UNIVERSITY OF ILLINOIS
BULLETIN

Vol. 45 | January 7, 1948 | No. 30

ENGINEERING EXPERIMENT STATION
BULLETIN SERIES No. 374

THE FREE SURFACE AROUND, AND INTERFERENCE BETWEEN, GRAVITY WELLS

BY
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PRICE: FIFTY CENTS
PUBLISHED BY THE UNIVERSITY OF ILLINOIS
URBANA

Published every five days by the University of Illinois. Entered as second-class matter at the post office at Urbana, Illinois, under the Act of August 24, 1912. Office of Publication, 358 Administration Building, Urbana, Illinois. Acceptance for mailing at the special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized July 31, 1918.

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THE FREE SURFACE AROUND, AND INTERFERENCE BETWEEN, GRAVITY WELLS

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PRICE: FIFTY CENTS

3000—1-48—36352

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ABSTRACT

The surface of seepage is a well-known phenomenon in the percolation of water through earth dams; but its existence, and that of the free surface, in water wells and in the ground adjacent thereto have not been widely recognized. The free surface is defined as the surface of water under atmospheric pressure. Its experimental determination is not easy, because of such factors as capillarity, surface tension, and rate of evaporation. The locus of points on the free surface of water surrounding a gravity well during pumping was found, by electrical analogy, in the laboratory and checked by observations on a large-scale sand model.

It is shown that applications of the Dupuit formula to conditions surrounding a gravity well will give results of value for most practical purposes, provided that the ratio of the drawdown in the well to the thickness of the ground-water stream penetrated is small, say less than about 0.2.

If the water table is sloping, so that the area of influence is not circular, or if the well is not in the center of the area of influence, the radius of a circle whose area is equal to the area of influence can be substituted in the Dupuit formula or in the formula for the free surface, without noticeable effect on the results within the limits of accuracy of the observations.

Much of the work reported herein is a substantiation of hypotheses and studies presented by other investigators. For example, this bulletin reports experimental substantiation of Kozeny's statement that the partial penetration of an aquifer by a well has but little effect on the Dupuit formula for the flow under conditions of full penetration, and also substantiation of Muskat's equations for the interference between two wells, for three wells in the pattern of an equilateral triangle, for three wells equally spaced in a straight line, for four wells on the corners of a square, and for four wells on the corners of a square with a fifth well in the center.

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THE FREE SURFACE AROUND, AND INTERFERENCE BETWEEN, GRAVITY WELLS

I. Introduction

1. *The Problem.*—In locating, designing, and operating wells and in predicting their probable future yield, knowledge of the characteristics of the flow of water into wells is valuable.

The present bulletin is confined to one important aspect of this broad subject. Solutions of problems concerning gravity wells are now based upon approximations. Although these approximations are in general satisfactory, yet there are conditions of flow to which they cannot be applied without danger of inexact, and hence misleading or worthless, solutions.

In particular, few applicable formulas make allowance for the phenomena of replenishment. Secondly, no valid equations for defining the free surface of the water in the sand adjacent to a well have been formulated. Furthermore, doubt has existed concerning the validity of various theories as to the interference effects in small groups of wells.

2. *Purpose and Scope of Investigation.* — It has therefore been the purpose of the present investigation to formulate the conditions of flow into gravity wells, with a view to increasing the precision possible in the solution of problems involving the flow of water into such wells. In the course of the investigation, data have been assembled concerning the shape of the free surface and the effect of the interference between wells on the flow of each well. Some of these data, and major facts relating to other aspects of the problem, have resulted from a comprehensive search of the pertinent literature published since 1898. It has been found to be rich in information, and points to the possibilities of practical applications of the results set forth in this bulletin.

3. *Acknowledgments.*—The investigation was conducted as part of the work of the Engineering Experiment Station. It was begun in an attempt to solve problems of the flow of polluted water underground, and was supported mainly from funds devoted to sewage research. As it advanced, the information gathered was applied to the solution of problems involving the hydraulics of gravity-flow wells.

Special acknowledgment is made to Dr. Max Suter for his helpful suggestions in the completion of the manuscript.

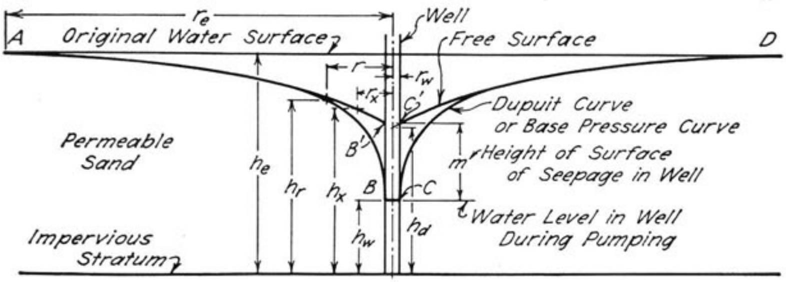


FIG. 1. CONDITIONS OF FLOW INTO A GRAVITY WELL

4. *Definitions.*—The type of well herein considered penetrates a water-bearing stratum in which the upper surface of the water — that is, the water table — is exposed to atmospheric pressure. This type of well is known as a gravity-flow, gravity, or "ordinary" well. Figure 1 shows the conditions of flow into such a well while the water is being pumped; it is assumed that the side of the well is porous and offers no resistance to the entrance of water.

If a well penetrates a water-bearing stratum in which water is confined under pressure between impervious strata, the water level in the well will be at a higher elevation than the top of the water-bearing stratum. This type of well is known as a pressure well or artesian well. Figure 2 shows the conditions of flow into such a well when it is being pumped.

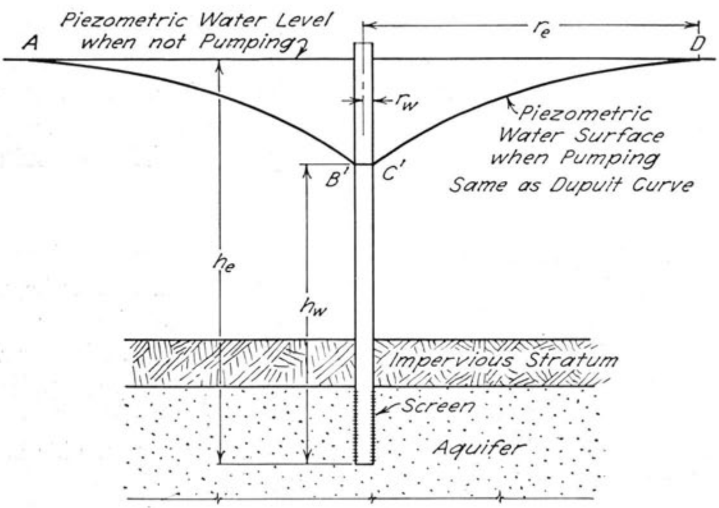


FIG. 2. CONDITIONS OF FLOW INTO A PRESSURE (ARTESIAN) WELL

5. *Underground Conditions Affecting Flow into a Gravity Well.*—Figure 1 illustrates hypothetical underground conditions surrounding a gravity well in which the entire surface of the well exposed to the aquifer will admit water into the well. If, before water is withdrawn from the well, a test bore hole is driven into the ground at any point on the same aquifer as the well, the water in the bore hole and in the well will stand at the original water surface (represented by AD).

After the well has been pumped for some time, water will stand in a series of test bore holes driven in a straight line passing through the center of the well, at elevations along the free surface (represented by $AB'C'D$), provided that the bore holes end at or very near the free surface. Because of capillarity, retention, or other phenomena, moisture will probably be found in the ground above the original water surface and the free surface. The original water surface and the free surface mark only elevations at which water will stand in bore holes. It is to be noted in Fig. 1 that the elevation of the water surface in the well while it is being pumped lies at some distance below the free surface outside the well and is immediately adjacent to it.

Figure 1 also illustrates the surface of seepage (explained in detail on pages 19 and 20). This is the vertical surface, or face, of the ground forming the outside surface of the well hole that is exposed between the surface of the water in the well and the free surface.

Underground flow occurs most commonly in saturated heterogeneous material, which may consist of sand, gravel, clay, or rock in any combination of size, proportion, and position of particles. The composition and arrangement of underground particles, of whatever size, will affect the shape of curves AB' and $C'D$ of Fig. 1, representing the intersection of a vertical plane passing through well and free surface.

Under the conditions of flow illustrated in Fig. 1, measurable characteristics are h_e, h_w, r_w, and Q, where Q is the rate of flow into the well and also the rate of discharge of the pump drawing water from the well. Under many conditions certain characteristics affecting flow cannot practically be measured; these include the porosity and perviousness of the water-bearing stratum, the size of the particles composing it, and the magnitude of r_e.

A condition which may affect underground flow into a well is such close proximity of two or more wells in an underground stratum that the flow from one well affects the flow into another or others. The wells are then said to interfere.

Formulas have been derived to express the relation between the measurable conditions surrounding a well and the rate of flow of water into it; but few if any make allowance for the manner in which the ground water adjacent to the well is replenished.

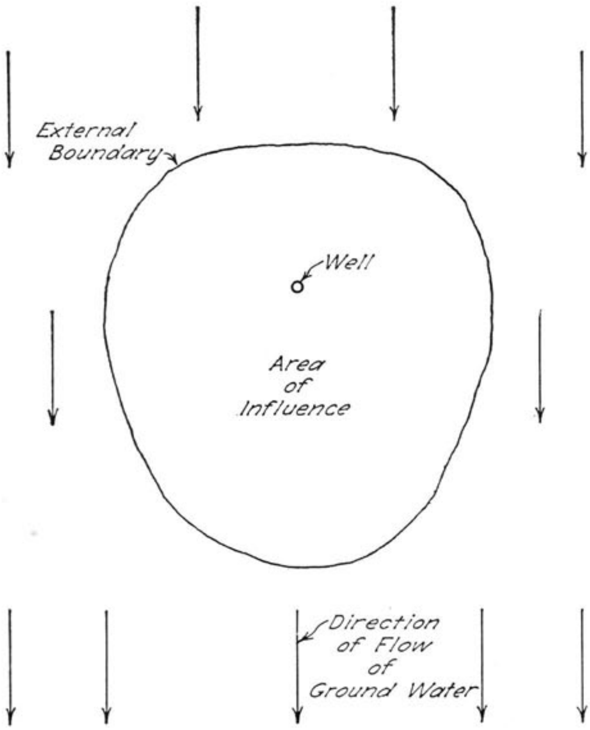


FIG. 3. CONDITIONS OF FLOW INTO A GRAVITY WELL FED FROM ONE SIDE ONLY

6. *Replenishment of Ground Water for Gravity Wells.* — The means of replenishing the ground water tributary to a well affects the shape of the "area of influence" illustrated in Figs. 3 and 4. The area of influence is a plane figure bounded by the intersection of the "free surface" with the original water surface; it is a circle only under unusual conditions. The means of replenishing the ground water tributary to a well affects also the shape of curves AB', $C'D$, AB, and CD of Fig. 1.

Replenishment of water withdrawn from a gravity well may occur entirely from the surface, entirely from one side of the well and within a channel of indefinite width, or entirely from one side of the well and within a channel of restricted width. Or there may be no replenishment, and the well may be surrounded by unlimited saturated strata.

In the first case, when the supply is entirely from the surface, and the rate of flow of water passing vertically through the area of influence is equal to the rate of discharge of the pump drawing water

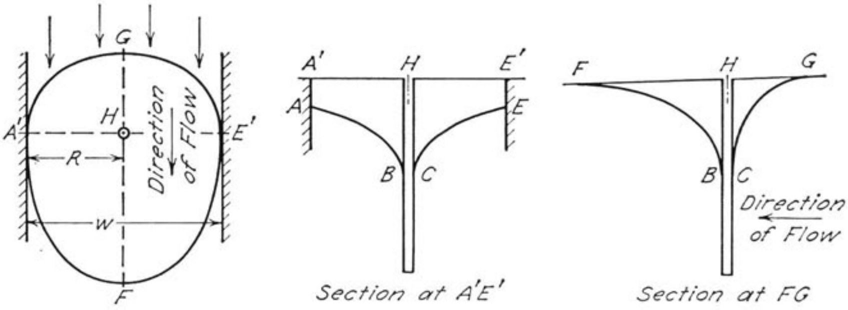


FIG. 4. CONDITIONS OF FLOW INTO A GRAVITY WELL FED FROM ONE SIDE ONLY AND IN AN UNDERGROUND CHANNEL OF LIMITED WIDTH

from the well, the area of influence will have reached a condition of equilibrium and the various radii, or lines drawn from the well to the boundary of the area of influence, will have reached a maximum and constant magnitude. Under such conditions the shape of the area of influence will be a circle only when the ground is composed of homogeneous, porous material and the rate of replenishment is uniform per unit area of the "area of influence." Replenishment from the surface is not always uniform. Causes of variation include infiltration, partly from a river bed or lake, and interruption by a lens of impervious material in the ground.

Conditions of flow when replenishment is entirely from one side of the well and within a channel of indefinite width are illustrated in Fig. 3.

Under the third condition named above, if the limiting width of the water-bearing channel is w, conditions of flow will appear somewhat as indicated by Fig. 4. Values of $A'A$, FH, and HG are dependent upon Q and the characteristics of the underground material.

Under the fourth condition, when there is no replenishment and when there are unlimited saturated strata surrounding the well, the area of influence will increase in size as long as pumping is continued.

Flow into a gravity well may occur under any one or a combination of the conditions set forth above, or the conditions in a well may change from one condition or combination of conditions to another.

7. *Previous Investigations.*—Darcy's analysis(1)*, made in 1856, of the conditions of the flow of water is one of the earliest attempts to solve the problem of hydraulic flow at low velocities. A well-known formula for underground flow has been given his name.

* The parenthesized index-numbers refer to the corresponding items in Appendix A.

Jules Dupuit[2], about 1863, was probably among the first to investigate scientifically the hydraulics of flow into wells. His mathematical development of the formula for flow into wells is widely quoted in textbooks and is used as the basis of most methods for computing such flows.

King[3] and Schlichter[4], both writing in 1896, summarized the work of their predecessors and presented mathematical analyses of flow conditions involving the effect of compaction of void space and the permeability of a porous material.

Muskat[5] published in 1937 "The Flow of Homogeneous Fluids Through Porous Media." This valuable treatise is comprehensive both theoretically and practically. Some of the theories and hypotheses presented by Muskat have been used as a basis in the course of the present investigation.

8. *Procedure.*—In conducting the investigation an effort has been made, wherever possible, to use the results of tests to confirm hypotheses and formulas resulting from the work of other investigators. Such hypotheses have been tested with small models representing hypothetical conditions, and the formulas either validated or disproved. All well models tested have represented wells with screens of length equal to the thickness of the aquifer.

9. *Nomenclature.*—The nomenclature used throughout this bulletin is given below. Any consistent set of units may be used. The combinations of length, time, or mass which apply to a particular quantity are given in parenthesis—(L^2), (M/L^2), etc.

A = Cross-sectional area (L^2).

C = Correction factor taking into account the partial penetration of the gravity well.

C_1, C_2 = Constants necessary to convert pressure to feet of water and to care for system of units used.

c = A constant.

C_x = A constant (see Fig. 12).

d = Effective diameter of sand grain, in mm.

d_o = Drawdown in original well (L).

d_t = Drawdown, total (L).

e = Hazen's effective size of sand, in mm. (L).

f = Specific yield, pure number.

F = Temperature, in deg. F.

g = Acceleration due to gravity (L/T^2).

h = Vertical distance from undisturbed ground-water surface to impervious stratum (L). See Fig. 22.

Δh = Head loss (L).

h_d = Height of free surface of water at the center of the well, above the bottom of the well (L). See Fig. 1.

h_e = Thickness of water-bearing medium at the external boundary, measured vertically between the bottom of the well and the undisturbed ground-water surface (L).

h_r = Head at any point in the water-bearing medium at a distance r from the center of the pumped well (L).

h_x = Depth of water in probe well referred to bottom of test well at a distance r_x from the center of the test well (L).

h_w = Depth of water in the well (L).

k = Permeability of a porous medium (L^3/TL^2).

$\bar{k}$ = Coefficient of permeability (L^2).

L = Length (L).

ΔL = Increment of length (L).

m = Length of surface of seepage (L). See Fig. 1.

M = Mass (M).

p = Well penetration expressed as a fraction of the total depth of the water-bearing sand, h_e/h, a ratio.

P = Pressure drop (M/LT^2).

ΔP = Increment of pressure drop (M/LT^2).

Q = Rate of flow (L^3/T).

R = Effective radius of area of influence (L).

r = Horizontal distance from center of pumped well to any point in water-bearing medium where head is h_r (L).

r_e = Radius of circle of influence (L).

r_w = Radius of pumped well (L).

r_x = Horizontal distance from center of pumped well to center of probe well, or to a point on the free surface whose head is h_x above the bottom of the pumped well (L).

S = Distance of travel of fluid (L).

ΔS = Increment of distance of travel of fluid (L).

v = Velocity of flow (L/T).

v_r = Horizontal velocity in a radial direction (L/T).

w = Any distance (L).

$\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\}$ = Directions of three-dimension coordinate axes (L).

δ = Horizontal distance of well from center of area of influence (L).

μ = Viscosity (M/LT).

ρ = Density (M/L^3).

II. Flow Through Porous Media

10. *Flow of Fluids Through Porous Media.*—Previous investigators have found that factors affecting the flow of fluids through porous media include porosity, permeability, the size of particles of porous material, the viscosity of the fluid and, in small models, the surface tension of the fluid. Since the viscosity of a given fluid is usually known at various temperatures, only the viscosity or the temperature needs to be observed.

Porosity, as used in this investigation, is defined as the ratio of the volume of the voids in the porous medium to the total volume of the medium which contains these voids. Because of the adsorption or adhesion of the fluid to the particles of the porous medium, the entire volume of voids is not available as a conduit through which flow will take place.

Of special interest in this investigation is the determination of the amount of water which will flow through, or penetrate, an aquifer. This may be called the permeability of the medium. It should not be confused with the specific yield (f) of the material, which is the ratio of the amount of fluid that will drain from a saturated material to the total volume of the material; nor with the specific yield of a well, which is the rate of flow for a drawdown of one foot. Among the factors affecting permeability are the sizes and size distribution of the grains, the shape of grains, and the degree of cementation of the material.

In the solution of problems of underground flow, the value of the coefficient of "permeability" must be known. It is expressed literally in Darcy's Law.

11. *Darcy's Law.*—In 1856 Darcy came to the conclusion, after studying the characteristics of the flow of water through sand filters, that the rate of flow, Q, of water through a filter is directly proportional to the area of the filter through which flow is taking place and to the difference between the head or fluid pressure at the inlet and outlet faces of the filter and inversely proportional to the viscosity, μ, and to the thickness, L, of the bed. He expressed the relationship as $Q = \frac{C_1 A \Delta h}{L}$, which can be converted to $Q = C_2 \frac{d^2 \Delta P}{2\mu \Delta S}$.

There is a maximum velocity beyond which Darcy's Law no longer holds. Flow to which the law is applicable is known as stream-line flow; flow at higher velocities, as turbulent flow. The velocity at which conditions change from stream-line to turbulent is sometimes called the critical velocity. Underground flow into wells usually occurs at velocities less than critical and under conditions to which Darcy's Law applies.

12. *Minimum Velocity of Flow.*—Measurements of normal velocities of underground flow through porous media[(6)] show such velocities to be measurable in units of a few feet per day. The velocity of underground flow into wells adjacent to the well screen can be demonstrated to be so slow as to give Reynolds numbers less than unity. It can safely be concluded, therefore, in the words of Muskat[(5)] (page 68): "In the great majority of flow systems, the flow underground will be strictly governed by Darcy's Law, except possibly in the very localized parts of the porous medium of very limited dimensions." If, therefore, Darcy's Law is true, there is no minimum velocity or minimum hydraulic gradient limiting the applicability of Darcy's formula. For a true fluid the law is valid down to the smallest measurable hydraulic gradient. Fishel has shown[(7)] that the velocity of flow varies directly as the hydraulic gradient for a slope as small as 1 in. per mi.

13. *Coefficient of Permeability.*—Because of the difficulty of measuring the average grain diameter and because the velocity of flow according to Darcy's formula varies directly as d^2 for any particular porous material of uniform size, it is convenient to combine the terms $C_2 d^2$ in the expression $Q = C_2 \dfrac{d^2 \Delta P}{2\mu \Delta S}$ into a constant, $\overline{k}$. This value of $\overline{k}$ may be termed the "coefficient of permeability."

Recalling that the constants C_1 and C_2 include the factors necessary to convert water pressure into feet of water as well as to include the system of units used, Darcy's Law for the flow of underground water can be rewritten as

$$V = \frac{\overline{k}\Delta P}{\mu \Delta S} = \frac{\overline{k}\rho g \Delta h}{\mu \Delta S} = k\frac{\Delta h}{\Delta S} = k\frac{dh}{dS}, \tag{1}$$

where dh/dS is the hydraulic gradient. Hazen[(8)] computed the values of k from the expression

$$k = cd^2\left(\frac{\mathrm{F} + 10}{60}\right). \tag{2}$$

The dimensions of k are those of velocity. If expressed as ft. per sec. the constant, c, is 0.038.

III. Location of Free Surface

14. *Dupuit's Formula.*—Hypothetical conditions of flow into a gravity well which penetrates the aquifer have been illustrated in Fig. 1 and discussed in Section 5. Various attempts have been made to formulate the factors affecting the flow into such a well. Probably the best known of such formulas is Dupuit's. In deriving his formula Dupuit assumed that the direction of underground flow toward a well is everywhere horizontal; that the velocity of flow at any point is proportional to the slope of the surface of the ground-water table vertically above that point; that there is a boundary to the sloping surface of the cone of depression (called "the circle of influence") where movement of water toward the well commences; and that the free surface of the water is at a height above the bottom of the well equal to the pressure head at the elevation of the bottom of the well. Despite these simplifying assumptions it has been shown that Dupuit's formula

$$Q = \frac{\pi k}{2.3} \cdot \frac{h_e^2 - h_w^2}{\log r_e/r_w} \tag{3}$$

can be used for almost all practical purposes. This is the equation of lines AB and CD in Fig. 1.

The use of Dupuit's formula for determining the shape of the water surface in sand has been less satisfactory, because the free surface is always found above the theoretical surface as calculated by this formula. Wenzel[(9)] has shown that the value of the coefficient of permeability, as determined by Dupuit's equation, depends on the distance of the probe well from the test well.

Muskat[(5)] proves also, by empirical investigation, the validity of Dupuit's equation for determining flow from a well that is being pumped, but points out that his own investigations failed to show any simple mathematical representation of the shape of the free water surface. He states (page 373): "While the Dupuit-Forchheimer theory explicitly gives an equation for the free surface, its failure to take into account the surface of seepage at the well should alone invalidate its implications with respect to the shape of the free surface."

From a theoretical standpoint the Dupuit equation should fail as h_w approaches zero, since the velocity of flow through the well screen into the well would approach an infinitely large value. It is physically possible for h_w to be zero—that is, it is possible to draw the water in

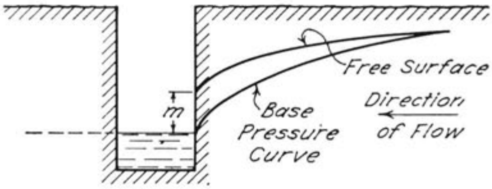


FIG. 5. CONDITIONS OF FLOW INTO ONE SIDE OF A GALLERY

the well down to the elevation of the bottom of the aquifer. The value is sometimes found to be zero in wells where the well penetrates an impervious stratum below the aquifer and the drawdown is below the bottom of the aquifer.

The explanation of this apparent failure is that the free surface enters the well at an elevation above the elevation of the water in the well. The surface of the sides of the well between these two elevations is the surface of seepage (shown in Fig. 1). The value of m is small for a small drawdown of the water in the well and large for a large drawdown.

15. *Surface of Seepage and Free Surface.*—If a horizontal trench is driven transversely across an underground stream, and the water is allowed to drain freely away from one end of the trench, the flow conditions will be somewhat as indicated in Fig. 5. The face of the trench for a height m represents the surface of seepage. The surface of seepage and the free surface generated when water flows into a gravity well are indicated in Fig. 1 as lines BB' and $B'A$, or CC' and $C'D$, respectively.

The surface of seepage is a well-known phenomenon in the percolation of water through earth dams; but its existence, and that of the free surface, in water wells and in the ground adjacent thereto have not been recognized by all investigators of the conditions of flow into gravity wells. In most practical problems the difference between surface of seepage and free surface can be ignored because of its negligible magnitude, particularly when the drawdown in the well is relatively small. Allowance for the magnitude becomes increasingly important, however, in gravity wells, where the ratio of the drawdown to the thickness of the aquifer is large. Hence, a completely satisfactory formula for the flow into a gravity well should make allowance for the surface of seepage.

The experimental determination of the free surface is not easy, because of such factors as capillarity, surface tension, and rate of evaporation. These make difficult the direct observation of the elevation of water surface in sand grains adjacent to a gravity well with fluctuating drawdown. However, it is possible to locate the free water surface by electrical analogy.

16. *Electro-Hydraulic Analogy to Gravity Flow.*—The analogy between the characteristics of flow of homogeneous fluids through porous media and certain other physical phenomena in engineering practice — e.g., heat, magnetism, electrostatics, elasticity, is well known.

A common analogy is that between the flow of water in pipes and the flow of an electric current on wires. Such an analogy can be drawn as follows: the head corresponds to voltage, the hydraulic gradient to the potential gradient, the permeability to the specific conductivity, equi-head lines to equipotential lines, stream lines to lines of flow, and Darcy's Law to Ohm's Law. The closeness of the analogy suggests the possibility of solving problems of underground flow electrically and of constructing an electrical model for the determination of the free surface. Analogous conditions in underground hydraulic flow and in electrical flow are shown in Table 1.

There is, however, no force which acting on flowing electricity will produce a "free surface" as the action of gravity does on flowing water in gravity systems. The free surfaces over which the pressure is uniform develop automatically in fluid systems under the influence of gravity but do not develop in the electrical model, the electrical lines of flow filling the complete model. In the electrical model the "free surface" (*DC* in Fig. 6) is the locus of points, or lines, along which the potentials vary linearly above the base of the system, analogously to the requirement that the head or energy of a particle

TABLE 1
ELECTRO-HYDRAULIC ANALOGY OF FACTORS IN ELECTRICAL TESTS

Factor	Electrical Description	Hydraulic Description
h_e	Vertical depth of model at anode	Depth of water-bearing sand at external boundary
h_w	Length of conductor at cathode of model	Depth of water in well
r_w	Thickness of cathode	Radius of well
r_e	Length of model	Radius of circle of influence

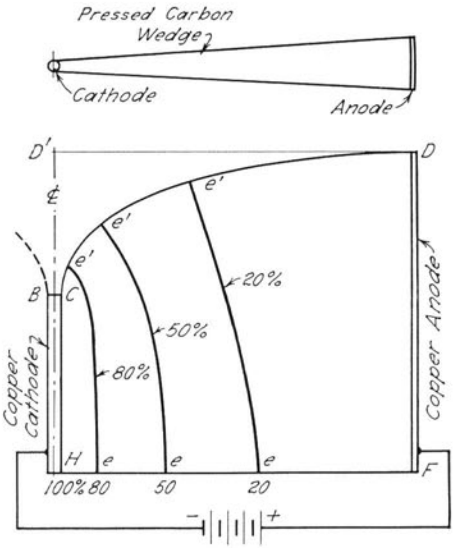


FIG. 6. MODEL SHOWING ELECTRICAL ANALOGY TO HYDRAULIC FLOW INTO A WELL

of water moving along the free surface in the fluid system be proportional to its elevation above the bottom of the porous layer, because the pressure over the free surface is uniform.

Since the electrical lines of flow completely fill the electrical model it is necessary to cut the model so that it will be bounded by a curve of the same shape as the free surface in the fluid system. The cutting can be done only by trial and error, because the shape of the free surface is not known initially.

The successive steps in the trial-and-error adjustment of the free surface for a typical model of a gravity well are shown in Fig. 7. Equipotential lines in the model before cutting are shown in Fig. 7a. The succeeding parts of the figure represent successive stages in the location of the equipotential and flow lines in the model after cutting along the lines determined from the dotted lines in the previous stage. The dotted lines in Fig. 7a – d, and the full line representing the free surface in Fig. 7e, were located by satisfying the requirement that the potential vary from electrode DF, the anode, to electrode CH, the cathode, in Fig. 6 and Fig. 8 as the vertical distance of the point from the base line HE. Usually three approximations were sufficient to find the final location of the free surface.

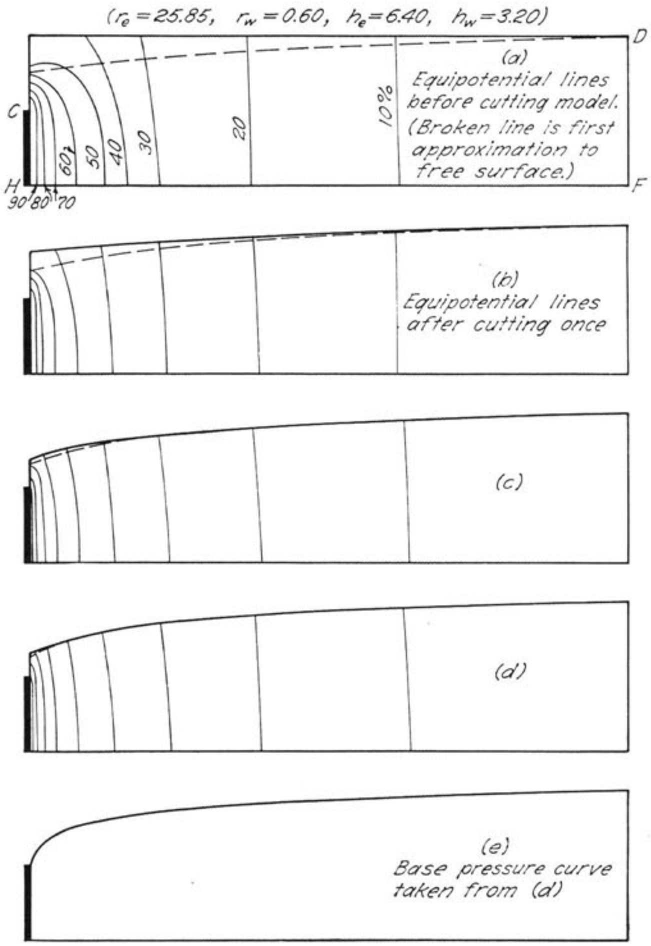


FIG. 7. STAGES IN DEVELOPMENT OF FREE SURFACE IN ELECTRIC MODEL

17. *Apparatus for Electro-Hydraulic Analogy.*—The apparatus used for the electrical tests consisted of a carbon model, a battery for supplying the current for testing, rheostats for adjusting the current, an ammeter, and a potentiometer for measuring the equipotential lines. The electrical connections are shown in Fig. 8.

It was found that a pressed carbon sheet, such as *HKDF* in Fig. 8, when supported on a bakelite back could be machined to the desired dimensions. The carbon sheets used were all 13 in. long and were brought to a precise taper by mounting the sheet, glued to a bakelite

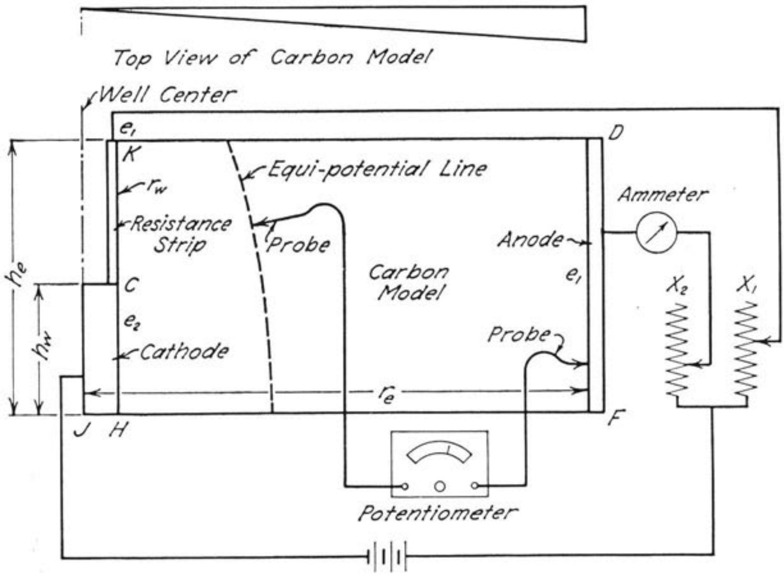


FIG. 8. ELECTRIC CIRCUITS AND CONNECTIONS USED ON MODEL

backing plate, on a previously planed steel block and planing to a thickness of about 0.005 in. at the thin end and of 0.25 in. at the thick end.

A 6-volt storage battery supplied current, and a standard Leeds and Northrup potentiometer was used to measure the potential drop and the equipotential lines.

The distribution of potentials analogous to the liquid system was achieved as follows. The inflow equipotential line is represented in Fig. 8 by the conducting strip *DF*, the anode, maintained at the potential e_1, and the constant potential outflow line (depth of water in the well) by the conducting strip *CH*, the cathode, maintained at potential e_2. The ratio CH/KH equals h_w/h_e, the ratio JH/JF equals r_w/r_e, and CH/KH represents the drawdown in a gravity well expressed as a fraction of the total sand thickness. A resistance strip *CK* is placed along the face of the well above the elevation of the water in the well—that is, above the cathode in the model—the terminals being kept at potentials e_1 and e_2, respectively. The resistance of strip *CK* is smaller than that of sheet *HKDF*. Hence the current flowing in the strip will greatly exceed the current flowing in the sheet, so that the distribution of potential is very nearly linear and thus corresponds to the uniform pressure above *HF*. Equipotential lines were located by steel needles connected to leads to the potentiometer. Such lines, indicating percentage of voltage drop from anode to cathode, are shown in Fig. 6.

18. *Procedure in Electrical Tests.*—Twelve complete tests were carried out with the carbon models. The procedure, which was the same in all tests, was as follows. The model to be tested was connected as shown in Fig. 8. Rheostat X_2 was then adjusted to give sufficient current through the model to produce a voltage drop of about 0.2 volt. The actual voltage drop from electrode *DF* to electrode *CH* was measured accurately with the potentiometer and recorded with the current. The potential at *K* was adjusted to the same value as that of *D* by means of rheostat X_1. The total voltage drop was then arbitrarily divided into ten equal parts, and the equipotential line was located which corresponded to each of the ten increments of voltage. Such lines are illustrated in Figs. 6 and 7. This procedure gave a map of the model, as indicated in Fig. 7a. It is evident that the horizontal line through *E*, in Fig. 7a, cannot be the free surface, since it must pass through point *C*. An approximation to the free surface was found by marking the point on each equipotential line which indicated the same fraction of the total drawdown as the corresponding voltage of the equipotential line was of the total voltage drop. The dotted line in Fig. 7a, joining these points, indicates the first approximation to the free surface. The model was then cut along this line, and the process repeated, as indicated in Fig. 7, until the true free surface was located, as shown in Fig. 7e. The final free surface was located in every case in three approximations at the most.

This procedure was followed for various drawdowns from 25 per cent to 100 per cent, and for various ratios of the depth of water stratum to the diameter of the circle of influence.

Factors considered in the electrical tests of models of underground flow are listed in Table 1.

By fixing values of h_w, r_w, h_e, and r_e, the remaining unknown left to be determined was the location and shape of the free surface. The free surfaces located in tests 1 – 4 and 7 are shown in Fig. 9. In this series of tests the ratio of h_e to r_e was held constant at 0.155.

Values of observations for tests 1 – 12 are entered in Table 2, columns 2 – 6. For comparative purposes it has been found convenient to express the various observations in terms of h_e, making the comparative results independent of the depth of the well. Column 7 shows

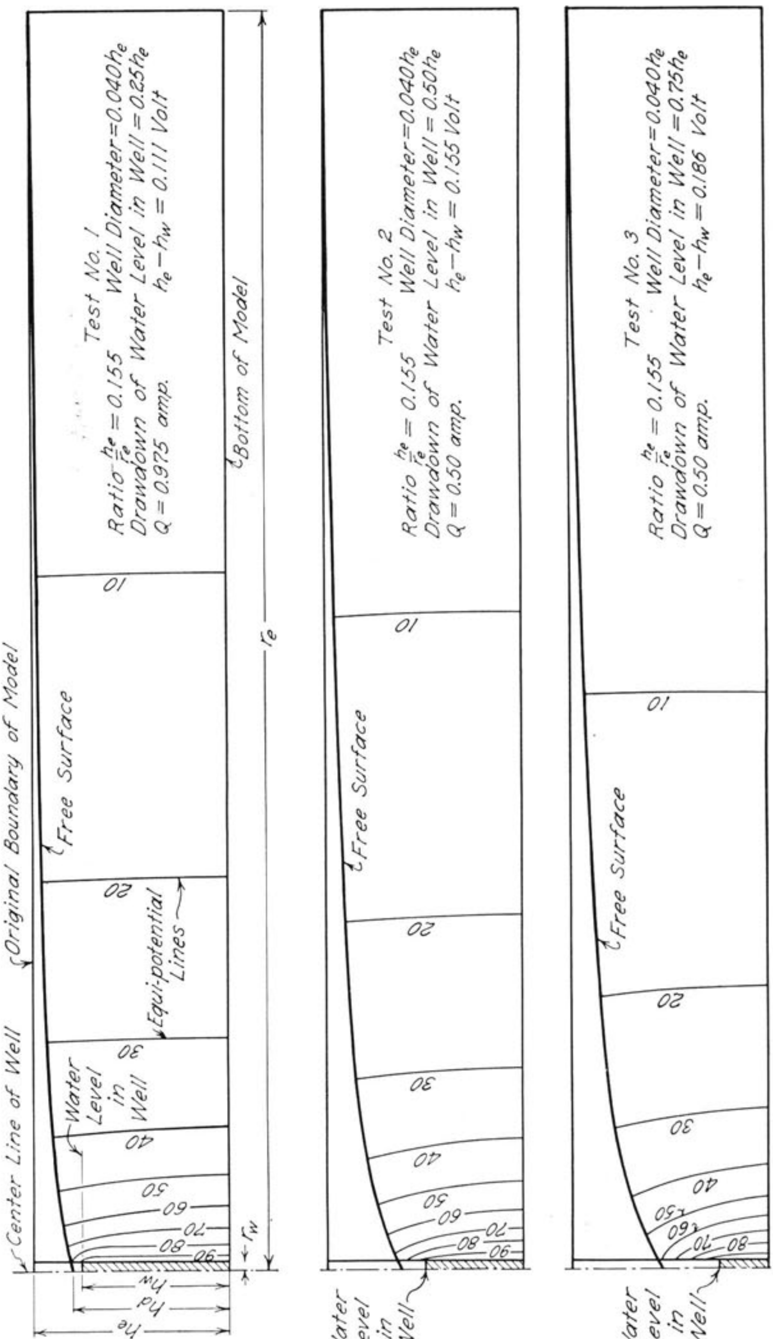


FIG. 9. FREE SURFACES DEVELOPED IN TESTS BY ELECTRICAL MEASUREMENTS — PART I, a, b, c

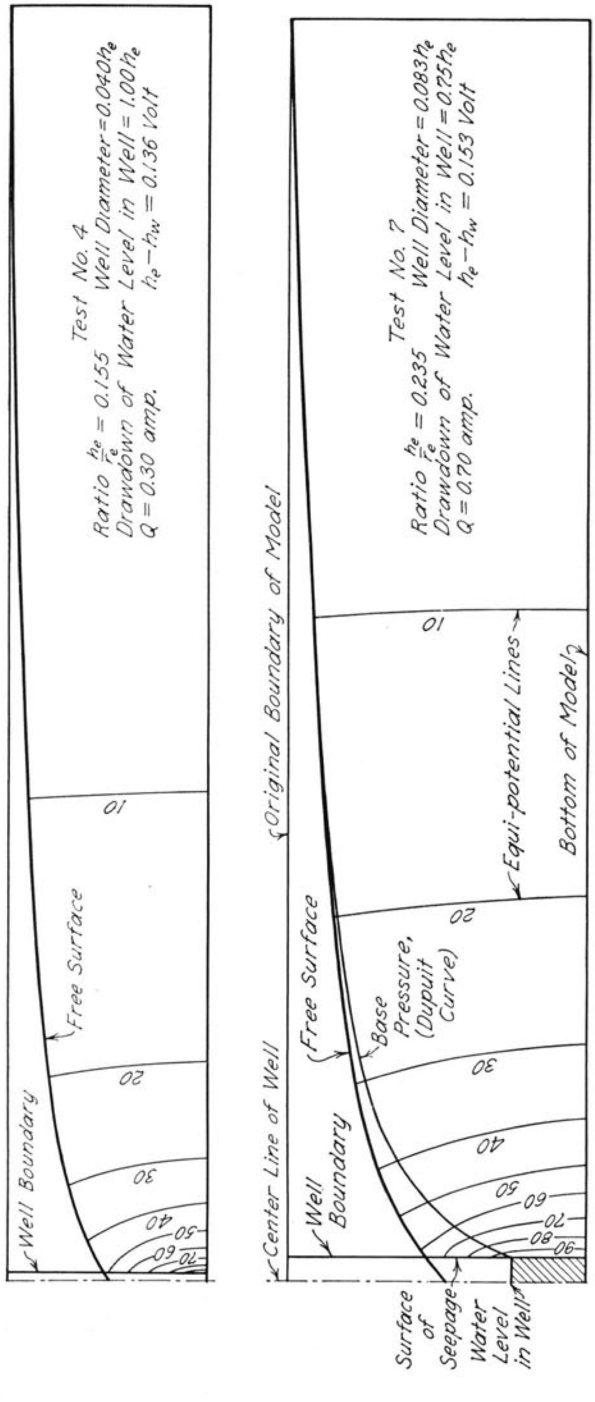


FIG. 9 (CONTINUED). FREE SURFACES DEVELOPED IN TESTS BY ELECTRICAL MEASUREMENTS — PART II, d, e

TABLE 2

OBSERVED AND CALCULATED QUANTITIES OF FLOW THROUGH ELECTRIC MODELS

Test Number	Ratio h_e/r_e	Well Radius, r_w/r_e	Drawdown of Water Level in Well, $\frac{h_e - h_w}{h_e}$	Drawdown of Free Surface Extended to Well Center, h_d/h_e	Observed Drawdown of Water Level in Well for a Well Radius of 1/10 h_e in Terms of h_e	Flow from Well Expressed as Per Cent of Maximum Flow: Observed	Flow from Well Expressed as Per Cent of Maximum Flow: Computed by Equation (3)	Flow from Well Expressed as Per Cent of Maximum Flow: Percentage of Error
1	2	3	4	5	6	7	8	9
1	0.155	0.040	0.25	0.20	0.21	38	38	0
2	0.155	0.040	0.50	0.375	0.43	68	68	0
3	0.155	0.040	0.75	0.46	0.61	85	85	0
4	0.155	0.040	1.00	0.51	0.73	93	93	0
5	0.235	0.083	0.25	0.23	0.24	43	42	−2
6	0.235	0.083	0.50	0.40	0.48	72	73	+1
7	0.235	0.083	0.75	0.52	0.72	95	92	−3
8	0.235	0.083	1.00	0.55	0.90	99	99	0
9	0.32	0.087	0.25	0.26	0.245	44	43	−2
10	0.32	0.087	0.50	0.44	0.49	75	74	−1
11	0.32	0.087	0.75	0.52	0.72	90	92	+2
12	0.32	0.087	1.00	0.56	0.91	99	99	0

the flow of current read on the ammeter, divided by the maximum flow observed. In order to find any possible relationship between rate of flow, Q, and drawdown of free surface the values were plotted in Fig. 10 for three different ratios of h_e/r_e. The points evidently fall on a straight line, establishing the relationship:

$$Q \text{ varies as } (h_e - h_d). \qquad (4)$$

This is an important conclusion, on which the development of a new equation for the discharge of a gravity well is based. The relationship is the same as that found by Dupuit between the discharge from an artesian well and the drawdown in the well.

In order to determine whether the shape of the free surface curve as found in the electrical model tests was the same in all cases, Fig. 11 was constructed. The drawdown at the well center was taken as 100 per cent of the drawdown at other points on the free surface computed in per cent of the drawdown at the well center. The drawdowns thus

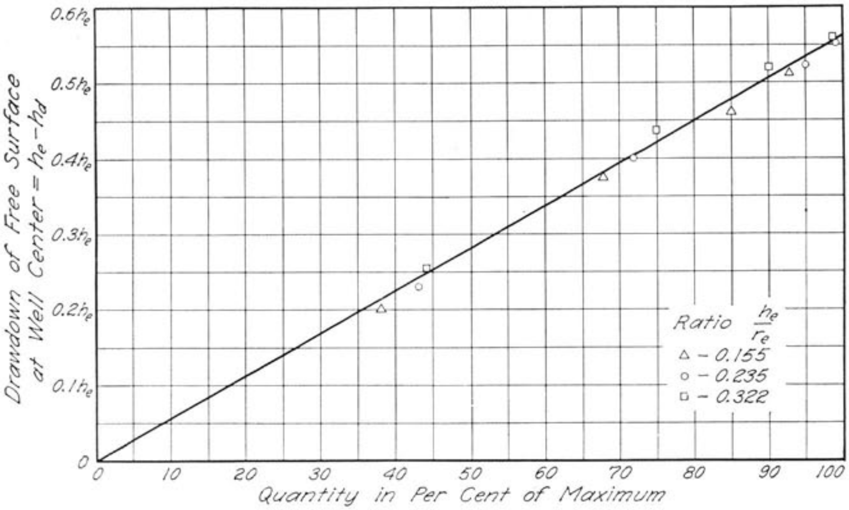


Fig. 10. Relation Between Rate of Flow and Drawdown of Free Surface in Electric Model

determined were plotted as ordinates and the ratios r_x/r_e as abscissas, r_x being the distance from the center of the well to the point in question and r_e the radius of the circle of influence. This procedure gave a figure independent of the physical dimensions of the model—i.e., the drawdown and the ratio h_e/r_e.

Figures 10 and 11 prove, so far as the electrical tests are concerned, that the properties of the free surface curve are independent of the

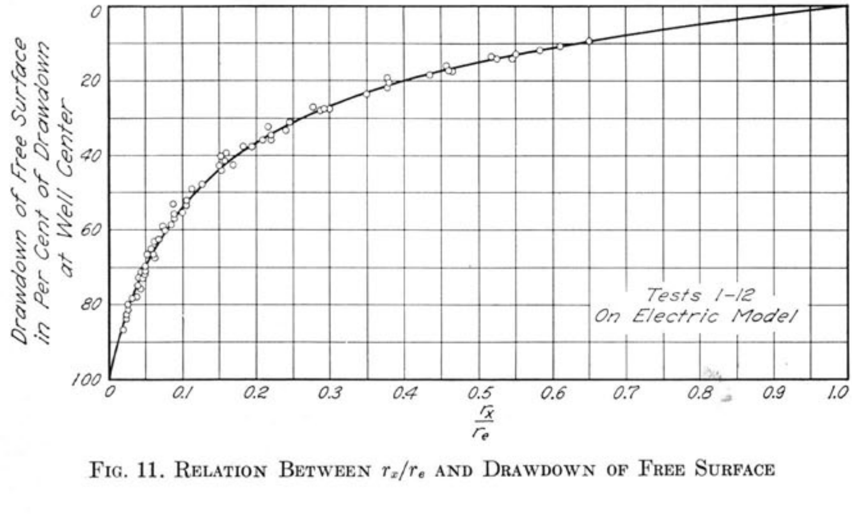


Fig. 11. Relation Between r_x/r_e and Drawdown of Free Surface

physical dimensions of the flow systems and depend only on the discharge from the well. For example, the physical dimensions of a flow system in which $h_e/r_e = 0.1$ are ten times deeper, for the same width or length, as a system in which $h_e/r_e = 0.01$. In both cases, where $h_w = 0.0$ the free surface will intersect the well face somewhere between the bottom of the well and the elevation of the undisturbed ground-water surface at the well. Since the shape of the free surface closely approximates the Dupuit curve at distances greater than h_e away from the well and since the shape of the Dupuit curve does not depend on the dimensions of the model, it follows that the shape of the free surface likewise does not depend on the dimensions of the model in regions at distances greater than h_e from the well center. It is known that the free surface lies above the Dupuit curve. Hence, in the area near the well the shape of the free surface cannot be far different for the different ratios of h_e/r_e. This fact is demonstrated in Fig. 10 for ratios of 0.155 to 0.322.

19. *Comparison of Observed and Calculated Results.*—A comparison of observed and calculated quantities obtained through tests on electrical models is shown in Table 2. When comparing results of tests on wells of different radii it is necessary to reduce the conditions to a common ratio of radius of well to depth of ground-water stream. Observations of depth of drawdown have been made in tests $1-12$ for a ratio of $r_w/h_e = 0.1$; they have been recorded in column 6 of Table 2. Column 7 shows the observed flow in each test, as measured on the ammeter and expressed as per cent of maximum flow. Column 8 shows the flow as computed by Dupuit's formula, Equation (3), using the values in columns 2, 3, and 4 and expressing the results as per cent of maximum.

Dupuit's formula is thus seen to hold remarkably well. It is evident also that Dupuit's formula is valid down to and including a value of $h_w = 0$, at which the maximum flow from the well will be reached; that is,

$$Q_{\max} = \frac{\pi k h_e^2}{2.3 \log r_e/r_w} . \tag{5}$$

Further corroboration of this formula was obtained from the results of sand model tests.

20. *Equation of Free Surface.*—The free surface of the water in the sand while the well is discharging at its maximum capacity can be

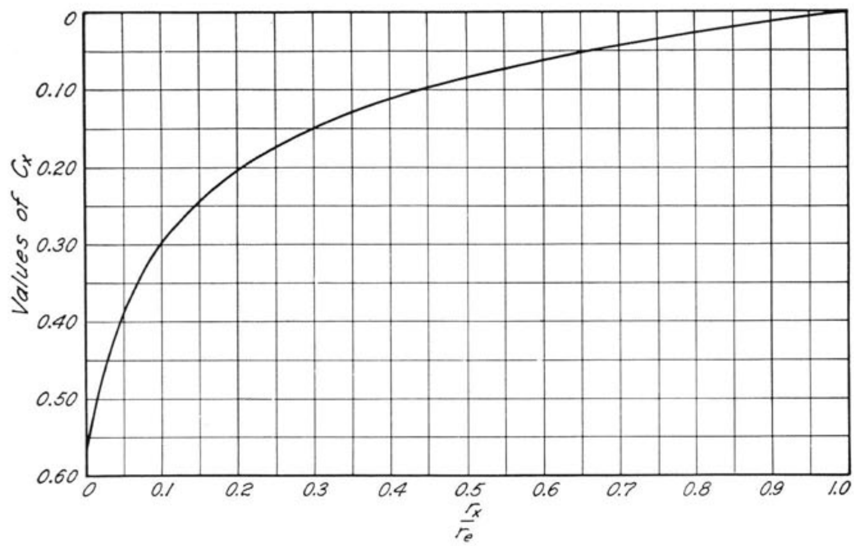


FIG. 12. RELATION BETWEEN r_x/r_e AND C_x IN EQUATION (7)

determined from Figs. 10 and 11. Figure 12 is a plotting of the free surface for a well of radius equal to $0.1h_e$ when discharging at its maximum rate—that is, according to Equation (5). The ordinate represents the decimal fraction of the total possible drawdown; it is called C_x. The subscript ($_x$) refers to any point within the area of influence whose distance from the center of the well is r_x. From Equation (4) the position of the free surface can be expressed in terms of another discharge, regardless of the dimensions of the well. For any discharge less than the maximum the drawdown to the free surface, $h_e - h_x$, will vary directly with the discharge, Q, as proved by Equation (4) and Fig. 9e. Hence

$$\frac{h_e - h_x}{C_x h_e} = \frac{Q}{\dfrac{\pi k h_e^2}{2.3 \log r_e/0.1h_e}}, \tag{6}$$

and, solving,

$$Q = \frac{\pi k h_e (h_e - h_x)}{2.3\ C_x \log r_e/0.1h_e}. \tag{7}$$

Equation (7) gives the discharge of any gravity well when expressed in terms of the height of the free surface above the bottom of the well. Values of C_x can be obtained from Fig. 12.

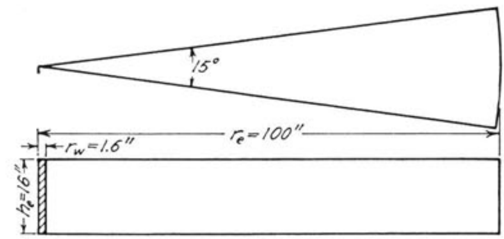


FIG. 13. SAND MODEL OF SECTOR OF AN AQUIFER

The value of 0.1h was chosen arbitrarily and was made of such magnitude that the drawdown in the model well would never cause the drawdown in the hypothetical well to be greater than h_e. The value chosen was found to give satisfactory results.

The equation of the free surface could be expressed directly in terms of r_x and h_x by expressing C_x in terms of r_x through Fig. 12. It was deemed best not to do so because of the empirical nature of that figure, whereas Equation (7) has some theoretical basis.

21. *Apparatus for Sand-Model Tests.*—To test Equation (7), a sand model of a sector of an aquifer tributary to a well was constructed as shown in Figs. 13 and 14. A 15-deg. sector was chosen, with an external-boundary radius of 100 in. measured from the point, or well center. The 16-in. depth of the sector-shaped tank was rigidly and carefully constructed to reduce tolerance in dimensions below $\frac{1}{64}$ in., the maximum error amounting to 2 per cent at the well boundary.

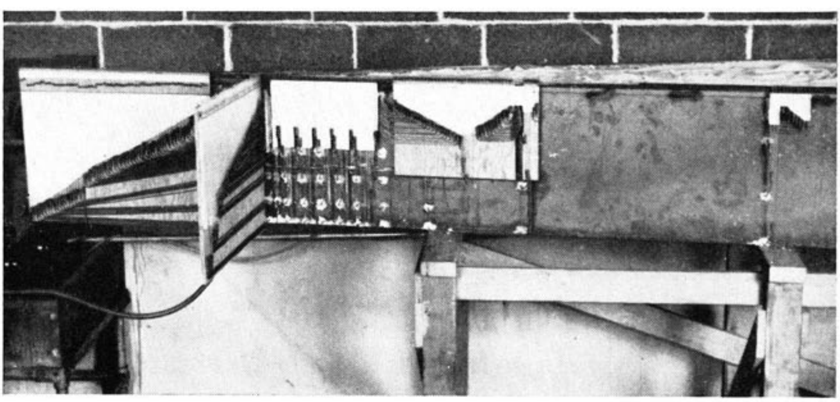


FIG. 14. CONNECTIONS OF PIEZOMETERS USED ON SAND MODEL

Sand used for filling the model was Ottawa standard silica sand of 0.42 mm. effective size and a uniformity coefficient of 1.40. The particles of sand were nearly spherical; hence the tendency for the horizontal permeability to be greater than the vertical was appreciably reduced. In order to exclude air bubbles, which greatly affect the permeability, the sand was placed and stirred in the tank while the tank was full of water.

In locating piezometers along the side of the tank the object was to permit later construction of the lines of equal head from the measurements taken. Four horizontal rows of piezometers were used. The lowest row was placed at the bottom of the tank, as shown in Figs. 14 and 15. The other three rows of piezometers were placed $2\frac{1}{2}$, $5\frac{1}{2}$, and $8\frac{1}{2}$ in. above the bottom of the tank. A fifth row, somewhat shorter than the rest, was placed $11\frac{1}{2}$ in. above the bottom. Since it was desired also to locate the free surface, five vertical columns of piezometers were used. In each column the piezometers were placed $\frac{3}{16}$ in. apart. In the location of the free surface the piezometer must be exactly at the free surface; only under this condition will the gage read the true elevation of the free surface. The reason is evident from an examination of any of the figures representing the results of the sand model tests, such as Fig. 16. It is seen in Fig. 16c that the heads as read in the row of piezometers along AA are less than the corresponding heads for the row of piezometers next above, BB, and those, in turn, less than the heads along row CC. It follows that the end of the piezometer must be on the free surface in order that the observed head may represent the free surface elevation. Since the elevation of the free surface was not known in advance and since, moreover, it was intended to conduct tests at various drawdowns in or discharges from the well, the vertical rows had to be constructed of closely spaced piezometers in order to assure that at least one of the piezometers would terminate at the free surface.

22. *Control of Water.*—During a test, water was added at the periphery of the model in such a manner as to maintain a constant depth at that point. The rate of flow of water from the model well was kept constant during a test by adjustment of the elevation of the end of a rubber hose connected to the bottom of the well. The model well consisted of a wire screen fine enough to exclude sand, forming a cylinder 1.6 in. in radius. The ratio of the radius of the well to the thickness of the aquifer was, therefore, $1.6/16 = 0.1$.

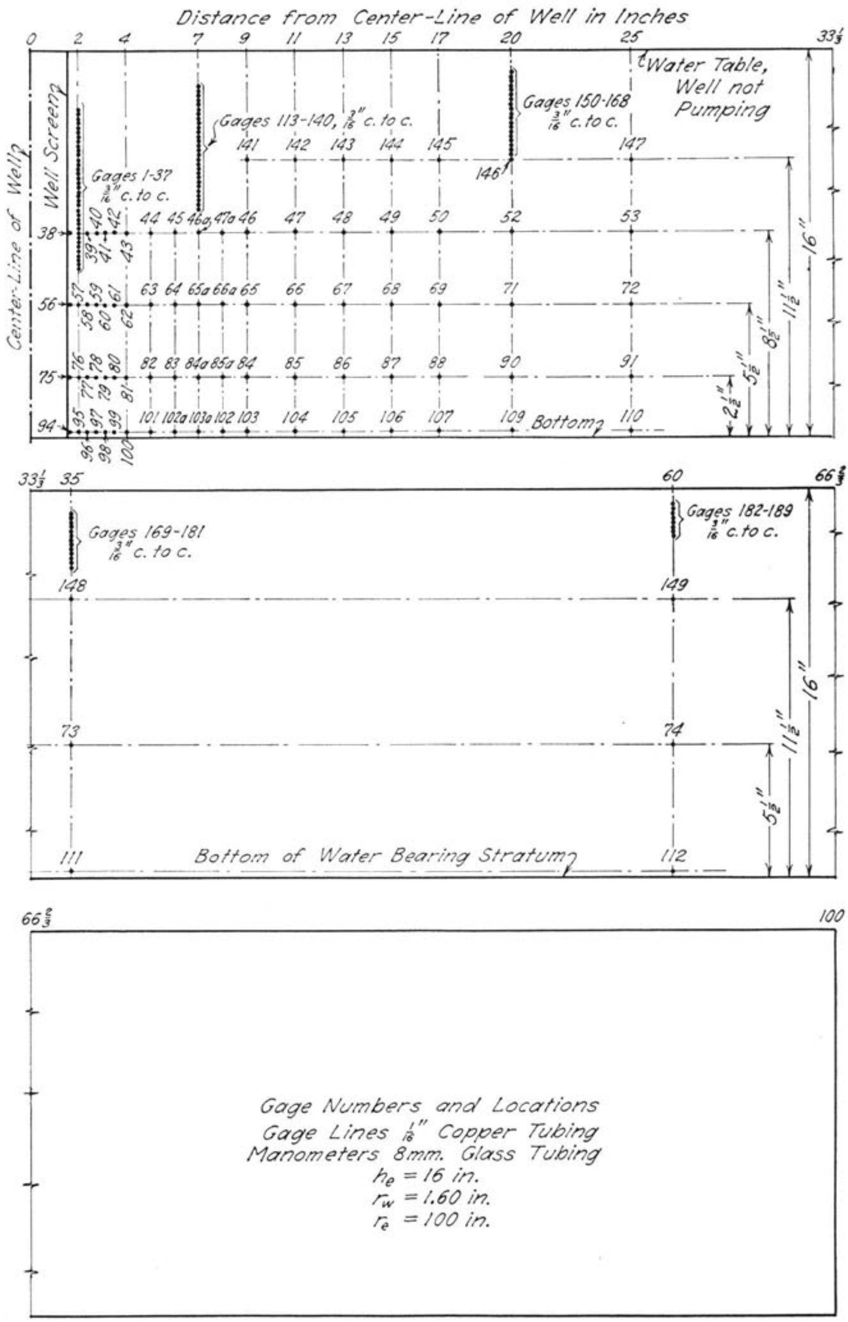


FIG. 15. CONNECTIONS (SHOWN DIAGRAMMATICALLY) OF PIEZOMETERS USED ON SAND MODEL

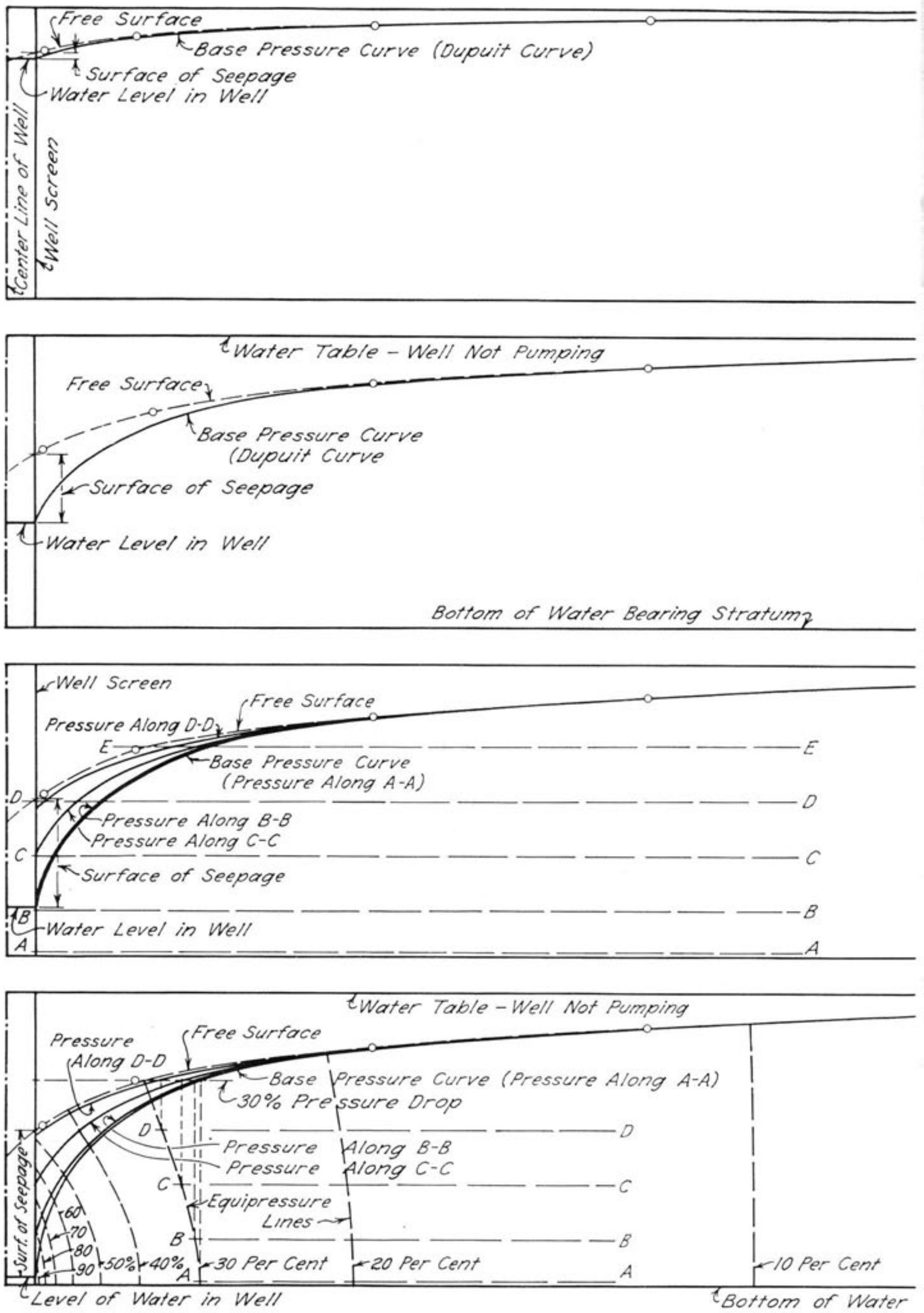
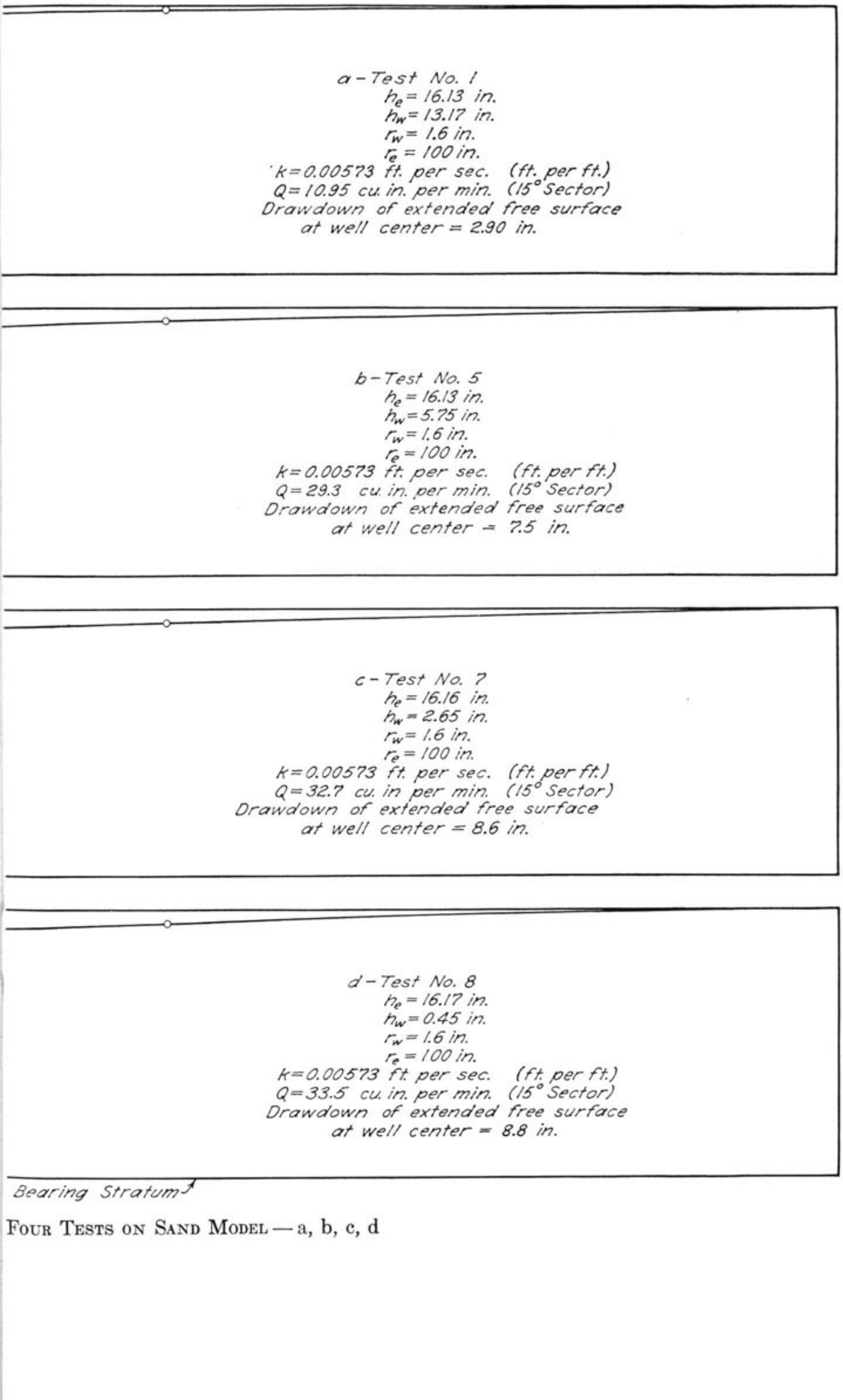


FIG. 16. FREE SURFACE DEVELOPED IN



FOUR TESTS ON SAND MODEL — a, b, c, d

23. *Observations of the Sand Model.*—A total of eight complete tests was made with the sand model shown in Fig. 14. After the model was filled with sand and water to exclude trapped air, the tests were run at various drawdowns, starting with the smallest and ending with the largest. After the water had been drawn below a point in the sand no tests were run where the water level was again above this point. Such a procedure was followed in order to avoid so far as possible the effects of entrapping air in the sand and thereby changing the permeability.

Typical observations are shown in Table 3. The numbers of the piezometers refer to their location as shown in Fig. 15. Plots of observation in tests 1, 5, 7, and 8 are shown in Fig. 16. Locations of the free surface, of the surface determined by the Dupuit curve, of

TABLE 3
DATA FROM SAND-MODEL TESTS

Test Number	Quantity Discharged from Well, ml.	Average Time, sec.	Rate of flow, cu. in. per min.	Temperature, deg. F.
1	1 000	334	263	72
2	1 000	261	336	72
3	1 000	180	488	72
4	1 000	134	610	72
5	1 000	125	704	72
6	1 000	118	745	72
7	1 000	112	785	72
8	1 000	109	805	72

Gage Number	Gage Readings (Italicized readings refer to observations on free surface)							
	1	2	3	4	5	6	7	8
1	*13.63*							
5	13.52	*12.96*						
12	13.42	12.61	*11.67*					
18	13.40	12.53	11.20	*10.60*				
22	13.36	12.51	10.73	10.08	*9.75*			
25	13.36	12.49	10.57	9.65	9.40	*9.38*		
27	13.36	12.48	10.55	9.40	9.10	9.05	*8.85*	
28	13.38	12.50	10.55	9.30	8.85	8.50	8.62	*8.65*
40	13.62	12.83	11.18	9.98	9.35	9.17	8.98	8.92
50	15.00	14.68	14.04	13.52	13.10	12.95	12.76	12.65
60	13.68	12.92	11.24	9.80	8.50	8.15	7.78	7.60
71	15.08	14.80	14.22	13.74	13.36	13.22	13.05	12.95
80	13.77	13.02	11.40	9.92	8.50	7.77	6.95	6.55
90	15.09	14.80	14.22	13.73	13.35	13.20	13.02	12.93
100	13.88	13.17	11.60	10.21	8.85	8.17	7.30	6.80
110	15.24	15.00	14.50	14.08	13.77	13.66	13.51	13.42
120	14.40	12.94	.....	.....	.....	.....	.....	.....
130	14.35	13.89	12.99	12.20	11.76	11.57	*11.32*	.....
140	14.35	13.85	12.90	12.00	11.55	11.38	11.10	10.97
150	15.11	*14.83*	.....	.....	.....	.....	.....	.....
160	15.09	14.80	14.24	13.77	13.41	13.27	*13.10*	.....
170	15.41	15.25	*14.90*	.....	.....	.....	.....	.....
180	15.42	15.24	14.87	15.56	14.33	14.26	14.15	14.09
190	13.17	12.20	9.98	7.93	5.75	4.45	.....	100%

the surface of seepage, and of lines of equal head or pressure are shown when space permits.

24. *Substantiation of Dupuit Equation.*—To obtain an average permeability coefficient of the sand in the model, Equation (3) was written as

$$k = \frac{Q}{\dfrac{\pi\,(h_e^2 - h_w^2)}{2.3 \log r_e/r_w}}. \tag{8}$$

The quantity Q was plotted as abscissas, and $\dfrac{\pi\,(h_e^2 - h_w^2)}{2.3 \log r_e/r_w}$ as ordinates in Fig. 17, and the value of k was determined from the reciprocal of the slope of the line as 0.00562 ft. per sec. This average value

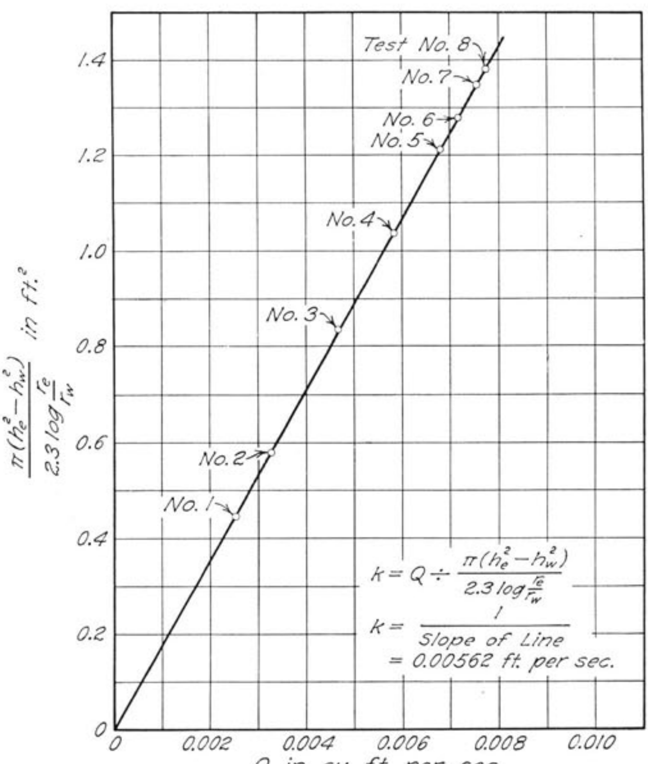


FIG. 17. DIAGRAM USED TO DETERMINE PERMEABILITY IN SAND MODEL

TABLE 4

COMPARISON OF OBSERVED AND CALCULATED VALUES FOR DISCHARGE FROM SAND MODEL

Calculated values from $Q = \frac{\pi k(h_e^2 - h_w^2)}{2.3 \log r_e/r_w}$. $R = 100$ in. $r_w = 1.6$ in. $k = 0.00564$.

Test Number	Undisturbed Depth of Water in Sand, in. h_e	Depth of Water in Well When Pumping, in. h_w	Discharge, cu. in. per min. Calculated	Observed	Percentage of Error
1	16.13	13.17	265	263	+0.8
2	16.13	12.20	342	336	+1.8
3	16.13	9.98	491	488	+0.6
4	16.13	7.92	607	610	−0.5
5	16.13	5.75	703	704	−0.1
6	16.15	4.45	743	745	−0.3
7	16.16	2.65	776	785	−1.3
8	16.17	0.45	806	805	+0.1

was used to compute the discharge from the well by means of the Dupuit equation. Agreement between the observed and calculated flows, which are compared in Table 4, is apparent and further substantiates the use of the Dupuit equation to determine flow from a gravity well.

25. *Relation of Free Surface and Rate of Discharge.*—From the results of various tests as plotted in figures similar to Fig. 16, it is possible to observe the drawdown of the free surface corresponding to a particular rate of discharge. Corresponding observations are plotted in Figs. 18 and 19. The straight line in Fig. 19 shows that the drawdown of the free surface varies directly as the rate of discharge. This fact confirms an important conclusion reached from studies of the electrical model.

26. *Location of Free Surface.*—In order to determine the validity of Equation (7), points were taken at random from the experimentally observed free surfaces in tests 1 – 8, plotted as shown in Fig. 16, and substituted in the equation. The drawdown of the free surface $(h_e - h_x)$ has been plotted in Fig. 20 against corresponding values of C_x taken from Fig. 12. The straight lines joining corresponding points in Fig. 20 validate Equation (7). Flow from the well was computed by substituting in Equation (7) the corresponding coordinates of observed points on the free surface in plots of observations similar to Fig. 16.

A comparison between observed and computed flows is shown in Table 5 (see page 41). The close agreement justifies the conclusion that Equation (7) correctly gives the shape of the free surface

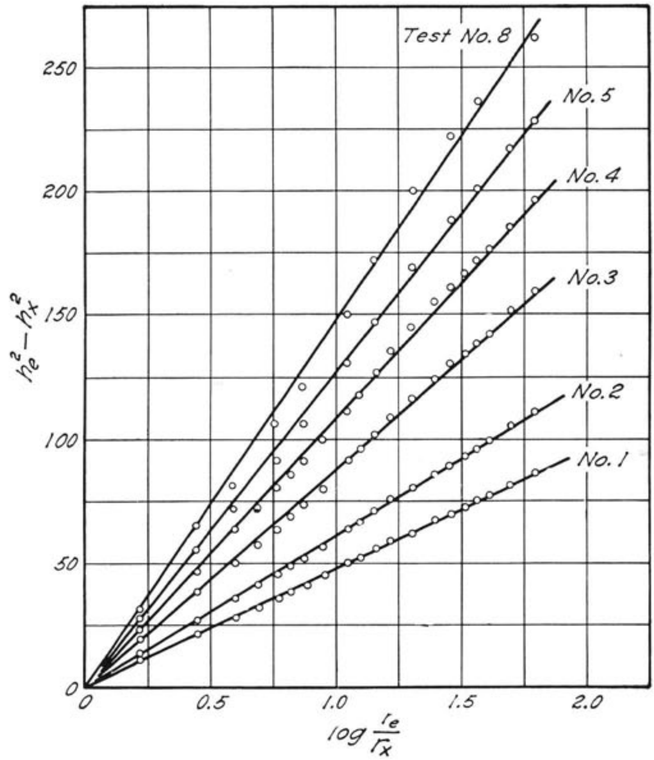


FIG. 18. RELATION BETWEEN r_e/r_x AND $h_e^2 - h_x^2$

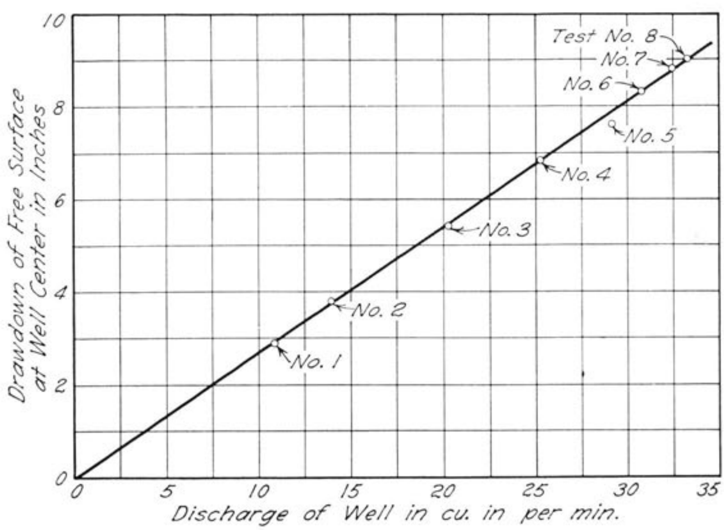


FIG. 19. RELATION BETWEEN RATE OF FLOW AND DRAWDOWN OF FREE SURFACE IN SAND MODEL

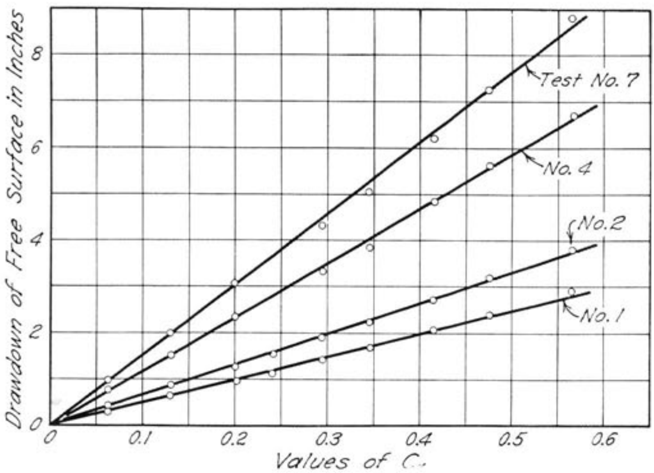


FIG. 20. RELATION BETWEEN C_x AND DRAWDOWN OF FREE SURFACE IN SAND MODEL

of flow into a gravity well, and shows correlation between observations made on the sand model and the equation developed from the electrical model. If observed values of $h_e - h_x$ at a point 2 in. from the well center, as shown in Table 5, are substituted in the Dupuit equation, the computed rate of flow and the percentage of error will be as shown at the bottom of the table.

TABLE 5

COMPARISON OF OBSERVED AND CALCULATED DISCHARGES FROM SAND MODEL OF GRAVITY WELL

Equation of Discharge $Q = \frac{\pi k h_e (h_e - h_x)}{2.3 C_x \log R/0.01 h_e}$.

Test Number	h_e, in.	$h_e - h_x$, in. (h_e − gage no. 182)	Discharge, cu. in. per min. Observed	Discharge, cu. in. per min. Calculated	Percentage of Error
Point x taken 60 in. from well center; $C_x = 0.063$, taken from Fig. 12; permeability = 0.00564 ft. per sec.					
1	16.13	0.33	265	262	−1.1
2	16.13	0.43	342	342	0
3	16.13	0.59	488	463	−4.1
4	16.13	0.75	610	595	−2.5
5	16.13	0.87	704	690	−2.0
6	16.15	0.92	745	730	−2.1
7	16.16	0.99	785	785	0
8	16.17	1.02	805	809	+0.5
Point x taken 35 in. from well center; $C_x = 0.130$ (h_e−gage no. 174)					
1	16.13	0.72	265	277	+4.5
2	16.13	0.88	342	338	−1.2
3	16.13	1.23	488	473	−3.0
4	16.13	1.53	610	588	−3.6
5	16.13	1.80	704	692	−1.7
6	16.15	1.89	745	727	−2.4
7	16.16	2.01	785	773	−1.5
8	16.17	2.08	805	800	−0.6
Point x taken 2 in. from well center; $C_x = 0.465$					
1	16.13	2.50	265	269	+1.5
2	16.13	3.17	342	342	0
3	16.13	4.53	488	488	0
4	16.13	5.53	610	596	−2.3
5	16.13	6.38	704	688	−2.3
6	16.15	6.80	745	733	−1.6
7	16.16	7.31	785	788	+0.4
8	16.17	7.48	805	806	+0.1
1	.....	....	265	242*	−8.7
2	.....	....	342	301*	−12.0
3	.....	....	488	410*	−16.0
4	.....	....	610	482*	−21.0
5	.....	....	704	539*	−23.4
6	.....	....	745	565*	−24.2
7	.....	....	785	597*	−24.0
8	.....	....	805	606*	−24.7

*Computed by Dupuit equation, using free surface extended to center of well.

IV. Area of Influence

27. *Area of Influence.*—That portion of the surface of a ground-water table the hydraulic gradient of which is affected by the flow into a well is called the area of influence. If conditions of withdrawal and of replenishment are uniform in all horizontal directions about the well the area of influence will be a circle, as illustrated in Fig. 3.

28. *Significance of Area of Influence.*—When a gravity well is pumped, the water drawn from the well is derived from the sand which is unwatered as the free surface is lowered. At the instant of starting the pump the radius of the circle of influence is zero. It increases gradually as the well is pumped. If the well were pumped indefinitely the radius of the circle of influence would eventually reach the boundary of the permeable sand, provided there were no vertical replenishment. When the rate of replenishment, either vertical or horizontal or combined, equals the rate of flow from the well, the area of influence becomes fixed in magnitude. When the area of influence covers the whole area of the water-bearing deposits it will no longer increase, but, provided there is no replenishment, there will be gradual lowering of the static water level due to continued pumping—a phenomenon known as "recession."

It has been observed in the field that after a period of continued pumping, most gravity wells tend to come to an apparent equilibrium; that is, the level of water in the well remains constant at a constant rate of pumping. This fact implies that the area of influence of the well has also become constant and that replenishment is equal to withdrawal. The magnitude of the radius of the area of influence after equilibrium has been established is generally unknown. It is usually assumed, to aid in the calculations of the discharge of the well.

From an examination of Equation (3), showing the discharge of a gravity well, it is seen that the discharge varies directly as the permeability, directly as $(h_e^2 - h_w^2)$, and inversely as the logarithm of r_e/r_w. Thus, other conditions being constant, the discharge of a gravity well varies inversely as the logarithm of the radius of the area of influence. The practical significance of this fact is that relatively large uncertainties in the value of r_e introduce relatively small uncertainties in the prediction of the discharge.

Where the ratio of $(h_e - h_w)$ to h_w is small, the discharge from a gravity well varies approximately as $h_e - h_w$. This relation is used extensively in practice for approximate computations of rate of discharge and drawdown.

29. *Effect of Sloping Water Table.*—In the field the surface of the water table is not horizontal, as has been assumed thus far in the study of the gravity well. Since the surface of the water slopes, there is some question of the correct value of h_e, the "static" water level, to use in the well formulas thus far derived, and of the applicability of such formulas to wells in sloping ground-water tables.

The static water level is the pressure at the external boundary of the well system. Under the assumption that the ground-water table is a plane surface, in the case of a sloping water table the average pressure over the external boundary, regardless of the value of r_e, is equal to the water level in the well when no water is being drawn from it. It should, therefore, be correct to apply Dupuit's equation to the determination of the flow from a gravity well in a sloping ground-water table, provided that the value of h_e is taken as the depth of water in the well when not being pumped, or after full recovery since previous pumpage.

30. *Effect of Non-Circular Area.*—The area of influence surrounding a gravity well in a sloping water table is not circular and, as indicated in Fig. 3, the well is not in the "center" of the area.

By means of Green's reciprocation theorem, Muskat[5] (pages 229 and 246) has proved that the flow from a well regardless of the shape of the region drained may be expressed as

$$Q = \frac{\pi k \, (h_e^2 - h_w^2)}{2.3 \log c/r_w}, \tag{9}$$

where c is a constant depending on the shape of the external boundary.

For a circular boundary of radius R with center at a distance δ from the well,

$$C = \frac{R^2 - \delta^2}{R}. \tag{10}$$

For a rectangular boundary of sides $2a$, $2b$, with the well at the center,

$$C = \frac{\sqrt{ab}}{\pi}. \tag{11}$$

In both cases it is shown that the values of C correspond to what can be called effective average radii of the external boundaries. It is reasonable to generalize and state that the discharge from a gravity well may be computed by Equation (9), using for C an estimate of an approximate average of the distance from the well to the external boundary.

31. *Well Not in Center of Area of Influence.*—The results set forth in the preceding section apply when the well is not in the center of the area of influence. The effect of the displacement of the well from the center of the area of influence is seen, from Equation (10), to be small even for displacements up to one-half of the value of R. For displacements of $R/2$ the increase in discharge of the well is less than 5 per cent (Fig. 21). The results of tests on the installation shown in Fig. 27 tend to show the validity of Fig. 21. In Fig. 27 a well displaced 17 in. from the center of a region, with R equal to 80 in., showed no measurable increase in capacity over a similar well in the center of the region.

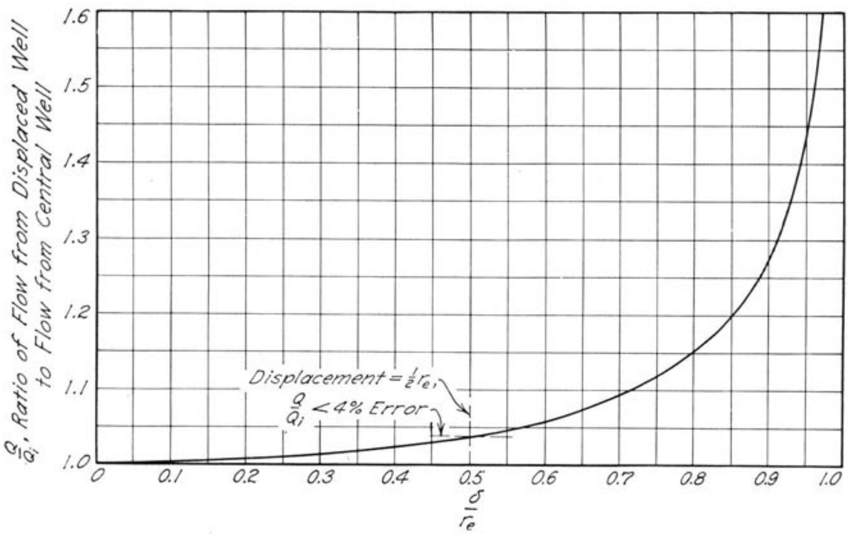


FIG. 21. EFFECT ON PRODUCTION CAPACITY OF WELL DUE TO ITS DISPLACEMENT δ FROM CENTER OF AREA OF INFLUENCE

V. Partial Penetration by a Gravity Well

32. *Equation of Flow into a Gravity Well Penetrating a Fraction of the Thickness of the Aquifer.*—If, as is frequently the case, the well does not penetrate to the bottom of the water-bearing sand, a correction must be applied to the computed well discharge. It is apparent that the discharge of the well will be increased from the contribution of the sand below the well. Figure 22 illustrates the case of a partially penetrating well.

It has been shown by Kozeny[10] that the equation

$$Q = \frac{\pi k\,(h_e^2 - h_w^2)}{2.3 \log r_e/r_w}\left[1 + 7\sqrt{\frac{r_w}{2h_e}}\cos\frac{\pi p}{2}\right] \qquad (12)$$

gives the discharge of such a well.

In using the above equation, the penetration of the well into the water-bearing part of the sand must be known in advance. Sometimes it is known from test borings or the logs of adjacent wells. If the penetration is small, even a rough estimate of the penetration does not introduce much error into the computed value of Q, as seen from Equation (12), since for small values of the fractional penetration p the term $7\sqrt{\frac{r_w}{2h_e}}\cos\frac{\pi p}{2}$ is very nearly equal to $7\sqrt{\frac{r_w}{2h_e}}$, a constant for the particular well.

33. *Relation of Flow from a Partially Penetrating Gravity Well to the Flow for Full Penetration.*—In using Equation (12) it has been found more convenient to express the effect of partial penetration in

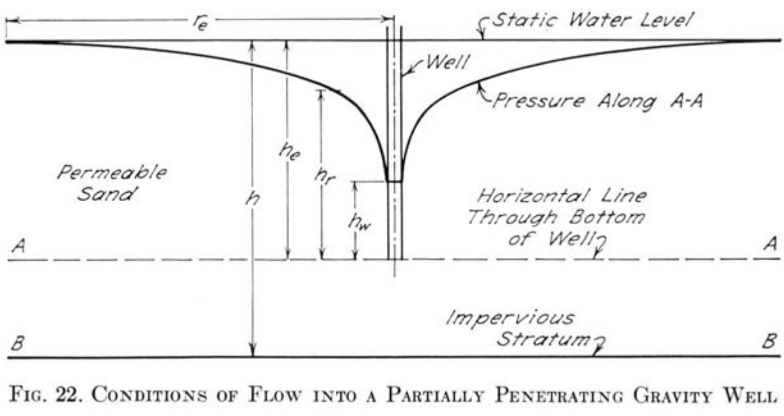


Fig. 22. Conditions of Flow into a Partially Penetrating Gravity Well

the form of a correction factor to be applied to the discharge from a fully penetrating well, thus:

$$Q_o = CQ, \tag{13}$$

where Q is the discharge of a partially penetrating well as given by Equation (12) and Q_o is the discharge of a fully penetrating well.

From Equations (5), (12), and (13),

$$C = \frac{1}{1 + 7\sqrt{\dfrac{r_w}{h_e}}\cos\dfrac{\pi p}{2}}. \tag{14}$$

Values of C computed by Equation (14) for various values of r_w/h_e have been computed and plotted in Fig. 23, with the per cent penetration as abscissas and the value of C as ordinates. The value of C as found from these curves is the proper correction factor to apply to the discharge of a fully penetrating well in order to obtain the discharge of the partially penetrating well.

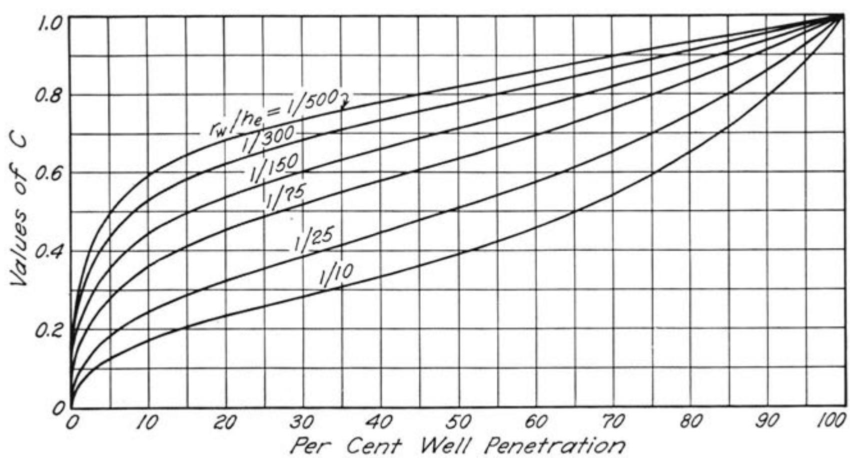


FIG. 23. CORRECTION FACTORS FOR PARTIALLY PENETRATING GRAVITY WELLS

VI. Interference Between Wells

34. *Muskat's Hypothetical Equations for Well Interference.*—By following Muskat's procedure[5], as set forth in his Chapter IX, equations can be developed to show interference between gravity wells in various combinations of positions. For example, for two interfering wells a distance w apart,

$$\bar{h}_w = c + \frac{2.3}{2\pi kh}\left[Q_1 \log r_w + Q_2 \log W\right] \tag{15}$$

$$\bar{h}_w = c + \frac{2.3}{2\pi kh}\left[Q_1 \log r_w + Q_2 \log r_w\right] \tag{16}$$

$$\bar{h}_w = c + \frac{2.3}{2\pi kh}\left[Q_1 \log R + Q_2 \log R\right], \tag{17}$$

in which $\bar{h}_w$ and $\bar{h}_e$ indicate average heads at the well and the external boundary. It can be shown that

$$\bar{h}_w = \frac{h_e^2 - h_w^2}{2h_e} \tag{18}$$

and that

$$\bar{h}_e = h_e. \tag{19}$$

Solving Equations (15), (16), and (17) simultaneously, Q_1 and Q_2 are found to be as follows:

$$Q_1 = Q_2 = \frac{2\pi kh\,(\bar{h}_e - \bar{h}_w)}{2.3 \log \dfrac{R^2}{r_w W}}, \tag{20}$$

in which Q_1 and Q_2 represent the flow into each well. If the values of $\bar{h}_w$ and $\bar{h}_e$ in Equations (18) and (19) are substituted in Equation (20), the final solution is

$$Q_1 = Q_2 = \frac{\pi k\,(h_e^2 - h_w^2)}{2.3 \log \dfrac{R^2}{r_w W}}, \tag{21}$$

where R is the radius of the area boundary, as shown in Fig. 24.

Following a similar procedure it can be shown that:

I. For three wells in the pattern of an equilateral triangle

$$Q_1 = Q_2 = Q_3 = \frac{\pi k\,(h_e^2 - h_w^2)}{2.3 \log \dfrac{R^3}{r_w W^2}}. \tag{22}$$

II. For three wells equally spaced in a straight line

$$Q_1 = Q_3 = \frac{\pi k\ (h_e^2 - h_w^2) \log W/r_w}{2.3\left(2 \log \frac{R}{W} \log \frac{W}{r_w} + \log \frac{W}{2r_w} \log \frac{R}{r_w}\right)}. \quad (23)$$

(outer wells)

$$Q_2 = \frac{\pi k\ (h_e^2 - h_w^2) \log W/2r_w}{2.3\left(2 \log \frac{R}{W} \log \frac{W}{r_w} + \log \frac{W}{2r_w} \log \frac{R}{r_w}\right)}. \quad (24)$$

(middle well)

III. For four wells in a square pattern

$$Q_1 = Q_2 = Q_3 = Q_4 = \frac{\pi k\ (h_e^2 - h_w^2)}{2.3 \log \frac{R}{\sqrt{2}\ r_w W^3}}. \quad (25)$$

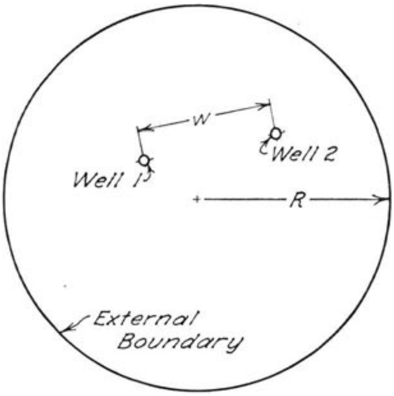


FIG. 24. TWO INTERFERING WELLS WITH RADIUS OF INFLUENCE EQUAL TO R

IV. For four wells in a square pattern with a fifth well in the center

$$Q_1 = Q_2 = Q_3 = Q_4 = \frac{\pi k\ (h_e^2 - h_w^2) \log \frac{W}{\sqrt{2}\ r_w}}{2.3\left(4 \log \sqrt{2}\ \frac{R}{W} \log \frac{W}{\sqrt{2}\ r_w} + \log \frac{R}{r_w} \log \frac{W}{4\sqrt{2}\ r_w}\right)}. \quad (26)$$

(corner wells)

$$Q_5 = \frac{\pi k\,(h_e^2 - h_w^2)\log\dfrac{W}{4\sqrt{2}\,r_w}}{2.3\left(4\log\sqrt{2}\,\dfrac{R}{W}\log\dfrac{W}{\sqrt{2}\,r_w} + \log\dfrac{R}{W}\log\dfrac{W}{4\sqrt{2}\,r_w}\right)}. \quad (27)$$

(center well)

35. *Experimental Verification of Interference Hypotheses.*—To test Equations (21)–(27) a sand model was constructed (Figs. 25 and 26). The sides were completely surrounded by water. The five wells, each 1.10 in. in diameter, were located as shown, approximately in the center of the field, and over short pipe nipples connected by 90-deg. ells to horizontal pipes embedded in the concrete bottom of the tank. Short lengths of rubber tubing were connected to the ends of the five horizontal discharge pipes, whose ends could be raised or lowered to adjust the rate of flow from the wells.

Piezometers for measuring the depth of water in the wells were made of ⅛-in. copper tubing connected by glass tubing to a gage panel at the front of the model.

Ordinary building sand was used. The model was first filled with water. The sand was then added, and stirred thoroughly to allow entrapped air to escape. The depth of water-bearing sand was 13 in.

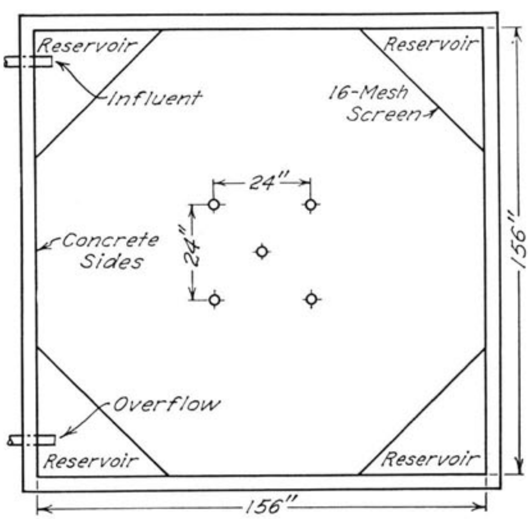


FIG. 25. LARGE-SCALE SAND MODEL USED IN TESTS (DIAGRAM)

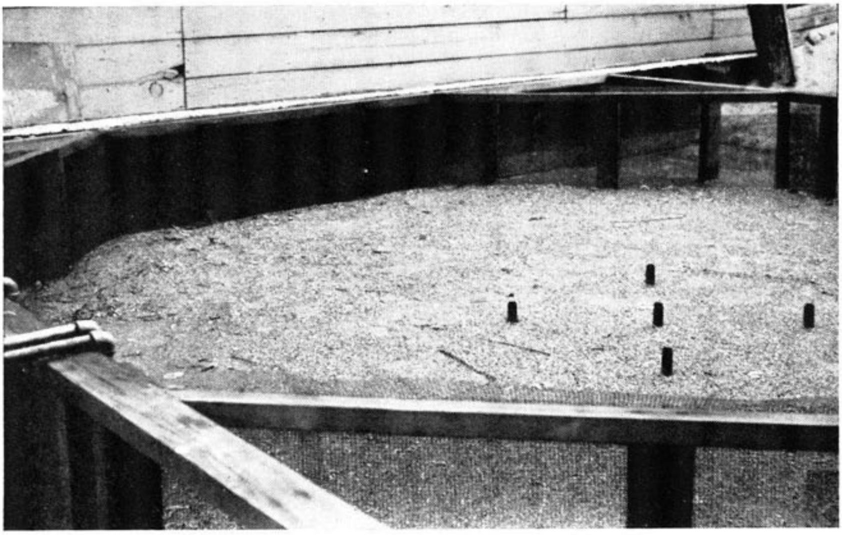


FIG. 26. LARGE-SCALE SAND MODEL USED IN TESTS (PHOTOGRAPH)

The level of water in the sand was kept constant, about $\frac{1}{4}$ in. below the sand surface, by an overflow pipe in one corner of the model while water was added at the opposite corner. Slight variations in reservoir water level were measured by a piezometer on the gage panel.

The arrangement of the wells was such that by operating different wells the investigator could obtain various combinations for interference. Thus, for the interference between two wells, three combinations were possible, each with a different spacing between the wells. The arrangement was suitable also for measuring the interference between three wells in a straight line, four wells in a square pattern, and five wells in a square pattern with one well in the center.

36. *Determination of Well Constants.*—In order to determine the permeability of the sand and the similarity of the five wells, each well was tested separately. The procedure consisted of measuring the rate of discharge of the well while at the same time observing the corresponding drawdown of the water level in the well. Discharge determinations were made by catching a volume of the discharge in a known period of time and measuring the volume. The rate could then be calculated.

The data for the tests of the wells singly are presented in Table 6 and are shown graphically in Fig. 27. It is apparent from the infor-

TABLE 6

DATA FROM TESTS ON SINGLE WELLS

Reservoir Reading, in.	Well Reading, in.	Drawdown, $h_e - h_w$	Quantity Collected, ml.	Time, sec.	Rate ml. per sec.	Rate cu. in. per min.
Well No. 5; Temp. 13 deg. C.; Date 10/10/41 – 10/11/41						
−0.05	1.62	1.67	898	60.2	14.9	54.5
−0.10	2.95	3.05	941	34.9	27.0	98.9
−0.10	4.27	4.37	988	26.7	37.0	136.0
−0.05	5.32	5.37	901	22.6	43.8	160.0
0.04	6.85	6.81	921	17.7	52.0	190.0
0.10	8.03	7.93	756	13.7	55.2	202.0
−0.03	4.97	5.00	1004	25.3	39.7	145.0
−0.03	6.50	6.53	928	18.7	49.6	182.0
0.02	7.98	7.96	996	17.7	56.3	206.0
0.03	9.13	9.10	916	15.5	59.1	216.0
0.00	9.88	9.88	970	15.5	62.5	229.0
Well No. 1; Temp. 12 deg. C.; Date 10/11/41						
−0.12	1.32	1.44	986	77.3	11.6	42.1
−0.12	2.24	2.36	982	47.2	20.8	76.1
−0.13	3.07	3.20	970	34.7	28.0	103.0
−0.12	4.00	4.12	1008	29.0	34.8	128.0
−0.10	5.20	5.30	980	22.8	43.0	158.0
−0.10	6.30	6.40	999	20.1	49.7	182.0
−0.07	7.62	7.69	1030	18.4	56.0	204.0
−0.06	9.67	9.73	997	15.5	64.3	235.0
Well No. 2; Temp. 12 deg. C.; Date 10/11/41						
−0.10	0.88	0.98	986	121.3	8.1	29.6
−0.13	1.65	1.78	971	63.7	15.3	56.0
−0.13	2.84	2.97	989	39.7	24.9	91.0
−0.12	3.78	3.90	1010	31.9	31.6	116.0
−0.10	5.84	5.94	962	21.7	44.3	162.0
−0.10	7.53	7.63	932	18.0	51.8	189.0
−0.05	10.05	10.10	965	15.8	61.0	223.0
−0.05	8.35	8.40	933	17.0	55.0	201.0
Well No. 4; Temp. 12.5 deg. C.; Date 10/11/41						
−0.10	1.75	1.85	972	62.8	15.5	56.7
−0.11	2.85	2.96	988	40.2	24.6	90.0
−0.10	4.22	4.32	983	28.6	34.4	126.0
−0.10	5.40	5.50	962	22.8	42.2	154.0
−0.10	6.86	6.96	870	17.8	48.9	183.0
−0.07	8.40	8.47	930	16.7	55.6	203.0
−0.04	10.22	10.26	970	15.8	61.5	225.0
Well No. 3; Temp. 13 deg. C.; Date 10/11/41						
−0.25	1.55	1.80	990	60.4	16.4	60.0
−0.75	2.52	2.77	975	39.0	25.0	91.5
−0.22	3.70	3.92	1000	29.6	33.8	124.0
−0.20	4.67	4.87	1002	24.4	41.0	150.2
−0.18	5.86	6.04	985	20.8	47.4	173.5
−0.17	5.88	6.05	970	20.8	46.7	171.0
−0.12	7.72	7.84	1030	18.3	56.3	206.0
−0.08	9.08	9.16	950	15.3	67.0	227.0
−0.05	9.42	9.47	920	15.1	61.0	223.5

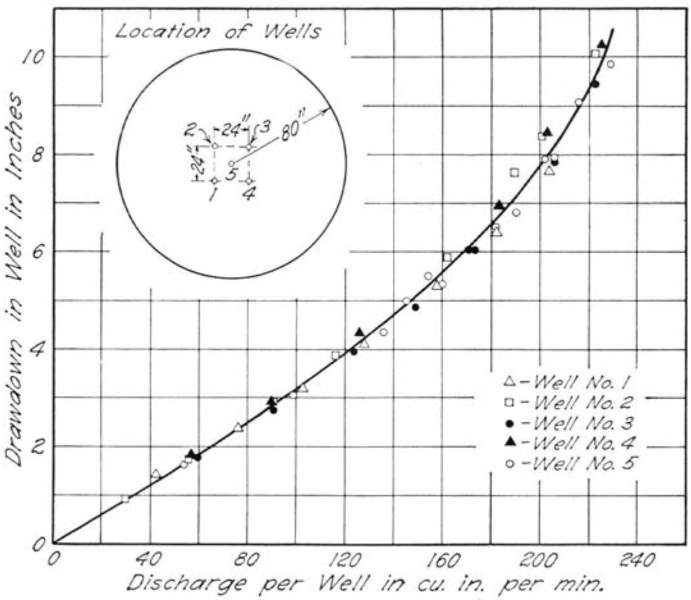


FIG. 27. RELATION BETWEEN DISCHARGE FROM WELLS AND DRAWDOWN OBSERVED IN WELLS

mation given in the figure that all five wells discharge at approximately the same rate for equal drawdowns. The solid line represents the average rate of discharge at any drawdown for a single well. The curve fits equation

$$Q = \frac{\pi k \,(h_e^2 - h_w^2)}{2.3 \log R/r_w}, \tag{28}$$

with

$$k = 0.0023 \text{ ft. per sec.}$$
$$h_e = 13 \text{ in.}$$
$$r_w = 0.55 \text{ in.}$$
$$R = 80 \text{ in.}$$

37. *Comparison of Computed and Observed Results.*—Observations of rates of flow from various combinations of wells in the model are recorded in Tables 7 and 8.

Rates of flow from four different wells have been observed under two different sets of conditions; they are recorded in Table 9. The variation of rates of flow observed from four different wells under ap-

TABLE 7
INTERFERENCE BETWEEN TWO WELLS

Well Numbers	Distance Apart, in.	Drawdown in Both Wells, in.	Rate of Flow, in cu. in. per min.		Variation of Computed Value from Observed Average, per cent
			Observed Ave. of Two Wells	Computed by Muskat's Formulas	
1 & 2	24	1.54	36.5	41.1	+15.3
		2.98	69.8	74.6	+6.9
		5.25	118	118	0
		6.93	141.5	144	+1.8
		8.97	167.5	166.4	−0.8
		11.09	179.5	181	+0.8
1 & 3	33.86	4.65	115.8	115.0	−0.7
		5.90	140.8	137.0	−2.7
		7.16	162.0	156.1	−3.6
		8.68	183.5	175.9	−5.2
		10.05	194.6	185.5	−4.7
		3.88	97.4	99.1	+1.7
		3.16	81.6	83.4	+2.2
		2.21	57.5	60.6	+5.4
		1.43	37.0	40.5	+9.5
1 & 4	24	5.95	131.8	130	−1.4
		9.65	176.0	172	−2.3
		9.85	176.0	174	−1.1
1 & 5	16.93	1.69	41.0	42.4	+3.4
		7.68	151	145.2	−3.8
		9.80	175	164	−6.3
		8.46	162	153	−5.6
		6.75	138	134	−2.9
		5.38	118	115	−2.5
		3.96	90.5	90.2	−0.3
		2.65	62.6	64.0	+2.2
		1.80	45.0	45.0	0
3 & 4	24	3.99	92.0	95.6	+3.9
		5.95	129.2	130	+0.6

parently identical conditions is as great as 12 per cent. In Table 10 are given similar data on five wells. The maximum variation reaches 11 per cent. Because of the uncertainty of the underground conditions in the sand model, due to a great extent to variations in permeability of the sand in various directions, no closer agreement between observed and computed data should be expected.

In general a gratifying agreement is to be noted between the observed flows and the flows computed by substitution in Muskat's formulas. A close agreement between observed and calculated values cannot be expected: the creation of a bed of sand with permeability uniform in all directions is impracticable because of the tendency of sand particles to settle with the long axis horizontal, thus making the horizontal permeability different from the vertical permeability. Air entrained in the sand would offer another cause of error in the measurement. A relatively small error or irregularity in the measurement of the well diameter would introduce a large error in the

TABLE 8

INTERFERENCE BETWEEN THREE, FOUR, AND FIVE WELLS

Well Numbers	Draw-down in All Wells, in.	Rate of Flow, in cu. in. per min. Observed Flow Well No. 1	Observed Flow Well No. 3	Observed Flow Well No. 5	Observed Flow Ave. of 1 & 3	Computed by Muskat's Formulas Well No. 5	Computed by Muskat's Formulas Ave. of 1 & 3	Variation of Computed Value from Observed Average, per cent (Muskat's Formulas) Well No. 5	Variation Ave. of 1 & 3
1, 3 and 5	2.00	41.7	45.0	31.8	43.4	36.8	46.0	+15.7	+6.0
	3.13	65.5	68.8	48.4	67.2	54.9	68.8	+13.4	+2.4
	4.37	85.0	92.6	65.9	88.8	71.2	89.2	+ 8.1	+0.4
	5.74	111.8	114.6	86.4	113.2	89.2	111.8	+ 3.2	−1.2
	7.77	141.2	131.9	112.0	136.6	108.8	136.3	− 2.9	+0.2
	10.03	161.8	160.8	132.9	161.3	123.0	154.2	− 7.5	−4.4
1, 2, 3 and 4					Ave. of All		Ave. of All		Ave. of All
	3.93	....	....	....	67.4	....	71.0	....	+5.3
	5.02	....	....	....	86.0	....	86.1	....	+0.1
	7.01	....	....	....	110.5	....	108.8	....	−1.5
	10.56	....	....	....	140.5	....	133.5	....	−5.0
1, 2, 3, 4 and 5				Well No. 5	Ave. of 1—4	Well No. 5	Ave. of 1—4	Well No. 5	Ave. of 1—4
	3.19	....	....	27.4	50.9	31.3	53.9	+14.3	+6.0
	4.83	....	....	41.2	70.3	44.5	75.2	+ 8.0	+7.1
	6.25	....	....	54.9	86.7	52.9	91.0	− 3.6	+5.0
	8.17	....	....	69.2	108.3	62.5	107.3	− 9.7	−0.9
	10.10	....	....	83.8	119.7	69.0	118.4	−17.7	−1.1

computed rate of flow. All these possibilities, added to possible variations in measurement of rates of flow and of time increments, lead to the conclusion that close agreement between observed and calculated results cannot be expected.

TABLE 9

DATA FROM TESTS ON FOUR WELLS

Well Nos. 1, 2, 3, 4; Temp. 16.5 deg. C.; Date 10/14/41

Reservoir Reading, in.	Well Reading, in.	Draw-down, $h_e - h_w$	Quantity, ml. No. 1	No. 2	No. 3	No. 4	Time, sec. No. 1	No. 2	No. 3	No. 4	Rate, ml. per sec. No. 1	No. 2	No. 3	No. 4	Ave. Rate, cu. in. per min.
−0.10	3.83	3.93	992	990	990	985	54.7	55.6	52.8	51.8	18.2	17.8	18.8	19.0	67.4
0.07	5.09	5.02	995	978	995	980	43.0	43.6	41.9	39.7	23.2	22.4	23.8	24.7	86.0
0.30	7.31	7.01	1000	870	1003	1030	33.2	30.4	33.3	32.1	30.1	28.6	30.1	32.0	110.5
0.17	10.72	10.55	1030	940	970	940	25.6	26.0	26.2	23.3	40.1	36.0	37.0	40.1	140.5

TABLE 10

DATA FROM TESTS ON FIVE WELLS

Well Nos. 1, 2, 3, 4, 5; Date 10/15/42

Reservoir Reading, in.	Well Reading, in.	Drawdown $h_e - h_w$	Quantity, ml. No. 1	No. 2	No. 3	No. 4	No. 5	Time, sec. No. 1	No. 2	No. 3	No. 4	No. 5	Rate, ml. per sec. No. 1	No. 2	No. 3	No. 4	No. 5	Ave. Rate, cu. in. per min. No. 1–4	No. 5
0.07	3.26	3.19	800	825	880	838	510	60.3	60.3	60.1	60.1	68.0	13.3	13.7	14.6	13.9	7.5	50.9	27.4
0.10	4.93	4.83	985	983	980	983	680	52.2	52.5	48.8	51.8	60.5	18.9	18.7	20.1	19.0	11.25	70.3	41.2
0.18	6.43	6.25	975	980	998	990	900	42.0	42.3	40.4	41.9	60.1	23.2	23.2	24.7	23.6	15.0	86.7	54.9
0.00	8.17	8.17	975	880	895	920	823	33.7	31.3	29.5	32.2	43.7	31.2	28.1	30.4	28.6	18.9	106.3	69.2
−0.05	10.05	10.10	905	727	808	900	865	26.9	23.0	25.5	26.2	37.7	33.4	31.5	31.7	34.4	22.9	119.7	83.8

VII. Summary

The significance of the coefficient of permeability has been discussed and various methods for measuring the permeability of a material have been presented.

The existing equations of flow from a single gravity well have also been discussed, and from experiments on electric models and on a sand model it is concluded that Dupuit's equation, Equation (3), can be used in computing the flow from a gravity well for all values of $h_e - h_w$ and that the maximum flow from the well will occur when $h_w = 0$.

It is shown that the Dupuit equation cannot be used to define the free surface of the water in the sand adjacent to the well. As a result of experimental analysis the equation for the free surface, Equation (7), has been developed.

The effect on the discharge from a gravity well of such factors as a sloping water table, non-circular area of influence, location of the well at a site other than in the center of the area of influence, vertical replenishment, and impervious boundaries to the water-bearing sand have been discussed, and methods for the solution of problems involving some of these conditions presented.

The general theory of the partial penetration of a water-bearing stratum by a single gravity well, adapted from Kozeny,[10] and a theory of interference effects in small groups of wells, adapted from Muskat,[5] have been set forth in brief form. Tests of interference effects of small groups of sand-model wells have been compared with the theoretical equations and, as a result, it has been concluded that the theoretical equations are valid.

APPENDIX A

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