TEMPORAL PRICE RELATION BETWEEN STOCK AND OPTION MARKETS AND A BIAS OF IMPLIED VOLATILITY IN OPTION PRICES

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Abstract

We show that if a particular temporal relation exists between the option and spot markets, the implied volatility in option prices can be biased depending on the level of the true volatility. The higher the true volatility, the more upward (downward) biased the implied volatility will be, if the option market leads (lags) the spot market. Using intraday data of the S&P 500 index options, we show that the option market leads the spot market at least in the sample. More importantly, the implied volatility is biased due to the lead-lag relationship, and the bias is more profound when the market is more volatile.

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I. INTRODUCTION

The volatility implied in an option's price reflects the market's assessment of the average volatility of the underlying security over the remaining life of the option.[1] As such, the implied volatility has been widely used as an ex ante estimate of future price perturbations, and its informational content has been documented in numerous previous studies.[2] This note shows that the implied volatility in option prices can be biased if a lead-lag relation exists between the stock and option markets and that the degree of the bias depends on the level of the true volatility.

Recently, the lead-lag relation between two securities markets has attracted a significant attention in the literature of market efficiency and microstructure.[3] Concerning the relation between the stock and option markets, the prevailing view was that the option market leads the stock market as evidenced in papers by Manaster and Rendleman (1982), Bhattacharya (1987) and Anthony (1988). This viewpoint was challenged by Stephan and Whaley (1990) who argued that the stock market leads the option market. More recently, Chan, Chung and Johnson (1993) interpreted the Stephan-Whaley results to conclude that neither market leads the other and that new information appears to be impounded in both markets simultaneously. On the other hand, Ostdiek, Fleming and Whaley (1993) show that index options lead cash

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[1]Feinstein (1989) has formalized this intuition in the context of the Black-Scholes valuation model with stochastic volatility.

[2]For instance, Latane and Rendleman (1976) show that the implied volatility predicts future volatility better than the estimators based on historical price data. Patell and Wolfson (1979) provide evidence that the implied volatility provides important information for annual earnings announcements. Day and Lewis (1988) show that stock index option prices reflect increases in the volatility of the underlying index at both quarterly and nonquarterly expiration dates, and that the behavior of the implied volatilities is consistent with an unexpected component of the increase in volatility. Fleming (1998) shows that the best available proxy for the stock return volatility is the volatility rate implied by the preceding day's option prices.

[3]For example, for the lead-lag relation between the stock and futures markets, see Finnerty and Park (1987), Kawaller, Koch and Koch (1987), Stoll and Whaley (1990), and Kleidon and Whaley (1992).

index by about five minutes, even after controlling for infrequent trading in the cash index, confirming the conventional viewpoint of the lead-lag relation between the two markets.

In section II, we show that if the option market leads(lags) the stock market, the implied volatility in option prices will be upward(downward) biased, and that the bias will be higher, the higher the true volatility. In section III, we present some empirical results based on the intraday data of the S&P 500 index options. We show that the option market leads the stock market at least in the sample, and that the implied volatility is significantly biased due to the lead-lag relation. Section IV contains a conclusion.

II. LEAD-LAG RELATION AND IMPLIED VOLATILITY

Suppose that the option market leads the stock market, as perceived in general. Assume that in the absence of any informational effects the option would be priced based on an implied volatility of σ. For simplicity, we just consider European options. The notation is as follows:

t: current time,

T: option expiration date,

S: observed stock price at time t,

r: risk-free rate,

K: exercise price,

σ: true implied standard deviation.

The Black-Scholes formula for a European call option is $C[S,K,r,\sigma,T-t]$. Since the option market is assumed to lead the stock market, the option price incorporates advance information on the stock price. Using a simple binomial model, the stock price is assumed to change to either $S + \Delta S$ or $S - \Delta S$, which are assumed to be equally likely.

In each case, we assume that the market price of the call option at time t will be, respectively,

$$C_u = C[S+\Delta S, K, r, \sigma, T-t]$$

$$C_d = C[S-\Delta S, K, r, \sigma, T-t].$$

If we compute the implied volatility based on C_u, we obtain an estimate σ_u which will be greater than σ. If we use C_d, the implied volatility, σ_d will be less than σ. Since the call option price is a convex function of the stock price, the average of these two estimates will exceed σ. The bias will be

$$\frac{\sigma_u + \sigma_d}{2} - \sigma. \tag{1}$$

The pair of equations for the implied volatility estimates is

$$C[S,K,r,\sigma_u,T-t] = C[S+\Delta S,K,r,\sigma,T-t] \tag{2}$$

$$C[S,K,r,\sigma_d,T-t] = C[S-\Delta S,K,r,\sigma,T-t]. \tag{3}$$

We use $f(S,\sigma)$ for the Black-Scholes price since we will be concerned with its dependence on S and σ. We expand the left-hand side of each equation as a Taylor series around σ and the right-hand side of each equation as a Taylor series around S.[4] After some simplification, we obtain

$$\frac{\sigma_u + \sigma_d}{2} = \sigma + \frac{1}{2}\Delta S^2 \frac{f_{ss}}{f_\sigma}$$

Note that the bias is positive since both f_{ss} and f_σ are positive. These correspond to gamma and vega, respectively. Furthermore, they have explicit forms in the Black-Scholes model (see Cox and Rubinstein (1985)). Hence, the bias in the implied volatility can be written as

[4]Since we use only the first few terms of the Taylor series expansions, this procedure involves error. Our exact computations in Table 1 show that the error is small for plausible parameter values.

$$\frac{\sigma_u + \sigma_d}{2} - \sigma = \frac{1}{2}\frac{\Delta S^2}{S^2}\frac{1}{\sigma T}. \qquad (4)$$

Equation (4) indicates that the magnitude of the bias depends on ΔS^2.

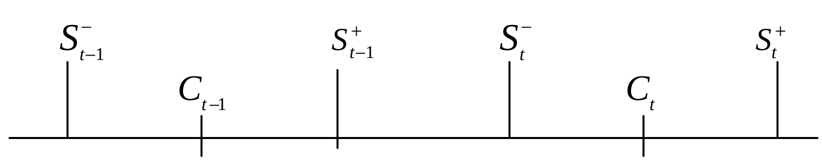
We now compare the values of the implied volatility based on the numerical solutions of equations (2) and (3) with the approximate values from equation (4) for different values of ΔS. The parameter values used are as follows: S = \$100, T = one month, r = 8%, K = \$100, and Δ = 25%. The results are shown in Table 1. The implied standard deviations (σ_u) in case of upward movements in the stock price are greater than the true standard deviation, 25 percent, and those in case of downward movements (σ_d) are smaller. Of particular interest is that as ΔS increases, the bias of the implied volatility increases. When ΔS is \$1.00, the bias is only 1 percent. When ΔS increases to \$2.00 and \$3,00, the bias increases to 3.8 percent and 8.4 percent, respectively. Thus, when the stock price is more volatile, the bias is larger, which is consistent with our theoretical prediction. Table 1 also shows that the error due to ignoring the high order terms of the Taylor series in equation (4) is very small. This confirms our conjecture based on equation (4) that the bias of the implied volatility will be larger when the stock price is more volatile.

III. EMPIRICAL ANALYSIS

A. Data

We randomly selected two contracts of the S&P 500 index options: August 1998 and December 1998 options. In this study, we consider only at-the-money options. As pointed out by Ostdiek, Fleming and Whaley (1992) and Wiggins (1987), at-the-money options provide the greatest accuracy in estimating implied volatility because their values are approximately linear in the volatility rate. Thus, the possible bias in implied volatility due to departure from the implicit assumption of non-stochastic volatility is minimized. Also, we use only nearby options except the expiration month. The exact time period is July 1-July 31, 1998 for August 1998 options and November 1-November 30, 1998 for December 1998 options. The time to expiration ranges from 17 to 45 days.

Intraday option prices, contemporaneous Treasury bill interest rates, and spot indexes are obtained from the *Bridge System*. This data source contains all reported transactions. For dividends, we use daily dividend yields provided by Datastream.[5] For every transaction on the S&P 500 index option, the contemporaneous spot index and the T-bill rate with the same time to maturity are matched using the records reported on average every 15 seconds. The time interval between option transactions ranges from 10 seconds to a few minutes. The exact transaction times are not usually simultaneous for options and spot indexes. Accordingly, whenever there is an option transaction, we record two available spot prices with recorded transaction times immediately preceding and following the option transaction. The following diagram shows our sampling procedure.



[5] The daily dividend yield is defined as current annualized dividend rate calculated on rolling 12-month basis and anticipated annual dividends.

In the diagram, C_t represents the option price whose transaction is reported at time t. We denote the spot price immediately preceding the option transaction of time t as S_t^- and the spot price immediately after the option transaction as S_t^+. The implied volatility (or implied standard deviation: ISD hereafter) for C_t is calculated based on S_t^-.

B. Implied Volatility and its Bias

Table 2 reports the summary statistics for the implied volatility, i.e., ISD and its *implied* bias[6]. In total, there are 1,829 observations for August 1998 call options, 1,807 observations for August 1998 put options, 1306 observations for December 1998 call options, and 1,235 observations for December 1998 put options. The average ISD is .185 for August call options and .189 for August put options. For December options, it is .202 and .208 for call options and for put options, respectively. The differences of average ISDs between call and put options are statistically significant[7]. These results are similar to those of Whaley (1986) for S&P 500

[6] The bias here occurs only due to the aforementioned lead-lag relation between the options and spot prices. Note also that the bias is estimated using implied volatility and other observable variables in the option pricing model. In the absence of the knowledge about the true volatility, the bias is called "implied bias" as opposed to the bias from the true volatility. However, for parsimony, it is referred to as just a bias without specification of "implied".

[7] The difference of means is .004 and the standard deviation of the difference is the sum of the mean standard deviations divided by the square root of the number of observations, assuming independent and identical distributions. The standard deviations of the differences are only .0012 and .0013 for August and December options, respectively.

index options, and to those of Harvey and Whaley (1992) for daily volatility series for S&P 100 index options.[8]

From equation (4), we obtain the bias of the ISD as follows:

$$Bias = \frac{\sigma_u + \sigma_d}{2} - \sigma = \frac{\sigma_u + \sigma_d}{4} - \sqrt{\left(\frac{\sigma_u + \sigma_d}{4}\right)^2 - \frac{1}{2}\frac{\Delta S^2}{S^2}\frac{1}{T}}$$

More specifically, we estimate the bias per percentage change in the spot price as follows:

$$\left\{\frac{\sigma_u + \sigma_d}{4} - \sqrt{\left(\frac{\sigma_u + \sigma_d}{4}\right)^2 - \frac{1}{2}\frac{\Delta S^2}{S^2}\frac{1}{T}}\right\} \Bigg/ \left(\left|\frac{\Delta S}{S}\right|\right). \qquad (5)$$

We use the bias of the ISD per percentage change in the spot price for three reasons. First, under the assumption of a lead-lag relationship between the option and spot markets, the bias is roughly in the same magnitude as the percentage change in the spot price divided by the annualized time to maturity. Thus, the size of the bias is directly related to the size of the percentage change in the spot price, not to the true volatility that is unknown and is assumed to be constant.[9] Second, the magnitude of the price change for a few seconds or minutes is very small and the square of this small number enters equation (4). Third, the bias would be more meaningful in the form of per percentage change in the spot price since the bias depends on the size of the spot price change which in turn varies with the length of time horizon. The bias in the form of equation (5) will be free from such a problem. The estimated bias is .0033 and .0025 for August options and December options, respectively, and both are statistically significant.

[8] Harvey and Whaley (1992) suggest a possible explanation: "purchase of index put is a convenient and inexpensive form of portfolio insurance. Excess buying pressure of puts (relative to calls) may cause price to increase, resulting in implied volatility estimates from put prices that are higher than those from calls."
[9] Note that unless the price changes, there would be no bias, according to equations (4) and (5).

The ISD series have significantly positive serial correlations, but the autocorrelations eventually decline to zero at longer lags[10]. The autocorrelation patterns are similar to those found in daily volatility series for S&P 100 index options (Harvey and Whaley (1992)), for Eurodollar options (Amin and Morton (1994)), and for foreign currency options (Campa and Chang (1995)). The partial autocorrelations and the results of Dickey-Fuller tests show that the ISD time series are stationary and mean reverting. The partial autocorrelations decline substantially after the first lag, suggesting that the implied volatility series follows an AR(1) process.

C. Lead-Lag Relation, Implied Volatility and the Spot Price Volatility

Equation (4) indicates that the implied volatility is directly and positively related to the spot price volatility. To investigate the relationship, we consider the following set of regressions:

$$ISD = \boldsymbol{a}_1 + \boldsymbol{b}_1 \sqrt{\left(\frac{S_t^+}{S_t^-} - 1\right)^2 \frac{1}{\Delta t}} + \boldsymbol{e}_1 \tag{6}$$

and

$$ISD = \boldsymbol{a}_2 + \boldsymbol{b}_2 \sqrt{\left(\frac{S_t^-}{S_{t-1}^+} - 1\right)^2 \frac{1}{\Delta t}} + \boldsymbol{e}_2 , \tag{7}$$

where Δt is the time interval between S_t^- (S_{t-1}^+) and S_t^+ (S_t^-).[11] We also estimate a regression of the ISD against both of the independent variables in equations (6) and (7) together. To

[10] The serial correlations are not reported in the paper to save space but will be available upon request from the authors.

[11] The approximate variance measure of the spot market is annualized by dividing it with Δt. In this way, we can account for the different time intervals between transactions and make the spot volatility measure comparable to the ISD. If S_t^- and S_{t-1}^+ are the same, the observation is excluded in the regression.

account for the AR(1) characteristics of the ISD series, we allow the first-order autoregressive errors and estimate the parameters with a modified Cochrane-Orcutt procedure as developed by Belsley and MacKinnon (1978).

The results are reported in Table 3. The slope coefficient estimate of equation (6), β_1, is .0333 and that of equation (7), β_2, is .0074 for the sample of August call options. For the sample of December call options, they are .0251 and .0093 for β_1 and β_2, respectively. In both cases, β_1 is strongly significant while β_2 appears to be mildly significant in statistical sense[12]. When we run a regression including both of the independent variables, β_2 becomes insignificantly different from zero for both August and December options. Therefore, it appears that the implied volatility is affected mostly by the spot price change following the option transaction. The results also suggest that the bias of the ISD depends on the level of the volatility, which is consistent with the theoretical implication of equation (4).[13]

Of particular interest is that the results also imply that the option market leads the spot market. If the option market acknowledges the arrival of new information prior to the spot market, the ISD calculated at time t should reflect the price change in the spot market from time

[12] Only call options are tested because the theoretical arguments in section II are for call options.
[13] We have actually run the following regressions to test the relationship between the ISD bias and the level of volatility: $Bias = a + b\left(\frac{S_t^+}{S_t^-} - 1\right)^2 + e$:(A), $Bias = a + b\left(\frac{S_t^-}{S_{t-1}^+} - 1\right)^2 + e$:(B)
The coefficient, β, is significantly positive in (A) whereas it is close to zero in (B). This result is quite expected given the results in Table 3. The independent variable in (A) and (B) is a measure of percentage change in the spot price, which is itself a major component of the estimated bias in Table 3. Since the bias is roughly equal to the percentage change in the spot price divided by the time to expiration, the positive and zero betas in (A) and (B), respectively, are expected, given the results in Table 3. The regression results of (A) and (B) indeed support it. The results are not reported in the paper but will be available from the authors upon request.

t to time t+1.[14] The results in Table 3 suggest that this is indeed the case. The ISD is significantly affected by the spot index price movement immediately following the option transaction rather than by the spot index movement prior to the option transaction. This result, in turn, indicates that the option market leads the spot market at least for the sample.

D. Does the Option Market Anticipate the Direction of Spot Price Change?

We have reported the possibility that the option market leads the spot market, and that the temporal price relation between the two markets may cause a systematic bias in the implied volatility of the option. One may then wonder further whether the option market can even anticipate the direction of the spot price movement. To test this possibility, we use the methods similar to those adopted by Harvey and Whaley (1991).

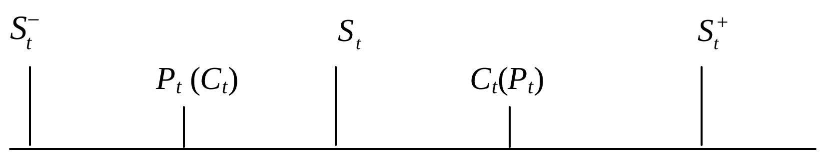
First, we consider volatility spread between call and put options. If the oncoming spot price movement is upward and the option market anticipates this, it will result in an increase in the call volatility and a decrease in the put volatility, thereby increasing the volatility spread. If the oncoming spot price movement is downward, however, it will cause the volatility spread to decrease. Therefore, we can test the temporal price relation by examining the relation between the change in the volatility spread and the change in the stock price.

Whaley (1986) reports that the put options on the S&P 500 index futures tend to have higher volatilities than the call options. Bates (1991) argues that the difference in implied volatilities between calls and puts might reflect investors' expectations about the probability of

[14] Note that the nature of information, whether positive or negative, does not need to be

market movements up or down. One caveat is that, due to nonsynchronous trading of the component stocks of the index, the reported index may lag the true index value while there is no significant lag in observed option prices. This means that the theoretical option price based on the reported index and the actual option price may be different whenever the prices of the index component stocks change fast. As a result, the option market may appear to anticipate the index price movement in advance. However, in this study we focus on whether a lead-lag relation exists between the two markets rather than trying to explore the sources of such a relation.

We select pairs of call and put option transactions that occurred within a minute. Again a diagram will help illustrate the sampling procedure.



In the above diagram, $P_t(C_t)$ is the put (call) premium at time t. We use a pair of P_t and C_t that occur in less than a minute apart but the first transaction can be either a put or a call. S_t is the spot price recorded between the two option transactions and closest to the middle.

To investigate whether the volatility spread predicts the upcoming change in the spot price, we estimate the following regression:

(..continued)
known in advance in the option market.

$$Spread = \boldsymbol{a}_3 + \boldsymbol{b}_3\left(\frac{S_t^+}{S_t} - 1\right) + \boldsymbol{e}_3, \tag{8}$$

where *Spread* is the volatility spread between call and put options. A positive $\boldsymbol{b}_3$ indicates that the option market anticipates the direction of the spot market movement. We also test whether the spread is influenced by the previous spot price change as well, using the following regression:

$$Spread = \boldsymbol{a}_4 + \boldsymbol{b}_4\left(\frac{S_t}{S_t^-} - 1\right) + \boldsymbol{e}_4 \tag{9}$$

The results are reported in Table 4. Overall, the results show that the volatility spread is indeed positively related to upcoming spot price movements for both August and December 1998 options. However, no significant relation is found between the volatility spread and the previous spot price change: the estimate of the coefficient, $\boldsymbol{b}_4$, is negative but not significantly different from zero.

In the above tests, we have used S_t as the contemporaneous spot price in calculating the volatility spread. Even though there is no reason to believe that there would be a systematic bias in our estimates by this practice, a concern may arise because S_t is not observable at the time of the first transaction of the option, P_t or C_t. To address this issue, we simultaneously estimate the implied volatility and the implied spot price using the call and put options together. There are two equations, one for the call and the other for the put, with two unknowns, so that we can derive unique solutions for the implied volatility and the implied spot price. Then, the following regression is estimated to test the predicting power of the implied spot price:

$$\left(\frac{S_t^+}{S_t^-} - 1\right) = a_5 + b_5\left(\frac{S_t^i}{S_t^-} - 1\right) + e_5, \tag{10}$$

where S_t^i is the implied spot price. If the implied spot price has any predicting power for the upcoming price level, b_5 would be positive.

The results are reported in Table 5. The slope coefficient estimates are significantly positive for both August and December options. However, the predictive power is not perfect, i.e., the coefficient estimates are far from one. The results suggest that the implied spot price only partially predicts the upcoming spot price.

IV. CONCLUSION

We show that if a particular temporal relation exists between the option and spot markets, the implied volatility in option prices can be biased depending on the level of the true volatility. The higher the true volatility, the more upward (downward) biased the implied volatility will be, if the option market leads (lags) the spot market. Using intraday data of the S&P 500 index options, we show that the option market leads the spot market at least in the sample. More importantly, the implied volatility is biased due to the lead-lag relationship, and the bias is more profound when the market is more volatile.

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Table 1

Values of Computed Implied Volatility When Option Market Leads Stock Market for Different Values of DS

ΔS	σ_u	σ_d	$(\sigma_u + \sigma_d)/2$	Equation (4)
$1	.3006	.2041	.2524	.2524
$2	.3558	.1630	.2594	.2596
$3	.4153	.1266	.2710	.2716

Table 2
Summary Statistics of Implied Volatility and Implied Bias

Intraday data of August and December 1998 S&P 500 index options are used in the estimation. Contemporaneous spot indexes (prices) and T-bill rates with the same times to maturity as the options are matched. Implied bias induced by the lead-lag relation is measured as $\left\{\frac{s_u + s_d}{4} - \sqrt{\left(\frac{s_u + s_d}{4}\right)^2 - \frac{1}{2}\frac{\Delta S^2}{S^2}\frac{1}{T}}\right\} \Big/ \left(\left|\frac{\Delta S}{S}\right|\right)$, i.e., the bias of the implied volatility per percentage change in spot index. The sample consists of 1,829 observations for August 1998 call options, 1,807 observations for August 1998 put options, 1306 observations for December 1998 call options, and 1,235 observations for December 1998 put options. Augmented Dickey-Fuller unit root tests are performed for different models but reported only for lag of 2 with trend and constant. All the models indicate rejection of the hypothesis of unit root.

	Mean	Standard Deviation	Partial Autocorrelation lag1	lag2	lag3	lag4	lag5	Dickey-Fuller unit root test
August 1998 Options								
Call Implied Volatility	.185	.022	.89	.28	.17	.15	.07	-9.90
Put Implied Volatility	.189	.024	.90	.40	.19	.13	.04	-11.16
Bias	.0033	.0018	.46	-.09	-.12	.14	.05	-12.11
December 1998 Options								
Call Implied Volatility	.202	.021	.80	.23	.15	.10	.04	-9.06
Put Implied Volatility	.208	.024	.91	.27	.11	.09	.05	-10.68
Bias	.0025	.0014	.21	.00	.02	.03	.03	-23.75

Table 3
Relationship between Implied Volatility and Spot Price Volatility

The following regressions are estimated using at-the-money August and December 1998 options.

$$ISD = a_1 + b_1 \sqrt{\left(\frac{S_t^+}{S_t^-} - 1\right)^2 \frac{1}{\Delta t}} + e_1$$

and

$$ISD = a_2 + b_2 \sqrt{\left(\frac{S_t^-}{S_{t-1}^+} - 1\right)^2 \frac{1}{\Delta t}} + e_2 ,$$

where ISD is the implied standard deviation, and t is the option transaction time. S_t^+ denotes the spot price immediately after the option transaction of time t and S_t^- the spot price just prior to the option transaction. The ISD is calculated based on S_t^-. Δt is the time interval between S_t^- (S_{t-1}^+) and S_t^+ (S_t^-). The parameter estimates are reported for the samples of August call and December call options separately. We also report the estimates from the regression of ISD on both of the independent variables. In the parenthesis are t-statistics. * represents significance at the level of 1%.

Sample	Observations	*Constant*	b_1	b_2
<u>August 1998 Call</u>	1,331	.1806 (259.9)*	.0333 (8.193)*	
		.1840 (294.7)*		.0074 (2.078)
		.1803 (245.9)*	.0327 (7.964)*	.0032 (.893)
<u>December 1998 Call</u>	1,032	.2254 (181.)*	.0251 (3.741)*	
		.2274 (207.6)*		.0093 (2.02)
		.2251	.0231	.0046

	(173.7)*	(3.284)*	(.957)

Table 4
Relationship between Implied Volatility Spread and the Spot Price Change

The following regressions are estimated using at-the-money August and December 1998 options:

$$Spread = a_3 + b_3\left(\frac{S_t^+}{S_t} - 1\right) + e_3,$$

and

$$Spread = a_4 + b_4\left(\frac{S_t}{S_t^-} - 1\right) + e_4$$

where *Spread* is the volatility spread between call and put options' implied volatilities. S_t is the contemporaneous spot price in calculating the volatility spread and S_t^+ denotes the spot price immediately after the option transaction of time *t* and S_t^- the spot price just prior to the option transaction. In the parenthesis are t-statistics. * represents significance at the level of 1%.

Sample	Observations	*Constant*	b_3	b_4
August 1998 options	1,429	.0010 (2.528)	3.9264 (3.193)*	
		.0011 (2.733)*		-.2552 (-.958)
		.0010 (2.578)	5.213 (2.865)*	-2.950 (-.893)
December 1998 options	1,171	.0018 (2.154)	3.121 (3.208)*	
		.0017 (2.133)		-.8521 (-1.642)
		.0018 (2.119)	4.252 (2.824)*	-1.689 (-1.480)

Table 5
Relationship between Implied Spot Price Changes and Actual Changes.

The following regressions are estimated using at-the-money August and December 1998 options:

$$\left(\frac{S_t^+}{S_t^-}-1\right)=a_5+b_5\left(\frac{S_t^i}{S_t^-}-1\right)+e_5,$$

where S_t^i is the implied spot price. S_t^+ denotes the spot price immediately after the option transaction at time *t* and S_t^- the spot price just prior to the option transaction. In the parenthesis are t-statistics. * represents significance at the level of 5%.

Sample	Observations	*Constant*	b_5
August option	479	-.0000 (-2.528)*	.2392 (2.193)*
December option	314	-.0000 (2.154)*	.1915 (1.908)*