Optimal Hedging Strategies for the U.S. Cattle Feeder

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ABSTRACT

Multiproduct optimal hedging is compared to alternative hedging strategies as applied to a Midwestern cattle feeder. One-period feeding margin hedge ratios are estimated using weekly cash and futures price data from a simulation of a custom feedlot for 1983 – 1995. Hedge ratios are estimated using the last 4 years, 6 years, or all prior data available at the moment of estimation; the ratios demonstrate less variability as the length of the underlying sample increases. Hypothesis of all hedge ratios being equal to each other, that leads to the proportional hedging model, is rejected. Means and variances of hedged feeding margins using the computed hedge ratios suggest that there is no consistent domination pattern among the alternative strategies. For the ratios computed based on all prior data available, all strategies are on the efficient frontier, leaving the hedging decision up to the agent's degree of risk aversion. All hedging strategies are shown to significantly reduce the feeding margin's means and variances compared to no hedging, with variance reduction always exceeding 50 percent. Whether a producer chooses multiproduct, single-commodity, or proportional hedge ratios is sensitive to the dataset and its size.

INTRODUCTION

The beef industry represents a major economic activity in the United States economy. Cattle and calves represent the largest segment of American agriculture. In terms of gross cash receipts, the $40 billion of cattle marketings represents almost a quarter of all agricultural marketings and 42 percent of livestock and poultry marketings for 1993. Cattle are produced by more than 1.2 million producers, doing business in a free-market system, in more states and regions of the country than any other commodity. During the last decade, the industry was characterized by wide swings in profits. According to Jones (1995), net returns from finishing yearling steers in Kansas ranged from a loss of $120 to a profit of $178 per head during the period from 1981 to 1994. This high volatility raises an obvious need for risk management.

The high profit volatility is due to several factors, which can be broadly defined as production-specific and market-specific. As with every producer, cattle feeders bear the risk of unfavorable price movements in both the input and output markets. The main inputs in the production process are feeder cattle and feed, usually corn and soybean meal, and the output is fed cattle. In addition, the feeder usually borrows money to cover the cost of the feeder animal and half of the feed, hence the interest paid on the loan can be interpreted as a cost component.

According to different authors, a considerable share of profit variability is explained by market price risks. For example, Trapp and Cleveland (1989) found that volatility in the market prices for both fed and feeder cattle explained 65.5% of the production margin volatility, compared to 22% attributable to production risks.

While profit fluctuations from production factors such as the cost of labor or cattle's response to weather changes are hard to anticipate and prevent, the component of production margin variance attributable to market price fluctuations can potentially be reduced by implementing various hedging strategies. Numerous articles reviewed below show that hedging some or all of the inputs and output provides a considerable advancement in risk management, resulting in improved and less variable production margin.

The previous studies usually used either a naive hedge (one-to-one), or hedge ratios, computed individually for each commodity. Thus far, no one has tried to estimate hedge ratios for the cattle feeding production complex as a whole, taking the covariances among different cash and futures prices into account. While the theory of optimal hedging has been widely discussed in the literature and applied to several production processes, it has not been applied to the cattle feeding industry. It is natural to inquire if it can provide any further advantages compared to other previously studied techniques.

This research focuses on evaluating hedging strategies for U.S. cattle producers, dealing in multiple commodities and futures contracts, in light of the recent developments in multiproduct hedging theory. While both multiproduct and optimal single-commodity hedging have been studied in a large number of publications, only a few papers have combined the two.

The primary objective of this research is to analyze whether the multiproduct hedging theory has any value for cattle producers, and can be practically implemented to reduce risks associated with cattle feeding. The first step is to establish a simulation of a custom feedlot using actual cash and futures prices over the period 1983 - 1995.

The next objective is to estimate hedge ratios from the simulated feeding using the optimal hedging theory, as well as various hedging strategies employed by the industry, including single-commodity hedges and proportional hedges with and without use of past information.

Finally, the research attempts to verify whether the optimal multiproduct hedge gives better results as compared to the simpler strategies used in the industry. Standard mean-variance framework is used to compare one-period optimal hedge against other strategies, including the naive hedge, no hedge, single-commodity hedge, and proportional hedge.

PREVIOUS STUDIES

Numerous studies have been completed on the economic effectiveness of hedging cattle with futures in the United States. Leuthold and Tomek (1979) provided a comprehensive review of the major findings in the early literature. They noted in their survey that studies of livestock hedging typically simulate a feedlot operation, with naive hedging being primarily used. Since then, several articles described below have refined this approach, varying the types of strategies. Only one article was devoted to establishing optimal cash and futures positions for a feedlot operator using a general portfolio framework. Fackler and McNew (1993) expanded and clarified the optimal hedging theory and its implementation for a certain kinds of production processes, however, the theory has never been applied to the cattle feeding industry. Thus, the previous research can be split into two distinct classes: studies of hedging fed cattle and sometimes feed as individual commodities in the multi-commodity framework (i.e., simultaneously), and studies of optimal hedging.

Earlier Studies: Hedging Individual Commodities

Shafer, Griffin, and Johnson (1978), and Leuthold and Mokler (1980) were the first to introduce simultaneous hedging for the cattle feeding industry. The hedge consists of taking long positions in input commodities (feeder animal and feed, usually corn) and a short position in the live cattle, thus locking in a profit margin. Leuthold and Mokler (1980) and Kenyon and Clay (1987) clearly demonstrated that hedging feed, feeders, and fed animals simultaneously is more efficient than no hedge and output-only hedge. The latter authors also demonstrated the importance of margin requirements and interest expenses. Gorman et al. (1982) demonstrated that even when hedging live cattle only, the operator could limit losses by almost 50%. Finally, Caldwell, Copeland, and Hawkins (1982), and Carter and Loyns (1985) showed details peculiar to hedging for a Canadian producer.

Leuthold and Mokler (1980) demonstrate that hedging inputs and output simultaneously enhances profits considerably along with reducing risks. In this study, daily futures prices, starting three months before feeding, were used to determine the profit margin being offered by the futures market for the “three-way” hedge, i.e. long hedge in feeder cattle and corn futures, and short hedge in live cattle futures. If the desired profit margin was found, appropriate hedges were placed in input and output markets. If a desired profit margin was not detected prior to feeding, cash corn and feeders were purchased and the search continued for a profitable feeding margin via a short live cattle hedge. The authors analyzed 234 feeding periods for a simulated Midwest feedlot during 1972-1976. The results showed an advantage of searching for a positive feeding margin before feeding begins in an attempt to establish a profitable feeding margin with a three-way hedge, as opposed to searching for only a single fed cattle hedge after feeding commences. The number of hedges placed depended on the predetermined margin, the highest profits were obtained when the operator waited for $5 per hundredweight margin, with either three-way or single hedges being placed in 74% of the feeding periods. The research also shows that multi-commodity hedges dominate hedging only live cattle for the production margins accepted by the industry, and always dominate no-hedge production in the mean-variance framework.

Kenyon and Clay (1987) conducted an analogous study, using daily data to simulate six years of a high-intensity hog producing unit. The authors analyzed two types of selective hedging strategies: locking in a margin whenever a predetermined fixed margin can be obtained, and using anticipated hog production to forecast expected cash margins and adjust desired hedge margin levels to reflect current conditions. Two main inputs, corn and soybean meal, were hedged using corn futures, and fed animals were hedged using live hog futures contracts. The results exhibited consistent positive feeding margin opportunities available via hedging hogs and corn simultaneously. As in the previous paper, most of the hedges were placed after feeding began and the additional income generated resulted from hedging hogs rather than feed costs. The authors discovered that hedging when the futures market offered a profit margin 60%-70% larger than the 2-quarter ahead predicted cash margin maximized average returns and reduced return variance compared to cash.

Gorman et al. (1982) analyzed data from a commercial feedlot with a one-time capacity of 15,000 head obtained over a period of 6.5 years, starting from 1971. The authors analyzed performance of unhedged positions and 5 different one-way hedging strategies, all of which were using one-to-one hedge ratios and varying the time the hedges were created and lifted. On average, the cash market loss was $24.50 per head, however, a monthly average strategy (trading rule), allowing the hedge to be lifted and placed several times during the feeding period, could have reduced the average loss on 747 pens examined by almost 50%. The losses can be explained by the wide fluctuations of cattle and feed prices during the period of study, besides its capturing only a part of a cattle price cycle. A longer period might produce a different result.

For the Canadian market, research has been published in papers by Caldwell, Copeland, and Hawkins (1982), and Carter and Loyns (1985). The former paper uses a simulator to test eight alternative strategies, hedging either inputs (feeder cattle and barley, which was cross hedged in corn), or the output, or both, depending on the market conditions within the mean-variance framework. While the paper does not take the exchange risk into account, it showed that certain strategies could increase a feedlot operator's income at the expense of its higher instability. The latter paper tests various strategies for hedging live cattle with primary data from western Canadian custom fed cattle over a nine-year period from 1972. It was found that primarily due to basis risk, the feedlot was generally better off without using the Chicago futures market for hedging. Hedging often reduced average returns and increased price risk, demonstrating insufficiency of U.S. instruments for the purposes of Canadian hedgers.

Recent Studies: Optimal Hedging

While the optimal hedging theory for a producer dealing in multiple futures and cash commodity markets was developed in the late 1970s and early 1980s (see, for example, Anderson and Danthine, 1980), it was never applied to the cattle feeding industry. In their 1987 paper, Peterson and Leuthold used a portfolio framework to find optimal cash and futures positions for a commercial feedlot operator. Myers and Thompson (1989) elaborated on a generalized approach to estimating optimal hedge ratios for a single-commodity case. Finally, Fackler and McNew (1993) expanded the approach of the previous article into multiple cash and futures positions, laying the framework for this study.

In their empirical study, Peterson and Leuthold (1987) applied the general portfolio theory to develop an optimal hedging strategy for a commercial cattle feedlot, handling multiple short and long positions in both cash and futures markets. The authors used a discreet nonlinear programming algorithm to generate true discrete optimal hedging solutions for two extreme levels of feedlot operator's risk aversion. Rounded-off continuous solutions were found to provide a very close approximation for the true discrete solutions. Partially-hedged or unhedged positions were optimal because the portfolio approach takes into account the full covariance matrix, as opposed to traditional hedges. Thus, the correlations among all possible combinations of futures and cash positions provided diversification benefits, capable of achieving the preferred degree of risk reduction while diminishing the amount of necessary hedging.

The fundamentals of optimal hedging for an agent dealing in multiple cash goods and futures were outlined by Anderson and Danthine (1980). The authors argued that output decision should be made jointly due to the price uncertainty, therefore, the hedging and production decisions were made simultaneously. The theory states that the optimal position depends on the covariances among the cash and futures prices and the covariances of the futures prices themselves, cash and futures positions being determined simultaneously. Another major result of their study is that the optimal hedge can be decomposed into two parts, namely the risk-minimizing position (pure hedge) and a speculative position.

In their study of generalized optimal hedge ratio for a single commodity, Myers and Thompson (1989) pointed out that the hedging decision should be made using a hedge ratio conditioned on the information available when the hedge is placed, as opposed to the traditional practice of regressing cash price levels on future price levels, leading to unconditional hedge ratio. This observation is especially true for agricultural commodities, and is incorporated into the current research.

Fackler and McNew (1993) clarified this theory to a multi-commodity case and applied it to verify effectiveness of optimal hedging for Central Illinois soybean processors, demonstrating significant advantages of the optimal hedge relative to simpler strategies. The authors estimated the optimal futures positions using the likelihood model for the joint cash and futures price process. Furthermore, they demonstrated that hedging and production decisions could be separated for a production process using inputs and outputs in fixed proportions. The optimal futures positions were determined based on the optimal cash position, using the SUR approach. The authors demonstrated that optimal hedging results in 78% risk reduction as compared to an unhedged position, where risk is measured as a percentage reduction in standard deviations of profit.

These results provide a persuasive reason to apply the above mentioned methodology to the cattle feeding industry. The next section describes the theoretical foundations for the optimal hedging theory, and its application to the case of U.S. cattle feeders.

OPTIMAL HEDGING FRAMEWORK

This section describes the optimal hedging theory, which serves as a theoretical basis for this research. The optimal hedging theory is outlined following the paper by Fackler and McNew (1993).

Most studies of optimal hedging use the mean-variance framework, where agents are maximizing the following objective function:

$$j = E(p) - \frac{l}{2} Var(p) , \qquad (1)$$

where *E()* and *Var()* are the expectation and variance operators, l is a measure of the agent’s degree of risk aversion, and p is a profit function.

U.S. cattle producers have at least five futures markets available to them, including futures contracts on fed and feeder cattle, corn and soybean meal, and T-bill futures. For the simplest case of one holding period, with the hedge placed at the beginning and lifted at the end, the company's profit function can be written as:

$$p = P'Q - C(Q) + (F - f)Z \ , \qquad (2)$$

where Q and Z are m*1 and p*1 vectors representing commodity and futures positions, respectively, P and F are random vectors of period-end cash and futures prices (with negative signs representing purchases for commodities or short positions for futures contracts), f is a vector of futures prices at the beginning of the period, and $C(Q)$ is a cost function.[1]

Using the mean-variance framework, the firm's problem can now be represented as follows:

$$\max_{Q,Z} j = Q'E(P) - C(Q) - \frac{l}{2}\left(Q'\Sigma_{PP}Q + 2Q'\Sigma_{PF}Z + Z'\Sigma_{FF}Z\right) \ . \qquad (3)$$

First-order necessary conditions for this system give the optimal production level if the explicit form of the cost function is given, and also express the optimal futures position Z in terms of the optimal cash position, Q:

$$Z = -\Sigma_{FF}^{-1}\Sigma_{FP}Q \ . \qquad (4)$$

It has become a tradition to express a hedge position in terms of the fraction of the cash commodity offset in the futures market. This implies that each cash commodity is offset with the corresponding futures contract, thus assuming m=p. Therefore, the m*1 vector of hedge ratios H can be written as:

$$H = -\left[diag(Q)\right]^{-1}Z = \left[diag(Q)\right]^{-1}\Sigma_{FF}^{-1}\Sigma_{FP}Q \ . \qquad (5)$$

[1] *C(Q)* represents cash goods production relations. It is interpreted as the cost of producing the cash positions *Q*, valued at the period-end. Assuming the production possibilities to be certain, *C(Q)* becomes non-stochastic.

In the case when all commodities are always held in fixed proportions, as it often is in the cattle feeding industry, the vector *Q* is always proportional to some vector *A*, determined by the technological process. This allows us to re-write the hedge ratios vector as:

$$H = \left[diag(A)\right]^{-1} \Sigma_{FF}^{-1} \Sigma_{FP} A \ . \tag{6}$$

It should be noticed that the theory discussed here assumes that the firm establishes both cash and futures positions at time 0, and has no opportunity to adjust those positions later.

As we can see from equation (6), the optimal hedge ratios vector depends on the term:

$$M = \Sigma_{FF}^{-1} \Sigma_{FP} \ . \tag{7}$$

To estimate *M*, let us consider a general case of multivariate normally distributed random variable *Y*, with the following log-likelihood function:

$$L = k - \frac{n}{2} \ln|\Sigma| - \frac{1}{2} \sum_{t=1}^{n} (Y_t - X_t b) \Sigma^{-1} (Y_t - X_t b)' \ , \tag{8}$$

where *Y* and *X* are row vectors. By splitting Y into two components, Y_1 and Y_2, one can decompose equation (8) into an expression for Y_1 conditional on Y_2 and *X* and expression for Y_2 conditional on X_2 alone:

$$\begin{aligned} L = k &- \frac{n}{2} \ln|\Gamma| \\ &- \frac{1}{2} \sum_{t=1}^{n} (Y_{1t} - X_{1t} b_1 - (Y_{2t} - X_{2t} b_2) M) \Gamma^{-1} (Y_{1t} - X_{1t} b_1 - (Y_{2t} - X_{2t} b_2) M)' \\ &- \frac{n}{2} \ln|\Sigma_{22}| - \frac{1}{2} \sum_{t=1}^{n} (Y_{2t} - X_{2t} b_2) \Sigma_{22}^{-1} (Y_{2t} - X_{2t} b_2)' \ , \end{aligned} \tag{9}$$

where $\Gamma = (\Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21})$ and $M = \Sigma_{22}^{-1} \Sigma_{21}$. Therefore, maximum likelihood estimates of M can be obtained using SUR, with Y_1 being regressed on Y_2 and X_1, and Y_2 being regressed on X_2. The coefficients associated with Y_2 can be used to construct an estimator of *M*.

In our case, P and F represent Y_1 and Y_2, respectively, X_{1t} and X_{2t} are the variables useful for predicting Y_{1t} and Y_{2t}, and M is a $p*m$ matrix of coefficients associated with F. Assuming that the futures prices are unbiased, that is, that $E(F)=f$, the estimation model is written:

$$P_t = \Delta F_t M + X_{1t} b_1 + e_t \quad , \tag{10}$$

where Δ is a difference operator, $\Delta F=F-f$. In the case when cash commodities are always held in fixed proportions ($Q\sim A$), one can use production margin $P_t^C = P_t A$ as a dependent variable in a single-equation estimation:

$$P_t^C = \Delta F_t h + X_{1t} b^* + e_t \quad , \tag{11}$$

where $b^* = b_1 A$. This may lead to the efficiency loss from using OLS rather than SUR, however, if the same set of regressors is used in every equation, SUR becomes equivalent to OLS, and thus no efficiency loss takes place.

METHODOLOGY AND DATA

This section discusses implementation of the optimal hedging theory for the case of a Midwestern feedlot operator. After establishing a simulated feeding scenario for a hypothetical feedlot operator, we discuss the empirical model for optimal hedging, criteria of optimality, and alternatives to the optimal hedging model. Next, we establish and discuss procedures for testing effectiveness of different models, and finally, review the data used for the simulation.

Feeding Scenario

For this research, we study only the final stage of the production cycle, when a yearling is fattened in a feeding program lasting about 4 months (120 days). During this period, a 700-pound steer bought by a feedlot operator consumes 42 bushels of corn and 100 pounds of soybean meal, which serves as a high-protein additive. Assuming the gain of

3.3 pounds a day, the animal's terminal weight is 1,100 pounds, which conforms to the USDA Choice requirements.

In this model, the feedlot operator buys one lot of feeder cattle and puts it on feed every week. One month prior to that, in order to fix the profits, s/he hedges both the inputs (feeder cattle, corn, and soybean meal) and output (fed cattle). Then, hedges are being lifted at the same time as cash transactions in the respective commodities are made. This implies that hedges on inputs are held for a month, until they are actually purchased, and that the live cattle hedge is held for 5 months, until fed cattle are sold. Since this is a one-period model, the futures positions are not adjusted during the life of the hedge. It is also assumed that the operator does not make any transactions with the lot of cattle while on feed, and that cattle are on feed for the full four months regardless of the situation in the market.

For example, on January 1st the operator makes a decision about feeding to begin on February 1st. S/he buys March futures on feeder cattle, corn, and soybean meal, and sells August live (fed) cattle futures. The operator is presumed to hedge in all cases, even if the production margin is negative. Then, at the beginning of February, animals and feed are purchased as respective hedges are lifted, and feeding begins. Finally, four months after feeding commenced, the fed animals are sold and the live cattle hedge is lifted. The same process is then repeated beginning on January 8, 15, 22, etc. The same contracts are used throughout the whole "decision-making" month, which helps avoid potential problems with price distortions from physical deliveries during delivery months, and simplifies data handling. Table 1 provides a detailed schedule of contracts used for hedging depending on the decision-making month.[2]

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[2] Hedging borrowing costs is theoretically appealing, however, hedging the interest expense that the producer incurs in this particular scenario is impractical because of the short hedging period combined with relatively low variability of short-term interest rates.

Table 1. Schedule of Contracts Used for Hedging Depending on the Decision-Making Month

Decision-Making Month	The Contract Month for Hedging			
	Feeder Cattle	Corn	SBM	Fat Cattle
Jan	3	3	3	8
Feb	4	5	5	8
Mar	5	5	5	10
Apr	8	7	7	10
May	8	7	7	12
Jun	8	9	8	12
Jul	9	9	9	2
Aug	10	12	10	2
Sep	11	12	12	4
Oct	1	12	12	4
Nov	1	3	1	6
Dec	3	3	3	6

Code: 1 - January, 2 - February, 3 - March, etc.

Empirical Implementation

For the purposes of this particular research, the second term in the right-hand side of the equation (11), can be expanded to include the lagged values of cash and futures prices in the commodities involved and monthly dummies to account for seasonal effects. This yields the following model:

$$P_t^C = \Delta F_t h + \sum_{j=1}^{q} p_{t-j} a_j + \sum_{j=1}^{q} f_{t-j} b_j + \sum_{j=1}^{12} D_{jt} f_j + e_t \ , \qquad (12)$$

where P^C is the feeder's production margin, $\Delta F_t = F_t - f_{t-1}$ is a vector of gains/losses from futures positions; the subsequent terms represent vectors of lagged values of cash and futures prices, monthly dummy variables, and a random normally distributed error term, respectively.

Alternatives to optimal hedging include either hedging each commodity individually, or hedging all of them in the same proportion. In the former case, the model can be stated as follows:

$$P_t^{(\alpha)} = \Delta F_t^{(\alpha)} h^{(\alpha)} + \sum_{j=1}^{q_\alpha} p_{t-j}^{(\alpha)} a_j^{(\alpha)} + \sum_{j=1}^{q_\alpha} f_{t-j}^{(\alpha)} b_j^{(\alpha)} + \sum_{j=1}^{12} D_{jt} \phi_j^{(\alpha)} + e_t \ , \tag{13}$$

where $P_t^{(a)}$ is cash price of α, with the rest of the terms becoming scalars instead of vectors while keeping their meanings, and α representing either feeder cattle, corn, soybean meal, or live cattle. In this case all equations are estimated independently.

Finally, in the case of hedging all commodities in the same proportion,[3] the model above is used to estimate the hedge ratio for the "commodity" $\alpha = f$ of feeding services, with the terms written for the cash and futures gross production margin. Thus, equation (13) takes the form of single-commodity hedge with the commodity being feeding services. Naive hedge (*h=1*) and no hedge (*h=0*) can be considered as special cases of this model.

Model Selection: Bayesian Information Criterion

For each of the above mentioned models, the optimal number of lags *q* used in the equations (12) and (13) is determined using the Bayesian information criterion, BIC. The standard MBIC procedure (i.e., the procedure of minimizing the value of BIC) is implemented; the respective model equations are over-fitted with lagged values of conditionals for *q*=1, 2, 3, … until the value of BIC is minimized.

$$BIC = n \log \sigma^2 + p \log n \ , \tag{14}$$

where *n* is the number of observations, σ^2 is the error term variance, and *p* is the number of parameters in the model. Another alternative approach is to use a similar procedure to minimize Akaike Information Criterion. However, as noted by Sawa (1978), as *p* becomes large, like in this case, the MBIC procedure is always more parsimonious than the procedure of minimizing AIC (MAIC) for a finite sample, increasingly discriminating against the inclusion of additional variables, while the opposite holds for MAIC.

[3] This model was originally formulated for similar-commodity complexes, such as soybean complex or oil complex. It is introduced here for its simplicity and in order to provide an additional benchmark for comparison.

Testing Hedging Effectiveness

To verify whether multiproduct hedging has advantages over other simpler approaches, it is compared to commonly used hedging techniques, using the mean-variance framework. Unlike Fackler and McNew, a strategy is considered superior to the other if, in addition to variance reduction in the hedged production margin, the strategy also provides a higher conditional mean for the hedged margin (that is, the standard mean-variance domination is employed).

Specifically, hedge ratios from alternative models, computed for every week are used to hedge the next week's feeding margin. Based on hedged margins series, one can compute the means and variances necessary for the comparison in the mean-variance framework. The ratios are computed using either all prior data available, or using only last 4 or 6 years of data.

Almost all alternative strategies used for comparison fall into a category of holding positions in each commodity proportional to one another, that is, hedge ratios being the same for each commodity. Fully hedged and unhedged positions are just special cases of this assumption. Another alternative strategy, single-commodity hedge, is to hedge each commodity with a corresponding contract, hedge ratios being independent of one another. The hedge ratios for these models are either known, or estimated according to equation (13).

Finally, following Fackler and McNew, we consider another measure of hedging effectiveness, variance reduction of the hedged position compared to unhedged. This popular measure is computed using the same margins as above, as follows:

$$a = 1 - \frac{Var(hedged)}{Var(unhedged)} \ . \qquad (15)$$

While this measure provides a convenience of a measurable risk reduction, using it as the only grounds for comparison presents only a limited picture, since it does not take the corresponding reduction in the hedged margin's means into consideration.

Data

Daily cash and futures prices for feeder cattle, corn, soybean meal, and live (fed) cattle were provided by the Office for Futures and Options Research database. To avoid capturing structural change in the cattle market, the sample covers the period from January 1, 1983 to December 31, 1994. Due to peculiar features of cash live cattle markets, the maximum trading volume is observed on Wednesdays; therefore, Wednesday data are used in the sample.[4] Thus, the sample contains 626 observations simulating putting cattle on feed between February 1, 1983, and January 31, 1995.

All prices are multiplied by the amounts of the respective commodities necessary to produce one animal, thus expressing the price data on the per head basis. For example, the live cattle price, expressed in $/cwt., is multiplied by 11.0 to yield the price of a fed animal. With prices expressed in these units, vector A becomes (-1, -1, -1, 1), facilitating direct hedge ratio computation. For each 626 feeding observation the producer's gross feeding margin is then computed as the difference between the price of fed animal and the cost of inputs (i.e., cost of feeder animal, corn, and soybean meal), with fixed production costs being ignored.

Corn and soybean meal prices in Central Illinois are widely available and used as the national benchmarks for both cash and futures prices. We assume that the feedlot has direct access to the both and can perform transactions with the prices available. Cash and futures prices in Oklahoma City for both feeder and fed cattle are widely available and acceptable, hence used in this study. Assuming fixed transportation costs for the period of this study, the spatial data distribution can be ignored by incorporating the transportation costs into the cost function *C(Q)*, thus enabling usage of Central Illinois grain and Oklahoma City cattle prices for the Midwestern operator.

Cash prices for a feeder, corn, soybean meal, and fed cattle, expressed in dollars per head,

[4] For a scenario planned in December 1994, cattle were put on feed in January 1995, and were sold in May 1995, with the June 1995 live cattle futures contract used for hedging. Therefore, the actual data sample extends through the first half of 1995.

are presented in Figure 1. The figure clearly shows that the primary cost component for a producer is the cost of the feeder animal, which is on average 5.8 times more than the cost of the next most important input, corn (see below). Finally, the cost of soybean meal additive is less than 2% of the cost of feeder.

All input prices exhibit high variability, with the feeder cattle price varying between $408 and $737 with an average of $578 per head, and the price of corn varying between $51 and $153 per head with an average of $101 per head. Both prices demonstrate seasonal variability, which is more profound in the case of corn.

The most noteworthy is the behavior of the output price, the price of fed cattle. While exhibiting the same general trends as the feeder cattle price, the fed cattle price is clearly more volatile, and has a clear seasonal pattern compared to the feeder cattle price. During the period of this study, the price ranges between $547 and $962 per head with the average of $758 per head.

The producer's gross feeding margin is shown on Figure 2. This is perhaps the most convincing argument why hedging should be important for cattle feeders. Exhibiting highly cyclical behavior, the margin varies from -$133 to $250 per head with the average of $70 per head. While the graph serves its purpose of demonstrating high variability of the gross feeding margin, the absolute values of the margin do not accurately reflect the profits actually earned by producers because they do not include other production costs.

RESULTS

Empirical Ratio Estimation

Following the procedure outlined in the previous section, the coefficients of the optimal hedging model were estimated for the equation (12) on the full sample containing 626 weekly feeding simulations, with the MBIC procedure yielding q=1. These estimates are summarized in Table 2. Notice the low value of t-ratio for the current soymeal futures hedge ratio, which is not significantly different from zero.

Figure 1. Cash Prices For Feeder Cattle, Corn, Soybean Meal, And Fed Cattle, In Dollars Per Head.

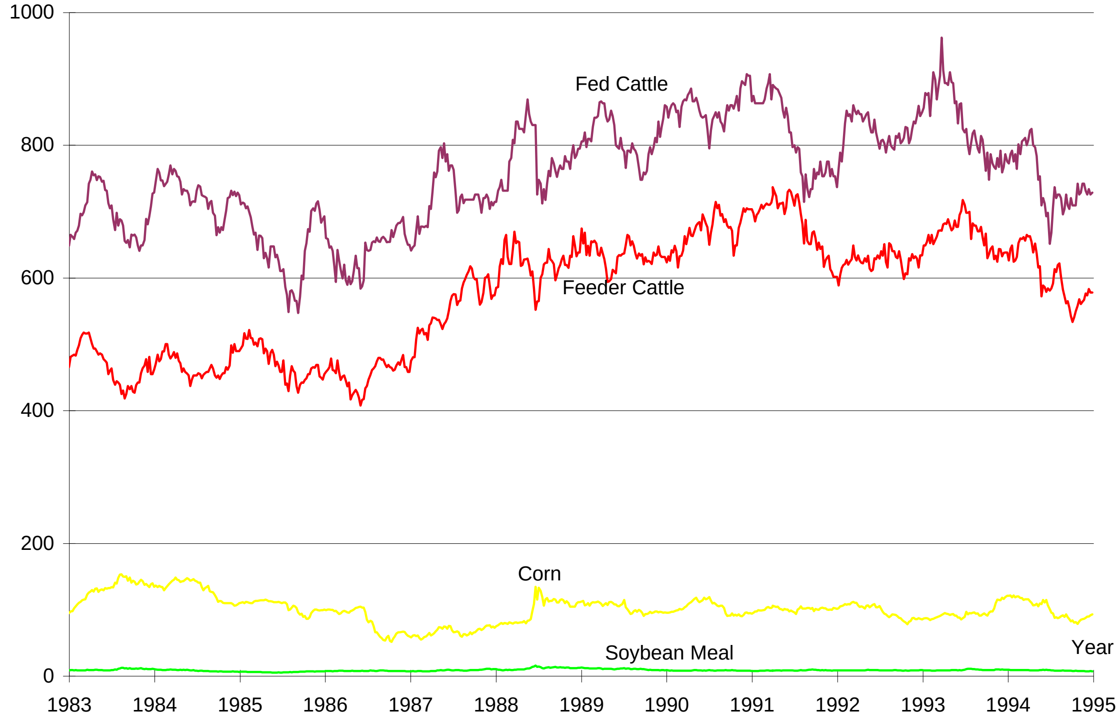
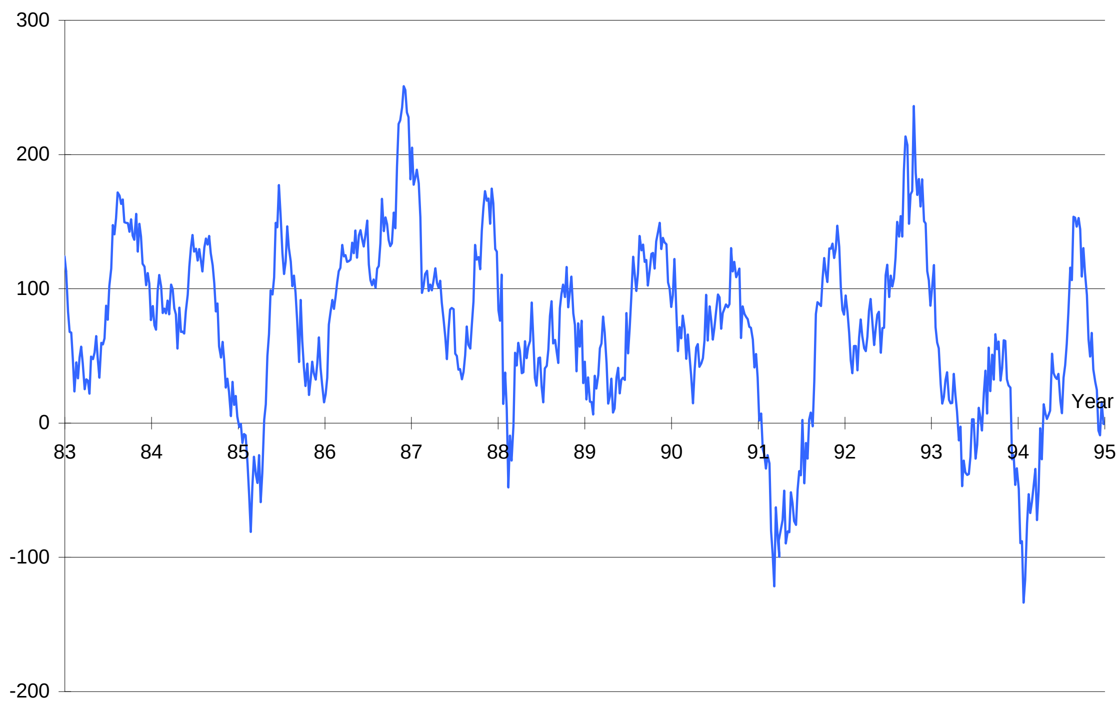


Figure 2. Production Margin, in Dollars Per Head.



One of the most notable results is the hedge ratio for feeder cattle (0.663) being significantly less than one, based on the 97.5% confidence interval *(-¥; 0.819)*. Therefore, the cattle feeder implementing the multiproduct optimal hedging model will be significantly less than fully hedged in feeder cattle. This result can be attributed to the positive correlation between feeder and fed cattle prices. Based on their confidence intervals, other ratios do not differ significantly from one, hence being indistinguishable from one-to-one hedges.

Note also the significance of monthly dummies, which show a profound seasonal trend, with values dropping during late winter and spring, and then going back up during summer and autumn. Finally, parameters on lagged cash and futures prices are less apparent, however, notice that among lag 1 futures, only the corn futures coefficient is significant, and fed cattle is the only insignificant cash conditional.

In order to compare these optimal ratios with those supplied by other models, alternative hedge ratios are summarized in Table 3. Along with the optimal ratios from the multiproduct optimal model, the table lists ratios from the single-commodity hedging and proportional hedging models.

For the single-commodity hedges the MBIC procedure yielded q=4 for feeder cattle, and q=1 for the other commodities. The single-commodity hedge ratios for both corn and soybean meal do not differ significantly from one, based on 95% confidence intervals. Based on the same intervals, the hedge ratio of 0.911 for feeder cattle is significantly less than one, while the ratio of 1.095 for live cattle is significantly greater than one. This suggests that someone interested in hedging feeder cattle alone will be slightly less than fully hedged, while someone hedging live cattle alone will be a little more than fully hedged.

For the proportional hedging model the MBIC procedure also yields q=1. The ratio is 0.97, and is not significantly different from 1. Therefore, if the hedger decides to hedge everything in the same proportion, s/he will be slightly less than fully hedged in all commodities.

Table 2. Estimated Parameter Values for the Multiproduct Optimal Model (1983-1994)

Variable	Coefficient	Standard Deviation	t-Ratio
Current Futures			
Feeders	-0.663	0.078	-8.488
Corn	-1.036	0.261	-3.963
Soymeal	-2.174	2.679	-0.811
Fed Cattle	1.011	0.034	29.437
Lag 1 Futures			
Feeders	0.137	0.143	0.962
Corn	-1.063	0.387	-2.747
Soymeal	-5.506	3.588	-1.535
Fed Cattle	0.076	0.111	0.682
Lag 1 Cash			
Feeders	-0.424	0.067	-6.280
Corn	0.830	0.308	2.697
Soymeal	6.612	2.979	2.220
Fed Cattle	0.030	0.034	0.881
Monthly Dummies			
MO01	175.553	21.224	8.272
MO02	158.498	21.125	7.503
MO03	137.339	20.894	6.573
MO04	142.616	20.830	6.847
MO05	142.517	21.870	6.517
MO06	146.915	21.870	6.718
MO07	173.112	21.927	7.895
MO08	177.197	22.377	7.919
MO09	183.503	23.145	7.928
MO10	170.207	23.500	7.243
MO11	201.980	21.952	9.201
MO12	191.753	21.679	8.845

Table 3 demonstrates the main difference among the models: while traditional models tend to have hedge ratios very close to 1, the multiproduct optimal hedging model suggests that the producer can be less than fully hedged in the feeder cattle, the principal input in the production process. Also, the hedge ratio for soybean meal exceeds two, but is not significant, suggesting that the other hedges carry the price risk in meal.

Table 3. Alternative Estimated Hedge Ratios for Feeder Cattle, Corn, Soybean Meal, and Live Cattle (1983-1994)

Model	**Hedge Ratios**			
	Feeder Cattle	**Corn**	**Soybean Meal**	**Live Cattle**
Multiproduct	0.663 (0.078) *8.488*	1.036 (0.261) *3.963*	2.174 (2.679) *0.811*	1.011 (0.034) *29.437*
Proportional	0.970 (0.033) *29.096*			
Single Commodity	0.911 (0.043) *21.425*	0.978 (0.029) *33.931*	0.983 (0.031) *31.291*	1.095 (0.033) *33.209*
			Key:	X (σ_X) *T-ratio*

Next we examine the optimal hedge ratios over time, continuously. Starting with a data sample covering the first 5 years (approximately one half of the beef cycle), and then extending the sample by one observation every week, we compute new hedge ratios based on the available information. Each ratio is computed based on all information available at the moment of computation. This results in a weekly series of hedge ratios computed over the period January 1, 1988 through January 1, 1995. The ratios for feeder cattle, corn, and live cattle are shown on Figures 3, 4, and 5, respectively. Since the hedge ratio for soybean meal is not significant, it is not included in this illustration.

One can argue that using archaic, and therefore, irrelevant information may lead to obtaining non-current hedge ratios, irrelevant in the current context. To investigate this aspect, hedge ratios for the alternative models were also computed based on the most recent 4 and 6 years of data; graphs of these ratios are provided in the Appendix.

Figures 3 – 5 show that for the primary contracts, namely feeder and live cattle, hedge ratios computed according to the multiproduct optimal model are consistently lower than the ones provided by the single-commodity hedging. Specifically, the optimal feeder cattle ratio consistently remains at the level of approximately 0.6, significantly lower than the single-commodity ratio. The single-commodity ratio for feeder cattle has an upward trend, and slowly grows from approximately 0.65 at the beginning of the sample to approximately 0.9 at the end of the sample.

The proportional hedge ratio, identical in each graph, also has an upward trend, and is asymptotically approaching 1 towards the end of the sample. This can possibly be explained by the growth of sample size, approaching the full period of the cattle cycle.

Unlike the feeder cattle ratios, the multiproduct ratio for corn (Figure 4) is higher than both the single commodity ratio and the proportional ratio. It is also less stable, but it asymptotically approaches 1 towards the end of the sample. Note that at the beginning of the graph the ratio is above 2.0 level, and then it drops to less than 1.3 by the year-end. This might be explained by sharp shocks in live cattle and corn prices at the end of the year, and by the smaller sample size at the beginning.

Finally, for the live cattle ratios, the single-commodity hedge ratio is consistently above 1, at the level of approximately 1.1, whereas the optimal hedge ratio is below 1 for the most of the sample, with an upward trend.

To check if the ratios are equal to each other, an F-test comparing the unrestricted system (equation 12) with restricted system (equation 13), with a being the cattle feeding services (it essentially assumes $h_{FC}=h_C=h_{SM}=h_{LC}$). The test statistic is distributed as F(3, 602) and has a value of 7.22, which exceeds 0.99 percentile of 3.78. Therefore, the hypothesis of hedging all commodities in the same proportion should be clearly rejected relative to the optimal model.

The Appendix demonstrates similarly computed ratios, based, however, on samples of fixed length. Figures A-1 through A-3 represent ratios computed based on the most recent 4 years of prior data, Figures A-4 through A-6 – based on the most recent 6 years of prior data. These graphs clearly show that hedge ratios computed on shorter sample sizes are much less stable compared to the ones computed on expanding samples. Specifically, the ratios computed based on a 4-year long sample exhibit more variability than the ratios computed on a 6-year long sample, which, in turn, are still more volatile than the ratios computed on all data available at the moment of computation (growing sample). This supports previously suggested effect of sample size on the hedge ratios stability.

Figure 3. Hedge Ratios for Feeder Cattle, Computed on All Prior Data Available

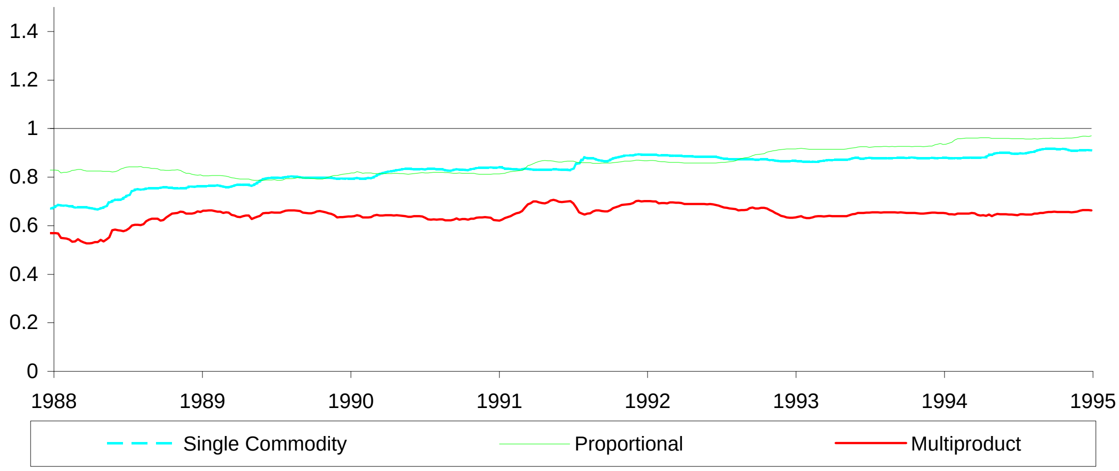


Figure 4. Hedge Ratios for Corn, Computed on All Prior Data Available

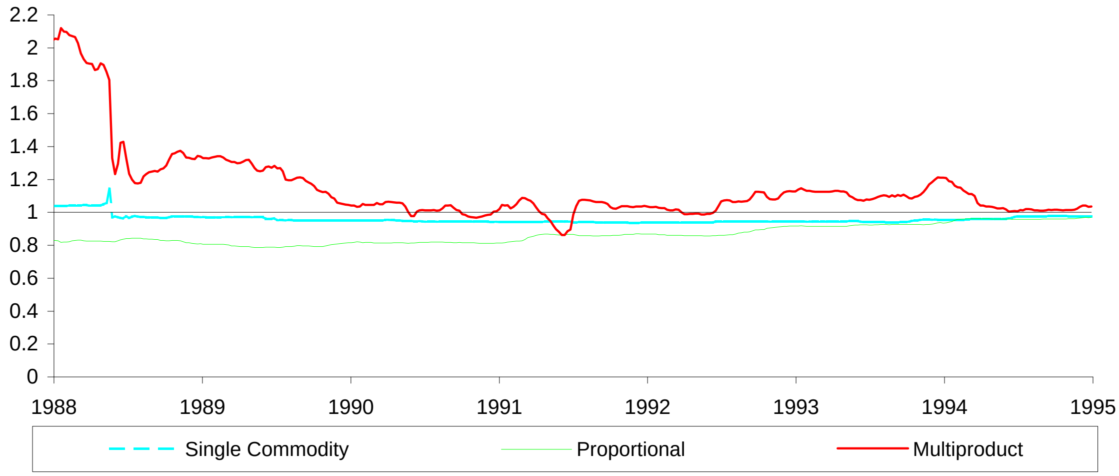
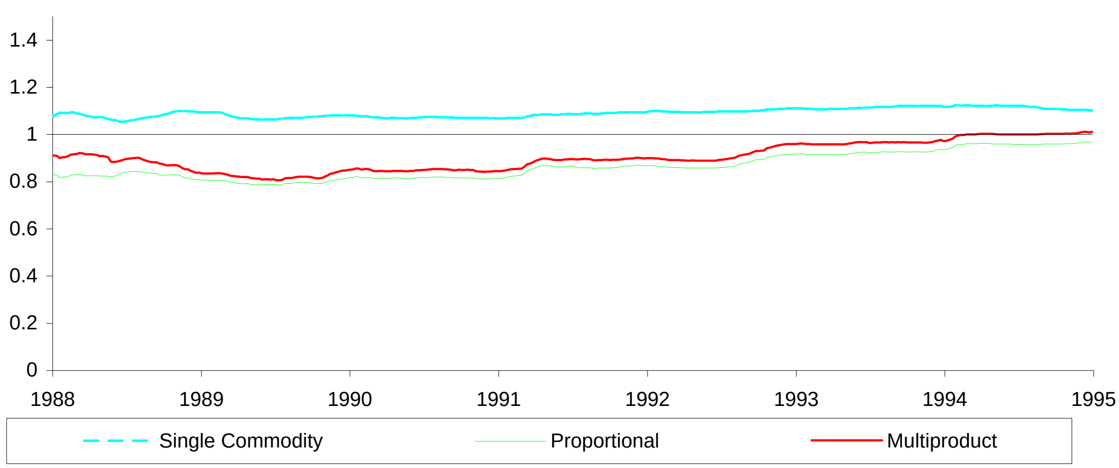


Figure 5. Hedge Ratios for Fed Cattle, Computed on All Prior Data Available



Effectiveness Testing

In this research, the variance reduction resulting from hedging is studied simultaneously with the corresponding reduction in profits, using out-of-sample testing. Hedge ratios computed for alternative models in the previous section were used to hedge one step-ahead margins, using the actual prices, from 1989 through 1994, for a total of 293 observations. That is, ratios computed at time *t-1* were used to hedge cattle that would be put on feed at time *t*. Values for the hedged production margin were calculated by adding gains or losses from the futures positions ΔFh to the producer's gross feeding margin P^C; as before, we ignored fixed costs. For all resulting series of differently hedged production margins means and standard errors were computed, and resulting values are shown in Figure 6.

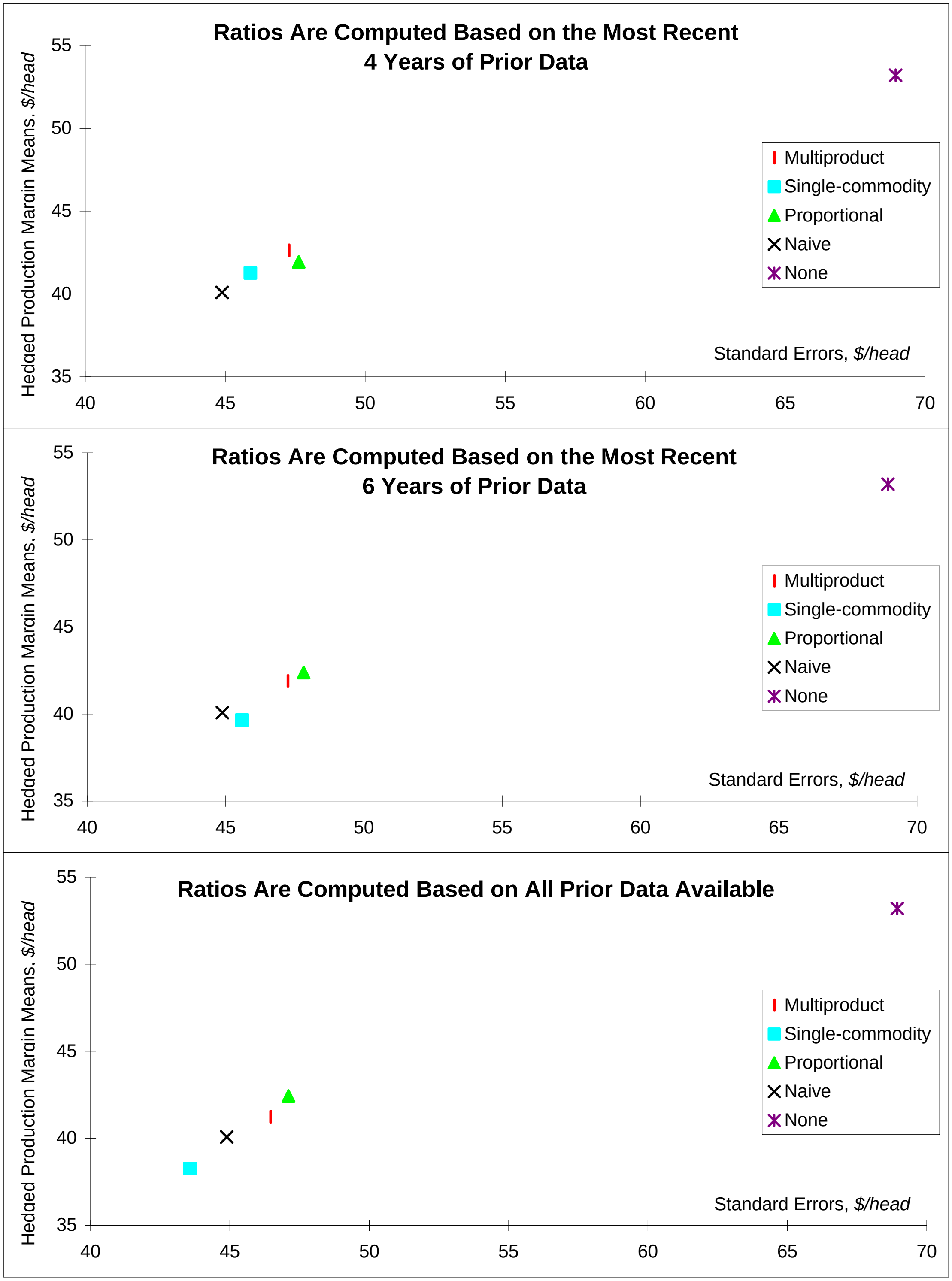
One must immediately notice that production margin means are lower for all hedging strategies relative to no hedging. This can be explained by the fact that for the testing period 1989-1994, the mean of ΔF_{LC} is almost $16 per head, resulting in a $16 loss for the naive hedge, and even higher losses for other hedges where fed cattle hedge ratios are above 1. These losses from fed cattle futures positions are partially offset by gains in the feeder cattle futures (see Table 4).

Table 4. Gains/Losses of Futures Positions and Gross Feeding Margin, 1989-1994 (n=293)

	P^C	ΔF_{FC}	ΔF_C	ΔF_{SM}	ΔF_{LC}
Minimum	-133.8	-46.0	-21.53	-1.58	-123.2
Maximum	236.1	48.5	13.44	1.95	122.1
Median	60.7	4.4	-0.53	-0.09	20.6
Mean	53.2	3.6	-0.78	-0.05	15.9
Standard Deviation	68.9	14.5	5.05	0.43	39.7

For the margins hedged using ratios based on the most recent 4 years of data, all strategies are efficient in the mean-variance framework, with the exception of proportional hedging, which is dominated by the multiproduct optimal hedging strategy, which has a higher mean and lower standard error. Naive hedge turns out to be least risky strategy, followed

Figure 6. Means vs. Standard Errors for Production Margins Hedged During 1989-1994, for Five Different Hedging Strategies



by single-commodity hedge, multiproduct optimal hedge, and no hedge.

For the margins hedged using ratios based on the most recent 6 years of data, naive hedge dominates single-commodity hedge, with the rest of the strategies being efficient. Again, naive hedge turns out to be least risky strategy, followed by multiproduct optimal hedge, proportional hedge, and no hedge.

Finally, for the margins hedged using ratios based on the growing sample, all strategies are efficient in the mean-variance framework, with none of them dominating the others. Note that the single-commodity hedging strategy turns out to be the least risky, followed by naive hedge, multiproduct optimal hedge, proportional hedge, and no-hedge strategy.

There is no consistent domination pattern in the Figure 6, suggesting high dependency on data and sample size. Even though the models can be ranked in terms of their riskiness, these ratings are inconsistent, and vary depending on the size of data sample, and probably which part of the cattle cycle is used for testing.

Whether each commodity should be hedged using only its corresponding contract can be determined by analyzing matrix *M* in equation (7). Single-commodity hedging implies that *M* is diagonal, therefore the role of cross hedges in risk reduction for the multiproduct optimal hedging model can be determined by the hypothesis of $M_{ij}=0$ for $i \neq j$. LM multipliers test has a test statistic associated with this hypothesis equal to λ_{LM} = 203.4, with a $\chi^2(6)$ distribution. The corresponding 0.995 percentile is 18.55, and therefore we clearly have to reject the null hypothesis of *M* being diagonal. Hence, cross hedges play a significant role in the optimal hedge risk reduction.

Variance reduction in hedged positions compared to the unhedged position, computed according to equation (15) is summarized in Table 5. Comparisons drawn from the table, however, can be misleading, because it implies that the models do not dominate each other in the mean-variance framework. For example, for the ratios computed on the most recent 6 years of prior data, the single-commodity hedge, with 56.3% variance reduction, should be considered the second-best alternative following the naive hedge, with its

57.6%. However, from Figure 6, it is obvious that single-commodity hedge should not be considered as an alternative at all, since the profits achieved by using the strategy are less than the ones achieved by the naive hedge. However, in the case of the hedge ratios based on all available data, the results drawn from the table are consistent with Figure 6, because in this case none of the models dominates any other in the mean-variance framework.

Table 5. Profit Variance Reduction for Alternative Models, 1989-1994

Hedging Model	Ratios Based on Prior 4 Years of Data		Ratios Based on Prior 6 Years of Data		Ratios Based on All Available Data	
	Variance	Reduction	Variance	Reduction	Variance	Reduction
No Hedge	4,753	0.0%	4,753	0.0%	4,753	0.0%
Naive	2,015	57.6%	2,015	57.6%	2,015	57.6%
Proportional	2,268	52.3%	2,288	51.9%	2,219	53.3%
Single-commodity	2,106	55.7%	2,078	56.3%	1,898	60.1%
Multiproduct	2,235	53.0%	2,232	53.0%	2,158	54.6%

Note that all models provide more than 50% reduction in profit margin variance. For the most part, other considerations absent, the hedging decision by a U.S. cattle feeder will depend on the feeder's degree of risk aversion.

Model's Implications and Limitations

Figure 6 represents a classical efficient frontier, with no-hedge being the most risky strategy, and single-commodity hedge being the least risky. As the usage of futures increases from zero for the unhedged position to the maximum levels for the single-hedged one, the points representing an efficient futures and cash portfolio move on the frontier from the top right to the bottom left corner. Therefore, the usage of futures is left to the operator depending on his or her risk aversion.

These conclusions should be treated with caution. First of all, we had to make some assumptions about agent behavior. One of the most significant underlying assumptions was that the operator would always hedge, while in reality hedgers do not always hedge expected negative profits. We also did not allow for position adjustment, that is, hedge ratio being non-stationary, during the period of the hedge. Finally, this framework does not allow multi-period hedging, a practice widely followed in industry.

In addition, there are a lot of uncontrolled factors and unpredictable factors, such as weather, which affects pasture conditions, grain crops and feed costs, animal health and performance; cattle marketings; and even consumer demand. Another example of an uncontrolled factor can be animal health – livestock in good health and condition gain weight more rapidly and efficiently, increasing profit opportunities.

These considerations call for a more in-depth model of hedging, which would allow the hedge ratios to have a stochastic nature, and at the same time integrate biological, human, and geophysical aspects of the production process.

SUMMARY AND CONCLUSIONS

This study compares multiproduct optimal hedging to alternative hedging strategies as applied to a Midwestern cattle feeder to assess whether the multiproduct hedging can be implemented to reduce risks associated with cattle feeding. Previous studies suggested that hedging inputs and output as individual commodities helps reduce the producer's margin variability, and that multiproduct optimal hedging can lead to further improvements.

The one-period multiproduct optimal hedging model, derived from a mean-variance framework, is used to estimate hedge ratios and analyze production margin variability. The model assumes that all cash positions are always held in the same proportions, that both cash and futures positions cannot be altered once the feeding begins, and that the producer always hedges all inventory, no matter what profit level s/he is establishing, including negative ones.

Hedge ratios were estimated for the alternative models using weekly data from a simulation of a custom feeding feedlot, using actual cash and futures prices for

1983 - 1995. For each model, a series of hedge ratios were estimated using either the prior 4 years, 6 years, or all prior data available at the moment of estimation. The ratios demonstrate less variability as the length of the underlying sample increases, with the ratios based on the most recent 4 years of data being least stable, and the ratios based on all available data being most stable. Hypothesis of all hedge ratios being equal to each other, that leads to the proportional hedging model, is rejected.

Computed hedge ratios from alternative models are used for one step-ahead hedging of production margins , resulting in gains/losses of futures and gross feeding margins, a sample of 293 feedlots. The results suggest that in the mean-variance framework there is no consistent domination pattern among the alternative strategies. For the ratios computed based on all prior data available, all strategies are efficient, leaving the hedging decision up to the agent's degree of risk aversion. All hedging strategies significantly reduce the production margin's means and variances compared to no hedge, with variance reduction always exceeding 50%. Comparison of the models based on variance reduction in hedged production margin versus unhedged margin is shown to have limited application in case of one strategy dominating another in the mean-variance framework. Whether a producer would choose multiproduct, single-commodity, or proportional hedge ratios is sensitive to the dataset and its size.

Clearly, following a regimented hedging plan reduces cattle feeding price margin risks considerably. These results are, however, limited to the assumptions of the model. A significant underlying assumption was that the operator would always hedge, while in reality hedgers would not always hedge negative profits. Related, feeding margins are clearly cyclical, so it may be possible to forecast this margin five months forward. The one-period model applied here does not allow for position adjustment, that is, for hedge ratio to be non-stationary during the period of the hedge. Some hedgers may want to adjust hedge ratios during the period, depending on the market conditions. These are topics for further research.

In addition, not including uncontrolled and unpredictable factors, such as weather, grain crops and feed costs, animal health and performance, cattle marketings, and consumer demand oversimplify the model. Possible improvement can be achieved from using a more in-depth model of joint hedging and production, which would allow the hedge ratios to have stochastic nature, and possibly take agent's behavior into account. This could also allow the producer to vary cash market positions, or level of production, depending upon the exogenous economic environment and conditions.

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APPENDIX
HEDGE RATIOS COMPUTED ON FIXED-LENGTH SAMPLES

Figure A-1. Hedge Ratios for Feeder Cattle, Computed on the Most Recent 4 Years of Prior Data

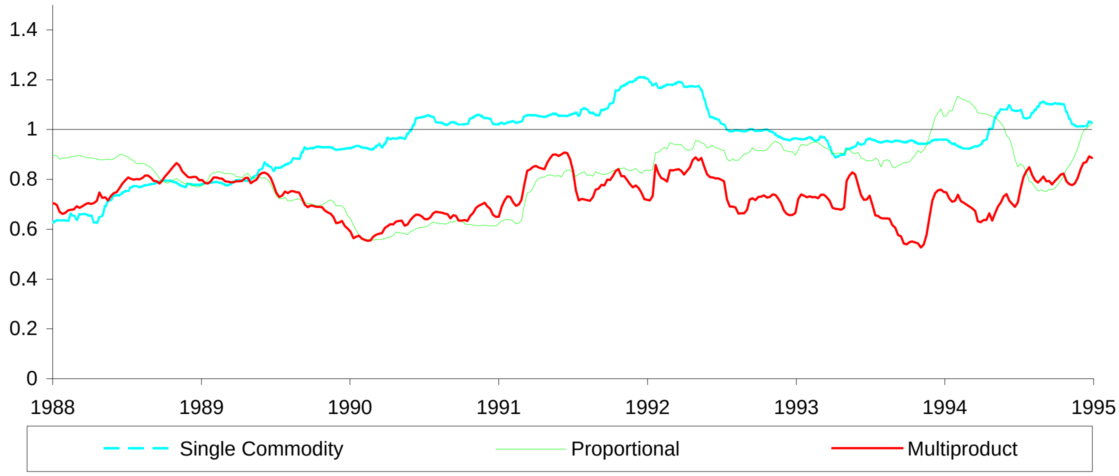

Figure A-2. Hedge Ratios for Corn, Computed on the Most Recent 4 Years of Prior Data

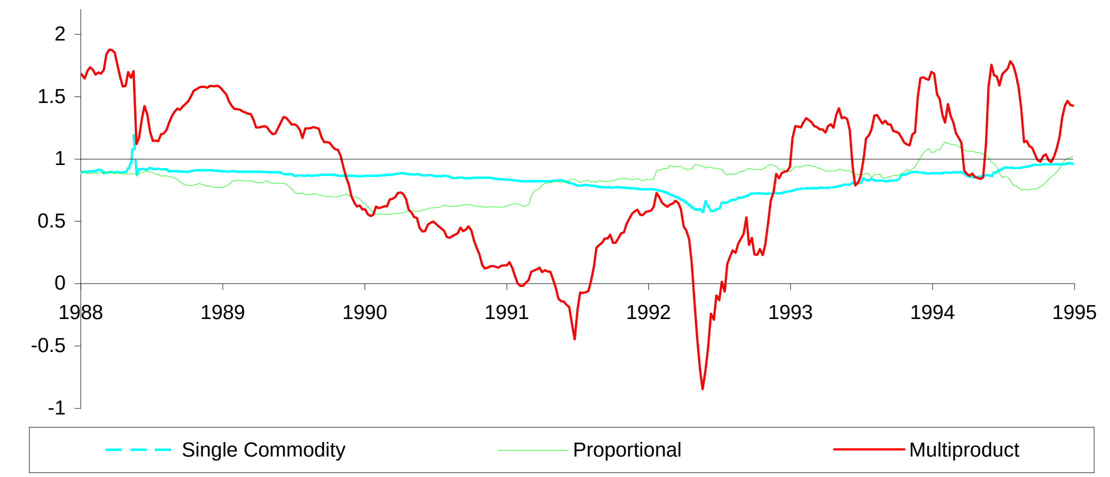

Figure A-3. Hedge Ratios for Fed Cattle, Computed on the Most Recent 4 Years of Prior Data

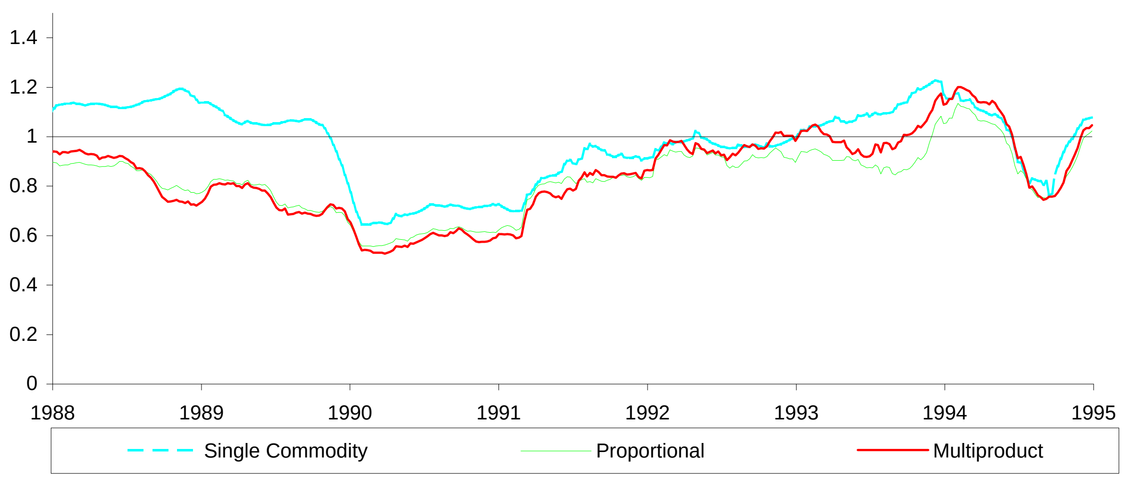

Figure A-4. Hedge Ratios for Feeder Cattle, Computed on the Most Recent 6 Years of Prior Data

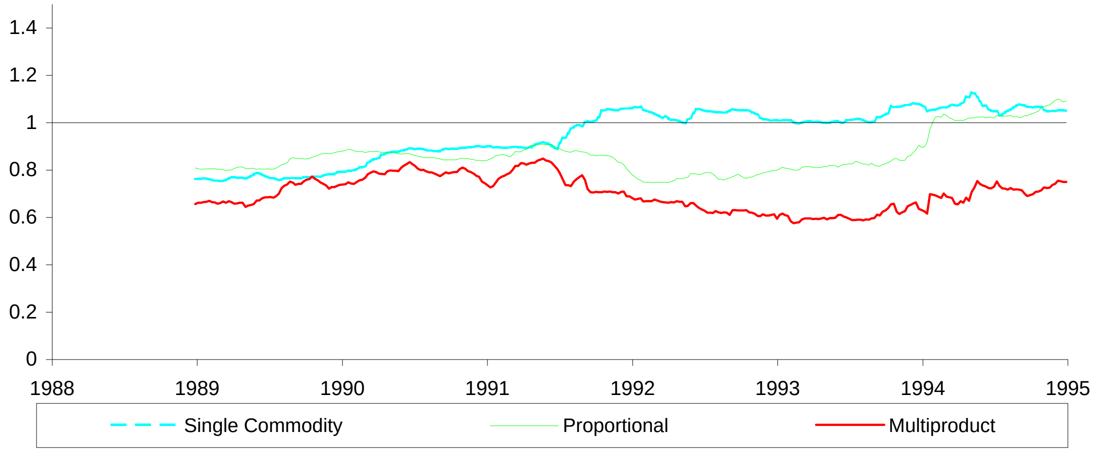

Figure A-5. Hedge Ratios for Corn, Computed on the Most Recent 6 Years of Prior Data

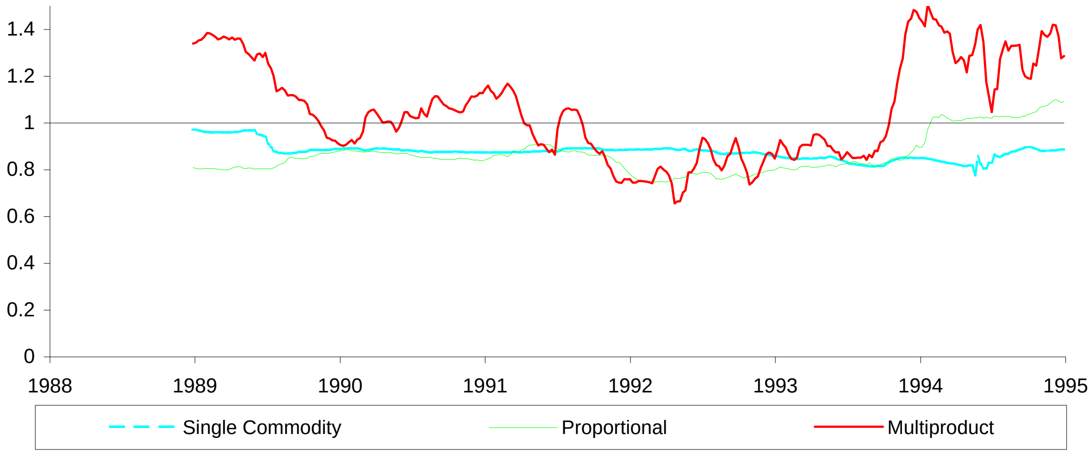

Figure A-6. Hedge Ratios for Fed Cattle, Computed on the Most Recent 6 Years of Prior Data