Long Agricultural Futures Prices: ARCH, Long Memory, or Chaos Processes?

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OFOR Paper Number 98-03
May 1998

Abstract

Price series that are 21.5 years long for six agricultural futures markets, corn, soybeans, wheat, hogs, coffee and sugar, possess characteristics consistent with nonlinear dynamics. Three nonlinear models, ARCH, long memory and chaos, are able to produce these symptoms. Using daily, weekly and monthly data for the six markets, each of these models is tested against the martingale difference null, one-by-one. Standard ARCH tests suggest that all series might contain ARCH effects, but further diagnostics show that the series are not ARCH processes, failing to reject the null. A long-memory technique, the AFIMA model, fails to find long-memory structures in the data, except for sugar. This allows chaos analysis to be applied directly to the raw data. Carefully specifying phase space, and utilizing correlation dimension and Lyapunov exponent together, the remaining five price series are found to be chaotic processes.

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1. INTRODUCTION

It is not uncommon that agricultural futures prices, like many other financial series: (1) are distributed nonnormally with the fat tails (Taylor 1986, Yang and Brorsen 1993), (2) possess autocorrelations that decay to zero very slowly, even for a very long time period (Taylor 1986), and (3) seem to have non-periodic cycles. Recently, three newly developed nonlinear models, i.e., autoregressive conditional heteroscedasticity (ARCH) process and its variants, long memory, and chaos, demonstrate good power to capture these characteristics.[1]

Often in agricultural futures markets, a large price change is followed by another large change, and a small change is followed by another small change. The volatility of markets is not constant over time. It also is observed that in futures trading the variance of prices will often increase as a contract gets closer to the maturity time. Commercial users trade more actively on those contracts which are nearby maturity due to more available information (Leuthold et al. 1989, p.10). This may suggest an ARCH process. A process with ARCH errors can be stationary with constant mean and finite and fixed variance, though its conditional variance is time dependent. Such processes often have fat tails in distributions and spikes in movements. Many empirical studies have found ARCH and its variants in financial markets.

A long-memory structure is a process characterized by long-term dependence and nonperiodic cycles (Fang et al. 1994).[2] The ARCH model considers that the nonlinear structure

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[1] From now on, "an ARCH model" refers to the autoregressive conditional heteroscedasticity process and its variants. "ARCH" will be used in a broad sense.

[2] See Beran (1994, p. 42) for a formal definition.

of a given series comes from the time-dependent conditional variance. In a long-memory process, the nonlinearity is the result of accumulated long-term dependence. Though these two models argue irregular price movements are an endogenous phenomenon of a market, they are stochastic models.

In contrast to ARCH and long-memory processes, chaos represents the stochastic behavior generated by deterministic systems (Tsonis 1992, p.3; Medio 1992, p.4). A chaotic series is called a nonlinear, dynamic, and deterministic process.[3] Though a chaotic orbit is generated by a deterministic model, the observed behavior can be described only by probability for two reasons: (1) a chaotic system is very sensitive to the starting point and coefficients, an error in starting point/coefficients will be accumulated exponentially, and (2) "disorders" or "disasters" will happen when the changes are accumulated to a certain point. The observed dynamics look like random processes, and conventional econometric methods tend to conclude they are random walks. However, they are the products of deterministic systems.[4]

The primary objective of this study is to conduct tests on prices from agricultural futures markets to see if their behavior can be characterized as chaotic processes. In empirical studies, it is not hard to find evidence to argue that the price series with random appearance might be nonlinear dynamic. But, the difficulty is to tell what kind of nonlinear dynamics. Common practice to distinguish chaos system from other nonlinear systems is to use other nonlinear models to filter data before pursuing chaos analysis, but, the simulations have proved that linear and nonlinear filters will distort potential chaotic structures. As shown later, most futures price series

[3] See Brock (1986) for a formal definition of a chaos system.

[4] As a good example, Cunningham (1993) applied the augmented Dickey-Fuller tests to a series generated by a very simple chaotic logistic model, and the null hypothesis, that a unit root is present, could not be rejected.

of this study are neither ARCH nor long-memory processes, providing a unique opportunity for conducting more robust chaos tests.

At the same time, many empirical chaos studies contain other methodological shortcomings, such as misuse of the BDS (Brock, Dechert, Schienlman 1987) test and arbitrary specification of the parameters for chaos estimation. This study corrects those mistakes and advances chaos methodology.

The next section reviews chaos literature. The third section presents the data and discusses its distribution, stationarity, and structure of autocorrelations, conducts ARCH tests, and tests the long memory of the price data series. The fourth section conducts chaos empirical analysis. The last section summarizes and discusses the significance and implications of the research.

2. PREVIOUS EMPIRICAL STUDIES

Many applications of chaos theory have been made in the studies of financial markets. For example, Scheinkman and LeBaron (1989) and Hsieh (1991) analyzed stock markets, and Peters (1991) and Vaidyanathan and Krehbiel (1992) investigated S&P 500 index. The literature is growing very fast. This review concentrates on all available known empirical research on commodity prices which have direct relevance to the present study.

Concerning the predictability of assets prices, Frank and Stengos (1989) used the correlation dimension (CD) and the Kolmogorov entropy[5] to analyze the rates of return of gold and silver prices for the period 1974 to 1986 at the frequencies of daily, weekly, and biweekly.[6] First, ARCH structures were identified in the series. Analyzing the residuals of the ARCH

[5] The definition of Kolmogorov entropy is very closely related to that of the Lyapunov exponent which is used in this study. They both measure the locally-diverging rate of a given series.

[6] In this study as well as in other studies discussed in this section, the time periods for different commodities are not always the same. A table showing the exact time period for each commodity in each study is available from the authors. Only the longest time period is mentioned here.

models, the study found the CD increases from values between 1 and 2 to values between 6 and 7 when the embedding dimension rises from 5,10, 15, until 25. At the same time, in the cases of computer generated numbers and reshuffled ARCH residuals, the CD increases from values between 2 and 4 to values between 11 and 14. Also, the Kolmogorov entropies are positive for all the series concerned. The study concluded that evidence of chaos had been found, which implies that asset price changes can be predicted in a very short period but not in a long run.

DeCoster et al. (1992) were concerned with the issue of whether there exists a chaotic structure in the behavior of futures prices. They investigated daily futures prices of silver, copper, sugar, and coffee for a period of about 20 years up to 1989. The CD analysis was the major tool of the study. When the embedding dimension increases from 4 to 40, for the residuals of ARCH processes, the CD increases from values between 1 and 3 to values between 12 and 15. The fact that the estimated CD values are always lower than those of shuffled data indicates the presence of chaos.

There is one common problem for both studies of Frank and Stengos (1989), and DeCoster et al. (1992). According to the chaos theory as discussed later, the estimates of the CD for a chaotic system will saturate at a certain value as the embedding dimension rises. However, in their studies the estimates of the CD keep on increasing as the embedding dimension rises. No saturation in the estimated CD's was observed.

Puzzled by the fact that no countercyclical production response has been made to earn extra profits in the pork market, though pork cycles were observed, Chavas and Holt (1991) suspected this has to do with chaos, since chaos produces nonperiodic cycles which can be observed but not predicted.

Chavas and Holt worked on the ratios of pork to corn quarterly prices from 1910 to 1984. GARCH structures were found in the series. They applied the BDS test to and calculated the

Lyapunov exponent (LE) for the residuals of GARCH processes, and found some evidence of chaos, which confirms their initial suspicion that because of chaos, market agents are not able to take actions to exploit profitable opportunities.

A particular problem with the Chavas and Holt (1991) study is the constructed phase space. In chaos theory, investigation of orbits is conducted in the dynamics of the phase space, i.e., the orbit is being examined in the different phase spaces with varying embedding dimensions to reveal chaotic properties of the data. However, only a single phase space with the embedding dimension 40 was constructed in Chavas and Holt (1991) study, on which the BDS test and the LE estimation were made.

Streips (1995) followed Chavas and Holt (1991) to search for the solution to the problem of the persistent hog price cycle. He analyzed the monthly hog-corn price ratios for the period 1910-1994 and confirmed the chaos findings of Chavas and Holt (1991). While he did not report how the phase space was constructed, he found by using the Poincare section that the chaotic structure of the hog-corn price ratios is very similar to a known chaotic function used for modeling the measles epidemics. This suggested existence of chaos, and argues that the complexity of chaos prevents traders from taking advantage of the persistent hog price cycle.

To search for the underlying generating processes in the futures markets of the S&P 500 index and soybeans, Blank (1991) estimated the CD and the LE and conducted the BDS test on the residuals of GARCH models since GARCH structures had been identified in these series. All the results were consistent with the existence of chaos.

Cromwell and Labys (1993) examined the monthly price behavior of sugar, coffee, cocoa, tea, and wheat from 1960-1992. It was found that an ARCH or GARCH model can represent well the nonlinearity of the series for sugar, coffee, cocoa, and tea, but not for wheat. The nonlinear dynamics is still visible in the residuals of a GARCH process of wheat. The estimates of the CD and the largest LE suggest that chaos is responsible for this remaining nonlinear dynamics in the wheat market. Except for the two problems discussed shortly about filters and the phase space, the research procedures of Blank (1991) and Cromwell and Labys (1993) are generally sound.

Yang and Brorsen (1992, 1993) analyzed nonlinear dynamics of daily cash and futures prices for some agricultural commodities (corn, soybeans, wheat, etc.) as well as some metals for the period of 1979-1988. Both studies suggested that GARCH rather than chaotic structures were the explanation of nonlinear dynamics existing in the series. The limitations of their conclusions regarding to GARCH effects will be discussed in section 3. Here, their research procedures of the chaos process are of concern.

In Yang and Brorsen's (1992, 1993) chaos research, only the CD analysis was adopted to examine the existence of chaotic structures. The estimated CD increases from values between 3 and 4 to values between 6 and 8 when the embedding dimension increases from 4 to 6 and finally to 8. Then, the conclusion was made that no chaos had been found. Unfortunately, the spread of the embedding dimension is too limited to allow for a reliable conclusion about whether the CD will saturate.

For example, both Frank and Stengos (1989) and Yang and Brorsen (1992, 1993) worked on silver and gold markets for the similar time period. In Frank and Stengos' analysis, when the embedding dimension goes from 5 to 25 in increments of 5, the CD saturates at values from 6 to 7. Then, chaos was concluded. In Yang and Brorsen's (1992, 1993) case, the estimated CD for daily silver and gold series reaches values between 5 to 8 when the embedding dimension goes to 8. It may be the case that when the embedding dimension goes beyond 8, the values of CD will stabilize. So the range of the embedding dimensions in Yang and Brorsen's study is too small. As discussed later, the reliable phase space has to have the embedding dimension, p, as

$p \geq 2U + 1$ where U is the correlation dimension of the series under study. Yang and Brorsen's analysis probably violated this rule.

Kohzadi and Boyd (1995) used the R/S analysis and the BDS test to detect the chaotic structure in monthly cash cattle prices from 1922 to 1990.[7] They found the evidence of chaos. This is a weak study, since the R/S analysis and the BDS test can not lead to the conclusive evidence of the presence of chaos. More thorough analyses of the CD and the LE should have been pursued.

There are two major concerns about the above empirical studies. First, all researchers filtered their data by either linear or nonlinear models, in most cases by ARCH-type models, then, conducted the chaos analysis on the residuals. The question is whether chaotic properties of the process are invariant to such transformations. Second, as discussed later, several important parameters of the phase space need to be set when conducting chaos research. However, most empirical studies did not discuss how these parameters were set.

Chen (1993) found that the correlation dimension is not invariant to a smooth coordinate transformation. The transformation of a moving average model introduces random noise into the original data, and may erase the fractal structure underlying the process. An autoregressive transformation will make the estimation of correlation dimension questionable since the probability density of the phase space has been changed. According to Chen, this was not addressed by Brock (1986) when Brock proved that linear filters would not change the dimensionally of the raw data.[8]

Nonlinear filters may also cause problems for chaos studies. Hsieh's (1991) Monte Carlo simulations showed that when the BDS test is applied to examine the residuals of GARCH and

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[7] R/S analysis is defined later in the paper.

EGARCH processes, the null hypothesis is rejected too infrequently, the asymptotic distribution of the BDS test does not approximate the finite sample distribution well. This implies that nonlinear filters may at least alter the probability density of the phase space if not the dimensionality of the raw data.

None of the above studies reported how the phase space was constructed. When constructing phase space $Y(p,q)t$ to evaluate the correlation dimension**,** the embedding dimension p should increase as required. The time lag between the orbits of the phase space, q, is defined by $q=G/p$**,** where G is the average length of non-periodic cycle of the data. Two mistakes could be made here. First, q is assumed arbitrarily; second q is fixed and is not the function of p. These two mistakes weaken the robustness of the chaos analysis. None of the above studies reported how these parameters had been determined. This study demonstrates more appropriate procedures.

In a word, using linear and nonlinear filters and not specifying the relevant parameters are two major shortcomings of the previous empirical research. The former makes it difficult to identify chaotic structures even when they exist; the latter makes the conclusion doubtful regarding having found or not found chaos.

3. DATA AND DATA CHARACTERISTICS

3.1 DATA SOURCES AND TRANSFORMATION

The futures prices of corn, soybeans, wheat, hogs, sugar, and coffee are selected. Choosing these six commodities covers different aspects of agricultural markets. Hogs as a livestock commodity are nonstorable, the other five are storable. Coffee has long production/adjustment periods, the other five have short ones. To the U. S. market, coffee and

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[8] Some empirical studies referred to this Brock study as the justification for using various filters.

sugar are mainly import goods, while the other four commodities are domestically produced and exports are important. Government policies and regulations have varying effects on these six commodity markets.

Nearby contracts are used to construct long time series.[9] Table 3-1 reports the contracts used for each commodity, as well as the markets where the prices were recorded.

The time period covers from January 1, 1974 to June 31, 1995. The beginning point of the data was set so as to avoid the collapse of Bretton Wood System in early 1970's. For each commodity, daily, weekly, and monthly prices are investigated. The monthly data are the prices of the last day of every month, the weekly data are the Friday prices of every week, the daily prices are closing prices of every trading day. The price series of three time frequencies for a given commodity essentially describe the same market. However, since chaos, long-memory and ARCH models are newly-growing fields of investigation, there are some aspects of these processes which remain unclear, and applying a method to the same market but at different time frequencies will help derive robust conclusions.

Table 3-1. Description of the Data

[9] Geiss (1995) discusses the biasness the various methods of constructing long future prices can create. In this study, the same empirical analysis of ARCH, long memory, and chaos have been applied to the three major transformations: differences, log differences, and the rate of returns. In general, the results remain unchanged with respect to the three transformations. The nonlinear models discussed in this study are not very sensitive to these specific data transformation procedures.

Commodity	Market[1]	Contracts Used	Daily Observations	Weekly Observations	Monthly Observations
Corn	CBOT	March, May, July, September, December	5422	1122	258
Wheat	CBOT	March, May, July, September, December	5422	1122	258
Soybeans	CBOT	January, March, May, July, August, September, November	5422	1122	258
Hogs	CME	February, April, May, July, August, October, December	5428	1122	258
Coffee	CSCE	March, May, July, September, December	5383	1122	258
Sugar	CSCE	March, May, July, September, October	5383	1122	258

1: CBOT: Chicago Board of Trade, CME, Chicago Mercantile Exchange, CSCE: Coffee, Sugar and Cocoa Exchange (New York).

Analyzing daily futures prices often creates a problem: the existence of limits for daily price changes, based on the closing market price of the previous day. Such series are truncated and that might distort nonlinear modeling. However, the analysis on the daily series is still necessary because of the following. (1) It seems that such a truncation has no significant impacts on the nonlinear dynamics of concern. Yang and Brorsen (1992 and 1993) utilized nonlinear modeling procedures on both cash and futures prices of corn, soybeans, and wheat for the similar time period, and the results were not significantly different.[10] (2) The daily prices of sugar and coffee used for this study contain few daily limits, or are essentially untruncated, and later comparisons between these two markets and the other four will show little or no affect on the results due to truncation. (3) Weekly and monthly series are analyzed for each market as well, and they provide the results of untruncated series for each market. (4) The truncation is the fact of these markets, and the results of analyzing daily series allow interpretation from truncated markets. (5) No other known nonlinear modeling on daily future prices has transformed the data

[10] Cash prices are not subject to the same daily price limits.

to avoid the effects of daily limits. The present study follows the same practice so that comparisons can be made between the results of this and other studies.

Heteroskedasticity is expected if examining a lifetime price series of a single contract since the variance of prices often increase as a contract gets closer to its maturity. However, if a price series is constructed by various nearby contracts and each contract contributes only the section of prices when it is heavily traded, over a long time period, such as 21.5 years in this study, the "maturity effect" as such might be avoided. Nevertheless, the question of whether the variances of the data are time-dependent remains for further investigation.

Constructing a price series of nearby contracts has one problem, the price "jumps" when changing contracts. This study adopts a specific "roll-over" procedure to avoid the jumps. When switching contracts, on the last day of the old contract, the difference between the old contract price and the new contract price is observed.[11] Then, all prices of the new contract are adjusted by this difference. For a 21.5 year series, many adjustments of this kind will be taken and in some cases prices even become negative. However, such data remain suitable for analysis because price changes are unaffected by the sign. Most importantly, unrealistic "jumps" are avoided.

3.2 NORMALITY

Table 3-2 reports descriptive statistics for all eighteen series under consideration. All means are not statistically different from zero if the standard deviation could be used to produce t-ratios.[12] However, such a standard t test could not be conducted because the unconditional distributions of all series, except hogs, are nonnormal--skewed and leptokurtic, as discussed below.

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[11] Contracts are rolled forward on the last trading day of the month preceding delivery month.

[12] The series studied here are price changes, not the rate of returns. The fact that the means of the series of price changes are equal to zero only implies that price level has not changed over the period under the study.

The coefficients of skewness g_1 and excess kurtosis g_2 quantify the deviation from a normal distribution and are defined by Smillie (1966). g_1 and g_2 are standard normal distributions with the mean of zero. Jarque and Bera (1980) developed an *O* statistic with a χ^2 distribution to summarize the deviation from a normal distribution.

Except for the hog series, the remaining fifteen series are far from normal distributions, the coefficients of skewness and kurtosis are strongly statistically significant and indicate that the distributions of the price change series are skewed and have fat tails.[13] Jarque-Bera's χ^2 statistics, which summarizes the deviation of the third and fourth moments from the parameters of a normal distribution, are strongly significant as a result. The significant deviation from normality can be a symptom of nonlinear dynamics (Fang et al. 1994).

Table 3-2. Descriptive Statistics of Price Changes

	Mean[1]	St. Dev[2]	g_1 (t-ratio)	g_2 (t-ratio)	*O*
Monthly					
Corn	-1.58	17.95	1.31 (8.73)	8.22 (27.43)	765.9
Soybeans	-3.80	54.22	0.66 (4.13)	7.46 (24.86)	587.7
Wheat	-2.14	25.16	-0.52 (-3.46)	3.08 (10.27)	107.8
Hogs	0.24	3.43	-0.22 (-1.47)	0.50 (1.66)	4.49
Coffee	0.94	15.30	1.11 (7.33)	4.29 (14.3)	239.6
Sugar	-0.13	2.25	0.68 (4.53)	10.18 (33.9)	1081
Weekly					
Corn	-0.317	7.62	0.14 (2.00)	4.89 (33.49)	1108.9
Soybeans	-0.798	24.31	-0.32 (-4.45)	5.08 (34.79)	1215.3
Wheat	-0.437	11.65	0.143 (1.96)	4.28 (29.31)	837.3
Hogs	0.062	1.46	-0.045	0.38	6.96

[13] This result differs from that of Taylor (1986) who found that the rate of return of 13 daily agricultural futures prices (corn, cocoa, coffee, sugar and wool) are approximately symmetric, though they have high kurtosis.

			(-0.62)	(2.53)	
Coffee	0.21	7.06	1.13 (17.94)	13.82 (94.56)	9079
Sugar	-0.02	1.08	-0.52 (-7.43)	14.38 (98.22)	9620
Daily					
Corn	-0.070	3.378	-0.072 (-2.18)	3.30 (50.06)	2464.3
Soybeans	-0.171	10.28	-0.171 (-5.27)	2.89 (43.82)	1910.1
Wheat	-0.100	5.44	-0.002 (-0.06)	2.92 (44.24)	1920.6
Hogs	0.011	0.68	-0.044 (-1.33)	-0.133 (-2.21)	5.83
Coffee	-0.046	3.10	1.098 (33.27)	21.022 (318.5)	99984
Sugar	-0.005	0.49	-0.415 (-12.57)	11.207 (169.8)	28262
Critical Value 5% sig.			*1.96*	*1.96*	*3.84*

1: The units for corn, soybeans, and wheat are cents per bushel, for hogs, coffee and sugar are cents per pound.
2: St. Dev: Standard deviation.

Across the time frequencies, daily data are further away from normal distributions than weekly and monthly data. Across the commodities, the deviation of coffee and sugar from a normal distribution are more severe than that of corn, wheat, and soybeans. Hog data are close to normal distributions at all three time frequencies, presumably due to nonstorability of hogs and that hog futures prices demonstrate more independence than those of storables.

3.3 STATIONARITY

Besides normality, another important property of data is stationarity. Of concern here is covariance stationarity or weak stationarity. To test the stationarity of a time series $Y_{t,}$ the conventional augmented Dickey-Fuller *t* test is used. However, since the price change series analyzed here likely contain heteroscedasticity, Phillips and Perron (1988) semi-parametric test which allows for serial correlation and heteroscedasticity is also used.

Table 3-3. The Augmented Dickey-Fuller (τ) and Phillips-Perron (*Z*) Tests

	Corn		Soybeans		Wheat		Hog		Coffee		Sugar	
	τ	*Z*	τ	*Z*	τ	*Z*	τ	*Z*	τ	*Z*	τ	*Z*

Monthly	-7.58	-16.3	-8.38	-16.7	-6.56	-13.9	-7.94	-16.2	-6.98	-14.9	-7.12	-11.3
Weekly	-13.3	-31.9	-13.3	-33.5	-12.4	-33.9	-13.5	-30.2	-13.6	-34.2	-11.8	-30.3
Daily	-23.4	-69.8	-23.7	-69.8	-23.4	-73.7	-22.1	-73.5	-21.6	-69.5	-23.9	-69.4

The critical value at 5% significance is 2.86. The nulls are that the coefficient equals zero.

The results from Table 3-3 show that all the series are stationary, do not contain unit roots. Phillips-Perron *Z* statistics, which relaxes the assumption that error terms have to be white noise, are usually more than twice as large as the Dickey-Fuller τ statistics.

The magnitude and *t*'s and *Z*'s are similar for all commodities for a given time frequency. The hog series, which looked more stationary than others, do not carry larger calculated statistics. For a given commodity, high time frequency series are more "stationary" than low time frequency series.

3.4 STRUCTURE OF AUTOCORRELATIONS

For a linear time series model, typically an autoregressive integrated moving average (ARIMA(p,d,q)) process, the patterns of autocorrelations and partial autocorrelations could indicate the plausible structure of the model. At the same time, this kind of information is also very important for modeling nonlinear dynamics. In Taylor's (1986) study, the long lasting autocorrelations of the data suggest that the processes are nonlinear with time-varying variances. The basic property of a long-memory process is that the dependence between the two distant observations is still visible. As Barnett and Chen (1988) and Chen (1993) demonstrated, many chaotic processes have long lasting autocorrelations though any individual one is not significant.

For six series of daily price changes, 200 autocorrelations and partial autocorrelations were estimated, i.e, j=1,...,200. For six series of weekly price changes, j=1,...,100. For six series of monthly price changes, j=1,..., 48.

Four features of the structures of partial autocorrelation and autocorrelations emerged for all eighteen series. First, the magnitude of autocorrelations and partial autocorrelations is very small. In terms of absolute values, the largest of autocorrelations and partial autocorrelations are about 0.06 for daily series, 0.10 for weekly series, and 0.20 for monthly series. For conventional linear models, this means the dependence among the elements is weak.

Second, the first autocorrelations and partial autocorrelations for all eighteen series are not significantly larger than the rest of others, and in most cases, they are not even the largest. The first several, usually the second, partial autocorrelations and autocorrelations slightly exceed the significant boundary defined as $1/T^{0.5}$. There are some partial autocorrelations and autocorrelations at much later time lags that exceed the significant boundary to the same extent. This indicates the dependence between nearby observations are not necessarily stronger than that between distant observations, or, the most recent market information is not necessarily more useful than older information.

Third, there is no evidence that the magnitude of partial autocorrelations and autocorrelations become small as the time lag, j, becomes large. The number of lags chosen here is large, yet the magnitude of partial autocorrelations and autocorrelations at the end of the above time lag sequences are almost as large as those at the beginning. Roughly it can be argued that the importance of market information does not decay as the time the information was collected spans.

Fourth, there are no clear patterns describing the fluctuation of partial autocorrelations and autocorrelations. No seasonal and other periodic cycles were observed.

3.5 THE ARCH TEST

All series for six commodity markets demonstrate uneven volatility over the whole sample period. In some time periods the series vary more dramatically than in other periods. This implies time-dependent conditional variances. Most series are found non-normally distributed with excessive skewness and kurtosis. Even so, the unit root tests suggest unconditional mean and variance of the data are finite and constant. The autocorrelation and partial autocorrelation analysis shows that the short-term dependence is obviously weak, but the autocorrelations, though they are very small, are very persistent. All of these symptoms potentially suggest nonlinear dynamics, as Taylor (1986) argued.

ARCH theory admits the nonnormality of unconditional distribution of the data. With the assumption of normality of the conditional distribution, an ARCH-type structure could be built to capture the time-dependent variances. Using such a variance function as an input the maximum likelihood estimates of mean become consistent and efficient. Financial series are typically found nonnormally distributed with the time-varying volatility. Therefore, ARCH models have become very popular in financial time series modeling.

To define an ARCH process, suppose a stochastic process Y_t is generated by an AR(p) process:

$$Y_t = a_0 + \sum_{j=1}^{P} a_j Y_{t-j} + e_t \, . \tag{3.1}$$

There exists an information set, $\Psi_{t-1} = \{Y_{t-1}, Y_{t-2}, ...\}$, such that:

$$e_t | \Psi_{t-1} \sim N(0, h) , \tag{3.2}$$

where

$$h_t = q_0 + \sum_{i=1}^{k} q_i e^2{}_{t-i} \tag{3.3}$$

with

$$q_0 > 0,\ q_i \ ^3 0,\ i = 1,..k, \tag{3.4}$$

to ensure the conditional variance is positive. The process Y_t is called AR(p) with ARCH(k) errors.

In equation (3.3), conditional variance of current error $\boldsymbol{e_t}$ is an increasing function of the magnitude of lagged errors, e_{t-i} , $i=1,...,k$. Hence, large errors (in terms of absolute values) tend to be followed by large errors, and small errors followed by small errors. k determines the length of time for which a shock persists in conditioning the variance of subsequent errors. The larger the value of k is, the longer a spiking period will last. Therefore, those spikes might not be the results of exogenous structural changes, but of the predictable nonlinear dependence.

The standard Lagrange Multiplier (LM) test is applied to all eighteen series to test whether there are ARCH(1) effects in the processes. Since ARCH(1) is the simplest structure of the ARCH and its variants, if ARCH(1) exists, further investigation of more suitable ARCH structures is encouraged. The LM test is conducted on the residuals of those AR models for ARCH(1) effects. Table 3-4 reports the results.

Table 3-4. LM Tests of ARCH(1) Effects

	Monthly	Weekly	Daily
Corn	10.7	145.5	952.6
Soybeans	20.1	116.1	941.9
Wheat	11.7	87.3	531.8
Hogs	4.9	7.7	60.8
Coffee	12.5	15.8	134.1
Sugar	7.7	54.2	857.9
Critical values(5%)	*3.84*	*3.84*	*3.84*

The null hypothesis is Y_t carries no ARCH. The alternative is Y_t carries ARCH.

The critical value is the χ^2 distribution with 1 degree of freedom. All calculated LM statistics are larger than the critical value. The null hypothesis has been rejected in all eighteen cases. And, the higher the time frequency is, the more the calculated LM statistics exceeds the critical value. Among these commodities, hogs are noticeably much less “ARCH” than others.

The previous analysis of autocorrelation and partial autocorrelation of all eighteen series indicated that short memory of the data is very weak and does not have clear patterns. As a preliminary effort, ARMA(1,1) is estimated for all eighteen series,[14] i.e.

$$Y_t = aY_{t-1} + e_t + be_{t-1}. \tag{3.5}$$

$$e_t \sim N(0, s).$$

Since this study utilizes price differences, i.e., Y_t is the series of price differences, the above ARMA(1,1) is equivalent to ARIMA(1,1,1). Further examination is pursued to determine the autocorrelation and partial autocorrelations of e^2 in the equation (3.5) in order to specify k and g of GARCH(k,g). Following Box-Jenkins procedures, the autocorrelations of e^2 are estimated to 24 lags and partial autocorrelations are estimated up to 12 lags. Table 3-5 describes the results.

For all eighteen series the magnitudes of autocorrelations and partial autocorrelations are noticeable, the maximum is 0.43. However except for monthly corn and soybean data, e^2 for other 16 series decay to zero very slowly. The autocorrelations and partial autocorrelations of many e^2 series are still significant even at 24 or 12 time lags, this especially true for weekly and daily series. And, most e^2 series do not have clear decay patterns, the values of autocorrelations and partial autocorrelations exceed the significant-level boundary randomly. As a reflection of

Table 3-5. The Structures of Autocorrelation and Partial Autocorrelations of e^2 Series

	No. of significant AC	Largest AC (lag)	No. of significant PAC	Largest PAC (lag)
Monthly				
Corn	1	0.14(1)	1	0.14(1)
Soybeans	2	0.28(1)	1	0.28(1)
Wheat	unclear	0.21(3)	unclear	0.20(3)
Hogs	unclear	0.17(14)	unclear	0.16(1)
Coffee	unclear	0.22(1)	unclear	0.22(1)

[14] Similar procedure was also done by Fang et al. (1994). Tthough no significant short-term dependence was found in 2,527 daily currency futures prices, they still used AR(3) as a filter.

Sugar	unclear	0.42(3)	unclear	0.38(3)
Weekly				
Corn	7	0.40(1)	5	0.40(1)
Soybeans	11	0.31(1)	7	0.31(1)
Wheat	unclear	0.29(1)	unclear	0.29(1)
Hogs	unclear	0.11(2)	unclear	0.10(2)
Coffee	unclear	0.41(2)	unclear	0.41(1)
Sugar	unclear	0.41(6)	unclear	0.27(6)
Daily				
Corn	unclear	0.43(1)	unclear	0.34(1)
Soybeans	unclear	0.42(1)	unclear	0.42(1)
Wheat	unclear	0.32(3)	unclear	0.32(3)
Hogs	unclear	0.17(14)	unclear	0.16(1)
Coffee	unclear	0.34(9)	unclear	0.29(9)
Sugar	unclear	0.43(2, 3)	unclear	0.40(1)

AC and PAC are autocorrrelation and partial autocorrelation, respectively.

this, for many e^2 series, the largest autocorrelations and partial autocorrelations are not necessarily located at lag 1. "Unclear" in Table 3-5 refers to either of the two or both situations: within 24 or 12 time lags autocorrelations or partial autocorrelations do not decay to zero, or/and autocorrelations and partial autocorrelations break significance boundaries randomly, and no judgment could be made that whether autocorrelations and partial autocorrelations have statistically decayed to zero.

This proposes a significant difficulty for specifying the structure of ARCH(k) or GARCH(k,g). For monthly corn and soybean data, GARCH(1,1) might be sufficient, but for remaining sixteen series, no structures are suggested. French et al. (1987) modeled 57 year (1928-84) daily S&P stock index data with 15,369 observations, GARCH(2,2) was found proper. For most financial data, GARCH(1,1) was sufficient (Bollerslev et al. 1992, Bera and Higgins 1995). The situation of the present data suggest that ARCH or GARCH is not a proper intepretation of nonlinear dynamics contained in the series under study.[15]

[15] It might be argued that wrongly-specified ARMA(1,1) is responsible for the unclear structure of the autocorrelation and partial autocorrelations of e^2 series, since ARMA(1,1) has little explanatory power in all cases. The series might be pure ARCH or GARCH processes where the conditional mean is simply $Y_t=e_t$. Accepting this reasoning, the structure of autocorrelation and partial autocorrelation of Y^2 has been analyzed (results are available upon request). However, the situation described in Table 3-5 remains the same.

3.6 LONG-MEMORY TEST

Autoregressive Fractally Integrated Moving Average (AFIMA) model is the newest development in long-memory process studies. For an AFIMA(p,d,q) model:

$$\Phi(B)(1-B)^d Y_t = \Psi(B)e_t, \tag{3.6}$$

where *0<d<1* and typically *0<d<0.5*, Φ(B) and Ψ(B) are the polynomials of the order *p* and *q*, respectively. The specification of *p* and *q* will affect the maximum likelihood estimates (MLE) of Φ(B) and Ψ(B). By examining the structure of autocorrelations and partial autocorrelations previously, it was found for all eighteen series the short memory was very weak, which suggests that both *p* and *q* should be specified as 0. The estimation will be based on the specification of AFIMA(0,d,0). Estimates of *d* and standard deviations, as well as the values of likelihood of the specifications are in Table 3-6.

Table 3-6. AFIMA Estimates of d

	AFIMA	Daily		Weekly		Monthly	
		D	likelihood	d	Likelihood	d	likelihood
corn	(0,d,0)	0.029	-14286	0.051	-3864	0.000	-1106
soybeans	(0,d,0)	0.036	-20321	0.005	-5167	0.000	-1390
wheat	(0,d,0)	0.000	-16874	0.020	-4342	0.052	-1192
hogs	(0,d,0)	0.004	-5629	0.084	-2012	0.000	-681
coffee	(0,d,0)	0.055	-13719	0.030	-3780	0.070	-1064
sugar	(0,d,0)	0.050	-3738	0.104	-1664	0.231	-566

In AFIMA(0,d,0) specification, in all but two sugar cases, estimates of d are very close to zero. The magnitude of *d* estimates for weekly and monthly sugar series, 0.104 and 0.231, respectively, are noticeable compared with the series of other markets, but the result of daily sugar series is not different from those of daily series of other commodities, where *d* is close to 0.

When $d=0$, AFIMA(0,d,0) becomes

$$Y_t = e_t ,$$

i. e., the series is white noise. The AFIMA model tells that except for the sugar market, the other five markets contain no long memory.

As a parametric statistical model, it is understandable that the AFIMA model might be more sensitive to the noise in the data than to its sample size if the sample size has exceeded certain thresholds. In Table 3-6 for sugar series, the value of *d* decreases as time frequency increases. That no long memory was found in the daily series is likely due to the fact that it has more noise than weekly and monthly series.

The interesting and important questions here are why the sugar market is different from other markets and what are the implications of these differences. The discussion on these questions will be made in section 5 after all three underlying hypotheses have been tested.

4. CHAOS TEST

The evidence reported above of nonnormal distributions, heteroscedasticity, and slowly-decaying autocorrelation in the data suggests nonlinear dynamics. The analysis in the previous section indicates that ARCH and long-memory processes are not capable of accounting for the possible nonlinear dependence in the data. This provides a unique opportunity for chaos tests, i.e., a chaos test can be conducted on the data directly without involving filters.

The correlation dimension (CD) is one of major tools in detecting the existence of chaos by measuring the dimension of a strange attractor. A pure stochastic process will spread all space as evolving, but the movements of a chaotic system will be restricted by an attractor. The concept of a attractor is central to chaos theory, most chaos modeling techniques are about the behavior of orbits around attractors. But, attractors are not unique to chaos systems, a convergent linear system might have a fixed point as its attractor, and a cointegrated system will have a linear function as its attractor. Also, a long-memory process might also have an attractor (Granger and Terasvirta 1993). Unique to chaotic processes is that attractors typically have noninteger dimensions which are one of the sources for the random appearance of series. However, because of noise, there are exceptions, a few chaos systems may have integer attractors.

This property proposes a difficulty in using CD to detect chaos. If the integer dimension for an attractor is provided by the CD analysis, it is not clear whether the system is chaos. Therefore, for testing the existence of chaos the CD analysis provides necessary but not sufficient conditions. On the one hand, the CD analysis should be complemented by other techniques, such as the Lyapunov exponent (LE), and on the other hand, a chaos study must be combined with other nonlinear dynamics analysis, such as long-memory and ARCH processes.[16]

[16] This does not mean to use other nonlinear models as filters for the reasons discussed later.

The LE measures local divergence or convergence of a pair of nearby orbits when the orbits are around the attractor. According to Wolf et al.'s (1985) algorithm, if the largest LE is positive, the system is chaos. However, a random walk exhibits divergence in any region of its space, and the largest LE for a random walk will be positive. Applying only the LE can not distinguish a chaotic from a pure stochastic process. This requires the support of the CD analysis.

Both CD and LE are nonparametric measurements, but the BDS statistic based on the CD has a well-defined distribution and is used widely in empirical work. Hsieh (1991) did extensive Monte Carlo simulations to examine the ability of the BDS statistic to detect the departure from identical independent distributions (IID) and obtained some very important results describing the strengths and weaknesses of the BDS statistic. Unfortunately, many applications of the BDS tests have ignored these results.

Hsieh's simulations showed that the BDS statistic has good power to detect at least four types of non-IID: linear dependence, nonstationarity, chaos, and nonlinear stochastic processes. The rejection of IID by a BDS test does not provide direct evidence of chaos. The process could be any of other three processes.

For this reason, many empirical studies have applied the BDS test on the residuals of ARIMA and ARCH-type filters. Hsieh's simulations showed that the asymptotic distribution of the BDS statistic is well preserved when using linear as well as some nonlinear filters. But, ARCH-type filters distort the distribution of the BDS statistic, and no robust statistical inferences can then be made.

It was also demonstrated by Hsieh that the BDS statistic is robust to lepkurtostic nonlinear series, but when facing asymmetric nonlinear processes, the BDS statistic can not reject the IID null hypothesis as frequently as it should. The BDS statistic has troubles detecting nonlinear processes with asymmetric distributions.

Section 3 showed that the data examined here carries "strange" variances and can not be represented by ARCH-type models. Also, all eighteen series are nonlinear processes with asymmetric distributions as reported in Table 3-2. Therefore, the BDS test is not proper for the present study.

4.1 THE CORRELATION DIMENSION ESTIMATION

The series of price differences of six commodities in this study contain no obvious linear structures, though evidence of nonlinear processes are strong. There are three nonlinear structures examined as alternatives for the possible nonlinear processes: ARCH type, long memory, and chaos. In section 3, it was shown that ARCH models failed in explaining the potential nonlinear structures. The modeling practices of long-memory theory has concluded that, except for sugar, the other five markets are not long-memory processes either. This provides a special stage for chaos tests, i.e., chaos analysis can be conducted directly on the data of these five commodities rather than the residuals of filters. The three sugar series are still analyzed for chaos for comparison purposes.

4.1.1 Determining the Parameters for Estimating the CD

The correlation dimension (CD) is estimated by

$$d(p) = \lim_{p\to\infty}\lim_{r\to 0}((\ln C_p(r)) / \ln(r)), \tag{4.1}$$

where, p is the dimension of phase space $Y(p,q)_t$, and r is an arbitrary small number. $C_p(r)$ is called the correlation integral, which is defined as:

$$C_p(r) = \lim_{T\to\infty} \#\left\{(i, j): \left\| M(p,q)_i - M(p,q)_j \right\| < r, 1 \le i \le T_{p,} 1 \le j \le T_r, i \ne j\right\} / (T_p^2 - T_p). \tag{4.2}$$

In order to execute the estimations of (4.1) and (4.2), the phase space, $Y(p,q)_t$, has to be specified. p, the embedding dimension, and q, the distance between the histories of the phase space, are the two parameters that need to be determined.

According to the Taken theorem, the embedding dimension of the phase space has to satisfied the condition, $p \geq 2U + 1$, in order to obtain a robust estimation, where *U* is Hausdorf dimension. Practically, the Hausdorf dimension (*U*) will be approximated by the correlation dimension (*d*). The above rule of determining the minimum embedding dimension is changed as $p \geq 2d + 1$. The task of the present study is to search for low-dimension chaos. Usually, chaos with a correlation dimension less than 5 is considered low-dimension chaos. Suppose the correlation dimension, *d*, is 5, according to $p \geq 2d + 1$, the minimum requirement for the embedding dimension, *p*, will be *11*. Then, this study sets the embedding dimension from 1 to 40 in increments of 1. That is enough for detecting low-dimension chaos.

None of the previously reviewed chaos empirical studies discussed the determination of *q*, which should be *G/p*, where *G* is the average length of nonperiodic cycles of the series. The classical R/S analysis provides support in identifying the average length of nonperiodic periods *G*.[17]

Another important parameter is the arbitrarily small number *r* in equations (4.1) and (4.2). It was observed in the preliminary tests that the estimates of CD are quite sensitive to the values of r. In the empirical studies, *r* is supposed to be a series from a given small number to a much smaller number, which is close to zero. If the minimum of *r* is set too close to zero, all correlation integrals will be *0* as r goes from a given value to zero. If the minimum of *r* is set too distant from zero, all correlation integrals will be *1* as *r* goes from a given value to zero. *r* should be set in such a way that when the embedding dimension goes to infinite and *r* goes to *0*, the correlation integral could range from *0* to *1*.

[17] R/S refers to rescaled range analysis, and relates to the Hurst exponent (see Peters 1991). R/S analysis is used in the current study, but details will be reported elsewhere.

After numerous experiments, Wolf et al. (1985) suggested the minimum *r* should be set as 10% of the range of the data, i.e. the distance between the minimum and maximum. The present study follows this rule of thumb. Table 4-1 reports *G* and the minimum of *r* determined for the present study, where G is suggested by the classical R/S analysis, and the minimum of *r* is directly derived from the data.

Table 4-1. The Parameters of the Correlation Dimension Estimation

	Daily		Weekly		Monthly	
	G	minimum of r	G	minimum of r	G	Minimum of r
Corn	250	4.6	52	10	12	16.8
Soybeans	250	14	52	26.6	12	53.7
Wheat	250	6	52	13	12	21
Hogs	250	0.3	52	1.1	12	2.5
Coffee	100	7.1	28	10	8	11.8
Sugar	250	0.7	52	1.7	12	25

4.1.2 The Correlation Dimension Estimates

Using the parameters defined in Table 4-1, equations (4.1) and (4.2) are estimated for all eighteen series. There are two estimation steps:

First, the embedding dimension, *p*, is set from 1 to 40 in increments of 1. For a given embedding dimension *p*, *r* is allowed to go to zero (the minimum of *r*) from a given value.[18] At each *r*, the correlation integral, *C(r)*, defined by equation (4.2), is estimated. Then, regressing log(C(r)) on log(r) produces the correlation dimension at the embedding dimension *p*, and is denoted *d(p).*

Then, observe the changes of *d(p)* as *p* goes from 1 to 40 in increments of 1. If *d(p)* appears to reach a fixed value as *p* increases, the "saturated" value is the final measurement of the

[18] The present calculation regime is adopted from Peters (1991), in which the "given value" equals 13 times the minimum of *r*.

correlation dimension, which could be evidence of chaos. However, if *d(p)* also increases as *p* rises from *1 to 40*, there is no proof that a strange attractor exists, and unlikely that chaos exists.

The results are reported in Figure 4-1, which consists of eighteen panels. Each of them describes the results of CD estimates for each corresponding series. The panels report the process of *d(p)* as the embedding dimension *p* goes from 1 to 40.

According to chaos theory, two aspects need to be examined when looking at the estimates of CD for a given series. The first is to see if the CD's saturate as the embedding dimension goes to infinity. Only the saturated CD values are considered final estimation of CD. The second is to see the magnitude of the saturated CD's. Practically, high CD's can not distinguish between the deterministic and purely stochastic processes. Low CD's, typically less than 5, interest economists since the chaos system with low CD's have better chances to be simulated.

As the embedding dimension goes from 1 to 40, for the six daily series the saturation of the CD is not very obvious in most cases except for hogs, where the CD's reach the peak at about 4 and then declines. For the series of corn, soybean, and wheat, the growth pace of CD's clearly slows down. The saturation is possible if the embedding dimension increases.[19]

For the six weekly prices, except coffee, the CD's tend to saturate as the embedding dimension goes from 1 to 40. Again, the CD's of hogs peaks at about 3.5 and then declines. For the six monthly series, except for sugar, the CD's stabilize in the range between 2 and 3. The coffee, hog, and wheat series tend to decline after that.

The estimates of the sugar series are very interesting. In the daily case, the CD approaches 2.5. But in weekly and monthly cases where saturation is observed, the CD stabilizes

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[19] The embedding dimension stops at 40 because of the sample size. Larger p requires very larger data sets.

at integers of 2 and 1, respectively. Because of the noise factor, it is likely that the estimates from weekly and monthly series are more reliable. Comparing with other five commodities, two aspects are noticeable. First, these saturated values are integer instead of noninteger of typical chaotic processes. This implies that the attractor for the sugar market, if there is any, is not ‘strange’ and is not a chaotic attractor. Second, these values are the lowest among the six commodities at each time frequency. This might imply that the sugar market is less “noisy” than other markets.

When the time frequency decreases, it is more likely to identify the correlation dimension. This is related to the noise in the data. Monthly data contain much less noise than daily data. It is advantageous for identifying chaos to have three series with different frequencies for the same market. Based on the above evidence, it could be suggested that all series except sugar contain stranger attractors with fractal correlation dimensions. Table 4-2 reports the approximated correlation dimension estimates for the monthly and weekly series of coffee, corn, hogs, soybeans and wheat. The estimates for daily series were not ensured because of “saturation” problems. The correlation dimensions of these five futures markets are about 2 to 4, which indicates low-dimension chaos.

Table 4-2. Approximated Correlation Dimension Estimates*

	Monthly	Weekly
Coffee	2.53	2.40
Corn	2.68	3.13
Hogs	2.84	3.58
Soybeans	2.37	2.84
Wheat	2.84	3.37

*The estimates are the averages of the five CD values located in “saturation region” in Figure 4-1.

However, the existence of low-dimension attractors is not a sufficient condition for the existence of chaos. Additional evidence needs to be searched for, the largest Lyaponuv exponent.

4.2 LYAPUNOV EXPONENT ESTIMATION

The Lyapunov exponent (LE) is considered as one of most useful diagnostics for chaotic systems. Wolf et al. (1985, p.285) illustrated the intuition of the LE as follows:

> "Lyapunov exponents are the average exponential rates of divergence or convergence of nearby orbits in phase space. Since nearby orbits correspond to nearly identical states, exponential orbital divergence means that systems whose initial differences we may not be able to resolve will soon behave quite differently - predictive ability is rapidly lost. Any system containing at least one positive Lyapunov exponent is defined to be chaotic, with the magnitude of the exponent reflecting the time scale on which system dynamics become unpredictable."

As in the case of CD estimation, some parameters need to be determined before initiating estimation.

4.2.1 Determining the Parameters for Estimating the LE

From Wolf et al. (1985), the largest Lyapunov exponent algorithm can be expressed as:

$$LE = (1/q)\sum_{j=1}^{p} \ln(L^{'}(t_{j+1}) / L(t_j)) . \tag{4.3}$$

The estimation is to be conduced in a phase space $Y(p,q)_t$, where the embedding dimension will be a constant, which should be the integer just larger than the actual correlation dimension of the strange attractor. Since the correlation dimensions for the monthly and weekly series of the six commodities were estimated between *2* and *4,* then $p=4$. For the daily data, the correlation dimension estimates lie between 3 and 5. But, considering the noise in the daily series, assuming p=4 is also reasonable for LE estimation for daily series. Once p is decided, $q=G/p$. G, the average length of nonperiodical cycles of a series, is also the distance between $L'(t)$ and $L(t)$. G was reported in Table 4-1.

The LE measures the rate at which two nearby orbits from the same series locally diverge away from each other. However, if the distance between the two orbits at a certain point is too large, it might not be the local divergence that has been measured. Similarly, if the distance between the two orbits at a certain point is too small, the divergence being measured might just be the effect of noise. Therefore, a maximum (*E*) and a minimum (*e*) need to be set such that:

$e < \left| L'(j+1) - L(j) \right| < E$ for any j=1,2,..p.

Following Wolf et al. (1985), *E* is set as the value which is 10% of the range of the series, *e* is 10% of *E*. Table 4-3 describes the parameters determined for eighteen series.

Table 4-3. The Parameters of Lyapunov Exponent Estimation

	Daily				Weekly				Monthly			
	G	q=G/p*	E	e	G	q=G/p*	E	e	G	q=G/p*	E	e
Corn	250	62	4.6	0.46	52	13	10	1	12	3	16.8	1.68
Soybeans	250	62	14	1.4	52	13	26.6	2.66	12	3	53.7	5.37
Wheat	250	62	6	0.6	52	13	13	1.3	12	3	21	2.1
Hogs	250	62	0.3	0.03	52	13	1.1	0.11	12	3	2.5	0.25
Coffee	100	25	7.1	0.71	28	7	10	1.0	8	2	11.8	1.18
Sugar	250	62	0.7	0.07	52	13	1.7	0.17	12	3	25	2.5

**p*, the embedding dimension, is equal to 4.

4.2.2 The Lyapunov Exponent Estimates

When two nearby local orbits are chosen, their locally-diverging behavior is being examined over time by equation (4.3). When the LE is being estimated, a given series, Y_t, will be regrouped into a series of incremental segments *S(i), i=1,2, ..., k*. *S(i)* is actually a segment of Y_t, the length of *S(i)* increases when *i* increases. And at the end, *S(k)* will be the original series Y_t as *k* is large enough. At each *S(i)* equation (4.3) is evaluated and a LE estimate is obtained. The equation will be re-evaluated as S(i) expands.

According to Wolf et al.'s (1985) algorithm, the largest LE could be identified only when the values of LE's saturate as S(i) expands. The saturated LE is the largest LE. However, if no saturation takes place, no chaos is found. At the same time, chaos is implied only when the value of the largest LE value is positive. Similar to the correlation dimension estimation, the estimation of LE in this algorithm is looking for: (1) whether the series of LE estimates stabilizes as S(i) expands, and (2) whether the saturated LE is positive.

Figure 4-2 reports the estimation of LE for all eighteen series. The estimation for daily series has two kinds of results, i.e., either there is no saturation of LE's to emerge, or the saturated LE is around 0. Since this algorithm is sensitive to noise in the data and daily series have more noise than weekly and monthly series, no solid results could be derived from the estimates of daily series. For a given commodity, the results of weekly and monthly series carry more credibility.

For all the weekly series, except sugar, all commodities have saturated LE estimates and all of them are greater than 0. Sugar's estimates are above 0, but no stabilization is observed. Similar results are generated from monthly series, the only difference here is that sugar's estimates did saturate, but the values are negative.

In the LE algorithm, only the sign of the largest LE matters. The absolute values might be very small. For example, a well-known univariate chaotic process, Mackey-Glass model, has the largest Lyapunov exponent of 0.0063 (Wolf et al. 1985, p. 289).

In order to show converging or diverging process of LE estimates as the series evolves, the scale of vertical axis in each panel of Figure 4-2 varies. According to these pictures, it is hard to compare the largest LE's among different series. If averaging last five LE estimates for each except the three sugar series, Table 4-4 shows the approximate of the largest LE estimates.

Table 4-4. Approximated Largest LE Estimates

	Monthly	Weekly	Daily
Corn	0.00974	0.00112	-0.00028
Soybeans	0.00068	0.00018	0.00026
Wheat	0.00032	0.00322	-0.00022
Hogs	0.00490	0.00402	0.00180
Coffee	0.00156	0.00156	-0.00040

4.3 FINAL ANALYSIS

Since the data for five out of six commodities are neither typical GARCH nor long-memory processes, chaos analysis can be applied directly to the raw data and avoid the distortion caused by various linear and nonlinear filters.

Utilizing the classical R/S analysis, the various parameters for the correlation dimension (CD) and the Lyapunov exponent (LE) study are carefully specified. Following closely chaos theory, the CD and LE are estimated for all six commodities. The estimates of CD show that the corn, soybean, wheat, hog, and coffee series carry strange attractors with low non-integer dimensions. The results of the LE analysis support this evidence of chaos, all weekly and monthly series have positive LE's.

The results from the CD and LE analyses are complementary. A low noninteger correlation dimension says, the series with random appearance does not move arbitrarily in every direction and fulfills all space, instead, the series is "attracted" by a closed set as it evolves. The positive LE adds that though the series will surely be trapped by its strange attractor, it will not converge to a fix set. The series will continuously fluctuate around its attractor.

A larger CD means a more complicated attractor. A larger LE indicates a more "chaotic" orbit. In the above chaos estimates, for a given commodity, if the estimated CD is large, correspondingly a large LE seems to be the result. For example, for weekly hog series, CD estimate is about 3.6, for weekly soybean series, about 2.8. The LE was estimated for weekly hog series as 0.00402, for weekly soybean series as 0.00018. The correlation between the estimates of the CD and the LE is positive. For monthly data, this correlation coefficient is 0.27, while for weekly series it is 0.71. Figure 4-3 demonstrates this close weekly correlation. Why the correlation is lower for monthly data is a question for further research.

The conclusion of finding chaos in the five markets is based on the weekly and monthly data. The fact that the analyses on daily series did not suggest chaos is very likely due to the noise in the data. The truncation introduced by the daily limits is not expected to alter the diagnostics of chaos significantly. The CD reflects the directions of price movements, the daily limit restricts the magnitude of price movements, but not direction. The LE analysis concerns only the sign of the largest LE and is to infer whether the series have local divergence. The daily limit is able to limit the magnitude of the local divergence, but can not change divergence into convergence. This might be the reason why some preliminary chaos analysis conducted on cash and futures markets lead to similar results.

Section 3 concluded that the sugar market is more likely to be characterized by a long-memory model. Why is the sugar market different from the other five, or why are sugar series long-memory processes and the other five markets chaotic systems?

A chaos model is a deterministic system but a long-memory model is a stochastic process. A low-dimension chaos model explicitly suggests that there are several deterministic forces shaping the price movements in markets, though the prices are not or nearly not predictable. A long-memory model can only imply that there is long-range dependence in markets, today's price is affected, or partially affected by the previous long price records. This long memory could either be the interactions of deterministic forces in the market or the effects of speculation, or both.

In world markets, sugar trade differs from the trade of the other five commodities in one major way, i.e., more countries participate in sugar trade, and they are diversified in terms of geographical locations and economic development levels. Sugar trade is more competitive and less likely to be dominated by one or a few superpowers.[20] In contrast, international corn, soybeans, wheat and coffee markets are dominated by a few major exporting and/or importing countries. However, live hogs are neither stored nor significantly traded internationally.

International Sugar Agreements have not been successful since the 1960's in terms of imposing quotas to restrict supply nor encouraging the release of stocks to increase supply. While sugar imports are relatively equally distributed among numerous countries, sugar exports come from a relatively large group of countries (Abbott 1990, Lord 1996). Since the late 1970's, there has been an emergence of a significant degree of potential world sugar production from many countries, and an increasing proportion of world sugar consumption has been accounted for by developing countries. Price elasticities of supply and demand in the sugar market seem larger than at least coffee (Harris 1987).

Given market structures, it is possible that the future prices of corn, soybeans, wheat, hogs, and coffee are more subject to several deterministic elements, consistent with what is suggested by the chaos analysis.[21] In contrast, many factors possibly act within the sugar markets, such that sugar futures prices may be more stochastic in the nature than the other future prices.

It was observed previously that the autocorrelations of all series analyzed here decay very slowly in an irregular fashion. The difference between sugar and the other five markets was

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[20] This argument benefited from discussions with Mr. Ron Lord, an economist of Economic Research Service, United States Department of Agriculture. According to Mr. Lord, The U.S.'s share in the world sugar market has been around 5-7%. The sugar futures market is international in nature, and the U.S. domestic sugar policy has impacts on the market, but the impacts are not substantial in most time periods.

noticeable. For the monthly and weekly series, the autocorrelations of sugar series decay slowly, but fall into a boundary and approach zero after certain time lags. However, the autocorrelations of other five commodities never show the tendency toward zero. (This is not seen in daily series, maybe due to noise effects.[22]) This is consistent with chaos and long-memory theories. A chaotic system is a deterministic process, an innovation occurring at a given time point will last forever. In contrast, an innovation in a long-memory process lasts very long, but not forever (the case of sugar).

5. SUMMARY AND CONCLUSIONS

Many economic and financial theories suggest the existence of nonlinear dependence in financial markets. Chartists accumulate nonlinear price patterns and advise traders for profit opportunities. Simple statistical screening on financial series often finds long-lasting autocorrelations and time-dependent variances, which are the symptoms of nonlinear dependence. The question is what type of nonlinear relationships they are, if they really exist.

This study focused on agricultural futures markets and explored the possible nonlinear relationships in the markets. Three competing nonlinear models, i.e., ARCH, long memory, and chaos were proposed. The three models share some nonlinear symptoms but have distinct generating mechanisms, and thus have totally different implications for understanding price behavior. Confusion on the relationships among these three nonlinear processes are common in previous empirical work, and many methodological shortcomings and limitations have been observed. This study advanced research methods and procedures toward determining which

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[21] It could be argued that those determinants of a chaotic system are dynamic processes themselves. This could be a topic for further research as a secondary-level question. The present study analyzes the chaotic system with the assumption that on average, or in the long run, these determinants behave like deterministic variables.

[22] It might also due to the truncation introduced by the daily price limits. But, it seems that such a truncation has no obvious impacts on either ARCH, or long-memory, or chaos processes.

model is most likely to explain the nonlinear dependence in the markets. This study goes beyond previous work by allowing these three models to compete with each other over the same data set.

5.1 FINDINGS

Price series that are 21.5 years long for six agricultural futures markets, corn, soybeans, wheat, hogs, coffee, and sugar, exhibit time-varying volatility, carry long-range dependence, and portray excessive skewness and kurtosis, though they are covariance stationary.[23] This suggests that the series contain nonlinear dynamics.

ARCH, long memory and chaos are the three nonlinear models that are able to produce these symptoms. Using each of these models as an alternative against the martingale difference null, three competitive hypotheses are formed for testing.

Though standard ARCH tests suggest that all series might contain ARCH effects, further diagnostics show that the series are not ARCH processes, since it has been found that the autocorrelations of the variances of the data decay to zero very slowly as the time span increases, and this is not a property of ARCH processes. Illustrating estimation of GARCH (1,1) on all series leads to the result that the unconditional second moments of the data are infinite.[24] However, all parametric Dickey-Fuller tests, semi-parametric Phillips-Perron tests, and newly-developed AFIMA models have convincingly concluded that the series are stationary with finite means and variances. In addition, all series exhibit obvious asymmetry that is out of the reach of regular ARCH processes. The martingale difference null can not be rejected by the ARCH model.

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[23] A process with heteroscedasticity can be stationary with finite and constant mean and variance (Bera and Higgins 1995).

[24] This study highlights the importance of unconditional moments of an ARCH process, often ignored in the literature. Unconditional moments help guide whether ARCH models fit the data. Typical ARCH processes have a zero unconditional third moment, implying symmetry.

The newly-developed long-memory technique, i.e., the AFIMA model, is applied to test the existence of long-memory process. An AFIMA model is able to circumvent the problem of a series containing a root very close to, but not exactly unity, by using a noninteger parameter. It was found that only the sugar market contains long-memory structures, the other five markets appear as stochastic processes.

The failure of ARCH and long-memory processes allows chaos analysis to be applied directly to the raw data. It has been found that various linear and nonlinear filters could distort potential chaotic structures in the data. Unfortunately, many previous empirical studies used ARCH-type models to filter the data first before pursuing a chaos test on the residuals of the filter.

Using the average length of nonperiodic cycles produced by the classical R/S analysis as an important input, the phase space is carefully specified for each series. With this constructed phase space, the correlation dimension (CD) and the largest Lyapunov exponent (LE) are estimated. For the corn, soybean, wheat, hog, and coffee series, the CD's lie in the range of 2-4, and the largest LE's are positive. These combined results infer that these five futures markets contain strange attractors that regulate the movements of the prices. They are chaotic processes.

Interestingly, the correlation dimensions of the sugar series are likely to be integer, but typically the correlation dimension of a chaotic process is noninteger. Also, the sugar series do not have positive Lyapunov exponents, which is another key property of chaos and indicates local-divergence of a time path. Therefore, the sugar market is not likely to be a chaotic process.

Hence, among three competitive hypotheses, for the five out of six markets examined, only the third hypothesis of chaos is valid and the martingale difference null can be rejected by the alternative.

This analysis has attempted to correct many inappropriate procedures found in the literature. As already indicated, filtering data first with an ARCH-type model can distort potential chaotic structures, and careful diagnostics are needed even before using ARCH-type models. An analysis of a chaotic structure must begin with constructing phase space, and the parameters of phase must be determined carefully either by scientific reasoning or rules of thumb. This procedure has seldom been followed. The correlation dimension for a chaos process can only be observed when the embedding dimension travels from a small value to a larger, noninfinite value. And, since correlation dimension and Lyapunov exponent each have strengths and weaknesses in detecting chaos, it is best to apply them together to take advantage of their complementarity. Finally, as Hsieh (1991) warned, the commonly used BDS statistic to test for chaos is not robust when applied to ARCH-type filtered data nor data asymmetrically distributed.

5.2 IMPLICATIONS OF CHAOS FINDINGS

5.2.1 Understanding Price Behavior

This study concludes that five out of six futures markets analyzed are most likely to be chaotic rather than ARCH or long-memory structures. This has important implications about how to understand observed nonlinear dynamics in the markets.

An ARCH model tends to say that the observed nonnormality, nonperiodic cycles, and spikes are due to the dependence in variances, or risk, of the price series. The interactions of various risk elements in the history of price changes lead to a path that exhibits irregular behavior. An investment decision should emphasize the factors that determine the level of market risk. In many cases, the factors affecting price levels are different from those affecting risk.

A long-memory model does not attribute irregular behavior of price changes to the time dependent variances. Rather, long-range dependence in the price series is responsible for the observed nonlinear dynamics. In a typical long-memory model, the observed time-varying volatility of the market is the product of long-range dependence. Here, the time-dependent market risk is a result, not a cause. Investors should focus on the elements that determine long memory of the prices. For example, the traders with long investment horizons are more likely to wait for a trend in a given market before taking a decision. If the share of long-run investors increases in a given market, the price movements are more likely to have persistent patterns.

The factors affecting the formation of long memory might be numerous and the interactions among them might be complicated. However, a low dimension chaotic process could be formed by a few variables with a simple functional representation. Because the value(s) of coefficient(s) fall into a particular range, or the reactions among different market forces happen to take particular forms and strength, a chaotic process with very simple functional form can give rise to the "chaotic" movements. Here, the attention should not be given to searching for some ambiguous or mystery variables, but for some fundamental factors, such as demand and supply forces. Examples of such studies are Chavas and Holt (1993) and Burton (1993).[25]

A chaos process tends to suggest that, without any exogenous shocks, a few deterministic forces interact with each other in such a way that a very complicated and volatile time path can be produced. For the futures markets analyzed, the volatile price waves for such a long time period may be caused by some basic forces, such as changes in demand and supply elasticities, and in government farm support systems. The amount of volatility that some random events, such as weather, could be responsible for might be much less than expected.

[25] Burton (1993) creates from a traditional utility function a classical chaos function. This so-called logistic map is able to produce very complicated chaotic behavior, though the functional form is not complicated. Savit (1988) details this type of map.

5.2.2 Predictability of Futures Prices

Sims (1984) argued that the price changes must be unpredictable over small time intervals. That is, given available information, the expectation for the variance of price changes is equal to the variance of the price itself. This means prediction error is the dominant component of price changes over small time intervals. This produces martingale differences used as the null hypothesis of this study.

For a martingale difference process, price changes are unpredictable in a short time period. However, this does not rule out the predictability over longer time periods. For example, Fama and French (1988), Poterba and Summers (1988) found pronounced negative long-term serial correlation in stock markets.

For a chaotic process, though the system is a deterministic one without any stochastic component and there are few explanatory variables, the series is still not predictable. Because a chaotic process is extremely sensitive to the initial value of the process and the value of coefficients, a difference at the fifth decimal point either in the initial value of the process or coefficient(s) of the function would lead to two totally different orbits after only a few iterations. Savit (1988) and Baumol and Benhabib (1989) illustrate this property clearly. If markets are, or nearly are chaotic, it is possible to make a very short-run prediction but not a long-run prediction. However, it is always far way from the exact when measuring economic activities and estimating plausible functions, therefore, even the very short-run prediction becomes impossible.[26]

5.2.3 Usefulness of Chaos Findings

If a market is concluded as a chaotic process but chaos is not predictable, what is the usefulness of such a conclusion? The conclusion here that chaos is not predictable (either in long

[26] In a sense, chaos is a compromise between the efficient market hypothesis and the belief that markets are regulated by fundamental economic forces.

run or short run) means it is almost impossible to build an accurate chaotic forecasting model based on empirical data, discouraging use of forecasts in financial trading. However, the diagnostic that a market is a chaotic structure opens the door for forecasting on the other side.

Chaos is a fractal structure, which in financial markets means the price movements in a short time interval are parts of the movements in a long time interval, superficially irregular but still related to the whole in form. This is called the property of self-similarity. In a financial market, a price wave is likely to be composed of smaller subwaves that correspond in dimension and character to the larger wave of which they are a part.

The challenge for traders is how to find the decision-making points that can be used as breakout points or stop-loss levels if the market reverses. Sorkin and Buyer (1996) proposed a three-step trading program, the cash flow analysis, to identify decision points. They identify that when trading volume converges around the same price in all time scales, market participants have reached consensus, and this signals a major move.

Related to the fractal property of chaos, Tvede (1992) found that the risk/reward properties for a market will be proportionally the same over different time frames. Accordingly traders can isolate the time frame that best suits their risk levels. The challenging part is to capture the low risk entry of the shorter time frame and exploit the profit opportunity of the long-term frame.

The current chaos findings are from very long time series, and only during a long time period, the impacts of single random events can be isolated and a few determinant factors are likely to dominate markets to produce chaotic price movements. Chaos is a long-run phenomenon. In the long run, it might be difficult to identify fractal structures and to use them for prediction. If such is the case, a passive strategy is suitable for managing assets over long investment horizons. Such a strategy suggests to establish a well-diversified portfolio of securities without attempting to find under- or overvalued financial assets. In futures markets, this implies that frequent buying and selling generate large brokerage fees but little financial gains, since markets are practically random walks. A buy- (or sell-) and-hold strategy is a more proper management method.

Actually, the index fund, as a practice of passive strategy, has been growing very fast since late 1970's in stock markets. Such a fund is designed to replicate the performance of a broad-based index in stock. Investors in a index fund have a very diversified portfolio but pay the minimum management fees, because portfolio turnover is kept low and there is no need to pay analysts to follow the markets (Bodie et al. 1996, p.348). Chaos theory justifies the investment of the index funds.

As a dynamic process, a chaos system is generated by a feedback mechanism, i.e., the output of a previous time point is the input of the present. If considering at some time points that markets are in essence living systems made up of the mass psychological behavior of their human participants, such a feedback mechanism will self-reinforce interdependent events and cause irregular behavior. Relative to the soybean market, the hog market has higher CD and LE, which means that the feedback mechanism of the hog market is stronger than that of the soybean market. Thus, it will not be surprising to observe much more random price movements in the hog market than in the soybean market. This irregular price behavior may mean that for investors who have higher risk bearing capabilities, their optimal points on the mean-variance efficiency frontier are more likely located in the hog market than in the soybean market.

5.3 FURTHER RESEARCH DIRECTIONS

Further research is very much needed at least in two directions: the theoretical and empirical relationships among ARCH, long-memory, and chaos processes, and more detailed nonlinear characteristics of individual futures markets.

While this study attempted to distinguish the three nonlinear models one from the other in agricultural futures market, there is no statistical theory, if not economic theories, which exist to explain the relationships among them. A long-memory process carries also the property of self-similarity as a chaotic process does, and a chaotic process exhibits long-range dependence as a long-memory process does. How are self-similarity and long-range dependence different between long-memory and chaotic structures?

Lo (1991) found that the modified R/S analysis can not detect long-range dependence in a tent map. A tent map is a well-known chaos model. As a deterministic structure, a tent map produces the orbits carrying obvious long memory. This infers that the long-range dependence in a chaotic structure is different from that in a long-memory structure. The question is why and how they are different.

This study found that the correlation dimensions of three sugar series, which have been identified as long memory processes, are integers, and decrease by a unit from weekly to monthly. At the same time, the sugar series can not be characterized by a Lyapunov exponent (no convergence was observed). These properties are the long-memory process in the view of chaos theory. No known theory is available to explain this phenomenon.

In the ARCH modeling practice, it was found that the variances of the series have slowly-decayed, though very small, autocorrelations. This suggests there might be long-memory components in the variances of the data. The long-range dependence of the variances is another type of heteroscedasticity. If using a long-memory, not an ARCH, process to capture the heteroscedasticity, then the efficiency of maximum likelihood of conditional mean function should be improved. There could be a new nonlinear model, a combination of an ARIMA mean model and a long-memory variance model. Interestingly, Hauser and Kunster (1994) recently developed a fractally difference model with ARCH errors, i.e., a combination of an AFIMA mean model and an ARCH variance model. Sengupta and Zheng (1995) found the market volatility of some of stock prices likely to be a chaotic process.

More work on the implications of nonlinear dynamics on trading practices is needed. It has been argued that the fractal property of a chaotic process can be used to uncover trading opportunities. The series studied here consist of many fractal structures. The average length of nonperiodic cycles indicates how long a single fractal structure lasts in the market. The magnitudes and shapes of those fractals could be useful for traders.

The chaos findings of the present study in the five markets are based on the weekly and monthly data. The results from the daily series are not clear regarding the existence of chaotic processes. Though there is some evidence that suggests this may more likely be due to noise in the daily data rather than the truncation introduced by daily price limits, further confirmation is needed, specially when the time period becomes shorter. In the short run, this truncation has more direct implications for trading activities than in the long run.

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Figure 4-1. Estimation of the Correlation Dimension

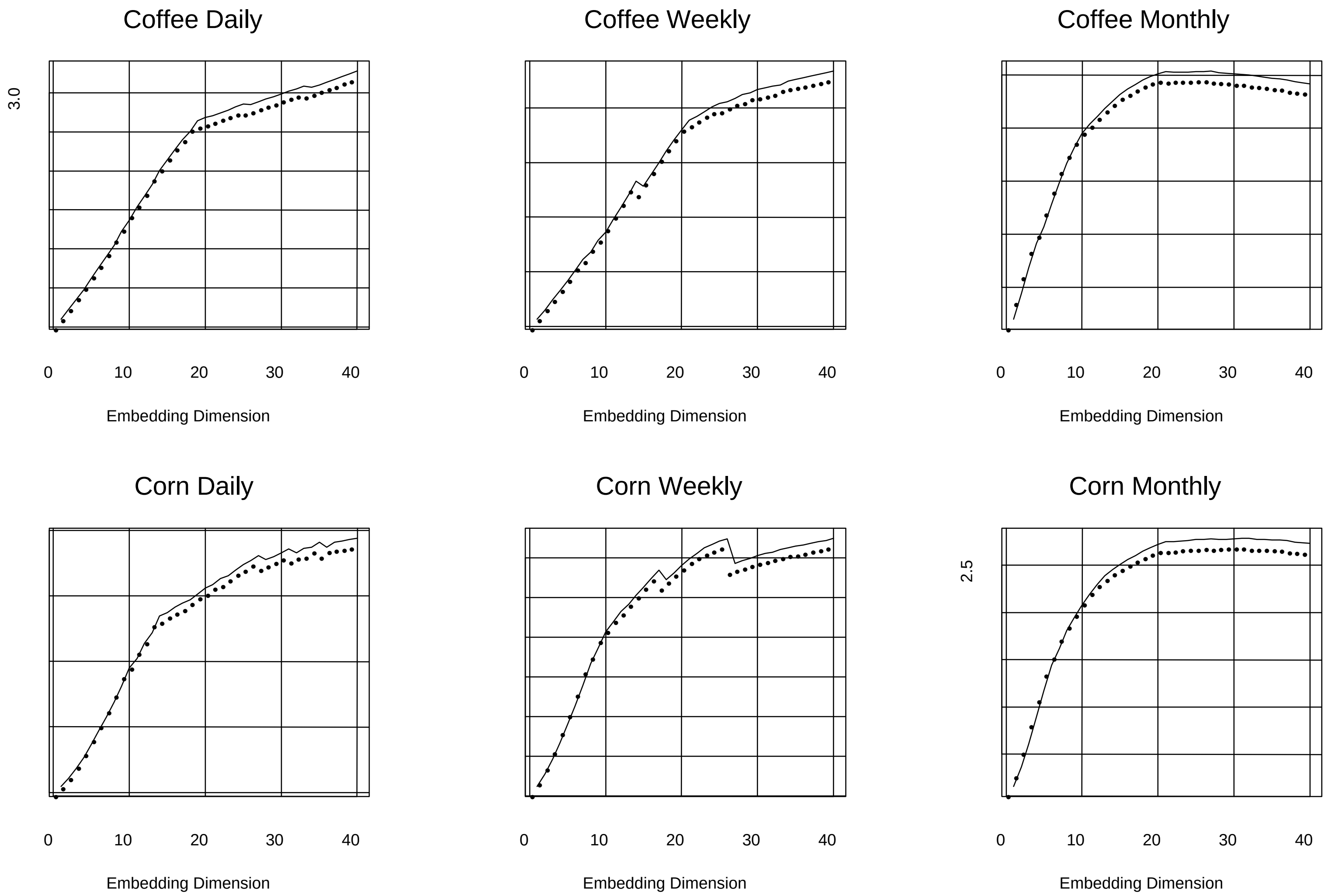


Figure 4-1(continued). Estimation of the Correlation Dimension

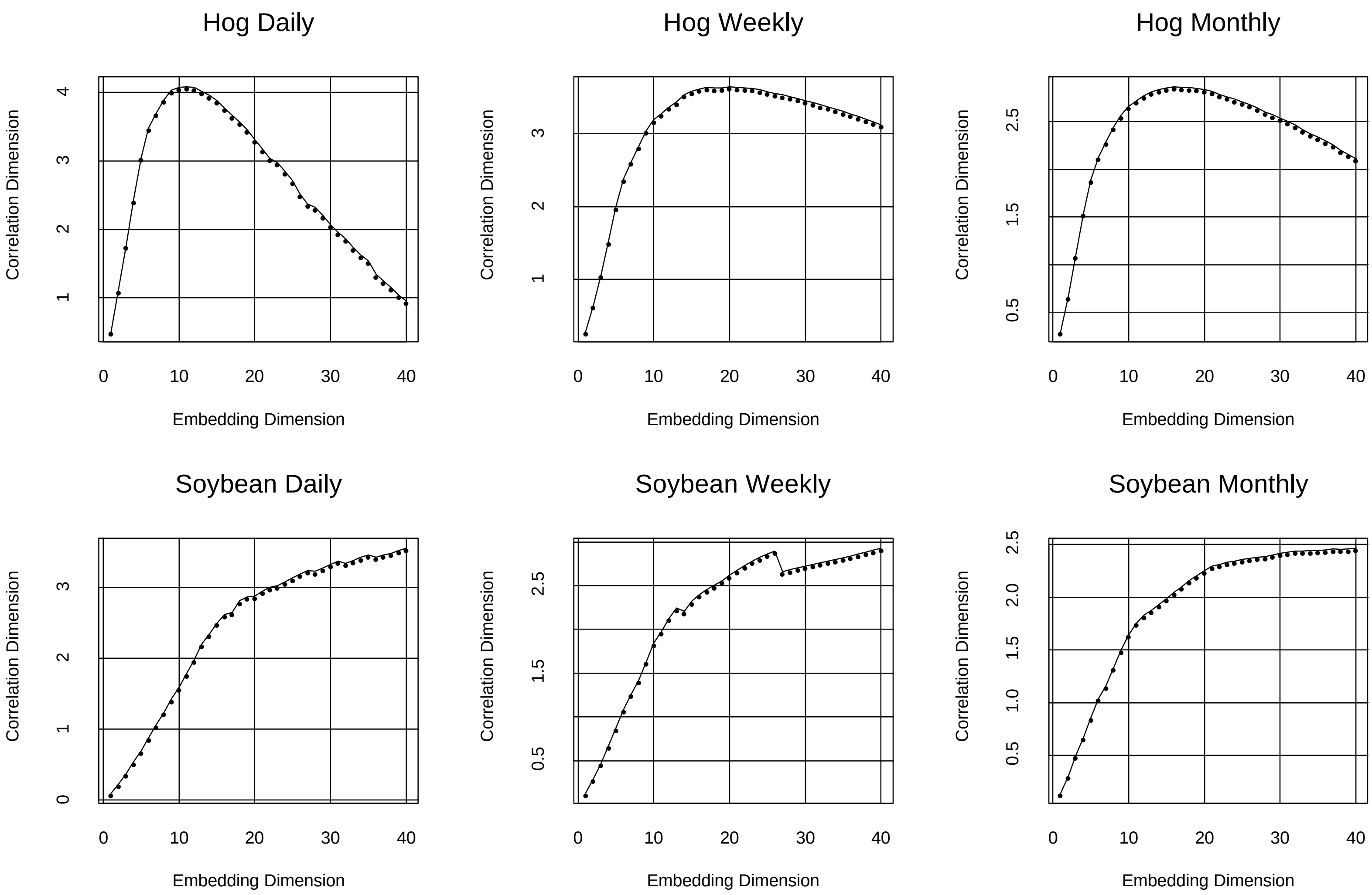


Figure 4-1(continued). Estimation of the Correlation Dimension

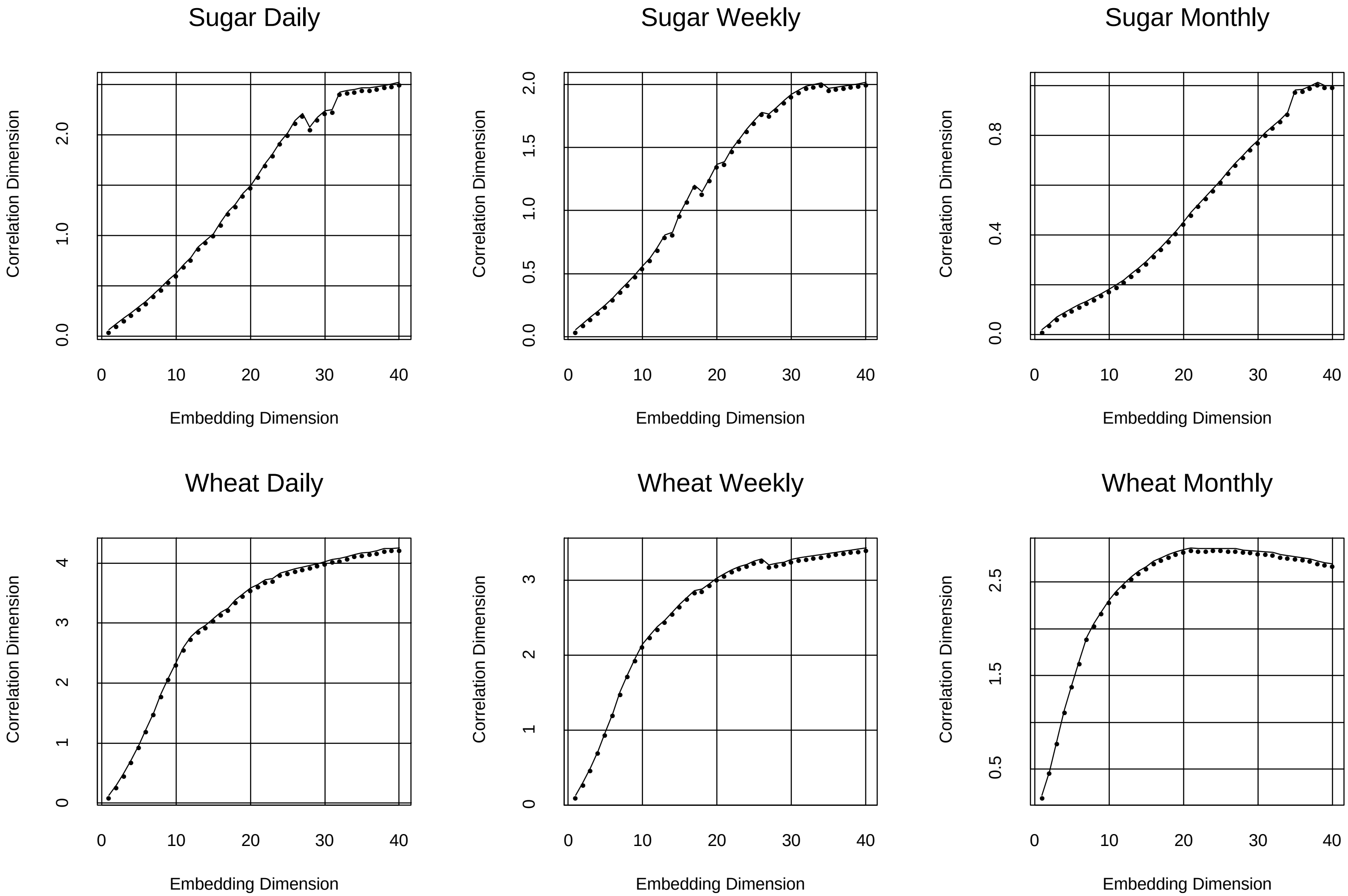
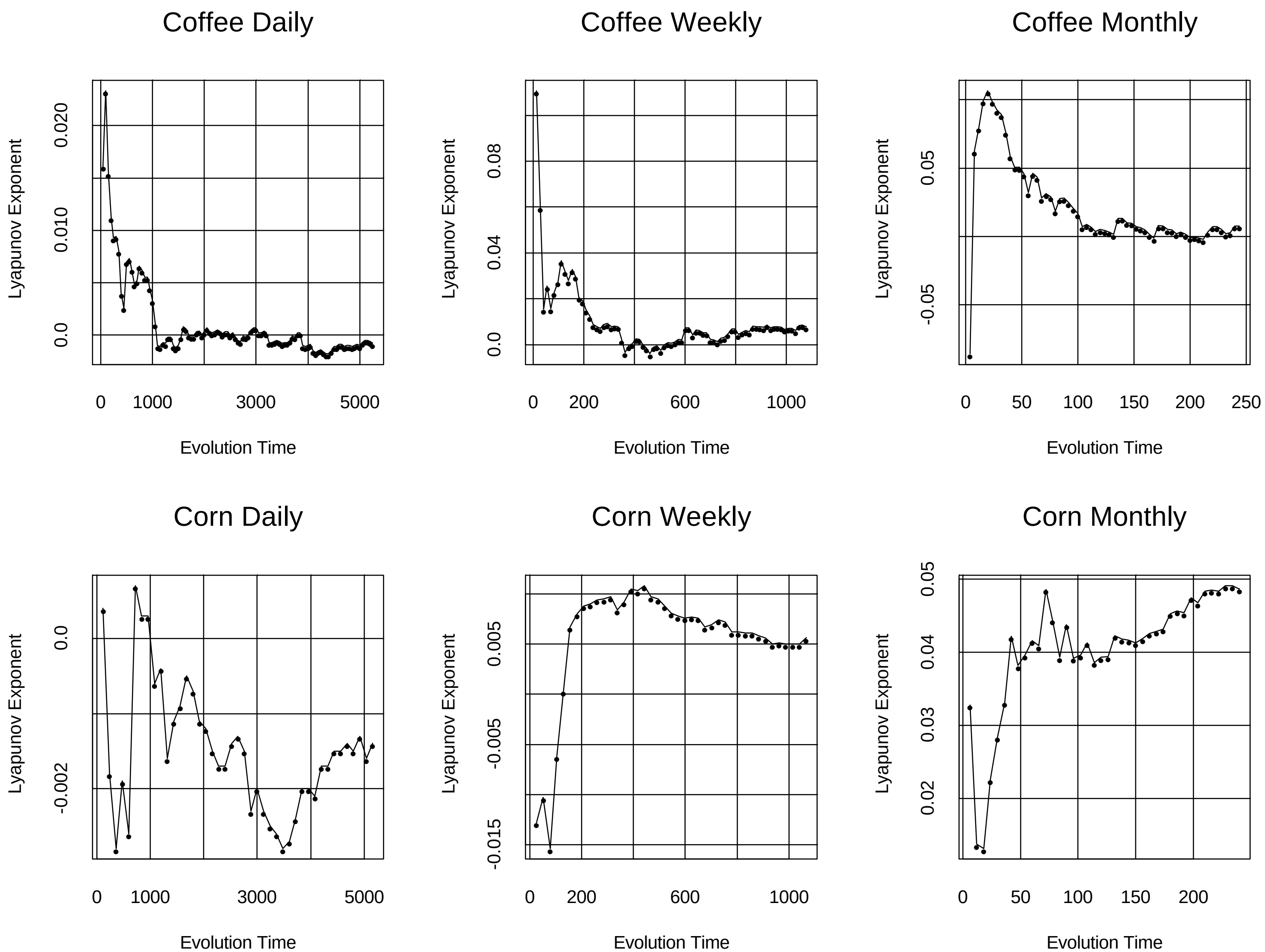


Figure 4-2. Estimation of the Lyapunov Exponent



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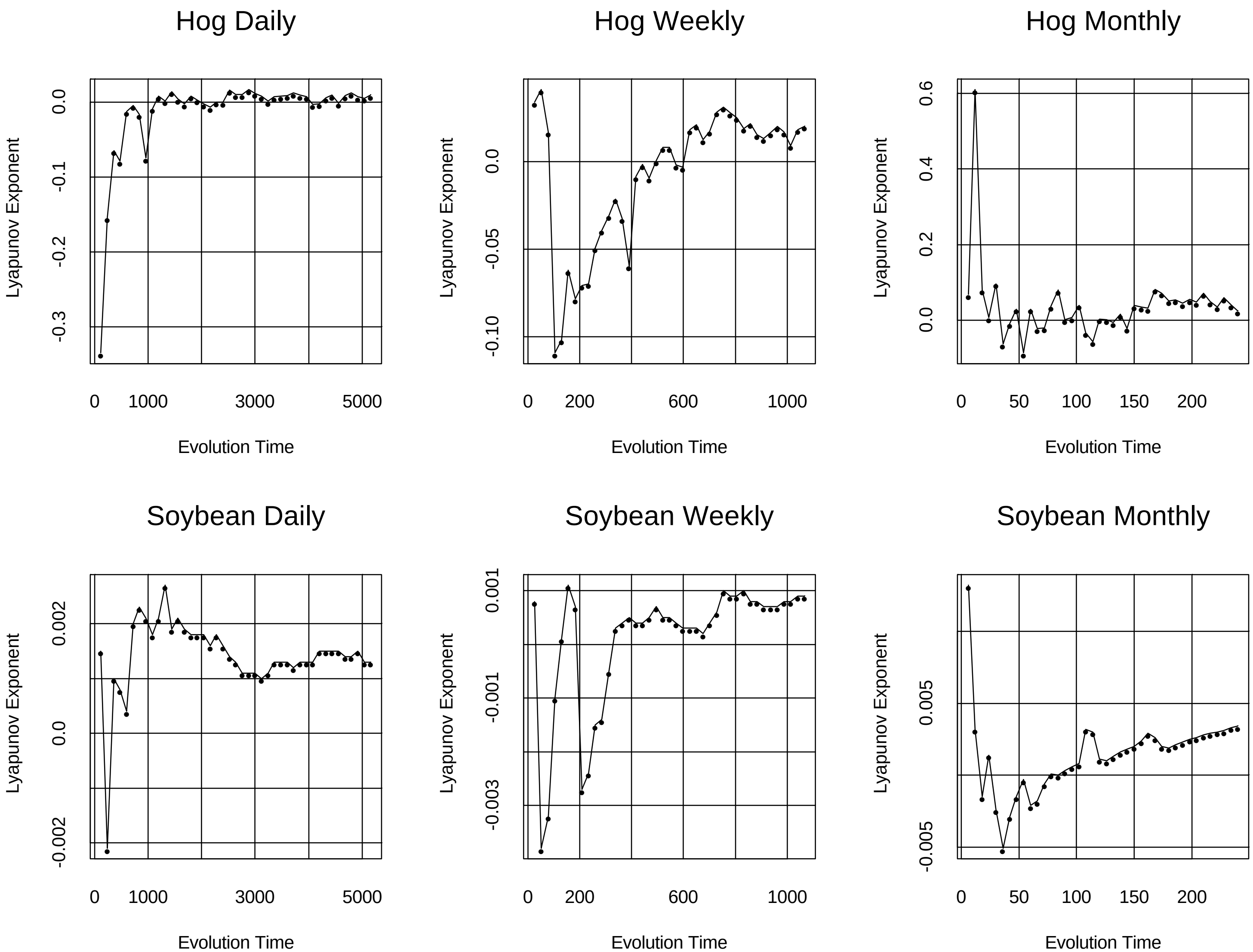

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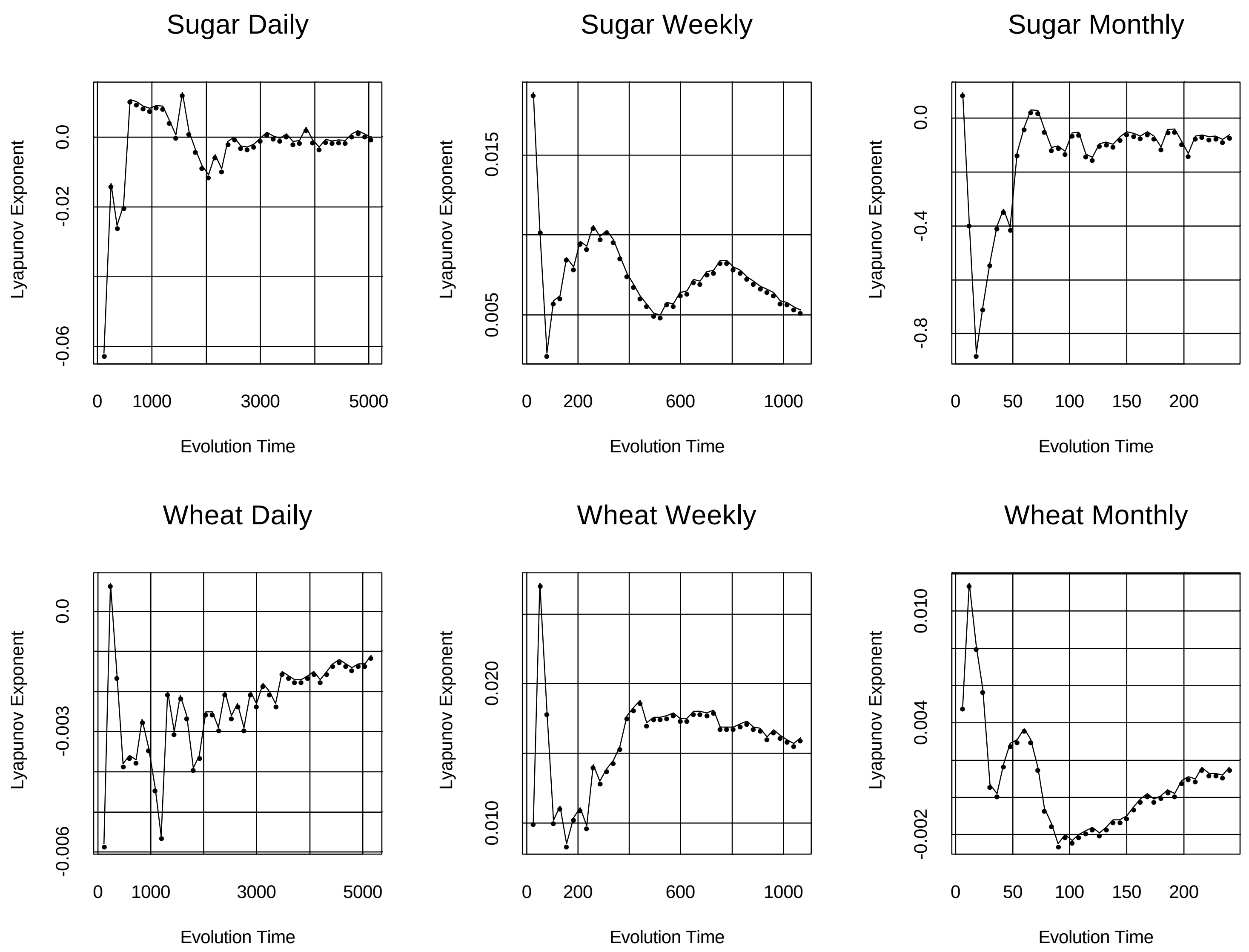

Figure 4-3. Relationship Between the CD and the LE Estimates (Weekly Data)