted, and the intersection gives the required value of T_e and the corresponding values of x and y .

The details of the calculation are shown in the illustrative example, p 32.

18. *Combustion Process (Diesel Cycle).*—In the Diesel cycle the combustion is at constant pressure. The procedure outlined in the preceding article will be modified slightly as follows:

In the energy equation (21) H_m , H_{CO} , H_{H_2} will be taken as the heats of combustion at constant pressure and at temperature T_2 ; and the difference $U'' - U'$ will be replaced by $I'' - I'$. Also in equation (22) the Δu 's will be replaced by Δi 's. Since the pressure is constant, equations (26) and (27) are applicable as they stand. Equation (29) may be written in the form

$$\log K_{p(CO)} = \log x - \log(1 - x) + \frac{1}{2} \log n_e - \frac{1}{2} \log n'_e - \frac{1}{2} \log P \quad (32)$$

Save for the preceding changes the procedure is exactly the same as for the Otto cycle.

In the case of the Otto cycle volumes V_3 and V_2 are equal; but in the case of the Diesel cycle volume V_3 is different from V_2 and must be known in order that the adiabatic process 3-4 may be calculated. From the gas equation

$$\begin{aligned} pV_2 &= n_2RT_2 \\ pV_3 &= n_3RT_3 \end{aligned}$$

Hence

$$V_3 = V_2 \frac{n_3 T_3}{n_2 T_2} \quad (33)$$

In these equations n_2 is the number of mols of initial air and n_3 the number of mols in state 3, that is, n_e in composition (b).

19. *Adiabatic Expansion.*—During the adiabatic expansion from the equilibrium temperature and pressure to the final temperature and pressure at the end of the stroke, it is assumed that the mixture is continually in equilibrium. On this assumption the reaction is reversible and further combustion of H_2 and CO does not give an increase of entropy. Under these conditions the entropy of the mixture remains constant throughout the expansion.

The deduction of the equation of the adiabatic involves tedious algebraic manipulation, but some labor may be saved by a careful arrangement of the notation employed. In the first place consider two compositions: (a) the composition of products on the assumption that combustion is complete; (b) the composition of the equilibrium mixture, which is obtained from the first mixture by dissociation of some of the CO_2 and H_2O .

(a)	(b)
Products of Complete Combustion	Mixture at Equilibrium
CO_2 n_1	CO_2 n_1x
H_2O n_2	CO $n_1(1 - x)$
O_2 n'_p	H_2O n_2y
N_2 n''	H_2 $n_2(1 - y)$
n_p	O_2 $n'_p + \frac{1}{2}n_1(1 - x) +$ $\frac{1}{2}n_2(1 - y) = n'_e$
	N_2 n''
	n_e

As in equation (20)

$$n'_s = n'_p + \frac{1}{2}n_1 + \frac{1}{2}n_2; n_s = n_p + \frac{1}{2}n_1 + \frac{1}{2}n_2$$

then

$$n'_e = n'_s - \frac{1}{2}n_1x - \frac{1}{2}n_2y; n_e = n_s - \frac{1}{2}n_1x - \frac{1}{2}n_2y$$

Now taking the expression $h - AR \log_e p$ for the entropy of 1 mol of a gas (see p. 9) the entropy of the equilibrium mixture may be determined as the sum of the entropies of the constituents. The total entropy S may be taken as the sum of two parts S' and S'' , of which the first is contributed by the terms involving the h 's and the second by the terms of the form $AR \log_e p$. These parts are evaluated separately.

$$\begin{aligned} S' &= n_1xh_{CO_2} + n_1(1-x)h_{CO} + n_2yh_{H_2O} + n_2(1-y)h_{H_2} \\ &+ (n'_s - \frac{1}{2}n_1x - \frac{1}{2}n_2y)h_{O_2} + n''h_{N_2} \\ &= n_1h_{CO} + n_2h_{H_2} + n'_sh_{O_2} + n''h_{N_2} \\ &- n_1x(h_{CO} + \frac{1}{2}h_{O_2} - h_{CO_2}) - n_2y(h_{H_2} + \frac{1}{2}h_{O_2} - h_{H_2O}) \end{aligned}$$

By an obvious transformation

$$S' = n_1(h_{\text{CO}} + \frac{1}{2}h_{\text{O}_2} - h_{\text{CO}_2}) + n_2(h_{\text{H}_2} + \frac{1}{2}h_{\text{O}_2} - h_{\text{H}_2\text{O}}) + n_1h_{\text{CO}_2} \\ + n_2h_{\text{H}_2\text{O}} + (n'_s - \frac{1}{2}n_1 - \frac{1}{2}n_2)h_{\text{O}_2} + n''h_{\text{N}_2} - n_1x(h_{\text{CO}} + \frac{1}{2}h_{\text{O}_2} \\ - h_{\text{CO}_2}) - n_2y(h_{\text{H}_2} + \frac{1}{2}h_{\text{O}_2} - h_{\text{H}_2\text{O}})$$

But

$$h_{\text{CO}} + \frac{1}{2}h_{\text{O}_2} - h_{\text{CO}_2} = \lambda_{\text{CO}}; h_{\text{H}_2} + \frac{1}{2}h_{\text{O}_2} - h_{\text{H}_2\text{O}} = \lambda_{\text{H}_2}$$

Therefore, finally

$$S' = n_1h_{\text{CO}_2} + n_2h_{\text{H}_2\text{O}} + n'_ph_{\text{O}_2} + n''h_{\text{N}_2} + n_1(1-x)\lambda_{\text{CO}} + n_2(1-y)\lambda_{\text{H}_2} \quad (34)$$

It should be observed that the first four terms of this expression involve precisely the factors in the composition (1) resulting from complete combustion.

For the part S'' the following expression is deduced from mixture (2).

$$S'' = -AR[n_1x \log_e p_{\text{CO}_2} + n_1(1-x) \log_e p_{\text{CO}} + n_2y \log_e p_{\text{H}_2\text{O}} \\ + n_2(1-y) \log_e p_{\text{H}_2} + (n'_s - \frac{1}{2}n_1x - \frac{1}{2}n_2y) \log_e p_{\text{O}_2} \\ + n'' \log_e p_{\text{N}_2}] \quad (35)$$

The partial pressures p_{CO} and p_{H_2} are eliminated by means of the relations

$$K_{p(\text{CO})} = \frac{p_{\text{CO}_2}}{p_{\text{CO}}p_{\text{O}_2}^{1/2}}, \quad K_{p(\text{H}_2)} = \frac{p_{\text{H}_2\text{O}}}{p_{\text{H}_2}p_{\text{O}_2}^{1/2}}$$

from which

$$\left. \begin{aligned} \log_e p_{\text{CO}} &= \log_e p_{\text{CO}_2} - \frac{1}{2} \log_e p_{\text{O}_2} - \log_e K_{p(\text{CO})} \\ \log_e p_{\text{H}_2} &= \log_e p_{\text{H}_2\text{O}} - \frac{1}{2} \log_e p_{\text{O}_2} - \log_e K_{p(\text{H}_2)} \end{aligned} \right\} (36)$$

The substitution of these expressions in equation (35) gives the equation

$$S'' = -AR[n_1 \log_e p_{\text{CO}_2} + n_2 \log_e p_{\text{H}_2\text{O}} + n'_p \log_e p_{\text{O}_2} + n'' \log_e p_{\text{N}_2} \\ - n_1(1-x) \log_e K_{p(\text{CO})} - n_2(1-y) \log_e K_{p(\text{H}_2)}] \quad (37)$$

The four partial pressures involved are given by mixture (2); thus

$$p_{\text{CO}_2} = \frac{n_1x}{n_e} P, \quad p_{\text{H}_2\text{O}} = \frac{n_2y}{n_e} P, \quad \text{etc.}$$

in which P is the varying pressure of the mixture and n_e the varying number of mols during the adiabatic expansion. From the gas equation

$$PV = n_e RT$$

or

$$\frac{P}{n_e} = \frac{RT}{V}$$

Hence

$$\begin{aligned} & n_1 \log_e p_{\text{CO}_2} + n_2 \log_e p_{\text{H}_2\text{O}} + n'_p \log_e p_{\text{O}_2} + n'' \log_e p_{\text{N}_2} \\ &= n_p \log_e \frac{RT}{V} + n_1 \log_e x + n_2 \log_e y + n'_p \log_e n'_e + n_1 \log_e n_1 \\ &+ n_2 \log_e n_2 + n'' \log_e n'' \\ &= n_p \log_e \frac{T}{V} + n_1 \log_e x + n_2 \log_e y + n'_p \log_e n'_e + C \end{aligned} \quad (38)$$

Combining the results given by equations (34), (37), and (38), the following expression is obtained for the entropy of the mixture:

$$\begin{aligned} S &= n_1 h_{\text{CO}_2} + n_2 h_{\text{H}_2\text{O}} + n'_p h_{\text{O}_2} + n'' h_{\text{N}_2} - AR n_p \log_e T \\ &+ n_1 (1-x) (\lambda_{\text{CO}} + AR \log_e K_{p(\text{CO})}) + n_2 (1-y) (\lambda_{\text{H}_2} + AR \log_e K_{p(\text{H}_2)}) \\ &+ AR n_p \log_e V - AR [n_1 \log_e x + n_2 \log_e y + n'_p \log_e n'_e] + C' \end{aligned} \quad (39)$$

The first line of the second member may be reduced as follows: The function h (p. 9) is defined by the equation

$$h = \int \frac{\gamma_p dT}{T} + s_0$$

and with

$$\begin{aligned} \gamma_p &= a + bT + cT^2 \\ h &= a \log_e T + bT + \frac{1}{2}cT^2 + \bar{s}_0 \end{aligned} \quad (40)$$

A second function ϕ is defined as follows (p. 19):

$$\phi = a' \log_e T + bT + \frac{1}{2}cT^2$$

Hence

$$\begin{aligned} h &= \phi + (a - a') \log_e T + s_0 \\ &= \phi + AR \log_e T + s_0 \end{aligned} \quad (41)$$

Therefore, the first line of equation (39) reduces to

$$n_1 \phi_{\text{CO}_2} + n_2 \phi_{\text{H}_2\text{O}} + n'_p \phi_{\text{O}_2} + n'' \phi_{\text{N}_2} + \Sigma n s_0$$

and $\Sigma n s_0$ is a constant that may be merged in the constant C' .

A further reduction of equation (39) is possible. Since the mixture is in equilibrium during the adiabatic expansion, the conditions of equilibrium (Bulletin No. 139, p. 30) impose the relations

$$\lambda_{\text{CO}} + AR \log_e K_{p(\text{CO})} = \frac{H_{p(\text{CO})}}{T} \quad (42)$$

$$\lambda_{\text{H}_2} + AR \log_e K_{p(\text{H}_2)} = \frac{H_{p(\text{H}_2)}}{T} \quad (43)$$

Making the changes in equation (39) the final equation for the entropy of the mixture is

$$\begin{aligned} S = & n_1 \phi_{\text{CO}_2} + n_2 \phi_{\text{H}_2\text{O}} + n_p' \phi_{\text{O}_2} + n'' \phi_{\text{N}_2} + n_1 (1-x) \frac{H_{p(\text{CO})}}{T} \\ & + n_2 (1-y) \frac{H_{p(\text{H}_2)}}{T} - AR [n_1 \log_e x + n_2 \log_e y + n_p' \log_e n_p' \\ & - n_p \log_e V] + C'' \end{aligned} \quad (44)$$

This equation enables one to calculate the temperature of the mixture of products at the end of the adiabatic expansion. Taking T_3 as the temperature at the point of maximum temperature and T_4 as the temperature at the end of expansion, the procedure is as follows:

The conditions at point 3, namely, T_3 , x_3 , y_3 have been determined by the methods of the preceding section. For the temperature T_3 the heats of combustion $H_{p(\text{CO})}$ and $H_{p(\text{H}_2)}$ are known. Consequently, the value of S from equation (44) can be calculated. The constant C'' is, of course, omitted since it is the same at both points.

Now several probable values of T_4 are assumed. The values of x_4 , y_4 are unknown, but y_4 is very nearly equal to 1. For each assumed T_4 values of x_4 and y_4 are so determined that the equilibrium conditions are satisfied. Then for each T_4 all the elements required for the calculation of S from equation (44) are present. The value of T_4 for which $S_3 = S_4$ is the value sought. This may be obtained by interpolation or by the intersection of curves.

The details of the calculation are shown in the illustrative problem, p. 33.

20. *Expression for Work.*—From the general equation (1) of Section 11 the work of the Otto cycle is the difference of energy $U_1 - U_4$. But the energy U_4 is partly thermal energy and partly chemical energy of the parts $(1 - x_4)$ of CO and $(1 - y_4)$ of H_2 still unburned. Furthermore, if part of the constituents are still unburned the number of mols of the products in state 4 is different from the number n_p for complete combustion.

Let the symbol U with a system of double subscripts denote the energy of a mixture for various assumed states. Thus

U_{i_1} = energy of initial mixture at temperature T_1

U_{p_1} = energy of mixture of products of *complete* combustion at temperature T_1

U_{p_4} = energy of mixture of products of *complete* combustion at temperature T_4

U_4 = energy of actual mixture at state 4

Now

$$A(W) = U_{i_1} - U_4 = (U_{i_1} - U_{p_1}) - (U_4 - U_{p_4}) - (U_{p_4} - U_{p_1}) \quad (45)$$

The first quantity in parenthesis is the heat of combustion H_v at constant volume and at the temperature T_1 . The second is the difference between the energy of the products in the state 4 and the energy of the products at the same temperature T_4 if the combustion were complete; it is therefore the chemical energy of the unburned CO and H_2 . Hence

$$U_4 - U_{p_4} = n_1(1-x)H_{v(\text{CO})} + n_2(1-y)H_{v(\text{H}_2)} \quad (46)$$

these heats of combustion being taken at the temperature T_4 . The third quantity in parenthesis is the heat required to raise the temperature of the products of complete combustion from temperature T_1 to temperature T_4 . Referring to the mixture (a) of products of complete combustion

$$U_{p_4} - U_{p_1} = n_1(u_4 - u_1)_{\text{CO}_2} + n_2(u_4 - u_1)_{\text{H}_2\text{O}} + (n'_p + n'')(u_4 - u_1)_D$$

or

$$U_{p_4} - U_{p_1} = n_1\Delta u_{\text{CO}_2} + n_2\Delta u_{\text{H}_2\text{O}} + (n'_p + n'')\Delta u_D \quad (47)$$

Hence, finally

$$A(W) = H_{vT_1} - n_1(1-x)H_{v(\text{CO})} - n_2(1-y)H_{v(\text{H}_2)} - [n_1\Delta u_{\text{CO}_2} + n_2\Delta u_{\text{H}_2\text{O}} + (n'_p + n'')\Delta u_D] \quad (48)$$

The same expression applies to the Diesel cycle.

21. *Mean Effective Pressure.*—Having the work $A(W)$ in B.t.u., the mean effective pressure of the ideal cycle is given by the equation

$$\text{m. e. p.} = \frac{778 A(W)}{144 (V_4 - V_2)} \quad (49)$$

22. *Otto Cycle with Insufficient Air.*—When the air supplied is insufficient for the complete combustion of the fuel certain modifications in the system of computation must be made.

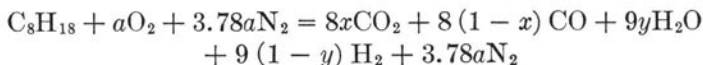
With sufficient air the products at the end of adiabatic expansion contain relatively little CO and H_2 ; therefore, the residual gas that is

mixed with the incoming charge may be considered as composed of CO_2 , H_2O , N_2 , and excess O_2 . If the air supply is insufficient the residual gas must contain in addition some CO and H_2 but no excess O_2 . In order to get approximately the composition of the residual gas the following procedure is adopted:

The gas is assumed to have the composition

$$\begin{aligned}\text{CO}_2 &= n_1x \\ \text{CO} &= n_1(1-x) \\ \text{H}_2\text{O} &= n_2y \\ \text{H}_2 &= n_2(1-y) \\ \text{N}_2 &= n''\end{aligned}$$

From the reaction equation may be found the values of n_1 , n_2 , n'' , and one relation between x and y . As an example, take the combustion of C_8H_{18} with a mols of O_2 , where $a < 12.5$. The reaction equation is



Comparing the number of atoms of oxygen on the two sides of the equation,

$$\begin{aligned}2a &= 16x + 8(1-x) + 9y \\ \text{or} \quad a &= 4(1+x) + 4.5y\end{aligned}\tag{50}$$

To get a second equation between x and y , we make use of the fact that the gas is in equilibrium during the expansion and therefore in the state 4 at the end of expansion. Equation (30) for the water-gas equilibrium is therefore valid. A probable temperature T_4 is assumed and the constant c in equation (30) is thus fixed. The solution of the two equations gives values of x and y , and the composition of the residual gas is thus determined.

Having the residual gas, the procedure is identical with that already developed until the adiabatic expansion is reached. Here an obstacle is encountered in the fact that n'_e , the oxygen in the expanding mixture, is vanishingly small, and equation (44) for the entropy of the mixture contains the term $n'_p \log_e n'_e$. However, the difficulty is readily overcome by eliminating n'_e from the equation by means of the relation

$$K_{p(\text{CO})} = \frac{x}{1-x} \sqrt{\frac{n_e}{n'_e P}}$$

23. *Sample Computation.*—The procedure of computing the efficiency and mean effective pressure of an ideal Otto cycle is shown in Table I. The computation is for case No. 9, for which the following are the data:

TABLE I
COMPUTATION FORM FOR OTTO CYCLE

Otto Cycle		Expansion to Suction Volume		
1 Compression Ratio = $\frac{V_1}{V_2} = 5$		Heat Losses : None		
Fuel : C_8H_{18} (Gasoline) with 100% theoretical air		2 Suction Pressure = Exhaust Pressure = 14.7 lb. per sq. in.		
$C_8H_{18} + 12.5O_2 + 47.25N_2 = 8CO_2 + 9H_2O + 47.25N_2$				
Fuel Mixture, Mols (with no residual gas)		Mixture of Products, Mols (with no residual gas)		
3 $C_8H_{18} = 1.00$		7 $CO_2 = 8.00$		
4 $O_2 = 12.50$		8 $H_2O = 9.00$		
5 $N_2 = 47.25$		9 $N_2 = 47.25$		
6 $n_1 = 60.75$		10 $n_2 = 64.25$		
Molar Specific Heat Equations:				
11 $\gamma_p(C_8H_{18}) = 38.327 + 38.00 \times 10^{-3}T$				
12 $\gamma_p(O_2, N_2, CO) = 6.93 + 0.0 \times 10^{-3}T + 0.120 \times 10^{-6}T^2$				
13 $\gamma_p(CO_2) = 7.15 + 3.9 \times 10^{-3}T - 0.60 \times 10^{-6}T^2$				
14 $\gamma_p(H_2O) = 8.33 - 0.276 \times 10^{-3}T + 0.423 \times 10^{-6}T^2$				
15 $[3] \times [11] = 38.327 + 38.00 \times 10^{-3}T$				
16 $([4] + [5]) \times [12] = 414.0675 + 0.0 \times 10^{-3}T + 7.17 \times 10^{-6}T^2$				
17 $\gamma_p(\text{fuel mixture}) = \frac{[15] + [16]}{[6]} = 7.4468 + 0.6255 \times 10^{-3}T + 0.1180 \times 10^{-6}T^2$				
18 $[7] \times [13] = 57.2 + 31.2 \times 10^{-3}T - 4.8 \times 10^{-6}T^2$				
19 $[8] \times [14] = 74.97 - 2.484 \times 10^{-3}T + 3.807 \times 10^{-6}T^2$				
20 $[9] \times [12] = 327.4425 + 0.0 \times 10^{-3}T + 5.670 \times 10^{-6}T^2$				
21 $\gamma_p(\text{products}) = \frac{[18] + [19] + [20]}{[10]} = 7.1535 + 0.4469 \times 10^{-3}T + 0.0728 \times 10^{-6}T^2$				
Assume $T_4 = 3740$ and $T_1 = 600$				
Mean temperature of fuel mixture = 560°F. abs.				
Mean temperature of products = 2170°F. abs.				
22 $\gamma_p(\text{fuel}) = [17]$ for $560^\circ = 7.8341$				
23 $\gamma_p(\text{products}) = [21]$ for $2170^\circ = 8.4661$				
24 $\beta = \frac{[23]}{[22]} = 1.0807$				
Computation of Initial Conditions				
Let $T_1 =$	630	640	650	From Fig. 4:
25 $\frac{rT_2}{T_1} = \frac{5 \times 3740}{T_1}$	29.68	29.22	28.77	$T_1 = 638^\circ\text{F.}$
26 $[25] - 1, (\text{Eq. 12})$	28.68	28.22	27.77	$p_1 = 14.7 \text{ lb. per sq. in.}$
27 $T_2 - T_1 = 3740 - T_1$	3110	3100	3090	$\frac{n_1}{n_2} = 28.28$
28 $T_1 - T_2 = T_1 - 520$	110	120	130	$n_c = \frac{[6]}{28.28} = 2.148 \text{ mols}$
29 $\frac{[27] \times [24]}{[28]}$, (Eq. 14)	30.55	27.92	25.69	residual gas
Composition of Residual Gas, Mols		Charge at End of Suction, Mols		
$CO_2 = 8 \times \frac{2.148}{64.250} = 0.267$		30 $C_8H_{18} = 1.000$		
$H_2O = 9 \times \frac{2.148}{64.250} = 0.301$		31 $CO_2 = 0.267$		
$N_2 = 47.25 \times \frac{2.148}{64.250} = 1.580$		32 $H_2O = 0.301$		
2.148		33 $N_2 = 48.830$		
		34 $O_2 = 12.50$		
		35 $n_1 = 62.898$		

TABLE I (Continued)
COMPUTATION FORM FOR OTTO CYCLE

<i>The Adiabatic Compression</i>				
Let $T_2 =$	1070	1080	1090	$T_1 = 638$
	34.562	34.610	34.657	31.961
	39.858	39.938	40.018	35.724
	44.206	44.267	44.327	40.888
36	294.16	294.88	295.60	258.95
37	2119.68	2122.61	2125.49	1960.15
38	10.64	10.66	10.69	9.54
39	13.31	13.32	13.34	12.31
	2437.79	2441.47	2445.12	2240.95
	196.84	200.52	204.17	
<i>(Eq. 17) $4.571 \times [35] \times \log_{10} 5 = 200.96$, which corresponds to 1081.2°</i>				
40	Take $T_2 = 1081^\circ \text{F.}$			
41	$p_2 = \frac{[11] \times [2] \times [40]}{T_1} = 124.5 \text{ lb. per sq. in.}$			
42	$= 8.47 \text{ atmospheres}$			
43	$V_2 = \frac{n_1 R T_1}{p_1} = \frac{[35] \times 1544 \times [40]}{[41] \times 144} = 5854 \text{ cu. ft.}$			
<i>The Combustion Process</i>				
<i>Equilibrium Mixture:</i>				
	$\text{CO}_2 = 8.267x$	44	$n_1 = 8.267$	
	$\text{CO} = 8.267(1-x)$	45	$n_2 = 9.301$	
	$\text{H}_2\text{O} = 9.301y$	46	$n'' = 48.830$	
	$\text{H}_2 = 9.301(1-y)$	47	$n'_o = 8.784$	
	$\text{N}_2 = 48.830$	48	$n_1 + n'_o + n'' = 65.881$	
	$\text{O}_2 = 8.784 - 4.134x - 4.65y$	49	$n_p = 66.398$	
43a	$n_e = 75.182 - 4.134x - 4.65y$			
<i>For $T_2 = 1081$, $H_{u_o} = 121410$, $H_{u_p} = 103370$, $H_m = 2151700$</i>				
<i>$n_1 H_{c_o} = 1003695$ $n_2 H_{h_2} = 961445$</i>				
<i>$n_1 H_{c_o} + n_2 H_{h_2} - H_m = -186560$</i>				
T_e	5040	5050	5060	
50	24 648	24 728	24 808	
51	23 974	24 048	24 122	
52	44 310	44 433	44 557	
53	39 647	39 804	39 962	
54	1 623 830	1 629 105	1 634 380	
55	114 626	114 998	115 370	
56	305 637	306 629	307 621	
57	222 982	223 670	224 359	
58	366 311	367 328	368 353	
59	368 757	370 217	371 687	
60	943 020	942 995	942 962	
61	660 250	1 666 215	1 672 180	
62	930 295	929 895	929 486	
	$\alpha = \frac{[60]}{[62]}$, (Equation 24)	1.0138	1.0142	1.0146
	$b = \frac{[61]}{[62]}$, (Equation 25)	1.7847	1.7919	1.7991

TABLE 1 (Continued)
 COMPUTATION FORM FOR OTTO CYCLE

T_e	5040	5050	5060	
$c = K_p$, (water gas)	7.8099	7.8317	7.8536	
$c(b-1)$	6.1284	6.2019	6.2758	
63 $c(b-1)-(a+b)$	3.3299	3.3958	3.4621	
64 $[63]^2$	11.0882	11.5314	11.9861	
65 $2a(c-1)$	13.8077	13.8574	13.9073	
66 $2bx2a(c-1)$	49.2852	49.6624	50.0412	
67 $[6a]+[66]$	60.3734	61.1935	62.0273	
68 $\sqrt{[67]}$	7.7700	7.8226	7.8756	
69 $[63]+[68]$	11.0999	11.2184	11.3377	
70 $x = \frac{[69]}{[65]}$	0.8039	0.8096	0.8152	
$1-x$	0.1961	0.1904	0.1848	
ax	0.8150	0.8211	0.8271	
$y = b-ax$	0.9697	0.9708	0.9720	
r'_s	8.7842	8.7842	8.7842	
$\frac{r'_s}{2}x$	3.3231	3.3466	3.3698	
$\frac{r'_s}{2}y$	4.5095	4.5146	4.5202	
$r'_e = r'_s - \frac{r'_s}{2}x - \frac{r'_s}{2}y$	0.9516	0.9230	0.8941	
$\frac{1}{2} \log r'_e$	7.98923	7.98260	7.97572	
$\log x$	7.90520	7.90827	7.91126	
$\log(1-x)$	7.29248	7.2967	7.26670	
$\frac{1}{2} \log \frac{r'_s T_s}{P_s} = \log \frac{[35]x[40]}{[42]}$	1.95224	1.95224	1.95224	
71 $R(xy) = \log x - \log(1-x) - \frac{1}{2} \log r'_e + \frac{1}{2} \log \frac{r'_s T_s}{P_s}$	2.57573	2.59824	2.62108	
$\frac{1}{2} \log T_s$	1.85121	1.85121	1.85121	
$\log K_{p(\text{CO})}$	0.76693	0.75670	0.74650	
72 $L(T) = \frac{1}{2} \log T_s + \log K_{p(\text{CO})}$	2.61814	2.60834	2.59857	
73 From Fig. 5, $T_s = 5053^\circ\text{F}$ $x_s = 0.8113$ $y_s = 0.9712$				
74 From [43a], $n_e = 75.182 - 3.354 - 4.516 = 67.312$ mols $P_s = \frac{[74] \times [73] \times [41]}{[35] \times [40]} = 622.95$ lb. per sq. in.				
The Expansion Process				
Let T_e be 3740°F . K_p (water gas) = 5.0720				
Let $x_e =$	0.972	0.974	0.976	0.978
cx	4.9300	4.9401	4.9503	4.9604
$cx+(1-x)$	4.9580	4.9661	4.9743	4.9824
$\frac{cx}{cx+(1-x)} = y_e$	0.9943	0.9948	0.9952	0.9956
r'_s	8.7842	8.7842	8.7842	8.7842
$\frac{r'_s}{2}x$	4.0179	4.0262	4.0345	4.0427
$\frac{r'_s}{2}y$	4.6239	4.6262	4.6281	4.6299
$r'_e = r'_s - \frac{r'_s}{2}x - \frac{r'_s}{2}y$	0.1424	0.1318	0.1216	0.1116
$\frac{1}{2} \log r'_e$	7.57676	7.55996	7.54247	7.52383
$\log x$	7.98767	7.98856	7.98945	7.99034
$\log(1-x)$	2.44716	2.41497	2.38021	2.34242
$\frac{1}{2} \log \frac{r'_s T_s}{P_s} \times \frac{V_e}{V_s}$	2.30172	2.30172	2.30172	2.30172
75 $R(xy) = \log x - \log(1-x) + \frac{1}{2} \log \frac{r'_s T_s}{P_s} \times \frac{V_e}{V_s} - \frac{1}{2} \log r'_e$	4.26547	4.31535	4.36849	4.42581

TABLE 1 (Continued)
COMPUTATION FORM FOR OTTO CYCLE

Assume T_4	3730	3740	3750	
$\frac{1}{2} \log T_4$	1.78585	1.78643	1.78701	
$\log K_p (co)$	2.58150	2.56279	2.54418	
$L(T) = \frac{1}{2} \log T + \log K_p (co)$	4.36735	4.34922	4.33119	
From Fig. 6, $x =$ $c = K_p$ (water gas)	0.9760	0.9753	0.9746	
cx	5.0501	5.0720	5.0938	
$y = \frac{cx}{cx + (1-x)}$	0.9951	0.9950	0.9949	$T_3 = 5053$
$1-x$	0.0240	0.0247	0.0254	0.1887
$n_1(1-x)$	0.1984	0.2042	0.2100	1.5501
$H_p (co)$	119.284	119.275	119.267	119.088
76 $\frac{n_1(1-x) \times H_p (co)}{T}$	6.3447	6.5123	6.6789	36.7681
$1-y$	0.0049	0.0050	0.0051	0.0288
$n_2(1-y)$	0.0456	0.0465	0.0474	0.2679
$H_p (H_2)$	107.331	107.327	107.323	104.978
77 $\frac{n_2(1-y) \times H_p (H_2)}{T}$	1.3121	1.3344	1.3566	5.5657
$\Phi_{(co, n_2, o_2)}$	41.5037	41.5214	41.5391	43.7020
$\Phi_{(co_2)}$	52.8829	52.9145	52.9460	56.5383
$\Phi_{(H_2O)}$	54.0958	54.1258	54.1559	58.1144
78 $n^* \Phi_{(co, n_2, o_2)} = \Phi_{(co_2)} \times [46]$	2026.6174	2027.4816	2028.3459	2133.9599
79 $n_1 \Phi_{(co)} = \Phi_{(co_2)} \times [44]$	437.2094	437.4706	437.7310	467.4304
80 $n_2 \Phi_{(H_2O)} = \Phi_{(H_2O)} \times [45]$	503.1396	503.4186	503.6986	540.5162
$\log x$	7.98945	7.98914	7.98883	7.90918
81 $n_1 \log x = [44] \times \log x$	-0.08722	-0.08978	-0.09235	-0.75085
$\log y$	7.99787	7.99782	7.99778	7.98722
82 $n_2 \log y = [45] \times \log y$	-0.01981	-0.02027	-0.02065	-0.11803
$\log V$	0.69897	0.69897	0.69897	0.0
83 $n_2 \log V = [49] \times \log V$	46.41028	46.41028	46.41028	—
84 $[81] + [82] - [83]$	-46.51731	-46.52033	-46.52328	-0.86888
85 $4.571 \times [84]$	-212.6306	-212.6444	-212.6579	-3.9716
$S = [76] + [77] + [78] + [79] + [80] - [85]$	3187.2538	3188.8619	3190.4689	3188.2119
86 $S_3 - S_4$	-0.9581	+0.6500	+2.2570	—

From Fig. 7, $T_6 = 3736^\circ F.$, $x_6 = 0.9756$, $y_6 = 0.9950$

From [43a], $n_4 = 75.182 - 4.033 - 4.627 = 66.522$

$$p_4 = \frac{n_4 \times T_4 \times [2]}{[35] \times T_1} = 91.04 \text{ lb. per sq. in.}$$

$$V_4 = \frac{n_4 \times 1544 \times T_4}{p_4 \times 144} = 29270 \text{ cu. ft.}$$

TABLE I (Concluded)
COMPUTATION FORM FOR OTTO CYCLE

<i>Work Done During the Cycle</i>	
$T_1 = 638, \quad T_4 = 3736$	
87	$H_{p(C_8H_{18})}$ at $T_1 = 2147500$
88	$n_1(1-x_4)H_{v(CO)}$ at $T_4 = 8.267 \times 0.0244 \times 115570 = 23312$
89	$n_2(1-y_4)H_{v(H_2)}$ at $T_4 = 9.301 \times 0.0050 \times 103620 = 4819$
90	$n_1 \Delta u_{C_8H_{18}} = 8.267(36201 - 4037) = 265900$
91	$n_2 \Delta u_{H_2} = 9.301(29130 - 4038) = 233380$
92	$n^* \Delta u_{O_2} = 48.830(20559 - 3166) = 849300$
$Work = [87] - [88] - [89] - [90] - [91] - [92] = 770790 \text{ B.t.u.}$	
<i>Efficiency of the Cycle</i>	
$H_{p(C_8H_{18})}$ at $520^\circ = 2143000$	
$\eta = \frac{770790 \times 100}{2143000} = 35.965\%$	
<i>Mean Effective Pressure</i>	
$V_4 - V_2 = 29270 - 5854 = 23416$	
$m.e.p. = \frac{work \times J}{V_4 - V_2 \times 144}$	
$= \frac{770790 \times 777.64}{23416 \times 144} = 177.77 \text{ lb. per sq. in.}$	

Fuel—octane (C_8H_{18})

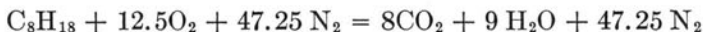
Compression ratio—5

Air supplied—100 per cent of theoretical

In the progress of the computation values for certain thermal magnitudes are required, such as heats of combustion of C_8H_{18} , CO, and H_2 at various temperatures, the energies of the gas constituents at various temperatures, the equilibrium constants K_p for the CO, H_2 and the water-gas equilibria, and the entropy function ϕ . Tables of these various magnitudes have been computed. Some of these tables are given in Appendix IV of Bulletin No. 139.

(a) The Initial Mixture

The chemical equation which represents the combustion process is



The nitrogen does not enter into the reaction but since the O_2 is obtained from the air the nitrogen must necessarily be considered. The ratio of nitrogen to oxygen in air is 3.78.

From the chemical equation the fuel mixture and the products mixture are obtained (Items 3 to 10). The amount of residual gas left in the cylinder is not considered at this time. From the mol specific heat equations for each of the constituents the mol specific heat equation of the fuel mixture (Item 17) and that of the products (Item 21) may be calculated.

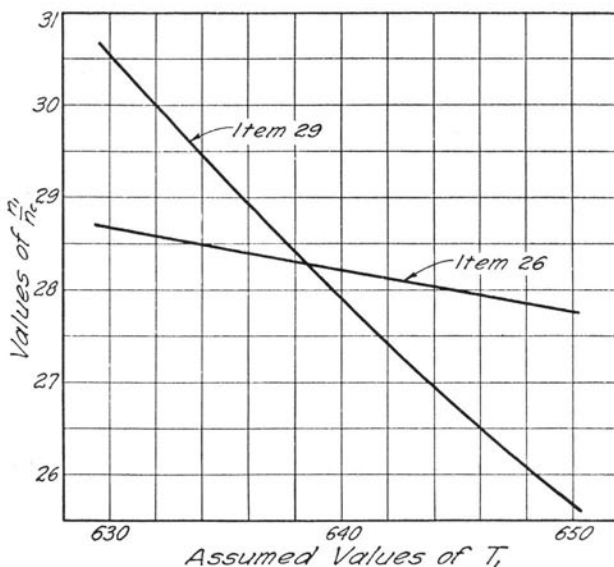


FIG. 4. COMPUTATION OF INITIAL CONDITIONS

It is now necessary to assume a temperature for the gases which are in the cylinder at the beginning of exhaust. If the end temperature found later does not agree closely with this assumption the calculations must be repeated. The temperature of the mixture of fuel and residual gas will be in the neighborhood of 600 deg. F. (abs.). If T_4 is taken as 3740 deg. (abs.) the mean temperature of the products is 2170 deg. F. The fuel enters the engine at 520 deg. F.; hence the mean temperature of the fuel mixture is 560 deg. F. The ratio β of the specific heat of the products to that of the fuel is 1.0807 (Item 24).

With the assumed value of T_4 , equations (12) and (14) are evaluated (Items 25 to 29). Items 26 and 29 are plotted against various values of T_1 (Fig. 4); it is found that T_1 is 638 deg. F. and that the amount of residual gas is equal to 2.148 mols. The pressure is taken as 14.7 lb. per sq. in.

The residual gas being divided into its constituents and these added to the theoretical fuel mixture the fuel mixture actually in the cylinder at the beginning of compression is obtained (Items 30 to 35).

(b) The Adiabatic Compression

T_1 being known and various values being assumed for T_2 , a value of T_2 may be found for which equation (17) is satisfied. These computations are made in Items 36 to 40 and it is found that T_2 is 1081 deg. F. From

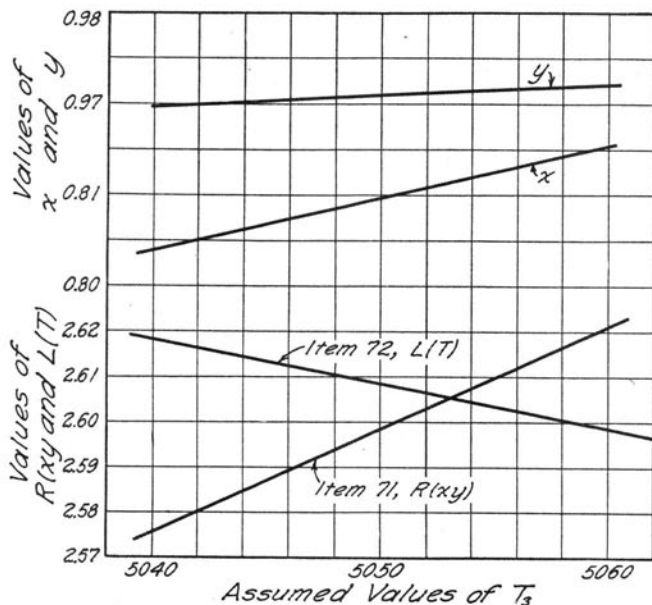


FIG. 5. COMPUTATION OF IGNITION CONDITIONS

the equation $p_2 = p_1 r \frac{T_2}{T_1}$ the value $p_2 = 124.5$ lb. per sq. in. or 8.47 atmospheres is determined.

The clearance volume may also be obtained at this point from the equation $V = \frac{n_2 RT_2}{p_2}$. It is found to be 5854 cu. ft. (Item 43).

(c) Adiabatic Combustion at Constant Volume

The system of computation has been indicated in Section 17. Items 44 to 49 give the values of the various n s that enter into the computation. The first step is the determination of the constants a and b in the energy equation (23). From a rough preliminary calculation the value of T_3 is known to lie in the neighborhood of 5050 deg. Hence values 5040, 5050, 5060 are assumed. Items 50–53 give the various Δu s (obtained from the tables); then Items 54–62 show the steps in getting the various terms in equations (24) and (25). Values of c , the water-gas equilibrium constant, are found in the tables, and Items 63–70 show the computation of x and y for each of the assumed values of T_3 , by making use of equation (31). Sufficient data are now at hand for the satisfaction of equation (29). Values of $R(x, y)$ are found (Item 71), likewise values of $L(T)$ (Item 72). These values are plotted against the corresponding values of T_3 , Fig. 5, and T_3 is found to be 5053 deg. The corresponding

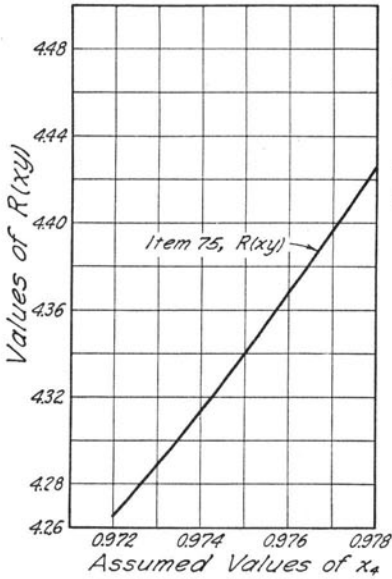


FIG. 6. COMPUTATION FOR x AT END OF EXPANSION

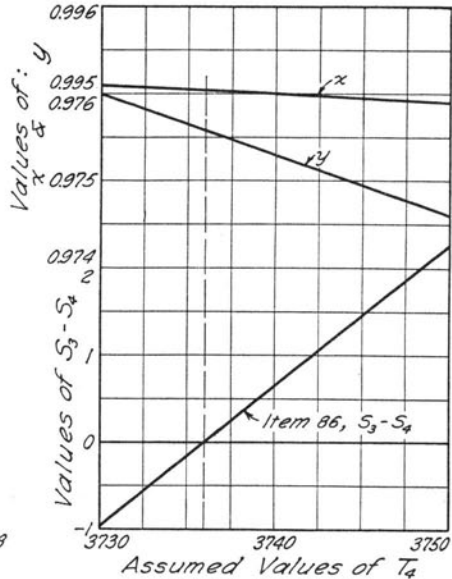


FIG. 7. COMPUTATION OF END CONDITIONS

values of x and y are plotted on the same figure, and from these curves the values $x = 0.8113$ and $y = 0.9712$ are found.

(d) The Adiabatic Expansion

A final temperature T_4 is assumed and the values of $R(x, y)$ (Equation 29) are calculated for various values of x . These values are plotted against x (Fig. 6). The expression $L(T)$, equation (29) is then evaluated for various assumed temperatures. Then from Fig. 6 we find the x corresponding to the assumed temperature. The error thus introduced by assuming one temperature for the $R(x, y)$ function is small because the function varies only slightly with changing temperature.

With the assumed T_4 and the corresponding values of x and y equation (44) is evaluated (Items 76 to 86). The difference in entropy between the beginning and end of expansion is plotted in Fig. 7. For adiabatic expansion this should be zero, and the zero on the curve gives the T_4 sought. Values of x and y are likewise plotted in Fig. 7, whereby x_4 and y_4 corresponding to T_4 are found.

(e) Efficiency and Mean Effective Pressure

The conditions at points 1 and 4 of the cycle being known it is possible to calculate the work done. Equation (48) is evaluated in

TABLE 2
RESULTS OF COMPUTATIONS FOR OTTO CYCLE, WITH GASOLINE (OCTANE) AS FUEL

Case No.	% Air	T_1	p_1	T_2	p_2	T_3	p_3	z_3	y_3	T_4	p_4	z_4	y_4	Work B.t.u.	η	m.e.p.
Compression Ratio 3.5 to 1																
1.....	75	683	14.7	1011	76.1	4715	369.3	0.4082	0.8310	3448	82.8	0.4570	0.7882	466.431	0.2176	145.7
2.....	90	703	14.7	1052	76.8	4993	396.4	0.6873	0.9442	3830	86.7	0.7532	0.9416	577.238	0.2693	147.0
3.....	100	715	14.7	1076	77.4	4979	383.2	0.8043	0.9690	3979	86.5	0.9527	0.9912	619.519	0.2891	140.2
4.....	110	710	14.7	1075	77.9	4872	373.6	0.8775	0.9818	3794	82.5	0.9894	0.9979	647.922	0.3023	134.3
5.....	125	705	14.7	1077	78.6	4654	355.6	0.9410	0.9913	3488	75.9	0.9981	0.9995	675.472	0.3152	123.5
6.....	150	700	14.7	1080	79.4	4279	326.1	0.9833	0.9974	3080	67.0	1.0000	1.0000	705.849	0.3293	107.7
Compression Ratio 5.0 to 1																
7.....	75	622	14.7	1029	121.6	4753	639.8	0.4070	0.8316	3176	85.5	0.4736	0.7733	578.725	0.2700	180.6
8.....	90	632	14.7	1062	123.5	5067	641.0	0.6911	0.9462	3549	89.4	0.7583	0.9370	711.080	0.3318	183.3
9.....	100	638	14.7	1081	124.5	5053	622.9	0.8113	0.9712	3736	91.0	0.9756	0.9950	770.790	0.3597	177.7
10.....	110	635	14.7	1085	125.6	4940	605.3	0.8838	0.9830	3521	85.6	0.9965	0.9992	802.883	0.3746	168.7
11.....	125	630	14.7	1088	126.8	4721	577.1	0.9440	0.9919	3218	78.4	0.9995	0.9998	832.327	0.3883	155.1
12.....	150	630	14.7	1101	128.4	4351	526.5	0.9835	0.9975	2830	68.4	1.0000	1.0000	865.485	0.4038	133.9

TABLE 2 (Concluded)
RESULTS OF COMPUTATIONS FOR OTTO CYCLE, WITH GASOLINE (OCTANE) AS FUEL

Case No.	% Air	T_1	p_1	T_2	p_2	T_3	p_3	x_3	y_3	T_4	p_4	x_4	y_4	Work B.t.u.	η	m.e.p.
Compression Ratio 6.5 to 1																
13.....	75	594	14.7	1066	171.5	4810	882.3	0.4058	0.8333	2995	84.5	0.4875	0.7620	649 832	0.3032	202.6
14.....	90	601	14.7	1098	174.6	5121	886.2	0.6925	0.9474	3355	89.0	0.7644	0.9317	799 822	0.3732	206.7
15.....	100	605	14.7	1117	176.4	5108	863.4	0.8150	0.9724	3553	91.3	0.9858	0.9969	874 648	0.4081	202.7
16.....	110	600	14.7	1119	178.2	4995	842.3	0.8868	0.9838	3327	85.6	0.9987	0.9997	907 518	0.4234	192.8
17.....	125	595	14.7	1123	180.3	4774	803.5	0.9453	0.9927	3029	78.2	1.0000	1.0000	938 016	0.4377	176.7
18.....	150	595	14.7	1139	182.9	4406	734.4	0.9835	0.9988	2655	68.0	1.0000	1.0000	972 715	0.4539	152.1
Compression Ratio 8.0 to 1																
19.....	75	577	14.7	1105	225.2	4829	1122.1	0.4050	0.8338	2837	82.4	0.5015	0.7487	700 045	0.3266	217.0
20.....	90	582	14.7	1137	229.7	5165	1136.0	0.6943	0.9480	3202	87.7	0.7689	0.9278	842 786	0.4047	225.0
21.....	100	586	14.7	1174	235.6	5150	1119.4	0.8178	0.9733	3405	90.3	0.9918	0.9980	949 274	0.4429	221.2
22.....	110	585	14.7	1167	234.6	5040	1072.5	0.8870	0.9640	3187	84.1	0.9895	0.9998	981 772	0.4581	208.0
23.....	125	580	14.7	1173	237.8	4821	1024.6	0.9454	0.9924	2890	76.5	1.0000	1.0000	1015 372	0.4738	190.7
24.....	150	580	14.7	1183	239.9	4451	947.2	0.9825	0.9971	2527	67.1	1.0000	1.0000	1052 102	0.4909	166.3

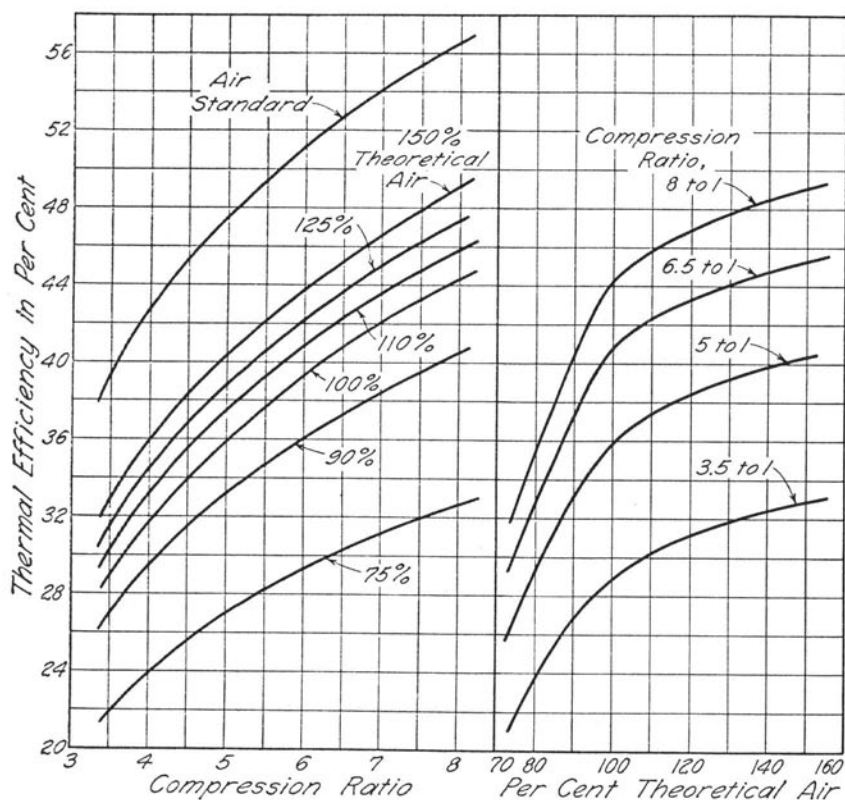


FIG. 8. VARIATION OF EFFICIENCY WITH COMPRESSION RATIO AND WITH MIXTURE STRENGTH FOR OTTO CYCLE

Items 87 to 92. The work delivered by the ideal cycle per mol of fuel is found to be 770 790 B.t.u.

As a basis for all the efficiencies in this work the heat of combustion of the fuel at 520 deg. F. abs., an average room temperature, was taken. The heat of combustion of C_8H_{18} at constant pressure at 520 deg. F. is 2 143 000. The efficiency of the ideal cycle is, therefore,

$770\,790 \div 2\,143\,000 = 0.3597$, or 35.97 per cent. From equation (49) the mean effective pressure is found to be 177.77 lb. per sq. in.

IV. RESULTS OF CALCULATIONS

24. *Otto Cycle. Efficiency and M.E.P.*—The system of calculation explained in the preceding section was applied to the ideal Otto cycle

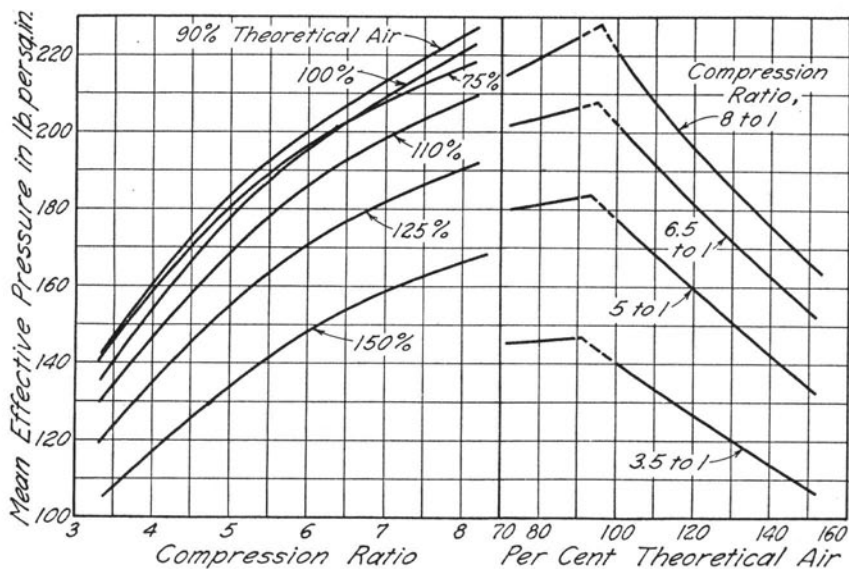


FIG. 9. VARIATION OF M.E.P. WITH COMPRESSION RATIO AND WITH MIXTURE STRENGTH FOR OTTO CYCLE

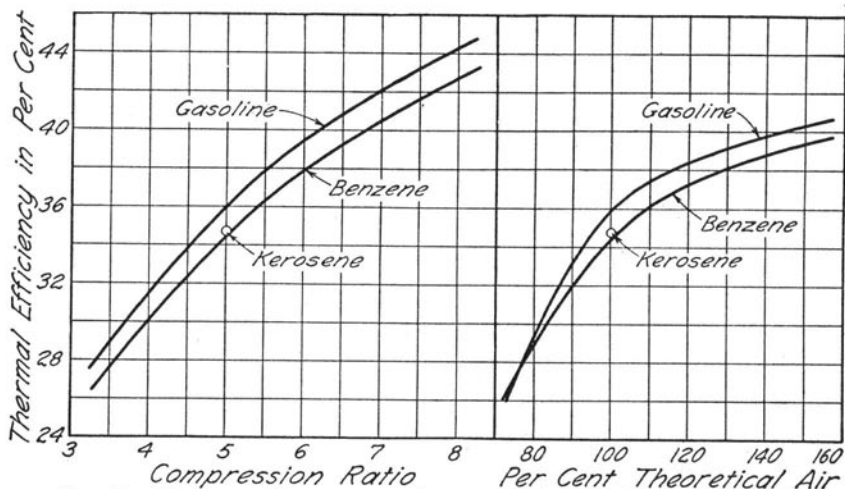


FIG. 10. EFFECT OF COMPRESSION RATIO AND MIXTURE STRENGTH UPON OTTO CYCLE EFFICIENCY USING VARIOUS FUELS

TABLE 3
RESULTS OF COMPUTATIONS FOR OTTO CYCLE, WITH BENZENE AS FUEL
Compression Ratio, 5 to 1. Various Percentages Theoretical Air

Case No.	% Air	T_1	p_1	T_2	p_2	T_3	p_3	z_3	y_3	T_4	p_4	z_4	y_4	Work B.t.u.	η	m.e.p.
.....	75	636	14.7	1096	126.7	5138	642.9	0.4378	0.8622	3476	86.9	0.4739	0.8020	368 433	0.2710	185.2
.....	100	655	14.7	1146	128.6	5237	609.6	0.7669	0.9649	3977	90.8	0.9559	0.9918	468 067	0.3443	173.6
.....	110	648	14.7	1141	129.4	5139	597.3	0.8385	0.9766	3768	86.6	0.9909	0.9981	491 453	0.3615	167.6
.....	125	642	14.7	1139	130.4	4931	573.9	0.9108	0.9873	3438	79.6	0.9985	0.9996	513 071	0.3774	155.4
.....	150	639	14.7	1142	131.4	4560	529.8	0.9700	0.9943	3013	69.9	1.0000	1.0000	535 775	0.3941	135.7

TABLE 4
RESULTS OF COMPUTATIONS FOR OTTO CYCLE, WITH BENZENE AS FUEL
100 Per Cent Theoretical Air. Various Compression Ratios

Case No.	r	T_1	p_1	T_2	p_2	T_3	p_3	z_3	y_3	T_4	p_4	z_4	y_4	Work B.t.u.	η	m.e.p.
.....	3.5	741	14.7	1141	79.2	5165	370.8	0.7590	0.9620	4197	84.9	0.9247	0.9867	375 130	0.2760	135.2
.....	5.0	655	14.7	1146	128.6	5237	609.6	0.7669	0.9649	3977	90.8	0.9559	0.9918	468 067	0.3443	173.6
.....	6.5	615	14.7	1180	183.3	5313	852.8	0.7704	0.9660	3801	92.3	0.9724	0.9945	533 165	0.3922	200.7
.....	8.0	595	14.7	1226	242.3	5354	1094.2	0.7681	0.9659	3660	91.8	0.9805	0.9959	580 959	0.4274	219.6

TABLE 5
RESULTS OF COMPUTATIONS FOR OTTO CYCLE, WITH VARIOUS FUELS
Compression Ratio, 5 to 1. 100 Per Cent Theoretical Air

Case No.	Fuel	T_1	p_1	T_2	p_2	T_3	p_3	T_4	p_4	y_3	T_4	p_4	z_4	y_4	Work B.t.u.	η	m.e.p.
.....	Gasoline	638	14.7	1081	124.5	5053	622.9	3736	91.0	0.9712	3736	91.0	0.9756	0.9950	770 818	0.3597	177.7
.....	Kerosene	640	14.7	1080	124.0	5309	643.4	4029	96.2	0.9609	4029	96.2	0.9489	0.9906	1 199 830	0.3472	187.6
.....	Benzene	655	14.7	1146	128.6	5257	609.6	3977	90.8	0.9669	3977	90.8	0.9559	0.9918	468 067	0.3443	173.6

TABLE 6
RESULTS OF COMPUTATIONS FOR DIESEL CYCLE, WITH KEROSENE AS FUEL

Case No.	r	100 Per Cent Theoretical Air											Work B.t.u.	η	m.e.p.		
		T_1	p_1	T_2	p_2	T_3	p_3	z_3	y_3	V_3	T_4	p_4				z_4	y_4
40	8	602	14.7	1360	266.1	4854	266.1	4286	115.9	0.9710	4286	115.9	0.9162	0.9849	86 797	0.2509	135.7
41	11	576	14.7	1472	413.2	4946	413.2	4062	111.6	0.9713	4062	111.6	0.8472	0.9904	102 552	0.3018	164.6
42	14	564	14.7	1582	502.3	5023	502.3	3881	108.8	0.9717	3881	108.8	0.8635	0.9929	117 281	0.3379	185.4
43	17	554	14.7	1674	755.1	5096	755.1	3802	108.5	0.9715	3802	108.5	0.8725	0.9945	124 746	0.3609	198.6
200 Per Cent Theoretical Air																	
44	8	585	14.7	1323	265.9	3556	265.9	2595	67.5	1.0000	2595	67.5	1.0000	1.0000	138 122	0.3996	109.8
45	11	561	14.7	1445	416.5	3663	416.5	2382	64.7	1.0000	2382	64.7	1.0000	1.0000	162 614	0.4705	130.6
46	14	553	14.7	1553	577.9	3759	577.9	2236	61.7	1.0000	2236	61.7	1.0000	1.0000	180 143	0.5212	144.3
47	17	546	14.7	1651	755.6	3844	756.1	2136	59.6	1.0000	2136	59.6	1.0000	1.0000	192 726	0.5576	154.6
300 Per Cent Theoretical Air																	
48	8	590	14.7	1333	265.7	2888	265.7	1885	48.1	1.0000	1885	48.1	1.0000	1.0000	154 886	0.4481	80.5
49	11	566	14.7	1448	413.7	3015	413.7	1745	46.3	1.0000	1745	46.3	1.0000	1.0000	179 399	0.5191	92.8
50	14	555	14.7	1559	578.1	3116	578.1	1640	44.5	1.0000	1640	44.5	1.0000	1.0000	195 218	0.5648	103.2
51	17	547	14.7	1654	755.6	3204	755.6	1563	43.0	1.0000	1563	43.0	1.0000	1.0000	207 784	0.6012	110.3

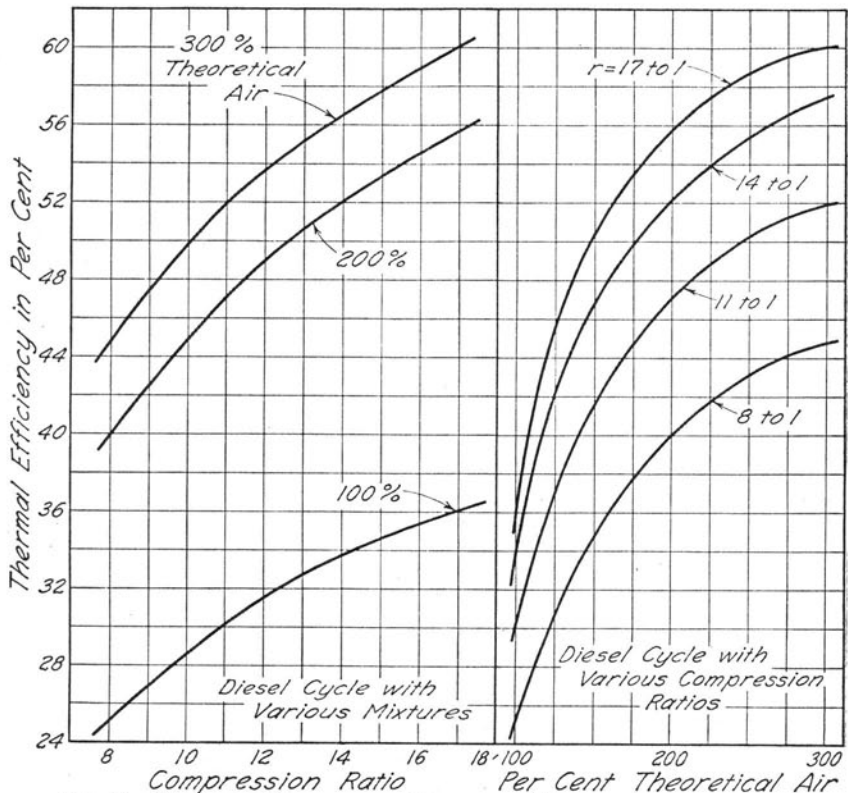


FIG. 11. EFFECT OF COMPRESSION RATIO AND MIXTURE STRENGTH UPON EFFICIENCY OF DIESEL CYCLE

with varying initial conditions. Four compression ratios were taken: namely, 3.5, 5, 6.5, 8. For each of these six different mixing ratios were assumed—75, 90, 100, 110, 125, and 150 per cent of theoretical air.

The results of the calculations are given in Table 2. Figure 8 shows the variation of efficiency with the compression ratio and with the mixture strength. It is instructive to compare these results with the air standard efficiencies, which are as follows:

Compression ratio r	3.5	5.0	6.5	8.0
Efficiency (per cent)	39.41	47.47	52.70	56.47

In Fig. 9 is shown the variation of the mean effective pressure with the compression ratio and with the mixture strength.

25. *Efficiency with Various Fuels.*—In the computations of Table 2, octane (C_8H_{18}) was taken as the fuel. This is considered a fair equivalent of gasoline. To determine the effect of the fuel on the efficiency,

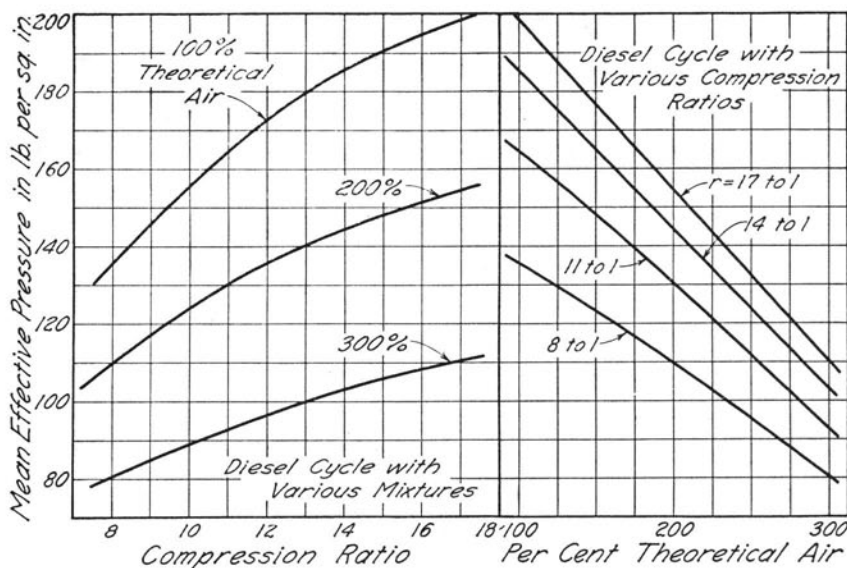


FIG. 12. EFFECT OF COMPRESSION RATIO AND MIXTURE STRENGTH UPON M.E.P. OF DIESEL CYCLE

the results given in Tables 3 and 4 were calculated with benzene as the fuel. Also one calculation was made with $C_{12}H_{26}$ which is assumed to represent kerosene. Table 5 gives a comparison of the efficiencies of the three fuels with 100 per cent of theoretical air and a compression ratio $r = 5$. The effect of a change of fuel is shown graphically in Fig. 10.

26. *Efficiency of Ideal Diesel Cycle.*—The results of the computation for the Diesel cycle are given in Table 6, and are shown graphically in Figs. 11 and 12.

27. *Temperatures and Pressures.*—The values of the temperature T_3 at the end of the combustion process are shown in Fig. 13; also the values of the maximum pressures p_3 . It will be observed that the maximum points for all the curves occur with a mixture strength corresponding to less than 100 per cent of theoretical air.

In Fig. 14 is shown a corresponding plot of the temperatures T_4 at the end of the adiabatic expansion. The value of T_4 falls rapidly as the amount of air is increased; and for the same air supply, T_4 is lower the higher the compression ratio.

A study of the values of p_4 given in Table 2 discloses the fact that for the same mixture the value of p_4 is nearly the same for all compression ratios. The deviations from a mean value are small and

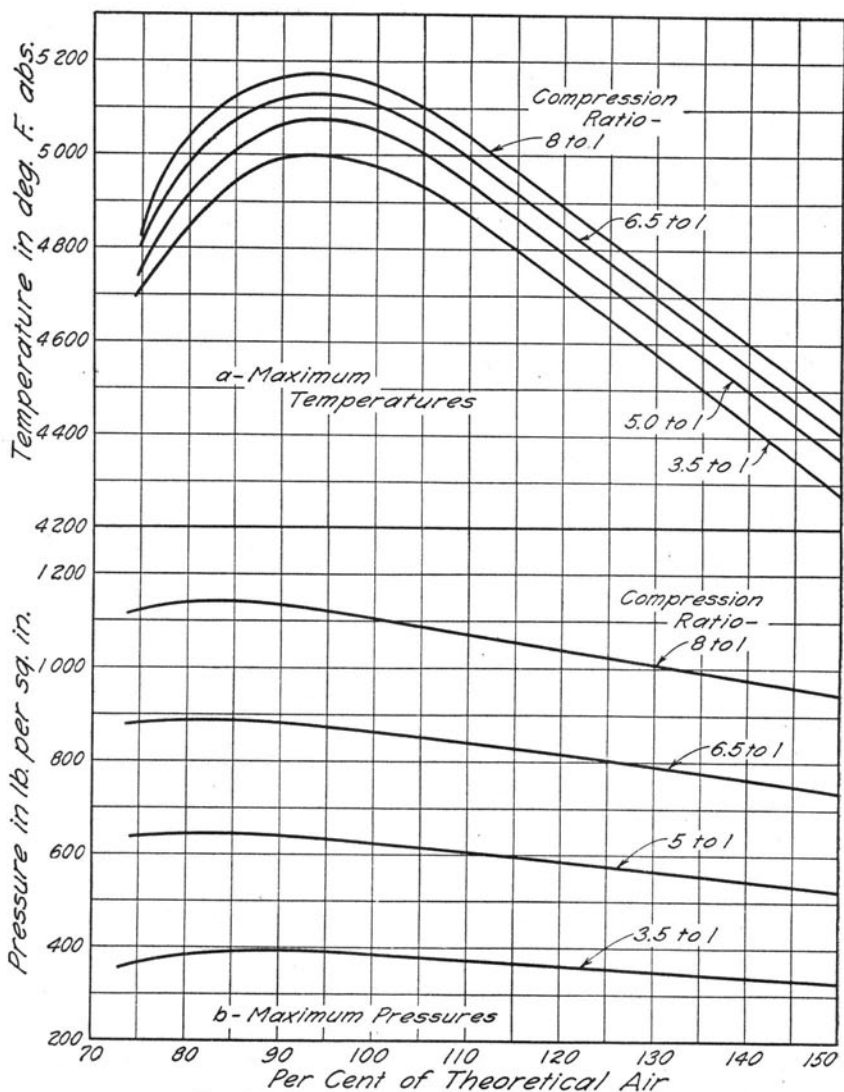


FIG. 13. CONDITIONS AT END OF COMBUSTION

irregular and may doubtless be ascribed to slight errors in the computations. The values of p_4 are plotted in curves b of Fig. 14. It is seen that p_4 has its maximum value at 100 per cent theoretical air and steadily drops as the air supply is increased.

28. *Unburned Gases at End of Expansion.*—The amount of H_2 and CO remaining unburned at the end of the adiabatic expansion may be

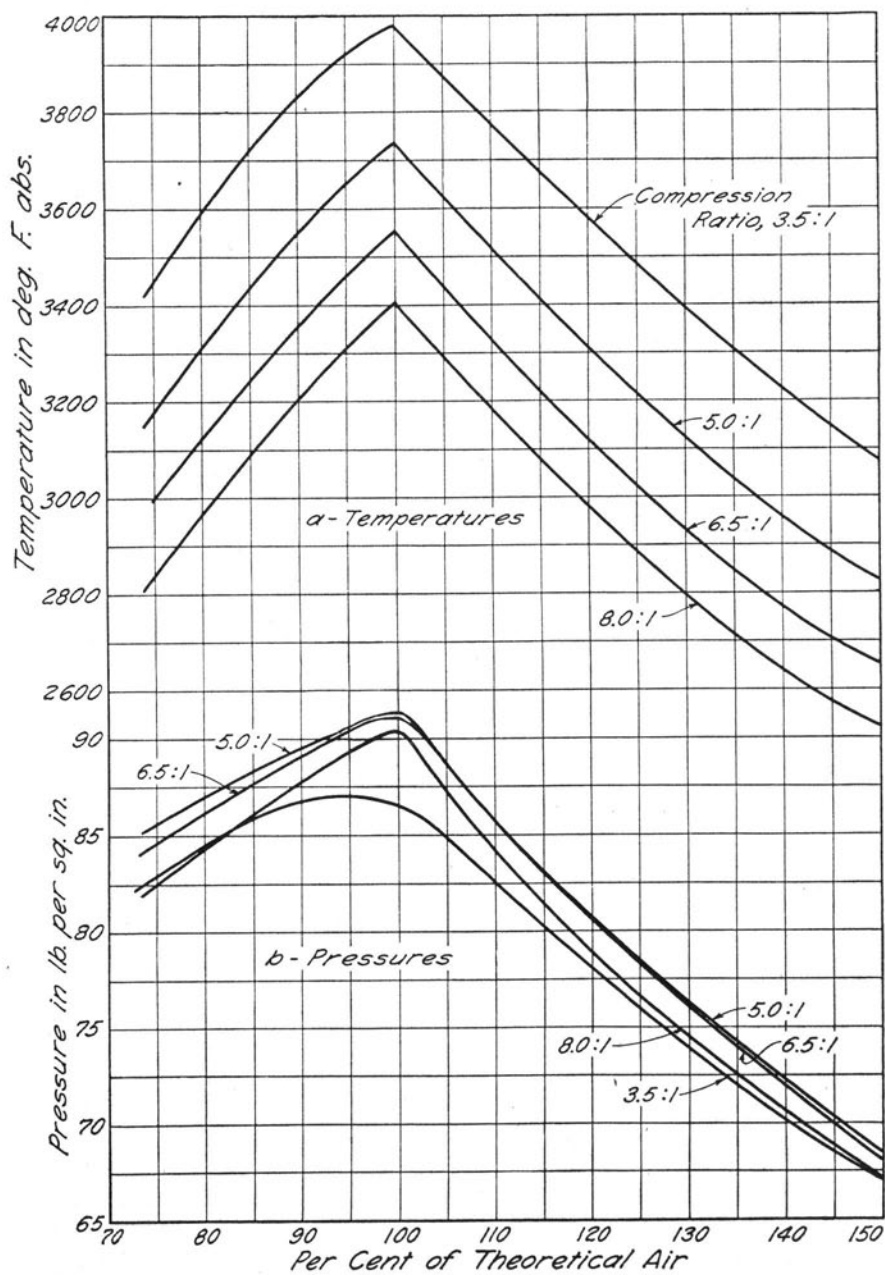


FIG. 14. CONDITIONS AT END OF EXPANSION

seen from the columns headed x_4 and y_4 in Table 2. When the percentage of air is less than 100 there is considerable unburned H_2 and CO, as would be expected. With 100 per cent air the figures are as follows:

Compression ratio . . .	3.5	5	6.5	8
Unburned H_2	0.0088	0.0050	0.0031	0.0020
Unburned CO	0.0473	0.0264	0.0142	0.0082

The amount of unburned H_2 is inappreciable; the amount of unburned CO is, however, appreciable at the lower compression ratios. With 10 per cent or more excess air the amounts of unburned gases are small at all compression ratios.

29. *Discussion of Results.*—An inspection of the results given in Tables 2 and 6 verifies certain conclusions that are already well established:

(1) The efficiency increases with the compression ratio r , that is, the higher the compression the higher the efficiency, other conditions remaining the same.

(2) For the same compression, the efficiency increases with the amount of air used. A lean mixture gives a higher efficiency than a rich mixture.

(3) The mean effective pressure, and therefore the power, is a maximum when the air supply is somewhat less than 100 per cent of the theoretical amount (Fig. 9). Thus the mixture for maximum power is a mixture of relatively low efficiency.

(4) The ideal efficiencies obtained from the various liquid fuels are practically the same. The small differences in the calculated values are without significance in the light of the probable inaccuracies of the assumed specific heats and heats of combustion of these fuels. The conclusion of Tizard and Pye that the ideal efficiency of the motor is independent of the kind of liquid fuel used is verified.

(5) The efficiencies of the Diesel cycle, as a group, range higher than the efficiencies of the Otto cycle. However, a comparison of the two efficiencies at the same compression ratio ($r = 8$) shows that the Otto cycle is inherently more efficient than the Diesel cycle. The superior efficiency of the Diesel cycle is due to the high compression ratio that is permitted by the system of operation.

The reason for the increase of the efficiency with the compression ratio and with the amount of air supplied should receive some attention. According to equation (1), Section 11, the work of the cycle is given by the difference $U_1 - U_4$. In the case of the Diesel cycle the subtractive term is also U_4 , equation (8). Now U_4 is the energy of the

mixture in the final state 4 at the opening of the exhaust; and this energy is made up of two parts: the chemical energy of the unburned CO and H₂, and the thermal energy of the mixture of the products at the temperature T_4 . With 100 per cent or more air the amount of unburned CO and H₂ is small and the chemical energy in the mixture is correspondingly small. Therefore, the energy U_4 will depend principally on the temperature T_4 . Any conditions that will result in a decrease of T_4 will likewise cause a reduction in U_4 and consequently an increase in efficiency. Consider now the effect of compression alone. With 100 per cent air, Fig. 13 shows that the increase of r from 3.5 to 8 causes T_3 to rise from 4980 to 5150 deg. F., or about 170 deg. But during the adiabatic expansion 3-4, the 3.5-fold expansion gives a drop of temperature from 4980 to 3980 deg. F., or 1000 deg., while the 8-fold expansion gives a drop from 5150 to 3405 deg. F., or 1745 deg. That is, while T_3 is higher for the high compression, T_4 is much lower, as shown in Fig. 14. The improved efficiency found with higher compression is due not to any effect of compression on combustion but solely to the more complete conversion of the energy of the products into work as the result of the more complete expansion.

Evidently, if the expansion could be made still more complete by some new arrangement of the cycle, the efficiency would be still further increased. The possibility of such an arrangement is discussed in Chapter VI of this bulletin.

The effect of increasing the air supply is seen in Figs. 13 and 14. With a greater supply of air the heat of combustion per unit weight of fuel (mol or pound) is required to raise the temperature of a larger weight of gas. Therefore, T_3 will be lower the more air supplied (Fig. 13). Consequently, T_4 and U_4 will be correspondingly decreased, and the efficiency will be increased.

V. EFFICIENCY STANDARDS

30. *Discussion of Engine Efficiencies.*—The efficiency of a heat engine is, in the first instance, defined as the ratio of the useful work obtained to the heat supplied. This ratio gives the low efficiency of 10 to 25 per cent in the case of the steam engine, and 20 to 40 per cent in that of the internal combustion engine. It is now customary to use as a basis for efficiencies not the total heat energy supplied, but only the *available part* of such heat energy. The unavailable part, the part that must inevitably be wasted in accordance with the second law of thermodynamics, is not charged to the engine. This second efficiency may be found from a comparison of two efficiencies of the first class. Thus

let η_a = actual efficiency of engine based on heat supplied

η_i = efficiency of an ideal engine operating under the same conditions but without losses of any kind.

Then the efficiency of the engine based on available energy is

$$\eta = \frac{\eta_a}{\eta_i} \quad (51)$$

The efficiency η_a is readily determined from the power and fuel consumption of the engine. In the case of the steam engine or turbine, the efficiency η_i of the ideal engine is also readily determined. The Rankine cycle is assumed and by the aid of a Mollier chart the ideal work is easily found.

In the case of the internal combustion engine the correct determination of η_i presents certain difficulties. If the air standard is used, the computation is easy, for η_i is given by the simple formula

$$\eta_i = 1 - \left(\frac{1}{r}\right)^n \quad (52)$$

with $n = 0.4$. The results are worthless, however, for the value of η_i for the air standard may be 30 per cent in excess of the true value. In order to get a true measure of η_i the following phenomena must be taken into account:

(1) The varying composition of the mixture during the phases of the cycle.

(2) The specific heats of the various gas mixtures.

(3) The dissociation of the products of combustion at high temperatures.

The calculation of η_i with a proper consideration of these phenomena is a laborious process, but the result is a value that gives a true indication of the extreme limit to which the efficiency of the actual engine may approach.

The values of η_i in Tables 2 to 6 are thus calculated. The accuracy of these values is limited only by the accuracy of the thermal data used in the computations.

31. *Empirical Formulas.*—The air standard formula for the efficiency η_i is given in equation (52). The same type of equation may be used to express the correct value of η_i as given in Table 2; but n instead of being a constant 0.4 will be a variable, and a function of the compression ratio and the air supply. Taking the values of η_i in Table 2, the corresponding values of n calculated from the preceding equation are those given in Table 7. Let a denote the per cent of theoretical air;

TABLE 7
VALUES OF EXPONENT n

Compression Ratio Per Cent Air Supplied.	n			
	100	110	125	150
3.5.....	0.2723	0.2874	0.3022	0.3189
5.....	0.2770	0.2916	0.3055	0.3214
6.5.....	0.2802	0.2942	0.3075	0.3232
8.....	0.2815	0.2964	0.3087	0.3248

thus $a = 100, 110, 125, 150$. Then the following empirical formula gives quite accurately the values of n in Table 7.

$$n = 0.3867 - \frac{6.5}{a - 35} - \frac{0.043}{r} \quad (53)$$

This formula applies for 100 per cent or more of the theoretical air. If the air supply is insufficient for complete combustion, the following formula gives a fair approximation:

$$n = 0.524 - \frac{24.6}{a} \quad (54)$$

Tizard and Pye* suggest the following formula for the ideal efficiency:

$$\eta_i = 1 - \left(\frac{1}{r}\right)^{0.295}$$

The value $n = 0.295$ is substantiated by Table 7 for an excess of air of from 10 to 25 per cent. The values for 150 per cent air are of theoretical interest only, as such an excess of air would probably give a non-explosive mixture.

A graphical comparison of calculated values of the efficiency η_i with the corresponding air-standard values is shown in Fig. 15. The large error of the air standard is quite evident.

For the ideal efficiency of the Diesel cycle, equation (52) may also be used with values of n given by the following empirical equation:

$$n = 0.434 - \frac{19.5}{a - r} - \frac{0.7}{r} \quad (55)$$

*The Automobile Engineer, Feb., 1921.

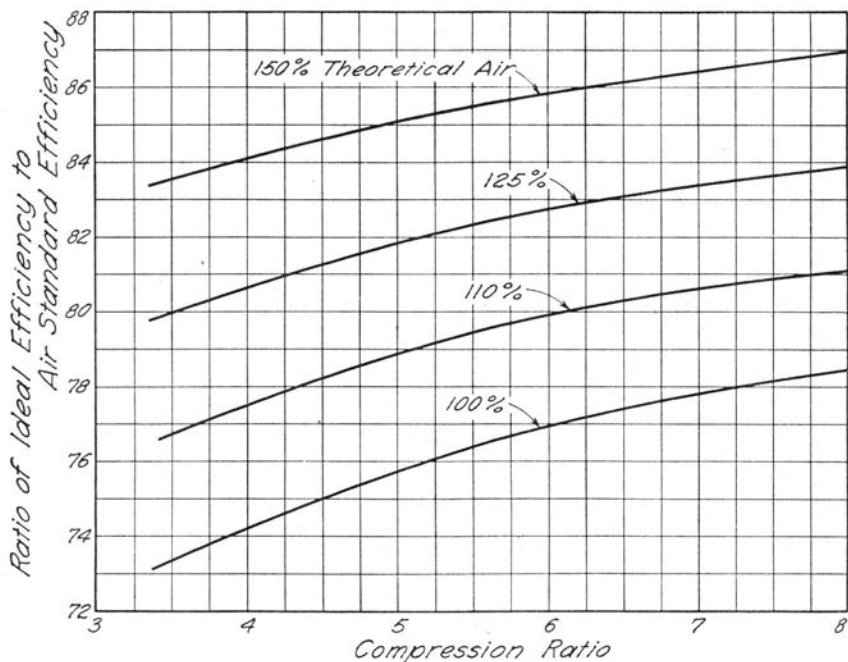


FIG. 15. RELATION BETWEEN THE IDEAL STANDARD EFFICIENCY AND AIR STANDARD

32. *Conclusion.*—As stated in the Introduction, the principal object of this investigation was the determination of a set of accurate values for the ideal efficiencies of the Otto and Diesel cycles, in order that such values may replace the usual air standard. This object is accomplished in the establishment of equations (52) to (55). While these equations apply specially to octane (C_8H_{18}), they may be used with small error for any ordinary liquid fuel. In general, the error should not exceed 2 or 3 per cent.

VI. THEORETICAL INVESTIGATION OF A MORE COMPLETE EXPANSION CYCLE

By ALBERT E. HERSHEY

33. *Introduction.*—The fact that the thermal efficiency of an internal combustion engine may be improved by increasing the compression is so well established, both by theoretical analysis and experimental investigation, that it may be accepted as one of the basic

principles of internal combustion engine design. However, it is also well known that several practical considerations establish an upper limiting value for the compression; the limit at the present time for automotive engines is the compression which corresponds to a compression ratio of about 5:1.

It is the purpose of this investigation to show, by means of the method of theoretical analysis developed previously in this bulletin, that in a more complete expansion cycle some of the advantages of high compression may be realized without the usual accompanying disadvantages. A more complete expansion cycle will be understood to be one in which the ratio of expansion of the products of combustion exceeds the ratio of compression of the combustible mixture. The theoretical thermal efficiency and indicated mean effective pressure of a more complete expansion engine and of an engine operating on a standard Otto cycle have been calculated. This has been done at both full and part loads under identical operating conditions for each engine, and from the results of the calculations the relative performance of each has been estimated.

In the discussion of the influence of increased compression on efficiency, p. 49, it was pointed out that this influence was somewhat indirect, being largely due to the direct effects of the accompanying increased expansion. Furthermore, most of the bad effects of high compression, such as detonation, over-heating, etc., are the direct results of high compression pressure on combustion. It would seem quite reasonable, therefore, to expect a more complete expansion cycle to have the same efficiency as a regular Otto cycle with higher compression and at the same time to be free from the latter's combustion difficulties.

34. *The Engine.*—More complete expansion engines may, in general, be divided into two classes, depending on the method employed in obtaining the increased expansion ratio. In the one class compound expansion is used, the gases after partial expansion in one cylinder being transferred to a second cylinder where the final expansion occurs; in the other class a variable stroke is used, the piston traveling through a greater distance on the expansion stroke than on the compression stroke. Although the more complete expansion engine considered here belongs strictly to neither of these groups, it is more nearly similar in its method of operation to the variable stroke engine since expansion is completed in a single cylinder and its expansion ratio is greater than its effective compression ratio.

In the development of a variable stroke engine the realization of either one or both of two objects is attempted: first, more complete

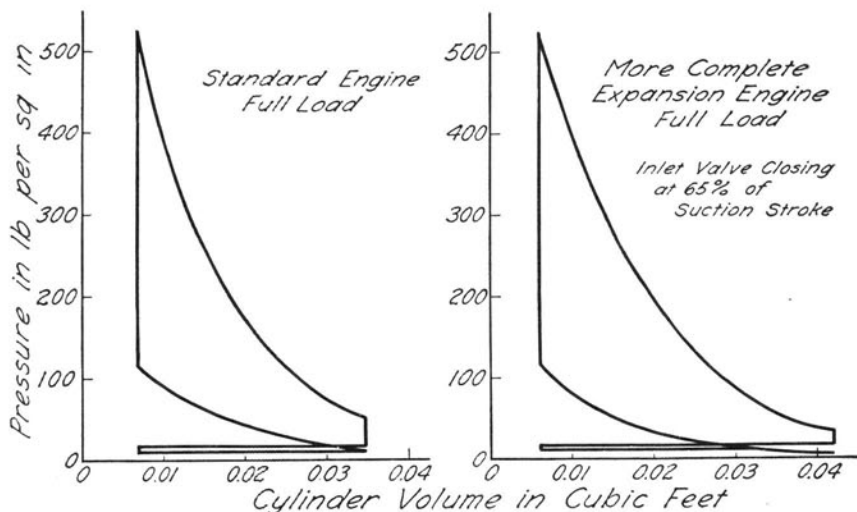


FIG. 16. INDICATOR DIAGRAMS FROM CALCULATED PRESSURES

expansion of the products of combustion, and second, more complete scavenging of these products during the exhaust stroke. In the engine investigated only the first aim, that of more complete expansion, is realized; but this is accomplished without the use of complicated linkage between the piston and the crank. The engine is unique in this respect, for the inherent defect in practically all variable stroke engines is the mechanical complication necessary to produce the variation in stroke. The closing of the inlet valve before the piston has completed the induction stroke is the simple expedient whereby the desired result is achieved. Effective compression, starting when the piston has reached this point on the return stroke, occurs during a portion of the total piston travel only, while expansion continues throughout the entire stroke.

For the purpose of showing more clearly the difference between the operation of this more complete expansion engine and that of a conventional engine, the indicator diagrams in Figs. 16 and 17 were plotted from calculated pressures and those in Fig. 18 traced from actual diagrams taken from such a more complete expansion engine. Figure 16 contains complete full load diagrams for both engines and the effect of the early closing of the inlet valve in the more complete expansion engine is at once apparent. The same thing is shown to better advantage in Fig. 17 where the lower part of full and half load diagrams for both engines are plotted on an enlarged scale. That these theoretical diagrams are substantially correct is evident from the diagrams in Fig. 18,

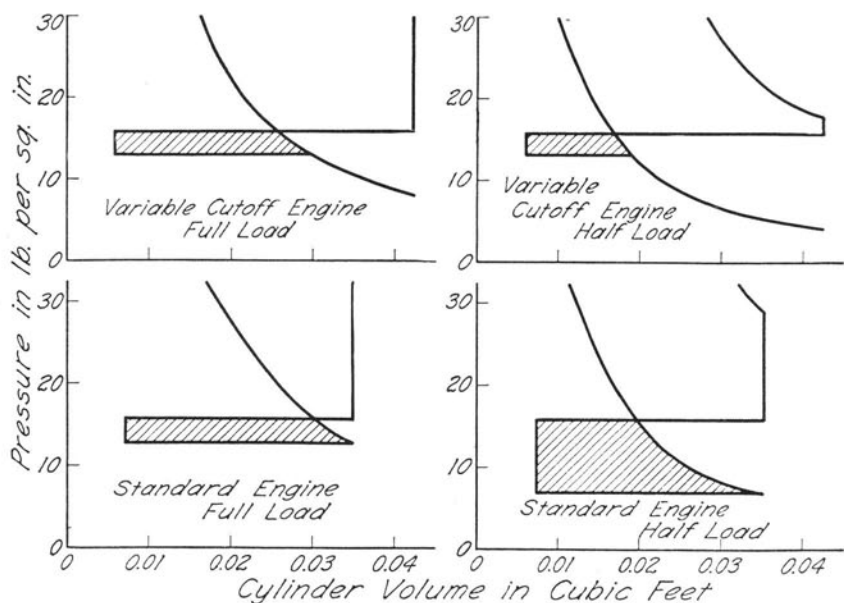


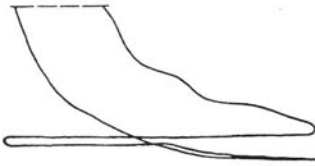
FIG. 17. SUCTION DIAGRAMS FROM CALCULATED PRESSURES

which are tracings of diagrams taken from a 10 by 20 inch more complete expansion engine.*

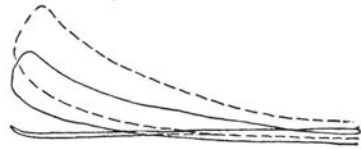
Thus it may be seen that gas is drawn into the cylinder during only a part of the induction stroke. When the inlet valve has closed this gas is expanded to some pressure lower than the induction pressure during the completion of the induction stroke. On the return stroke the gas is compressed until the induction pressure is again reached; at this time the piston will have returned to the point of inlet valve closure. This expansion and compression occurs under such conditions that, without sensible error, each may be regarded as adiabatic. The work done by the gas in expanding is, therefore, practically the same as that done on the gas in compressing it up to the induction pressure and hence this part of the cycle may be disregarded entirely. Compression continues to the end of the stroke, the final pressure depending upon the relation between the clearance volume and the cylinder volume at the time of inlet valve closure.

The remainder of the cycle is the same as the regular Otto cycle, combustion taking place at constant volume and expansion continuing throughout the entire piston travel. In an engine operating on such a

*Sargent, C. E., "The Complete Expansion Engine," Trans. A. S. M. E., vol. 22, 1901, p. 312.



Taken with light indicator spring.



Two cards taken at an interval of five seconds to show action of governor in controlling point of inlet valve closure.

FIG. 18. INDICATOR DIAGRAMS FROM A MORE COMPLETE EXPANSION ENGINE

more complete expansion cycle the ratio of expansion may be made considerably greater than the ratio of compression, resulting in a release pressure and temperature which should be proportionally lower than those of a standard engine with the same compression ratio.

Two modifications of this more complete expansion engine have been considered in the investigation. In the first type the power output is controlled by changing the point at which the inlet valve closes, so that, since the valve is open a relatively shorter time, a smaller charge of mixture is drawn into the cylinder at light loads than at heavy loads. In the second type the point of cutoff is fixed, the power output being controlled by a throttling valve in the usual manner. The two types will be referred to as the variable-cutoff and throttle-controlled engines, respectively. The indicator diagrams in Fig. 18 were taken from a variable cutoff engine in which the point of inlet valve closure was controlled by a governor. The shifting of this point of closure at different loads and the corresponding effect on the power output is shown very well by the right-hand diagram in this figure.

35. *Procedure.*—It was necessary to fix standard operating conditions for all three engines as the first step in determining their efficiencies and mean effective pressures. The full load effective compression ratio was taken as 5 to 1 in every case, and the point of maximum cutoff for the more complete expansion engines was fixed at 65 per cent of the stroke. With the exception of inlet valve closure in the more complete expansion engines all valve operation was assumed to be instantaneous and at the end of the stroke. Complete combustion at constant volume of a mixture of gasoline vapor with the theoretical amount of air was also assumed, the chemical formula for the gasoline

being taken as C_8H_{18} ,* and the initial temperature of the mixture being chosen as 559.6 deg. F. (abs.) At full load the induction pressure for all three engines was assumed to be 13 lb. per sq. in. At part loads the induction pressure for the variable-cutoff engine was also 13 lb. per sq. in. while the point of inlet valve closure was taken at 50 per cent, 35 per cent, and 20 per cent, respectively, of the induction stroke. Corresponding to these three load conditions the induction pressures of the throttling engines were taken to be 10, 7, and 5 lb. per sq. in., respectively.†

Since the temperature of the exhaust gases remaining in the clearance volume at the end of the exhaust stroke will be different for each engine and load, it was necessary to assume a reasonable temperature from which preliminary calculations were made to determine the corresponding exhaust temperature at release; with this temperature as a basis, a second approximation for the residual exhaust gas temperature was made, and from this the final results were calculated. Variations in this temperature of as much as 100 deg. affect the final results but little so that this method of procedure was found to be entirely satisfactory. A simplifying assumption was made in dealing with the mixing of the incoming charge and the residual exhaust gas, namely, that the complete charge at the assumed initial temperature and pressure was taken into the cylinder before any mixing with the residual exhaust gas began. This simplifies the calculation to a marked degree and introduces errors of inappreciable magnitude.

All heat loss was assumed to take place during combustion and expansion, the total loss being taken as 35 per cent of the available heat in every instance—10 per cent during combustion and 25 per cent during expansion. Pumping losses were obtained from the theoretical indicator diagrams without considering a diagram factor. These diagrams were plotted from the calculated pressures and the pumping losses calculated from the area of the cards lying below the assumed back pressure line at 15.7 lb. per sq. in.

36. *Results.*—The calculated results which are of most importance for such a comparison as that undertaken here, are the compression pressure, the release pressure and temperature, the indicated thermal efficiency, and the indicated mean effective pressure. These items for each of the three engines at different loads are arranged in Table 8 and are represented graphically in Figs. 19 to 23.

In Fig. 19 are curves showing the variation of the compression ratio and compression pressure with changes in load. The compression

*Wilson and Barnard, Jour. S. A. E., Vol. 9, 1921, p. 313.
†Rosecrans, C. Z., Automotive Industries, vol. 53, 1925, p. 1053.

TABLE 8
CALCULATED RESULTS FOR STANDARD AND MORE COMPLETE EXPANSION ENGINES

Point of Cutoff % stroke	Compression Ratio r_c :1	Conditions at Beginning of Cycle			End of Compression		End of Combustion		End of Expansion		Load Condition % Full Load	Indicated m.c.p. lb. per sq. in.	Indicated Thermal Eff. %	Pumping Loss B.t.u. per Cycle
		Exhaust Gas Temp. deg. F. abs.	Mixture Temp. deg. F. abs.	Pres. lb. per sq. in.	Temp. deg. F. abs.	Pres. lb. per sq. in.	Temp. deg. F. abs.	Pres. lb. per sq. in.	Temp. deg. F. abs.	Pres. lb. per sq. in.				
100	5.00	1500	628.5	13.0	1069	110.4	4828	524.1	2444	53.1	100	115.9	28.14	212.3
100	5.00	1450	608.2	10.0	1038	85.3	4712	406.6	2367	40.8	75.1	87.0	27.47	503.4
100	5.00	1400	572.1	7.0	982	60.1	4542	290.9	2263	29.0	49.6	57.5	25.91	893.3
100	5.00	1350	528.3	5.0	913	43.2	4260	210.3	2077	20.5	35.3	40.9	25.81	1240.9
Standard Engine with Throttling Control.														
More Complete Expansion Engine with Variable Cutoff.														
65	5.00	1300	619.8	13.0	1056	110.7	4776	526.2	2180	33.6	100	89.1	33.30	210.7
50	4.08	1250	631.6	13.0	1008	84.7	4635	408.2	2052	25.3	81.4	72.5	35.22	207.2
35	3.15	1200	649.6	13.0	954	60.2	4441	292.9	1944	17.9	58.4	52.0	36.09	207.5
20	2.23	1000	655.5	13.0	850	37.6	3818	175.2	1596	10.2	35.5	31.9	38.70	588.8
More Complete Expansion Engine with Throttling Control.														
65	5.00	1300	619.8	13.0	1056	110.7	4776	526.2	2180	33.6	100	89.1	33.30	210.7
65	5.00	1250	595.7	10.0	1019	85.5	4608	405.6	2030	25.0	78.9	70.4	34.18	501.8
65	5.00	1200	555.2	7.0	955	60.2	4409	290.6	1919	17.7	52.9	47.2	32.75	901.4
65	5.00	1100	500.2	5.0	854	42.7	4071	211.7	1724	12.5	37.2	33.1	32.19	1340.7

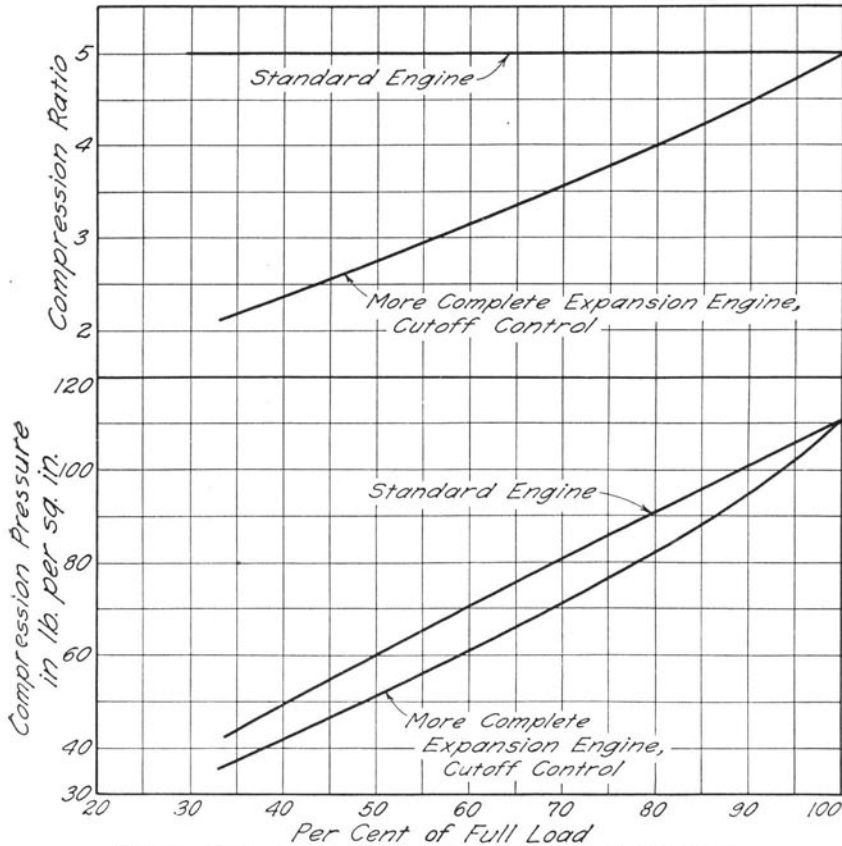


FIG. 19. VARIATION OF COMPRESSION RATIO AND COMPRESSION PRESSURE WITH CHANGES IN LOAD

ratio for each of the throttling engines is the same at all loads, being 5 to 1. Since the cylinder volume at the beginning of compression in the variable-cutoff engine is different for each load, the compression ratio, which is 5 to 1 at full load, was found to decrease to 2.23 to 1 at about one-third load. At full load the compression pressure found for each of the three engines is the same; and while this pressure was also found to decrease uniformly with the load in every case, the drop in pressure found for the variable-cutoff engine is slightly greater than that found for the throttling engines. The lower compression pressure found at part loads for the former is due to starting the effective compression so late in the stroke. In the case of the throttle-controlled engines it is due to throttling the incoming charge at light loads.

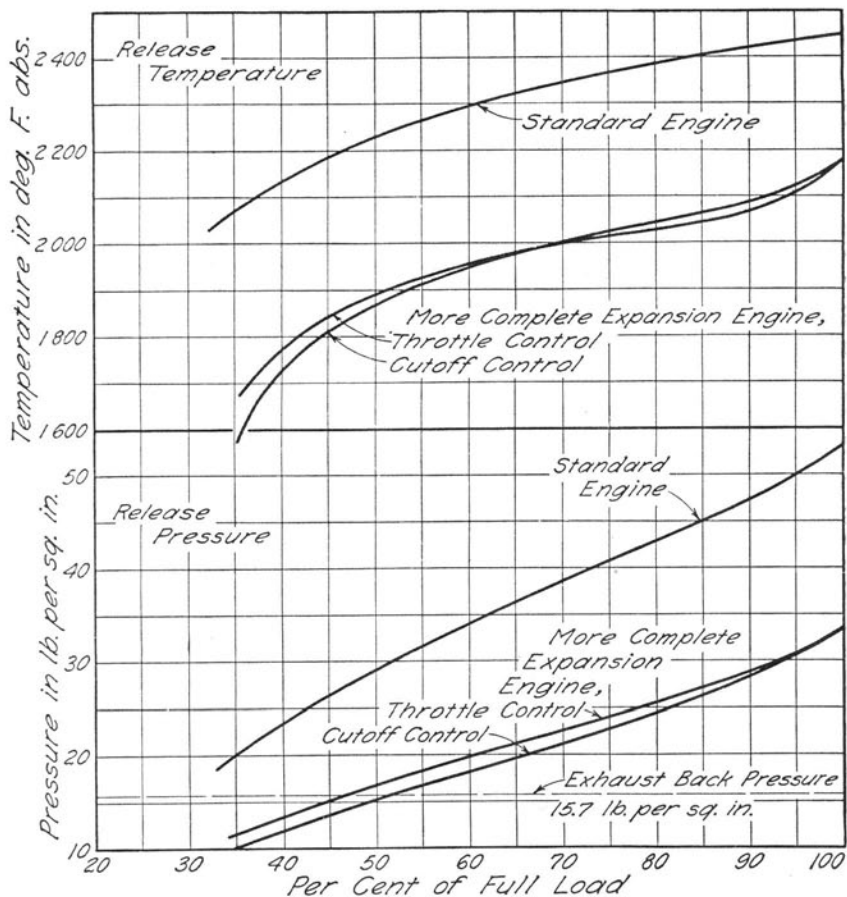


FIG. 20. VARIATION OF RELEASE TEMPERATURE AND PRESSURE WITH CHANGES IN LOAD

The effects of the greater expansion of the more complete expansion engines on release pressures and temperatures are evident from the curves in Fig. 20. Thus the release pressure of both the throttle-controlled and the variable-cutoff more complete expansion engines was found to be below the assumed back pressure of 15.7 lb. per sq. in. at all loads below 45 per cent of full load; while the release pressure of the standard engine was always higher than this back pressure. What is of greater significance, however, is that the release temperature of the more complete expansion engines was found to be considerably lower than that of the standard engine under corresponding load conditions, this difference being 260 deg. F. at full load and increasing to 470 deg. F. at one-third load.

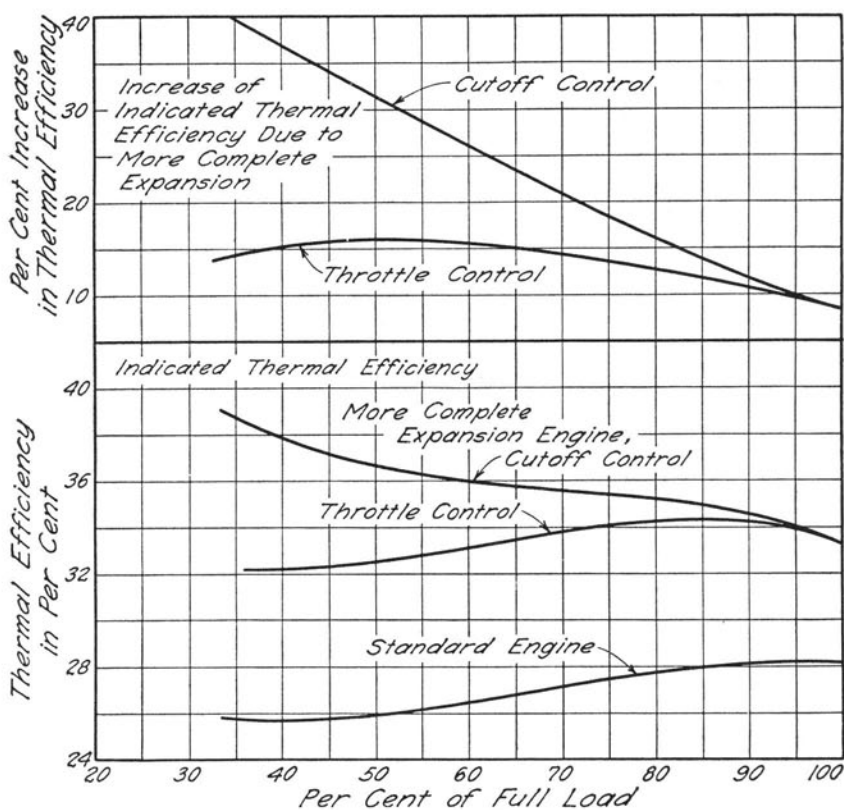


FIG. 21. VARIATION OF INDICATED THERMAL EFFICIENCY WITH CHANGES IN LOAD

The lower release temperatures found for the more complete expansion engines are the direct result of converting more of the internal energy of the products of combustion into useful work. The gain in indicated efficiency as shown by the curves in Fig. 21, is also a result of this more complete conversion of energy. Thus the throttle-controlled more complete expansion engine was found to show a fairly constant improvement in efficiency over the standard engine at all loads, the percentage of gain being 18.5 per cent at full load and 24.5 per cent at about one-third load. The advantage of the variable-cutoff engine, in this respect, was found to increase as the load decreased. The calculated efficiency of the variable-cutoff engine is 33.3 per cent at full load and 38.7 per cent at one-third load, these figures representing increases of 18.5 and 50 per cent, respectively, over the calculated efficiencies of the conventional engine. The explanation of the somewhat unusual

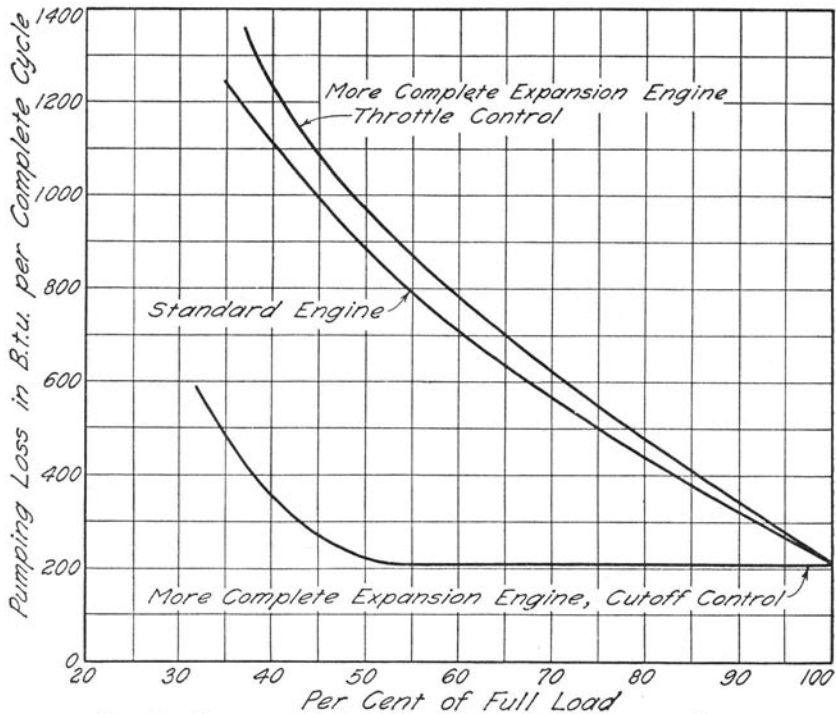


FIG. 22. VARIATION OF PUMPING LOSS WITH CHANGES IN LOAD

condition of an engine operating with higher efficiency at part load than that realized at full load is found when the variation of the pumping loss with changes in load is investigated. The calculated values of this loss for each of the three engines are plotted in Fig. 22. From these curves it is evident that the pumping loss found for the variable cutoff engine is practically constant for all loads above 50 per cent full load. In fact, for this engine the pumping loss was found to increase only when the release pressure fell below the exhaust back pressure and there was a negative loop at the toe of the indicator diagram. On the other hand, the pumping losses of both the throttling engines were found to increase rapidly as the load fell off, due, of course, to the throttling of the incoming charge at part loads.

From a consideration of the calculated mean effective pressures of the three engines it seems evident that the gain in thermal efficiency, which was found as one of the results of more complete expansion, is accompanied by a decrease in mean effective pressure. The curves in Fig. 23 represent these calculated results graphically, and from them it is apparent that the mean effective pressures found for the more com-

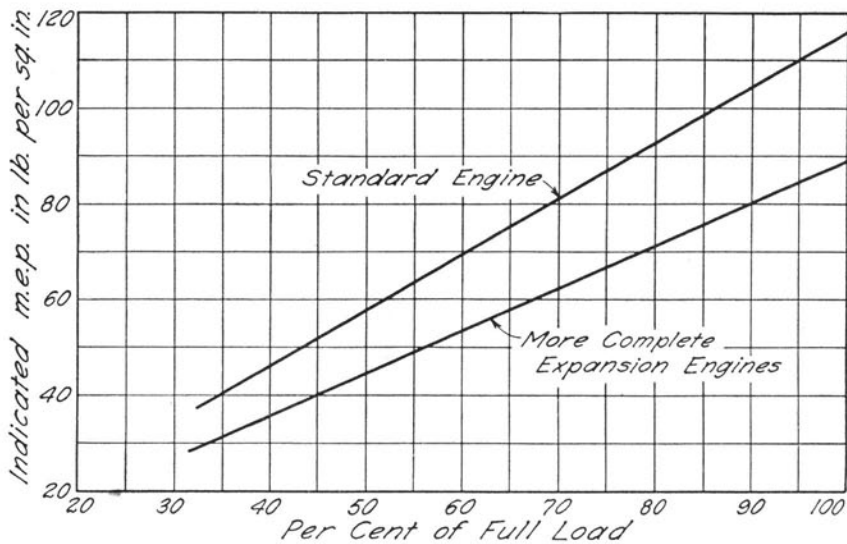


FIG. 23. VARIATION OF INDICATED MEAN EFFECTIVE PRESSURE WITH CHANGES IN LOAD

plete expansion engine are about the same with either type of control. At full load the value found for this engine was 26.8 lb. per sq. in. below that found for the standard engine and at one-third load this difference was found to be 7.8 lb. per sq. in. From the calculated mean effective pressures may be estimated the relative displacements of a more complete expansion engine and a standard engine, each of which would develop the same power. Thus, since the standard engine was found to have calculated mean effective pressures which are about 30 per cent higher at all loads than those found for the more complete expansion engines, the latter, in order to develop the same torque, would require at least 30 per cent greater displacement than the former. As a concrete example, suppose the standard engine to be a six cylinder automobile engine with a bore of 3 inches and a stroke of 5 inches, the displacement of such an engine being 288.6 cu. in. Then a more complete expansion engine of either type having maximum cutoff at 65 per cent of the stroke and developing the same power as this standard engine would have, on the basis of these calculated mean effective pressures, a displacement of 377.3 cu. in. Assuming a weight of 2.43 lb. per cu. in. of displacement,* the respective weights of two engines with these displacements would be 701 and 917 pounds.

*Average of a number of commercial automotive engines of similar dimension.

Without doubt the calculated results, which have here been summarized, represent the maximum values that can be expected, and in some instances they are probably in excess of those realizable in actual engines. Nevertheless, since every effort has been made to keep the procedure uniform for all engines considered, the results may be taken as a fairly reliable basis of comparison for estimating the relative efficiency and performance of the more complete expansion engines.

37. *Discussion of Results.*—It may be well to emphasize by repetition that this investigation is purely theoretical, the only purpose being to determine whether or not the more complete expansion cycle possesses sufficient inherent advantages to warrant experimental investigation. Thus indicated efficiencies and mean effective pressures have been calculated with no attempt at estimating mechanical losses. These, it was felt, could be properly determined only by actual tests, and for the present, at least, interest is confined to a consideration of the advisability of such tests. Some mechanical complication would be involved in changing the point of inlet valve closure; the extent and disadvantages of such complication as well as its influence on the mechanical losses can likewise only be determined experimentally. Hence, these and other similar questions are allowed to remain unanswered for want of accurate information.

There is available, however, some experimental data from an engine operating on the more complete expansion cycle under consideration. This is a 10 by 20 inch, two cylinder, double acting, tandem engine which was built and operated a number of years ago.* The indicator diagrams in Fig. 18, as well as the results tabulated in Table 9, are from tests which were made on the engine shortly after it was constructed. While the tests were not as complete as could be desired, since all loads are so nearly the same that no conclusions as to the variation of the thermal efficiency with changes in load may be reached, there is reasonably close agreement between the full load test efficiencies, whose average value is 34.1 per cent, and the calculated efficiency of 33.3 per cent obtained in the theoretical analysis. Such agreement between experimental and theoretical results offers good evidence in favor of the validity of the methods employed and justification of the assumptions made in arriving at the calculated results.

The possibility of the indicated thermal efficiency of an internal combustion engine increasing as the load decreases is by no means remote, this being one of the outstanding characteristics of the Diesel engine. In Fig. 24 are curves showing the variation of indicated thermal

*Sargent, C. E., "The Complete Expansion Engine," A. S. M. E. Trans. Vol. 22, 1901, p. 312.

TABLE 9
EXPERIMENTAL RESULTS WITH MORE COMPLETE EXPANSION ENGINE

I. H. P.	Gas Consumed cu. ft. per I.H.P.-hr.	Heating Value B.t.u. per cu. ft.	Available Heat B.t.u. per I.H.P.-hr.	Indicated Thermal Efficiency per cent
69.5.....	8.49	877.26	7450	34.1
68.6.....	8.66	895.00	7750	32.8
65.8.....	8.27	895.00	7400	34.4
64.8.....	8.79	844.85	7430	34.2
62.3.....	9.12	774.58	7060	36.0

efficiency with load for two Diesel engines†. Each of these curves is very similar to the efficiency-load curve found for the cutoff controlled more complete expansion engine. That such similarity is to be expected follows from the fact that both the Diesel engine and the variable cutoff engine are controlled by varying the quantity of fuel without throttling the incoming charge.

The two factors, usually considered in comparing internal combustion engine performance, are thermal efficiency, or power developed per unit of available heat energy, and mean effective pressure, or power developed per unit of engine displacement. Since these two factors depend upon principles of design that are more or less antithetic, it is difficult if not impossible to design an engine in which both thermal efficiency and mean effective pressure attain maximum values. The nature of the service expected from the engine must determine whether fuel economy or bulk economy shall have preponderate consideration. By comparing a more complete expansion engine with a conventional engine, calculated values of these two factors being taken as a basis of comparison, it should be possible to form a fairly accurate estimate of the type of service in which the former could be used to better advantage than the latter.

38. *Conclusion.*—From the results of this analysis it would seem quite reasonable to expect a more complete expansion engine with throttle control to show a higher thermal efficiency and greater fuel economy than a similar engine operating on the standard Otto cycle. If advantage is taken of the further improvement in efficiency due to varying the point of cutoff, constant efficiency at all loads, or, possibly, higher efficiency at part load than at full load may be obtained. In order,

†Lucke, C. E., "Large Oil Engines," A.S.M.E. vol. 46, p. 1052

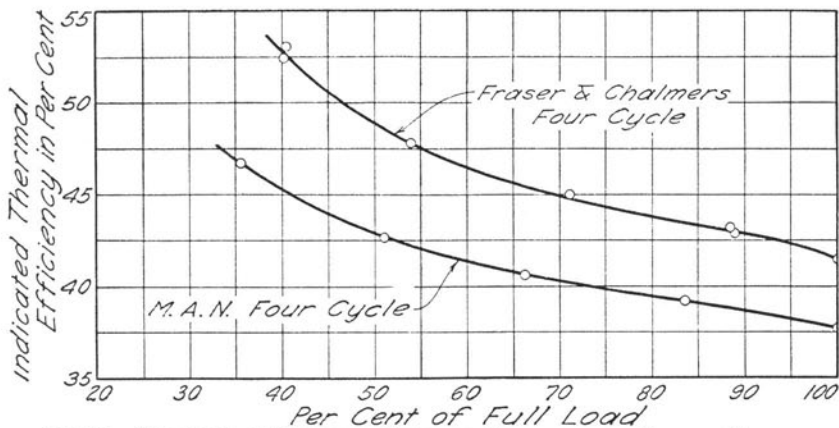


FIG. 24. VARIATION OF INDICATED THERMAL EFFICIENCY OF TYPICAL DIESEL ENGINES WITH CHANGES IN LOAD

however, to have an equal power output from both engines the more complete expansion engine would require a somewhat greater displacement, the increase in displacement depending on the point of maximum cutoff.

For stationary or marine engines such an increase in displacement with its attendant increase in weight would not entail any great disadvantage other than the consequent increase in first cost, and this would probably be compensated for by the lower operating cost due to better fuel economy. In this respect, however, the requirements of automotive transportation are quite different. While an improvement in fuel economy which is greater at part load than at full load is highly desirable, since engines in this service operate at loads above 50 per cent of their maximum such a small part of the time, the lower torque, or increased engine size for the same torque, as compared with the standard engine is a decided disadvantage in so far as present requirements and tendencies are considered. It would, of course, be possible to fix the point of maximum cutoff of a variable cutoff more complete expansion engine at the same point at which the inlet valve in the standard engine closes, i.e., 30 to 60 degrees after lower dead center. With this arrangement the full load, or maximum cutoff, efficiency and mean effective pressure of the more complete expansion engine would be about the same as those of a standard engine of equal size, while the part load characteristics would be similar to those found for the variable cutoff engine considered in this investigation. An engine which thus combines the full load torque of the standard engine with the part load efficiencies of

the more complete expansion engine would certainly offer possibilities in the automotive field particularly for taxi, bus, and truck service. The demands of the aeroplane place such great emphasis on light weight that the more complete expansion engine can scarcely be considered in this connection. Furthermore, the aeroplane engine operates so continuously at nearly full load that the advantage of high efficiency at part load, the outstanding characteristic of the variable cutoff more complete expansion engine, would be of little or no consequence.

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