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STRESS CONCENTRATION AT FILLETS, HOLES, AND KEYWAYS AS FOUND BY THE PLASTER-MODEL METHOD

BY

FRED B. SEELY

AND

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BULLETIN NO. 276

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I. Introduction

1. *The Plaster-Model Method.*—The mathematical method of determining stresses in structural and machine members due to external forces or loads may frequently be inadequate or impracticable, particularly when the member has abrupt changes in section that produce what are called stress concentrations or localized stresses. Further, even though a mathematical analysis may yield values of the stresses at any point in the member within the elastic action of the material, the significance of these stresses in determining the nearness to damage to the member, or the proportion of the utilizable strength of the member that is drawn on, under the conditions of loading, may not be revealed by such an analysis.

To supplement the mathematical method, or to obtain a solution (often approximate) for the stresses in a member when a mathematical analysis is not possible or feasible, various methods involving models have been used. One such model method is the brittle-material method, in which plaster-of-Paris or pottery plaster may be used; the method is then frequently called the plaster-model method. The brittle-material method may be explained briefly as follows: From a material that is brittle, and that has a nearly straight stress–strain diagram up to the ultimate strength of the material, a model is made of the member of irregular shape in which the stress is to be found; from the same material is also made a specimen of simple shape in which the stress due to a given load can be computed satisfactorily. The specimen of simple shape and the model of complex shape are then tested to destruction. The test of the simple shape gives approximately the ultimate tensile strength of the material, and the test of the model gives, with a fair degree of accuracy, the load which produced the same ultimate stress in the most highly stressed fiber of the model; from these values the relation between load and maximum tensile stress in the model of irregular shape is obtained.

For example, let it be assumed that the breaking load for a beam that is made of brittle material and that has an abrupt change of section is found to be one-half the breaking load for a similar beam without the abrupt change in section; the maximum stress produced in the beam that has the abrupt change in section, by any given load, is then twice that in the beam of constant cross-section when subjected to the same load, and this latter stress can be computed from the flexure formula.

The ideal material for use in the brittle-material method of determining stresses is one in which the compressive and shearing strengths are sufficiently high to cause the material to fail in tension when tested in tension or in torsion, and one in which the tensile stress-strain diagram is straight until rupture occurs. For such a material the proportional limit in tension is the ultimate strength, and the material would be expected to fail whenever the tensile stress at any point in a member exceeded the tensile proportional limit of the material; likewise the plane of rupture would be expected to be perpendicular to the direction of the maximum tensile stress. Although such an ideal material is not available, "pottery plaster" meets the requirements fairly well, and it was used in the tests herein reported.

With certain limitations, to be discussed later, the same ratio is assumed to hold between load and maximum stress in any member made of elastic non-brittle (ductile) material (such as low carbon steel, etc.) as is found for a similarly-loaded brittle-material member, provided that the stress does not exceed the proportional limit of the ductile material.

2. *Purpose of Investigation.*—The plaster-model method has been shown (7)* to be fairly satisfactory for determining the maximum stress in a member where the stress is not highly localized or concentrated. Some work (13, 6) has also been done in applying the method to the determination of stress concentration at fillets and holes in beams. The object of the tests herein recorded was to obtain further information on the reliability of the method for determining highly localized stresses. For this purpose tests were made to determine the effects of fillets, holes, and keyways in cylindrical members subjected to bending alone and also to torsion alone.

3. *Acknowledgment.*—A portion of the tests herein reported was made by MR. H. R. LISSNER as a graduate student at the University of Illinois in satisfying the requirement for thesis work for the degree of Master of Science in Theoretical and Applied Mechanics. Another portion of the tests was made by MR. R. W. SEELY as a senior student in Mechanical Engineering.

*The numbers in parentheses refer to the list of references in the bibliography at the end of the bulletin.

The investigation herein reported was made as part of the work of the Engineering Experiment Station of which Dean M. L. Enger is the director, and of the Department of Theoretical and Applied Mechanics of which Prof. F. B. Seely is the head.

II. Material and Specimens

4. *General Properties of Material.*—The material used for the plaster models was a high grade pottery plaster. This plaster is a commercial product, and has nearly the same chemical composition as has plaster of Paris, but is somewhat slower in setting, thereby giving ample time for working and placing.

As in many other cements, the strength and quality of pottery plaster may be greatly affected by the method of handling. Within limits, the density and strength increase as the amount of mixing water decreases, and the time of setting decreases materially as the amount of mixing water decreases. The tensile and compressive moduli of elasticity and the modulus of rupture in bending of specimens made in the proportions of 70 lb. water to 100 lb. plaster, as used in this investigation, were approximately 1 000 000 and 600 lb. per sq. in., respectively. The time of blending, that is, the time elapsing between putting the plaster in the water and the beginning of the stirring operation, is also an important factor affecting the strength and quality of the specimen or casting. The strength increases with the blending time up to fourteen minutes when 70 per cent of water is used for mixing, and begins to decrease rapidly as the blending time exceeds sixteen minutes.

It is desirable that the stirring be continuous and of such a character that air bubbles will be carried up from the bottom and liberated without entrained air being carried into the mixture. Violent stirring is neither necessary nor desirable.

The strength of the plaster increases rapidly with the length of time of curing in moist air up to two days; after that length of time in the moist room further increase in strength is scarcely perceptible.

Set plaster is very easily "burned" (dehydrated) in drying, especially after it has become rather dry. The maximum drying temperature cannot be above 120 deg. F. without danger of burning; burned plaster is chalky and very weak. Repeated wetting of the plaster after it has set, especially after once being dried, causes the strength to decrease markedly.

5. *Method of Mixing, Pouring, and Curing.*—The mixture used was in the proportion of 70 lb. of water to 100 lb. of plaster. After

the two ingredients were weighed, the plaster was poured carefully into the water and the mixture was allowed to stand for 10 minutes before being disturbed. After this 10-minute period, stirring was begun by using a small propeller on one end of a steel rod, the other end being driven by an electric drill motor. The propeller blades were bent so that the material was moved in an upward direction to help liberate air bubbles.

The stirring was continued steadily for 5 minutes, and immediately afterwards the mixture was poured into the molds. The molds were cylindrical in shape and consisted of two parts that were clamped together, the clamp being released after the material had set in the mold. As the plaster set there was a slight expansion which caused all parts of the mold to fill nicely. After about 15 minutes the material began to give off heat and felt quite warm to the hand. At this time the casting was sweating and a film of water was formed between the casting and the oiled surface of the mold. When in this condition the molds were most easily removed. After the specimens were taken from the molds they were marked and placed in the moist room for curing.

All specimens were left in the moist room at least two days. They were then brought out and dried for several weeks in air at about 70 deg. F. (a minimum of three weeks was used, although more rapid drying is possible by use of a controlled oven); this procedure produced hard specimens with a distinctly metallic ring when struck, and with stress–strain diagrams that were straight lines all the way to rupture. If the specimens are not thoroughly dried they do not have a straight stress–strain diagram, and they exhibit appreciable flow under a steady load.

6. *Form and Dimensions of Specimens.*—The cylindrical molds had an inside diameter of 4 in. and a length of 16 to 18 in. All the specimens were turned in a lathe to give the cylindrical shapes and dimensions shown in Fig. 1. The holes in the specimens were drilled in a drill press, and the keyways were formed in a milling machine. Views of some of the specimens are shown in Figs. 9, 11, 12, and 13.

The specimens with fillets were easily machined to a smooth surface with forming tools made from medium carbon steel, which can be readily shaped accurately by grinding and filing to the desired form. The specimens were most readily machined before they were thoroughly dried. The speed of cutting may vary considerably, but the depth of cut cannot be large without danger of breaking the specimen.

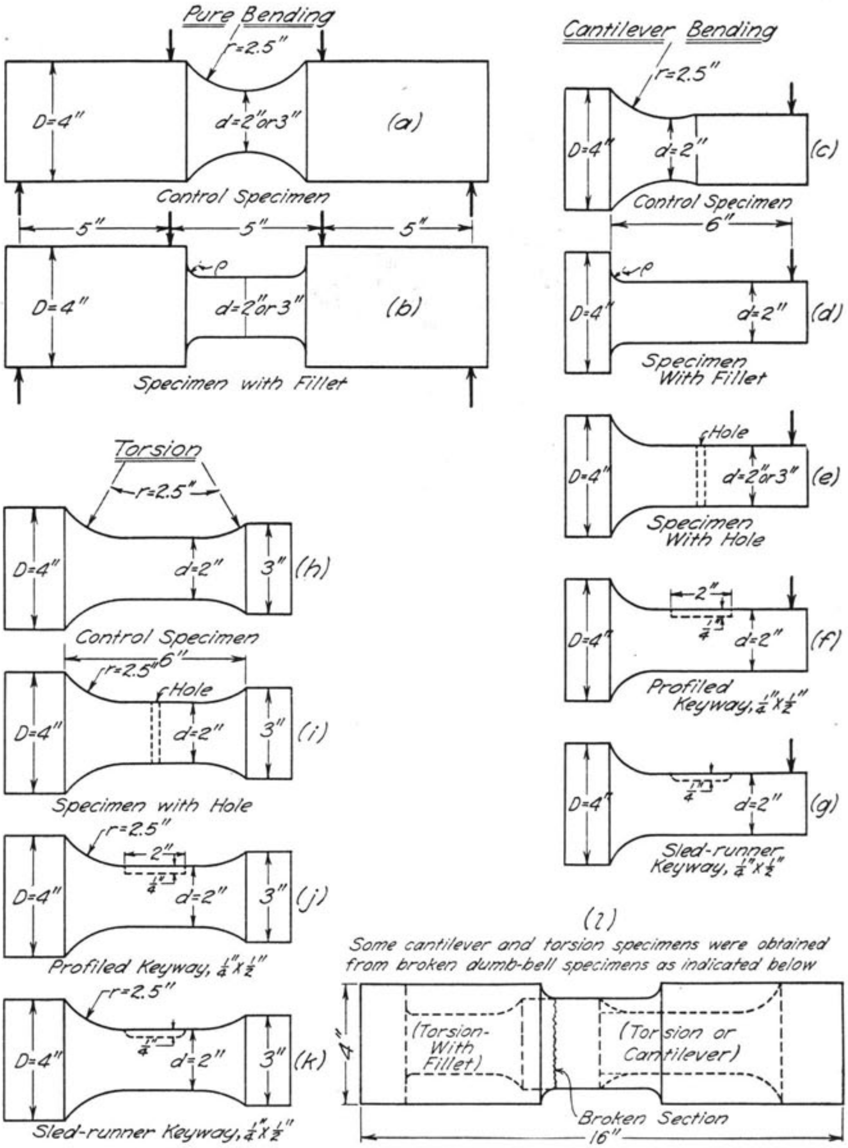


FIG. 1. FORM AND DIMENSIONS OF CYLINDRICAL SPECIMENS USED

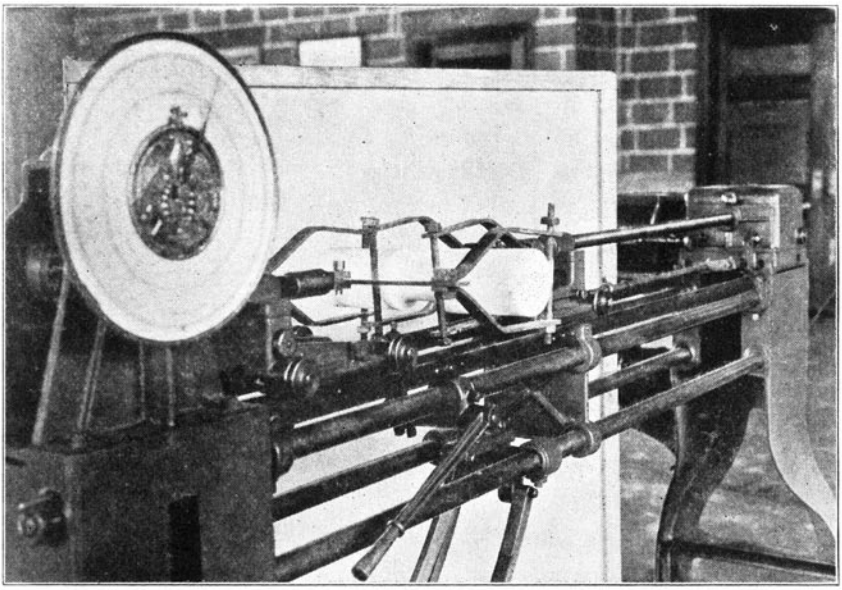


Fig. 2. Apparatus for Testing Dumb-bell Shaped Specimens in Bending

III. Method of Testing

7. Testing Machines Used.—

Bending Tests

The dumb-bell shaped specimens, Fig. 1(a) and (b), were tested in a Scott horizontal testing machine of 1000 lb. capacity (see Fig. 2). The larger portion of the broken specimen was then turned down to form a cantilever beam specimen (Fig. 1) and tested with dead weights as shown in Fig. 3. Some of the specimens had fillets, others holes, and still others keyways. More than one specimen was obtained from the broken 3-in. specimens as is indicated in Fig. 1(l). The control specimens had a change of section (see Fig. 1(a) and (c)) which was considered to be sufficiently gradual to make the stress concentration negligible.

Torsion Tests

The specimens were tested in torsion in the apparatus shown in Fig. 4. By means of this apparatus a pure twisting moment was applied to the specimen which could be easily measured from the weight applied and the dimensions of the apparatus.

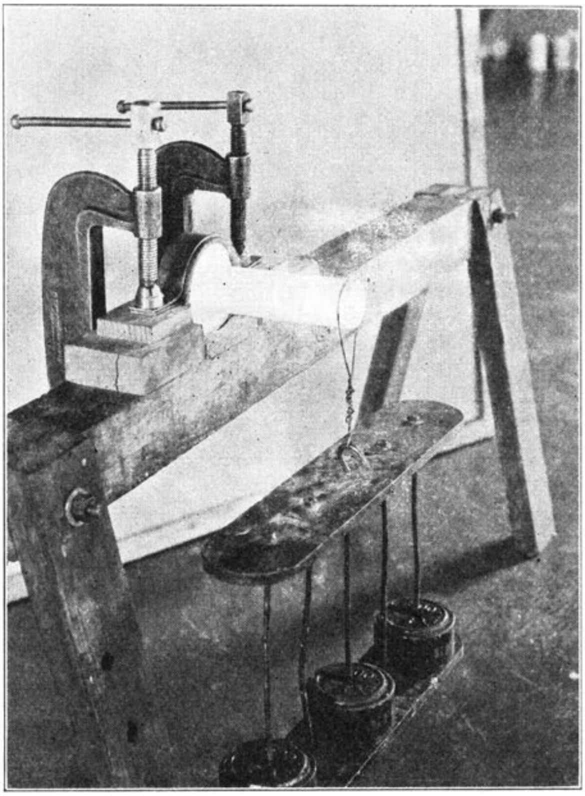


FIG. 3. APPARATUS FOR TESTING CANTILEVER SPECIMENS IN BENDING

IV. RESULTS AND DISCUSSION

8. *Meaning of Stress-Concentration Factor.*—As found by the plaster-model method, the stress-concentration factor for a fillet, hole, keyway, or similar abrupt change in section in a member subjected to bending (or torsion) is the ratio of the bending (or torsional) moment required to break a similar member or specimen having no abrupt change in section to the moment required to break the specimen having the abrupt change in section. For example, in Fig. 5 (a) the maximum or breaking stress is given satisfactorily by the flexure formula $s = \frac{M_1 c}{I}$ where M_1 is the moment that causes the specimen to break, but the specimen in Fig. 5 (b) breaks at the same stress as caused the specimen in Fig. 5 (a) to break, since the material has

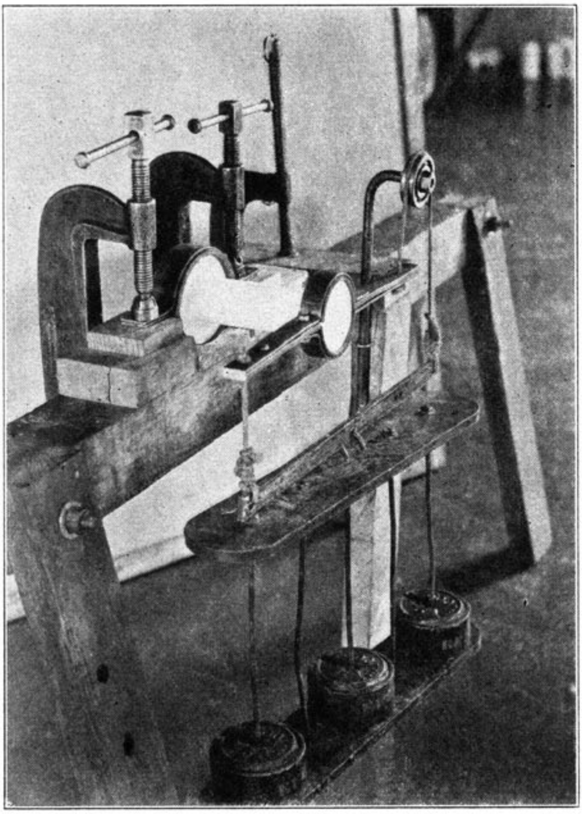


FIG. 4. APPARATUS FOR TESTING SPECIMENS IN TORSION

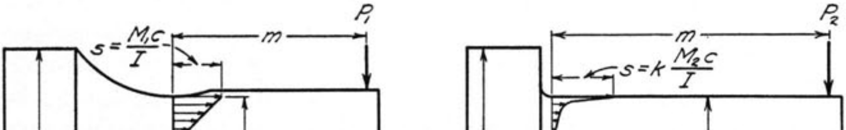


FIG. 5. MEANING OF STRESS-CONCENTRATION FACTOR

a straight stress-strain diagram until the material ruptures, and the beam fails when the most highly stressed fiber reaches the ultimate strength. Hence

$$s = \frac{M_1 c}{I_1} = k\,\frac{M_2 c}{I_2} \quad \text{or} \quad k = \frac{M_1}{M_2}\,\frac{I_2}{I_1} = \frac{M_1}{M_2}$$

in which, as noted above, M_1 is the moment causing the control specimen to break, and M_2 is the moment causing the specimen with the abrupt change in section to break, the moments of inertia I_1 and I_2 for the two specimens being assumed to be equal. For some forms with abrupt changes in section the value of I_2 is sometimes corrected for the area removed in making the abrupt change in section (this has been done for holes in the bending specimens), whereas in other forms such a correction is impossible or undesirable.

For a few of the tests only two specimens were made from a given mix, one specimen containing the abrupt change in section (fillet, hole or keyway) and the other (the control specimen) being free from stress concentration. However, most of the specimens were made from a mix composed of 100 lb. of plaster and 70 lb. of water. From this mix a large number of specimens could be made, several specimens being used for the control specimens, and several for each of the different desired abrupt changes in section. The results of the tests of the specimens free from abrupt changes in section are given in Table 1.

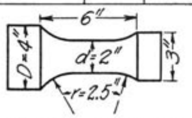
9. *Ideal and Effective Stress Concentration.*—A distinction should be made between the stress concentration caused by a change in form in an ideal material as determined by the mathematical theory of elasticity, and the stress concentration that is effective or significant in producing damage to the actual material in the member as used in service.

The ideal mathematical stress concentration is due only to change in form of the member, whereas the stress concentration determined by the plaster-model method, and also by the repeated-stress (fatigue) method, is influenced not only by the abrupt change in section, but to some (unknown) extent by inherent defects in the material, by slight readjustment of the material due to yielding or flow, and by the orientation of the structural units (crystalline grains in the case of metals) composing the material. In addition, the effective stress concentration may be influenced by so-called skin effect, or surface tension effect, at points of high stress concentration, although little is

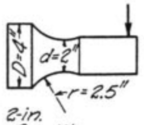
TABLE 1

MOMENTS CAUSING FRACTURE AND MODULI OF RUPTURE OF THE PLASTER CONTROL SPECIMENS

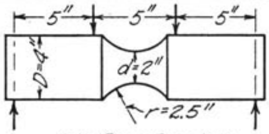
Dia. of Specimen, d (in.)	Ratio D/d	Bending Tests: Average Value Bending Moment M, (lb-in.)	Bending Tests: Average Value Mod. of Rupture lb. per in.²	Bending Tests: Max. Value of M	Bending Tests: Min. Value of M	Bending Tests: No. of Tests Averaged	Torsion Tests: Average Value Twisting Moment T, (lb-in.)	Torsion Tests: Average Value Mod. of Rupture lb. per in.²	Torsion Tests: Max. Value of T	Torsion Tests: Min. Value of T	Torsion Tests: No. of Tests Averaged
2 Cantilever Specimens	2	505	641	544	465	5	743	472	809	672	6
2 Pure Bending Specimens	2	577	735	680	463	9					
3 Pure Bending Specimens	1.33	1630	612	1970	1330	5					



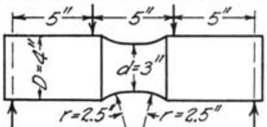
Torsion Specimen



2-in. Cantilever Specimen



2-in. Pure Bending Specimen



3-in. Pure Bending Specimen

definitely known about this effect. The photo-elastic model method gives values of stress concentration which are affected much less by internal defects, orientation of structural units, etc., than are the results of the plaster-model and fatigue-model methods, and hence, if geometric similitude is maintained, the results of the photo-elastic model method approach more closely to the ideal values given by mathematical analysis. There is, however, always some doubt as to the values of the stresses at the sharply curved edge or boundary of the discontinuity because of the technique used in the photo-elastic method, and also a question as to the correctness of applying the results of a two-dimensional stress situation (as in the photo-elastic method) to a three-dimensional state of stress in a material with widely different values of modulus of elasticity and Poisson's ratio.

10. *Values of Stress-Concentration Factors.*—

Fillets in Bending

The values of the stress-concentration factors for fillets obtained from bending tests of plaster models are shown in Fig. 6 and in Table 2. These values are averages of the results on cantilever and pure bending (dumb-bell) specimens. The values of the ratio of the

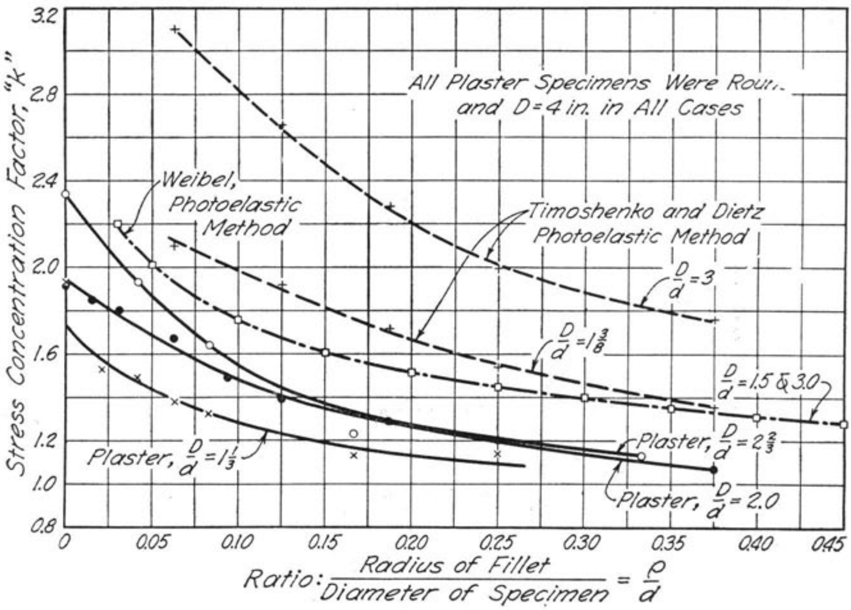


FIG. 6. VALUES OF STRESS CONCENTRATION AT FILLETS IN BENDING

radius, ρ, of the fillet to the diameter, d, of the smaller part of the specimen varied from 0 (a square corner) to 0.375, and three values of the ratio of the larger diameter D to the smaller diameter d were used, namely, $1\frac{1}{3}$, 2, and $2\frac{2}{3}$, the value of D in all cases being 4 in. If the ratios $\frac{\rho}{d}$ and $\frac{D}{d}$ for two specimens are maintained constant, the specimens are geometrically similar, at least for the same type of loading or straining action.

The results of photo-elastic experiments on flat bending specimens having fillets (two dimensional state of stress) obtained by Timoshenko and Dietz (5) and also by Weibel (17) for approximately the same values of $\frac{D}{d}$ as those for the plaster models are also shown in Fig. 6 for comparison. It will be observed that the values of the stress-concentration factors found by Timoshenko and Dietz are considerably higher than those found by Weibel, and that their results are different for different values of $\frac{D}{d}$, whereas Weibel's results are the same for both values of $\frac{D}{d}$. Each of the plotted values

TABLE 2

STRESS-CONCENTRATION FACTOR FOR FILLETS IN BENDING AND TORSION SPECIMENS

Radius of Fillet ρ (in.)	Ratio $\frac{\rho}{d}$	Stress Concentration Factor k			Number of Values Averaged	Radius of Fillet ρ (in.)	Ratio $\frac{\rho}{d}$	Stress Concentration Factor k			Number of Values Averaged	Radius of Fillet ρ (in.)	Ratio $\frac{\rho}{d}$	Stress Concentration Factor k			Number of Values Averaged
		Av. Value	Max. Value	Min. Value				Av. Value	Max. Value	Min. Value				Av. Value	Max. Value	Min. Value	
Bending Tests of Cylindrical Specimens																	
Diameter of Specimen = d = 3″ $\frac{D}{d}$ = 1.33						Diameter of Specimen = d = 2″ $\frac{D}{d}$ = 2.0						Diameter of Specimen = d = $1\frac{1}{2}$″ $\frac{D}{d}$ = 2.67					
0	0	1.93	2.19	1.64	9	0	0	1.92	2.40	1.40	13	0	0	2.34	2.61	1.93	5
$\frac{1}{32}$						$\frac{1}{32}$	0.015	1.85	2.00	1.60	8	$\frac{1}{32}$					
$\frac{1}{16}$	0.021	1.53	1.76	1.34	10	$\frac{1}{16}$	0.031	1.80	1.96	1.63	9	$\frac{1}{16}$	0.042	1.93	2.00	1.83	5
$\frac{1}{8}$	0.042	1.49	1.80	1.27	10	$\frac{1}{8}$	0.062	1.67	2.46	1.38	12	$\frac{1}{8}$	0.083	1.64	1.69	1.55	4
$\frac{3}{16}$	0.062	1.38	1.44	1.23	6	$\frac{3}{16}$	0.094	1.49	1.61	1.31	8	$\frac{3}{16}$	0.125				
$\frac{1}{4}$	0.083	1.32	1.60	1.17	9	$\frac{1}{4}$	0.125	1.39	1.59	1.17	14	$\frac{1}{4}$	0.167	1.23	1.38	1.12	5
$\frac{3}{8}$						$\frac{3}{8}$	0.187	1.29	1.54	1.00	12	$\frac{3}{8}$					
$\frac{1}{2}$	0.167	1.13	1.40	1.00	9	$\frac{1}{2}$	0.250	1.20	1.47	1.02	9	$\frac{1}{2}$	0.333	1.13	1.32	1.00	4
$\frac{3}{4}$	0.250	1.14	1.29	1.00	4	$\frac{3}{4}$	0.375	1.07	1.41	0.95	6	$\frac{3}{4}$	0.500				
Torsion Tests of Cylindrical Specimens, d = 2″, $\frac{D}{d}$ = 2.0																	
						0	0	1.30	1.40	1.25	4						
						$\frac{1}{8}$	0.062	1.13	1.94	1.01	6						

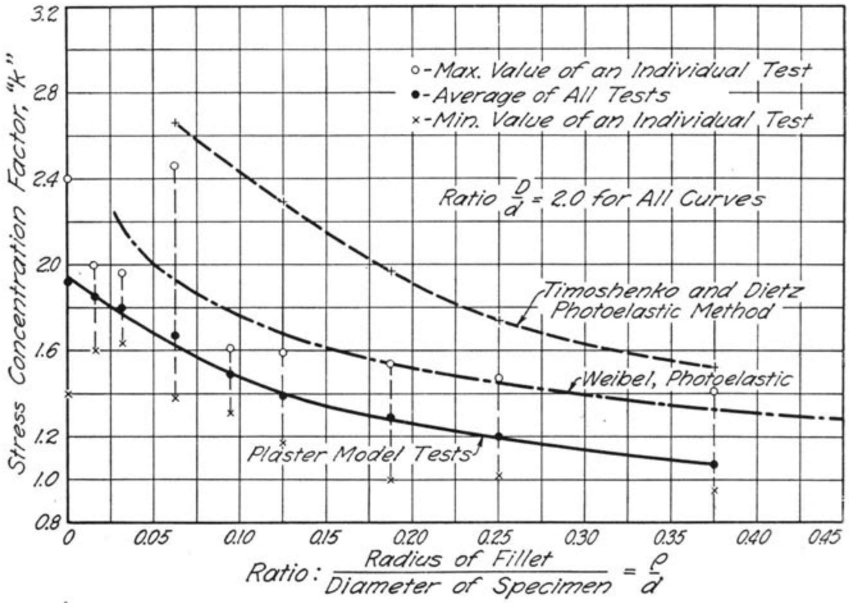


FIG. 7. RANGE OF VALUES OF STRESS-CONCENTRATION FACTORS FOR FILLETS IN BENDING

in Fig. 6 for the results of the plaster-model tests are the average of several tests, as indicated in Table 2. It will also be observed that the curves drawn through these plotted points are approximately parallel to, but considerably below, those obtained by the photo-elastic method. The question as to whether the flat specimens used in the photo-elastic experiments give results applicable to cylindrical specimens (involving a three-dimensional state of stress) has frequently been raised. This question is discussed briefly under Section 11. It should be noted that the specimens listed as having a square corner ($\rho = 0$) actually had a fillet with a radius slightly less than 0.005 in. as measured on a longitudinal section through the specimen by means of a metallurgical microscope at a magnification of 200x. Figure 9 shows bending specimens with typical fractures at the fillets.

The need for using the average of a number of tests is emphasized in Fig. 7, where the range of values of the stress-concentration factors for each value of $\frac{\rho}{d}$ as well as the average value* from the tests are plotted for one value of $\frac{D}{d}$, namely, $\frac{D}{d} = 2$. The results found from

*The standard deviations of the stress-concentration factors for the various values of ρ/d ranged from 0.025 corresponding to $\rho/d = 0.0313$ to 0.282 for $\rho/d = 0.0625$.

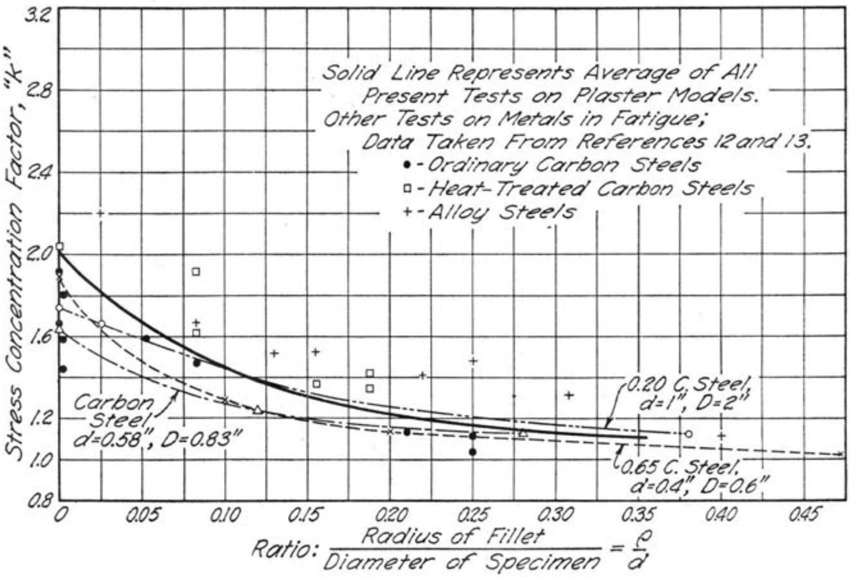


FIG. 8. COMPARISON OF STRESS-CONCENTRATION FACTORS FOUND FROM PLASTER MODELS WITH THOSE FOUND FROM FATIGUE SPECIMENS

the photo-elastic method by Timoshenko and Dietz, and by Weibel, are also shown in Fig. 7 for comparison. The reasons for the variation in the plaster-model results are not entirely clear, but some of the factors possibly involved in the action are discussed in Section 11.

Note should be made of the fact that the average values of the stress-concentration factors for fillets as found by the plaster-model method are in fair agreement with those found by the repeated stress (fatigue) method for similar fillets in carbon steel specimens of small size, as indicated in Fig. 8. For alloy-steel and heat-treated specimens, however, the agreement is not so close, and there is some evidence (11) indicating that the agreement is also not so close for carbon steel specimens of relatively large size. These observations seem to emphasize the fact, previously noted, that the effective or significant stress concentration for any type of external discontinuity of form of a metal member depends not only on the form of the discontinuity, but also on the internal condition, constitution and structure of the metal, etc. Hence the results of model tests (of all types) must be correlated with the results of the tests of the actual metal members under conditions simulating as closely as possible those met in service, before the full significance of model tests can be determined.

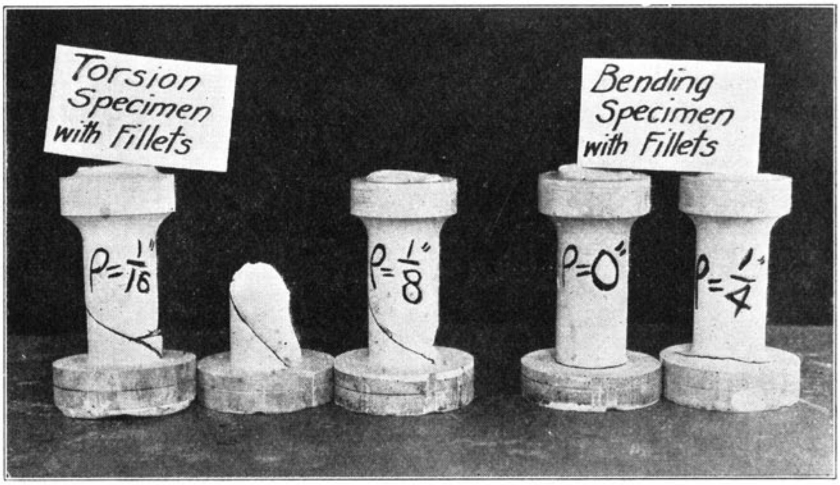


FIG. 9. VIEW OF SOME OF THE BROKEN SPECIMENS HAVING FILLETS

When a good correlation is obtained between plaster-model results and the results of tests of actual members or of tests of other types of models, it is not known to what extent the correlation is due to geometric similarity and to what extent to similarity in internal effects (orientation of crystalline structure, inherent defects, etc.). This statement also applies to the results of other model tests, and therefore a broad interpretation of the significance of the results of model tests is difficult to justify, although model tests frequently may yield very useful results.

Fillets in Torsion

The results of torsion tests of plaster models having fillets indicated very small stress concentration at the fillets. In other words, the specimens with fillets resisted nearly as large twisting moments as did the specimens that had a very gradual change in section (see Fig. 1). It is not quite clear why there appears to be so little stress concentration in these specimens, for, according to the results of Jacobsen's electric analogy method (4), rather large shearing stress-concentration factors are found for fillets in torsion. And, since at any point in a bar under torsion the tensile (45-degree diagonal) stress is equal to the shearing stress, the tensile stress at the fillet might also be expected to show considerable stress concentration. Apparently the strengthening effect of the enlarged section on one side of the fillet (the diagonal area on which the maximum tensile stress causes rupture is increased) offset the effect of the high stress at the fillet, and restrained the material from breaking at the point of high stress. Figure 9 shows the typical failure of the torsion specimens with fillets; the specimen shattered somewhat, and did not give a clean-cut fracture on a 45-degree plane like the fracture of the torsion control specimens, and those with holes.

The values of stress-concentration factors, k, found by Armbruster (9) by the repeated-stress (fatigue) method for fillets in two different grades of steel subjected to completely reversed torsional stresses are as follows:

Designation of Steel	$\frac{D}{d}$	$\frac{\rho}{d}$	k
N	1.4	0.17	0.96
N	1.4	0.03	0.99
E	1.4	0.17	1.11
E	1.4	0.03	1.66

(The diameter d in each case was 10 mm.)

The steel designated as "N" had an endurance limit of $16.0 \frac{\text{kg}}{\text{mm.}^2}$ (22,750 lb. per sq. in.) and that designated by "E" had an endurance limit of $29.5 \frac{\text{kg}}{\text{mm.}^2}$ (41,950 lb. per sq. in.). Apparently, in the steel designated as N, the fillet had no effect in causing stress concentration, which is consistent with the results of the plaster-model tests as already noted.

Holes in Bending Specimens

The stress-concentration factor for holes in cylindrical specimens tested in bending (see Fig. 1e) are shown in Fig. 10 and Table 3. Three values of the ratio of the diameter, w, of the hole to the diameter, d, of the specimen were used, namely, $\frac{1}{16}$, $\frac{1}{8}$, and $\frac{1}{4}$, and two sizes for the diameter of the specimen, namely, 2 in. and 3 in. Some of the broken specimens are shown in Fig. 11. In determining the values of the stress-concentration factors for holes in the specimens subjected to bending, allowance was made for the decreased strength caused by the reduction in the moment of inertia due to the area of cross-section removed by the holes. The stress-concentration factors in Fig. 10, therefore, are based on the net area of the section through the hole.

In Fig. 10 are also shown the stress-concentration factors obtained by Wahl and Beeuwkes (16) from photo-elastic experiments on flat specimens with holes when the specimens were subjected to an axial tensile load (not bending). The results found by mathematical analysis by Howland (8) are in close agreement with the results obtained

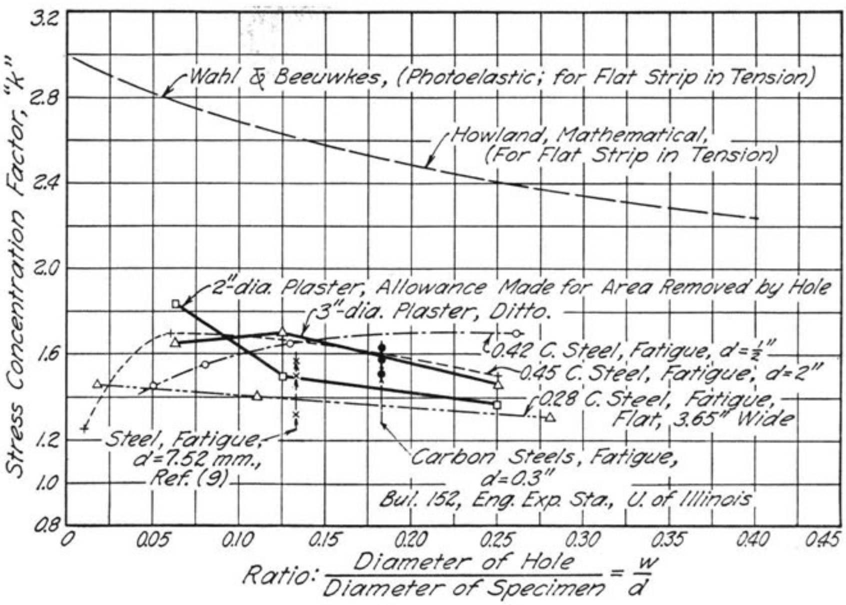
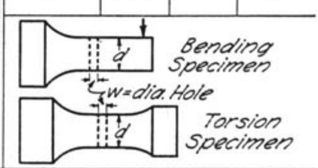


FIG. 10. VALUES OF STRESS-CONCENTRATION FACTORS FOR HOLES IN CYLINDRICAL BENDING SPECIMENS

TABLE 3

STRESS-CONCENTRATION FACTOR FOR HOLES IN BENDING AND TORSION SPECIMENS

Dimensions			Bending Tests				Torsion Tests			
Dia. of Specimen, d (in.)	Dia. of Hole, w (in.)	Ratio w/d	Av. Value of Stress Concentration Factor k*	Max. Value of k*	Min. Value of k*	Number of Values Averaged	Av. Value of Stress Concentration Factor k	Max. Value of k	Min. Value of k	Number of Values Averaged
2	1/8	1/16	2.06 1.84	2.21 1.98	1.88 1.68	4	1.86	1.98	1.75	5
2	1/4	1/8	1.89 1.50	2.45 1.95	1.60 1.27	7	1.89	2.05	1.68	5
2	1/2	1/4	2.36 1.37	2.55 1.47	2.16 1.25	4	2.12	2.50	1.84	8
3	3/16	1/16	1.85 1.65	2.21 1.97	1.56 1.39	5				
3	3/8	1/8	2.16 1.70	3.04 2.39	1.86 1.46	4				
3	3/4	1/4	2.54 1.46	2.99 1.72	2.09 1.20	5				

* Upper values take no account of reduced moment of inertia of section due to hole; lower values make allowance for the area removed. In torsion no allowance is made for area of hole removed.

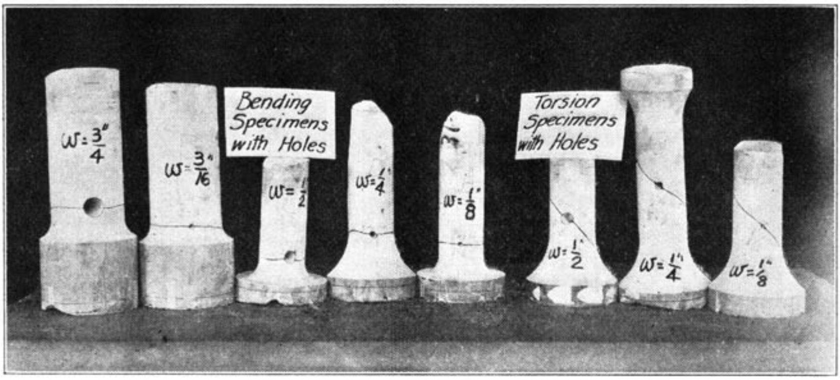


FIG. 11. VIEW OF SOME OF THE BROKEN SPECIMENS WITH HOLES

by Wahl and Beeuwkes for the same type of specimens and loading, and these values are considerably larger than those found from the plaster model tests. But judging from Weibel's (17) results on fillets (where he found smaller stress-concentration factors in bending than in tension on the same specimens) the stress-concentration factors for holes in flat specimens subjected to bending probably should not be expected to check the values of the factors obtained with the same specimens subjected to direct tensile stress, and cylindrical specimens probably introduce additional modifying influences.

In Fig. 10 are also shown the values of stress-concentration factors obtained from fatigue tests (9, 12) of small specimens of carbon steel. As with fillets, the correlation between the results of the fatigue tests and those of the plaster-model tests is reasonably good.

Holes in Torsion Specimens

The specimens subjected to torsion failed in tension as shown in Fig. 11. The ideal or mathematical value (3) of the maximum tensile stress at the edge of a small hole in a torsion specimen is four times the value that would exist at the point if no hole were present. That is, the maximum ideal stress-concentration factor is nearly four, dependent somewhat on the size of the hole in relation to the diameter of the specimen. The values found by the plaster-model method are given in Table 3. These values are the ratios of the twisting moments required to break the specimens without holes to those required to break the specimens with holes. No allowance is made for the material removed by the hole, because no mathematical analysis indicating how the shearing stress in such a section varies with the dimensions of the area is available. The results in Table 3 indicate that the stress at the hole is only about one-half of the theoretical value; in other words, the effective stress-concentration factor is about 2 instead of about 4.

It is not clear to what extent this difference is due to the support given to the material at the edge of the hole by the under-stressed material beneath the point of maximum stress, and to what extent slight yield or flow of the material reduces the concentration of stress.

In fatigue tests at the University of Illinois the value of 1.30 was found for the stress-concentration factor for a hole in steel cylindrical specimens subjected to complete reversals of torsional stress. The specimens were made from hot-rolled low carbon (S.A.E. 1020) steel, and the ratio of the diameter of the hole to the diameter of the specimen was $\frac{w}{d} = \frac{1}{10}$, the diameter, d, of the specimen being 0.40 in. Evidently this steel is not very sensitive in repeated stress to the stress increase caused by a hole. The results of the plaster model tests would be decidedly on the safe side as applied to holes in this grade of steel for resisting fatigue, at least for small specimens.

Armbruster (9) found fatigue stress-concentration factors of 1.31 to 1.82 for specimens with transverse holes, and made of three grades of steel; the specimens were subjected to complete reversals of torsional stress. The diameter of the hole was 1½ mm. and the diameter of the specimen was 10 mm. $\left(\frac{w}{d} = \frac{1.5}{10} = 0.15\right)$. The larger value of stress concentration applies to the high strength steel (steel designated as E in the previous table) and the lower value to the lower strength steel (designated as N in the previous table). The results of the plaster model tests in torsion as indicated in Table 3 are in agreement with the result found by Armbruster for the higher strength steel.

Keyways in Bending Specimens

Two types of keyways were used, namely, the sled-runner and the profiled type. The form and dimensions of the specimens containing keyways are shown in Fig. 1. Some of the broken specimens are shown in Fig. 12, and it will be observed that the fracture occurred, as would be expected, near the end of the keyway where the abrupt change of section occurs.

As shown in Table 4, the stress-concentration factor found for the profiled type of keyway is somewhat greater than for the sled-runner type, the average values being 1.46 and 1.23, respectively, if no allowance is made for the area of keyway removed. It will be observed

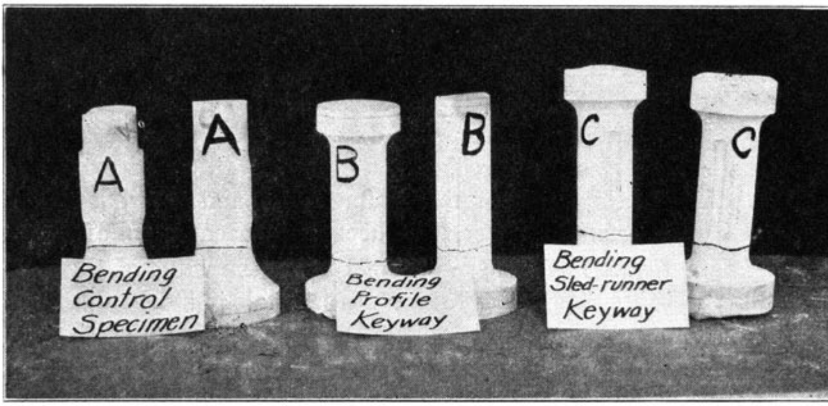


FIG. 12. VIEW OF SOME OF THE BROKEN BENDING SPECIMENS HAVING KEYWAYS

TABLE 4

STRESS-CONCENTRATION FACTOR FOR KEYWAYS IN BENDING AND TORSION SPECIMENS

Plaster Model Method					Repeated Stress Method for Steel		Static Tests of Steel	
Type of Keyway	Stress Concentration Factor, k			Number Values Averaged	Stress Concentration Factor k	Kind of Steel	Stress Concentration Factor k	Kind of Steel
	Average Value	Max. Value	Min. Value					
Bending Tests								
					From R.E. Peterson:			
Sled-Runner, Fig. 1, (k)	*1.08 1.23	1.44	1.14	3	*1.35 1.61	Chromium-Nickel (Heat Treated)		
					*1.11 1.32	Medium-Carbon (Normalized)		
Profiled, Fig. 1, (j)	*1.28 1.46	1.50	1.44	4	*1.74 2.07	Chromium-Nickel (Heat Treated)		
					*1.35 1.61	Medium-Carbon (Normalized)		
Torsion Tests								
					From H. J. Gough:		From H. F. Moore:	
Sled-Runner, Fig. 1, (k)	1.44	1.64	1.28	6			1.30	Cold-Rolled Shafting
							From H. J. Gough:	
Profiled, Fig. 1, (j)	1.68	2.09	1.39	7	1.27	0.65% Carbon	1.32	0.65% Carbon
					1.14	Armco Iron	1.28	Armco Iron

* In upper values allowance has been made for removal of the area due to the keyway, while no such allowance has been made in the lower values.

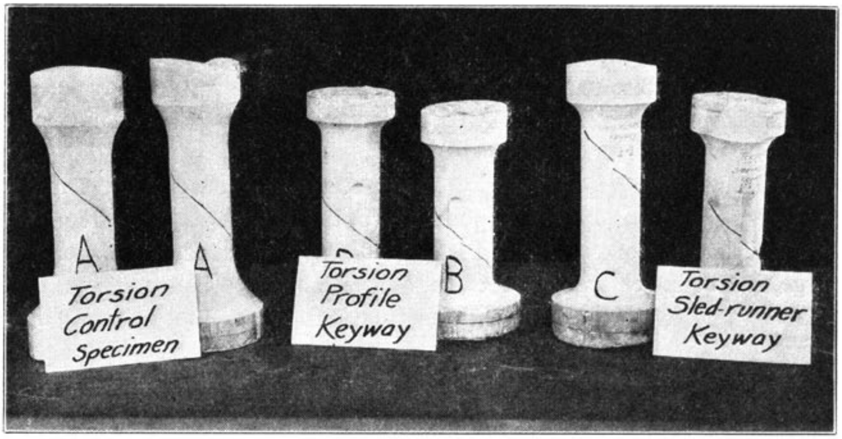


FIG. 13. VIEW OF SOME OF THE BROKEN TORSION SPECIMENS HAVING KEYWAYS

that these are in good agreement with fatigue stress-concentration factors for similar keyways in medium carbon steel specimens of small size, but are considerably less than those found for similar keyways in heat-treated alloy steel.

Keyway in Torsion Specimens

The keyways in the torsion specimens were the same size as those in the bending specimens, and the specimens had the same diameter, namely, 2 inches. Some of the broken specimens are shown in Fig. 13. The stress-concentration factors found from the plaster-model tests are given in Table 4. The results given in Table 4 show that the stress-concentration factor for the profiled keyway in torsion is greater than that for the sled-runner type, and that the value for the profiled type is considerably greater than the fatigue stress-concentration factor obtained for the profiled type, both for very low carbon steel (Armco Iron) and for 0.65 per cent carbon steel specimens, as reported by H. J. Gough (2). The stress-concentration factors obtained from the plaster-model tests are somewhat greater than those found from static tests of steel and based on the elastic strength of the specimens, as reported by Moore (1) and Gough (2).

11. *Discussion of Results.*—The results reported in the preceding tables and curves indicate, as other investigations employing pottery plaster have indicated, that the stress as determined from the results of the test of one pair of specimens cannot be considered to be reliable or significant but that a number of pairs of specimens must

be tested to obtain consistent results, at least with the methods of mixing, curing, and drying that have so far been tried. The cause of variation in test results is not at present apparent, but may be due in part to pin holes in the plaster which may not be evenly dispersed through the mass.

It is also not yet clear why this material, which gives a straight stress–strain curve up to the point of rupture in a specimen free from abrupt changes of section, fails to indicate stress concentrations as large, at abrupt changes of section, as those obtained by mathematical analysis and by the photo-elastic method. It seems likely that a slight amount of yielding or flow may occur in the plaster at the small region of localized stress resulting in a more favorable distribution of stress. The suggestion has been made also that there may be a so-called skin effect analogous to surface tension of the surface layer of atoms at a point of high localized stress such as at a fillet of small radius, etc.; further, it may be that the strength properties of material under high concentration of stress surrounded by a much larger mass of understressed material is not reliably indicated by the tests of the usual laboratory control specimens. Under what conditions these ideas are valid, if at all, is not known. In this connection it may be noted that the higher values of stress-concentration factors obtained for fillets by the plaster model method are in fair agreement with the values found by the photo-elastic method (see Fig. 7).

12. *Significance of Model Experiments.*—It is clear from recently published papers and discussions (14, 16) that there is not unanimity of opinion on the value or significance of the results of tests of models such as photo-elastic, repeated-stress (fatigue), and plaster-model tests. If the model is mechanically similar to the member or prototype, the results of the model tests should be reliable for predicting the stress in the member. Specimens that are mechanically similar are geometrically similar and in addition have, at corresponding points, similar internal structures and orientations. Geometric similitude alone does not, of course, insure mechanical similitude. For example, consider a repeated-stress specimen (small model); even though the specimen or model is made from a portion of the actual (large) member and is geometrically similar to the actual member (including the manner of loading) it may not be mechanically similar, because in the model the small number of crystalline grains at the point of high stress concentration may not permit the same mechanical action or offer the same opportunity for failure to start as would occur at the point or region of high stress concentration in the large member

where there are a larger number of crystals at which failure may start. In other words, the number of crystals involved and the orientations of the crystals will, in general, be different.

Likewise there is reasonable doubt as to whether the stress distribution in a two-dimensional state of stress as shown by the photo-elastic method in a flat transparent model is a reliable picture of the damaging stresses in a three-dimensional state of stress in a metal shaft or body of varying thickness, even though the transparent model conforms to ideal conditions; namely, that it is made as far as possible similar to the metal member, is free from initial stresses, follows Hooke's law, and is stressed within the proportional limit of the material.

Similarly, a plaster-model that is geometrically similar to a metal member may not indicate the damaging stress in the metal member in which the stress is highly concentrated, even though the plaster material gives a straight stress–strain diagram up to rupture in a control specimen free from abrupt change in section. The values for Poisson's ratio for the two materials are different, and the type and degree of adjustment that occurs in the two materials, particularly at points of highly localized stress, are not necessarily the same. In members with less highly concentrated stress, a closer correlation between the results of plaster models and those of mathematical analysis of ideal material would be expected and has been found (7).

The results of model tests, therefore, would apparently need careful scrutiny before their significance and worth can be determined. Their correlation with the results of tests on the actual member is the only convincing evidence. Obviously such direct evidence is seldom possible to obtain, and hence model tests may justifiably be resorted to, but they should be used with discretion.

The results herein reported of tests on cylindrical plaster models in which highly localized stresses occur show that the values of stress concentration are decidedly below those indicated by mathematical analysis of flat homogeneous, isotropic continuous material, and less than those obtained by the photo-elastic method on flat models.

The average values of stress-concentration factors found from the plaster-model method appear to be in fairly good agreement with those found from fatigue tests of non-heat-treated plain carbon steel specimens of small size.

If the results of plaster-model tests for certain types of abrupt changes of section and for certain types of straining action (such as bending, etc.) have been compared with the results of tests of actual members or of tests of other models whose significance is known, the plaster-model method affords an inexpensive and simple means of obtaining approximate but useful values for stresses in bodies of irregular shapes.

V. Conclusion

13. *Conclusion.*—The main object of the tests herein recorded was to determine whether the plaster-model method is a reliable method for obtaining the significant, effective, or damaging stress in a body of irregular shape, especially if the stress is highly localized or concentrated at an abrupt change in section.

The conclusion is reached that the plaster-model method affords a relatively quick, inexpensive, and simple means of obtaining approximate but useful values of the effective stress concentrations in bodies of irregular shapes, provided that the average of the results of the tests of a number of pairs of specimens is used (seven pairs of specimens would usually be sufficient).

The results of plaster-model tests, however, should be used with considerable judgment. The real significant or damaging effect of the stress concentrations found by the plaster-model method, like the significance of the results of all models tests in which the model material is different from the material in the actual member, depends largely on the relation that has been shown to exist between the model test results and those found from tests, or from service records, with the actual material. However, for bodies of irregular shapes for which there are no available data concerning the stress concentration caused by the abrupt changes in section, the plaster-model method affords a valuable means of obtaining very useful, though approximate, results.

14. *Summary.*—Abrupt changes in section such as occur at fillets, keyways, screw threads, holes, etc. in members that resist loads are usually accompanied by relatively high stresses in small portions of the members where the changes in sections occur. These abrupt changes in section are frequently referred to as "stress raisers" and the stresses caused by such stress raisers are frequently called localized stresses or stress concentrations.

Various methods making use of models for determining stress concentrations are in use, including the photo-elastic, repeated-stress (fatigue), elastic-membrane (soap-film), brittle-material (plaster-model), etc., methods. The plaster-model method was used in the investigation herein described to determine the stress concentrations at fillets, holes, and keyways.

A summary of the main facts concerning the investigation may be given as follows:

(1) Pottery plaster, if properly cured and dried, has a straight stress–strain diagram up to the ultimate or breaking stress; its behavior up to rupture, therefore, is approximately the same as the elastic behavior of a ductile metal, like steel.

(2) The plaster-model method of determining the maximum stress in a member caused by given loads may be stated very briefly as follows: A pair of specimens of pottery plaster are made, one specimen being a model of the member in which the stress is to be found, and the other a model of simple form for which the relation between load and stress is known. The two specimens are tested to rupture. From the known relation of load and stress in the specimen of simple shape and the ratio of the loads (or bending moments) causing rupture of the two specimens, the ratio between load and maximum stress in the model of complex shape is found. The same ratio is assumed to hold in a similarly-shaped member made of ductile material, provided that the stresses are within the proportional limit of the material.

(3) The effects of fillets, holes, and keyways in increasing the stress in cylindrical specimens that are subjected to either bending or torsion were found by use of plaster-model tests. The stress-raising effect of each type of abrupt change in section is expressed by a value called the stress-concentration factor, which is the factor by which the stress as calculated without regard to the abrupt change in section must be multiplied in order to obtain the maximum stress that exists at the section. Values of the stress-concentration factors for each of the three forms of sudden change of section are given in the form of tables and curves.

(4) Attention is called to the difference between the ideal stress concentrations obtained from mathematical analysis, and the effective or significant stress concentrations that are associated with damage to the actual material under service conditions, and which are affected by the internal structure of the material, etc., in addition to the external geometric form of the discontinuity.

(5) Comparisons of the results of the plaster-model tests are made, where results are available, with the stress-concentration factors obtained for the same types of external or geometric discontinuities by other model methods, such as the photo-elastic and repeated-stress (fatigue) methods, and by mathematical analysis. In general the values of stress-concentration factors found from the plaster-model

tests were considerably smaller than those obtained by the photoelastic method and by mathematical analysis, but they agreed fairly well with those found from fatigue tests of small specimens of non-heat-treated plain carbon steel.

(6) The limitations of the use of models are briefly discussed, and the need for a knowledge of the relation of the results of model tests with those of tests that simulate the conditions of actual members in service are emphasized as a warning in interpreting the engineering significance of the results of model tests.

APPENDIX A

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