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EFFECT OF TIME YIELD IN CONCRETE UPON DEFORMATION STRESSES IN A REINFORCED CONCRETE ARCH BRIDGE

A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH

THE UNITED STATES BUREAU OF PUBLIC ROADS

BY

WILBUR M. WILSON

AND

RALPH W. KLUGE

BULLETIN No. 275
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EFFECT OF TIME YIELD IN CONCRETE UPON DEFORMATION STRESSES IN A REINFORCED CONCRETE ARCH BRIDGE

I. Introduction

1. *Object and Scope of Investigation.*—The stresses in a statically indeterminate structure are of two kinds: (1) load stresses, which are due to the direct effect of the loads that the structure carries, and (2) deformation stresses, which are due to a movement of the supports. Stresses essentially the same as deformation stresses are also produced if the lengths of certain members change, due to a change in temperature or other causes, even though the supports remained fixed.

The length of a reinforced concrete arch rib changes (a) because of the immediate strain accompanying the application of a load, usually designated as rib shortening, (b) because of the time yield in the concrete, (c) because of temperature changes, and (d) because of the shrinkage in the concrete that takes place as the concrete dries. All changes in the length of the rib of an arch with fixed abutments change the shape of the rib, thereby producing flexual stresses similar to the flexual stresses that are produced by moving the abutments without changing the length of the rib. Since the stress is due to a change in shape and not due to a load, it is a *deformation stress,* and, since the changes, except rib shortening, take place slowly, it would appear possible that the time-yield property of the concrete might enable the rib to assume its new shape without incurring the stresses that would be produced if the same changes in length occurred quickly.

In view of this possibility, tests were made to determine the change in reactions at the springings of an arch due to the shrinkage of the concrete, due to the combined effect of shrinkage and time yield due to dead load, and due to changing the span by an amount equivalent to a change in temperature of 100 deg. F. In order to simulate the gradual temperature trends throughout a season, the change in span was made in five equal increments; each increment was equivalent to a change of 20 deg. F. and the changes were made at intervals of approximately 30 days. Readings were taken just before and after each change in span to determine the immediate change in the reactions. Subsequent readings were taken at intervals of approximately one week to determine the change in reactions due to time yield.

A series of tests was also made to determine the effect of the gradual yielding of the abutments upon the load-carrying capacity of the arch. The initial readings were taken when the design dead load was on the structure and when the abutments were in their normal position relative to each other. Subsequent readings were taken after each of a number of live loads had been added, and after each of a number of movements of the abutments. In general, a period of one week was allowed to elapse between successive abutment displacements to permit time yield of the concrete to occur.

The temperature of the laboratory remained very constant, seldom varying more than one degree F. during a single set of observations. The temperature of the air was recorded for all tests, and a standard temperature was designated at the beginning of each series. Subsequent tests were made on days when the temperature of the air in the laboratory was found to vary by not more than one or two degrees from the standard temperature. Moreover, the ventilation of the building was controlled so that the temperature of the air did not vary by more than one or two degrees for a period of two or three hours immediately preceding a test. It is believed that there are no appreciable errors in the results due to temperature changes.

2. *Acknowledgments.*—The tests described in this report are a part of the research work sponsored by the Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers. They also constitute a part of the investigation resulting from the coöperative agreement entered into by the Engineering Experiment Station of the University of Illinois, of which DEAN M. L. ENGER is the director, and the United States Bureau of Public Roads, of which THOMAS H. MACDONALD is Chief of Bureau. The tests were planned by the authors in consultation with MR. E. F. KELLEY, Chief of Division of Tests, and A. S. GEMENY, Senior Structural Engineer, both of the United States Bureau of Public Roads; and with PROF. GEORGE E. BEGGS, E. H. HARDER, A. C. JAMMI, and PROF. CLYDE T. MORRIS, members and chairman, respectively, of the Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers. The experimental work was done by RALPH KLUGE and NATHAN NEWMARK, Research Assistants in Civil Engineering.

The tests were made in the Materials Testing Laboratory of the University of Illinois. The direct expenses of the tests were paid from funds provided by United States Bureau of Public Roads, American Society of Civil Engineers, Engineering Foundation, Universal Atlas Cement Company, Illinois Steel Company, American Bridge Company, Jones and Laughlin Steel Corporation, Interstate

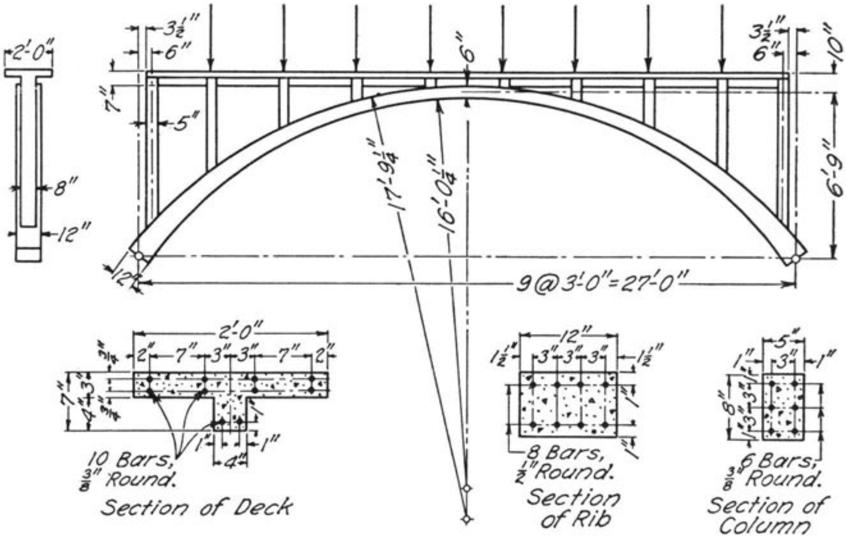

FIG. 1. DETAILS OF ARCH

Sand and Gravel Company, Fairbanks, Morse and Company, Lincoln Sand and Gravel Company, and Neal Sand and Gravel Company.

II. SPECIMENS AND TESTING APPARATUS

3. *Description of Specimen.*—The specimen consisted of an open-spandrel reinforced concrete arch with a deck so low as to be integral with the rib at the crown. The dimensions of the arch, given in Fig. 1, were the same as for one span of the three-span arch bridge with a deck which was previously tested.* The span was 27 feet, and the rise 6 feet 9 inches. The aggregate and the mix were as nearly as possible like the aggregate and mix used for the three-span structures. The mix was 1:3:3 by volume with a 1.2 water-cement ratio. The quantities for a batch were determined by weight, correction being made for the moisture content of the aggregate. Each batch was mixed at least four minutes, and eight 6-in. by 12-in. control cylinders were made, one from each of eight batches.

The arch was poured June 24 and the forms were removed July 5 and 6, 1933. The surfaces of the concrete were wet when the forms were removed, and the concrete probably had not shrunk an appreciable amount at that time.

The stress-strain diagrams for the control cylinders are given in

*See Bulletin 270, Engineering Experiment Station, University of Illinois.

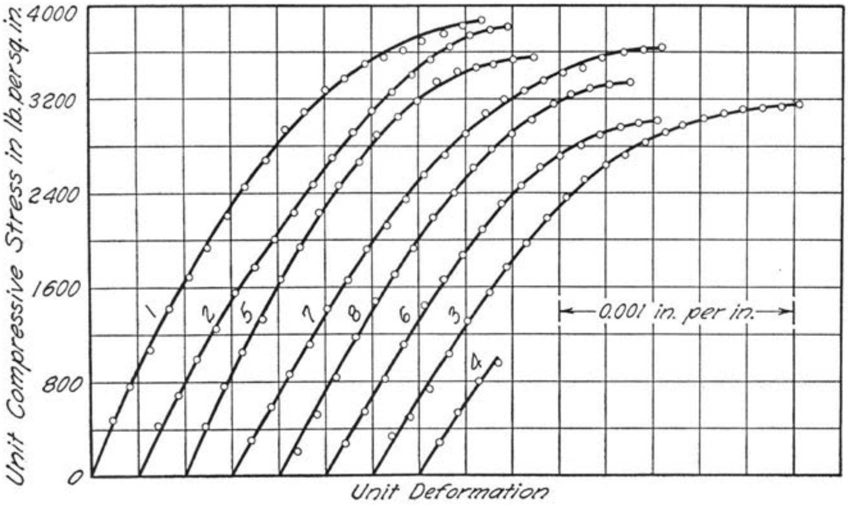

FIG. 2. STRESS-STRAIN DIAGRAMS FOR CONTROL CYLINDERS

Fig. 2, and the physical properties of the concrete are given in Table 1. Figure 3 shows the general appearance of the arch.

4. *Description of Control Specimens.*—Four control specimens were made for use in studying the magnitude of shrinkage and time yield in a straight rectangular block of concrete. Each specimen was four feet long and had a transverse section 9 in. by 12 in., which is the average of the sections at the crown and springing of the arch rib. Each control specimen was reinforced with eight ½-in. plain round steel rods, the same as the reinforcement in the arch rib. The control specimens were poured on the same day and of the same

TABLE 1
PHYSICAL PROPERTIES OF CONCRETE FROM CONTROL CYLINDERS

Cylinder No.	Ultimate Stress lb. per sq. in.	Modulus of Elasticity E 10^6 lb. per sq. in.
1	3860	4.3
2	3810	4.0
3	3180	3.3
4	2780	3.1
5	3540	4.2
6	3080	3.4
7	3640	3.4
8	3350	3.5
Av.	3400	3.7

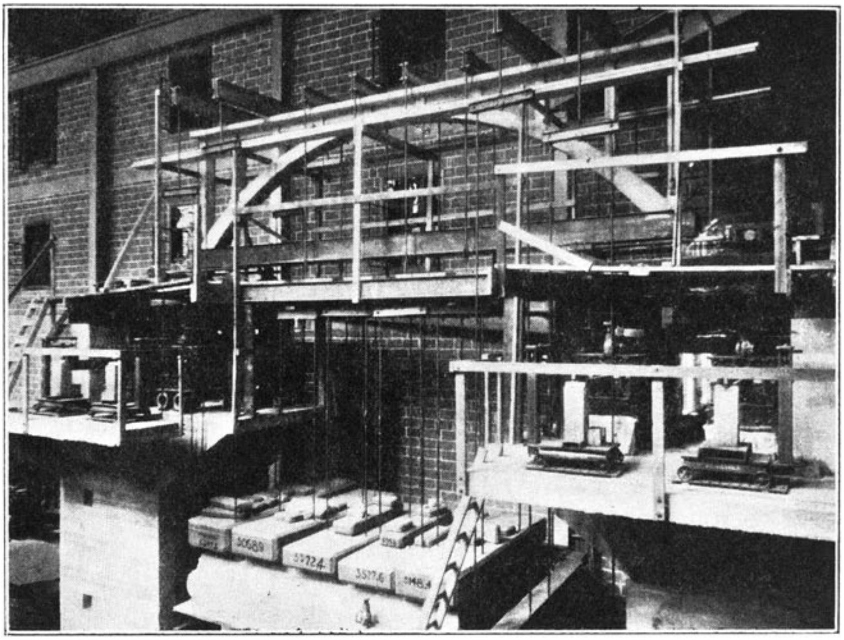

FIG. 3. GENERAL VIEW OF ARCH

concrete as the arch, and they were stored in the laboratory near the arch where the atmosphere was dry and warm. Two specimens were unloaded throughout the tests; the other two were subjected to an axial thrust of 240 lb. per sq. in., approximately the average dead-load thrust on the arch rib. The change in length of these control specimens was measured on four reinforcing bars, two on the top and two on the bottom, with a 24-in. Berry strain gage.

5. *Description of Apparatus.*—Figure 3 shows the general appearance of the specimen. The apparatus was designed and built for the three-span arch bridge that had previously been tested.*

The load upon the structure was produced by suspending concrete blocks of known weight at the load points immediately over each spandrel column. For each load point there was one large concrete block suspended by four steel rods, as shown in Fig. 3, which served as a loading platform on which to place smaller blocks that, in combination with the loading platform, constituted the total load to be applied at a particular point. When the arch was unloaded, the

*See Bulletin 269, Engineering Experiment Station, University of Illinois.

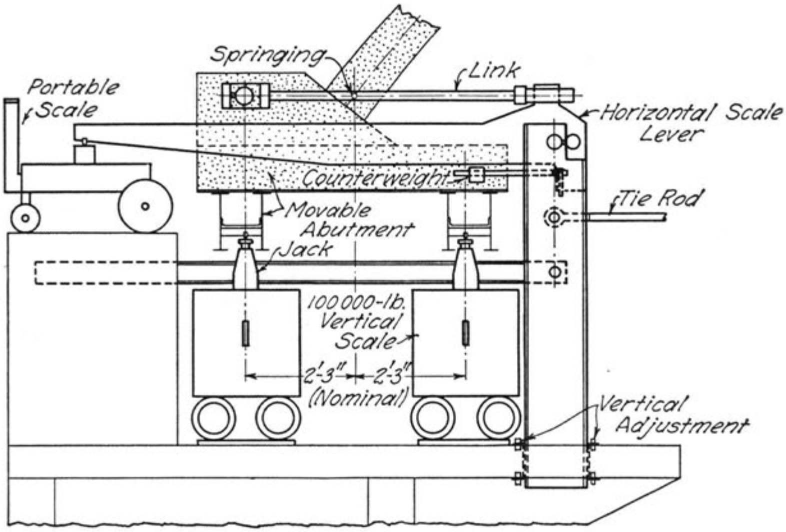

FIG. 4. WEIGHING APPARATUS AND SUPPORTS FOR ABUTMENTS

loading platform rested upon supports provided for the purpose, and the loading beam was carried on the steel suspension rods acting as struts. To apply the load the turnbuckles in the suspension rods were turned, shortening the rods, until the loading beam came into contact with the loading shelf. The turnbuckles were then turned, successively, by small amounts in order, back and forth from one end of the structure to the other, thereby transferring the weight of the loading platforms gradually from the supports to the arch. Each load point on the arch was capped with a steel plate containing a ¾-inch steel ball located in its top surface and at the point of application of the load. The loading beam had a small steel block attached at the center of its bottom flange. A depression in the bottom of this block fitted over the top of the ball in the steel plate on the loading shelf, thus accurately locating the point of application of the load.

The load at a given load point for a particular test was obtained by placing concrete blocks of known weight upon the loading platform until the desired load had been obtained. Each abutment was supported on two vertical-load scales, as shown in Fig. 4. The load was transmitted from the abutment to the scales by means of jacks, two for each scale, one on the north and the other on the south side of the abutment. The contact between the abutments and jacks was through knife-edges embedded in the abutment, so that the line of action of the vertical forces weighed by each scale was accurately known. An abutment could be raised or lowered without rotation by extending or depressing all jacks by the same amount; or an abutment could be rotated about a horizontal north-and-south axis by extending both the east jacks and depressing both the west jacks, or the reverse. The vertical-load scales were mounted on carefully-machined steel rollers 10 inches in diameter that ran upon a carefully-machined track, so the vertical scales offered practically no resistance to horizontal motion.

The horizontal reaction of each abutment was measured by means of a horizontal-load scale, as shown in Fig. 4. This scale consisted of two right-angle levers (bell cranks), one on the north and the other on the south side of the abutment, that received the horizontal thrust through links, and converted it into a vertical force that was delivered to a platform scale. The bell crank had a nominal multiplication ratio of 10 to 1, the actual multiplication ratio being determined for each. The link connecting an abutment and horizontal scale had knife-edge contacts at both ends. The link was maintained in a horizontal position, and the line of action of the horizontal reaction was determined from the position of the knife-edge embedded in the abutment. An abutment could be moved horizontally by turning the link since it had a right-hand thread at one end and a left-hand thread at the other.

The strain in the concrete was measured with an 8-in. Berry strain gage. Readings were taken on two gage lines on the intrados and on two on the extrados at the section midway between each pair of adjacent load points.

The angular position of the abutments was determined by means of extremely sensitive level bubbles, one for each abutment. Each bubble was attached to a steel rod embedded in the concrete in the longitudinal central plane of the rib, and normal to the axis at the springing.

The vertical movement of the abutments was measured with two hook gages, one at each abutment, connected to the same pipe line in such a manner that the water surface was at the same level for both gages at any instant. The vertical movement of the points in the rib immediately below the spandrel columns was measured with reference to two steel I beams, one on each side of the arch, supported on pins projecting from the sides of the rib at the springings. A ½-in. vertical rod embedded in the concrete, and projecting below the rib on its center line directly below each spandrel column, had a small hole in the lower end. The tops of the I beams were connected with steel battens directly beneath each spandrel column, and each batten had a small hole at its center, and directly below the rod projecting from the bottom of the rib. An instrument, consisting of an Ames dial having a conical point on its plunger and mounted on a rod having a conical end, was used to measure the distance from the lower ends of the projecting rods to the top of the battens, thereby giving the movement of the points on the arch rib relative to the I beams.

The I beams from which the deflection was measured were also used to measure changes in the span of the arch. The beams were attached to the arch at the east springing, but they were supported on rollers at the west springing. An Ames dial was attached to the west end of each beam in such a manner that its plunger bore upon a steel pin projecting from the side of the arch at the west springing. This dial indicated changes in the span of the arch.

The apparatus that has been described made it possible to give the abutments any movement that was desired. That is, each abutment could be given separately each of the three components of movement (X, Y, or θ). Moreover, each component of movement could be measured. Also all three components of each abutment reaction (H, V, and M) could be determined in magnitude and position from the scale readings.

III. Results of Tests

6. *Tests to Determine Abutment Reactions Due to Shrinkage.*—Previous to erecting the forms for the specimens, the scales supporting the abutments were read when they carried only the steel supporting the abutments. These readings were used as a base in computing the reactions at the springings of the arch from subsequent readings of the scales.

The arch was poured June 24, and the forms were removed July 5 and 6. After the forms had been removed and the apparatus attached to the arch, the abutments were brought to their normal position relative to each other. In making this adjustment, the span was changed until the horizontal thrust at the springing, as indicated by the horizontal scales, was equal to the value of the thrust as computed from the weight of the structure. This was approximately 3075 lb. Likewise, in making the adjustment for the angular position of an abutment, the abutment was rotated until the moment at the springing, as indicated by the vertical scales, was equal to the value of the moment at the same point as computed from the weight of the structure. This was approximately 16 000 in. lb. The bubble tubes attached to the arch at the springings were then adjusted so as to bring the bubbles to their mid-position, thus preserving the normal angular position of the abutments. The readings of the dials, indicating the normal span, and of the hook gages, indicating the normal relative elevation, were recorded and used, together with the bubbles, as a basis for maintaining the abutments in their normal position relative to each other during the subsequent tests.

These basic readings were taken July 8 when the arch was 14 days old, and two days after the forms had been removed. Inasmuch as the surface of the concrete was wet when the forms were removed, the volume of the concrete had probably changed very little at the time the basic readings were taken.

The arch carried no load except its own weight during the period from July 7 to August 11. The abutments were retained in their normal position and the scale readings were recorded at frequent intervals to determine the change in the reactions at the springings due to the shrinkage of the concrete. The results of the observations are presented in the left-hand portion of Fig. 5. These diagrams indicate that, for the period of 34 days beginning when the arch was 16 days old, the horizontal thrust fell off 540 lb. and the moment fell off 23 600 in. lb. at the springings. During the same period the control specimens shrank 0.00010 in. per in., the average value shown by the diagram of Fig. 6. The shrinkage of the concrete in the arch rib, as determined from strain-gage readings at sections midway between the spandrel columns, at the intrados and extrados, was 0.000092 in. per in., the average of the values for all sections. This corresponds to a change in the length of the 324-in. span of 0.0298 in. Using this value as the change in span *that was prevented,* the observed changes in reactions are equivalent to a horizontal thrust of 18 100 lb., and a moment at the springing of 800 000 in. lb. per inch change in span. These observations cover a period from the time the arch was 16 days old until it was 50 days old. The tests made to determine the immediate change in the reactions due to a unit change in span, described in Section 8, gave a change in thrust of 25 400 lb., and a change in moment of 1 313 000 in. lb. per inch change in span. These observations covered a period from the time the arch was 216 days old until it was 415 days old. The ratio of the latter value of the thrust to the former is 1.40, and the ratio of the latter value of the moment to the former is 1.64. The fact that these ratios exceed unity may have been due in part to the low value of the modulus of elasticity of the con-

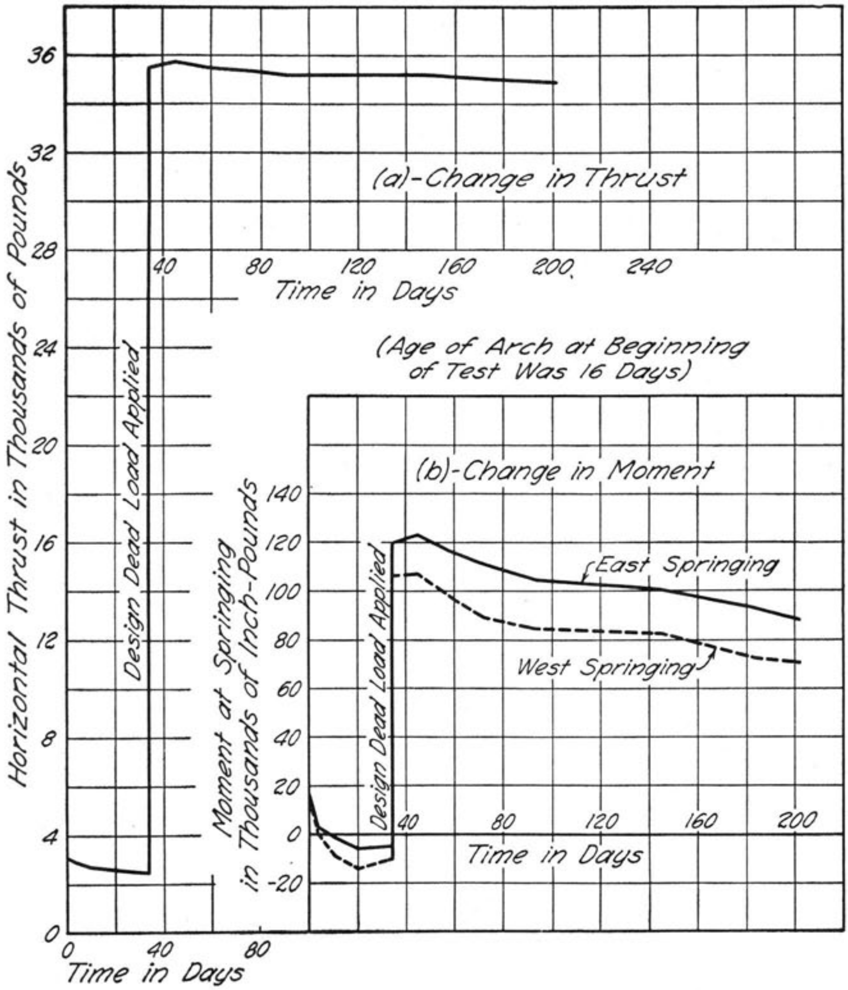

FIG. 5. CHANGE IN REACTION COMPONENTS ACCOMPANYING SHRINKAGE AND TIME YIELD

crete during these tests when the concrete was green, or it may have been due to the fact that the time yield in the concrete reduced the flexural stress due to shrinkage. But, whatever the cause, the unit stress due to shrinkage was much less than the stress due to a sudden change in span equivalent to the unit shrinkage and based upon a value of E for the aged concrete.

The deflection of the arch rib due to shrinkage of the concrete is shown by the broken lines of Fig. 7. Inasmuch as a deflection dia-

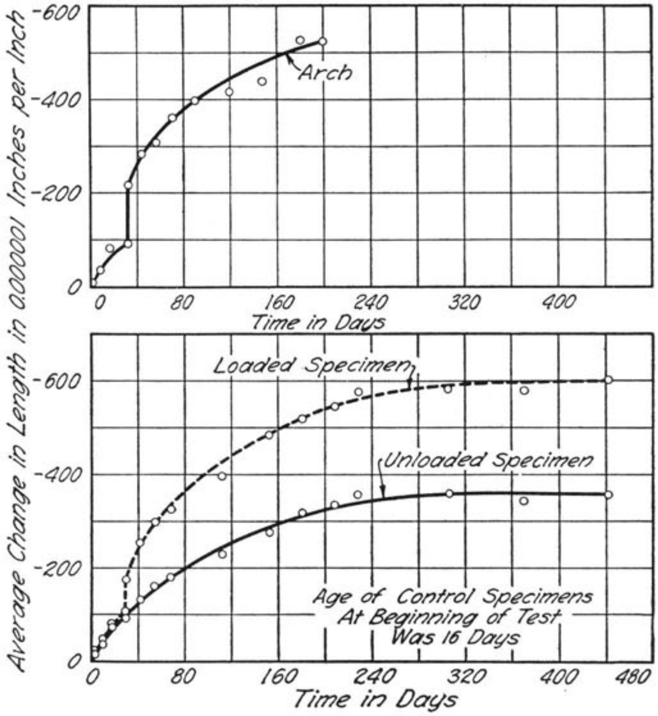

FIG. 6. UNIT CHANGE IN LENGTH OF ARCH RIB AND CONTROL SPECIMENS DUE TO SHRINKAGE AND TIME YIELD

gram is an influence line for horizontal thrust, there should be a definite relation between the lines representing the deflection due to shrinkage, the influence line for horizontal thrust, and the unit shrinkage of the concrete in the rib. These relationships will be indicated for load points 4 and 5, the two adjacent to the crown. The experimentally-determined influence ordinate for horizontal thrust for these points is 0.88.*

The vertical movement due to shrinkage is made up of two parts. One part is due to the direct vertical shrinkage and is equal to the product of the unit shrinkage and the vertical projection of the portion of the arch considered.† The other part is due to the flexural effect of changing the length of the arch axis without changing the span, and is equal to the influence ordinate for the horizontal thrust at the point considered, 0.88 for load points 4 and 5, multiplied by the change

*See Table 9, Bulletin 270, Engineering Experiment Station, University of Illinois.
†"Temperature Deformations in Concrete Arches," by Professor Hardy Cross, Engineering News-Record, Feb. 4, 1926, p. 190.

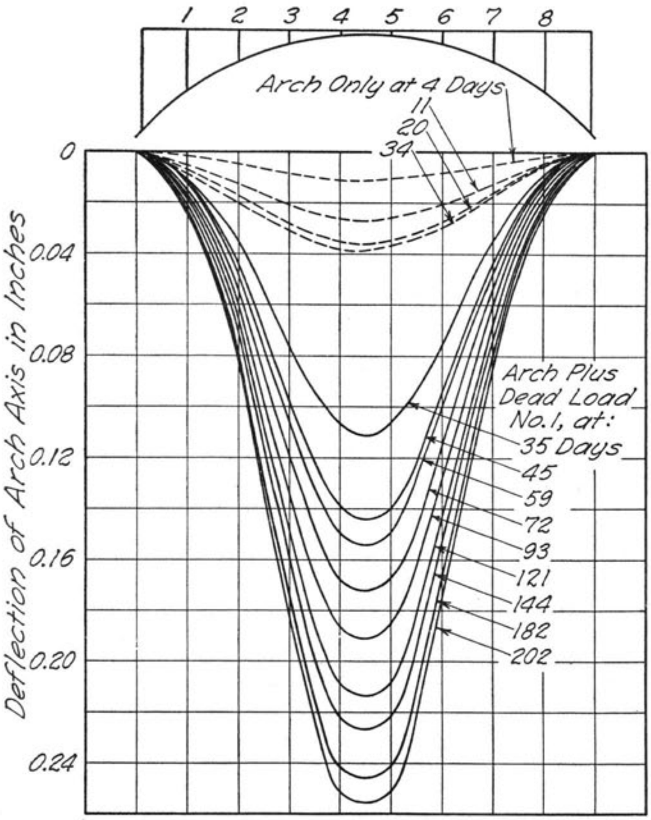

FIG. 7. DEFLECTION OF ARCH AXIS ACCOMPANYING SHRINKAGE AND TIME YIELD

in span that would have been produced by the shrinkage if the abutments had been free. The sum of these two parts should be equal to the vertical movement of load points 4 and 5. The average unit shrinkage at the end of the 34-day period, as determined by the strain gage, was 0.000092 in. per in., corresponding to a deflection in the 324-in. span of 0.0264 in. To this should be added the vertical movement due to direct vertical shortening, which for this case is $0.000092 \times 80 = 0.0075$ in. The total computed value of the vertical movement is therefore $0.0264 + 0.0075 = 0.0339$ in.; the measured value, the average for the points 4 and 5, was 0.0380 in. The computed and the measured values, respectively, of the vertical movement at load points 4 and 5 at the end of 2-day, 9-day, and 18-day periods are 0.003 in. and 0.01 in.; 0.014 in. and 0.026 in.; and 0.032 in.

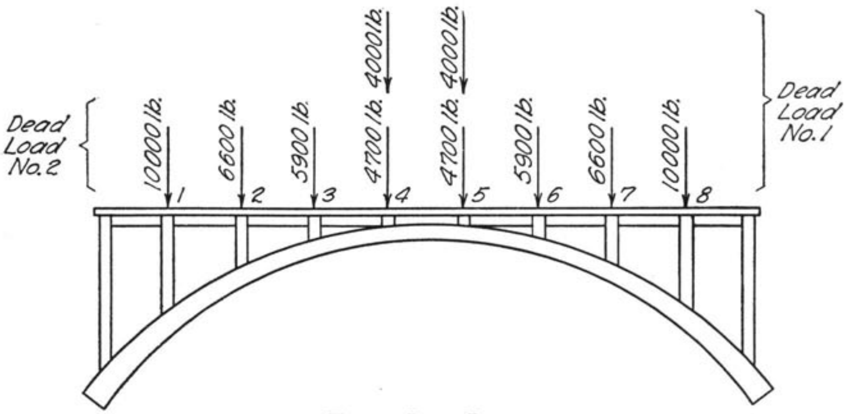

FIG. 8. DEAD LOAD

and 0.035 in., respectively. The measured value exceeds the computed value in every instance; the difference is about 10 per cent for the 18-day and 34-day periods, the only ones for which the values are great enough to be measured satisfactorily. This difference between measured and computed values is large, relatively, but the quantities themselves are so small that the absolute differences are not greater than the probable tolerance in the measurements, and they are not considered as having any significance.

7. *Tests to Determine Effect of Time Yield and Shrinkage of Concrete Upon Abutment Reactions; Dead Load on Structure.*—After the tests to determine the effect of shrinkage in concrete upon the reactions, described in Section 6, had been completed, the same structure was used in a series of tests to determine the effect of combined shrinkage and time yield of the concrete upon the reactions at the springings when the arch carried the dead load. The dead load used in this test, No. 1, Fig. 8, was purposely chosen so as to produce a large moment at the springing.

Preliminary to the tests, the abutments were brought to their normal position as indicated by the instruments provided for the purpose. The scales, the strain, and the vertical position of the points on the arch rib below each spandrel column were read when the structure carried no load except its own weight. The dead load was then applied to the arch, the abutments returned to their normal position, and the readings were again recorded. The changes in the reactions at the springings due to the application of the dead load are shown in Fig. 5, and the vertical movement of points on the arch axis is shown in Fig. 7. The application of the dead load changed the moment on the east abutment from − 5000 in. lb. to + 120 000* in. lb., and on the west abutment from − 10 000 in. lb. to + 106 000 in. lb. The resulting change in moment at the springing, the average of the two ends, is 113 000 in. lb. The horizontal thrust at the springing increased from 2500 lb. to a little over 35 500 lb. The stress across the section at the springing, computed from the weighed reactions on the basis that concrete takes tension, varied from 44 lb. per sq. in. tension at the intrados to 620 lb. per sq. in. compression at the extrados.

The dead load remained on the arch for a period of 167 days, and observations were made at frequent intervals during this time. For these subsequent readings the abutments were returned to their normal position relative to each other so that, insofar as it was possible to bring it about, the only factors acting to change the abutment reactions were the time yield and the shrinkage of the concrete. Both of these act to shorten the axis of the rib. The shrinkage, by shortening the axis when the abutments are fixed, reduces the horizontal thrust and introduces a bending moment at all sections. The time yield of the concrete also shortens the arch axis. In addition to this direct effect of shortening the axis, some engineers hold that time yield reduces the dead-load flexural stress at sections where this portion of the stress is large. It had been hoped that the test would throw some light upon this question, but, if time yield did reduce dead-load flexural stresses, the reduction was so small that it could not be detected.

The diagrams of Fig. 5 show that both the horizontal thrust and the moment at the springing decreased as the test continued. The question of interest is, are the reductions in the reactions as great as they would have been if the shortening of the arch axis had occurred in a few hours instead of extending over a period of several months?

The change in length of the control specimens, described in Section 4, is given in Fig. 6. Two of the specimens, represented by the lowest curve, remained unstressed, and the changes in length are attributed to the shrinkage of the concrete. Two other specimens, represented by the middle curve, were loaded axially by means of calibrated springs to a unit stress equal, approximately, to the average unit dead-load stress in the arch rib. The change in length of the specimens is thus due partly to shrinkage and partly to time yield.

*A plus (+) moment is one that produces tension at the intrados.

The upper curve represents the change in length of the rib of the arch as determined from strain readings on four 8-in. gage lines at sections midway between each pair of adjacent spandrel columns. The upper and the middle curve indicate that the control specimens and the arch rib shortened by nearly the same amount during the period of this test. The unit shortening of the arch axis was 0.00031 in. per in.; this is equivalent to a shortening of 0.100 in. for the 324-in. span. The tests reported in Section 8 indicate that a quick change in span of 0.10 in. without any other movement of the abutments and without any change in load, produces a change in moment at the springing of 131 300 in. lb. This is comparable to the reduction in the moment of 33 000 in. lb. which actually did take place as determined from the weighed reactions. The tests of Section 8 also indicate that a quick change in span of 0.10 in. produces a change in the horizontal thrust of 2540 lb. This is comparable to the reduction in thrust of 640 lb. which actually did take place. That is, the change in the length of the arch axis that occurred in a little less than six months, beginning when the arch was a little over one month old, caused a change in both the horizontal thrust and in the moment at the springing only one-fourth as great as would have been produced if the change in the length had taken place quickly. As in the tests of Section 6, it is not clear whether the reduction in stress is due entirely to time yield or partly to time yield and partly to the fact that the value of E for the concrete was less at this time than later when the elastic constants were determined by a quick change in span. But the arch had been in a warm dry atmosphere for more than a month when the tests of this series were begun. Moreover, the tests from which the change in thrust and moment due to a quick change in span were determined were made immediately after this series was completed. It would therefore appear safe to assume that E did not vary greatly between the time of the tests of this series and the time when the elastic constants were determined by a quick change in span.

8. *Tests to Determine Effect of Time Yield Upon Reactions Accompanying Change in Span.*—After the tests described in Section 7 had been completed the load upon the structure was changed as indicated in Fig. 8 by removing 4000 lb. from each of load points 4 and 5, the resulting load being designated as Dead Load No. 2. This change was made so that the dead-load moment at the springings would be small when the abutments were in their normal position, thereby making possible a considerable change in span without cracking the rib of the arch.

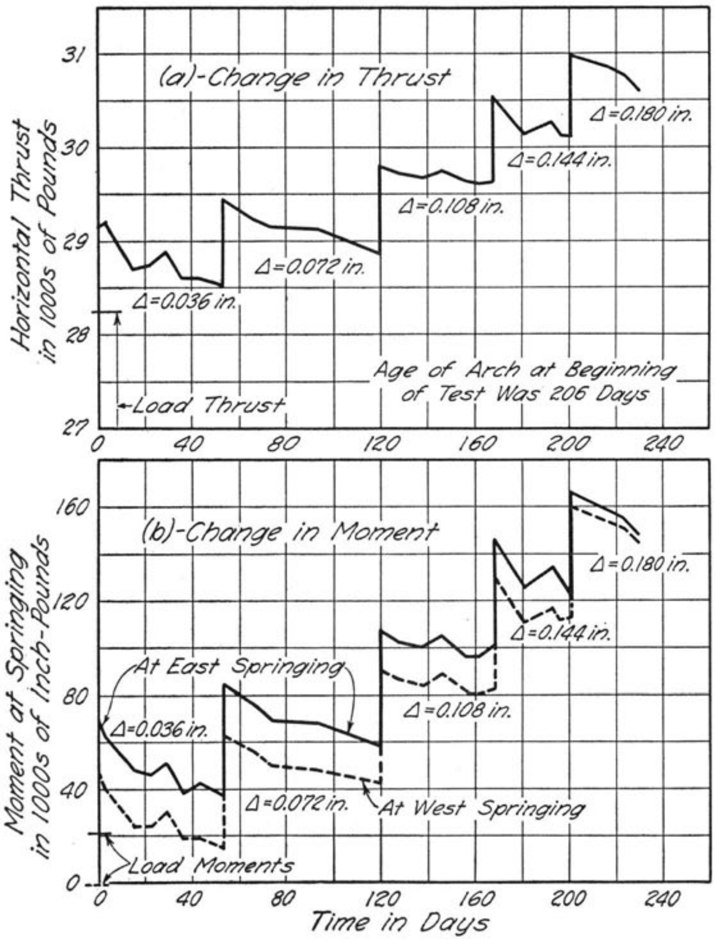

FIG. 9. CHANGE IN REACTION COMPONENTS ACCOMPANYING SHRINKAGE, TIME YIELD, AND CHANGES IN SPAN

The abutments were brought to their normal position relative to each other and a complete set of readings was recorded. The span of the arch was then reduced 0.036 in., which is equivalent to a rise in temperature of 20 deg. F., and a second set of readings was recorded. Subsequent readings were taken at intervals of approximately a week for a period of about a month, the span, load, position of the abutments, and the temperature in the laboratory being the same for all readings. The span was then reduced another 0.036 in. and the observations were repeated. Five reductions in span were made in this manner, each equivalent to a rise in temperature of 20 deg. A complete set of readings was recorded immediately after each change, and at intervals of approximately a week during the periods between changes, which were about a month apart.

The changes in the reactions are shown in Fig. 9. The straight vertical portions of these diagrams represent the changes that occurred immediately when the span was changed; the portions of the diagrams that slope downward to the right represent the changes that occurred during the period between the changes in span. The latter changes in the reactions are attributed to time yield and shrinkage in the concrete. The concrete was 206 days old when the series began. The unstressed control specimens contracted 0.000025 in. per in. during this period, and this shortening is attributed to shrinkage. The stressed control specimens contracted 0.000059 in. per in. The corresponding free contraction of the arch rib would shorten the span 0.02 in.

The diagrams of Fig. 9 indicate that a quick change in span of 1 in. is accompanied by a change in the horizontal thrust of 25 400 lb. and a change in the moment at the springing of 1 313 000 in. lb. The total shortening of the span in the period of the test was 0.18 in., of which 0.02 in. may be considered as being due to the free contraction of the rib. The remainder, or 0.16 in., would produce flexure. If this shortening had taken place quickly it would, according to the values just given for unit change in span, have been accompanied by a change in horizontal thrust of 4060 lb., and a change in the moment at the springing of 210 000 in. lb. The actual change in the thrust was 2350 lb. and in the moment was 136 800 in. lb. That is, for this test, the effect of time yield in the concrete upon the horizontal reaction and the moment at the springing due to a change in span corresponding to a change in temperature of 100 deg. F. in a period of 230 days was as follows: The horizontal thrust was decreased 42 per cent and the moment at the springing was decreased 35 per cent. The fact should be noted, however, that if time yield in concrete decreases with age and with repeated applications of load,* then the fact that time yield caused an appreciable reduction of the deformation stresses in this arch does not necessarily mean that it would have an equal effect in an arch several years old.

* "Flow of Concrete Under Action of Sustained Loads," by Raymond E. Davis and Harmer E. Davis, Proc. Am. Con. Inst., Vol. 27, 1931, p. 837.

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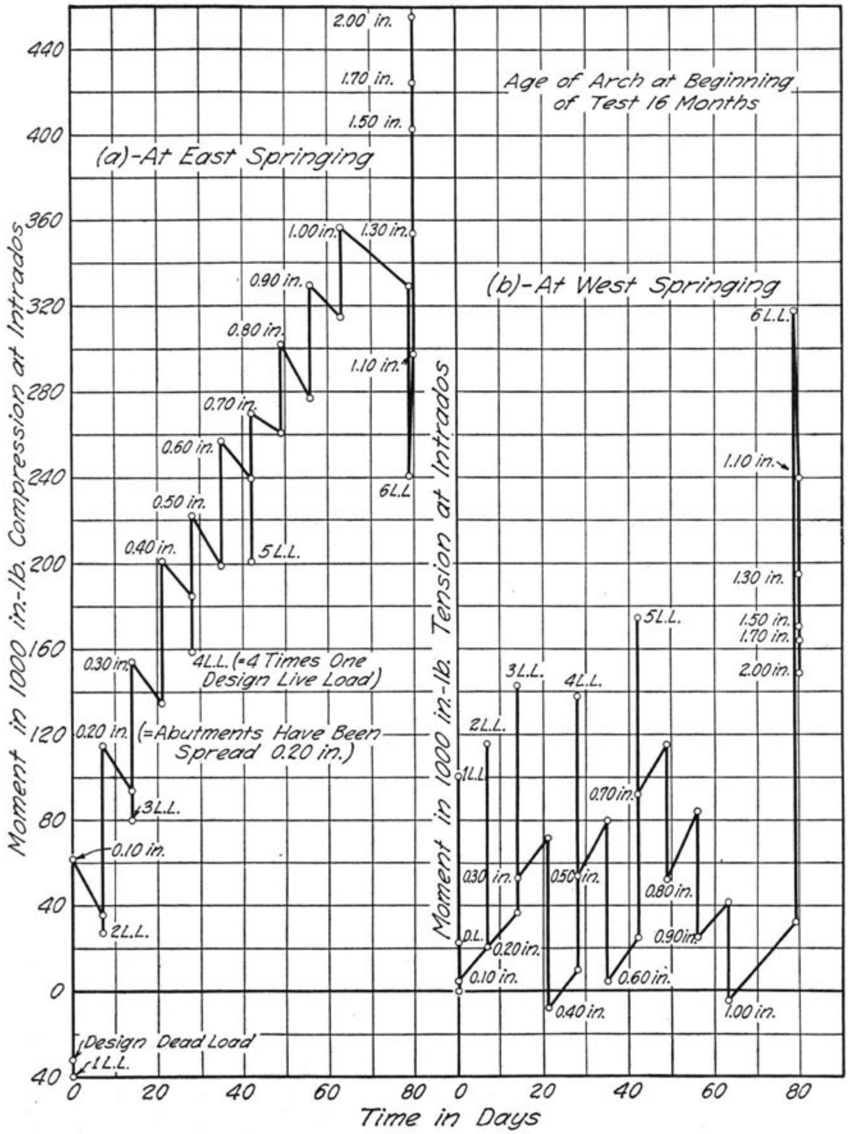

Fig. 10. Relation Between Time and Moment at East and West Springings

9. *Tests to Determine Effect of Time Yield Upon Reactions Accompanying Combined Live Load and Abutment Movements.*—After the tests described in Section 8 had been completed the abutments were returned to their normal position preliminary to a series of tests planned to determine the combined effect of live load and abutment movements. The arch was about 480 days old when this series of tests began. Expansion joints were cut in the deck in the panels adjacent to and outside of load points 4 and 5. This was necessary as the apparatus was not strong enough to break the structure if the deck remained continuous. After the expansion joints had been cut in the deck the weakest section of the rib was at load point 4, where the moment due to a change in span is large, and its sense the same as the moment due to the live load when the latter is distributed for maximum stress at the section. The design live load used in the tests consisted of 960 lb. at load point 3 and 2760 lb. at load point 4. The two loads together were designated as "one live load."

The initial readings were taken when design dead load No. 2, Fig. 8, was on the structure, and when the abutments were in their normal position relative to each other. The second set of readings was taken after one live load had been applied to the arch and the abutments had been returned to their normal position. The abutments were then allowed to spread 0.10 in. without rotation or vertical movement and without change in load. Readings were taken immediately after these changes had been made, and again at the end of a week. Changes were made in this manner, increasing the number of live loads and the spread of the abutments, until the arch carried six times the design live load and the abutments had been spread 2.0 in. Readings were taken just before and after each change, and again in seven days, and, for some changes, a third set of readings was taken two weeks or more after the change in span or load. The reactions of the abutments were determined from the scale readings, and include the effect of the weight of the structure as well as the superimposed load.

The relation between time and the moment at the springings is shown in Fig. 10; and the relation between time and the moment at load point 4, the point of maximum unit stress, is shown in Fig. 11. In these figures the legend, 0.30 in. and 0.40 in., indicates that the abutments have been allowed to spread 0.30 in. and 0.40 in., respectively; the legend, 2 LL and 3 LL, indicates that the arch carried two times and three times the design live load, respectively.

The moment at the east abutment was negative (compression at

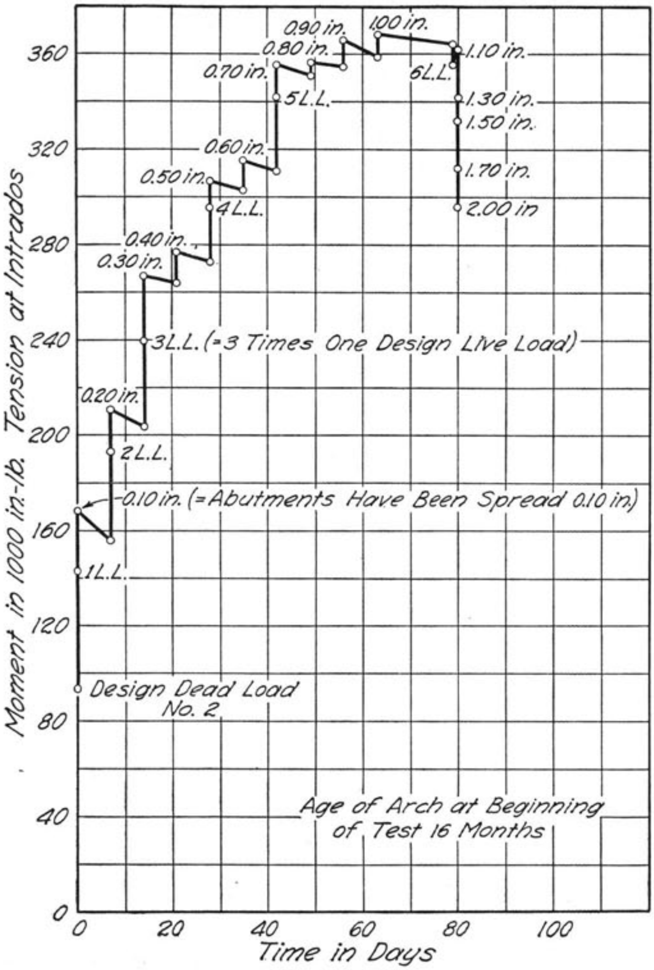

FIG. 11. RELATION BETWEEN TIME AND MOMENT AT LOAD POINT 4

the intrados) and at the west abutment was positive, except in two instances. The effect of the time yield was to reduce the negative moment at the east abutment and increase the positive moment at the west abutment. The rate of change was approximately 19 000 in. lb. per week, and had about the same value at the end as at the beginning of the 80-day period covered by this series. It should be noted, however, that the magnitude of the moment was much greater near the end of the period than at the beginning. The total change in the moment at the springing due to time yield was about 170 000 in. lb., and the absolute moment on the morning of the 80th day, when the arch carried the design dead load and six times the live load and the abutments were spread 1.00 in., was 240 000 in. lb. for the east abutment and 320 000 in. lb. for the west abutment.

The time yield in the concrete produced a smaller reduction in the moment at load point 4 than at the abutments. The total reduction during the 80-day period was 52 000 in. lb., whereas the absolute moment on the morning of the 80th day was 356 000 in. lb. The increase in moment at load point 4 due to each live load decreased as the load and the spread increased, the increase in moment for each additional design live load from 1 to 6 being, in 1000 in. lb., 49, 37, 36, 23, 31, and − 9, respectively. Likewise the increase in moment due to each 0.10 in. increase in span decreased, somewhat irregularly, as the load and spread increased, the increase in moment for each additional increment in span from 0 to 1.1 in. being, in 1000 in. lb., 25, 18, 27, 13, 11, 13, 14, 6, 11, 10 and 6. The moment at load point 4 actually decreased as the spread increased beyond 1.1 in., due to the fact that the concrete had spalled so badly that the section functioned as a hinge.

If each increment of live load had produced the same moment and thrust as the first increment, then the application of six times the live load in addition to the design dead load and the weight of the structure would have produced a unit stress equal to 4535* lb. per sq. in. If each 0.10 in. of spread had produced the same moment and thrust as the first increment, the spread of 2.0 in. would have produced a maximum unit stress at the section through load point 4 of 5075* lb. per sq. in. This added to the hypothetical value of 4535 due to load would make a total of 9610 lb. per sq. in., a value greatly in excess of the strength of the concrete. Nevertheless the arch carried six times the live load for several days after the abutments had been spread 2 in. It is true, however, that there were several large cracks in the rib, deck, and columns, and the concrete in the rib at load point 4 was badly spalled. The behavior of the arch under these extreme conditions was of interest, primarily, as demonstrating the punishment that a concrete arch can sustain and still carry a large load.

The condition of the structure when carrying four times the design live load with the abutments spread 0.60 in. is of interest. The

*The unit stress was determined from the thrust and moment on the basis that concrete takes tension. The authors realize that this method is not satisfactory but, in their opinion, no other method is any more satisfactory for a badly cracked section.

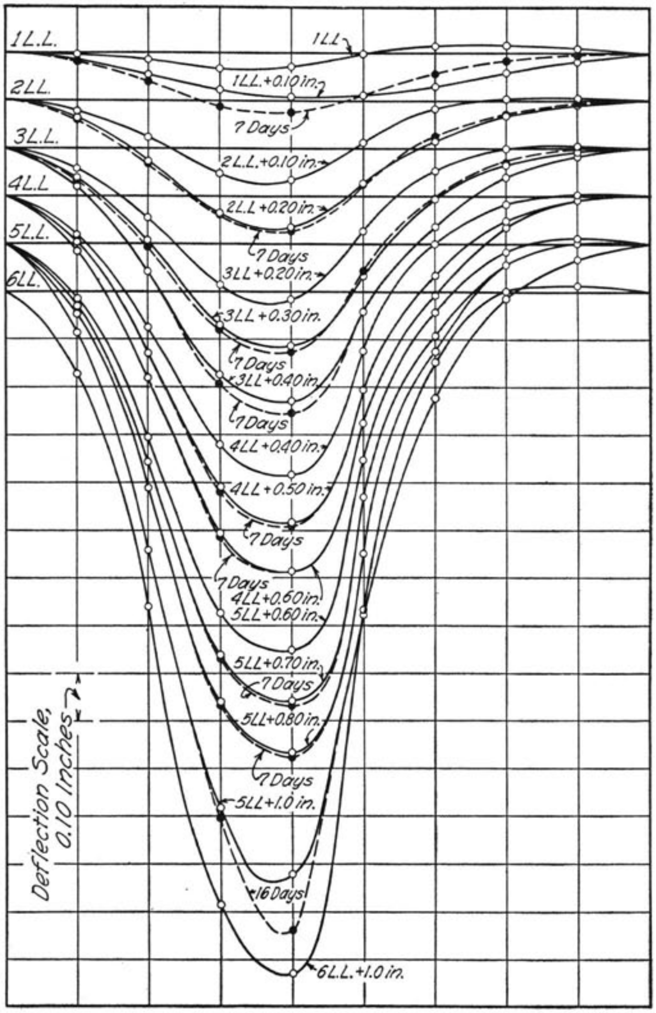

Fig. 12. Relation Between Time and Deflection of Arch

stress due to load, based upon the assumption that each live load produced the same increment of stress as the first live load, is 3640 lb. per sq. in.; and the stress due to the spread of the abutments, based upon a corresponding assumption, is 1520 lb. per sq. in., a total of 5160 lb. per sq. in., which is considerably in excess of the strength of the control cylinders. Nevertheless the structure was not seriously cracked.

The maximum stress at load point 4 as computed from the meas-

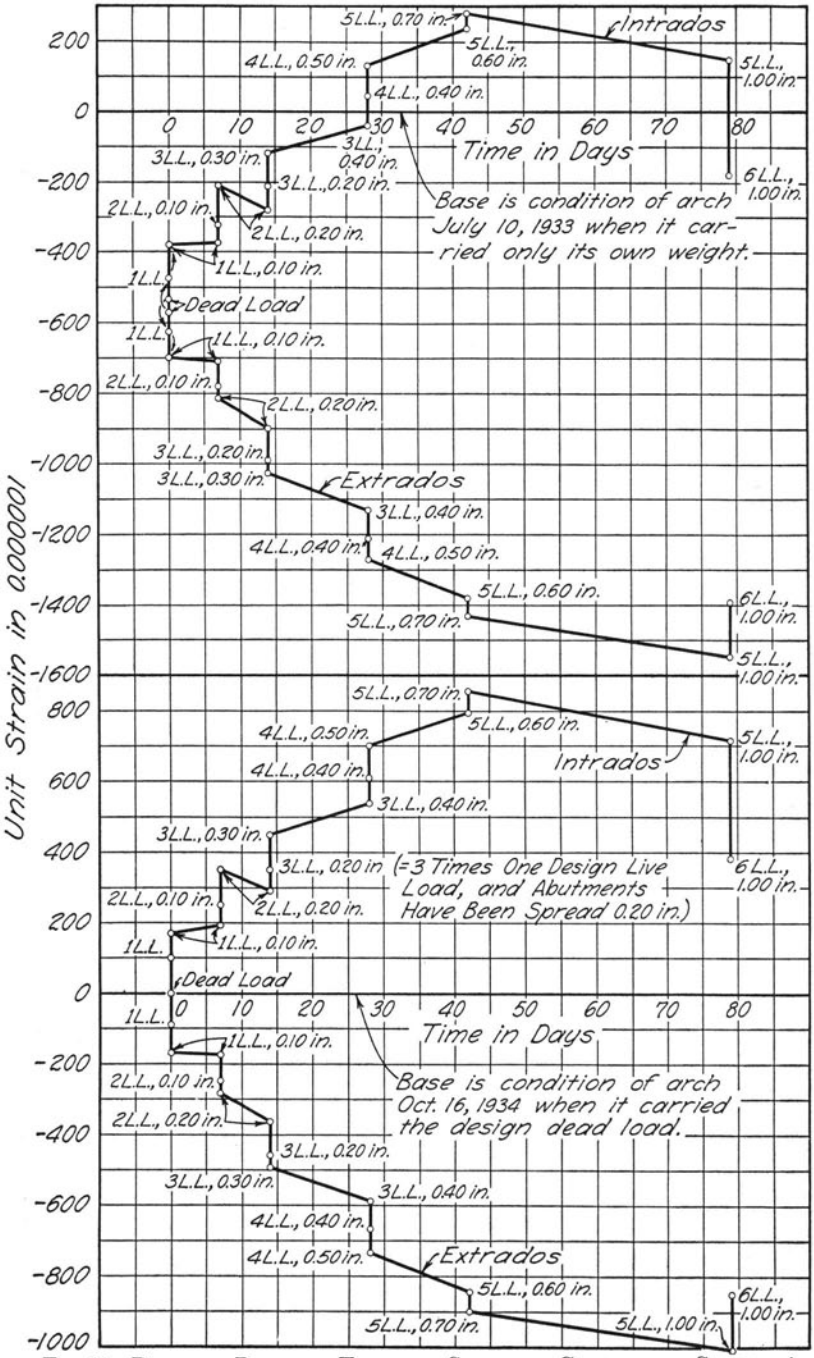

FIG. 13. RELATION BETWEEN TIME AND STRAIN IN CONCRETE AT SECTION 4

ured abutment reactions occurred when the abutments were spread 1 inch and the arch carried five times the live load in addition to the design dead load and the weight of the structure. The maximum unit stress under these conditions, computed from the measured reactions on the basis that concrete takes tension, was 4480 lb. per sq. in.

The vertical movements of the load points on the arch rib are shown in Fig. 12. The various deflections are due to changes in load and changes in span as indicated. The deflection represented by the difference between a full line and the adjacent broken line is due to time yield.

The strain in the concrete was measured on two gage lines at the intrados and two at the extrados on a section midway between each pair of adjacent load points. For most sections there were cracks in the concrete that made the strain gage readings of little value, but there were no cracks across the gage lines at section 4 in the panel just east of load point 4 at any time during the test, and the strains on these lines are reported.

Figure 13 shows the deformation in the concrete at section 4 during the 80-day period covered by the capacity test. The upper part of the figure shows the total deformation relative to conditions July 10, 1933; and the lower part the deformation that occurred during the capacity tests, from Oct. 16, 1934 to Jan. 4, 1935. The deformation at the beginning of the period was − 0.000536 in. per in. at the extrados and − 0.000567 in. per in. at the intrados. This deformation is due to shrinkage, dead load, and time yield. It is interesting to note that, on Oct. 16, when the arch was approximately 16 months old and carried the design dead load in addition to its own weight, the average unit deformation in the concrete at section 4 was 0.000552 and nearly uniform over the section. Assuming that the deformation in the steel was the same as in the adjacent concrete, the compression in the steel due to shrinkage, dead load and time yield was 16 500 lb. per sq. in.

IV. Conclusions

10. *Summary of Conclusions.*—A single test has little statistical significance and has value primarily in that it may indicate the manner in which a structure functions and direct attention to phenomena that affect its behavior. The fact that time yield in concrete quickly eliminates a large part of the stress in the concrete of an arch that would otherwise accompany early volume changes (shrinkage, initial drop in temperature, and time yield), has been anticipated by ration-

al methods.* This test supports the results of analysis, and it would appear safe to conclude that a considerable part of the early volume changes have little effect upon the stresses in the concrete of an arch rib. Although time yield of concrete reduces deformation stress in the concrete of an arch, it increases the stress in the steel, a phenomenon that is recognized in the design of reinforced concrete columns. It is believed that the high compression in the steel is not a serious source of weakness, providing the rods are supported against buckling. If time yield in concrete diminishes with age and the number of applications of a load,† it is possible that temperature stresses in the concrete of arches a few years old may not be greatly relieved by time yield.

The fact that the large gradual change in the span of the arch did not greatly affect its load-carrying capacity has been attributed to the fact that near the ultimate load a large gradual increase in strain is accompanied by a small increase in stress.

*"Plastic Flow in Concrete Arches," by Lorenz G. Straub, Assoc. Mem. A.S.C.E., Trans. A.S.C.E., Vol. 95, 1931, p. 613.

"Plain and Reinforced Concrete Arches," by Charles S. Whitney, Mem. A.S.C.E., Proc. Am. Con. Inst., Vol. 28, 1932, p. 479.

†"Flow of Concrete Under Sustained Compressive Stress," by Raymond E. Davis, Proc. Am. Conc. Inst., Vol. 24, 1928, p. 303.

"Flow of Concrete Under Sustained Compressive Stress," by Raymond E. Davis and Harmer E. Davis, Proc. A.S.T.M., Vol. 30, 1930, p. 707.

"Flow of Concrete Under Action of Sustained Loads," by Raymond E. Davis and Harmer E. Davis, Proc. Am. Conc. Inst., Vol. 27, 1931, p. 837.

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