VNIVERSITAT Ð ΌVALÈNCIA

Theory and phenomenology of Dirac neutrinos: Symmetry breaking patterns, flavour implications and Dark Matter

Tesi Doctoral
Programa de Doctorat en Física

Salvador Centelles Chuliá

Director: Dr. José Wagner Furtado Valle
Co-Director: Dr. Rahul Srivastava
Tutora: Dra. María Amparo Tórtola Baixauli

IFIC - CSIC/Universitat de València
Department de Física Teòrica

València, Junio 2021

A mis padres por ayudarme y apoyarme siempre.
A Olaia porque contigo todo es más fácil.
A mis hermanos aunque crean que estoy tronao.
A mi abuelita porque dirá que si lo explico yo se entiende todo y a mi abuelito que le hubiera encantado leerla.
A Ravioli por ayudarme a escribir la tesis y a Daphne por intentar sabotearla.

Dr. José Wagner Furtado Valle,

Profesor de investigación del Consejo Superior de Investigaciones Científicas (CSIC),

Dr. Rahul Srivastava,

Assistant professor, Indian Institute of Science Education and Research - Bhopal,

CERTIFICAN:

Que la presente memoria "Theory and phenomenology of Dirac neutrinos: Symmetry breaking patterns, flavour implications and Dark Matter" ha sido realizada bajo su dirección en el Institut de Física Corpuscular, centro mixto del CSIC y de la Universitat de València, por Salvador Centelles Chuliá y constituye su Tesis para optar al grado de Doctor en Física.

Y para que así conste, en cumplimiento de la legislación vigente, presenta en el Departament de Física Teòrica de la Universidad de Valencia la referida Tésis Doctoral, y firma el presente certificado.

Paterna (Valencia), a 22 de Agosto de 2021.

Dr. José Wagner Furtado Valle

Bhopal (India), a 22 de Agosto de 2021.

Dr. Rahul Srivastava

Acknowledgements

This thesis wouldn't have been possible without the amazing people that surrounded me in the last few years. For all of you, you have my deepest gratitude.

Thanks to Jose for guiding me and making me read Schechter-Valle 1980 when I was in physics kindergarten, always opening opportunities for me in terms of assistance to schools, conferences, summer positions and financial support even before starting my PhD, and for directing my bachelor and master's thesis. In the same line, thanks to Rahul for patiently explaining everything I needed to know everytime I bugged you with simple questions, physics or otherwise, and I am happy to call you friend. I cannot think of better PhD advisors that you both and your guidance has been paramount to my career.

Similarly, it was a pleasure to collaborate with so many great scientists and persons: Ricardo, Werner, Gui-Jun, Peng, Eduardo, Ulises, Manfred, Matthias, Björn, Christian, Avelino, César, Andreas, Pedro and Ernest. I have learned a lot from all of you but most importantly we've had a lot of fun discussing physics.

I also want to dedicate some words to my office mates over the years Pedro, Lucho, Pablo I, Ricardo, Christoph, Ivania and Jacopo, as well as the members of the AHEP group in Valencia and the MPIK in Heidelberg. All the good moments will not be forgotten, nor Martin's monday cakes.

I feel the need to remember the IFIC Secretaría people who helped me navigate the bureaucracy seas: Amparo, Teresa, Luis, Pilar and Elena.

Out of the physics realm I have to foremost thank my extended family and specially my parents. Their constant support, love and guidance have made me who I am, and their advice proved to be good even when I ignored it. I also have to thank my childhood friends for being how they are, because when we all took diverging paths, friendship continued and we still find time to converge.

My final thank you goes to Olaia, because enduring me for all these years is a PhD in itself.

This thesis is also yours. So long and thanks for all the fish.

Contents

Chapter 1

List of scientific publications

1 Publications included in the thesis

This thesis is based on the following scientific publications:

1. *Dirac Neutrinos and Dark Matter Stability from Lepton Quarticity* [1]

   Salvador Centelles Chuliá (Valencia U., IFIC), Ernest Ma (UC, Riverside), Rahul Srivastava (Ahmedabad, Phys. Res. Lab), José W. F. Valle (Valencia U., IFIC)

   e-Print: 1606.04543 [hep-ph]

   DOI: 10.1016/j.physletb.2017.01.070

   Published in: Phys.Lett.B 767 (2017), 209-213

2. *Generalized Bottom-Tau unification, neutrino oscillations and dark matter: predictions from a lepton quarticity flavour approach* [2]

   Salvador Centelles Chuliá (Valencia U., IFIC), Rahul Srivastava (Valencia U., IFIC), José W. F. Valle (Valencia U., IFIC)

   e-Print: 1706.00210 [hep-ph]

   DOI: 10.1016/j.physletb.2017.07.065

Published in: Phys.Lett.B 773 (2017), 26-33

3. *Seesaw roadmap to neutrino mass and dark matter* [3]

   Salvador Centelles Chuliá (Valencia U., IFIC), Rahul Srivastava (Valencia U., IFIC), José W.F. Valle (Valencia U., IFIC)

   e-Print: 1802.05722 [hep-ph]

   DOI: 10.1016/j.physletb.2018.03.046

   Published in: Phys.Lett.B 781 (2018), 122-128

4. *Seesaw Dirac neutrino mass through dimension-six operators* [4]

   Salvador Centelles Chuliá (Valencia U., IFIC), Rahul Srivastava (Valencia U., IFIC), José W.F. Valle (Valencia U., IFIC)

   e-Print: 1804.03181 [hep-ph]

   DOI: 10.1103/PhysRevD.98.035009

   Published in: Phys.Rev.D 98 (2018) 3, 035009

5. *Dark matter stability and Dirac neutrinos using only Standard Model symmetries* [5]

   Cesar Bonilla (Munich, Tech. U.), Salvador Centelles-Chuliá (Valencia U., IFIC), Ricardo Cepedello (Valencia U., IFIC), Eduardo Peinado (Mexico U.), Rahul Srivastava (Valencia U., IFIC)

   e-Print: 1812.01599 [hep-ph]

   DOI: 10.1103/PhysRevD.101.033011

   Published in: Phys.Rev.D 101 (2020) 3, 033011

6. *The inverse seesaw family: Dirac and Majorana* [6]

   Salvador Centelles Chuliá (Valencia U., IFIC), Rahul Srivastava (IISER, Bhopal), Avelino Vicente (Valencia U., IFIC and Valencia U.)

2 Other publications not included in the thesis

1. *CP violation from flavour symmetry in a lepton quarticity dark matter model* [7]

   Salvador Centelles Chuliá (Valencia U., IFIC), Rahul Srivastava (Ahmedabad, Phys. Res. Lab and IMSc, Chennai), José W.F. Valle (Valencia U., IFIC)

   e-Print: 1606.06904 [hep-ph]

   DOI: 10.1016/j.physletb.2016.08.028

   Published in: Phys.Lett.B 761 (2016), 431-436

2. *Neutrino oscillations from warped flavour symmetry: predictions for long baseline experiments T2K, NOvA and DUNE* [8]

   Pedro Pasquini (Campinas State U. and Valencia U., IFIC), S.C. Chuliá (Valencia U., IFIC), J.W.F. Valle (Valencia U., IFIC and Valencia U.)

   e-Print: 1610.05962 [hep-ph]

   DOI: 10.1103/PhysRevD.95.095030

   Published in: Phys.Rev.D 95 (2017) 9, 095030

3. *Neutrino Predictions from Generalized CP Symmetries of Charged Leptons* [9]

Peng Chen (Ocean U.), Salvador Centelles Chuliá (Valencia U., IFIC), Gui-Jun Ding (Hefei, CUST), Rahul Srivastava (Valencia U., IFIC), José W.F. Valle (Valencia U., IFIC)

e-Print: 1802.04275 [hep-ph]

DOI: 10.1007/JHEP07(2018)077

Published in: JHEP 07 (2018), 077

4. *Realistic tribimaximal neutrino mixing* [10]

   Peng Chen (Ocean U.), Salvador Centelles Chuliá (Valencia U., IFIC), Gui-Jun Ding (Hefei, CUST), Rahul Srivastava (Valencia U., IFIC), José W.F. Valle (Valencia U., IFIC)

   e-Print: 1806.03367 [hep-ph]

   DOI: 10.1103/PhysRevD.98.055019

   Published in: Phys.Rev.D 98 (2018) 5, 055019

5. *CP symmetries as guiding posts: revamping tri-bi-maximal mixing. Part I.* [11]

   Peng Chen (Ocean U.), Salvador Centelles Chuliá (Valencia U., IFIC), Gui-Jun Ding (Hefei, CUST), Rahul Srivastava (Valencia U., IFIC), José W.F. Valle (Valencia U., IFIC)

   e-Print: 1812.04663 [hep-ph]

   DOI: 10.1007/JHEP03(2019)036

   Published in: JHEP 03 (2019), 036

6. *Systematic classification of two loop $d = 4$ Dirac neutrino mass models and the Diracness-dark matter stability connection* [12]

Salvador Centelles Chuliá (Valencia U., IFIC), Ricardo Cepedello (Valencia U., IFIC), Eduardo Peinado (Mexico U.), Rahul Srivastava (Valencia U., IFIC)

e-Print: 1907.08630 [hep-ph]

DOI: 10.1007/JHEP10(2019)093

Published in: JHEP 10 (2019), 093, JHEP1910(2019)093

7. *Scotogenic dark symmetry as a residual subgroup of Standard Model symmetries* [13]

Salvador Centelles Chuliá (Valencia U., IFIC), Ricardo Cepedello (Valencia U., IFIC), Eduardo Peinado (UNAM, Mexico), Rahul Srivastava (Valencia U., IFIC and IISER, Bhopal)

e-Print: 1901.06402 [hep-ph]

DOI: 10.1088/1674-1137/44/8/083110

Published in: Chin.Phys.C 44 (2020) 8, 083110

8. *CP symmetries as guiding posts: Revamping tribimaximal mixing. II.* [14]

Peng Chen (Zhejiang Ocean U.), Salvador Centelles Chuliá (Valencia U., IFIC), Gui-Jun Ding (Hefei, CUST), Rahul Srivastava (Valencia U., IFIC), José W.F. Valle (Valencia U., IFIC)

e-Print: 1905.11997 [hep-ph]

DOI: 10.1103/PhysRevD.100.053001

Published in: Phys.Rev.D 100 (2019) 5, 053001

9. *Two-Higgs-doublet models with a flavoured $\mathbb{Z}_2$ symmetry* [15]

S. Centelles Chuliá (Valencia U., IFIC), W. Rodejohann (Heidelberg, Max Planck Inst.), U.J. Saldaña-Salazar (Heidelberg, Max Planck Inst.)

e-Print: 1911.06824 [hep-ph]

DOI: 10.1103/PhysRevD.101.035013

Published in: Phys.Rev.D 101 (2020) 3, 035013

10. *Asymmetric tri-bi-maximal mixing and residual symmetries* [16]

    Salvador Centelles Chuliá (Valencia U., IFIC), Andreas Trautner (Heidelberg, Max Planck Inst.)

    e-Print: 1911.12043 [hep-ph]

    DOI: 10.1142/S0217732320502922

    Published in: Mod.Phys.Lett.A 35 (2020) 35, 2050292

11. *Natural axion model from flavour* [17]

    Salvador Centelles Chuliá (Valencia U., IFIC), Christian Döring (Heidelberg, Max Planck Inst.), Werner Rodejohann (Heidelberg, Max Planck Inst.), Ulises J. Saldaña-Salazar (Heidelberg, Max Planck Inst.)

    e-Print: 2005.13541 [hep-ph]

    DOI: 10.1007/JHEP09(2020)137

    Published in: JHEP 09 (2020), 137

12. *Gravitational wave induced baryon acoustic oscillations* [18]

    Christian Döring (Heidelberg, Max Planck Inst.), Salvador Centelles Chuliá (Valencia U., IFIC), Manfred Lindner (Heidelberg, Max Planck Inst.), Björn Malte Schaefer (Heidelberg, Astron. Rechen Inst.), Matthias Bartelmann (Heidelberg U.)

    e-Print: 2107.10283 [gr-qc]

    Sent for publication to SciPost Physics

Chapter 2

Introduction

The building of the Standard Model (SM) of particle physics has been a tremendous joint effort between theorists and experimentalists [19–25], culminating with the discovery of the Higgs boson in 2012 [26,27]. While the success of the theory cannot be denied, there are still some shortcomings and open questions in particle physics. An incomplete list includes neutrino masses, Dark Matter (DM) [28], the hierarchy problem [29] and the strong CP problem [30–32]. In this dissertation we will focus on the first, with special attention to the nature of neutrinos: they may be Dirac particles like the rest of the fermions of the Standard Model, in contrast to the canonical expectations that they are Majorana particles [33, 34], meaning that the neutrino mass Eigenstates are their own self conjugate fields.

While a detailed exposition of the SM is out of the scope of this work it can be found in standard quantum field theory textbooks, reviews and textbooks (see for example [35]). However, since the focus of this work will be directed towards neutrino masses, we will first give a short overview on the mass mechanism in the SM, i.e. the so-called Englert-Brout-Higgs mechanism [20–22] in Sec. 1. While the SM predicts massless neutrinos, neutrino oscillation experiments show that they are indeed massive particles. We will briefly review neutrino oscillations in

Sec. 2. To conclude the introduction chapter, in Sec. 3 we will cover the seesaw completions of the SM which predict Majorana neutrino masses.

1   Fermion masses in the Standard Model

The unbroken $SU(3)_{QCD} \times SU(2)_L \times U(1)_Y$ gauge structure of the SM can qualitatively reproduce the particle interactions seen in collider experiments. A big drawback is that it automatically predicts massless fermions and massless gauge bosons, all of which are experimentally known to be massive [36]. The reason is that their Lagrangian mass terms are forbidden by the gauge symmetry. For example, take the fermionic mass term for the electron:

$$\mathcal{L}_{m_e} = m_e \bar{e}_L \, e_R \tag{2.1}$$

The field e_L forms part of an $SU(2)_L$ doublet, $L_e = (e_L, \nu_{L,e})^T$ (analogously, the left handed quarks form part of a different $SU(2)_L$ doublet: $Q_u = (u_L, d_L)^T$, and part of a colour triplet) and therefore transforms non-trivially under an $SU(2)_L$ local transformation. In contrast, e_R is a singlet, as well as the other right-handed fermions in the theory, u_R and d_R. We could repeat the exercise for any fermion and reach the same conclusion. Traditionally ν_R is not added to the model as during the early days of the construction of the SM neutrinos were thought to be massless.

Since we know that the fermions are massive, we know that the symmetry must be broken. The bare mass terms like Eq. 2.1 are forbidden by $SU(2)_L \times U(1)_Y$, but do not break the $SU(3)_{QCD} \times U(1)_{QED}$ symmetry group since i.e. left and right chiralities transform in the same manner. Moreover, the associated gauge bosons of the unbroken group, the gluons and the photon, are known to be massless [36].

These are strong hints that point towards the need of a breaking of the electroweak group as

$$SU(2)_L \times U(1)_Y \to U(1)_{QED} \tag{2.2}$$

while respecting the color group $SU(3)_{QCD}$. Finally, let us note that an explicit breaking, for example simply adding the term in 2.1 to the lagrangian, would spoil the desired features of the gauge framework and render the model non-renormalizable [25]. Thus, the solution is to **spontaneously** break the symmetry, meaning that the Lagrangian is invariant under a certain symmetry group G, but the lowest energy state, the vacuum, is only invariant under a subgroup $F \subset G$. For a more detailed exposition of this mechanism we direct the reader to the vast literature in the topic and proceed to sketch how it generates masses for the SM fermions and gauge bosons.

In order to achieve Spontaneous Symmetry Breaking (SSB), to the fermion and gauge inventory of the massless SM we now add a complex scalar ϕ. The scalar potential will be given by

$$-\mathcal{L}_\phi = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \tag{2.3}$$

These two terms will *always* be allowed by the gauge symmetry irrespective of the charges that we assign to ϕ and since its hypercharge is non-zero they will be the *only* allowed terms. Let us now study a simplified case in which ϕ is an $SU(2)_L$ singlet. If the mass parameter μ^2 is negative (i.e. μ is purely imaginary) then we can see that the minimum of the Hamiltonian in the classical theory is not found in $\phi = 0$ but in $|\phi_{\min}| = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$. This is the lowest energy state of the theory and perturbations must be made around this point (and not around $\phi = 0$). We can thus rewrite ϕ as $\frac{1}{\sqrt{2}}(v + \phi_1 + i\phi_2)$ and immediately see that ϕ_2 is a massless field. This is a direct consequence of the Goldstone theorem [37–39].

Since we want to break $SU(2)_L \times U(1)_Y \to U(1)_{QED}$ we must give the scalar ϕ a non-trivial charge under this group, while it must be a singlet of the color group in order to respect $SU(3)_{QCD}$. Moreover, we must ensure that the fields which couple to the photon do not get a vacuum expectation value (vev) in order to respect the subgroup $U(1)_{QED}$. If ϕ is a doublet of $SU(2)_L$ with hypercharge $Y = 1/2$, then we can write

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.4}$$

Where the names ϕ^+ and ϕ^0 are given according to their electromagnetic interactions: ϕ^0 does not couple with the photon at tree level and the superscripts represent the electric charge. Since only ϕ^0 gets a vev and not ϕ^+ and therefore the photon remains massless. If we rewrite

$$\phi = \frac{1}{\sqrt{2}} e^{\frac{i}{2}\sigma_i \theta_i} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \tag{2.5}$$

where σ_i are the three Pauli matrices, now the 4 degrees of freedom are in the three real fields θ_i and h instead of in the two coplex fields ϕ^+ and ϕ^0. Taking advantage of the $SU(2)_L$ local symmetry of the lagrangian we can rotate away any dependence on θ_i. This is the so-called unitary gauge.

We turn our focus to the following Yukawa terms, which are allowed by the gauge symmetry

$$\mathcal{L}_Y = Y_e \, \bar{L} \, \phi \, e_R \, + \, Y_u \, \bar{Q} \, \phi^c \, u_R \, + \, Y_d \, \bar{Q} \, \phi d_R + h.c. \tag{2.6}$$

Where the sum over flavour indices is understood, $\tilde{\phi} = i\sigma_2 \phi^*$. Now expanding the $SU(2)_L$ indices in the unitary gauge we find

$$\mathcal{L}_Y = \frac{m_e}{v} \, (v + h) \, \bar{e}_L \, e_R \, + \, \frac{m_u}{v} \, (v + h) \, \bar{u}_L \, u_R \, + \, \frac{m_d}{v} \, (v + h) \, \bar{d}_L \, d_R + h.c. \tag{2.7}$$

Where we have explicitly rewritten the Yukawa matrices into mass matrices $m_x = \frac{Y_x v}{\sqrt{2}}$. We can now see that the fermions are massive thanks to the interaction with the Higgs boson. Additionally, an important prediction arises: the Yukawa interaction of a given fermion with the physical scalar h will be proportional to the fermion mass.

The last step is to find the fermion mass Eigenstates. If we perform a unitary transformation on flavour space for each fermion type and chirality we have

$$f_L \to U_L^f f_L^{(m)}, \qquad\qquad f_R \to U_R^f f_R^{(m)}, \qquad\qquad f \in \{e, u, d\}, \qquad (2.8)$$

where $U_{L/R}^f$ are unitary matrices which we call mixing matrices and $f_{L/R}^{(m)}$ are chiral mass Eigenstates. The right-handed mixing matrices are unphysical, but the left-handed quark mixing matrices enter in the $W^\pm$ interactions in the combination

$$U_{CKM} \equiv U_L^u \left(U_L^d \right)^\dagger \qquad (2.9)$$

U_{CKM} has been measured with great precision [36]. Analogously, in the lepton sector of a SM extension where neutrinos are massive we have the lepton mixing matrix given by

$$U_{lep} \equiv \left(U_L^l \right)^\dagger U_L^\nu, \qquad (2.10)$$

which has been measured in neutrino oscillations [40], see Sec. 2.

The Higgs mechanism also gives mass to the gauge bosons, this time through the covariant derivative $D^\mu \phi = \left(\partial^\mu + ig\tilde{W}^\mu + ig' y_\phi B^\mu \right) \phi$. Since our focus is on fermion masses we will not expand on this further.

2   The status of neutrino oscillations

We can now note that neutrinos are massless in the SM simply by construction: since the right handed neutrino ν_R is missing from the model, the Higgs mechanism that gives mass to the other fermions in the SM cannot work for neutrinos. This was a natural choice at the time. In the 60 and the 70s, when the SM was being built, there was no experimental indication that neutrinos where massive and only upper limits could be set on the neutrino mass. Today we know that at least two neutrinos are massive thanks to the discovery of neutrino oscillations [41, 42]. Irrespective of the underlying mass generation mechanism, if at least one neutrino is massive and non-degenerate and there is a mixing between gauge states to form the massive Eigenstates then there will be neutrino oscillations, which can and have been measured with great precision [40]. What is more, the L/E dependence of the oscillations cannot be explained, to the best of our knowledge, by any other alternative mechanism [1]. We now briefly lay out the neutrino oscillations mechanism in terms of simple quantum mechanics. For a more complete, quantum field theory derivation see for example [44–46], yielding the same result.

We start by assuming that some underlying mechanism gives mass to neutrino fields. Moreover, the gauge states $|\nu_\alpha\rangle$, $\alpha \in \{e, \mu, \tau\}$ mix to give the mass eigenstates $|\nu_i\rangle$, $i \in \{1, 2, 3\}$ as:

$$|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle \tag{2.11}$$

---

[1]If one only has access to a single measurement, for example Davis experiment [43] in which a neutrino deficit is observed, possible alternative explanations to neutrino oscillations could be, for example, non-standard interactions. However these alternative explanations cannot explain the detailed L/E profile we have measured today and have been ruled out by other observations.

In a neutrino beam of fixed trimomenta $\vec{p}$ the Hamiltonian Eigenstates will be simply the mass Eigenstates. Their time evolution is, therefore, trivial

$$|v_i\rangle (t) = |v_i\rangle (t = 0)e^{-iE_i t} \tag{2.12}$$

And the energy E_i is given in the ultra relativistic approximation as $E_i = \sqrt{m_i^2 + \vec{p}^2} \approx |\vec{p}| + \frac{1}{2}\frac{m_i^2}{E}$. Then, the energy difference between two mass Eigenstates will be

$$E_j - E_i = \frac{1}{2}\frac{m_j^2 - m_i^2}{|\vec{p}|} \equiv \frac{1}{2}\frac{\Delta m_{ij}^2}{|\vec{p}|} \equiv \Delta E_{ij} \tag{2.13}$$

Now let's assume that a neutrino of a given flavour α is produced, for example in a proton-proton fusion (which is the main production mechanism of electron neutrinos in the sun). The time evolution of such a state will be given by

$$|\psi\rangle (t = 0) = |\nu_\alpha\rangle, \qquad |\psi\rangle (t) = U_{\alpha i}e^{-iE_i t} |\nu_i\rangle \tag{2.14}$$

And the probability of detecting a given flavour β is then

$$P_{\alpha \to \beta} = |\langle \nu_\beta | \psi \rangle|^2 = U_{\beta j}^* U_{\alpha j} U_{\beta k} U_{\alpha,k}^* e^{-i\Delta E_{kj} t} \tag{2.15}$$

Since neutrinos are ultrarelativistic we can replace $t \to L$, $|\vec{p}| \to E$ and the final expression becomes

$$P_{\alpha \to \beta} = U_{\beta j}^* U_{\alpha j} U_{\beta i} U_{\alpha,i}^* e^{-i\frac{1}{2}\Delta m_{ij}^2 \frac{L}{E}} \tag{2.16}$$

Two important features can be immediately extracted. First, the aforementioned dependence of the oscillations on L/E. Second, the dependence not on the absolute mass of neutrinos but on the square mass differences.

It is common to parametrize the mixing matrix U in a convenient way. Let's start with some mathematical considerations. An $n \times n$ general complex matrix

has $2n^2$ independent real parameters (n^2 moduli and n^2 phases). If the matrix is unitary then the equation $U^\dagger U = I$ imply n^2 conditions and therefore the number of independent parameters is n^2. Note that the equation $UU^\dagger = I$ does not impose any new independent constraint. Coming to physics and assuming 3 generations of light neutrinos we get 9 parameters, of which 3 can be rotated away thanks to the the $U(1)$ symmetry of the lepton doublets. Then we are left with 3 angles and 3 phases. 2 of these phases are irrelevant to neutrino oscillations and are actually unphysical if neutrinos are Dirac particles. Let us define the matrices

$$U_{12}(\theta,\delta) \equiv \begin{pmatrix} \cos\theta & e^{-i\delta}\sin\theta & 0 \\ -e^{i\delta}\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{13}(\theta,\delta) \equiv \begin{pmatrix} \cos\theta & 0 & e^{-i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\tag{2.17}$$

$$U_{23}(\theta,\delta) \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & e^{-i\delta}\sin\theta \\ 0 & -e^{i\delta}\sin\theta & \cos\theta \end{pmatrix}, \quad P(\delta_1,\delta_2,\delta_3) \equiv \begin{pmatrix} e^{i\delta_1} & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_3} \end{pmatrix}$$

$$\tag{2.18}$$

The parametrization used by the Particle Data Group (PDG) [36] is given by

$$U_{PDG} = U_{23}(\theta_{12},0)U_{13}(\theta_{13},\delta_{CP})U_{12}(\theta_{12},0)P(\alpha,\beta,0) \tag{2.19}$$

Where the parameters θ_{ij} and δ_{CP} are relevant for neutrino oscillations. The phases α and β are unphysical in the Dirac neutrino case and irrelevant for neutrino oscillations in any case, but appear in neutrinoless double beta decay calculations. Alternatively one could use the so-called symmetric parametrization [34,47] which is more transparent from a theoretical and model building point of view:

$$U_{\text{symmetric}} = U_{23}(\theta_{23},\delta_{23})U_{13}(\theta_{13},\delta_{13})U_{12}(\theta_{12},\delta_{12}) \tag{2.20}$$

Which is related to U_{PDG} by $\delta_{CP} = \delta_{13} - \delta_{12} - \delta_{23}$.

These oscillations parameters are now measured with great precision by many complementary experiments. A global fit to all the oscillation parameters can be found in [40]. The relevant experiments for each of the parameters, the best-fit values and the χ^2 profiles are shown in Fig. 2.1

3 Majorana seesaw completions

We know that neutrinos are massive, as discussed in Sec. 2, and that the SM predicts massless neutrinos simply by construction. We can then naively think of adding ν_R and generate a Yukawa term of the form

$$\mathcal{L}_{Y\nu} = Y_\nu \, \bar{L} \phi^c \nu_R + h.c. \rightarrow \frac{m_D}{v} \left(v + h \right) \bar{\nu}_L \, \nu_R + h.c. \tag{2.21}$$

Where $m_D = \frac{Y_\nu v}{\sqrt{2}}$ and the subindex D stands for 'Dirac'. Note however that if we don't extend the symmetry inventory of the SM the following term is also allowed by the gauge symmetries

$$\mathcal{L}_M = \frac{1}{2} M \bar{\nu}_R^c \nu_R + h.c. \tag{2.22}$$

Where we have defined the conjugate of a field as $\psi^c = \mathcal{C} \bar{\psi}^T$, $\mathcal{C}$ is the charge conjugation matrix given by

$$\mathcal{C} = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} \tag{2.23}$$

and M is a symmetric complex matrix. Note that this term violates lepton number symmetry, which is accidentally conserved in the SM [2]. The presence of this term implies that the neutrino mass Eigenstates will be Majorana particles

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[2]An accidental symmetry is one which is not explicitly imposed upon the construction of the model but is conserved anyway.

parameter	best fit $\pm$ 1σ	2σ range	3σ range
$\Delta m_{21}^2 [10^{-5}\text{eV}^2]$	$7.50^{+0.22}_{-0.20}$	7.12–7.93	6.94–8.14
$|\Delta m_{31}^2| [10^{-3}\text{eV}^2]$ (NO)	$2.55^{+0.02}_{-0.03}$	2.49–2.60	2.47–2.63
$|\Delta m_{31}^2| [10^{-3}\text{eV}^2]$ (IO)	$2.45^{+0.02}_{-0.03}$	2.39–2.50	2.37–2.53
$\sin^2\theta_{12}/10^{-1}$	3.18 ± 0.16	2.86–3.52	2.71–3.69
$\theta_{12}/°$	34.3 ± 1.0	32.3–36.4	31.4–37.4
$\sin^2\theta_{23}/10^{-1}$ (NO)	5.74 ± 0.14	5.41–5.99	4.34–6.10
$\theta_{23}/°$ (NO)	49.26 ± 0.79	47.37–50.71	41.20–51.33
$\sin^2\theta_{23}/10^{-1}$ (IO)	$5.78^{+0.10}_{-0.17}$	5.41–5.98	4.33–6.08
$\theta_{23}/°$ (IO)	$49.46^{+0.60}_{-0.97}$	47.35–50.67	41.16–51.25
$\sin^2\theta_{13}/10^{-2}$ (NO)	$2.200^{+0.069}_{-0.062}$	2.069–2.337	2.000–2.405
$\theta_{13}/°$ (NO)	$8.53^{+0.13}_{-0.12}$	8.27–8.79	8.13–8.92
$\sin^2\theta_{13}/10^{-2}$ (IO)	$2.225^{+0.064}_{-0.070}$	2.086–2.356	2.018–2.424
$\theta_{13}/°$ (IO)	$8.58^{+0.12}_{-0.14}$	8.30–8.83	8.17–8.96
δ/π (NO)	$1.08^{+0.13}_{-0.12}$	0.84–1.42	0.71–1.99
$\delta/°$ (NO)	194^{+24}_{-22}	152–255	128–359
δ/π (IO)	$1.58^{+0.15}_{-0.16}$	1.26–1.85	1.11–1.96
$\delta/°$ (IO)	284^{+26}_{-28}	226–332	200–353

,

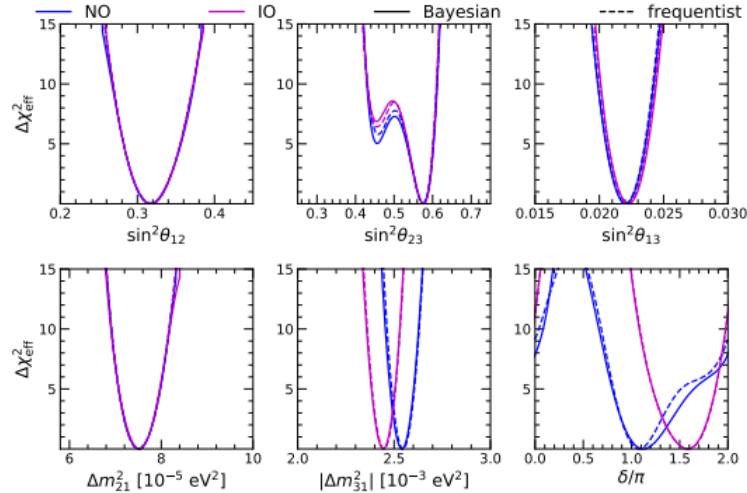| Parameter	Main contribution from	Other contributions from
Δm_{21}^2	KamLAND	SOL
$|\Delta m_{31}^2|$	LBL+ATM+REAC	-
θ_{12}	SOL	KamLAND
θ_{23}	LBL+ATM	-
θ_{13}	REAC	(LBL+ATM) and (SOL+KamLAND)
δ	LBL	ATM
MO	(LBL+REAC) and ATM	COSMO and $0\nu\beta\beta$

,

,

Figure 2.1: **Up:** Global fit to the oscillation parameters. **Middle:** Sensitivity of the different oscillation experiments to each parameter. **Down:** χ^2 profiles for the 6 oscillation parameters. All tables taken from [40] with their kind permission.

and it is thus conceptually extremely important. The existence of such lepton number violating terms may imply the existence of $\Delta L = 2$ low energy processes. There are several ongoing and planned experiments searching for this phenomena, such as neutrinoless double beta decay [48–53] or $\mu^- \to e^+$ conversion in muonic atoms [54] as well as high energy processes that could be detected in colliders [55–57]. However, the most conceptually important implication is the reverse, and comes from the Schechter-Valle black box theorem [58]. This theorem states that if neutrinoless double beta decay is present in nature then neutrinos must be Majorana particles, see Fig.2.2 for a diagramatic proof.
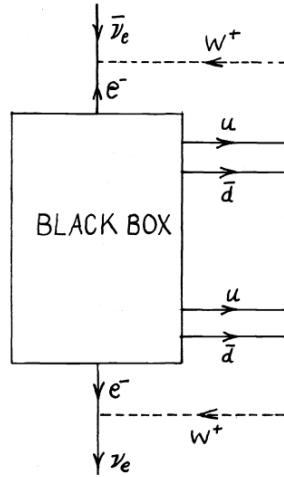

Figure 2.2: Diagram representing the black box theorem. If neutrinoless double beta decay exists in nature (even if it is not mediated by neutrinos) then the 'black box' mechanism generating it can be 'dressed' with W bosons, becoming a Majorana mass for neutrinos. Original figure taken from [58] with their kind permission.

Note however that the black box theorem cannot provide a quantitative prediction for the 'Majorananess' of neutrino masses [59]. As an example scenario, let's assume that neutrinos are Dirac at tree level. Lepton number is broken in the Lagrangian through a leptoquark which mediates the neutrinoless double beta decay process, see [60] as an example. Then, the black box theorem ensures that there will be a loop induced Majorana mass through the black-box loop' depicted

in Fig.2.2, but this contribution will necessarily be small: at least 4-loop. Neutrinos will thus be quasi-Dirac [61]. For an updated study of quasi-Dirac neutrinos phenomenology see for example [62, 63]. An extension of the black-box theorem to other lepton number violating processes is given in [64].

Coming back to our discussion, adding the two neutrino mass Lagrangians and writing in matrix form we get after SSB

$$\mathcal{L}_{m_\nu} = \mathcal{L}_{Y\nu} + \mathcal{L}_M \to \text{SSB} \to \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^\dagger & M \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c. \qquad (2.24)$$

Where we have ignored the interaction with the physical Higgs. Now we perform some transformations and definitions

$$\nu \equiv \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}, \qquad\qquad M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix} \qquad\qquad (2.25)$$

$$\nu \to U\nu_m, \qquad \text{Relation between gauge and mass Eigenstates} \qquad (2.26)$$

$$\bar{\nu}^c M_\nu \nu \to \bar{\nu}_m^c U^T M_\nu U \nu_m \qquad\qquad (2.27)$$

Where we impose the condition that $U^T M_\nu U$ is diagonal, real and positive. In [34] it is beautifully shown that this U will always exist. The diagonal entries will be the neutral mass Eigenvalues.

In this discussion we are avoiding writing explicitly the flavour indices. Therefore we are not commited to a certain number of generations. Let us now, in the spirit of [34], define n as the number of generations of 'active' neutrinos, i.e. those coming from the lepton doublet L, and m as the number of 'sterile' neutrinos, i.e. those coming from the gauge singlet ν_R. While it may be useful to perform some conceptual calculations in the $n = m = 1$ limit, realistic models need $n = 3$ since we have measured the number of active neutrinos through the Z boson decay [65]

and $m \geq 2$, since we know from oscillations that at least 2 neutrinos are massive [40, 66–69], see Sec. 2. The matrix sub blocks m_D and M are of dimensions $n \times m$ and $m \times m$ respectively, while the symmetric mass matrix M_ν and the unitary transformation matrix U are both $(n + m) \times (n + m)$.

The beauty of the mass matrix in 2.24 is that it can explain the smallness of neutrino masses without the need to impose extremely small values for the Yukawa couplings. If we assume the hierarchy $M \gg m_D$, then we can approximately diagonalize the matrix M_ν in a perturbative way. Following [70] we start with the Ansatz

$$U = e^{iH} V \tag{2.28}$$

$$H = \begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix}, \quad V = \begin{pmatrix} V_{\text{light}} & 0 \\ 0 & V_{\text{heavy}} \end{pmatrix} \tag{2.29}$$

The matrix e^{iH} block diagonalizes the matrix M_ν:

$$e^{iH^T} M_\nu e^{iH} = \begin{pmatrix} m_{\text{light}} & 0 \\ 0 & m_{\text{heavy}} \end{pmatrix} \tag{2.30}$$

where m_{light} and m_{heavy} are general symmetric complex matrices of dimensions $n \times n$ and $m \times m$, respectively, which are diagonalized by

$$V_i^T m_i V_i = \text{real, positive, diagonal}, \quad i \in \{\text{light}, \text{heavy}\} \tag{2.31}$$

If we define the 'parameter order' $\epsilon \sim O\left(\frac{m_D}{M}\right)$. This is more an intuitive definition than a rigurous one, since the order of magnitude of a matrix is not well defined. However its meaning is clear. If $\epsilon << 1$ we can assume $S \sim O(\epsilon)$ and we can make a perturbative expansion in the parameter ϵ keeping only the terms of order ϵ^2 or lower and, after some algebra, find

$$S = -i m_D^* M^{-1}, \qquad m_{\text{light}} = -m_D M^{-1} m_D^T \tag{2.32}$$

Particularizing in the one generation limit we obtain

$$m_\nu \propto Y^2 \frac{v^2}{M} \tag{2.33}$$

The celebrated 'seesaw formula'. While we have showed the method to block diagonalize the type I seesaw mass matrix, it can be easily adapted to different seesaw scenarios [34, 71–75]. As a benchmark scenario of order of magnitude values we can consider $m_\nu \sim 0.1eV$, $Y \sim 0.1$, $v \sim 100GeV$ we get $M \sim 10^{12}GeV$. We can compare this scenario with the one obtained taking $M = 0$. In this case neutrinos will be purely Dirac particles, lepton number is conserved and neutrino masses come solely from the Higgs mechanism $m_\nu \sim Y_\nu v \rightarrow Y_\nu \sim 10^{-12}$. While this choice is not unnatural in the 't Hooft sense [29], because the radiative corrections of Y_ν are proportional to themselves, it is considered unaesthetical. However, it is indeed unnatural in the Dirac sense [76].

The drawbacks of both limiting scenarios are immediate: there are no phenomenological predictions that can be realistically tested. In the Majorana case, the reason is the suppresion of the phenomenological signatures by the high scale $M \sim 10^{12}GeV$. In the Dirac case, the suppresion comes from the small Yukawas $Y_\nu \sim 10^{-12}$. These two cases are not the only possibilities in order to give neutrino masses. In what follows we will give a quick overview on the Majorana mass generation mechanisms. Before starting the discussion, let us define what we mean by mass models and mass mechanisms.

Neutrino Mass Models: A proper neutrino mass model should be capable of generating neutrino masses and should be renormalizable. It is also highly desirable, though not essential, that the mass model also provides an "explanation" for the non-zero yet so tiny masses of neutrinos when compared to masses of all the other fermions in the SM.

Neutrino Mass Generation Mechanisms: A mechanism is a class of models which generates the neutrino masses in the same or very closely related ways. For example, various variants of the canonical type-I seesaw model can be classified together as type-I seesaw mechanism.

The model-building opportunities for neutrino masses are vast and the quantity and quality of the bibliography on the subject in the past few decades gives proof. We refer the interested reader to the cited references and the references within for more details on neutrino model building.

From a model-independent perspective, if only SM fields are present, there is a unique dimension-5 operator that gives rise to neutrino masses, the well-known Weinberg operator [77],

$$\frac{C}{\Lambda}\, L\phi\phi L \tag{2.34}$$

where Λ represents the cutoff scale. Above the cutoff scale the Ultra-Violet (UV) complete theory is at play, involving new 'messenger' fields, whose masses lie close to the scale Λ.

There are three different ways of contracting the relevant fields at tree level and the UV completions of these three different contraction possibilities correspond to the so-called type I, II and III seesaw mechanism:

$$\underbrace{\underbrace{\bar{L}^c \otimes \phi}_{1} \otimes \underbrace{\phi \otimes L}_{1}}_{\text{Type I}} \;,\quad \underbrace{\underbrace{\bar{L}^c \otimes L}_{3} \otimes \underbrace{\phi \otimes \phi}_{3}}_{\text{Type II}} \;,\quad \underbrace{\underbrace{\bar{L}^c \otimes \phi}_{3} \otimes \underbrace{\phi \otimes L}_{3}}_{\text{Type III}} \tag{2.35}$$

In (2.35) the underbrace denotes a $SU(2)_L$ contraction of the fields involved. The number under the brace denotes the $SU(2)_L$ transformation of the contracted fields. Although not explicitly written, the global contraction should always be a singlet of the full SM gauge group. The UV complete realization of these

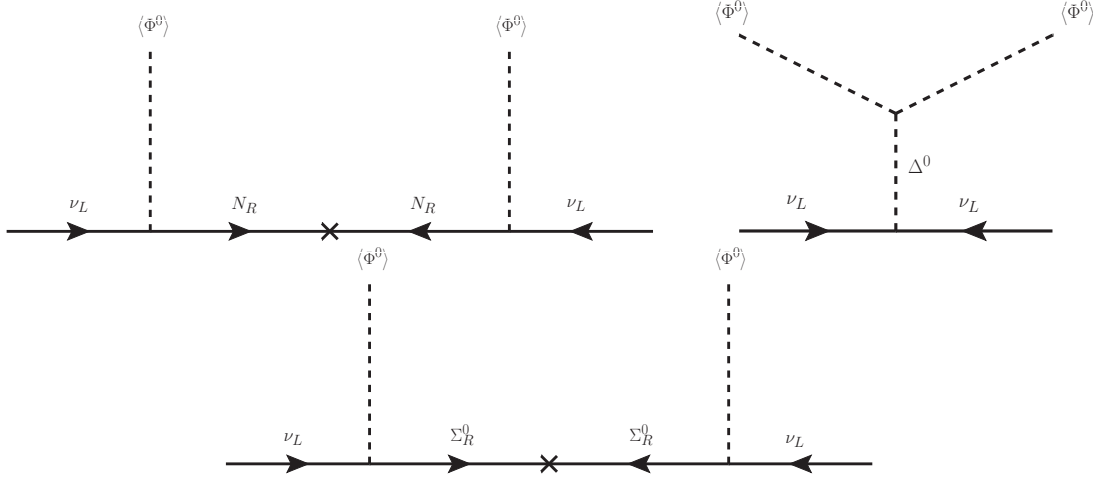contractions results into the three well known seesaw variants as shown in a diagramatical way in Figure 2.3.

Figure 2.3: Feynman diagram generating Majorana masses in Type I, II and III seesaw mechanism.

Figure 2.3 illustrates three UV–complete seesaw realizations of the same Weinberg operator of (2.35), differing from each other in the nature of the messenger fields involved. In the left–panel diagram of Fig. 2.3, corresponding to the type-I seesaw explained before, the field N_R is a heavy fermion which transforms as a singlet under $SU(2)_L$ and carries no $SU(3)_C$ or $U(1)_Y$ charge. In the middle diagram corresponding to type-II seesaw, the field Δ^0 is the neutral component of a heavy scalar multiplet transforming as triplet under $SU(2)_L$. In the right–panel diagram corresponding to type-III seesaw, Σ_R^0 is the neutral component of the heavy fermion multiplet transforming as triplet under $SU(2)_L$ symmetry.

Notice that the three possible messenger fields and their $SU(2)_L$ transformation properties arise from the different possibilities of field contractions of the Weinberg operator as shown in (2.35).

The possibility where both LL and $\phi\phi$ contract to a singlet is forbidden, since $\phi\phi$ is symmetric, while the singlet contraction is antisymmetric, and therefore

vanishes. Even in the presence of another Higgs doublet, the singlet contraction would vanish due to electric charge conservation.

Weinberg's dimension-5 operator is the lowest one which can generate Majorana neutrino masses. In general, using only Standard Model fields, i.e. a single Higgs doublet ϕ, it is easy to show that the $SU(2)_L$ symmetry implies that Majorana masses can only be generated by odd-dimensional operators. This means that they can only arise from operators involving even number of Higgs doublets. The general operator allowed by $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry is

$$\frac{1}{\Lambda^{2n+1}} \, \bar{L}^c \phi^2 \left(\phi^\dagger \phi \right)^n L, \ \ n \in \{0, 1, 2, 3...\} \tag{2.36}$$

A systematic classification of all the possibilities for operators of dimension 5 up to three loops, dimension 7 up to one loop and dimensions 9, 11 and 13 at tree-level have been discussed [78–88], see Fig. 2.4. Moreover, a plethora of UV complete models can be found in the literature. We will not develop this further and move on to the traditionally understudied possibility that neutrinos are Dirac particles.
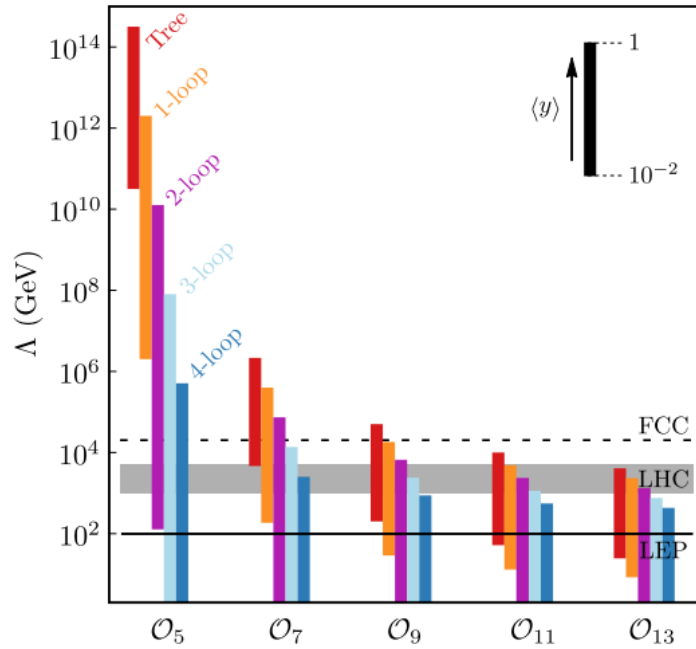
Figure 2.4: Typical new physics scale that reproduce the correct neutrino mass for different operator dimension and dominant loop level. Figure taken from [86] with their kind permission.

Chapter 3

Dirac neutrino mass models

We now turn our focus to the possibility that neutrinos are Dirac particles. Traditionally this option has received less attention than the Majorana option from a theoretical point of view. Several seesaw [34,70–73,89,90] and loop [91–94] mass generation mechanism for Majorana neutrinos have been known for a long time. Furthermore, a systematic classification of all Majorana neutrinos mass mechanisms at a given operator dimensionality and up to certain number of loops also exist in the literature [78–88]. The reason is twofold: first, Dirac neutrinos necessarily need a right-handed partner of the neutrinos, ν_R, to be added to the SM. If it is added without further symmetry conditions then the Majorana mass term is automatically allowed too, as explained in Sec. 3. A protecting symmetry that forbids Majorana mass terms is needed if neutrinos are to be Dirac, see Sec. 1. While the obvious choice is lepton number, already present in the SM itself as a global accidental symmetry, this raises the second concern: why is the neutrino mass so low compared to the electroweak scale? If neutrinos acquire a mass via the SM Higgs mechanism then Yukawas must be 'ad-hoc' of order 10^{-12}. This model is consistent and natural in 't Hooft's sense, but utterly unaesthetical and has no distinguishing phenomenology. These concerns can be addressed with the addition of new fields and symmetries. Then, we conclude that the simplest

Majorana mass model, the type I seesaw [34], is simpler in terms of new fields and symmetries with respect to the Standard Model than the simplest Dirac model with naturally small masses, the Dirac type I seesaw [1]. However in more involved scenarios the Dirac option may be as appealing as the Majorana one, as we will argue in what follows.

Most importantly, from an observational point of view the nature of neutrinos is still a major unknown. As explained in Sec. 3, the chief process for distinguishing Dirac and Majorana neutrinos is neutrinoless double beta decay [48–53] thanks to the black-box theorem [58, 59]. Other related lepton number violation processes are also being attracting experimental attention [54–57]. On the other hand, the fact that the weak interaction is V-A turns this quest into a major challenge [95, 96]. Indeed, the 'Dirac-Majorana confusion theorem' [97–99] states that the difference between Dirac and Majorana neutrinos must vanish in the massless limit. Still, some works have tried to find alternative ways to distinguish the neutrino nature, see [100–104] and the references within. As of now the nature of neutrinos remains as mysterious as the mechanism responsible for generating their small masses. Little is known regarding the nature of its associated messenger particles, the underlying mass scale or its flavour structure [105], currently probed only in neutrino oscillation experiments [40].

For these reasons, in the last few years there has been a renewed interest in looking at natural, elegant and predictive mass models for Dirac neutrinos. In this direction, several seesaw mechanisms have been proposed [1, 2, 7, 106–119], as well as loop models for Dirac neutrinos [5, 6, 120–140]. Other mechanisms for light Dirac neutrinos can be seen in models with extra dimensions [8, 141, 142], where the ν_R states are required for the consistent high energy completion of the theory, GUT theories [143, 144], or natural explanations of the SM fermion mass hierarchy [17]. Other Dirac neutrino works without an explicit mass mechanism

are, for example, [145–148]. Regarding the symmetries utilized to protect the Dirac nature of neutrinos, in the literature we can find several $U(1)$ lepton number symmetries [106, 107, 120] or some of its discrete Z_n subgroups [1–3, 5, 7, 106–108, 122, 130, 135, 136, 149–151], but also by invoking the presence of additional new symmetries [132], including non-abelian flavour groups [146].

Finally, Dirac neutrino masses can be found in different new physics scenarios which have the need of extra symmetries. Examples are Peccei-Quinn mechanisms with Dirac neutrinos [17, 152–155], which require a 'PQ symmetry', Dirac neutrinos and dark matter models [1, 2, 5, 7, 12, 135], which require a 'dark matter symmetry', Dirac neutrinos arising from Grand Unification Theories [143, 144] etc.

Due to the increasing interest in Dirac neutrino mass models, a classification of such tree-level and one-loop models at dimension 4 [156], dimension 5 [3, 157], dimension 6 [4, 158] and two-loops [12] have also been considered. Moreover, in some recent works it has been argued that in a full theory with the weak gravity conjecture neutrinos are expected to be Dirac fermions [159].

1 Symmetry requirements

Before beginning our discussion on possible Dirac neutrino mass models, let us mention certain generic conditions which must be satisfied in order to have Dirac neutrinos. By definition, a Dirac fermion can be viewed as two chiral fermions, one left handed and other right handed, having exactly degenerate masses [34]. Thus in order to have massive Dirac neutrinos one must extend the Standard Model particle content by adding the right handed partners of the known neutrinos, ν_R, being singlets under $\mathrm{SU}(3)_\mathrm{C} \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_\mathrm{Y}$ gauge symmetry. Second, owing to the color and electric charge neutrality of neutrinos, an additional exactly conserved symmetry beyond the Standard Model gauge symmetries is required

to protect the Diracness of neutrinos by forbidding the Majorana masses of the neutrino states - and any neutral fermion that mixes with them. While not the only option, from a theoretical point of view, the issue of the Dirac/Majorana nature of neutrinos is intimately connected with the $U(1)_{B-L}$ symmetry of the SM and its possible breaking pattern [145, 150]. If the $U(1)_{B-L}$ symmetry is conserved in nature, like it is in the Standard Model, then the neutrinos will be Dirac fermions. We do not know if physics Beyond the Standard Model (BSM) will break the lepton or baryon number symmetries or, if they are broken, if some non-trivial residual subgroups will survive the spontaneous symmetry breaking. However, if it is broken to a residual $\mathcal{Z}_m$ subgroup with $m \in \mathbb{Z}^+$ and $m \geq 2$, with $\mathbb{Z}^+$ being the set of all positive integers, then the Dirac/Majorana nature will depend on the residual $\mathcal{Z}_m$ symmetry provided that the SM lepton doublets $L_i = (\nu_{L_i}, l_{L_i})^T$ do not transform trivially under it. Thus, we have

$$U(1)_{B-L} \rightarrow \mathcal{Z}_m \equiv \mathcal{Z}_{2n+1} \text{ with } n \in \mathbb{Z}^+$$

$$\Rightarrow \text{ neutrinos are Dirac particles}$$

$$U(1)_{B-L} \rightarrow \mathcal{Z}_m \equiv \mathcal{Z}_{2n} \text{ with } n \in \mathbb{Z}^+ \tag{3.1}$$

$$\Rightarrow \text{ neutrinos can be Dirac or Majorana}$$

If the $U(1)_{B-L}$ is broken to a $\mathcal{Z}_{2n}$ subgroup, then one can make a further classification depending on how the L_i transform,

$$L_i \begin{cases} \not\sim \omega^n & \text{under } \mathcal{Z}_{2n} \Rightarrow \text{Dirac neutrinos} \\ \sim \omega^n & \text{under } \mathcal{Z}_{2n} \Rightarrow \text{Majorana neutrinos} \end{cases} \tag{3.2}$$

where $\omega^{2n} = 1$.

Irrespective of the choice of the symmetry that will forbid the Majorana mass terms for the neutrinos at all orders, throughout this work we will call such a

symmetry the 'Diracness symmetry'. Any Dirac neutrino mass model must feature this (unbroken) symmetry protection.

Additionally, it is clear that the renormalizable dimension 4 operator $Y\bar{L}\phi^c\nu_R$ is allowed by the gauge symmetries of the Standard Model, giving rise to a Dirac neutrino mass at tree level through the Higgs mechanism 1. If this tree level Lagrangian term is present it will give a contribution to Dirac neutrino masses of $m_\nu = Yv$. Therefore, except for the case of extremely fine tuned cancellations with other new physics sector, if neutrinos are Dirac and the bare mass term is present then the Yukawa coupling Y should be very small, of order 10^{-12} to reproduce the observed neutrino masses. Therefore in order to have 'natural' (in the Dirac sense) small neutrino masses we must ensure that this Lagrangian term is forbidden by some symmetry, which we call 'seesaw symmetry'. The simplest choice is a simple Z_2 [1] under which the Standard model transform trivially but $\nu_R \sim -1$, forbidding the tree level mass term. Another option is to choose chiral lepton number charges: If the left and right chiralities transform differently under lepton number, the tree level is automatically forbidden [5, 106–108, 135]. Furthermore, one could also choose a non-abelian symmetry [2, 7], leading to flavour predictions. See Sec. 1 for more details.

Dirac neutrino masses can also be found in different new physics scenarios which have the need of extra symmetries. Some examples are Peccei-Quinn symmetry, dark matter stability, GUT models etc.

Note, however, that it is not necessary to have a different symmetry group for each of these purposes. In many of the cited works one symmetry group is fulfilling two or more requirements at the same time. Following this minimalistic approach, in [5] and in Sec. 4 we develop an elegant mechanism with a single symmetry group, a $U(1)_{B-L}$ with chiral charges for the ν_R which plays the role of

the 'Diracness symmetry', 'seesaw symmetry' and 'dark matter symmetry' at the same time. This $U(1)_{B-L}$ is also anomaly free and thus can be gauged, leading to a richer phenomenology.

2   The Dirac seesaw formula

We will now develop the analogue of the Majorana seesaw formula given in Eq. 2.33. As explained in Sec. 1, the tree level Yukawa term $\bar{L}\phi^c\nu_R$ must be forbidden by some 'seesaw symmetry' in order to have naturally small neutrino masses and Majorana mass terms are forbidden for neutrino fields by some 'Diracness symmetry'. As the simplest seesaw scenario, consider the Standard Model with the addition of $n \geq 2$ gauge singlets ν_R and m vector-like (VL) pairs[1], which are also gauge singlets and we call them N_L and N_R. Since they are vector-like pairs, their number is unconstrained by theory in the sense that they won't contribute to gauge anomalies [34]. The neutrino mass Lagrangian is then given by

$$\mathcal{L}_{m_\nu} = Y_\phi \bar{L}\phi^c N_R \,+\, Y_\chi \bar{N}_L \chi \nu_R \,+\, M N_L N_R \,+\, h.c. \tag{3.3}$$

where χ is a SM singlet carrying a vev $\langle 0 | \chi | 0 \rangle = u$. The mass mechanism can be diagramatically portrayed as Fig. 3.1.
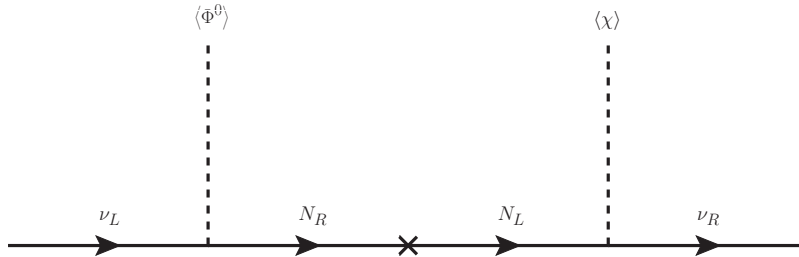

Figure 3.1: Feynman diagram representing the Dirac Type I seesaw.

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[1]'Vector-like fermions' here means simply that the left and right chiralities transform in the same way under the symmetries considered, as opposed to the term 'chiral fermions' where the symmetry transformations are chiral-dependent.

After spontaneous symmetry breaking the Lagrangian becomes

$$\mathcal{L}_{m_\nu} \to \text{SSB} \to \begin{pmatrix} \bar{\nu}_L & \bar{N}_L \end{pmatrix} \begin{pmatrix} 0 & Y_\phi v \\ Y_\chi u & M \end{pmatrix} \begin{pmatrix} \nu_R \\ N_R \end{pmatrix} + h.c. \qquad (3.4)$$

Where we have ignored the interaction with the physical Higgses. Similar to the Majorana case, we define

$$n_L \equiv \begin{pmatrix} \nu_L \\ N_L \end{pmatrix}, \qquad n_R \equiv \begin{pmatrix} \nu_R \\ N_R \end{pmatrix}, \qquad M_\nu = \begin{pmatrix} 0 & Y_\phi v \\ Y_\chi u & M \end{pmatrix} \qquad (3.5)$$

$$n_L \to U_L n_L^{(m)}, \qquad n_L \to U_R n_R^{(m)}, \qquad n_{L/R}^{(m)} \text{ are the mass Eigenstates} \qquad (3.6)$$

$$\bar{n}_L M_\nu n_R \to \bar{n}_L^{(m)} U_L^\dagger M_\nu U_R n_R^{(m)} \qquad (3.7)$$

Where we impose the condition that $U_L^\dagger M_\nu U_R$ is diagonal, real and positive. Under these conditions the unitary matrices U_L and U_R are unique modulo rearranging the mass Eigenvalues in the diagonal matrix [2]. The diagonal entries will be the masses of the neutral states. Now me make the Ansatz

$$U_L = e^{iH_L} V_L, \qquad U_R = e^{iH_R} V_R \qquad (3.8)$$

$$H_L = \begin{pmatrix} 0 & S_L \\ S_L^\dagger & 0 \end{pmatrix}, \qquad V_L = \begin{pmatrix} V_{\text{light}L} & 0 \\ 0 & V_{\text{heavy}L} \end{pmatrix} \qquad (3.9)$$

$$H_R = \begin{pmatrix} 0 & S_R \\ S_R^\dagger & 0 \end{pmatrix}, \qquad V_R = \begin{pmatrix} V_{\text{light}R} & 0 \\ 0 & V_{\text{heavy}R} \end{pmatrix} \qquad (3.10)$$

$$(3.11)$$

We impose that the matrices e^{iH_L} and e^{iH_R} block diagonalize the matrix M_ν:

$$e^{iH_L \dagger} M_\nu e^{iH_R} = \begin{pmatrix} m_{\text{light}} & 0 \\ 0 & m_{\text{heavy}} \end{pmatrix} \qquad (3.12)$$

---

[2]Note that failing to impose the reality and positivity of the diagonal entries breaks the uniqueness of U_L and U_R

where m_{light} and m_{heavy} are general complex matrices of dimensions $n \times n$ and $m \times m$, respectively, which are diagonalized by

$$V_{iL}^{\dagger} m_i V_{iR} = \text{real, positive, diagonal}, \quad i \in \{\text{light}, \text{heavy}\} \qquad (3.13)$$

And once again we define the 'parameters order' $\epsilon_1 \sim O\left(\frac{Y_\phi v}{M}\right)$ and $\epsilon_2 \sim O\left(\frac{Y_\chi u}{M}\right)$. If $\epsilon_1, \epsilon_2 << M$ and if we assume $S \sim O(\epsilon)$ we can make a perturbative expansion in the ϵ parameters keeping only the terms of order ϵ^2 or lower and, after some algebra, find

$$S_L = -iv Y_\phi M^{-1}, \qquad S_R = -iu Y_\chi M^{-1}, \qquad m_{\text{light}} = -Y_\phi v M^{-1} Y_\chi u \quad (3.14)$$

At first order this implies for the left handed mixing matrix

$$U_L = e^{iH_L} V_L \approx (I_{n+m} + iH_L) V_L = \begin{pmatrix} I_n & v Y_\phi M^{-1} \\ -v(M^{-1})^{\dagger} Y_\phi^{\dagger} & I_m \end{pmatrix} \begin{pmatrix} V_{\text{light}L} & 0 \\ 0 & V_{\text{heavy}L} \end{pmatrix} =$$

$$(3.15)$$

$$\begin{pmatrix} V_{\text{light}L} & v Y_\phi M^{-1} V_{\text{heavy}L} \\ -v(M^{-1})^{\dagger} Y_\phi^{\dagger} V_{\text{light}L} & V_{\text{heavy}L} \end{pmatrix}$$

$$(3.16)$$

Where I_k is the identity matrix of dimension $k \times k$. We can now note two relevant features. The matrix $V_{\text{light}L}$ will diagonalize the light neutrino mass matrix from the left, and can thus be identified with the lepton mixing matrix of Eq. 2.20 (modulo charged lepton contributions to the mixing). $V_{\text{light}L}$ is a unitary matrix and the U_L as written above is unitary to first order. We can conclude that non-unitarity effects on the lepton mixing matrix which affect neutrino oscillations come at order $\sim (v/M)^2$. Non-unitarity bounds can then be translated into lower limits in the mass scale M. Moreover, the mixing between the ν and N states comes at order $\sim v/M$. This mixing will generate interactions between SM

particles and the heavy neutrinos which can be tested for example in Charged Lepton Flavour Violation (LFV) processes, see Sec. 3. As a side note, the right-handed neutrino mixing will instead come at u/M order, but U_R is unphysical.

In the one generation limit we find for the light eigenstates

$$m_\nu = Y_\phi Y_\chi \frac{v\,u}{M},\tag{3.17}$$

in clear analogy to Eq. 2.33, where now the vev of the singlet u appears multiplying a single power of the SM vev v. Again, like in the Majorana case this formula applies to more involved escenarios than the simple one depicted here, as will be explained in Sec. 3. The $SU(2)_L \times U(1)_Y$ charges can be generalized and textures can be applied to the mass matrices.

3 The Dirac seesaw zoo

We will now generalize the previous result to a class of models which can elegantly accomodate naturally small Dirac neutrino masses in the seesaw spirit. Many of these models are analogous to some popular Majorana seesaw models, but some are completely new. Let us start with the simplest case, the dimension 4 operator

$$y_\nu \, \bar{L}\Phi^c \nu_R \tag{3.18}$$

where y_ν is the Yukawa coupling constant. Although this possibility is allowed on theory grounds, it leaves the smallness of neutrino masses unexplained, implying the need for a tiny Yukawa coupling for neutrinos ($y_\nu \sim \mathcal{O}(10^{-13})$). Therefore, this operator is assumed to be forbidden by the 'seesaw symmetry' discussed in Sec. 1. Moreover, the Weinberg operator and any other operator for Majorana neutrino masses is assumed to be forbidden by the 'Diracness symmetry'. A more

attractive possibility would be to obtain naturally small neutrino masses through generalized Weinberg operators or their higher dimensional counterparts.

In general, using only the Standard Model Higgs doublet Φ, the Standard Model gauge symmetry implies that the only allowed dimensions for the operators that can induce Dirac neutrino masses are even, i.e. operators involving odd number of Higgs doublets, namely

$$\frac{1}{\Lambda^{2n}} \bar{L} \Phi^c \left(\Phi^\dagger \Phi \right)^n \nu_R, \;\; n \in \{0, 1, 2, 3, 4...\} \tag{3.19}$$

Therefore, after the dimension 4 Yukawa term of (3.18), the next allowed operator involving only Standard Model Higgs would be of dimension 6. We will first study the dimension 5 operators, which necesarily imply the existence of new scalar fields.

3.1   Generalized Weinberg operators

We will now build the possibilities for the dimension-5 operators in the Weinberg spirit. We start by constructing the generalized dimension 5 Weinberg operator for Dirac neutrinos, given by

$$\frac{1}{\Lambda} \bar{L} \otimes X \otimes Y \otimes \nu_R \tag{3.20}$$

where X and Y are scalar fields transforming as some n-plets of $SU(2)_L$ with appropriate $U(1)_Y$ charges.

Invariance of (3.20) under $SU(2)_L$ symmetry implies that, if X transforms as a n-plet under $SU(2)_L$, then Y must transform either as a $n + 1$-plet, or a $n - 1$-plet under $SU(2)_L$ symmetry. For example, if we take X to be a singlet then Y should be a doublet. If we take X to be a doublet then Y can only be a singlet (equivalent to the previous case) or a triplet.

Each of these cases leads to different $SU(2)_L$ contractions which, as we will see shortly, will lead to different seesaw UV completions. For example, for the case $X = \chi$ i.e. a singlet under $SU(2)_L$ symmetry and $Y = \Phi$ i.e. a doublet under $SU(2)_L$ symmetry, we have the following possible contractions, which can be viewed as Dirac analogues of the type I, II and III Majorana seesaw mechanism,

$$\underbrace{\underbrace{\bar{L} \otimes \Phi^c}_{1} \otimes \underbrace{\chi \otimes \nu_R}_{1}}_{\text{Type I analogue}} \ , \quad \underbrace{\underbrace{\bar{L} \otimes \nu_R}_{2} \otimes \underbrace{\Phi^c \otimes \chi}_{2}}_{\text{Type II analogue}} \ , \quad \underbrace{\underbrace{\bar{L} \otimes \chi}_{2} \otimes \underbrace{\Phi^c \otimes \nu_R}_{2}}_{\text{Type III analogue}} \ , \quad (3.21)$$

where again the underbrace denotes a contraction of the fields involved and the number under the brace denotes the n-plet contraction of $SU(2)_L$ to which the fields contract. Note that invariance under $U(1)_Y$ requires that Φ^c should appear in this operator. The global contraction should be a singlet in order that the operator is allowed by $SU(2)_L$. The three possible seesaw completions of this operator are shown in figure 3.2.
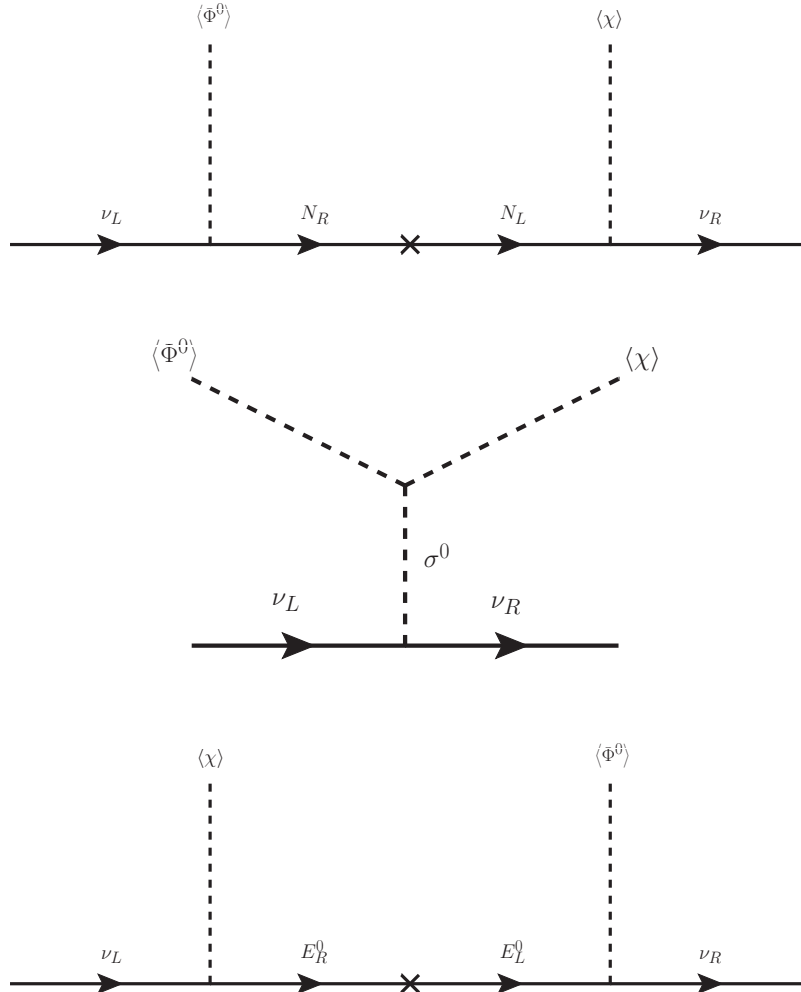
Figure 3.2: Feynman diagrams representing the Dirac Type I, II and III seesaw analogues.

Here the fields N_L and N_R are heavy fermions transforming as singlets under $SU(2)_L$, σ^0 is the neutral component of a $SU(2)_L$ doublet scalar, and E_L^0 and E_R^0 are the neutral components of the heavy vector like fermions transforming as doublets under $SU(2)_L$. Note that the new $SU(2)_L$ doublet scalar σ in the Type-II case must be a new scalar, and cannot be identified with the Standard model Higgs Φ, as such an identification will also imply presence of a tree-level Dirac neutrino mass term we assumed to be forbidden by symmetries.

Some of the corresponding UV–complete theories have been discussed in the literature, while others have not. For example, explicit models employing type I

Dirac seesaw have already been realized in [1, 2, 7, 106], while explicit models for type II Dirac seesaw where considered in [109, 110, 113, 149]. In contrast, to the best of our knowledge, a full–fledged UV–complete theory using the Dirac type III seesaw has so far not been explicitly developed.

Going beyond singlets and doublets opens up still more possibilities. For example, taking $X = \Phi$ and $Y = \Delta$, where Δ is an $SU(2)_L$ triplet [3], yields

$$\underbrace{\underbrace{\bar{L} \otimes \nu_R}_{2} \otimes \underbrace{\Phi \otimes \Delta}_{2}}_{\text{Type II like}} \ , \quad \underbrace{\underbrace{\bar{L} \otimes \Delta}_{2} \otimes \underbrace{\Phi \otimes \nu_R}_{2}}_{\text{Type III like}} \ , \quad \underbrace{\underbrace{\bar{L} \otimes \Phi}_{3} \otimes \underbrace{\Delta \otimes \nu_R}_{3}}_{\text{Type III like}} \tag{3.22}$$

The underbrace denotes, as before, field contraction, and the number under the brace denotes the n-plet contraction of $SU(2)_L$ to which the fields reduce. The global contraction should be an $SU(2)_L$ singlet. Note that for this operator we have two possibilities for $U(1)_Y$ charge of Δ. Apart from the operator in (3.22) (which has Δ with $U(1)_Y = -2$) another operator namely $\bar{L}\Phi^c\Delta_0\nu_R$ with Δ_0 carrying $U(1)_Y = 0$ is also possible. The diagrams for this case will be identical to those discussed here but the hypercharges of the intermediate fields will be different. Note that one can always induce the vevs of either χ or Δ with the coupling to a pair of Φ's. Such operators will have dimension 6 and will be discussed in Sec. 3.2. The diagrams leading to the seesaw completion of (3.22) are shown in Figure 3.3.

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[3]$SU(2)_L$ multiplets of order higher than 2 face stringent phenomenological constraints. In particular, their vev must be much smaller than the EW vev and there must not exist a massless Goldstone boson associated to it. Both problems can be solved by inducing the vev and thus generating a dimension 6 operator for neutrino mass, see Sec. 3.2.

Figure 3.3: Feynman diagrams for Type-II and Type-III Dirac seesaw mechanism.

where, as before, the field Δ^0 is the neutral component of the $SU(2)_L$ triplet, Σ_L^0 and Σ_R^0 are the neutral components of the heavy $SU(2)_L$ triplet fermions, σ^0 is the neutral component of an $SU(2)_L$ doublet scalar σ, while E_L^0 and E_R^0 are the neutral components of the heavy vector–like fermions transforming as $SU(2)_L$ doublets. As before, owing to the symmetry requirements, σ must be a new $SU(2)_L$ doublet, distinct from the Standard model Higgs Φ. There have been so far no dedicated study of UV–complete theories in literature corresponding to these new possibilities.

Going yet to higher multiplets of $SU(2)_L$ will open up novel ways to generate Dirac neutrino mass at the dimension 5 level. These can easily be realized following our procedure in a straightforward way, so here we skip the details. Furthermore, as already discussed, Dirac neutrino masses can also be generated at higher dimensions. The general operator for such a scenario involves several scalar fields X_i; $i = 1, \cdots, n$ which can be different $SU(2)_L$ multiplets, carrying appropriate $U(1)_Y$ charges, as follows

$$\frac{1}{\Lambda^{n-1}} \, \bar{L} \otimes X_1 \otimes \cdots \otimes X_n \otimes \nu_R \tag{3.23}$$

Where some of the X_i may coincide and the overall combination $X_1 \otimes \cdots \otimes X_n$ must transform as an $SU(2)$ doublet of hypercharge $-1/2$.

3.2 Dimension 6 operators

We now turn our discussion to operators involving only Standard Model scalar doublet Φ and discuss the various possible UV-complete models for this case. As has been argued in [3] and in Eq. 3.19, for Dirac neutrinos, after the Yukawa term, the lowest dimensional operator involving only the Standard Model scalar doublet appears at dimension-6 and is given by

$$\frac{1}{\Lambda^2} \bar{L} \otimes \Phi^c \otimes \Phi^c \otimes \Phi \otimes \nu_R \tag{3.24}$$

where L and Φ denote the lepton and Higgs doublets, ν_R is the right-handed neutrino field and Λ represents the cutoff scale. Above Λ the UV complete theory is at play, involving new "messenger" fields, whose masses lie close to the scale Λ. Recently, this operator has also been studied in [158] and our results agree.

Before starting our systematic classification of the UV-complete seesaw models emerging from this operator, we stress again that, in order for this operator to

give the leading contribution to naturally small Dirac neutrino masses, the lower dimensional Yukawa term $\bar{L}\Phi^c\nu_R$ should be forbidden by the 'seesaw symmetry'. The operator in (3.24) can lead to several different UV-complete seesaw models, depending on the field contractions involved, which can be arranged into five distinct topologies for the Feynman diagrams of neutrino mass generation. For lack of better names, we are calling these five topologies as T_i, $i \in \{1, 2, 3, 4, 5\}$. These topologies are shown in Fig. 3.4.

Each topology involves new "messenger fields" which can be either new scalars (φ), new fermions (Ψ) or both. The masses of these heavy messenger fields lying typically at or above the cutoff scale Λ of the dimension-6 operators. The first topology T_1 involves two messenger fields, a scalar (φ) and a Dirac fermion (Ψ). The scalar φ gets a small induced vev through its trilinear coupling with Y and Z scalars. Topology T_2 is very similar to T_1 and also involves two messenger fields, a scalar φ and a Dirac fermion Ψ. However, the small vev of φ in T_2 is induced by its trilinear coupling with X and Y scalars. The third topology T_3 is distinct from the first two and only involves scalar messengers φ and φ'. The scalar φ' gets a small induced vev through its trilinear coupling with Y and Z scalars. The other scalar φ subsequently gets a "doubly-induced vev" through its trilinear coupling with φ' and X scalars. The fourth topology T_4 only involves a single messenger scalar φ which gets an induced vev through its quartic coupling with scalars X, Y and Z. The final fifth topology involves only fermionic Dirac messengers Ψ and Ψ', as shown in Fig. 3.4. The $SU(2)_L \otimes U(1)_Y$ charges of the messenger fields in all topologies will depend on the details of the operator under consideration and the contractions involved. We will discuss all such possibilities in the following sections. Each topology leads to different estimates for the associated light neutrino mass generated in each case. Neglecting
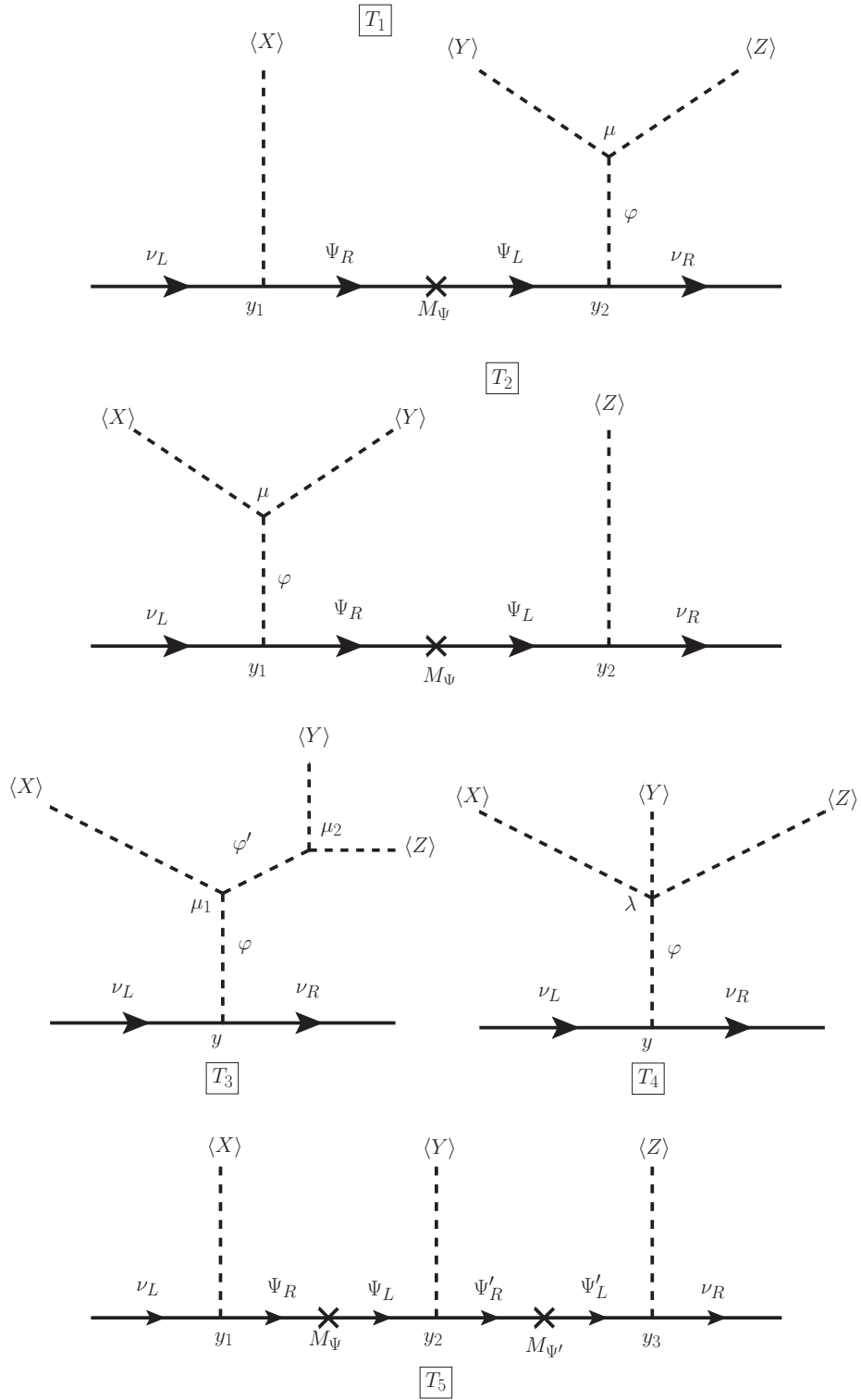
Figure 3.4: Feynman diagrams representing the five different topologies, T_1, T_2, T_3, T_4, T_5, respectively.

the three generation structure of the various Yukawa coupling matrices in family space, the resulting formulas for the neutrino masses are listed in Table 3.1.

Topology	Messenger Fields	Neutrino Mass Estimate
T1	Ψ, φ	$\frac{\mu y_1 y_2 \, v_X v_Y v_Z}{M_\psi M_\varphi^2}$
T2	Ψ, φ	$\frac{\mu y_1 y_2 \, v_X v_Y v_Z}{M_\psi M_\varphi^2}$
T3	φ, φ'	$\frac{\mu_1 \mu_2 y \, v_X v_Y v_Z}{M_\varphi^2 M_{\varphi'}^2}$
T4	φ	$\frac{y \lambda \, v_X v_Y v_Z}{M_\varphi^2}$
T5	Ψ, Ψ'	$\frac{y_1 y_2 y_3 \, v_X v_Y v_Z}{M_\psi M_{\psi'}}$

Table 3.1: Possible topologies and messengers leading to light neutrino masses, and the associated estimates for each topology.

There are fifteen inequivalent ways of contracting the operator in Eq. 3.24, each of which will require different types of messenger fields for UV-completion. Out of these 15, 9 are very similar to the 'pure' type-I (1), type-II (3) and type-III (5) Dirac seesaws described in Sec. 3.1, with the particularity that the singlet or triplet external scalars will now acquire an 'induced' vev, which in turn can provide a further suppression to neutrino mass. Moreover, the cubic coupling between the scalar S (be it singlet or triplet) with two Higgs doublets will also remove the massless Goldstone boson associated to the pseudo scalar neutral component of S. This is not mandatory in the singlet case, since a singlet Goldstone can scape the experimental constraints such as stellar cooling in many appealing scenarios, see Sec. 3.3, but it is key in the triplet case, since a Goldstone triplet is ruled out by collider signatures [160]. We will study these 9 induced versions of the type I, II and III Dirac seesaws in Sec. 3.2.1. The remaining 6 diagrams generate new models and will be shown in Sec. 3.2.2

3.2.1 Type I, II and III Dirac seesaw mechanisms with induced vev

We start by laying out the simplest contraction of the dimension-6 operator of (3.24), which is as follows:

$$\underbrace{\underbrace{\bar{L} \otimes \Phi^c}_{1} \otimes \underbrace{\Phi^c \otimes \Phi}_{1} \otimes \underbrace{\nu_R}_{1}}_{1} \qquad (3.25)$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{Type I with induced vev Fig. 3.5}}$$

Like in previous sections, in (3.25) the under-brace denotes a $SU(2)_L$ contraction of the fields involved, whereas the number given under it denotes the transformation of the contracted fields under $SU(2)_L$ (note that the other possible contraction in which $\Phi^c \otimes \Phi^c$ goes to a singlet is simply 0). Although not made explicit, we take it for granted that the global contraction leading to a UV-complete model where the neutrino mass is generated by the diagram shown in Fig. 3.5 should always be an $SU(2)_L$ singlet.
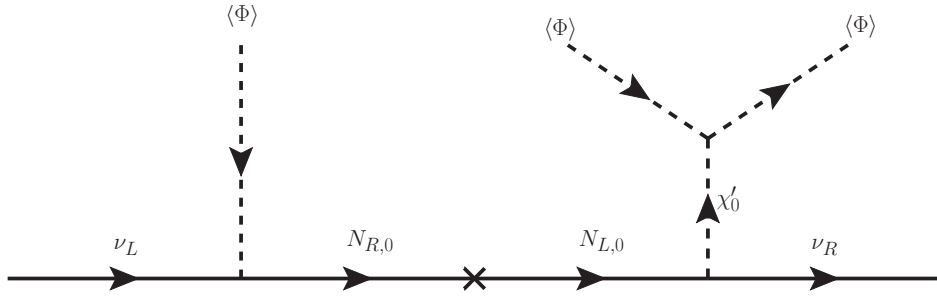

Figure 3.5: Feynman diagram representing the Dirac Type-I seesaw with an induced vev for χ_0'.

The diagram in Fig. 3.5 involves two messenger fields, a vector-like neutral fermion N_0 and a scalar χ_0', both singlets under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group. As listed in Table 3.1, the light neutrino mass is doubly suppressed first by the mass of the fermion N_0 and also by the small induced vev for χ_0'. In contrast to the type I Dirac seesaw diagram of Fig. 3.2, here the messenger field χ_0' required for the UV-completion gets a small induced vev via its cubic coupling

with the Standard Model Higgs doublet, also removing the massless Goldstone boson associated to it.

The three possibilities for this case are shown in (3.26).

$$\underbrace{\underbrace{\bar{L} \otimes \nu_R}_{2} \otimes \underbrace{\Phi^c \otimes \Phi}_{1} \otimes \underbrace{\Phi^c}_{2}}_{\underbrace{}_{2}}, \quad \underbrace{\underbrace{\bar{L} \otimes \nu_R}_{2} \otimes \underbrace{\Phi^c \otimes \Phi}_{3} \otimes \underbrace{\Phi^c}_{2}}_{\underbrace{}_{2}}, \quad \underbrace{\underbrace{\bar{L} \otimes \nu_R}_{2} \otimes \underbrace{\Phi^c \otimes \Phi_3^c}_{2} \otimes \underbrace{\Phi}_{2}}_{\text{Type II induced Fig. 3.6}}$$

$$\text{Type II induced Fig. 3.6} \qquad \text{Type II induced Fig. 3.6}$$

(3.26)

These three contraction possibilities lead to three different UV-completions, as illustrated in Fig. 3.6. All diagrams in Fig. 3.6 belong to the T_3 topology, and
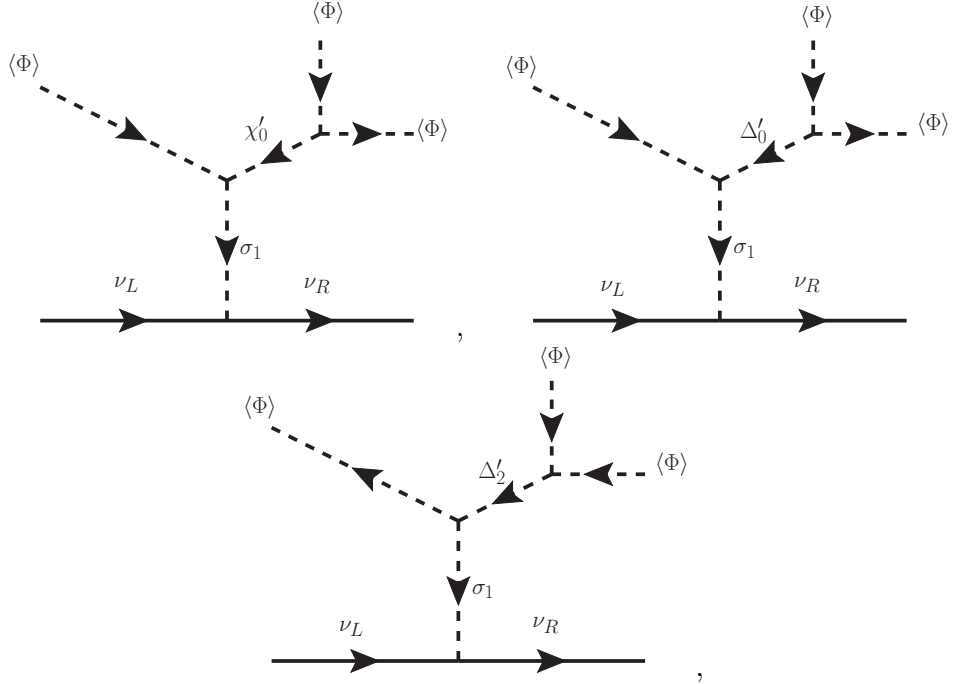

Figure 3.6: Feynman diagrams representing the three realizations of the Dirac Type-II seesaw with an induced vev for χ_0' or Δ_i'.

require two scalar messengers. The diagram on the left requires a $SU(2)_L$ singlet χ_0' and a new doublet σ_1 (different from the Standard Model Higgs doublet) with $U(1)_Y = 1$. The middle one requires an $SU(2)_L$ triplet Δ_0' (with $U(1)_Y = 0$) and an $SU(2)_L$ doublet σ_1 (with $U(1)_Y = 1$) scalar messengers. The third diagram is identical to the second, exchanging $\Phi \leftrightarrow \Phi^c$ in two external legs. Note that the

hypercharges of the intermediate fields Δ_0' and Δ_2' are different so that, although the UV-completions share the same topology, the underlying models are different. The associated light neutrino mass estimate is given in Table 3.1.

The operator of (3.24) also leads to five distinct type-III like seesaw possibilities with induced vevs [4]. The various possible contractions leading to such possibilities as shown in (3.27).

$$\underbrace{\underbrace{\bar{L}}_{2} \otimes \underbrace{\Phi^c \otimes \Phi}_{1} \otimes \underbrace{\Phi^c \otimes \nu_R}_{2}}_{2}, \quad \underbrace{\underbrace{\bar{L}}_{2} \otimes \underbrace{\Phi^c \otimes \Phi}_{3} \otimes \underbrace{\Phi^c \otimes \nu_R}_{2}}_{2}, \quad \underbrace{\underbrace{\bar{L}}_{2} \otimes \underbrace{\Phi^c \otimes \Phi^c}_{3} \otimes \underbrace{\Phi \otimes \nu_R}_{2}}_{2}$$

$$\text{Type III induced Fig. 3.7} \qquad \text{Type III induced Fig. 3.7} \qquad \text{Type III induced Fig. 3.7}$$

$$\underbrace{\underbrace{\bar{L} \otimes \Phi^c}_{3} \otimes \underbrace{\Phi^c \otimes \Phi}_{3} \otimes \underbrace{\nu_R}_{1}}_{1}, \quad \underbrace{\underbrace{\bar{L} \otimes \Phi}_{3} \otimes \underbrace{\Phi^c \otimes \Phi^c}_{3} \otimes \underbrace{\nu_R}_{1}}_{1} \qquad (3.27)$$

$$\text{Type III induced Fig. 3.7} \qquad \text{Type III induced Fig. 3.7}$$

The UV-completions of each of these possible contractions involve different messenger fields, leading to five inequivalent models. The neutrino mass generation in these models is shown diagrammatically in Figure 3.7.

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[4] We denote all diagrams with T_1 or T_2 topologies as type-III seesaw-like if they involve fermions transforming non-trivially under $SU(2)_L$.
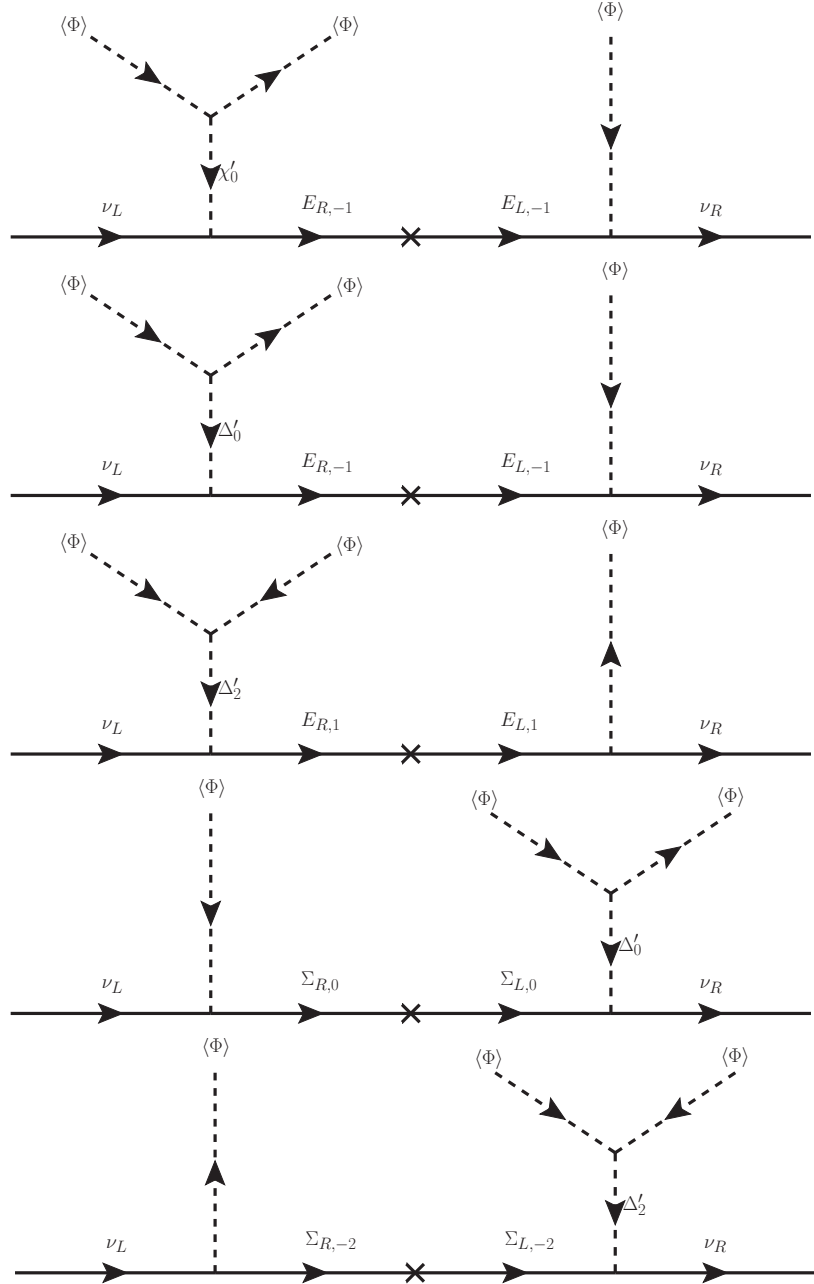
Figure 3.7: Feynman diagram representing the five realizations of the Dirac Type-III seesaw with an induced vev for χ_0' or Δ_i'.

The first diagram in Figure 3.7 involves, as messenger fields, scalar singlet χ_0' and vector-like fermions E_{-1} transforming as an $SU(2)_L$ doublet with $U(1)_Y = -1$. The second diagram involves hypercharge-less $SU(2)_L$ triplet scalars Δ_0' and the same $SU(2)_L$ doublet vector-like fermion E_{-1} as messenger fields. The third diagram is identical to the second one, but with exchange $\Phi \leftrightarrow \Phi^c$ in the external

legs. This leads to a different hypercharge $U(1)_Y = 2$ for the intermediate scalar

triplet Δ_2', as well as for the $SU(2)_L$ doublet vector fermion E_1, with $U(1)_Y = 1$.

The fourth and fifth diagrams again are related to each other by exchanging

$\Phi \leftrightarrow \Phi^c$ in two external legs. They involve, as messenger fields, $SU(2)_L$ triplet

scalars Δ_i'; $i = 0, 2$ together with vector-like $SU(2)_L$ triplet fermions Σ_i; $i = 0, -2$.

The hypercharges of Δ_i' are $U(1)_Y = 0, 2$ and of Σ_i are $U(1)_Y = 0, -2$ respectively.

The first three diagrams belong to T_2 topology, while the fourth and fifth diagrams

have the topology T_1 and the associated light neutrino masses for T_1 and T_2 are

given in Table 3.1. Notice that, in contrast to the type III like Dirac seesaw

diagrams discussed in Fig. 3.2, here the χ_0' and Δ_i' both get induced vevs from

their cubic interaction terms with the Standard Model Higgs doublet.

3.2.2 New diagrams

Apart from the above diagrams, there are also six new ones which have no

dimension-5 analogues listed in Sec. 3.1. The first of these possibilities arise from

the field contraction shown in (3.28).

$$\underbrace{\underbrace{\bar{L} \otimes \nu_R}_{2} \otimes \underbrace{\Phi^c \otimes \Phi \otimes \Phi^c}_{2}}_{\text{Type II as in 3.8}}, \tag{3.28}$$

This particular contraction of the operators leads to a UV-complete model where

the neutrino mass arises from the Feynman diagram shown in Fig. 3.8 involving

a single scalar messenger field σ_1 transforming as $SU(2)_L$ doublet with $U(1)_Y =$

1. The field σ_1 gets a small induced vev through its quartic coupling with the

Standard Model Higgs doublet. This diagram belongs to the T_4 topology and the

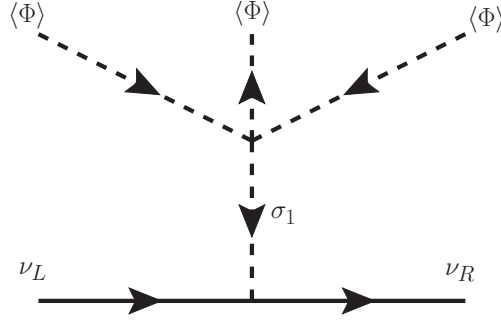resulting light neutrino mass estimate is given in Table 3.1.

Figure 3.8: Feynman diagram representing the new possible UV completion belonging to topology T_4.

Finally, there are five other field contractions of the dimension-6 operator, as shown in (3.29) and (3.30).

$$\underbrace{\underbrace{\bar{L} \otimes \Phi^c}_{1} \otimes \underbrace{\Phi^c}_{2}}_{2} \otimes \underbrace{\Phi \otimes \nu_R}_{2}, \qquad \underbrace{\underbrace{\bar{L} \otimes \Phi^c}_{1} \otimes \underbrace{\Phi}_{2}}_{2} \otimes \underbrace{\Phi^c \otimes \nu_R}_{2} \qquad (3.29)$$
$$\underbrace{\text{Double Dirac seesaw } 3.9} \qquad\qquad \underbrace{\text{Double Dirac seesaw } 3.9}$$

$$\underbrace{\underbrace{\bar{L} \otimes \Phi^c}_{3} \otimes \underbrace{\Phi^c}_{2}}_{2} \otimes \underbrace{\Phi \otimes \nu_R}_{2}, \quad \underbrace{\underbrace{\bar{L} \otimes \Phi^c}_{3} \otimes \underbrace{\Phi}_{2}}_{2} \otimes \underbrace{\Phi^c \otimes \nu_R}_{2}, \quad \underbrace{\underbrace{\bar{L} \otimes \Phi}_{3} \otimes \underbrace{\Phi^c}_{2}}_{2} \otimes \underbrace{\Phi^c \otimes \nu_R}_{2}$$
$$\underbrace{\text{Double Dirac seesaw } 3.9} \qquad \underbrace{\text{Double Dirac seesaw } 3.9} \qquad \underbrace{\text{Double Dirac seesaw } 3.9}$$
$$(3.30)$$

Notice that the UV-completions of these five field contractions lead to the neutrino mass generation through topology T_5 [5], as shown diagrammatically in Fig. 3.9.

All these UV complete models only involve fermionic messengers. One sees that two different types of fermionic messengers are needed. In the first and second diagrams the N_0 is a vector-like gauge singlet fermion, whereas the vector fermions E_1 or E_{-1} are $SU(2)_L$ doublets. In the first diagram the field E_1 carries a hypercharge $U(1)_Y = 1$ while in the second diagram E_{-1} carries a hypercharge $U(1)_Y = -1$. The last three diagrams in Fig. 3.9 also involve two types of

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[5]This is a 'double' Dirac seesaw: neutrino masses are doubly suppressed by the two fermion propagators.

Figure 3.9: Feynman diagram representing the five realizations of the topology T_5 diagrams.

fermionic messengers the vector-like fermions E_1 or E_{-1} are $SU(2)_L$ doublets, while the vector-like fermions Σ_i transform as triplet under the $SU(2)_L$ symmetry. In the third diagram Σ_0 carries no hypercharge while E_1 has $U(1)_Y = 1$. In the fourth diagram Σ_0 carries no hypercharge, but E_{-1} has $U(1)_Y = -1$. In the fifth diagram Σ_{-2} has hypercharge $U(1)_Y = -2$, while E_{-1} again has $U(1)_Y = -1$. The light neutrino mass expected for all these diagrams is the same as that given in Table 3.1 for T_5 topology.

3.3 Dirac inverse seesaws

The inverse seesaw is a popular approach for the generation of neutrino masses with the mediator masses potentially close to the electroweak scale. It is characterized by the presence of a small mass parameter, generally denoted by μ, which follows the hierarchy of scales

$$\mu \ll v \ll \Lambda \,, \tag{3.31}$$

with v the Higgs VEV that sets the electroweak scale and Λ the neutrino mass generation scale, determined by the masses of the seesaw mediators. The μ-parameter suppresses neutrino masses as $m_\nu \propto \mu$, allowing one to reproduce the observed neutrino masses and mixing angles with large Yukawa couplings and light seesaw mediators. This usually leads to a richer phenomenology compared to the standard high-energy seesaw scenario.

In this section we will explore the various types of inverse seesaws possible for Dirac neutrinos. We first define the criterion to determine which models can be classified as belonging to the inverse seesaw mechanism:

1. **Presence of a Small Symmetry Breaking Parameter :** The first and foremost condition for a model to be classified as an inverse seesaw model is the requirement of a "small" symmetry breaking "μ-parameter". The μ-parameter has to be such that the limit $\mu \to 0$ enhances the symmetry of the Lagrangian. This crucial feature implies that in the absence of the μ-parameter, the model would have a conserved symmetry group $\mathcal{G}$, which gets broken by $\mu \neq 0$ as

$$\mathcal{G} \xrightarrow{\ \mu\ } \mathcal{G}' \,. \tag{3.32}$$

Here $\mathcal{G}' \supset \mathcal{G}$ is a residual symmetry.[6] Therefore, the limit $\mu \to 0$ enhances the symmetry of the model, making it natural in the sense of 't Hooft [29] and protecting the small value of μ from quantum corrections. Note that here smallness[7] of the μ-parameter is with respect to other parameters in the model under consideration, for example the electro-weak scale.

2. **μ-parameter from Explicit/Spontaneous Symmetry Breaking:** The μ-parameter can either be an explicit symmetry breaking term or a spontaneously induced symmetry breaking term. However, to classify as a genuine inverse seesaw, the μ-parameter should be a "soft term". In particular, this means that if the μ-parameter is an explicit symmetry breaking term, then it should have a positive power mass dimension.

3. **Neutrino Mass Dependence on μ-parameter:** The neutrino mass at leading order must be directly proportional to the μ-parameter.

4. **Extended Fermionic Sector:** A genuine inverse seesaw model should always have an extended fermionic sector directly participating in the neutrino mass mechanism. This means fermions beyond the fermionic content of the SM should be involved in neutrino mass generation.

5. **The μ-parameter need not be unique:** In cases where there are different μ_i-parameters, all should be soft and in the limit of $\mu_i \to 0 \ \forall \ i$, the symmetry of Lagrangian should be enhanced. Also, at least one μ_i-parameter should be directly involved in the neutrino mass generation mechanism. Furthermore, all the μ_i-parameters directly involved in neutrino mass generation should satisfy all the other conditions listed above.

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[6]It can happen that the μ-parameter completely breaks the symmetry group $\mathcal{G}$. In such a case $\mathcal{G}' \equiv \mathcal{I}$ i.e. the trivial Identity Group.

[7]We leave the "How small should be considered small?" question to the model creator's taste.

An example of a model which satisfies all these features is the canonical Majorana inverse seesaw model [161]. The SM field inventory is extended to include a Vector Like (VL) fermion transforming as a singlet under the gauge group. The explicit Majorana mass term (a soft term) for this new fermion will break lepton number in two units explicitly and thus its smallness is protected by a symmetry. In this notation, $\mathcal{G} = \mathrm{U}(1)_\mathrm{L}$ while $\mathcal{G}' = \mathbb{Z}_2$ and μ is the explicit Majorana mass term. However, here we will concentrate in the Dirac variants.

Let us emphasize again that the μ-parameter can be explicitly introduced in the Lagrangian, as a symmetry-breaking mass term, or spontaneously generated by the VEV of a scalar. In the rest of the section we will concentrate on the latter case. This is particularly convenient for our discussion, since the identification of the broken symmetry becomes more transparent. Furthermore, the smallness of the μ-parameter can be more easily justified in extended models that generate it spontaneously. We note, however, that scenarios with an explicit μ-term would lead to analogous conclusions, just replacing a VEV by a bare mass term. [8]

While these remarks also apply to Majorana generalizations of the original model [161], see [6] for a complete discussion, here we will focus on the Dirac neutrino case. As explained in Sec. 1, we need a 'Diracness symmetry' and a 'seesaw symmetry' in order to have consistent naturally small Dirac neutrino masses. In the particular case of the inverse seesaw, we also have the 'inverse seesaw symmetries' of Eq. 3.32. These three symmetry requirements need not lead to three different symmetry groups and, in particular, in this Section we will stick to an elegant solution which satisfies all the requirements with a single symmetry group.

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[8]Note that this analogy is only true if the μ-term does not break any of the SM gauge symmetries. Otherwise, explicit violation is forbidden while scenarios with spontaneous violation are in principle allowed, provided $\mu \ll v$, as generally assumed in the inverse seesaw setup. Therefore, the spontaneously broken scenario is in a sense more general than the explicitly broken one. Of course, electric charge and color should remain as conserved charges in either case.

We will take $\mathcal{G} = \mathrm{U}(1)_{\mathrm{B-L}}$ and $\mathcal{G}'$ can be any of its $\mathbb{Z}_n$; $n > 2$ subgroups [150]. In this section we take the simplest possibility of $\mathcal{G}' = \mathbb{Z}_3$. This $\mathrm{U}(1)_{\mathrm{B-L}}$ symmetry and its residual $\mathbb{Z}_3$ subgroup are enough to play all these roles if the right-handed neutrinos are chiral. Moreover, the $U(1)_{B-L}$ group is anomaly free in the so-called *445 chiral solution* [106, 107, 162]. In this case, the right-handed neutrinos carry $(\nu_{R1}, \nu_{R2}, \nu_{R3}) \sim (-4, -4, 5)$ charges under $\mathrm{U}(1)_{\mathrm{B-L}}$. Being anomaly free, the $\mathrm{U}(1)_{\mathrm{B-L}}$ symmetry can also be gauged, leading to a richer phenomenology. While this is not the only possible symmetry solution [5] it is just a particularly elegant one. This chiral $\mathrm{U}(1)_{\mathrm{B-L}}$ symmetry is playing the roles of the 'Diracness symmetry' (forbidding the Majorana mass terms), the 'seesaw symmetry' (forbidding the tree level neutrino mass term) and the 'inverse seesaw symmetry' by protecting the smallness of the μ parameters from higher order corrections. Usually, the minimal versions of these type of models will lead to just two massive neutrinos, since $\nu_{R3} \sim 5$ will decouple from the rest of the particle content. More involved scenarios can give mass to the third neutrino and potentially explain the atmospheric and solar mass splitting scales.

3.3.1 The simplest Dirac inverse seesaw

We begin the discussion on Dirac versions of the inverse seesaw mechanism with a very minimal realization. In this simple case, the leading effective operator for neutrino masses is $\bar{L}H^c\chi\nu_R$, where χ is a scalar gauge singlet. In the full UV complete theory, the particle content and symmetry transformations are shown in Tab. 3.2, while the relevant Lagrangian terms for the generation of neutrino masses are given by

$$\mathcal{L}_{\mathrm{Min}} = Y\,\bar{L}\widetilde{H}N_R \,+\, \lambda\,\bar{N}_L\chi\nu_R \,+\, M\,\bar{N}_L N_R \,+ \mathrm{h.c.}. \qquad (3.33)$$

	Fields	$\mathrm{SU(2)_L \otimes U(1)_Y}$	$\mathrm{U(1)_{B-L}} \rightarrow \mathbb{Z}_3$
Fermions	L_i	$(\mathbf{2}, -1/2)$	$-1 \; \rightarrow \; \omega^2$
	ν_R	$(\mathbf{1}, 0)$	$(-4, -4, 5) \; \rightarrow \; \omega^2$
	N_L	$(\mathbf{1}, 0)$	$-1 \; \rightarrow \; \omega^2$
	N_R	$(\mathbf{1}, 0)$	$-1 \; \rightarrow \; \omega^2$
Scalars	H	$(\mathbf{2}, 1/2)$	$0 \; \rightarrow \; \omega^0$
	χ	$(\mathbf{1}, 0)$	$3 \; \rightarrow \; \omega^0$

Table 3.2: Particle content of the minimal model implementing the Dirac inverse seesaw. All quarks transform as $1/3$ (ω^1) under $\mathrm{U(1)_{B-L}}$ $(\mathbb{Z}_3)$, while their $\mathrm{SU(3)_C \otimes SU(2)_L \otimes U(1)_Y}$ charges are identical to those in the SM. Here $\omega = e^{2\pi i/3}$ is the cube root of unity with $\omega^3 = 1$. Moreover, with this choice of charges the $\mathrm{U(1)_{B-L}}$ symmetry is anomaly free.

The scalar acquire VEVs

$$\langle H \rangle = v \quad , \quad \langle \chi \rangle = u \,. \tag{3.34}$$

and break the electroweak and $\mathrm{U(1)_{B-L}}$ symmetries, respectively. The VEV of χ induces the small symmetry breaking μ-term.

$$\mu = \lambda \, u \,. \tag{3.35}$$

Also, note that the VEV of the singlet scalar χ breaks $\mathrm{U(1)_{B-L}}$ in three units, leaving a residual $\mathbb{Z}_3$ symmetry under which all scalars transform trivially while all fermions (except quarks) transform as ω^2, with $\omega = e^{2i\pi/3}$; $\omega^3 = 1$ being the cube root of unity. This symmetry forbids all Majorana terms and therefore protects the Diracness of light neutrinos. The H and χ VEVs also induce *Dirac* masses proportional to the Y and λ Yukawa couplings. These, in the Dirac basis
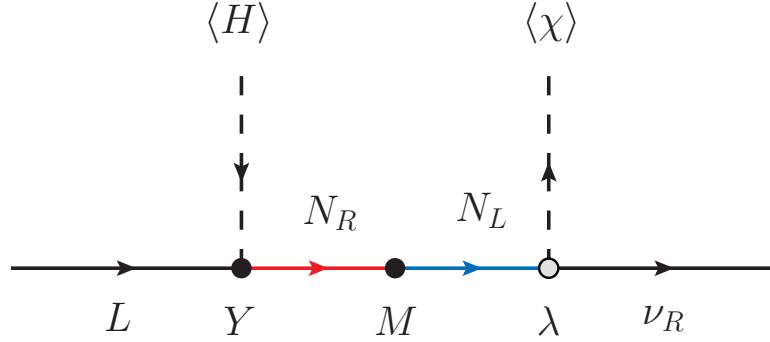
Figure 3.10: Neutrino mass generation in the minimal Dirac inverse seesaw.

$\left(\bar{\nu}_L\ \bar{N}_L\right)$ and $(\nu_R\ N_R)^T$, gives rise to the mass matrix

$$\mathcal{M} = \begin{pmatrix} 0 & Y\,v \\ \mu & M \end{pmatrix} \tag{3.36}$$

where, as mentioned before, $\mu = \lambda\,u$ will be naturally small as it is the $\mathrm{U(1)_{B-L}}$ symmetry breaking term.

Thanks to the residual $\mathbb{Z}_3$ symmetry, the neutrinos are Dirac particles whose masses in the inverse seesaw limit $M \gg Yv \gg \mu$ are given by

$$m_\nu = Y\,v\,\frac{\mu}{M}\,, \tag{3.37}$$

as diagramatically shown in Fig. 3.10.

Since in the limit $\mu \to 0$ the $\mathrm{U(1)_{B-L}}$ symmetry is restored, the smallness of the μ-term is protected. Its smallness will not be altered by higher order corrections, and therefore is perfectly natural in the 't Hooft sense. For example, if we take a small $\mu \sim 10$ eV, we can obtain $m_\nu \sim 0.1$ eV for $Y \sim 0.1$ and $M \sim 1$ TeV. We should also point out that the mass matrix in Eq. (3.36) is similar to the mass matrix obtained in the Dirac type-I seesaw [1, 3, 106, 107]. However, in the type-I seesaw case, the off-diagonal terms in Eq. (3.36) are taken to be

comparable to each other i.e. $Y v \approx \lambda u \ll M$. Thus, for Dirac neutrinos, the minimal inverse seesaw and the type-I seesaw are just two limits of the same mass matrix.

Some final comments are in order. First, since the $U(1)_{B-L}$ symmetry is anomaly free, it can be a gauge symmetry. This eliminates the Goldstone boson associated to its breaking and leads to a much richer phenomenology. In contrast, the canonical Majorana inverse seesaw is not anomaly free and hence cannot be gauged. Second, the Dirac inverse seesaw model is also relatively simple in terms of new fields added to the theory. In addition to the Standard Model particles, we have just added the right-handed neutrinos ν_R, a VL fermionic pair N_L and N_R and an extra singlet scalar χ, which is needed only if the spontaneous symmetry breaking is desired.

3.3.2 Generalized Dirac inverse seesaw

We will now generalize the $SU(2) \times U(1)_Y$ multiplicity of the Dirac inverse seesaw. The Lagrangian of the model would be formally identical to the one in Sec. 3.3.1, but with higher $SU(2)_L$ multiplets and replacing $H \to \phi$ and $\chi \to \varphi$, where ϕ is now a generic scalar which doesn't break $U(1)_{B-L}$, which can be identified with the SM Higgs doublet H in certain special scenarios. On the other hand, φ is the scalar whose vev breaks $U(1)_{B-L}$ spontaneously, generating the μ term. The charges under $U(1)_{B-L}$ will also be identical and therefore the model would share the same appealing features. The general $SU(2)_L \times U(1)_Y$ charges can be seen in Tab. 3.3 and diagramatically in Fig. 3.11.

Let us list a few viable examples in Tab. 3.4. The general strategy to build these models is as follows. We first fix the $SU(2)_L \otimes U(1)_Y$ charge of the scalar ϕ. Depending on the ϕ charges, the N_L and N_R fields can have only one option for their $SU(2)_L \otimes U(1)_Y$ transformations. Finally, given the charges of both ϕ and

	Fields	$SU(2)_L \otimes U(1)_Y$	$U(1)_{B-L} \to \mathbb{Z}_3$
Fermions	L_i	$(\mathbf{2}, -1/2)$	$-1 \to \omega^2$
	ν_R	$(\mathbf{1}, 0)$	$(-4, -4, 5) \to \omega^2$
	N_L	$(\mathbf{n} \pm \mathbf{1}, Y - 1/2)$	$-1 \to \omega^2$
	N_R	$(\mathbf{n} \pm \mathbf{1}, Y - 1/2)$	$-1 \to \omega^2$
Scalars	H	$(\mathbf{2}, 1/2)$	$0 \to \omega^0$
	φ	$(\mathbf{n} \pm \mathbf{1}, Y - 1/2)$	$3 \to \omega^0$
	ϕ	$(\mathbf{n}, Y)$	$0 \to \omega^0$

Table 3.3: Particle content of the generalized Dirac inverse seesaw. As in the Majorana case, here also ϕ can be identified with the SM Higgs under certain conditions. Again, φ breaks the symmetry and generates the μ-term. The particle charges under the residual $\mathbb{Z}_3$ symmetry are given by cube roots of unity with $\omega = e^{2\pi I/3}$; $\omega^3 = 1$.
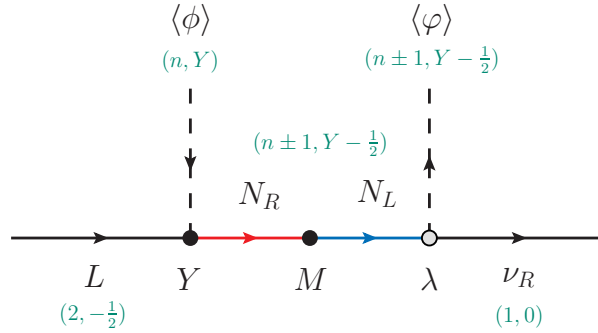

Figure 3.11: Generalized Dirac inverse seesaw.

the N_L, N_R fields, the viable options for the φ gauge charges can be obtained. The final list of viable models for the Dirac inverse seesaw with generalized multiplets are listed in Tab. 3.4.

In Tab. 3.4 we have restricted outselves to $n = 4$ i.e. upto the case in which ϕ transforms as a quadruplet under $SU(2)_L$. Nevertheless, the generalization to higher $n > 4$ is rather straightforward. We have named the "type" of the model based on the $SU(2)_L$ transformation of the N_L, N_R fermions. The subclass numbering is based on the $SU(2)_L$ transformation of ϕ as well as the $SU(2)_L \otimes U(1)_Y$ transformation of the φ field.

Name of the model	N_L and N_R	ϕ	φ (μ-term)
Standard Dirac Inverse seesaw	$(\mathbf{1}, 0)$	$(\mathbf{2}, 1/2)$	$(\mathbf{1}, 0)$
Type-III Dirac Inverse seesaw	$(\mathbf{3}, 0)$	$(\mathbf{2}, 1/2)$	$(\mathbf{3}, 0)$
Type-III Dirac Inverse seesaw variant I	$(\mathbf{3}, -1)$	$(\mathbf{2}, -1/2)$	$(\mathbf{3}, -1)$
Type-III Dirac Inverse seesaw variant II	$(\mathbf{3}, 1)$	$(\mathbf{4}, 3/2)$	$(\mathbf{3}, 1)$
Exotic or Type IV Dirac inverse seesaw	$(\mathbf{4}, Y - 1/2)$	$(\mathbf{3}, Y = 0, \pm 1)$	$(\mathbf{4}, Y - 1/2)$
Type-V Dirac Inverse seesaw	$(\mathbf{5}, 0)$	$(\mathbf{4}, 1/2)$	$(\mathbf{5}, 0)$
Type-V Dirac Inverse seesaw variant I	$(\mathbf{5}, 1)$	$(\mathbf{4}, 3/2)$	$(\mathbf{5}, 1)$
Type-V Dirac Inverse seesaw variant II	$(\mathbf{5}, -1)$	$(\mathbf{4}, -1/2)$	$(\mathbf{5}, -1)$
Type-V Dirac Inverse seesaw variant III	$(\mathbf{5}, -2)$	$(\mathbf{4}, -3/2)$	$(\mathbf{5}, -2)$

Table 3.4: A few examples of the generalized Dirac inverse seesaw.

3.3.3 Dirac "Double" Inverse Seesaw

To build the double Dirac inverse seesaw, we start with the field and symmetry inventory of the minimal Dirac inverse seesaw of Section 3.3.1. To it, we add new VL fermions, $S_{L,R}$, and a new singlet scalar χ_2. The only modification in the B − L charges is that we will take (χ_1, χ_2) to transform as $(\chi_1, \chi_2) \sim (6, -9)$. Remember that in the previous example we had $\chi \sim 3$. Moreover, we take the new VL fermion to transform as $S_{L,R} \sim 5$, while the rest of the fields share their transformation properties with the previous model as shown in Tab. 3.5.

With the above choice of $U(1)_{B-L}$ charges it is easy to check that the model is anomaly free and can be gauged if desired. Also, the B − L charges of the scalars are chosen in such a way that their VEVs break $U(1)_{B-L} \to \mathbb{Z}_3$. This residual $\mathbb{Z}_3$ symmetry remains unbroken thus ensuring the Dirac nature of neutrinos. Note that the fields $\nu_{R,3}$ and S_R share the same transformation properties and are therefore indistinguishable, but we call them differently to follow the notational conventions of the previous and following sections.

	Fields	$SU(2)_L \otimes U(1)_Y$	$U(1)_{B-L} \to \mathbb{Z}_3$
Fermions	L_i	$(\mathbf{2}, -1/2)$	$-1 \;\to\; \omega^2$
	ν_R	$(\mathbf{1}, 0)$	$(-4, -4, 5) \;\to\; \omega^2$
	N_L	$(\mathbf{1}, 0)$	$-1 \;\to\; \omega^2$
	N_R	$(\mathbf{1}, 0)$	$-1 \;\to\; \omega^2$
	S_L	$(\mathbf{1}, 0)$	$5 \;\to\; \omega^2$
	S_R	$(\mathbf{1}, 0)$	$5 \;\to\; \omega^2$
Scalars	H	$(\mathbf{2}, 1/2)$	$0 \;\to\; \omega^0$
	χ_1	$(\mathbf{1}, 0)$	$6 \;\to\; \omega^0$
	χ_2	$(\mathbf{1}, 0)$	$-9 \;\to\; \omega^0$

Table 3.5: Particle content of the Dirac analogue of the inverse seesaw. The $U(1)_{B-L}$ charges of the fermions are fixed by an anomaly cancellation condition while the $U(1)_{B-L}$ charges of the scalars are chosen such that the residual $\mathbb{Z}_3$ symmetry remains unbroken and the leading contribution to neutrino mass is induced by the double inverse seesaw.

With all these ingredients, the Lagrangian of the model relevant to neutrino mass generation is given by

$$\mathcal{L}_{\text{Dir}} = Y \, \bar{L} \, \widetilde{H} \, N_R \, + \, \lambda_2 \, \bar{S}_L \, \chi_2^* \, \nu_R \, + \, \lambda_1 \, \bar{N}_L \, \chi_1^* \, S_R \, + \, \lambda_1' \, \bar{S}_L \, \chi_1 \, N_R \tag{3.38}$$

$$+ \, M_N \, \bar{N}_L N_R \, + \, M_S \, \bar{S}_L S_R \, + \text{h.c.} . \tag{3.39}$$

Symmetry breaking is triggered by the scalar VEVs

$$\langle H \rangle = v \quad , \quad \langle \chi_1 \rangle = u_1 \quad , \quad \langle \chi_2 \rangle = u_2 , \tag{3.40}$$

which lead to the following μ-terms,

$$\mu_1 = \lambda_1 \, u_1 \quad , \quad \mu_1' = \lambda_1' \, u_1 \quad , \quad \mu_2 = \lambda_2 \, u_2 . \tag{3.41}$$
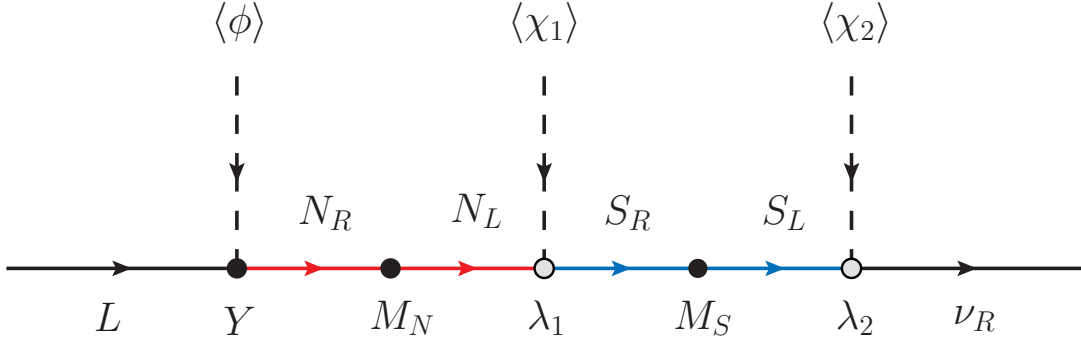
Figure 3.12: Neutrino mass generation in the Dirac double inverse seesaw. The μ-terms can be either explicit or spontaneously generated by the VEV of the scalars χ_1 and χ_2.

After symmetry breaking, Eq. (3.38) leads to the mass Lagrangian in matrix form

$$
\mathcal{L}_m = \begin{pmatrix} \bar{\nu}_L & \bar{N}_L & \bar{S}_L \end{pmatrix} \begin{pmatrix} 0 & Y_N\,v & 0 \\ 0 & M_N & \mu_1 \\ \mu_2 & \mu_1' & M_S \end{pmatrix} \begin{pmatrix} \nu_R \\ N_R \\ S_R \end{pmatrix}. \tag{3.42}
$$

Again, the natural hierarchy among the parameters of the model is

$$
\mu_1, \mu_1', \mu_2 \ll Y\,v \ll M_N, M_S\,, \tag{3.43}
$$

which leads to the light neutrino mass matrix

$$
m_\nu = \begin{pmatrix} Y\,v & 0 \end{pmatrix} \begin{pmatrix} M_N & \mu_1 \\ \mu_1' & M_S \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \mu_2 \end{pmatrix}. \tag{3.44}
$$

For one generation of light neutrinos, this is equivalent to

$$
m_\nu = Y_N \, \frac{v\,\mu_1\,\mu_2}{\mu_1\mu_1' - M_N M_S} \simeq -Y_N\,v\,\frac{\mu_1\mu_2}{M_N\,M_S}\,. \tag{3.45}
$$

This result is illustrated in the Feynman diagram shown in Fig. 3.12.

As it is clear from Eq. (3.45), neutrino masses in this case are suppressed by two μ-terms and hence the name Dirac *double* inverse seesaw. Note that the effect of μ' is subleading in the neutrino mass generation. Finally, we should remark that the spontaneous breaking of $U(1)_{B-L}$ by the u_1 and u_2 VEVs leaves a residual $\mathbb{Z}_3$ symmetry. As in the minimal Dirac model of Section 3.3.1, here too all scalars transform trivially under this residual symmetry, while all fermions (except quarks which transform as ω) transform as ω^2. Again, this symmetry forbids all Majorana mass terms and protects the Diracness of light neutrinos.

We finally point out that in order to induce a triple Dirac inverse seesaw and beyond one would just need to sequentially add a new VL fermion alongside a new scalar with a judicious choice of symmetry breaking charges for the scalars. To obtain an nth order Dirac inverse seesaw as the leading contribution we need to add n VL fermionic pairs and n scalars. Of course appropriate symmetries, with particles carrying appropriate charges under them, are required to ensure that neutrinos are Dirac particles with the leading contribution to their mass given by the nth order Dirac inverse seesaw as

$$m_\nu \approx Y \, v \prod_{i=1}^{i=n} \frac{\mu_i}{M_i} \,, \tag{3.46}$$

A combination of both generalization methods, i.e. higher order multiplets of $SU(2)_L$ in double or more Dirac inverse seesaws is straightforward and will not be pursued here.

4 Radiative masses and the 'scoto' connection

At present, a plethora of cosmic observations all indicate that the bulk of matter in the Universe is in the form of dark matter, a hitherto unknown form of matter which interacts gravitationally, but has little or no electromagnetic inter-

action [163]. These observations point to one of the most serious shortcomings of
the Standard Model (SM) since in the SM there is no viable candidate for dark
matter. Thus, they inarguably point to the presence of new physics beyond the
SM and it is a topic of active theoretical and experimental research.

To explain dark matter, the particle content of the SM needs to be extended.
Furthermore, to account for dark matter stability new explicit [94, 164] or acci-
dental symmetries [165] beyond those of the SM are also invoked.

There are particularly attractive scenarios that connect dark matter to neu-
trino physics in an intimate manner. The scotogenic model is one such model
where the 'dark sector' participates in the loop responsible for neutrino mass
generation [94]. This work has been followed up in multiple ocasions over the last
two decades, analyzing the phenomenology of the scotogenic model and variants
of it in multiple dimensions as well as connecting the neutrino mass generation
and dark matter with other new physics scenarios, both for Majorana and Dirac
neutrinos [5, 13, 120, 125, 127, 128, 132, 136, 139, 166–178, 178–219].

Recently, a relation between the Dirac nature of neutrinos and dark matter
stability has also been proposed [1]. Furthermore, it has been shown that this
relation is independent of the neutrino mass generation mechanism [3]. It utilizes
the SM lepton number $U(1)_L$ symmetry[9] or its appropriate $\mathcal{Z}_n$ subgroup, to forbid
Majorana mass terms of neutrinos as well as to stabilize dark matter [1]. In this
approach, the Dirac nature of neutrinos and the stability of dark matter are
intimately connected, having their origins in the same lepton number symmetry.

In this section we aim to combine and generalize these two approaches and
develop a general formalism where the following conditions are satisfied:

I. Neutrinos are Dirac in nature.

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[9]One can equivalently use the anomaly free $U(1)_{B-L}$ symmetry.

II. Naturally small neutrino masses are generated through finite loops, forbidding the tree-level neutrino Yukawa couplings.

III. The dark sector participates in the loop. The lightest particle being stable is a good dark matter candidate.

Usually one needs at least three different symmetries besides those within the Standard Model to achieve this [122]. However, we show that all of these requirements can be satisfied without adding any extra explicit or accidental symmetries. In our formalism we employ an anomaly free chiral realization of the $U(1)_{B-L}$ spontaneously broken to a residual $\mathcal{Z}_n$ symmetry and show that just the $U(1)_{B-L}$ already present in the SM is sufficient.

Before going into the details of the formalism, let us briefly discuss the possibility of chiral solutions of a gauge $U(1)_{B-L}$ and the anomaly cancellation conditions. It is well known that the accidental $U(1)_B$ and $U(1)_L$ symmetries of the SM are anomalous, but the $U(1)_{B-L}$ combination can be made anomaly free by adding three right-handed neutrinos ν_{R_i} with $(-1, -1, -1)$ vector charges under $U(1)_{B-L}$. However, chiral solutions to $U(1)_{B-L}$ anomaly cancellation conditions are also possible. The particularly attractive feature of chiral solutions is that by using them one can automatically satisfy conditions I and II, as shown in [106,107], using the chiral solution $\nu_{R_i} \sim (-4, -4, 5)$ under $U(1)_{B-L}$ symmetry.

Our general strategy is to use the chiral anomaly free solutions of $U(1)_{B-L}$ symmetry to generate loop masses for Dirac neutrinos and also have a stable dark matter particle mediating the aforementioned loop. Then, after symmetry breaking, once all of the scalars get a vacuum expectation value (vev), the $U(1)_{B-L}$ symmetry will be broken down to one of its $\mathcal{Z}_n$ subgroups, such that the dark matter stability and Dirac nature of neutrinos remain protected. This scheme is shown diagrammatically in Fig. 3.13.

In Fig. 3.13 the SM singlet fermions N_{Li}, N_{Ri}, as well as the right-handed neutrinos ν_R, have non-trivial chiral charges under $U(1)_{B-L}$ symmetry.[10] In order to generate the masses of these chiral fermions we have also added SM singlet scalars χ_i which also carry $U(1)_{B-L}$ charges. To complete the neutrino mass generation loop, additional scalars φ, η_i are required. After spontaneous symmetry breaking (SSB) of the $U(1)_{B-L}$ symmetry, all of the scalars χ_i will acquire vevs that break the $U(1)_{B-L} \to \mathcal{Z}_n$ residual symmetry. The fermions N_{Li}, N_{Ri} obtain masses through the vevs of the scalars χ_i, while the neutrinos acquire a naturally small n-loop mass as shown in Fig. 3.13.

(a) General $U(1)_{B-L}$ charge assignment.

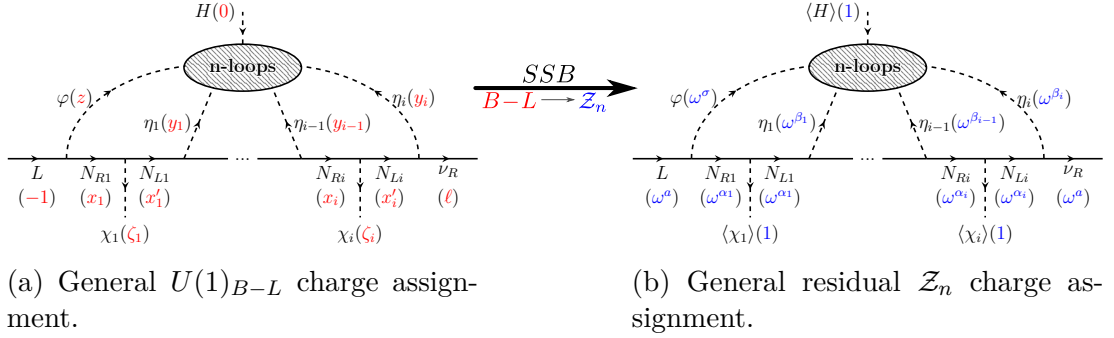(b) General residual $\mathcal{Z}_n$ charge assignment.

Figure 3.13: General charge assignment for any topology and its spontaneous symmetry breaking pattern.

In order to satisfy all of the requirements listed above, several conditions must be applied. First of all, the model should be anomaly-free:

- The chiral charges of the fermions must be taken in such a way that the anomalies are canceled.

In order to obtain non-zero but naturally small Dirac neutrino masses we impose the following conditions:

- The tree-level Yukawa coupling $\bar{L}H^c\nu_R$ should be forbidden. This implies that apart from the SM lepton doublets L_i no other fermion can have

---

[10]It is not necessary that all fermions N_{Li}, N_{Ri} be chiral under $U(1)_{B-L}$ symmetry.

$U(1)_{B-L}$ charge of ± 1. Furthermore, to ensure that the desired loop diagram gives the dominant contribution to the neutrino masses, all lower loop diagrams should also be forbidden by an appropriate choice of the charges of the fields.

- The operator leading to neutrino mass generation, i.e. $\bar{L}H^c\chi_1 \ldots \chi_i \nu_R$, should be invariant under the SM gauge symmetries as well as under $U(1)_{B-L}$. Following the charge convention of Fig. 3.13, i.e. $L \sim -1$, $\chi_i \sim \zeta_i$ and $\nu_R \sim \ell$, the charges of the vev carrying scalars χ_i should be such that $\sum_i \zeta_i = -1 - \ell$.

- All of the fermions and scalars running in the neutrino mass loop must be massive. Since the fermions will be in general chiral, this mass can only be generated via the coupling with a vev carrying scalar. For example, in the diagram in Fig. 3.13 we should have $-x_i + x_i' + \zeta_i = 0$.

- To protect the Diracness of neutrinos, all of the Majorana mass terms for the neutrino fields at all loops must be forbidden in accordance with Eq. (3.2).

Additionally, for dark matter stability, we impose the following conditions:

- After SSB, the $U(1)_{B-L}$ symmetry is broken down to a $\mathcal{Z}_n$ subgroup. Only even $\mathcal{Z}_n$ subgroups with $n > 2$ can protect dark matter stability. The odd $\mathcal{Z}_n$ subgroups invariably lead to dark matter decay.[11] The symmetry breaking pattern can be extracted as follows. First all the $U(1)$ charges must be rescaled in such a way that all the charges are integers and the least common multiple (lcm) of all of the rescaled charges is 1. Defining n

---

[11]For odd $\mathcal{Z}_n$ subgroups, there will always be an effective dark matter decay operator allowed by the residual odd $\mathcal{Z}_n$ symmetry. Even then it is possible that such an operator cannot be closed within a particular model, thus pinpointing the existence of an accidental symmetry that stabilizes dark matter. Another possibility is that the dark matter candidate decays at a sufficiently slow rate. Thus for residual odd $\mathcal{Z}_n$ symmetries, one can still have either a stable dark matter stabilized by an accidental symmetry or a phenomenologically viable decaying dark matter. In this work we will not explore such possibilities.

as the least common multiple of the charges of the scalars χ_i, it is easy to see that the $U(1)$ will break to a residual $\mathcal{Z}_n$. This n must be taken to be even as explained before, i.e. $n \equiv \mathrm{lcm}(\zeta_i) \in 2\mathbb{Z}$.

- Dark sector particles should neither mix with nor decay to SM particles or to vev carrying scalars.

- There are two viable dark matter scenarios depending on the transformation of the SM fermions under the residual symmetry.

  – When all SM fields transform as even powers of ω, where $\omega^n = 1$, under the residual $\mathcal{Z}_n$, the lightest particle transforming as an odd power will be automatically stable, irrespective of its fermionic or scalar nature. We will show an explicit example of this simple yet powerful idea later.

  – In the case in which all SM fermions transform as odd powers of the residual subgroup, it can be shown that all of the odd scalars and the even fermions will be stable due to a combination of the residual $\mathcal{Z}_n$ and Lorentz symmetry.

Given the long list of requirements, most of the possible solutions that lead to anomaly cancellation fail to satisfy some or most of them. Still we have found some simple one-loop and several two-loop solutions that can satisfy all of the conditions listed above.

In what follows we demonstrate the idea for a simple solution in which the $U(1)_{B-L}$ symmetry is broken down to a residual $\mathcal{Z}_6$ symmetry. However, in general, many other examples with different residual even $\mathcal{Z}_n$ symmetries can be found by applying the given framework.

Realistic example

	Fields	$SU(2)_L \otimes U(1)_Y$	$U(1)_{B-L}$	$\mathcal{Z}_6$
Fermions	L_i	$(\mathbf{2}, -1/2)$	-1	ω^4
	ν_{R_i}	$(\mathbf{1}, 0)$	$(-4, -4, 5)$	$(\omega^4, \omega^4, \omega^4)$
	N_{L_l}	$(\mathbf{1}, 0)$	$-1/2$	ω^5
	N_{R_l}	$(\mathbf{1}, 0)$	$-1/2$	ω^5
Scalars	H	$(\mathbf{2}, 1/2)$	0	1
	χ	$(\mathbf{1}, 0)$	3	1
	η	$(\mathbf{2}, 1/2)$	$1/2$	ω
	ξ	$(\mathbf{1}, 0)$	$7/2$	ω

Table 3.6: Charge assignment for all of the fields. $\mathcal{Z}_6$ is the residual symmetry in this example, with $\omega^6 = 1$.

Let us consider an extension of the SM by adding an extra Higgs singlet χ with a $U(1)_{B-L}$ charge of 3, along with an scalar doublet η, a singlet ξ and two vector-like fermions N_{L_l} and N_{R_l}, with $l = 1, 2$, all carrying non-trivial $U(1)_{B-L}$ charges as shown in Table 3.6 and depicted in Fig. 3.14a.

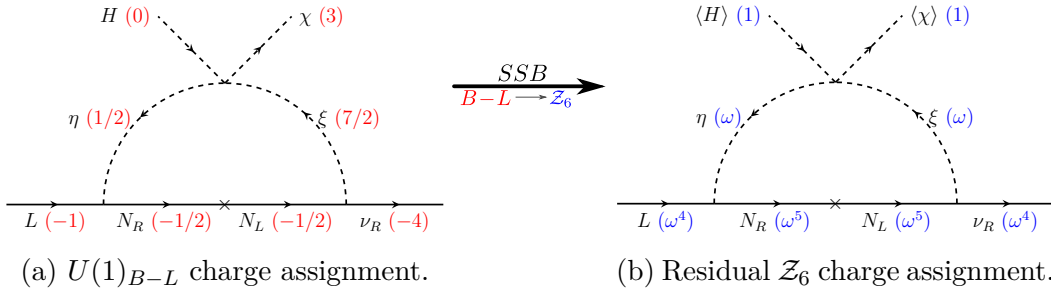(a) $U(1)_{B-L}$ charge assignment. (b) Residual $\mathcal{Z}_6$ charge assignment.

Figure 3.14: Charge assignment for the example model and its spontaneous symmetry breaking pattern.

The neutrino interactions are described by the following Lagrangian,

$$\mathcal{L}_\nu = y_{il}\bar{L}_i\tilde{\eta}N_{R_l} + y'_{li}\bar{N}_{L_l}\nu_{R_i}\xi + M_{lm}\bar{N}_{R_l}N_{L_m} + h.c., \qquad (3.47)$$

where $\tilde{\eta} = i\tau_2\eta^*$, with the indices $i = 1, 2, 3$ and $l, m = 1, 2$. The relevant part of the scalar potential for generating the Dirac neutrino mass is given by

$$\mathcal{V} \supset m_\eta^2\eta^\dagger\eta + m_\xi^2\xi^\dagger\xi + (\lambda_D H^\dagger\eta\chi\xi^* + h.c.), \qquad (3.48)$$

where λ_D is a dimensionless quartic coupling.

After spontaneous symmetry breaking of $U(1)_{B-L}$, the scalar χ gets a vev $\langle\chi\rangle = $ u, giving mass to two neutrinos through the loop depicted in Fig. 3.14. Note that only ν_{R_1} and ν_{R_2} can participate in this mass generation due to the chiral charges $(-4, -4, 5)$, i.e. $y'_{l3} = 0$ in Eq. (3.47). The third right-handed neutrino ν_{R_3} remains massless and decouples from the rest of the model, although it is trivial to extend this simple model to generate its mass.

The neutral component of the gauge doublet η and the singlet ξ are rotated into the mass eigenbasis with eigenvalues m_i^2 in the basis of (ξ, η^0). The neutrino mass matrix is then given in terms of the one-loop Passarino-Veltman function B_0 [220] by,

$$(M_\nu)_{\alpha\beta} \sim \frac{1}{16\pi^2}\frac{\lambda_D \text{vu}}{m_\xi^2 - m_\eta^2} y_{\alpha k} y'_{k\beta} M_k \sum_{i=1}^{2}(-1)^i B_0(0, m_i^2, M_k^2), \qquad (3.49)$$

where M_k $(k = 1, 2)$ are the masses of the Dirac fermions N_k and $\langle H \rangle = $ v the Standard Model vev.

As a benchmark point, we can take the internal fermion to be heavier than the scalars running in the loop, one of which will be the dark matter candidate. Then, Eq. (3.49) can be approximated by,

$$m_\nu \sim \frac{1}{16\pi^2}\frac{vu}{M}yy'\lambda_D. \qquad (3.50)$$

For comparison, we can take the Yukawa couplings to be of order 10^{-2} and the quartic coupling $\lambda_D \sim 10^{-4}$, like in the original scotogenic model [94]. We can also take neutrino masses to be of order 0.1 eV and $u \sim v$. With these choices, we can find the mass scale of the neutral fermions,

$$M \sim \frac{1}{16\pi^2}\frac{vu}{m_\nu}yy'\lambda_D \sim 10^4 \text{GeV}. \qquad (3.51)$$

Compared with the type-I seesaw scale $M \approx y^2 \frac{v^2}{m_\nu} \sim 10^{10}$ GeV we can see a five order of magnitude suppression coming from the loop and the possibility of a broader parameter space.

It is worth mentioning that since the $U(1)_{B-L}$ is anomaly free, it can be gauged. Then the physical Nambu-Goldstone boson associated to the dynamical generation of the Dirac neutrino mass, the Diracon [118,149], is absent.

Regarding dark matter stability in this particular model, we can see that the lightest particle inside the loop is stable. This is true for both the fermionic and scalar dark matter candidates. As can be seen in Fig. 3.14b, all of the internal loop particles are odd under the remnant $\mathcal{Z}_6$, while all of the SM particles are even. Therefore any combination of SM fields will be even under the residual subgroup, forbidding all effective operators leading to dark matter decay as shown graphically in Fig. 3.15.
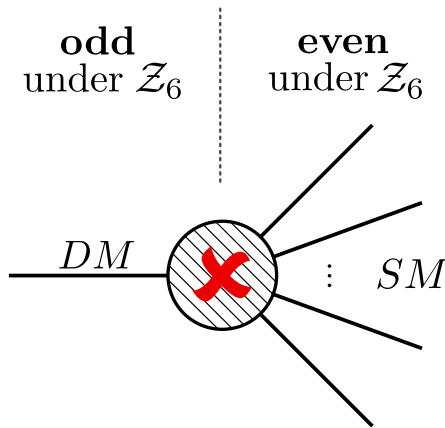

Figure 3.15: The decay of dark matter (odd under $\mathcal{Z}_6$) to SM particles (all even under $\mathcal{Z}_6$) is forbidden by the residual $\mathcal{Z}_6$ symmetry. This argument can be generalized to any even $\mathcal{Z}_n$ symmetry.

To summarize, we have shown that by using the $U(1)_{B-L}$ symmetry already present in the Standard Model, it is possible to address the dark matter stability and relate it to the smallness of Dirac neutrino masses. We have described a general framework in which these features are realized by exploiting the anomaly free chiral solutions of a global $U(1)_{B-L}$. This framework can be utilized in a wide

variety of scenarios. We have presented a particular simple realization of this idea where neutrino masses are generated at the one-loop level and the $U(1)_{B-L}$ symmetry is broken spontaneously to a residual $\mathcal{Z}_6$ symmetry. The framework can also be used in models with higher-order loops as well as in cases where $U(1)_{B-L}$ symmetry is broken to other even $\mathcal{Z}_n$ subgroups. Since the $U(1)_{B-L}$ is anomaly free, it can be gauged in a straightforward way, giving a richer phenomenology from the dark matter and collider point of view.

Chapter 4

Phenomenology of Dirac neutrino mass models

As argued in Chapter 3, the 'Dirac-Majorana confusion theorem' [97–99] states that phenomenological differences between Dirac and Majorana neutrinos will be proportional to m_ν, posing an experimental challenge that is not expected to be solvable in the foreseeable future. Model independent analysis must therefore deal with the conclusions of this theorem [100–104]. While the detection of neutrinoless double beta decay would indoubtedly imply Majorana neutrinos, the reverse is not true and the non-observation of this process sheds no light on the mistery. However, as argued in [150], a weak black box theorem can be formulated for the case of Dirac neutrinos: if neutrinoless quadrupole beta decay [221] is detected before the double one, then neutrinos are most probably Dirac in nature. The contrary would imply extreme fine tuned cancellations.

We direct the interested reader to the references above and within them for more details on model independent proof of Dirac neutrinos. In this chapter we will briefly review the phenomenology of some appealing classes of testable Dirac neutrino mass models. In Sec. 1 we will focus on predictive flavour models, which

can be tested in upcoming oscillation experiments, while in Sec. 3 we will study
how low scale Dirac neutrino mass models feature lepton flavour violation.

1   Flavour predictive Dirac neutrino models

One of the mysteries of particle physics is the understanding of the pattern
of fermion masses and mixings from first principles. Indeed, the charged fermion
mass pattern is not described in the theory: the SM only allows us the freedom
to fit the observed charged fermion masses, while lacking the masses of neutri-
nos altogether. An approach towards addressing, at least partially, the charged
fermion mass problem, is the possibility of relating quarks and lepton masses as
a result of a flavour symmetry [222], i.e.

$$\frac{m_b}{\sqrt{m_d m_s}} = \frac{m_\tau}{\sqrt{m_e m_\mu}} \ . \tag{4.1}$$

Notice that this mass relation constitutes a consistent flavour-dependent gen-
eralization of the conventional bottom-tau $SU(5)$ prediction, but does not re-
quire grand-unification. It provides a partial solution to the charged fermion
mass problem, which can be shown to hold in some theories of flavour based
on the A_4 [222–224]. T_7 [225] and $SO(10)$ [226] symmetries. Several alterna-
tive ideas for flavour predictions have come out, invoking non-Abelian flavour
symmetries [7, 9–11, 14, 15, 17, 227–274], both in UV complete models or in a
model-independent way and covering both possibilities for neutrino nature. On
the other hand, specially tantalizing is the idea that the stability of dark mat-
ter can be directly traced to the Dirac nature of neutrinos. One way to realize
this idea is by means of a Z_4 Lepton Quarticity symmetry [1, 7]. Within such
approach the same Z_4 discrete lepton number symmetry ensures the stability of
dark matter and the absence of all the Majorana mass terms. Thus owing to

Lepton Quarticity, the Dirac nature of neutrinos and the stability of dark matter are intimately related: the breakdown of this symmetry will simultaneously imply loss of dark matter stability as well as the *Diracness* of neutrinos.

As an example of an elegant and predictive Dirac neutrino mass model we will flesh out the model in [2], along the lines of the Lepton Quarticity models of dark matter [1,7]. Its testability potential in DUNE has been analyzed in [275]

2   Sample model: Bottom-Tau unification and neutrino oscillations

The particle content of our model along with the $SU(2)_L \otimes Z_4 \otimes A_4$ charge assignments of the particles are given in Table 4.1. Note that in Table 4.1 the

Fields	$SU(2)_L$	A_4	Z_4	Fields	$SU(2)_L$	A_4	Z_4
L_i	**2**	**3**	**z**	$\nu_{e,R}$	**1**	**1**	**z**
$N_{i,L}$	**1**	**3**	**z**	$\nu_{\mu,R}$	**1**	**1′**	**z**
$N_{i,R}$	**1**	**3**	**z**	$\nu_{\tau,R}$	**1**	**1″**	**z**
$l_{i,R}$	**1**	**3**	**z**	$d_{i,R}$	**1**	**3**	**z³**
$Q_{i,L}$	**2**	**3**	**z³**	$u_{i,R}$	**1**	**3**	**z³**
Φ_1^u	**2**	**1**	**1**	χ_i	**1**	**3**	**1**
Φ_2^u	**2**	**1′**	**1**	η	**1**	**1**	**z²**
Φ_3^u	**2**	**1″**	**1**	ζ	**1**	**1**	**z**
Φ_i^d	**2**	**3**	**1**				

Table 4.1: Charge assignments for leptons, quarks, scalars (Φ_i^u, Φ_i^d and χ_i) as well as 'dark matter sector' (ζ and η). Here **z** is the fourth root of unity, i.e. $\mathbf{z}^4 = 1$.

$L_i = (\nu_i, l_i)^T$, $i = e, \mu, \tau$ denote the lepton doublets, transforming as indicated under the flavour symmetry.

Apart from the Standard Model fermions, the model also includes three right–handed neutrinos $\nu_{i,R}$ which are singlets under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge group, singlets under A_4, but carry charge **z** under Z_4. We also add three

gauge singlet Dirac fermions $N_{i,L}, N_{i,R}$; $i = 1, 2, 3$ transforming as triplets of A_4 and with charge $\mathbf{z}$ under Z_4, as shown in Table 4.1. Notice that in the scalar sector we have two different sets of fields Φ_i^u, Φ_i^d; $i = 1, 2, 3$, which are all doublets under the $\mathrm{SU}(2)_L$ gauge group, both sets transforming trivially under Z_4. Under the A_4 flavour symmetry, Φ_i^d transforms as a triplet, while Φ_i^u transform as singlets. In addition to the above symmetries we also impose an additional Z_2 symmetry[1]. Under this Z_2 symmetry, all the fields transform as 1 except for Φ_i^d, $l_{i,R}$ and $d_{i,R}$, which transform as -1. The role of this Z_2 symmetry is to prevent the Higgs doublets Φ_i^d from coupling the up-type quarks and neutrino sector, and the Φ_i^u Higgs doublets from the down-type quarks and charged leptons.

In addition we need scalar singlets, for example the χ_i, $i = 1, 2, 3$. These are gauge singlets transforming as a triplet under the A_4 and trivially under Z_4. We also add two other gauge singlet scalars ζ and η both of which transform trivially under A_4 but carry Z_4 charges $\mathbf{z}$ and $\mathbf{z}^2$ respectively. Notice that, since under the Z_4 symmetry the field η carries a charge $\mathbf{z}^2 = -1$, it follows that η can be taken to be real.

As discussed in [1, 7] the lepton quarticity symmetry Z_4 serves a double purpose. It not only ensures that neutrinos are Dirac particles, but also guarantees the stability of the scalar particle ζ, making it a viable dark matter WIMP. If the quarticity symmetry is broken either by an explicit soft term or spontaneously, through non-zero vacuum expectation values (vevs) to any of the scalars η, ζ which carry a non-trivial Z_4 charge, then both the Dirac nature of neutrinos and stability of dark matter is simultaneously lost.

We now turn our attention to the Yukawa sector of our model. In the neutrino sector the Yukawa terms relevant for generating masses for the neutrinos and the

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[1]This additional Z_2 symmetry is only required in a non-supersymmetric variant. Clearly the model can be easily supersymmetrized, in which case this additional Z_2 symmetry is no longer required.

heavy neutral fermions N_L, N_R are given by

$$
\mathcal{L}_{\mathrm{Yuk},\nu} = y_1 \left[\begin{pmatrix} \bar{L}_e \\ \bar{L}_\mu \\ \bar{L}_\tau \end{pmatrix}_3 \otimes \begin{pmatrix} N_{1,R} \\ N_{2,R} \\ N_{3,R} \end{pmatrix}_3 \right]_1 \otimes (\Phi_1^u)_1 + y_2 \left[\begin{pmatrix} \bar{L}_e \\ \bar{L}_\mu \\ \bar{L}_\tau \end{pmatrix}_3 \otimes \begin{pmatrix} N_{1,R} \\ N_{2,R} \\ N_{3,R} \end{pmatrix}_3 \right]_{1''} \otimes (\Phi_2^u)_{1'}
$$

$$
+ y_3 \left[\begin{pmatrix} \bar{L}_e \\ \bar{L}_\mu \\ \bar{L}_\tau \end{pmatrix}_3 \otimes \begin{pmatrix} N_{1,R} \\ N_{2,R} \\ N_{3,R} \end{pmatrix}_3 \right]_{1'} \otimes (\Phi_3^u)_{1''} + y_1' \left[\begin{pmatrix} \bar{N}_{1,L} \\ \bar{N}_{2,L} \\ \bar{N}_{3,L} \end{pmatrix}_3 \otimes \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}_3 \right]_1 \otimes (\nu_{e,R})_1
$$

$$
+ y_2' \left[\begin{pmatrix} \bar{N}_{1,L} \\ \bar{N}_{2,L} \\ \bar{N}_{3,L} \end{pmatrix}_3 \otimes \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}_3 \right]_{1''} \otimes (\nu_{\mu,R})_{1'} + y_3' \left[\begin{pmatrix} \bar{N}_{1,L} \\ \bar{N}_{2,L} \\ \bar{N}_{3,L} \end{pmatrix}_3 \otimes \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}_3 \right]_{1'} \otimes (\nu_{\tau,R})_{1''}
$$

$$
+ M \left[\begin{pmatrix} \bar{N}_{1,L} \\ \bar{N}_{2,L} \\ \bar{N}_{3,L} \end{pmatrix}_3 \otimes \begin{pmatrix} N_{1,R} \\ N_{2,R} \\ N_{3,R} \end{pmatrix}_3 \right]_1 + c_1 \left[\begin{pmatrix} \bar{N}_{1,L} \\ \bar{N}_{2,L} \\ \bar{N}_{3,L} \end{pmatrix}_3 \otimes \left[\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}_3 \otimes \begin{pmatrix} N_{1,R} \\ N_{2,R} \\ N_{3,R} \end{pmatrix}_3 \right]_{3S} \right]_1
$$

$$
+ c_2 \left[\begin{pmatrix} \bar{N}_{1,L} \\ \bar{N}_{2,L} \\ \bar{N}_{3,L} \end{pmatrix}_3 \otimes \left[\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}_3 \otimes \begin{pmatrix} N_{1,R} \\ N_{2,R} \\ N_{3,R} \end{pmatrix}_3 \right]_{3A} \right]_1 + h.c. \tag{4.2}
$$

where $3S$ and $3A$ denote the symmetric and antisymmetric A_4 triplet combinations obtained from the tensor product of two A_4 triplets. Notice also that $3S$ and $3A$ are not two different irreducible representations of A_4, which only has one triplet, but simply different contractions with the same transformation rule. Also, $y_i, y_i', c_1, c_2; i = 1, 2, 3$ are the Yukawa couplings which, for simplicity, are taken to be real. The parameter M is the gauge and flavour-invariant mass term for the heavy neutral fermions. Here we like to highlight the important role played by the A_4 flavour symmetry. Owing to the A_4 charges of the left and right handed neutrinos, a tree level Yukawa coupling between them of type $y_\nu \bar{L}_L \nu_R \Phi_i^u$

is forbidden. Thus neutrino masses can only appear through type-I Dirac seesaw mechanism as we now discuss.

After symmetry breaking the scalars χ_i and Φ_i^u acquire vevs $\langle \chi_i \rangle = u_i$; $\langle \Phi_i^u \rangle = v_i^u$; $i = 1, 2, 3$. The invariant mass term M can be naturally much larger than the symmetry breaking scales, i.e. $M \gg v_i^u, u_i$. In this limit, for any numerical purpose the last two terms in Eq. 4.2 can be safely neglected. Under this approximation the 6×6 mass matrix for the neutrinos and the heavy neutral fermions in the basis $(\bar{\nu}_{e,L}, \bar{\nu}_{\mu,L}, \bar{\nu}_{\tau,L}, \bar{N}_{1,L}, \bar{N}_{2,L}, \bar{N}_{3,L})$ and $(\nu_{e,R}, \nu_{\mu,R}, \nu_{\tau,R}, N_{1,R}, N_{2,R}, N_{3,R})^T$ is given by

$$M_{\nu,N} = \begin{pmatrix} 0 & 0 & 0 & a_1' & 0 & 0 \\ 0 & 0 & 0 & 0 & a_2' & 0 \\ 0 & 0 & 0 & 0 & 0 & a_3' \\ y_1' u_1 & y_2' u_1 & y_3' u_1 & M & 0 & 0 \\ y_1' u_2 & \omega y_2' u_2 & \omega^2 y_3' u_2 & 0 & M & 0 \\ y_1' u_3 & \omega^2 y_2' u_3 & \omega y_3' u_3 & 0 & 0 & M \end{pmatrix} \tag{4.3}$$

where ω is the third root of unity, with $\omega^3 = 1$ and

$$a_1' = y_1 v_1^u + y_2 v_2^u + y_3 v_3^u$$

$$a_2' = y_1 v_1^u + \omega y_2 v_2^u + \omega^2 y_3 v_3^u$$

$$a_3' = y_1 v_1^u + \omega^2 y_2 v_2^u + \omega y_3 v_3^u \tag{4.4}$$

As mentioned before, owing to the A_4 symmetry, a direct coupling between ν_L and ν_R is forbidden, leading to the vanishing of all entries in the upper left quadrant of Eq. 4.3. The mass matrix in Eq. 4.3 can be rewritten in a more

compact form, as

$$M_{\nu,N} = \begin{pmatrix} 0 & \mathrm{diag}(a'_1, a'_2, a'_3) \\ \mathrm{diag}(u_1, u_2, u_3)\sqrt{3}U_{\mathrm{m}}\mathrm{diag}(y'_1, y'_2, y'_3) & M\mathrm{diag}(1,1,1) \end{pmatrix} \qquad (4.5)$$

Where U_{m} is the usual magic matrix,

$$U_{\mathrm{m}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} . \qquad (4.6)$$

Note that, in the limit $M \gg v_i^u, u_i$ the mass matrix in Eq. (4.3) can be easily block diagonalized by the perturbative seesaw diagonalization method given in Ref. [70]. The resulting 3×3 mass matrix for light neutrinos can be viewed as the Dirac version of the well known type-I seesaw mechanism. The above mass generation mechanism can also be represented diagramatically as shown in Fig. 4.1.

Figure 4.1: Feynman view of type-I Dirac seesaw mechanism in the model where the indices $i, j, k, m, l = 1, 2, 3$.

The 3×3 matrix for the light neutrinos is

$$M_\nu = \frac{1}{M}\mathrm{diag}(a_1, a_2, a_3)\sqrt{3}U_{\mathrm{m}}\mathrm{diag}(y'_1, y'_2, y'_3) , \qquad (4.7)$$

where $a_i = a'_i u_i$. We take the alignment $u_1 = u_2 = u_3 = u$ for the vev of the A_4 triplet scalars χ_i similar to [227,231]. In this alignment limit of A_4 triplet scalars we have

$$a_1 = (y_1 v_1^u + y_2 v_2^u + y_3 v_3^u)u$$

$$a_2 = (y_1 v_1^u + \omega y_2 v_2^u + \omega^2 y_3 v_3^u)u$$

$$a_3 = (y_1 v_1^u + \omega^2 y_2 v_2^u + \omega y_3 v_3^u)u \qquad (4.8)$$

which simplifies the notation, although it does not change the form of the neutrino mass matrix in Eq. (4.7). Notice that we have not imposed any alignment for the vevs of the A_4 singlet scalars Φ_i^u [2] The light neutrino mass matrix of Eq. 4.7 with the simplified a_i of Eq. 4.8 can be diagonalized by a bi-unitary transformation

$$U_\nu^\dagger M_\nu V_\nu = D_\nu, \qquad (4.9)$$

where D_ν is diagonal, real and positive. Owing to the A_4 flavour symmetry, the resulting rotation matrix acting on left handed neutrinos U_ν in the standard parametrization (for both hierarchies), leads to $\theta_{23}^\nu = \frac{\pi}{4}$ and $\delta^\nu = \pm\frac{\pi}{2}$ while the other two angles can be arbitrary. Thus, owing to the A_4 symmetry, U_ν in standard parameterization leads to following mixing angles

$$\theta_{23}^\nu = 45°, \qquad \delta^\nu = \pm 90°$$

$$\theta_{12}^\nu = \text{arbitrary} \qquad \theta_{13}^\nu = \text{arbitrary} \qquad (4.10)$$

---

[2]Doing so for the different A_4 singlet scalar vevs is not very natural. Indeed, unlike the case of A_4 triplet scalars, a priori the vevs of different A_4 singlet scalars have no reason to obey any mutual alignment.

Similar features of maximal θ_{23} and δ have been obtained previously in the context of Majorana neutrinos [231, 276]. Although the angles θ_{12}^{ν} and θ_{13}^{ν} can independently take any value, they are strongly correlated with each other.

We have performed an extensive numerical scan for both type of hierarchies in the whole parameter range taking all Yukawa couplings in the perturbative range of $[-1, 1]$. We find that in the whole allowed range for either type of ordering, one cannot simultaneously fit both θ_{12}^{ν} and θ_{13}^{ν} in the current global experimental range obtained from neutrino oscillation experiments [40]. This implies that in our model U_{ν} alone cannot explain the current neutrino oscillation data.

However, the lepton mixing matrix U_{LM} which is probed by neutrino oscillation experiments is the product of the charged lepton rotation matrix U_l with the neutrino transformation matrix U_{ν} [34] i.e.

$$U_{LM} = U_l^{\dagger} U_{\nu} \tag{4.11}$$

In our model the charged lepton mixing matrix U_l is also non-trivial and contributes to the full leptonic mixing matrix U_{LM}. We now move to discuss the structure of mass matrices and mixing matrices for charged leptons as well as the up and down type quarks.

We now turn to the discussion with up type quark mass matrix. The invariant Yukawa Lagrangian relevant to generating up type quark mass matrix is given

by

$$
\mathcal{L}_{\text{Yuk,u}} = y_1^u \left[\begin{pmatrix} \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{Q}_3 \end{pmatrix}_3 \otimes \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}_3 \right]_1 \otimes (\Phi_1^u)_1 + y_2^u \left[\begin{pmatrix} \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{Q}_3 \end{pmatrix}_3 \otimes \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}_3 \right]_{1''} \otimes (\Phi_2^u)_{1'}
$$

$$
+ y_3^u \left[\begin{pmatrix} \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{Q}_3 \end{pmatrix}_3 \otimes \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}_3 \right]_{1'} \otimes (\Phi_3^u)_{1''} + \ h.c. \tag{4.12}
$$

where y_i^u; $i = 1, 2, 3$ are the Yukawa couplings which for simplicity we take to be all real. After spontaneous symmetry breaking Eq. 4.12 leads to a diagonal mass matrix given by

$$
M_u = \begin{pmatrix} y_1^u v_1^u + y_2^u v_2^u + y_3^u v_3^u & 0 & 0 \\ 0 & y_1^u v_1^u + \omega\, y_2^u v_2^u + \omega^2\, y_3^u v_3^u & 0 \\ 0 & 0 & y_1^u v_1^u + \omega^2\, y_2^u v_2^u + \omega\, y_3^u v_3^u \end{pmatrix}.
$$

$$\tag{4.13}$$

On the other hand, the Yukawa Lagrangian relevant to down type quarks mass generation is given by

$$
\mathcal{L}_{\text{Yuk,d}} = y_1^d \left[\begin{pmatrix} \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{Q}_3 \end{pmatrix}_3 \otimes \left[\begin{pmatrix} q_{d,R} \\ q_{s,R} \\ q_{b,R} \end{pmatrix}_3 \otimes \begin{pmatrix} \Phi_1^d \\ \Phi_2^d \\ \Phi_3^d \end{pmatrix}_3 \right]_{3S} \right]_1
$$

$$
+ y_2^d \left[\begin{pmatrix} \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{Q}_3 \end{pmatrix}_3 \otimes \left[\begin{pmatrix} q_{d,R} \\ q_{s,R} \\ q_{b,R} \end{pmatrix}_3 \otimes \begin{pmatrix} \Phi_1^d \\ \Phi_2^d \\ \Phi_3^d \end{pmatrix}_3 \right]_{3A} \right]_1 + \ h.c. \tag{4.14}
$$

where y_i^d; $i = 1, 2$ are the Yukawa couplings which for simplicity are taken to be real. The resulting mass matrix for down type quarks after spontaneous symmetry breaking is given by

$$M_d = \begin{pmatrix} 0 & a_d\alpha & b_d \\ b_d\alpha & 0 & a_d r \\ a_d & b_d r & 0 \end{pmatrix}. \tag{4.15}$$

where $\langle \Phi_i^d \rangle = v_i^d$; $i = 1, 2, 3$ and $a_d = (y_1^d - y_2^d)v_2^d$, $b_l = (y_1^d + y_2^d)v_2^d$. Moreover, α and r are ratios of the vevs of Φ_i^d and are given as $\alpha = {v_3^d}/{v_2^d}$ and $r = {v_1^d}/{v_2^d}$.

Finally, the invariant Yukawa terms for the charged leptons is given by

$$\mathcal{L}_{\text{Yuk,l}} = y_1^l \left[\left[\begin{pmatrix} \bar{L}_1 \\ \bar{L}_2 \\ \bar{L}_3 \end{pmatrix}_3 \otimes \left[\begin{pmatrix} l_{e,R} \\ l_{\mu,R} \\ l_{\tau,R} \end{pmatrix}_3 \otimes \begin{pmatrix} \Phi_1^d \\ \Phi_2^d \\ \Phi_3^d \end{pmatrix}_3 \right]_{3S} \right]_1 \right.$$

$$+ y_2^l \left[\begin{pmatrix} \bar{L}_1 \\ \bar{L}_2 \\ \bar{L}_3 \end{pmatrix}_3 \otimes \left[\begin{pmatrix} l_{e,R} \\ l_{\mu,R} \\ l_{\tau,R} \end{pmatrix}_3 \otimes \begin{pmatrix} \Phi_1^d \\ \Phi_2^d \\ \Phi_3^d \end{pmatrix}_3 \right]_{3A} \right]_1 + h.c. \tag{4.16}$$

where y_i^l, $i = 1, 2$, are the Yukawa couplings which, for simplicity, we take to be real. After symmetry breaking the charged lepton mass matrix is given by

$$M_l = \begin{pmatrix} 0 & a_l\alpha & b_l \\ b_l\alpha & 0 & a_l r \\ a_l & b_l r & 0 \end{pmatrix}. \tag{4.17}$$

where, just as in the down quark case, here also $a_l = (y_1^l - y_2^l)v_2^d$, $b_l = (y_1^l + y_2^l)v_2^d$. The parameters α, r which are the ratios of the vevs of Φ_i^d i.e. $\alpha = {v_3^d}/{v_2^d}$ and $r = {v_1^d}/{v_2^d}$ are the same as those defined after Eq. 4.15. This matrix is completely

analogous to the down-type quark mass matrix. Note that while α and r are the same both in the quark and in the lepton sector, as they are simply ratios between the vevs of Φ_i^d, while a_f and b_f, $f \in \{l, q\}$, are different.

These mass matrices for charged leptons and down-type quarks correspond to those discussed in [222–225] and lead to the generalized bottom-tau relation of 4.1. In section 2.0.4 we show that there is enough freedom to fit the charged lepton and down type quark masses within their 1-σ range. Apart from fitting all the masses as well as leading to the generalized bottom-tau relations, the charged lepton mass matrix 4.17 also leads to non-trivial charged lepton rotation matrix U_l. As we show in section 2.0.4 this non-trivial contribution from U_l results in a lepton mixing matrix U_{LM} consistent with the current global fits to neutrino oscillation data [40]. The lepton mixing matrix obtained from our model also implies normal ordering for neutrino masses and leads to an interesting correlation between the atmospheric mixing angle and CP violating phase, the two most ill–determined parameters in leptonic mixing matrix.

2.0.4   Flavour predictions: numerical results

In this section we discuss the phenomenological implications of our model. The important predictions emerging in our model are:

- The flavour-dependent bottom-tau unification mass relation of Eq. (4.1).

- A correlation between the two poorly determined oscillation parameters: the atmospheric angle θ_{23} and δ_{CP}.

- A normal ordering for the neutrinos.

In this section we discuss these numerical predictions in some detail, given the experimentally measured 'down-type' fermion masses, solar and reactor mixing angles as well as neutrino squared mass differences.

2.0.5   Charged lepton and down-type quark masses

We start our discussion by looking in more detail at the down type quark and charged lepton mass matrices discussed previously in Eqs. 4.15 and 4.17. This structure for the down type quark and charged lepton mass matrices has been previously discussed in several works [222–225]. In this section for illustration purpose we first discuss the results obtained in previous works by closely following the approach taken in previous works like in [277]. Subsequently, we will generalize the analysis of previous works and discuss how the same results can be obtained using a more general setup and more detailed considerations.

We start from the charged lepton mass matrix obtained in Eq. 4.17. The correct charged lepton masses are reproduced if the vevs of the A_4 triplet fields Φ_i^d satisfy the alignment limit $v^d(1, \epsilon_1, \epsilon_2)$, where $v^d \gg \epsilon_1, \epsilon_2$. Then, in similar notation and spirit as in Ref. [277], we extract the three invariants of the Hermitian matrix $S = M_l M_l^\dagger$: $Det(S)$, $Tr(S)$ and $Tr(S)^2 - Tr(S^2)$. We then compute their values in the diagonal basis in terms of the charged lepton masses, m_e, m_μ and m_τ. The equations are

$$\mathrm{Det}S = (m_e m_\mu m_\tau)^2$$

$$\mathrm{Tr}S = m_e^2 + m_\mu^2 + m_\tau^2$$

$$(\mathrm{Tr}S)^2 - \mathrm{Tr}S^2 = 2m_e^2 m_\mu^2 + 2m_e^2 m_\tau^2 + 2m_\mu^2 m_\tau^2 \tag{4.18}$$

The expressions in Eq.4.18 can be readily solved in the vev alignment limit $v^d(1, \epsilon_1, \epsilon_2)$ discussed before. This amounts to the approximation

$$v_1^d \gg v_3^d \quad \text{and} \quad \frac{v_1^d}{v_2^d} \gg \frac{y_1^l + y_2^l}{y_1^l - y_2^l} \gg 1,$$

or equivalently

$$r \gg \alpha \quad \text{and} \quad r \gg \frac{b_l}{a_l} \gg 1.$$

The solutions for r, a_l and b_l are given as

$$r = \frac{m_\tau}{\sqrt{m_e m_\mu}} \sqrt{\alpha} \tag{4.19}$$

$$a_l = \frac{m_\mu}{m_\tau} \sqrt{\frac{m_e m_\mu}{\alpha}} \tag{4.20}$$

$$b_l = \sqrt{\frac{m_e m_\mu}{\alpha}} \tag{4.21}$$

Owing to the A_4 symmetry, the charged lepton mass matrix in Eq. 4.17 and down type quark mass matrix in Eq. 4.15 have the same structure. As a result the down quark mass matrix can also be decomposed using equations analogous to Eq. 4.18. For down type mass matrix of 4.15 we obtain

$$r = \frac{m_b}{\sqrt{m_d m_s}} \sqrt{\alpha} \tag{4.22}$$

$$a_d = \frac{m_s}{m_b} \sqrt{\frac{m_d m_s}{\alpha}} \tag{4.23}$$

$$b_d = \sqrt{\frac{m_d m_s}{\alpha}} \tag{4.24}$$

Note that the parameters α and r are common for both the charged lepton sector as well as in the down-type quark sector, as they are simply ratios between vevs of the fields Φ_i^d. Thus comparing Eqs. 4.19 and 4.22 we obtain the following mass relation

$$\frac{m_\tau}{\sqrt{m_e m_\mu}} = \frac{m_b}{\sqrt{m_s m_d}} \tag{4.25}$$

The procedure sketched above can be performed in a more general way by solving the equations numerically. The relevant equations for the case of charged

leptons are

$$(m_e m_\mu m_\tau)^2 = a_l{}^6 r^2 \alpha^2 + 2a_l{}^3 b_l^3 r^2 \alpha^2 + b_l{}^6 r^2 \alpha^2$$

$$m_e^2 + m_\mu^2 + m_\tau^2 = (a_l{}^2 + b_l^2)(1 + r^2 + \alpha^2)$$

$$2m_e^2 m_\mu^2 + 2m_e^2 m_\tau^2 + 2m_\mu^2 m_\tau^2 = (a_l{}^2 + b_l^2)^2(1 + r^2 + \alpha^2)^2 - (a_l{}^2 + b_l^2 r^2)^2 \quad (4.26)$$

$$- (b_l^2 + a_l{}^2 \alpha^2)^2 - (a_l{}^2 r^2 + b_l^2 \alpha^2)^2 \quad (4.27)$$

Taking as input parameters the best fit values (at M_Z scale) for the charged lepton masses [278] and imposing $r > \frac{b}{a}$, there is a one-parameter family of solutions to these equations. These are related to the approximate solution described before. We build the functions $r(\alpha)$, $a_l(\alpha)$ and $b_l(\alpha)$ taking α as a free parameter. In the correct range for the parameter α, the unique solution is found to be near the limit $r \gg \dfrac{b}{a} \gg 1$ and therefore it again leads to the mass relation in Eq. 4.1. Since the (α, r, a_l, b_l) are solutions of Eqs. 4.26, the charged lepton masses are fitted exactly to their best-fit values. In order to underpin the relevant solution for down type quark masses, we also need to take into account not only the mass relation 4.1 and the charged lepton masses, but also the constraints for the experimental measurements (along with renormalization group evolution to M_Z scale) of all the down-type quark masses [278]. Here, we will impose the rather stringent 1-σ bounds[3] on the down-type quarks masses at Z boson energy scale [278].

Then, for each valid $(\alpha, r, a_l, b_l,)$ we take a_q and b_q as

$$a_q = \frac{m_s}{m_b} \sqrt{\frac{m_d m_s}{\alpha}} (1 + \epsilon_1) \quad (4.28)$$

$$b_q = \sqrt{\frac{m_d m_s}{\alpha}} (1 + \epsilon_2) \ , \quad (4.29)$$

---

[3]Imposing 1-σ is in fact rather stringent, and can easily be relaxed to a more conservative criterium e.g. 3-σ. We have deliberatively imposed the stringent 1-σ bound in other to highlight the high precision obtained from our results.

where ϵ_1 and ϵ_2 are expected to be small.

Using this procedure gives sets of parameters $(\alpha, r, a_l, b_l, a_q, b_q)$ which give at the same time best fit values for charged lepton masses, down-type quarks inside the 1σ range and the mass relation in Eq. 4.1. In figure 4.2 we show the family-dependent bottom-tau mass prediction of our model for the s and d masses, along with their allowed 1-σ ranges.
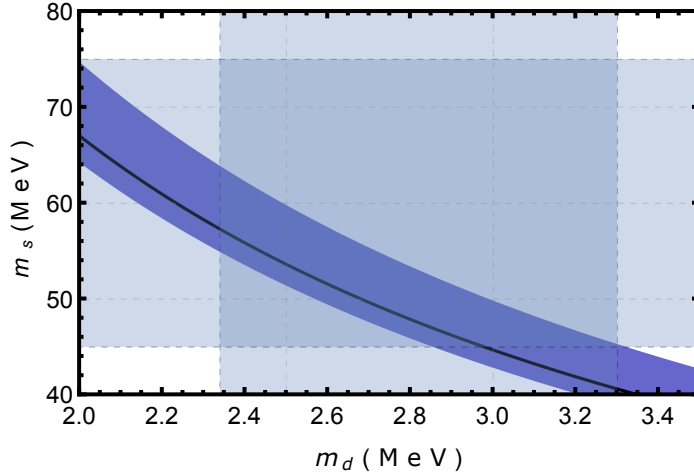

Figure 4.2: Prediction for the s and d quark masses (at M_Z scale) in our model. The dark blue area is the allowed region from our model for the s and d quark masses, while varying the mass of the b quark in its 1-σ range. The light blue area is the allowed 1-σ range (at M_Z scale) for the mass of the quarks s and d [278].

2.0.6   The charged piece of the lepton mixing matrix U_l

The charged lepton mass matrix Eq. 4.17 not only leads to correct lepton masses but also to non-trivial charged lepton rotation matrix U_l as we discuss now. Just like the neutrino mass matrix, the charged lepton mass matrix can also be diagonalized by a bi-unitary transformation as

$$U_l^\dagger M_l V = \text{diag}(m_e, m_\mu, m_\tau) \qquad (4.30)$$

The charged lepton mixing matrix U_l in standard parameterization can be written as

$$U_l = P U_{23}(\theta_{23}^l, 0) U_{13}(\theta_{23}^l, \delta^l) U_{12}(\theta_{12}^l, 0) P' \tag{4.31}$$

where P and P' are diagonal matrix of phases and U_{ij} is the usual complex rotation matrix appearing in the symmetric parametrization of fermion mixing given in [34], e.g.

$$U_{12}(\theta_{12}, \delta) = \begin{pmatrix} \cos\theta_{12} & e^{-i\delta}\sin\theta_{12} & 0 \\ -e^{i\delta}\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4.32}$$

with an analogous definitions for U_{13} and U_{23}. For the charged lepton mass matrix 4.17, we find that

$$\sin\theta_{12}^l = \sqrt{\frac{m_e}{m_\mu}}\frac{1}{\sqrt{\alpha}} + \mathcal{O}(\frac{1}{\alpha^2}) \approx \mathcal{O}(\lambda_C)$$

$$\sin\theta_{13}^l = \frac{m_u}{m_\tau^2}\sqrt{m_e m_\mu}\frac{1}{\sqrt{\alpha}} + \mathcal{O}(\frac{1}{\alpha^2}) \approx \mathcal{O}(10^{-5})$$

$$\sin\theta_{23}^l = \frac{m_e m_\mu^2}{m_\tau^3}\frac{1}{\alpha} + \mathcal{O}(\frac{1}{\alpha^2}) \approx \mathcal{O}(10^{-7}) \tag{4.33}$$

Where $\lambda_C \approx 0.22$ is the sine of the Cabbibo angle. In order to reproduce adequate values for the CKM matrix elements we may introduce a vector-like quark mixing with the up-type quarks, along the lines followed recently in [279].

The diagonal phases in P and P' are all are exactly 0 except for one which is π. Performing the numerical computation reconfirms the results obtained in Eq. 4.33 for the charged lepton mass matrix i.e. θ_{12}^l is finite and its value depends on the value of α in an inverse way, while θ_{13}^l and θ_{23}^l are both negligible (in particular, $\theta_{13}^l \sim 10^{-5}$ and $\theta_{23}^l \sim 10^{-7}$. Then, the charged lepton mixing matrix for our

model is given as

$$
U_l \approx \begin{pmatrix} \cos\theta_{12}^l & \sin\theta_{12}^l & 0 \\ -\sin\theta_{12}^l & \cos\theta_{12}^l & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.34}
$$

Thus in our model the lepton mixing matrix $U_{LM} = U_l^\dagger U_\nu$ receives significant charged lepton corrections which have interesting phenomenological consequences as we discuss in next section.

2.0.7   The lepton mixing matrix and neutrino mass ordering

As mentioned before in Section 2, the light neutrino mass matrix in Eq. 4.7 leads to the neutrino mixing matrix U_ν which in standard parameterization [34] leads to

$$
U_\nu = P U_{23}\left(\pi/4, 0\right) U_{13}\left(\theta_{13}^\nu, \pi/2\right) U_{12}\left(\theta_{12}^\nu, 0\right) P' \tag{4.35}
$$

As mentioned before, owing to the A_4 symmetry, we have that $\theta_{23}^\nu = 45°$ and $\delta_{CP}^\nu = 90°$ for both types of mass ordering: normal ordering (NO) or inverted ordering (IO). Since neutrinos in our model are Dirac fermions, the phases in the right in Eq. 4.35 i.e. P', are unphysical, while θ_{13}^ν and θ_{12}^ν are strongly correlated between each other. This result is completely general and follows from the A_4 symmetry, independently of the mass ordering, NO or IO. However, the behavior of the correlation between θ_{12}^ν and θ_{13}^ν does depend on the choice of NO or IO.

Taking into account the results in the previous sections, the lepton mixing matrix is

$$
U_{LM} = U_l^\dagger U_\nu = U_{12}\left(\theta_{12}^l, 0\right)^\dagger \cdot U_\nu \tag{4.36}
$$

One can regard the matrix U_l as a correction to the neutrino mixing parameters obtained just by diagonalizing the neutrino mass matrix. For the NO case, the angle θ_{12}^l has to be big enough ($\sim> 15°$) so as to account for the correct mixing angles of the lepton mixing matrix, but at the same time it has to remain controlled ($< 20°$) otherwise the down-type quark masses cannot be fitted. This means that the parameter α has to be between 0.04 and 0.08 in order to fit in the 3σ allowed range of the neutrino oscillation parameters. This lepton mixing matrix can fit the neutrino oscillation parameters within 3σ at the same time as the mass matrices fit the down-type quarks and the neutrino squared mass differences in the 1σ range and the charged lepton masses to their best fit value. Once the lepton mixing matrix is written in the standard parametrization, two interesting features arise. On the one hand, $\theta_{23} > 45°$ and, on the other, a strong correlation appears between the atmospheric angle θ_{23} and δ_{CP}, as shown in figure 4.3.

For IO, a different scenario arises. As in the case for NO, lepton corrections cannot be very big otherwise the down-type quark masses will not be fitted. However, in this case the structure of the correlation between θ_{12}^ν and θ_{13}^ν implies that for allowed charged lepton corrections, the reactor angle θ_{13} is always outside the 3σ allowed range. Note that the model does not include any a priori theoretical bias in favour of normal ordering but it is a prediction of the model once one impose experimental constraints.

To summarize, we have proposed a $A_4 \otimes Z_4 \otimes Z_2$ flavour extension of the Standard Model with naturally small Dirac neutrino masses. Our lepton quarticity symmetry simultaneously forbids Majorana mass terms and provides dark matter stability. The flavour symmetry plays a multiple role, providing :

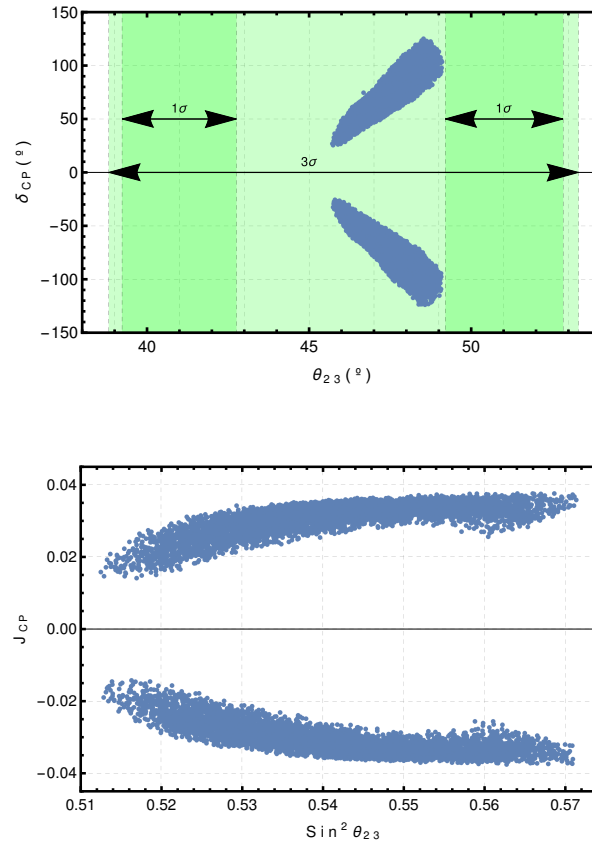- A generalized family-dependent bottom-tau mass relation, Eq. (4.1) and Fig. 4.2.

Figure 4.3: CP violation and θ_{23} predictions within the model. Left panel: δ_{CP} vs θ_{23}. The green regions are the 1σ (dark) and 3σ (light) regions for θ_{23} from current oscillation fit. Right panel: Same correlation, now showing J_{CP} vs $\sin^2\theta_{23}$ and zooming in the region allowed by the model, fully consistent in the 2σ experimental range.

- A natural realization of the type-I seesaw mechanism for Dirac neutrino masses, as the tree level Dirac Yukawa term between left and right handed neutrinos is forbidden.

- A very predictive flavour structure to the lepton mixing matrix. The latter directly correlates the CP phase δ_{CP} and the atmospheric angle θ_{23}, as shown in Fig. 4.3. CP must be significantly violated in neutrino oscillations, and the atmospheric angle θ_{23} lies in the second octant. (v) Only the normal neutrino mass ordering is consistet at 3σ.

Our approach provides an adequate pattern of neutrino mass and mixing as well as a viable stable dark matter. This is achieved while providing testable predictions concerning the currently most relevant oscillation parameters, the atmospheric angle θ_{23} and the CP phase δ_{CP}, as well as a successful family generalization of bottom-tau unification, despite the absence of an underlying Grand Unified Theory. Our lepton quarticity approach can also potentially lead to other interesting phenomena such as neutrinoless quadruple beta decay ($0\nu4\beta$), which has recently been searched for by the NEMO collaboration [221]. The intimate connection between the Dirac nature of neutrinos and dark matter stability constitutes a key feature of our model. Other phenomenological implications will be taken up elsewhere.

3 Charged Lepton Flavour Violation

Lepton Flavour Violation (LFV) processes such as $l_\alpha \to l_\beta\gamma$ or $l_\alpha \to 3l_\beta$ and closely related quantities like the lepton anomalous magnetic moments have received a lot of attention for a few decades. In particular, a long standing discrepancy between the measured muon anomalous magnetic moment [280, 281] and the SM calculated prediction [282–286] hints towards some unknown new physics that can explain the SM deviation or a need to improve the already very precise SM calculations, in particular hadronization contributions. On the other hand, the SM with massless neutrinos predicts exactly a 0 rate for LFV processes [4], but models with massive neutrinos need to factor in stringent experimental upper bounds [289–291]. Most importantly, models that try to explain the muon or electron anomalous magnetic moment will typically generate LFV. The bibliography on this topic is vast and we direct the reader to the existing reviews

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[4]Some extensions of the SM still generate sizeable LFV processes even in the massless limit [287, 288]

and references within for more complete information [292–296]. In this section we show how a class of low scale Dirac neutrino mass models can generate observable LFV rates. The fact that LFV can proceed even in the limit of massless neutrinos was noted a long time ago [297–299] and hence it can be probed in experiment. It was also proposed that in inverse seesaw schemes, the mediators of neutrino mass generation can be searched for at high energy colliders [287, 288]. The latest restrictions come for the ATLAS and CMS experiments [300]. The phenomenology of charged LFV can be probed at high energies and also high intensity facilities [301–305].

Here we present an illustration showing that, as expected, similar result holds for inverse Dirac seesaw models.

As a paradigmatic example of a LFV process in a low scale Dirac neutrino model, let us estimate the $\mu^- \to e^- \gamma$ decay rate in the simplest Dirac inverse seesaw, described in Sec. 3.3.1 and [6]. In this simple scenario the neutrino mass for the one generation limit is given by

$$m_\nu = Y_\phi Y_\chi \frac{v\,u}{M} \tag{4.37}$$

In order to get $m_\nu \sim 0.1$ eV with Yukawas of order $Y_i \sim 1$ and $v \sim 100$ GeV we find the relation $u/M \sim 10^{-12}$. In the inverse seesaw limit we take $u \ll v$ and M to be not far from the electroweak scale, for example $M \sim 1$ TeV and $u \sim 1$ eV. The 6×6 lepton mixing matrix is given by

$$U_L = \begin{pmatrix} U_l & U_{\mathrm{mix}} \\ U_{\mathrm{mix}}^\dagger & U_{\mathrm{heavy}L} \end{pmatrix} \tag{4.38}$$

Where U_l is the light neutrino mixing matrix, measured in oscillation experiments, while $U_{\mathrm{mix}} \sim \mathcal{O}(v/M)$ is the mixing matrix between the light and heavy states. Since oscillation experiments yield no evidence for the non-unitarity of U_l [306], this mixing must be small.

In the scalar sector of the model, there will be a tree level mixing between the physical scalars. The Lagrangian terms $-\mu_\phi^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 - \mu_\chi^2(\chi^\dagger\chi) + \lambda_\chi(\chi^\dagger\chi)^2 + \lambda_{\text{mix}}(\phi^\dagger\phi)(\chi^\dagger\chi)^2$ lead to a scalar mass matrix after spontaneous symmetry breaking of the form

$$m_h = \begin{pmatrix} \lambda v^2 & \lambda_{mix}u\,v \\ \lambda_{mix}u\,v & \lambda_\chi u^2 \end{pmatrix} \tag{4.39}$$

Which can be diagonalized by a standard 2×2 rotation matrix of angles

$$\tan 2\alpha = \frac{\lambda_{\text{mix}}uv}{\lambda v^2 + \lambda_\chi u^2} \approx \frac{\lambda_{\text{mix}}}{\lambda}\frac{u}{v} \sim 10^{-11} \tag{4.40}$$

Owing to the small mixing, the heavy scalar h will be be SM like and thus $m_h \sim 125\text{GeV}$. The lightest scalar H, on the other hand, will have a mass of order $u = 1$ eV. The coupling of H with the electrons will come at tree level via the term $\bar{L}\phi e_R$ and will be proportional to $\sin\alpha$, the mixing between h and H, and will therefore be small enough to safely escape constraints like those coming from stellar cooling [307]. The mixing matrix of the pseudoscalar part is simply a matrix of 0s. The pseudoscalar doublet will be 'eaten' by the Z^0 boson and the singlet will remain as 'pure' singlet without mixing. Therefore, the coupling between electrons and the massless pseudoscalar D, the Goldstone boson of the theory or the 'Diracon' [149], will come at the loop level and proportional to the mixing between the light and heavy neutrinos $\sim v/M$. Therefore, processes like $\mu^- \to e^- H$ are strongly suppressed. The leading LFV processes will be $\mu^- \to e^- D$ and $\mu^- \to e^- \gamma$ which both happen at the one loop level via exchange of a W boson. We will concentrate on the latter, depicted in Fig. 4.4.

The amplitude $G_{\mu e}$ can be calculated as [302, 303]

$$G_{\mu e} = \sum_k (U_L)_{ek}^* (U_L)_{ek} G_\gamma\left(\frac{M_{N_k}^2}{M_W^2}\right), \qquad G_\gamma(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3}{2(1-x)^4} \tag{4.41}$$
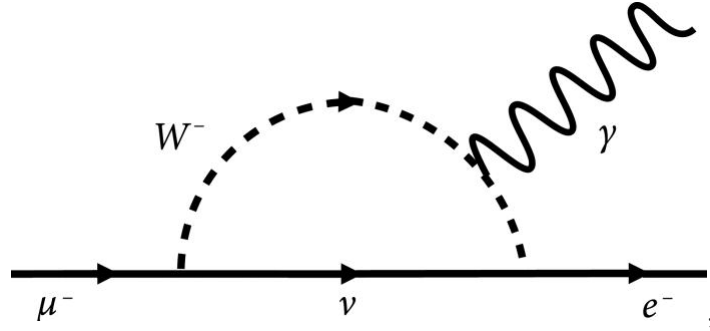
,

Figure 4.4: Leading contribution to $\mu \to e\,\gamma$ in the Dirac inverse seesaw model. Note that the neutrino mass Eigenstates, including the heavy neutral states, are running inside the loop. Therefore the amplitude of this process is dependent on the particular neutrino mass model under study, but independent on the neutrino nature.

As a proof of concept, let us for now perform a rough approximation to estimate the orders of magnitude of the magnitued involved. Since the light neutrino states are small, $m_\nu/M_W \to 0$ and $G_\gamma(0) \to 0$. Therefore we will neglect the light neutrino contribution. If the heavy states have a mass of the order TeV, then $M_N \gg M_W$ and $G_\gamma(x) \to 1/2$. On the other hand, the mixing between heavy and light states given by Eq. 4.38 is of order $(U_L)_{ek} \sim Yv/M_N \sim 10^{-2}$ for $k \in \{4, 5, 6\}$. This yields an amplitude of order

$$G_{\mu e} \sim \frac{3}{2} \left(\frac{Yv}{M_N} \right)^2 \sim 10^{-3} \tag{4.42}$$

In order to compare with experimental constraints we must compute the $\mu \to e\,\gamma$ branching ratio given by

$$\Gamma(\mu \to e\,\gamma) = \frac{\alpha_W^3 s_W^2}{256\pi^2} \left(\frac{m_\mu}{M_W} \right)^4 \frac{m_\mu}{\Gamma_\mu} |G_{\mu e}|^2 \sim 10^{-12} \tag{4.43}$$

by taking the PDG values [36] and the result above. The 2016 experimental limit given by MEG [291] is

$$\Gamma(\mu \to e\,\gamma) < 4.2 10^{-13} \tag{4.44}$$

Thus showing that this class of models has the potential to predict high LFV rates, testable in the close future. A more detailed phenomenological work following these lines is ongoing, so we will not expand further.

Chapter 5

Resumen de la tesis

En esta sección realizaremos un resumen en español de la tesis junto a la metodología empleada, los resultados más importantes y la contribución personal del estudiante a cada uno de los artículos en los que se basa la tesis.

1 Introducción y objetivos

El Modelo Estándar de interacciones Electro-Débiles ha sido un gran éxito desde un punto de vista tanto teórico como experimental. Si bien este éxito no se puede negar, es hora de avanzar y abordar las preguntas que el Modelo Estándar deja sin respuesta, como las masas de neutrinos, la naturaleza de la materia oscura, el problema de la jerarquía o la no observación de la violación de CP en el sector de las interacciones fuertes, entre otros.

De hecho, el Modelo Estándar predice neutrinos sin masa. Sin embargo, los experimentos de oscilación de neutrinos muestran claramente que al menos dos neutrinos son masivos y arrojan luz sobre su patrón de mezcla. Estos experimentos son múltiples, la evidencia es sólida y las mediciones son muy precisas, alcanzando el nivel de error menor al 1% en algunos observables. Sin embargo, las oscilaciones de neutrinos no pueden obtener información acerca de cuestiones más

profundas respecto a la naturaleza de los neutrinos: ¿cuál es la escala absoluta de las masas de neutrinos? Las oscilaciones de neutrinos son sensible solo a las diferencias cuadráticas de las masas de los autoestados masivos. ¿Su naturaleza es Dirac o Majorana? No hay diferencia en el fenómenos de las oscilaciones en uno u otro caso. Se necesitan observaciones complementarias para responder a estas preguntas, y ya se está haciendo con experimentos en funcionamiento en el presente y también experimentos proyectados para el futuro.

Tras una breve introducción sobre el modelo estándar, las oscilaciones de neutrinos y los mecanismos de generación de masa de neutrinos de Majorana, el mecanismo 'seesaw', comenzamos la tesis dando una definición general de un fermión de Dirac: un fermión cuya antipartícula tiene la misma masa pero cargas de signo contrario. Un ejemplo paradigmático sería el electrón, cuya antipartícula de idéntica masa y carga positiva, el positrón, fue descubierta experimentalmente en 1932. En realidad, todos los fermiones cargados conocidos hasta el momento (electrones, quarks y sus versiones pesadas de distintas generaciones) son partículas de Dirac y debido a su carga eléctrica y/o de color no pueden ser de otra manera. Sin embargo, en esta descripcción hemos dejado de lado a los neutrinos: el único fermión neutro cuya existencia conocemos. Puesto que el neutrino es neutro bajo el grupo gauge conservado, el grupo del electromagnetismo multiplicado por el grupo de color, surge una posibilidad adicional: que los neutrinos sean de tipo Majorana, es decir, que dicha partícula sea idéntica a su antipartícula. Aunque solamente pueden distinguirse estas posibilidades experimentalmente, la comunidad suele asumir que son de Majorana debido a su neutralidad.

Desde el punto de vista teórico, está claro que el Modelo Estándar debe ampliarse para incluir masas de neutrinos. La forma más sencilla de hacerlo es agregar el compañero diestro del campo de neutrinos. En ese caso, si no se amplía el inventario de simetría del Modelo Estándar, se viola el número leptónico en dos

unidades y los neutrinos resultan ser de tipo Majorana. Este es el llamado mecanismo del balancín ('seesaw') tipo I, que explica de manera elegante las masas de neutrinos y su pequeñez con respecto a las demás escalas de masas del Modelo Estándar. Se han desarrollado muchos mecanismos de masas de neutrinos diferentes, incluidas múltiples variantes del mecanismo del balancín (tipo II, tipo III, tipo lineal, tipo inverso...), así como modelos radiativos. Sin embargo, la comunidad generalmente asume que los neutrinos son campos de Majorana, y la opción de que sean de Dirac ha sido relativamente poco estudiada.

El objetivo de esta tesis será centrarse en la posibilidad, abierta experimentalmente, de que los neutrinos sean partículas de Dirac. En la tesis damos los requisitos generales de simetría necesarios para tener neutrinos de Dirac. En este sentido, es inmediato ver que el patrón de ruptura del número leptónico es un concepto central para determinar la naturaleza de los neutrinos. Sin embargo, es importante tener en cuenta que la naturaleza Dirac de los neutrinos se puede imponer con una simetría distinta del número leptónico, incluso utilizando grupos no abelianos. Esta simetría, sea una variante del número leptónico o alguna otra simetría a priori no relacionada, puede desempeñar al mismo tiempo diferentes roles: puede ser la simetría de Peccei-Quinn, estabilizar la materia oscura, explicar la jerarquía de masa de fermiones del Modelo Estándar, etc.

Después procedemos a revisar el zoológico de mecanismos del balancín ('seesaw') para neutrinos de Dirac. Análogamente a los mecanismos conocidos en la literatura para neutrinos de Majorana, estos son mecanismos elegantes que pueden explicar naturalmente la pequeñez de las masas de neutrinos. Algunos de ellos requieren de mediadores a escalas muy altas, lejos del alcance de los colisionares de partículas actuales, pero otros, como los mecanismos del balacín inversos o pueden tener mediadores de escala TeV y fenomenología observable.

Alternativamente, la pequeñez de las masas de neutrinos puede explicarse por su origen radiativo si se generan a través de bucles cuánticos. Es en este marco que la conexión de la simetría de número leptónico (o más precisamente, la combinación número bariónico menos numero leptónico) con la materia oscura se vuelve más clara: esta simetría puede al mismo tiempo estabilizar la materia oscura, que corre en el bucle que da masa a los neutrinos, protege la naturaleza Dirac de los mismos y explica la pequeñez de su masas al prohibir el término de masa a nivel de árbol, además de permitir ser una simetría local al estar libre de anomalías. Esta es la contribución más importante y elegante de la tesis.

Finalmente, para contrarrestar la noción generalizada en la comunidad de que los modelos de masas de neutrinos no tienen fenomenología testable en experimentos, mostramos dos modelos de contraejemplo. En uno de ellos utilizamos una simetría de sabor encima de un mecanismo de balancín de Dirac de tipo I. Mientras que el mecanismo del balancín de tipo I tiene mediadores de masas mucho más elevadas que la escaal electrodébil, y por tanto inaccesibles en experimentos de colisionadores, es importante hacer notar que esta característica es compartida por el mecanismo tipo I de Majorana. Sin embargo, al imponer simetrías de sabor extra obtenemos predicciones muy fuertes en este sector: una relacón entre las masas de los quarks y los leptones, una correlación muy fuerte entre los parámetros de mezcla de los neutrinos y una predicción para el ordenamiento de sus masas, que deberá ser ordenamiento normla, excluyendo el ordenamiento inverso. El segundo modelo de ejemplo es un mecanismo de balancín inverso. Puesto que en este tipo de modelos la escala de los mediadores puede reducirse hasta la escala electrodébil, es posible obtener predicciones testables en colisionadores. Además, al encontrarse la nueva física en una escala relativamente baja (orden TeV) habrá procesos visibles a bajas energías como por ejemplo procesos de violación del número leptónico de sabor. En esta sección describimos un mod-

elo de esta clase y estimamos las ratios de decaimiento de un muón en un electrón y un fotón, proceso que no debería darse en el modelo estándar pero que diversos experimentos están buscando.

2 Metodología

El estudio teórico en profundidad de la posibilidad de que los neutrinos sean partículas de Dirac requiere de un conocimiento adecuado de ciertas herramientas tanto teóricas como computacionales. En concreto, una lista incompleta de las herramientas de software necesarias son:

- Wolfram Mathematica para ayudar con manipulaciones analíticas de expresiones y ecuaciones matemáticas. Algunos de los paquetes que hemos utilizado están integrados en Mathematica, como Susyno, Sym2Int, Sarah, SPheno y MicrOMEGAS. Usaremos los dos primeros para facilitar las partes de simetría de construcción de modelos de nuestros proyectos y los tres últimos para calcular los observables de la física de partículas a partir de ellos: secciones transversales, masas físicas, relaciones entre parámetros, abundancia de reliquias de materia oscura, etc.

- Python, ya que algunos de los paquetes usan este lenguaje. En particular, CosmoTransitions se basa en Python y se utiliza para calcular información relacionada con las transiciones de fase de un modelo de física de partículas determinado. Esto es clave para determinar si un modelo contará con una transición de fase de primer orden, qué tan fuerte será y nos permitirá calcular si las ondas gravitacionales producidas serán observables o no en experimentos futuros.

- GAP es un programa muy poderoso para realizar cálculos de teoría de grupos. Esto es extremadamente útil en el análisis de simetría CP, así como en la construcción de modelos cuando la simetría es complicada.

Por otro lado, será necesario tener conocimientos avanzados en, entre otros,

- Mecánica cuántica. La mecánica cuántica es la teoría fundamental que describe las interacciones de partículas subatómicas a una escala microscópica cuando las energías involucradas son bajas.

- Relatividad especial. La relatividad especial es el marco teórico en el que se describen los fenómenos que implican velocidades altas, cercanas a la velocidad de la luz.

- Teoría cuántica de campos. La teoría cuántica de campos unifica la relatividad especial y la mecánica cuántica, es decir, que es el marco teórico adecuado para describir las interacciones entre partículas a altas energías.

- Teoría de grupos, que es la hierramienta matemática necesaria para predecir diversos fenómenos físicos gracias al teorema de Noether.

- Cosmología, que estudia la evolución del universo y tiene relación directa con la física de partículas, en particular con la física de neutrinos y la materia oscura, además de con transiciones de fase en sectores de Higgs extendidos.

Además, será importante consultar la bibliografía especializada en cada tema de interés.

3 Resultados y contribución personal

En esta sección resumiremos los resultados más importantes en cada uno de los artículos que forman parte de la tesis, además de la contribución personal

del estudiante en el desarrollo de los mismos, que ha sido sustancial en todos los casos. Por supuesto, todos los proyectos son colaborativos y los demás coautores también aportaron una parte importante en el desarrollo de las ideas subyacentes, los cálculos, la interpretación y la escritura del manuscrito. Aunque es difícil especificar que tareas concretas realizó cada persona (todos los coautores están involucrados en todos o casi todos los pasos), intentaremos resaltar los puntos donde la contribución del estudiante fue más destacada.

3.1 Dirac Neutrinos and Dark Matter Stability from Lepton Quarticity

En este artículo [1] encontramos un mecanismo elegante para obtener masas de neutrinos pequeñas con lo que se conoce como 'parámetros naturales', es decir, sin la necesidad de ajustar los parámetros del modelo de forma muy fina. Esto se puede conseguir de forma muy sencilla con un mecanismo al 'seesaw' tipo I, con la particularidad de que los neutrinos son de Dirac en vez de Majorana. Esto requiere la adición al modelo de una pareja de fermiones pesados, con una masa por encima de la escala electrodébil. La inversa de dicha masa genera la escala correcta de las masas de los neutrinos. A su vez, el modelo tiene algunas condiciones de simetría, como se argumentó a lo largo de la tesis. Por un lado, es necesario prohibir el operador de dimensión 4 $\bar{L}\phi^c\nu_R$, ya que de existir daría la contribución principal a las masas de los neutrinos y sería necesario tomar 'a mano' un acoplamiento pequeño para dicho operador (orden 10^{-12}). Esto lo conseguimos simplemente con una simetría Z_2, que a su vez está espontáneamente rota por el singlete escalar χ, que permite el operador de dimensión 5 $\bar{L}\phi^c\chi\nu_R$. Por otro lado, si queremos que los neutrinos sean de Dirac debe existir una simetría que proteja su naturaleza de Dirac prohibiendo los términos de Majorana como $\bar{\nu}_R^c\nu_R$. Esto lo conseguimos con un subgrupo de la simetría de número leptónico

Z_4. La belleza de dicha simetría es que automáticamente permite la existencia de un mecanismo para la estabilidad de la materia oscura.

La contribución personal del estudiante a este artículo fue hallar los grupos de simetría mínimos necesarios para conseguir el objetivo expuesto y las cargas de cada campo bajo dichos grupos de simetría. También realizó los cálculos necesarios en el sector escalar para encontrar los bosones físicos, sus masas y sus mezclas, así como el acoplamiento con la materia oscura via 'portal de Higgs' y los cálculos sencillos, además de contribuir a la redacción del manuscrito.

3.2 Generalized Bottom-Tau unification, neutrino oscillations and dark matter: predictions from a lepton quarticity flavour approach

En [2] tomamos una idea similar al anterior artículo, pero esta vez cambiando la simetría Z_2 por un grupo no abeliano: A_4. Al extender dicha simetría también para los quarks obtenemos una serie de predicciones muy interesantes, en particular una relación entre las masas de los leptones y los quarks, $\frac{m_b}{\sqrt{m_d m_s}} = \frac{m_\tau}{\sqrt{m_e m_\mu}}$, fuertes correlaciones entre algunos ángulos de mezcla (en concreto la mejor predicción se da entre θ_{23} y δ_{CP}) y, debido a la estructura de sabor, el ordenamiento inverso de las masas de neutrinos está excluido a más de 3σ, mientras que el ordenamiento normal sobrevive. Es, por tanto, un modelo de sabor altamente predictivo. Además este modelo conserva las propiedades del anterior, sobre el que se construye: los neutrinos son partículas de Dirac, la pequeñez de sus masas se explica de forma natural con un mecanismo 'seesaw' y existe una relación entre la naturaleza Dirac de los neutrinos y la estabilidad de la materia oscura.

La contribución personal del estudiante fue encontrar las cargas del grupo de simetría no abeliano A_4 que llevaran a las predicciones más fuertes. Además, el estudiante se dio cuenta, a partir de la bibliografía presente, que extendiendo

la simetría A_4 a los quarks podía obtenerse la relación de masas. El estudiante realizó todos los cálculos de teoría de grupos, obtuvo los parámetros de mezcla a partir de las cargas de simetría y realizó los gráficos correspondientes, además de contribuir a la redacción del manuscrito.

3.3 Seesaw roadmap to neutrino mass and dark matter

En [3] obtenemos todas las realizaciones a nivel árbol de los operadores de masas de neutrinos de dimesnión 5 tomando escalares que forman parte de distintos multipletes de $SU(2)_L$ hasta tripletes, siendo fácil su generalización para multipletes mayores. Empezamos por el bien conocido operador de Weinberg $LL\phi\phi$, que da lugar a los tres modelos de masas de neutrinos de Majorana más sencillos y populares: los tipos I, II y III del mecanismo 'seesaw'. Después procedemos a estudiar los diferentes operadors de masas de Dirac, y aunque hay muchos más, el caso más interesante es el del operador $\bar{L}\phi^c\chi\nu_R$, donde ϕ es el doblete de Higgs del modelo estándar y χ es un singlete. Este operador da lugar a los análogos de Dirac directos de los mecanismos 'seesaw' I, II y III. Además, es posible obtener variantes de dichos mecanismos simplemente modificando el multiplete de $SU(2)_L$ de los escalares involucrados en la generación de masas de neutrinos.

El estudiante desarrolló de forma sistemática todas las contracciones posibles de todos los operadores de dimensión 5 y realizó los diagramas respectivos. Como en todos los demás trabajos de investigación, colaboró con la discusión e interpretación de resultados y la escritura del manuscrito.

3.4   Seesaw Dirac neutrino mass through dimension-six operators

Construyendo sobre los resultados del artículo anterior, en [4] pasamos a los operadores de dimensión 6 para masas de neutrinos de Dirac a nivel árbol. El número de diagramas y operadores se multiplica al aumentar la dimensionalidad, por lo que el análisis es más largo y complejo. Algunos de los diagramas pueden indentificarse con casos de dimensión 5 en los que una de las patas externas obtiene un valor esperado en el vacío inducido por otros dos campos. Esta inducción es, de hecho, necesaria cuando una pata externa no es un singlete, ya que de lo contrario en el modelo existirá un bosón de goldstone sin masa que no forme parte mayoritaria de un singlete. Esto está prohibido por las observaciones, ya que al no ser un singlete acoplará con los bosones gauge y sería difícil 'esconder' su efecto. Además, algunos casos aparentemente podrían de forma inocente identificarse con una versión espontánea de los modelos de Dirac 'seesaw inverso' como ya existen para el caso Majorana. Sin embargo, evitamos una mención explícita a esto pues ese caso es más sutil y lo relegamos a un trabajo futuro, [6].

De nuevo, la aportación principal del estudiante fue obtener sistemáticamente todas las contracciones posibles de todos los operadores de dimensión 6 y realizar los diagramas respectivos, participar de la redacción del manuscrito, de la interpretación de los resultados y las discusiones.

3.5   Dark matter stability and Dirac neutrinos using only Standard Model symmetries

[5] es probablemente el artículo más completo de todos los presentados en esta tesis. En él se unifican y pulen todos los conceptos desarrollados en trabajos anteriores, para obtener un resultado extremadamente elegante y simple. Primero

demostramos que los criterios de simetría generales necesarios para obtener (a) neutrinos de Dirac, (b) masas naturalmente pequeñas (c) estabilidad de la materia oscura y (d) la materia oscura forma parte de la generación de masas de los neutrinos via 'loop', pueden obtenerse todas ellas con un solo grupo de simetría. Además, aunque no es estrictamente necesario para obtener un modelo consistente y fenomenológicamente correcto, la solución cumple (e) que esta simetría no tiene anomalías cuánticas, por lo que puede promocionar de simetría global a simetría local consiguiendo una fenomenología más rica y eliminar el bosón Goldstone de la teoría.

Todo ello se consigue simplemente con la simetría $U(1)_{B-L}$ en la que los neutrinos ν_R transforman con unas cargas exóticas $(-4, -4, 5)$, prohibiendo el término $\bar{L}\phi^c\nu_R$ en el lagrangiano. El singlete escalar χ rompe la simetría desde $U(1)$ hasta Z_6, que es la que estabiliza al candidato de materia oscura. Aunque el mecanismo es completamente general mostramos un ejemplo particularmente sencillo donde la masa de los neutrinos se obtiene a través de un mecanismo 'scotogénico' a nivel 1 'loop'.

La contribución del estudiante en este caso es más difícil de identificar puesto que es un artículo bastante más conceptual que se desarrolló a través de múltiples conversaciones y debates entre los coautores. El estudiante participó activamente de dichas discusiones aportando ideas y demostraciones, además de contribuir a la redacción del manuscrito.

3.6   The Inverse Seesaw Family: Dirac And Majorana

En [6] desarrollamos una definición general y rigurosa que permite diferenciar los modelos que podrían entrar en la categoría de 'seesaw' inverso y cuales no. Aunque este concepto ha sido utilizado en muchas ocasiones en la bibliografía, su significado general no había sido definido hasta ahora. Utilizamos dicha definición

para construir el clásico y bien conocido modelo de 'seesaw' inverso para el caso de Majorana y procedemos a generalizarlo en dos direcciones: por un lado construimos el 'seesaw' inverso doble, triple etc, donde la supresión de las masas de los neutrinos es mútiple, y por otro lado generalizamos los multipletes de $SU(2)_L$ que pueden utilizarse. Después realizamos el mismo ejercicio para el caso Dirac. Sorprendentemente, el modelo de 'seesaw' inverso más sencillo para neutrinos de Dirac es formalmente el mismo que el 'seesaw' tipo I: la diferencia estriba en el espacio de parámetros y, en concreto, en la escala de ruptura del número leptónico, ya que para una escala muy por encima de la electrodébil tendremos el seesaw de alta escala tipo I y para una escala de ruptura pequeña (orden 1 eV) tendremos un 'seesaw' inverso.

La gran ventaja de todos estos modelos es, como en el modelo 'seesaw' inverso clásico, que al reducir la escala de los mediadores de la masa de los neutrinos hasta O(TeV) se puede obtener una fenomenología más rica que en el caso de un 'seesaw' de escala alta. Además, en el caso de Dirac, puede sumarse a la fenomenología de numero leptónico local, puesto que la simetría no es anómala al contrario que para el caso Majorana. A costa de complicar el modelo un poco más pueden obtenerse más predicciones siguiendo las generalizaciones propuestas: multipletes de orden mayor implicarán mayor probabilidad de detección en experimentos de colisionador y al hacer los mecanismos 'doble', 'triple' etc se consigue una supresión aún mayor a la masa de los neutrinos, pudiendo de este modo reducir la escala de los mediadores.

La contribución del estudiante estuvo en la discusión de la primera parte del artículo, más conceptual, y en hallar las posibles generalizaciones del mecanismo para Dirac y Majorana, así como las fórmulas generales para la masa de los neutrinos y la redacción del texto. Además, el estudiante realizó cálculos

fenomenológicos para los modelos más simples aunque al final no se incluyeran en el manuscrito final.

4 Conclusiones

Los neutrinos son una puerta entreabierta hacia nueva física. Sabemos que son masivos, cuando el modelo estándar predice que no lo sean, pero a día de hoy no conocemos nada sobre el mecanismo que les otorga masa. Además, los neutrinos podrían dar la explicación a diversos problemas aparentemente no relacionados en otros sectores físicos. En la bibliografía hay miles de artículos con propuestas que podrían explicar la masa de estas partículas, asumiendo la mayoría de ellos que son partículas de Majorana, es decir, que son su propia antipartícula. En esta tesis partimos desde el punto de vista contrario, es decir, que sean partículas de Dirac, y exploramos las consecuencias de dicha suposición. Es claro que para tener neutrinos de Dirac es necesario extender el inventario de simetrías del modelo estándar. Esto puede verse como una oportunidad, ya que dicha simetría extra puede servir otros propósitos: por ejemplo, estabilizar un candidato de materia oscura. Estudiar el patrón de ruptura del número leptónico es por tanto clave para dilucidar esta materia. Así, la contribución más relevante de la tesis es la presentación de un modelo radiativo (donde la masa de los neutrinos sucede no a nivel árbol sino a nivel 'loop') donde los neutrinos son de Dirac y el patrón de ruptura de la simetría 'número bariónico menos número leptónico' consigue al mismo tiempo, sin necesidad de simetrías extra a) estabilidad de la materia oscura b) neutrinos de Dirac naturalmente ligeros c) inexistencia de simetrías accidentales adicionales y d) un inventario de materia nueva minimal.

Además de discutir los posibles patrones de ruptura de simetría, en esta tesis describimos sistemáticamente una clase de mecanismos elegantes de generación de masas de neutrinos de Dirac: los mecanismos del balancín (o seesaw), en analogía

a los ya existentes para Majorana. También discutimos en más detalle algunos modelos fenomenológicamente más interesantes en el sector de sabor.

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