AGRICULTURAL FUTURES PRICES AND LONG MEMORY PROCESSES

Anning Wei and Raymond M. Leuthold

Anning Wei
Economist
Rabobank International

And

Raymond M. Leuthold
T.A. Hieronymus Professor
Department of Agricultural and Consumer Economics
University of Illinois at Urbana-Champaign

OFOR Paper Number 00-04
April 2000

AGRICULTURAL FUTURES PRICES AND LONG MEMORY PROCESSES

ANNING WEI
Economist
Rabobank International

Raymond M. Leuthold
T.A. Hieronymus Professor
Department of Agricultural and Consumer Economics
University of Illinois at Urbana-Champaign

April 2000

AGRICULTURAL FUTURES PRICES AND LONG MEMORY PROCESSES

Abstract

Price series that are 21.5 years long for six agricultural futures markets, corn, soybeans, wheat, hogs, coffee, and sugar, exhibit time-varying volatility, carry long-range dependence, and portray excessive skewness and kurtosis, though they are covariance stationary. This suggests that the series contain nonlinear dynamics. ARCH and long memory are the two stochastic nonlinear models that are able to produce these symptoms. Though standard ARCH tests suggest that all series might contain ARCH effects, further diagnostics show that the series cannot be ARCH processes. The martingale difference null cannot be rejected by the ARCH model. Three long memory techniques, the classical R/S analysis, the modified R/S analysis, and the AFIMA model, are applied to test the martingale difference null against the long memory alternative. The nonparametric method, the classical R/S analysis, suggests there might be long memory structures in the series. However, two other more robust tests, the modified R/S analysis and the AFIMA model, confirm the case of sugar, but reject this proposition for the other five markets.

1. DEFINITIONS AND OBJECTIVES

Long memory models are relatively new to applied economists. Though its origin can go back at least to Mandelbrot's (1969) work, it was not until the 1980's that researchers began to apply the rescaled range analysis, one of the tools in long memory theory, to financial markets and macroeconomic prices. In 1991 Lo modified the classical R/S method. Also, the autoregressive fractally integrated moving average (AFIMA) process began to be accepted. In just recent years have most applications of the AFIMA model appeared. So far, no AFIMA modeling of agricultural prices exists.

Based on Beran (1994, pp. 41-66), a stationary process with long memory has the following qualitative features:

- Certain persistence exists. In some periods the observations tend to stay at high levels, in some other periods, the observations tend to stay at low levels.
- During short time periods, there seem to be periodic cycles. However, looking through the whole process, no apparent periodic cycles could be identified.
- Overall, the process looks stationary.

Quantitatively, for a stationary process, these features could be described as:

- When adding more observations, the variance of the sample mean, var($\overline{Y}$), decays to zero at a slower rate than n^{-1} which is the rate at which a white noise decays, and is asymptotically equal to a constant g times n^{-a} for some $0<a<1$.
- The correlation r_j decays to zero slowly and is asymptotically equal to a constant c times j^{-a} for some $0<a<1$.

More rigorously, the following definition exists (Beran 1994, p. 42):

"Let Y_t be a stationary process for which the following holds. There exists a real number a $\hat{I}(0,1)$ and a constant $c>0$ such that

$$\lim_{j \to \infty} r(j) / cj^{-a} = 1. \qquad (1.1)$$

Then Y_t is called a stationary process with long memory or long-range dependence or strong dependence, or a stationary process with slowly decaying or long-range correlation."

Besides its heteroscedasticity long-range dependence, long memory processes have other certain unique properties. Mandelbrot and Wallis (1969) and Mandelbrot (1972) showed a long-range dependence process could demonstrate itself as a highly non-Gaussian time series with large skewness and kurtosis, and carries nonperiodic cycles. A long memory process could allow conditional heteroscedasticity (Fung et al. 1994), which could be the explanation of nonperiodic cycles. It seems a long memory model is more flexible than an ARCH model in terms of capturing irregular behavior.

From the definition of a long memory process, *a* is the critical parameter to characterize the process, and also could be expressed by *H* (called Hurst exponent, which will be explained later) in the fashion: $a=2-2H$ (Beran 1994, p. 42). In the literature, there are three main methods existing to estimate *a* or *H*: the classical rescaled range (R/S) analysis, the modified R/S analysis, and the AFIMA model. The first two methods are mostly concerned with whether long-range dependence exists in the process being examined. An AFIMA model is the extension of an ARIMA model, and is able to measure the strength of long-range dependence.

These three methods could complement each other and allow a comparison of the robustness of the results. And furthermore, one method reveals unique information the others are not able to. Therefore, all three techniques are applied to analyze the data in this study.

Like many financial time series, agricultural futures prices exhibit irregular behavior. When conventional linear models tend to conclude that these types of price series are, or nearly are, random walks, many economic and financial theories suggest that irregular behavior might be due to nonlinear dependence in the markets.

For example, Tomek (1994) argued that most agricultural production has obvious seasonality, but consumption continues throughout the year and stocks are always nonnegative. Thus, there exists a nonlinear relationship between prices and inventories. Herner (1983) proposed a competence-difficulty gap theory. The gap between an economic agent's competence to make an optimizing decision and the difficulty of the decision problem suggest the agent follow rule-governed behavior that can be smooth sometimes and erratic at other times. Dynamically, the price movements contain nonperiodic regularities. Peters (1994) proposed the fractal market hypothesis. The large variations in agents' investment horizons produce ample liquidity in trade, which maintains the stability of markets. Short-term investors are more sensitive to technical factors in the market, but long-term investors rely more on fundamental information. When an event makes the fundamental information questionable, the investment horizons of various agents tend to unify in the short term, then the market becomes unstable and price volatile.

However, the economic theories about nonlinear dependence suggest only plausible nonlinear specifications, and the structure of nonlinear dependence is not clear. It is not uncommon that agricultural futures prices, like many other financial series: (1) are distributed nonnormally with the fat tails (Taylor 1986, Yang and Brorsen 1993), (2) possess autocorrelations that decay to zero very slowly even for a very large time period (Taylor 1986), and (3) seem to have cycles but the cycles are not periodic. As discussed above, a long memory process demonstrates good power to capture these symptoms.

Among many empirical studies of long memory, Booth et al. (1982) and Helms et al. (1984) accepted the hypothesis that the long memory process is the explanation of irregular cyclical patterns of certain financial series. On the other hand, Lo (1991) and Cheung and Lai (1993) identified little evidence of long memory in certain stock prices and gold market returns. Granger and Joyeux (1980) and Hosking (1981) developed an autoregressive fractally integrated (AFIMA) model and provided a parametric tool for long memory analysis. Fang et al. (1994) applied this method to analyze four currency futures and concluded that "statistically significant evidence of fractal structure is found in three out of four currency futures return series considered" (p. 179). However, Fung et al. (1994) observed no consistent pattern of long memory in S&P 500 index futures prices by the same method.

This study will conduct long memory tests on eighteen futures price series of six agricultural commodities, corn, soybeans, wheat, hogs, coffee, and sugar, traded at the Chicago and New York markets from January 1974 through June 1995. Each commodity has three series at daily, weekly, and monthly frequencies.

The primary objective of this study is to investigate if the price behavior in these agricultural futures markets can be characterised by long memory models. It is not hard to find evidence to argue that the price series with random appearance might be nonlinear dynamic. But, the difficulty is to tell what kind of nonlinear dynamics. Another commonly used stochastic model, the autoregressive conditional heteroscedasticity process (ARCH model) and its variants[1], shares similar symptoms with

---

[1] From now on, unless illustrated specifically, "an ARCH model" usually refers to the autoregressive conditional heteroscedasticity process and its variants. "ARCH" will be used in a broad sense.

long memory models, such as nonnormality and heteroscedasticity, but they have totally different generating mechanisms and implications[2].

A time series with the ARCH property typically has two components, a conditional mean and a conditional variance function. The nonlinearity of the series comes from the nonlinearity of conditional variances. An ARCH model that fits the data well could improve the prediction of the variances of prices but not the price itself (Bera and Higgins 1995)[3]. A long memory model approaches nonlinearity by noninteger differencing. A long memory model is a single mean equation (system) and has a flexible structure. It represents short and long memory simultaneously. This study will conduct ARCH tests before pursuing long memory analyses.

Utilizing a long time price series, such as 21.5 years, is an important attribute of the present study. Financial markets, especially those underlying agricultural markets, are very vulnerable and sensitive to exogenous shocks, such as weather changes. Therefore, there exists a tendency that many unexplained price spikes are attributed to exogenous shocks and are kept out of modeling practices. ARCH and long memory models have proven that certain stochastic behavior previously considered random shocks can be generated by well-defined processes. If the data represent a long time period, such generating processes have more chances to repeat themselves in one way or another, there are more chances for modeling practices to succeed[4].

[2] A chaotic process, a deterministic structure, also captures these symptoms. A separate study, Wei and Leuthold (1998), was dedicated to chaos tests.

[3] A process could be a pure ARCH process with the conditional mean represented by a white noise.

[4] The trade-off of using long price series is that there might be structural changes during the period of study. It has always been an empirical question whether a particular change can be considered a structural change and whether such a change has a significant impact on a particular generating mechanism. In this study, except the

This paper is organized as follows. The next section reviews the literature of long memory modeling. The third section presents the data and discusses its distribution, stationarity, and structure of autocorrelations, and then conducts ARCH tests. The fourth section investigates long memory theory. The fifth section applies long memory theory to analyze the series under study. The last section summarizes and concludes the study by discussing the significance and implications of the present research.

2. LITERATURE REVIEW

Admitting that no formal financial theory explains long memory, Helms et al. (1984) applied rescaled range (R/S) analysis to detect the existence of long memory in the futures prices of the soybean complex (soybeans, soybean oil, and soybean meal). With the Hurst exponent in the range of 0.5 to 1 indicating long memory, these authors found the Hurst exponents ranges from 0.558 to 0.711 for daily prices of two futures contracts of the soybean complex in 1976, and from 0.581 to 0.627 for intraday prices of five soybean contracts in 1977-78. Milonas et al. (1985) endorsed Helms et al.'s (1984) efforts in using the new method to model nonperiodic cycles in financial series, but pointed out that Helms et al. (1984) did not check the stationarity of the data, which reduced the credibility of the findings of Hurst exponents.

Fung and Lo's (1993) long memory study analyzed the prices of two interest rate futures markets, Eurodollars and Treasury bills. The results from the classical R/S analysis and Lo's (1991) modified R/S analysis provide no evidence of the existence of long memory and support for the weak

---

sugar market, it is not clear that the other five markets have experienced significant structural changes that are able to alter the nonlinear generating processes of concern. This issue is subject to further research.

form efficient market hypothesis[5]. Why did Helms et al. (1984) find long memory in the commodity futures markets? Fung and Lo (1993) argued, relative to interest rate futures markets, commodity futures markets have low liquidity and less active trade, which may lead to long memory.

Turning their attention to the prices of intraday stock index futures, Fung et al. (1994) examined long memory by using variance ratios[6], R/S, and autoregressive fractally integrated moving average (AFIMA) models[7]. All three types of analyses concluded that no long term memory exists in the data. Interestingly, the authors tested the impact of liquidity on the existence of long memory and found no evidence for it. Differing from their 1993 study, this time Fung et al. (1994) suspected the long memory found by Helms et al. (1984) in commodity futures prices comes from the seasonality of agricultural prices.

The AFIMA model searches for a non-integer parameter, d, to difference the data to capture long memory. The existence of non-zero d is an indication of long memory and its departure from zero measures the strength of long memory. Long memory is also called fractal structure because of non-integer d. Fang et al. (1994) investigated four daily currency futures price series, each series lasting from January 1982 through December 1991 with 2,527 observations. The estimates of d for three out of four series are significantly different from zero and fractal dynamics is concluded. It is worthy noting

[5] The modification will be discussed in next section.

[6] The variance ratio test investigates the ratio of the variance of q-differences of the series to the variance of its first differences. The test statistic could distinguish dependence from random walks and is robust to heteroscedasticity, but cannot tell short memory from long memory.

[7] The AFIMA model will be discussed in next section.

the authors admitted “little is known about the possible effects of chaotic dynamics on the statistical procedure” (p. 179).

Similar studies have been pursued on stock markets (Lo 1991, Chow et al. 1995), inflation rate (Scacciavillani 1994, Hassler and Wolters 1995), gold prices (Cheung and Lai 1993), foreign exchange rate (Booth et al. 1982), and spot and forward metals prices (Fraser and MacDonald 1992). The results are mixed, but all authors agreed that identification of long memory is very significant in at least two senses: (1) the time span and strength of long memory will be an important input for investment decisions regarding investment horizons and composition of portfolios; and (2) prediction of price movements will be improved. It is also noticeable that research methodologies have developed very fast. In the 1980’s, the classical R/S analysis was the major tool. Entering the 1990’s, the methods are being diversified with the modified R/S analysis and the AFIMA model as new techniques.

The long memory study on agricultural futures markets is at the beginning. The empirical work of Helms et al. (1984) is the only one known to us, which analyzed the short series (about 230 observations) of one commodity (the soybean complex) using only the classical R/S techniques. The present study will take advantage of the new developments in statistical methods to analyze much longer time series for six agricultural commodities at three time frequencies.

3. DATA AND DATA CHARACTERISTICS

The procedures for collecting and transforming data affect any serious statistical modeling. Also, before initiating sophisticated statistical analysis, it is important to analyze the basic properties of the data with simple methods. Therefore, this section first presents the data used in the study, and then discusses normality, stationarity, and the structure of autocorrelations and partial autocorrelations of the data. The importance of normality and stationarity is shared by many empirical studies, while the structure of autocorrelations has special significance in nonlinear dynamics modeling.

3.1 DATA SOURCES AND TRANSFORMATION

The futures prices of corn, soybeans, wheat, hogs, sugar, and coffee are selected. Choosing these six commodities covers different aspects of agricultural markets. Hogs, a livestock commodity, are nonstorable, while the other five are storable. Coffee has long production/adjustment periods, the other five have short ones. To the U.S. market, coffee and sugar are mainly import goods while the other four commodities are domestically produced and exports are important. Government policies and regulations have varying effects on these six commodity markets.

Nearby contracts are used to construct long time series[8]. Table 1 reports the contracts used for each commodity, as well as the markets where the prices were recorded. The prices are supplied by the Office for Futures and Options Research, University of Illinois at Urbana-Champaign.

[8] Geiss (1995) discussed the biasness the various methods of constructing long future prices can create. In the present study, the same empirical analysis of ARCH, long memory, and chaos have been applied to the three major transformations: differences, log differences, and the rate of returns. (See Wei and Leuthold (1998) for the chaos study.) In general, the results remained unchanged with respect to the three transformations. It seems that the nonlinear models discussed in this study are not very sensitive to these specific data transformation procedures. This nonsensitivity remains to be confirmed in a future study by adopting Geiss's method.

Table 1. Sources of the Data

Commodity	Market[1]	Contracts Used	Daily Observations	Weekly Observation s	Monthly Observations
Corn	CBOT	March, May, July, September, December	5422	1122	258
Wheat	CBOT	March, May, July, September, December	5422	1122	258
Soybeans	CBOT	January, March, May, July, August, September, November	5422	1122	258
Hogs	CME	February, April, May, July, August, October, December	5428	1122	258
Coffee	CSCE	March, May, July, September, December	5383	1122	258
Sugar	CSCE	March, May, July, September, October	5383	1122	258

1: CBOT: Chicago Board of Trade, CME, Chicago Mercantile Exchange, CSCE: Coffee, Sugar and Cocoa Exchange (New York).

The time period covers from January 1, 1974 through June 31, 1995. The beginning point of the data was set so as to avoid the collapse of Bretton Wood System in early 1970's. For each commodity, daily, weekly, and monthly prices are all investigated. The monthly data are the prices of the last day of every month, the weekly data are the Friday prices of every week, and the daily prices are closing prices of every trading day. While it is well known that a market becomes more noisy as the time frequency gets higher, the price series of three time frequencies for a given commodity essentially reflect the same market. Since chaos, bng memory and ARCH models are newly-growing fields of investigation, there are some aspects of these processes which remain unclear, applying a method to the same market but at different time frequencies will help derive robust conclusions.

Utilizing daily prices often runs encounters one problem, the limits for daily price changes, based on the closing market price of the previous day[9]. Therefore, the series are truncated and that might distort nonlinear modeling. However, the analysis on the daily series is still necessary because of the following. (1) It seems that such a truncation has no significant impacts on the nonlinear dynamics of concern. Wei (1997) and Yang and Brorsen (1992, 1993)[10], have conducted nonlinear modeling procedures on both cash and futures prices of corn, soybeans and wheat with results are not significantly different between cash and futures prices[11]. (2) The daily prices of sugar and coffee used for this study contain few daily limits, or are essentially untruncated, and later comparisons between these two markets and the other four will show little or no affect on the results due to truncation. (3) Weekly and monthly series are analyzed for each market as well, and they provide the results of untruncated series for each market. (4) Truncation is a fact of these markets and truncated markets need to be researched. The results of daily series will be interpreted as those from truncated markets. (5) All other known nonlinear modeling of daily future prices did not transform the data to avoid the effects of daily limits. The present study follows the same practice so comparisons can be made between the results of this study and those of other studies.

Heteroscedasticity is expected when examining a lifetime price series of a single contract since the variance of prices typically increases as a contract gets closer to maturity. However, if a price series

---

9 The daily price limits are 10 cents for corn, 30 cents for soybeans, 20 cents for wheat, and 150 cents for hogs (Leuthold et al. 1989, p. 35), 6 cents for coffee, and 0.5 cents for sugar (institutional database of Futures Industry Institute). The daily limits for coffee and sugar started in 1980 and do not apply to the nearest two months of a particular contract.

10 Yang and Brorsen (1992, 1993) analyzed by GARCH and chaos models the daily prices of corn, soybeans, and wheat, among other commodities, for both cash and futures markets for the period of 1979-1988.

11 Cash prices are not subject to the same daily price limits.

is constructed by various nearby contracts and each contract contributes only the section of prices when it is heavily traded, over a long time period, such as 21.5 years in this study, the "maturity effect" as such might be avoided. Nevertheless, the question of whether the variances of the data are time dependent remains for further investigation. Also, since the constructed series used here excludes the time when a contract is thinly traded, the series contains more market information.

The price series of nearby contracts has one problem, the price "jumps" when changing contracts. This study adopts a specific "roll-over" procedure to avoid the jumps. When switching contracts, on the last day of the old contract, the difference between the old contract price and the new contract price is observed[12]. Then, this difference is added or subtracted to all prices of the new contract. Table 2 illustrates this procedure by assuming March and May contracts switch at the end of February and beginning of March.

Table 2. Illustration of Rollover Procedures

	Feb. 26	Feb. 27	Feb. 28	March 1	March 2	March 3
March contract price	250	265	270			
May contract price			320	321	310	335
Adjusted series	***250***	***265***	***270***	***271***	***260***	***285***

For a 21.5-year series, many adjustments of this type take place and in some cases prices become negative. Overall, the series of price levels is not meaningful. However, price changes are accurate and without jumps, and are suitable for analysis. For example, the prices on three consecutive dates are 32, 30, and 36, respectively. Here, the two price changes are -2 and 6. If 40 were subtracted from the three price records because of contract switching, they become -8,

---

[12] Contracts are rolled forward on the last trading day of the month preceding delivery month.

-10, and -4. But the two price changes are still -2 and 6. Hence, when analyzing price changes, negative prices in levels have no impact on the analysis.

3.2 NORMALITY

After the contract rollover adjustments just described, the price changes of all six commodities, each of them with three frequencies of daily, weekly, and monthly, are produced. The term "the series" from now on always refers to the series of price changes.

Table 3 reports descriptive statistics for all eighteen series under consideration. All means are not statistically different from zero, if the standard deviation could be used to produce t-ratios[13]. However, such a standard t test could not be conducted because the unconditional distributions of all series except hogs are nonnormal--skewed and leptokurtic as discussed below.

The coefficients of skewness g_1 and excess kurtosis g_2 quantify the deviation from a normal distribution and are defined by Smillie (1966). g_1 and g_2 are standard normal distributions with the mean of zero. Jarque and Bera (1980) developed an O statistic with a χ^2 distribution to summarize the deviation from a normal distribution.

Except for the hog series, the remaining fifteen series are far from the normal distributions. The coefficients of skewness and kurtosis are strongly statistically significant and indicate that the

[13] The series studied here are price changes, not the rate of returns. The fact that the means of the series of price changes are equal to zero only implies that the price level has not changed over the period under study. No reference could be derived to say whether the expected return from futures trading is zero.

distributions of the price change series are skewed and have fat tails[14]. Jarque and Bera's (1980) χ^2 statistics, which summarize the deviation of the third and fourth moments from the parameters of a normal distribution, are strongly significant as a result. The significant deviation from normality can be a symptom of nonlinear dynamics (Fang et al. 1994).

Across the time frequencies, daily data depart further from a normal distribution than weekly and monthly data. Across the commodities, the deviation of coffee and sugar from a normal distribution are more severe than that of corn, wheat, and soybeans. Hog data are close to normal distributions at all three time frequencies. Hogs are the only nonstorable commodity here. As discussed by Leuthold et al. (1989, pp. 45-60), markets of nonstorables have no storage costs to hold or link the prices of spot and futures together, and to hold or link prices of different futures together, which differs from the markets of storables. The movements of prices of nonstorables are expected to demonstrate more independence than those of storables.

3.3 STATIONARITY

Besides normality, another important property of the data is stationarity. As common practice, covariance stationarity or weak stationarity is of concern here. The conventional augmented Dickey-Fuller test (***t*** statistic) is used first. The weakness of the augmented Dickey-Fuller test is that error terms of the test model are assumed a white noise process (Phillips and Perron 1988). It has been observed that the price change series under study very likely contain heteroscedasticity. Phillips and

14 This result differs from that of Taylor (1986). In his study, Taylor found that the rate of return of 13 daily agricultural futures prices (corn, cocoa, coffee, sugar and wool) are approximately symmetric, though they have high kurtosis.

Perron (1988) proposed a semi-parametric test (*Z statistic)* that allows for a wide range of serial correlation and heteroscedasticity. The Phillips-

Table 3. Descriptive Statistics of Price Changes

	Mean[1]	St. Dev[2]	g_1 (t-ratio)	g_2 (t-ratio)	*O*
Monthly					
Corn	-1.58	17.95	1.31 (8.73)	8.22 (27.43)	765.9
Soybeans	-3.80	54.22	0.66 (4.13)	7.46 (24.86)	587.7
Wheat	-2.14	25.16	-0.52 (-3.46)	3.08 (10.27)	107.8
Hogs	0.24	3.43	-0.22 (-1.47)	0.50 (1.66)	4.49
Coffee	0.94	15.30	1.11 (7.33)	4.29 (14.3)	239.6
Sugar	-0.13	2.25	0.68 (4.53)	10.18 (33.9)	1081
Weekly					
Corn	-0.317	7.62	0.14 (2.00)	4.89 (33.49)	1108.9
Soybeans	-0.798	24.31	-0.32 (-4.45)	5.08 (34.79)	1215.3
Wheat	-0.437	11.65	0.143 (1.96)	4.28 (29.31)	837.3
Hogs	0.062	1.46	-0.045 (-0.62)	0.38 (2.53)	6.96
Coffee	0.21	7.06	1.13 (17.94)	13.82 (94.56)	9079
Sugar	-0.02	1.08	-0.52 (-7.43)	14.38 (98.22)	9620
Daily					
Corn	-0.070	3.378	-0.072 (-2.18)	3.30 (50.06)	2464.3
Soybeans	-0.171	10.28	-0.171 (-5.27)	2.89 (43.82)	1910.1
Wheat	-0.100	5.44	-0.002 (-0.06)	2.92 (44.24)	1920.6
Hogs	0.011	0.68	-0.044 (-1.33)	-0.133 (-2.21)	5.83
Coffee	-0.046	3.10	1.098	21.022	99984

			(33.27)	(318.5)	
Sugar	-0.005	0.49	-0.415 (-12.57)	11.207 (169.8)	28262
Critical Value 5% sig.			*1.96*	*1.96*	*3.84*

1: The units for corn, soybeans, and wheat are cents per bushel, for hogs, coffee and sugar are cents per pound.
2: St. Dev: Standard deviation.

Perron *Z* test has the same asymptotic distribution as does the Dickey-Fuller ***t*** test, and both share the same critical values.

In both cases, augmented Dickey-Fuller and Phillips-Perron tests, the length of lag of the series needs to be determined to ensure the serial correlation of the process can be removed. Diebold and Nerlove (1990) found that the integer part of $T^{0.25}$ works well in determining the length of lag in practice[15]. Since there are 5,383 to 5,427, 1,121, and 257 observations available for daily, weekly, and monthly data, respectively, *p* and *l* will be 8, 5, and 4 for daily, weekly, and monthly series[16].

Table 4. Augmented Dickey-Fuller (t) and Phillips-Perron (*Z*) Tests*

	Corn		Soybeans		Wheat		Hogs		Coffee		Sugar	
	τ	***Z***	τ	***Z***	τ	***Z***	τ	***Z***	τ	***Z***	τ	***Z***
Monthly	-7.58	-16.3	-8.38	-16.7	-6.56	-13.9	-7.94	-16.2	-6.98	-14.9	-7.12	-11.3
Weekly	-13.3	-31.9	-13.3	-33.5	-12.4	-33.9	-13.5	-30.2	-13.6	-34.2	-11.8	-30.3
Daily	-23.4	-69.8	-23.7	-69.8	-23.4	-73.7	-22.1	-73.5	-21.6	-69.5	-23.9	-69.4

The critical value at 5% significance is 2.86.
* τ and ***Z*** are t-type calculated statistics (see Wei 1997).

15 Actually AIC's and SC's are very flat when varying *p* and *l* in the experimental estimation.

16 The experimental estimations had been conducted, where *p* and *l* were specified from 3 to 12, but the results reported in Table 4 remained unchanged, i.e., the null hypotheses of the existence of unit roots could be rejected at 5% significance level.

The results from Table 4 indicate all the series are stationary and do not contain unit roots. Phillips-Perron *Z* statistics, which relaxes the assumption that error terms have to be white noise, are usually more than twice as large as the Dickey-Fuller τ statistics.

The magnitudes and t's and Z's are similar for all commodities for a given time frequency. The hog series, which looks more stationary than others, do not carry larger calculated statistics than others. For a given commodity, high time frequency series are more "stationary" than low time frequency series.

3.4 STRUCTURE OF AUTOCORRELATIONS

For a linear time series model, typically an autoregressive integrated moving average (ARIMA(p,d,q)) process, the patterns of autocorrelations and partial autocorrelations could indicate the plausible structure of the model. At the same time, this kind of information is also very important for modeling nonlinear dynamics. In Taylor's (1986) study, the long lasting autocorrelations of the data suggest that the processes are nonlinear with time-varying variances. The basic property of a long memory process is that the dependence between the two distant observations is still visible.

For six series of daily price changes, 200 autocorrelations and partial autocorrelations were estimated, i.e., j=1,...,200. For 6 series of weekly price changes, 100 autocorrelations and partial autocorrelations were estimated, i.e., j=1,...,100. For 6 series of monthly price changes, 48 autocorrelations and partial autocorrelations were estimated, i.e., j=1,..., 48.

Four features of the structures of autocorrelations and partial autocorrelations emerge for all eighteen series. First, the magnitude of autocorrelations and partial autocorrelations is very small. In terms of absolute values, the largest autocorrelations and partial autocorrelations are about 0.06 for daily series, 0.10 for weekly series, and 0.20 for monthly series. For conventional linear models, this means the dependence among the elements is weak.

Second, the first autocorrelations and partial autocorrelations for all eighteen series are not significantly larger than the remaining coefficients, and in most cases, they are not even the largest. The first several, usually the second, autocorrelations and partial autocorrelations slightly exceed the significant boundary defined as $1/T^{0.5}$. There are some coefficients at much later time lags that exceed the significant boundary to the same extent. This indicates the dependence between nearby observations is not necessarily stronger than that between distant observations, or the most recent market information is not necessarily more useful than the information from a while ago.

Third, there is no evidence that the magnitude of autocorrelations and partial autocorrelations become small as the time lag, j, becomes large. 200 days, 100 weeks, and 48 months are significant time lags for daily, weekly and monthly series, respectively. Even so, the magnitude of autocorrelations and partial autocorrelations at the end of the above time lag sequences are almost as large as those at the beginning. Roughly it can be argued that the importance of market information does not decay as the time the information was collected spans.

Fourth, there are no clear patterns describing the fluctuation of autocorrelations and partial autocorrelations. No seasonal and other periodic cycles were observed.

To demonstrate the above features, Tables 5 through 7 report the 10 largest (in terms of absolute values for negative estimates) autocorrelations of each series and their time lags. The tables demonstrate the irregular patterns of autocorrelations of the data as discussed above, i.e., their magnitudes are small and relatively independent of the length of time spans, they do not decay exponentially over time span, and they show no clear periodic patterns. The partial autocorrelations have the same characteristics as just described, and the results are not shown.

Table 5. The Largest 10 Autocorrelations of Daily Series

	Lag	Corn	Lag	Soybeans	Lag	Wheat	Lag	Hogs	Lag	Coffee	Lag	Sugar
Largest 5	145	-0.050	147	-0.045	2	-0.051	2	-0.053	29	-0.051	143	-0.078
negative	2	-0.044	89	-0.043	52	-0.039	129	-0.040	55	-0.049	68	-0.070
values[1]	128	-0.036	116	-0.042	143	-0.038	68	-0.032	104	-0.044	17	-0.068
	60	-0.035	63	-0.042	60	-0.035	167	-0.030	37	-0.041	37	-0.063
	39	-0.033	60	-0.038	120	-0.032	92	-0.030	62	-0.038	88	-0.056
Largest 5	155	0.035	141	0.031	187	0.033	43	0.038	8	0.052	12	0.050
positive	61	0.035	61	0.032	180	0.034	38	0.041	26	0.052	24	0.052
values	12	0.036	18	0.036	29	0.035	14	0.042	1	0.054	61	0.056
	1	0.053	43	0.038	62	0.036	191	0.049	44	0.054	1	0.059
	7	0.063	1	0.054	84	0.048	4	0.051	9	0.074	8	0.061

1: In terms of absolute values.

Table 6. The Largest 10 Autocorrelations of Weekly Series

	Lag	Corn	Lag	Soybeans	Lag	Wheat	Lag	Hogs	Lag	Coffee	Lag	Sugar
Largest 5	30	-0.072	15	-0.076	14	-0.080	63	-0.078	13	-0.076	31	-0.120
negative	49	-0.061	68	-0.070	30	-0.080	66	-0.074	51	-0.063	9	-0.112
values[1]	67	-0.059	30	-0.067	35	-0.075	68	-0.074	4	-0.060	46	-0.108
	79	-0.052	18	-0.066	67	-0.073	14	-0.070	20	-0.052	43	-0.106
	31	-0.050	14	-0.063	62	-0.072	54	-0.069	68	-0.051	30	-0.080
Largest 5	82	0.048	89	0.045	38	0.069	11	0.074	44	0.055	15	0.090
positive	1	0.053	42	0.046	42	0.073	41	0.078	9	0.074	37	0.091
values	32	0.053	32	0.049	55	0.073	48	0.083	7	0.085	39	0.102
	46	0.057	9	0.059	2	0.089	1	0.104	59	0.093	1	0.109
	3	0.089	3	0.081	5	0.091	50	0.109	2	0.161	2	0.111

1: In terms of absolute values.

Table 7. The Largest 10 Autocorrelations of Monthly Series

	Lag	Corn	Lag	Soybeans	Lag	Wheat	Lag	Hogs	Lag	Coffee	Lag	Sugar
Largest 5	7	-0.118	3	-0.140	6	-0.124	15	-0.160	43	-0.146	6	-0.196
negative	16	-0.096	36	-0.133	39	-0.120	13	-0.153	15	-0.128	23	-0.107
values[1]	15	-0.078	7	-0.125	7	-0.118	25	-0.125	12	-0.108	7	-0.101
	35	-0.073	24	-0.107	34	-0.101	24	-0.122	26	-0.078	5	-0.089
	39	-0.072	35	-0.094	48	-0.084	16	-0.118	21	-0.075	10	-0.085
Largest 5	43	0.049	21	0.056	5	0.093	19	0.102	8	0.086	3	0.059
positive	44	0.060	43	0.058	1	0.111	9	0.120	20	0.101	36	0.064
values	2	0.069	2	0.063	20	0.129	41	0.120	9	0.102	20	0.075
	29	0.085	32	0.072	9	0.133	11	0.136	2	0.125	4	0.127
	31	0.101	29	0.121	10	0.209	21	0.179	28	0.152	1	0.330

1: In terms of absolute values.

3.5 SUMMARY OF BASIC DATA CHARACTERISTICS

The data used in this study are 21.5 years of futures prices of six agricultural commodities (corn, soybeans, wheat, hogs, coffee, and sugar) at monthly, weekly, and daily frequencies. For each commodity, various nearby contracts are used to construct a long series and a special rollover procedure is followed to remove jumps when switching contracts. The series of price differences are the modeling targets of the study.

All series for six commodity markets demonstrate uneven volatility over the whole sample period. In some time periods the series vary more dramatically than in other periods. This implies time-dependent conditional variances. Most series are found non-normally distributed with excessive skewness and kurtosis. Even so, the unit root tests suggest unconditional mean and variance of the data are finite and constant. The autocorrelation and partial autocorrelation analysis shows that the short-term dependence is obviously weak, but the autocorrelations, though they are very small, are very persistent. All of these symptoms potentially suggest nonlinear dynamics, as Taylor (1986) argued.

The conclusion of stationarity in the above is subject to one risk, i.e., the series contain the roots that are, though not exactly units, close to units, and the unit root tests conducted here are not powerful enough to differentiate. However, long memory analysis to be conducted disproves this possibility.

3.6 ARCH TESTS

ARCH theory admits nonnormality of the unconditional distribution of the data. With the assumption of normality of the conditional distribution, an ARCH-type structure could be built to capture the time-dependent variances. Using such a variance function as an input, the maximum likelihood estimates of mean become consistent and efficient. Financial series are typically found non-normally distributed with the time-varying volatility. Therefore, ARCH models have become very popular in financial time series modeling.

Volatility clustering and nonnormality were uncovered from the data, which may lead to the ARCH structures. Further, screening the plots of all eighteen series of price changes for six commodity futures, it is obvious that the volatility is not stable. In certain time periods, large price changes are followed by large price changes, irrespective of sign, and form of spikes. In some other time periods the markets are rather quiet, i.e., small price changes are followed by small price changes. This phenomenon seems well represented by an ARCH process as defined below.

Suppose a stochastic process Y_t is generated by an AR(p) process:

$$Y_t = \alpha_0 + \sum_{j=1}^{P} \alpha_j Y_{t-j} + \varepsilon_t \, . \tag{3.1}$$

There exists an information set, $\Psi_{t-1} = \{Y_{t-1}, Y_{t-2}, \ldots..\}$, such that:

$$\varepsilon_t | \Psi_{t-1} \sim N(0,h), \tag{3.2}$$

where

$$h_t = \theta_0 + \sum_{i=1}^{k} \theta_i \varepsilon^2{}_{t-i} \tag{3.3}$$

with

$$\theta_0 > 0,\ \theta_i \geq 0,\ i=1,..k, \tag{3.4}$$

to ensure the conditional variance is positive. The process Y_t is called AR(p) with ARCH(k) errors.

The conditional variance function might be complicated for a long series lasting for 21.5 years, which means k in equation (3.3), the lag of the conditional variance ε^2, is large. If so, the computation becomes burdensome and interpretation becomes difficult. Bollerslev (1986) proposed generalized ARCH (GARCH):

$$h_t = \theta_0 + \sum_{i=1}^{k} \theta_i \varepsilon^2_{t-i} + \sum_{j=1}^{g} \delta_j h_{t-j} \tag{3.5}$$

where h_{t-j} are lagged unconditional variances, and

$$\theta_0 > 0,\ \theta_i \geq 0,\ i=1,..k,\ \delta_i \geq 0,\ j=1,..g, \tag{3.6}$$

to ensure positive variances. Actually, GARCH(k,g) is an infinite order ARCH process with a rational lag structure imposed on the coefficients. The equation (3.5) is readily interpreted as an ARMA model for ε^2. In practice, identification of k and g follows Box-Jenkins procedure to specify p and q in ARMA(p,q).

The standard Lagrange Multiplier (LM) test is applied to all eighteen series to test whether there are ARCH(1) effects in the processes. Since ARCH(1) is the simplest structure of ARCH and its variants, if ARCH(1) exists, further investigation of more suitable ARCH structures is encouraged. In the case of the stationarity test, AR(4), AR(5), and AR(8) are specified to capture autocorrelations for monthly, weekly, and daily series, respectively. The LM test is conducted on the residuals of those AR models for ARCH(1) effects. Table 8 reports the results.

Table 8. LM Tests of ARCH(1) Effects

	Monthly	Weekly	Daily
Corn	10.7	145.5	952.6
Soybeans	20.1	116.1	941.9
Wheat	11.7	87.3	531.8
Hogs	4.9	7.7	60.8
Coffee	12.5	15.8	134.1
Sugar	7.7	54.2	857.9
Critical values (5%)	*3.84*	*3.84*	*3.84*

The null hypothesis is, Y_t carries no ARCH. The alternative is, Y_t carries ARCH.

The critical value is the χ^2 distribution with 1 degree of freedom. All calculated LM statistics are larger than the critical value. The null hypothesis has been rejected in all eighteen cases. And, the higher the time frequency, the more the calculated LM statistics exceeds the critical value. Among the six commodities, hogs are noticeably much less "ARCH" than others. Therefore, GARCH modeling proceeds.

The analysis of autocorrelation and partial autocorrelation of all eighteen series indicates that short memory of the data is very weak and does not have clear patterns. As a preliminary effort, ARMA(1,1) is estimated for all eighteen series[17], i.e.

$$Y_t = \boldsymbol{a}Y_{t-1} + \boldsymbol{e}_t + \boldsymbol{be}_{t-1}. \qquad (3.7)$$
$$\boldsymbol{e}_t \sim N(0, \boldsymbol{s}).$$

Since this study employs price differences, i.e., Y_t is the series of price differences, the above ARMA(1,1) is equivalent to ARIMA(1,1,1).

The estimation of (3.7) is shown in Table 9.

[17] Similar practice was also done by Fang et al. (1994), though in their case, no significant short-term dependence is found in 2,527 daily currency futures prices, so AR(3) is still used as a filter.

Table 9. Estimates of ARMA(1,1)

	α		β		R^2	Likelihood
	Coefficient	t-value	Coefficient	t-value		
Monthly						
Corn	-0.52	-0.38	-0.49	-0.35	0.00	-1102
Soybeans	-0.92	-21.39	-0.93	-2.14	0.06	-1378
Wheat	-0.52	-2.10	-0.66	-3.04	0.05	-1183
Hogs	0.51	0.06	0.50	0.06	0.02	-676
Coffee	0.54	1.25	0.45	0.98	0.01	-1061
Sugar	-0.01	-0.09	-0.41	-2.85	0.13	-553
Weekly						
Corn	0.69	3.37	0.64	2.88	0.01	-3860
Soybeans	-0.09	0.00	-0.09	0.00	0.00	-5163
Wheat	-0.57	-0.89	-0.55	-0.83	0.01	-4336
Hogs	0.54	2.72	0.44	2.09	0.00	-2011
Coffee	-0.57	-2.01	-0.51	-1.72	0.00	-3776
Sugar	0.73	7.72	0.68	6.71	0.08	-3739
Daily						
Corn	-0.68	-6.89	-0.73	-7.97	0.00	-14276
Soybeans	-0.24	-1.06	-0.29	-1.32	0.00	-20315
Wheat	-0.51	-0.94	-0.52	-0.98	0.00	-16872
Hogs	-0.59	-1.65	-0.61	-1.74	-0.01	-5627
Coffee	-0.14	-0.58	-0.19	-0.81	0.00	-13718
Sugar	-0.24	-1.06	-0.29	-1.32	0.00	-20315

Not surprisingly, model (3.7) performs very poorly. Either across three time frequencies or across six commodities, the model has little explanatory power as indicated by the R^2's, which are all around zero though the estimates of α and β are statistically significant for some series. Other more complicated ARMA(p,q) structures have been experimented with, and the results remain similar. It seems that the mean function of the series is white noise. Further examination is pursued to determine the autocorrelations and partial autocorrelations of e^2 in equation (3.7) in order to specify k and g of GARCH(k,g). Following Box-Jenkins procedures, the autocorrelations of e^2 are estimated to 24 lags and partial autocorrelations are estimated up to 12 lags. Table 10 displays the results.

Table 10. The Structures of Autocorrelations and Partial Autocorrelations of e^2 Series

	No. of significant AC	Largest AC (lag)	No. of significant PAC	Largest PAC (lag)

Monthly				
Corn	1	0.14(1)	1	0.14(1)
Soybeans	2	0.28(1)	1	0.28(1)
Wheat	unclear	0.21(3)	unclear	0.20(3)
Hogs	unclear	0.17(14)	unclear	0.16(1)
Coffee	unclear	0.22(1)	unclear	0.22(1)
Sugar	unclear	0.42(3)	unclear	0.38(3)
Weekly				
Corn	7	0.40(1)	5	0.40(1)
Soybeans	11	0.31(1)	7	0.31(1)
Wheat	unclear	0.29(1)	unclear	0.29(1)
Hogs	unclear	0.11(2)	unclear	0.10(2)
Coffee	unclear	0.41(2)	unclear	0.41(1)
Sugar	unclear	0.41(6)	unclear	0.27(6)
Daily				
Corn	Unclear	0.43(1)	unclear	0.34(1)
Soybeans	Unclear	0.42(1)	unclear	0.42(1)
Wheat	Unclear	0.32(3)	unclear	0.32(3)
Hogs	Unclear	0.17(14)	unclear	0.16(1)
Coffee	Unclear	0.34(9)	unclear	0.29(9)
Sugar	Unclear	0.43(2, 3)	unclear	0.40(1)

AC and PAC are autocorrelation and partial autocorrelation, respectively.

First of all, for all eighteen series the magnitudes of autocorrelations and partial autocorrelations are noticeable, the maximum is 0.43. However, except for monthly corn and soybean data, e^2 for the other 16 series decay to zero very slowly. The autocorrelations and partial autocorrelations of many e^2 series are still significant even at 24 or 12 time lags, especially for weekly and daily series. And, most e^2 series do not have clear decay patterns, the values of autocorrelations and partial autocorrelations exceed the significant-level boundary randomly. As a reflection of this, for many e^2 series, the largest autocorrelations and partial autocorrelations are not necessarily located at lag 1. Table 10 summarizes these observations, in which "unclear" refers to either of two or both situations: within 24 or 12 time lags autocorrelations or partial autocorrelations do not decay to zero, or/and autocorrelations and partial autocorrelations break significance boundaries randomly, and no judgment can be made about whether autocorrelations and partial autocorrelations have statistically decayed to zero.

This proposes significant difficulty for specifying the structure of ARCH(k) or GARCH(k, g). For monthly corn and soybean data, GARCH(1,1) might be sufficient, but for remaining sixteen series, no structures are suggested. French et al. (1987) modeled 57 years (1928-84) daily S&P stock index data with 15,369 observations, and GARCH(2,2) was found proper. For most financial data, GARCH(1,1) has proved to be sufficient (Bollerslev et al. 1992, Bera and Higgins 1995). The situation of the present data causes suspicion about whether either ARCH or GARCH is a proper alternative.

It might be argued that wrongly-specified ARMA(1,1) is responsible for the unclear structure of the autocorrelations and partial autocorrelations of the e^2 series, since ARMA(1,1) has little explanatory power in all cases. The series might be pure ARCH or GARCH processes where the conditional mean is simply $Y_t = e_t$. Accepting this reasoning, the structure of autocorrelation and partial autocorrelation of Y^2 has been analyzed.[18] However, the pictures described in Table 10 remain the same.

The present study is not alone with the frustration of ARCH/GARCH models in analyzing long agricultural price series. Yang and Brorsen (1992) analyzed daily cash prices of corn, pork bellies, soybeans, sugar and wheat, plus gold and silver from January 1979 through December 1988. Though they concluded that the GARCH model is a better explanation of the observed nonlinear dynamics, they admitted "though GARCH models reduced serial dependence and leptokurtosis, they are not a well-calibrated model of the process that generated the sample observations in spot commodity markets" (p. 714). Similarly, Yang and Brorsen (1993) applied GARCH and other nonlinear models to 15 daily future prices including corn, soybeans, wheat, coffee and oats. The data period is from January 1979

[18] These results are available from the authors upon request, or in Wei (1997).

through December 1988. The conclusion is similar to their study about cash markets. They found GARCH(1,1) was rejected by Kolmogorov-Smirnov test of fit for all cases, and they felt a higher order GARCH process may be appropriate for some prices.

This study's data period doubles the length of Yang and Brorsen's sample, which may introduce many new features into the data. One of them might be that the autocorrelations of the e_t^2 process of the series decay so slowly that a GARCH model finally becomes totally inadequate to the data generating processes.

Also, as Bera and Higgins (1995, p. 224) discussed, the third moment for a regular ARCH process is zero, therefore unconditional distributions of the series must be symmetric. The conventional ARCH or GARCH model is able to capture the excessive kurtosis, but not asymmetry.

The series under study demonstrate obvious asymmetry. The asymmetric distribution was considered related to "leverage effects" in financial markets, i.e., the return of a financial asset is negatively associated with its volatility. Volatility tends to rise in response to lower returns than expected, and tends to fall in response to higher returns than expected (Black 1976, Christie 1982). The conventional ARCH/GARCH process is not able to capture such an asymmetric phenomenon.

Nelson (1990) developed an exponential GARCH model to allow asymmetry. Kang and Brorsen (1995) applied the exponential GARCH method to daily futures prices of wheat for the period of January 1980 through September 1990, and found the model fit the data to certain degrees. But, with the "strange" ε^2_t process as described in Table 10, the exponential GARCH has little role to play.

4. THEORY OF THE LONG MEMORY MODEL

The plots of all eighteen series of price changes of the agricultural futures markets exhibit distinct, but nonperiodic cycles. In a long memory model, nonperiodic cycles are considered the effects of long-range dependence in the data, i.e., the autocorrelations of a given series decay to zero very slowly as the time lag increases, and the dependence is still visible even after a very large time span, though individually all measurements of dependence are quite small. After examining general properties of the data and rejecting the ARCH hypothesis, now the long memory hypothesis will be tested:

H_o: $E[\Delta Y_t \mid Y_{t-j},\ j \geq 1\ ]=0$, where $\Delta Y_t = Y_t - Y_{t-1}$
H_a: $Y_t = f_2(I_t)+\varepsilon_t$, where, f_2 is a long memory structure defined by equations (1.1). I_t is the information set at t and for a univariate series is typically Y_{t-j}, ε_{t-j}, $T \geq j \geq 0$.

4.1 THE CLASSICAL AND MODIFIED RESCALED RANGE ANALYSIS

H, a Hurst exponent, is produced by the rescaled range analysis, or R/S, analysis which was established by hydrologist H. E. Hurst in 1951, further developed by B. Mandelbrot in the 1960's and 1970's, and applied to economic price analysis by Booth et al. (1982), Helms et al. (1984), Peters (1989), and others in the 1980's. For a given time series, the Hurst exponent measures the long-term nonperiodic dependence, and indicates the average duration the dependence may last.

For a time series with total observations of *T*, there is a integer *n*, $n \leq T$. The R/S analysis, first estimates the range *R* for a given *n*:

$$R(n) = (\underset{1\leq j\leq n}{Max}\sum_{j=1}^{n}(Y_j - \overline{Y}) - \underset{1\leq j\leq n}{Min}\sum_{j=1}^{n}(Y_j - \overline{Y})), \tag{4.1}$$

where *R(n)* is the range of accumulated deviation of *Y(t)* over the period of *n*. Let *S(n)* be the standard deviation of Y_t over the period of *n*, i.e.:

$$S(n) = (\frac{1}{n}\sum_{j=1}^{n}(Y_j - \overline{Y})^2)^{0.5}.$$

For a given *n*, there exists a statistic

$$Q(n) = R(n)/S(n)\ . \qquad (4.2)$$

It is clear that *n* is the time scale to split total observations *T* into *int[T/n]* segments where *int[.]* denotes the integer part of *[.]*. There will be *int[T/n]* estimates of *R(n)/S(n)* for a given *n*. The final *R(n)/S(n)* is the average of *int[T/n]'s R(n)/S(n)*. As *n* increases, the following holds:

$$R(n)/S(n) = \alpha n^H \text{ or,}$$

$$\log(R(n)/S(n) = \log \alpha + H\log(n)\ . \qquad (4.3)$$

It is clear that *H* is a parameter that relates mean R/S values for subsamples of equal length of the series to the number of observations within each equal length subsample. *H* is always greater than 0. When 0.5<H<1, the long memory structure exists. If H≥1, the process has infinite variance and is nonstationary. If 0<H<0.5, anti-persistence structure exists. If H=0.5, the process is white noise.

While estimating *H*, equation (4.3) also generates the average length of nonperiodic cycles by varying the size of *n*.

Equation (4.2) is first estimated for the small sample in which *n* goes from an initial value to a small number, then a *H* is recorded. The second round estimation is conducted with a larger sample, i.e., *n* goes from an initial value to a larger number, and a new *H* is recorded. This process continues until *H* reaches a peak and begins to decline. When *H* reaches the peak, the ending point of sample size, that particular n would be an indication of the average length of nonperiodic cycles, let's say G, and the saturated H is the final estimation of the Hurst exponent.

Further, Peters (1994) proposed another method, still based on the R/S analysis, to identify the average length of nonperiodic cycles. For various values of n, there is a corresponding series of V, which is defined as

$$V(n)=(R(n)/S(n))/n^{0.5}. \tag{4.4}$$

For a long memory process, to plot $V(n)$ against $log(n)$ will find a turning point in the V curve from where the slope of V becomes zero or even negative. Here only visual screening is applied. Peters (1994) showed V statistics are more noise-resistant than the procedure of H estimation in performing this function.

The greatest advantage of the R/S analysis is that the measure is independent of the distribution assumption for a given series. The robustness of results remains unaffected regardless whether the distribution is normal or nonnormal. The dependence the Hurst exponent captures is the nonlinear relationships inherent in the structure of the series (Peters 1991).

Lo (1991) and Chueng and Lai (1993) found that Hurst and Mandelbrot's R/S analysis (called the classical R/S analysis hereafter) will generate a biased Hurst exponent when any combination of the following happens: (1) the series contains short-term memory; (2) the series is characterized with heterogeneities, and (3) the series is nonstationary. Lo (1991) modified the above R/S method and produced a new statistic that is robust to (1) and (2), and with a well-defined distribution.

Lo's (1991) modification was made to *S(n)* in equation (4.2), i.e., *Q(n) = R(n)/S(n)*. In Lo's algorithm, *S(n)* is replaced by $S(n)_q$.

$$(S(n)_q)^2 = (S(n))^2 + (\frac{2}{n}\sum_{i=1}^{q} w_i(q)\{\sum_{j=i+1}^{n}(Y_j - \overline{Y})(Y_{j-i} - \overline{Y})\}, \tag{4.5}$$

where $w_i(q) = 1 - \frac{i}{q+1}$, q<n.

Modified statistic is :

$$Z(n)=R(n)/S(n)_q. \tag{4.6}$$

Both *S(n)* and $S(n)_q$ measure the standard deviation of the partial sum, but in $S(n)_q$*'s* regime, the variance of the partial sum is not simply the sum of the variance of individual observations, but also includes the weighted autocovariances up to lag q. Weight $w_i(q)$ is suggested by Newey and West (1987) to ensure positive $S(n)_q$. q is the optimal lag of autocovariances and is typically determined by Andrew's (1991) data dependent rule:

$$q = \text{int}[(3T/2)^{1/3}\{2d/(1-d^2)\}^{2/3}], \tag{4.7}$$

where *int[.]* denotes the integer part of *[.]* and d is the first-order autocorrelation coefficient of the data.

Therefore, comparing with the classical R/S statistic *Q(n)*, the modified statistic *Z(n)* is able to isolate short-range dependence when detecting a long-range one. Also *Z(n)* has the well defined $F_V(v)$ distribution, which could lead to statistical inference (Lo 1991, pp. 1291-93).

However, in *Z(n)'s* estimation, $n=T$, and the H exponent as well as the average length of nonperiodic cycle, G, could not be derived. The classical R/S analysis contains more information than Lo's new method.

4.2 THE AFIMA MODEL

The AFIMA model approaches the long memory process from different perspectives. In an ARIMA(p,d,q) process,

$$\Phi(B)(1-B)^d Y_t = \Psi(B)e_t, \tag{4.8}$$

d is an integer, and typically either 0 or 1. When Y_t is integrated of the order 1, i.e. $d=1$, the process is said to be nonstationary and its autocorrelation decays to zero linearly. When Y_t is integrated of the order 0, i.e., $d=0$, the process is said to be stationary and its autocorrelation decays to zero exponentially. Hence, observations separated by long-time spans are independent.

Eighteen series of price differences in this study satisfy the assumption of stationarity according to the unit root tests, as shown earlier. However, all eighteen series exhibit dependence between distant observations that, although small, is by no means negligible. There are two possibilities here. First, the series do not contain exact unit roots, but the roots they have are very close to a unit. Unit root tests have been fooled. Second, the results of unit root tests are correct in that the series contain no unit roots, but the series carry long-range dependence, which can hardly be represented by regular ARMA processes. If one differences the series again, the series will be over-differenced.

An AFIMA model is able to distinguish these two possibilities. In this case, Granger and Joyeux (1980) and Hosking (1981) suggested *d* in equation (4.8) to be extended to a non-integer.

$\Phi(B)(1-B)^d Y_t = \Psi(B)e_t$, where *d* is noninteger, and

$$(1-B)^d = 1 - dB - \frac{d(1-d)}{2!}B^2 - \frac{d(1-d)(2-d)}{3!}B^3 - \ldots \quad . \tag{4.9}$$

For $0<d<0.5$, it can be shown that the autocorrelations of the process decay to zero hyperbolically, i.e., at a much slower rate than the exponential decay of an ARMA(d=0) process. Then, AFIMA(p,d,q) is able to represent existing long-memory structure without over-differencing.

If $d \geq 0.5$, the process has infinite variance and is nonstationary. The nonstationarity missed by conventional unit root tests is captured by the AFIMA model. If $-0.5<d<0$, an anti-persistence structure exists. If $d=0$, the process is white noise.

Lo (1991, p. 1285) used the following Table 11 to illustrate the property of the long memory process, in which AR(1) is compared with two AFIMA processes. For AFIMA(0,d=1/3,0), the dependence, indicated by autocorrelation *r(j)*, lasts for very long; for AFIMA(0,d=-1/3,0), *r(j)* decays to zero very fast like a regular AR process. For example, at lag 10, *r(j) of* AFIMA(0,d=1/3,0) is as high as 0.235, *r(j)* of AFIMA(0,d=-1/3,0) and AR(1) are -0.005 and 0.001, respectively. At lag 100, *r(j) of* AFIMA(0,d=1/3,0) is still as high as 0.109, but *r(j) of* AFIMA(0,d=-1/3,0) and AR(1) are not different from 0.

Table 11. The Comparison Between AR(1) and Two AFIMA Processes

Lag	ρ(j)	ρ(j)	ρ(j)
j	AFIMA(0, d=1/3, 0)	AFIMA(0, d=-1/3, 0)	AR(1) ρ=0.5
1	0.500	-0.250	0.500
2	0.400	-0.071	0.250
3	0.350	-0.036	0.125
4	0.318	-0.022	0.063
5	0.295	-0.015	0.031
10	0.235	-0.005	0.001
25	0.173	-0.001	$2.98x10^{-8}$
50	0.137	$-3.24x10^{-4}$	$8.88x10^{-16}$
100	0.109	$-1.02x10^{-4}$	$7.89x10^{-31}$

Source: Lo (1991, p.1285, Table 4.1)

Many estimators have been suggested to estimate d (see Beran (1994) for a comprehensive review). The most robust estimator is the maximum likelihood estimator suggested by Haslett and Raftery (1989). Suppose an AFIMA(p,d,q) is defined by (4.9) with innovations ε_t being independent Gaussian random variables. Let

$$\hat{Y}_t^{t-1} = E(Y_t|Y_1,\ldots,Y_{t-1,}\boldsymbol{f}_1,\ldots,\boldsymbol{f}_p,d,\boldsymbol{j}_1,\ldots,\boldsymbol{j}_q)$$

denote the conditional mean one-step-ahead prediction of Y_t. Let

$$\boldsymbol{s}^2 f_t = \mathrm{var}\,(Y_t|Y_1,\ldots,Y_{t-1,}\boldsymbol{f}_1,\ldots,\boldsymbol{f}_p,d,\boldsymbol{j}_1,\ldots,\boldsymbol{j}_q)$$

denote the conditional variance of $\hat{Y}_t^{t-1}$, where $\boldsymbol{s}^2$ is the variance of ε_t. Maximizing the following concentrated likelihood function by choosing the certain values of $\boldsymbol{f}_1,\ldots,\boldsymbol{f}_p,d,\boldsymbol{j}_1,\ldots,\boldsymbol{j}_q$:

$$\log L(Y_1,\ldots,Y_t) = constan\,t - \frac{1}{2}(\sum_{t=1}^{T}\log f_t + \sum_{t-1}^{T}(Y_t - \hat{Y}_t)^2)\,.$$

Practically, having a long series it is not affordable for CPU time to numerically maximize the above likelihood function (for example, a single evaluation of the likelihood takes about three hours of CPU time on a VAX 11/780). Haslett and Raftery (1989) proposed an excellent approximation method in which (see Haslett and Raftery 1989, pp. 12-14 for details):

1. the conditional mean and variance could be accurately approximated by using the partial autocorrelations for the AFIMA(0,d,0) process;
2. f_t could be approximated analytically, and concentrated likelihood is a function only of $\phi_1,\ldots,\phi_p,d,\varphi_1,\ldots,\varphi_q$;
3. the simplified likelihood function is maximized.

5. EMPIRICAL RESULTS OF LONG MEMORY TESTS

The classical R/S analysis is able to capture long memory and reveal the average length of nonperiodical cycles. But it is vulnerable to distortion from the existence of short memory and heteroscedasticity, and not subject to statistical inference. The modified R/S method is robust to the possible distortion from short memory and heteroscedasticity, and provides a well-defined statistical test for the existence of long memory. However, it is not able to tell the average length of nonperiodical cycles, and it presents only long memory. While the AFIMA model is still not able to indicate the average length of nonperiodical cycles, it parametrically models short and long memory simultaneously. Therefore, the results of three long memory models complement each other.

5.1 The Classical and Modified R/S Analysis

In this study, when equation (4.1) is estimated for daily, weekly, and monthly price differences, suppose w is a series of natural integers, such as *1,2,3,* and T is total observations. For monthly prices, n series was set as $3, 6, \ldots, w3 \leq$ *integer of* $T/2$; for weekly series, n is the series of $4, 8, \ldots, w4 \leq$ *integer* *of T/2*; and for daily series, n is the series of *5, 10, ..., w5£ integer of T/2*. *3, 4,* and *5* are chosen for the factors of constructing subsamples because *3* months form a quarter, *4* weeks a month, and *5* days a working week, which are convenient assumptions. Experiments showed that the estimates of the *H* exponent are not sensitive to the factors differing from 3, 4, and 5.

While (4.1) was applied to estimate *H* exponents, the average length of nonperiodic cycles was identified, labeled as *L* in Table 12. At the same time, the estimation of *L* was supplemented by *V* statistics as suggested by equation (4.4). The results are in Table 12.

Table 12. The Hurst Exponent (H) and Average Lengths of Non-periodic Cycles Identified by Model (4.1) (L) and by V statistics (V)

Markets	Daily			Weekly			Monthly		
	H	L	V	H	L	V	H	L	V
Corn	0.62	265	215	0.68	52	44	0.73	11	11
Soybeans	0.59	235	260	0.62	60	60	0.66	10	9
Wheat	0.58	235	255	0.65	56	56	0.68	13	12
Hogs	0.61	265	235	0.67	56	48	0.70	13	13
Coffee	0.62	100	125	0.65	28	28	0.68	8	7
Sugar	0.60	250	210	0.66	52	44	0.78	12	13

All *H* estimates are above *0.5,* which indicates the existence of long memory in the series since *H=0.5* implies an independent process. From daily to weekly and to monthly time frequencies, the estimated *H* increases for each of commodities, especially in the case of sugar. As time frequencies increase, the series become more noisy, which will reduce *H*. Roughly, for daily series, estimates are around *0.60*, for weekly around *0.65*, for monthly around *0.70*.

Among the six commodities, corn seems to have the highest *H* estimates except in the case of monthly frequency where it is the second largest after sugar. The variation among the six commodities in *H* estimates increases as the time frequency decreases. The ranges between the maximum and minimum are 0.04, 0.06, and 0.10 for daily, weekly and monthly series, respectively. This implies that as noise decreases, the different commodity markets begin to demonstrate their own characteristics.

The average length of nonperiodic cycles was investigated by the process of *H* estimations and *V* statistics, as reported by the columns with headings *L* and *V* in Table 12, respectively. It is noticeable that the average length of nonperiodic cycles indicated by *L* and *V* statistics are roughly the same.

Except coffee, the other five markets have roughly 1 year as their average length of nonperiodic cycles. It is about half year for coffee. This time length stands for all three time frequencies. Though certain differences in time length exists among three time frequencies, the differences are not substantial.

The above results from the classical R/S analysis remain to be clarified further since the classical R/S analysis is sensitive to the heterogeneities, which have been found in all eighteen series. Also, the distribution of *H* exponents is not well defined and no statistical inference could be pursued.

Lo's (1991) modifications to the classical R/S analysis attack these shortcomings, which includes the weighted sum of autocovariance into the denominator of *R/S* ratio. Lo (1991) analytically proved that the modification provides robustness to short memory. Further, Cheung and Lai (1993) showed by Monte Carlo simulations that the modified R/S analysis is robust to ARCH effects and the shifts in variances.

Table 13 reports the estimates of $Z(n)$ statistics and the order of autocovariance, q, that has been identified by Andrew's (1991) rule, i.e., equation (4.7).

Table 13. Z(n) of Lo's Modified R/S Analysis and Lag of Autocovariances Included (q)

Commodity	Daily		Weekly		Monthly	
	Z(n)	q	Z(n)	q	Z(n)	q
Corn	1.109	4	1.032	2	0.907	0
Soybeans	0.919	4	0.861	0	0.719	1
Wheat	1.398	0	1.386	0	1.166	2
Hogs	1.365	0	1.266	4	1.202	0
Coffee	1.565	4	1.509	1	1.351	1
Sugar	1.994**	4	1.790**	4	1.324	5

The null hypothesis is, Y_t carries no long memory. The alternative is, Y_t carries long memory.
Critical values (Lo 1991, p. 1288): 10%: 1.620, 5%: 1.747
** Significant at 5% level.

In general, the results are opposite of Table 12 where the evidence of long memory was found. All *Z(n)* estimates except two (daily and weekly sugar series) are below the critical value of 10% significance. For those series, the existence of long memory is not confirmed though suggested by the classical R/S analysis.

The special feature of the *Z(n)* statistic is its inclusion of the weighted sum of autocovariances of the data, which is to correct the biasness caused by the existence of short memory as well as heteroscedasticity. The values of *q* tell roughly to what extent this correction effort is needed. The question could be asked whether the results reported by Table 13 are sensitive to different *q*'s.

The various values of *q*, differing from those identified by Andrew's (1991) rule and reported in Table 13, were assumed and *Z(n)* statistics were re-estimated to examine the stability of the results. It was found that changing *q* influence the estimates of *Z(n)*, but not substantially. For example, when *q* is equal to *3 or 5*, the Z(n) for daily sugar is *2.055 or 1.943*, when *q* is equal to *4 or 5*, the *Z(n)* for monthly sugar is *1.345 or 1.357*, and when *q* is equal to *1 or 2, the Z(n)* for daily hogs is *1.345 or 1.357*. This indicates that the results in Table 13 are robust to the variation of *q*.

The *Z(n)* estimates are quite similar for a given commodity among three time frequencies except sugar. The *Z(n)* estimates for daily series tend to be highest while for monthly tend to be the lowest, but the differences are not very big. The *Z(n)* estimates for corn, soybeans, wheat and hogs are similar in terms of magnitude, but the estimates for coffee, though not significant, are more close to those for sugar.

Sugar daily and weekly series contain long memory structure, as suggested by *Z(n)* estimates, but sugar monthly data do not[19]. The sugar market is the only one here containing long-range memory. Though the classical R/S analysis tends to suggest that all eighteen series are long memory processes, the modified R/S analysis confirmed only two of eighteen cases, i.e. daily and weekly sugar prices. The remaining sixteen series, after taking account of short memory and heterogeneities and subjected to statistical inferences, are not long memory processes. This conclusion can be verified further by the AFIMA model, which follows.

5.2 THE AFIMA MODEL

For the AFIMA(p,d,q) model:

$$\Phi(B)(1-B)^d Y_t = \Psi(B)e_t,$$

where $0<d<1$ and typically $0<d<0.5$, Φ(B) and Ψ(B) are the polynomials of the order p and q, respectively. The specification of p and q will affect the maximum likelihood estimates (MLE) of Φ(B) and Ψ(B). By examining the structure of autocorrelations and partial autocorrelations given earlier, it

[19] It will be discussed in the last section why long memory has been found in daily and weekly, but not monthly series, of the same sugar market.

was found for all eighteen series that short memory was very weak, which suggests that both p and q should be specified as 0. The estimation is then based on the specification of AFIMA(0,d,0). Estimates of d and standard deviations, as well as the values of likelihood of the specifications, are in Table 14.

Table 14. AFIMA Estimates of d

	AFIMA	Daily		Weekly		Monthly	
		d	likelihood	d	likelihood	d	likelihood
Corn	(0,d,0)	0.029	-14286	0.051	-3864	0.000	-1106
Soybeans	(0,d,0)	0.036	-20321	0.005	-5167	0.000	-1390
Wheat	(0,d,0)	0.000	-16874	0.020	-4342	0.052	-1192
Hogs	(0,d,0)	0.004	-5629	0.084	-2012	0.000	-681
Coffee	(0,d,0)	0.055	-13719	0.030	-3780	0.070	-1064
Sugar	(0,d,0)	0.050	-3738	0.104	-1664	0.231	-566

In the AFIMA(0,d,0) specification, in all but the sugar cases, estimates of d are very close to zero. The magnitude of d estimates for weekly and monthly sugar series are noticeable compared with the series of other markets, 0.104 and 0.231, respectively, but the result of the daily sugar series is not different from daily series results of the other commodities, where d is close to 0.

When $d=0$, AFIMA(0,d,0) becomes

$$Y_t = e_t ,$$

i.e., the series is white noise. The AFIMA model tells that except for the sugar market the other five markets contain no long memory. This confirms the finding of the modified R/S analysis.

Lo (1991) conducted Monte Carlo simulations to evaluate the size and power of the $Z(n)$ statistic of the modified R/S analysis. $Z(n)$ is sensitive to the sample size. When the sample size gets smaller, $Z(n)$ has lower power to reject a wrongly specified null hypothesis against the long memory alternative. This is especially true for a sample size below 250. The monthly series in this study have 251 observations. Considering the noise the data contains, it is anticipated that the modified R/S analysis on monthly data might not identify long memory even if it actually exists, such as the case of sugar. For the daily sugar series that have more than 5,000 observations, the modified R/S analysis uncovered long memory.

As a parametric statistical model, it is understandable that the AFIMA model might be more sensitive to the noise in the data than to its sample size if the sample size has exceeded certain thresholds. In Table 14 for sugar series, the value of *d* decreases as time frequency increases. That no long memory was found in the daily series is very likely due to the fact that daily series have much more noise than do weekly and monthly series.

Though the evidence of long memory produced by the classical R/S analysis is not reliable, the average length of nonperiodical cycles identified by the classical R/S analysis is still meaningful. Since nonperiodical cycles are not unique to long memory processes, they are also observed in chaotic systems. Peters (1994) conducted many simulations on the robustness of the classical R/S analysis in uncovering the average length of nonperiodical cycles, and the results are very positive.

It has been troublesome to conventional unit root tests for a long time that a series contains a root that is very close to a unit but not exactly a unit. Now the AFIMA model is able to avoid this trap by using noninteger "*d*" to indicate the stationarity. A *d* that is larger than 0.5 implies nonstationary.

6. CONCLUSIONS

Many economic and financial theories suggest the existence of nonlinear dependence in financial markets. Chartists accumulate nonlinear price patterns and advise traders for profit opportunities. Simple statistical screening on financial series often finds long-lasting autocorrelations and time-dependent variances, which are the symptoms of nonlinear dependence. The question is what type of nonlinear relationships they are, if they really exist.

Price series that are twenty-one and half years long for six agricultural futures markets, corn, soybeans, wheat, hogs, coffee, and sugar, exhibit time-varying volatility, carry long-range dependence, and portray excessive skewness and kurtosis, though they are covariance stationary[20]. This suggests that the series contain nonlinear dynamics. ARCH and long memory are the two stochastic nonlinear models that are able to produce these symptoms. Though standard ARCH tests suggest that all series might contain ARCH effects, further diagnostics show that the series are not ARCH processes, since it has been found that the autocorrelations of the variances of the data decay to zero very slowly as the time span increases, and this is not a property of ARCH processes. In addition, all series exhibit obvious asymmetry that is out of the reach of regular ARCH processes. The martingale difference null can not be rejected by the ARCH model.

Three long memory techniques, i.e., the classical R/S analysis, the modified R/S analysis, and the AFIMA model, are applied to test the martingale difference null against the long memory alternative. The nonparametric method, the classical R/S analysis, suggests there might be long memory structures in the series. However, the other two more robust tests, the modified R/S analysis and the AFIMA model, confirm this in the case of sugar, but reject this proposition for the other five markets.

Why are sugar series long memory processes while the other five markets are not? A long memory model can only imply that there is long-range dependence in markets, today's price is affected, or partially affected, by the previous long price records. This long memory could either be the interactions of deterministic forces in the market or the effects of speculation, or both.

In world markets, sugar trade differs from the trade of the other five commodities in one major way. Sugar trade is participated in by more countries, which are diversified in terms of geographical locations and economic development levels, and is more competitive and less likely to be dominated by one or a few superpowers[21].

Live hogs are costly for international trade. Futures prices of live hogs, compared with the other five commodities, are much more dependent on US domestic supply and demand, and also heavily influenced by corn prices. For the time period 1974 to 1995, the US has been a dominating market power in the international corn and soybean trade by having about 70% of the world exports (Lin et al. 1996, Ash et al. 1996). In the international wheat market the US is still the biggest exporter in the world with about 30-40% of the world exports since 1970 (other big players are Canada, Argentina, Australia, and EU for exports; USSR and China for imports). This share is much less than those in corn

---

[20] A process with heteroscedasticity can be stationary with finite and constant mean and variance (Bera and Higgins 1995).

[21] This argument and related discussion benefited from conversations with Mr. Ron Lord, an economist of Economic Research Service, United State Department of Agriculture. According to Mr. Lord, the US's share in the world sugar market has been around 5-7%. The sugar futures market is international in nature, the US domestic sugar policy has impacts on the market, but the impacts are not substantial in most time periods.

and soybean trade, but is significant enough to lead price fluctuations in international markets (Hoffman et al. 1996).

The dominating force in the international coffee market is Brazil, which had about 40% world exports in 1970's and about 25% in 1990's. (Other important exporters are Columbia, Indonesia and Cote d'Ivoire. US and EU are the most important importers.) And more interesting and important, the international coffee trade from 1962 to 1989 (which includes a major portion of the series examined in this study) was structured by the International Coffee Organization (ICO). ICO successfully controlled the flow of coffee from exporting member countries and consequently stabilized and elevated the level of prices (Farmer 1994).

International Sugar Agreements have never been a single success since the 1960's in terms of imposing quotas to restrict supply and urging the release of stocks to increase supply. While sugar imports are relatively equally distributed among numerous countries, sugar exports are led by a group of countries (EU, Australia, Brazil, Cuba, Dominican Republic, Thailand, Philippines, etc.). The number of countries in this group and the share of each country have changed over time (Abbott 1990, Lord 1996). In the late 1970's, and especially since 1980, there has been an emergence of a significant degree of potential world sugar production, which can swiftly be converted into actual production, and the increasing proportion of world sugar consumption has been accounted for by developing countries. Price elasticities of supply and demand in the sugar market seem larger than at least those for coffee (Harris 1987).

With the above market structures, for the time period as long as 21.5 years, the future prices of corn, soybeans, wheat, hogs, and coffee are more likely to be subject to several deterministic elements, such as US agricultural and trade policies and regulations, supply and demand of US economy, as well as ICO's regulations and the Brazilian coffee economy. In contrast, too many factors are acting within the sugar markets, such that sugar futures prices are more likely to be stochastic in nature than the other future prices.

The international sugar market has many more players and less dominating forces than the corn, soybean, wheat, hog, and coffee markets. This appears to suggest that the prediction of the sugar market might be more difficult than that of the other five markets. The present study has concluded that the sugar market contains a long memory structure, and the other five markets do not. A few determinants in these five markets react to each other in such a way that the produced price movements are very volatile. In the end, the prices in these five markets are not very predictable. In the sugar market, economic and political factors affecting prices are many and each of them is not significant enough, hence price movements are more likely to be smooth and continuous, and thus easier to predict.

A long memory model does not attribute irregular behavior of price changes to the time dependent variances. Rather, long-range dependence in the price series is responsible for the observed nonlinear dynamics. In a typical long memory model, the observed time-varying volatility of the market is the product of long-range dependence. Here, the time-dependent market risk is a result, not a cause. Investors should focus on the elements that determine long memory of the prices. For example, traders with long investment horizons are more likely to wait for a trend in a given market before taking a decision. If the share of long-run investors increases in a given market, the price movements are more likely to have persistent patterns.

Long memory models, especially the modified R/S and AFIMA analyses, have not been widely used in agricultural market studies. This study has reviewed three major tools in long memory theory, and discussed the weaknesses and strengths of each one. Among the three models, the AFIMA model is endowed with a flexible structure to capture short and long memory, regular and irregular behavior at the same time.

REFERENCES

Abbott, G.C. 1990. *Sugar*. Routledge Press. London and New York.

Andrews, D.W.K. 1991. "Heteroscedasticity and Autocorrelation Consistent Covariance Estimation." *Econometrica*. 59:817-858.

Ash, M., G. Douvelis, J. Castaneda, and N. Morgan. 1996. *Oilseeds: Background for 1995 Farm Legislation.* Agricultural Economic Report Number 715. Economic Research Services, United States Department of Agriculture. Washington, D.C.

Bera, A. and M. Higgins. 1995. "On ARCH Models: Properties, Estimation, and Testing." In: *Surveys in Econometrics,* edited by L. Oxley, D.A.R. George, C.J. Roberts, and S. Sayer. Blackwell Press. Oxford of UK and Cambridge of USA. Pp. 171-215.

Beran, J. 1994. *Statistics for Long-Memory Processes.* Chapman and Hall Publishing Inc. New York.

Black, F. 1976. "Studies of Stock Market Volatility Changes." *1976 Proceedings of the American Statistical Association*. Business and Economic Statistics Section. Pp.177-181.

Bollerslev, T. 1986. "A Generalized Autoregressive Conditional Heteroskedasticity." *Journal of Econometrics*. 31:307-327.

Bollerslev, T., R.Y. Chou, and K.F. Kroner. 1992. "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence." *Journal of Econometrics*. 52:5-59.

Booth, G.G., F.R. Kaen, and P.E. Koveos. 1982. "R/S Analyses of Foreign Exchange Rates under Two International Monetary Regimes." *Journal of Monetary Economics*. 10:407-415.

Cheung, Y.W., and K.S. Lai. 1993. "Do Gold Market Returns Have Long Memory?" *The Financial Review*. 28:181-202.

Chow, K.V., K.C. Denning, S. Ferris, and G. Noronha. 1995. "Long-Term and Short-Term Price Memory in the Stock Market." *Economic Letters*. 49:287-293.

Christie, A.A. 1982. "The Stochastic Behavior of Common Stock Variances: Value, Leverage, and Interest Rate Effects." *Journal of Financial Economics*. 10:407-432.

Diebold, F.X. and M. Nerlove. 1989. "Unit Roots in Economic Time Series: A Selective Survey". In: *Advances in Econometrics*, edited by G.F. Rhodes and T.B. Fomby. JAI Press. Greenwich, CT. 8:3-69.

Fang, H.K., S. Lai, and M. Lai. 1994. "Fractal Structure in Currency Futures Price Dynamics." *The Journal of Futures Markets.* 169-181.

Farmer, A. 1994. "Information Sharing with Capacity Uncertainty: The Case of Coffee." *Canadian Journal of Economics*. 27:415-432.

Fraser, P. and R. MacDonald. 1992. "Spot and Forward Metals Prices: Efficiency and Time Series Behavior." *The Review of Futures Markets.* 11:24-34.

French, K.R., G.W. Schwert, and R.E. Stamburgh. 1987. “Expected Stock Returns and Volatility.” *Journal of Financial Economics*. 19:3-30.

Fung, H.G. and W.C. Lo. 1993. “Memory in Interest Rate Futures.” *The Journal of Futures Markets*. 13:865-872.

Fung, H.G., W.C. Lo, and J.E. Peterson. 1994. “Examining the Dependence in Intra-Day Stock Index Futures.” *The Journal of Futures Markets*. 14:405-419.

Geiss, C.G. 1995. “Distortion-Free Futures Price Series.” *The Journal of Futures Markets*. 15:805-831.

Granger, C.W.J. and R. Joyeux. 1980. “An Introduction to Long-Memory Time Series Models and Fractal Differencing.” *Journal of Time Series Analysis*. 1:15-29.

Harris, S. 1987. “Current Issues in the World Sugar Economy.” *Food Policy*. 12:127-145.

Haslett, J. and A.E. Raftery. 1989. “Space-Time Modeling with Long Memory Dependence: Assessing Ireland’s Wind Power Resources.” *Applied Statistics*. 38:1-50.

Hassler, U. and J. Wolters. 1995. “Long Memory in Inflation Rates: International Evidence.” *Journal of Business and Economic Statistics*. 13:37-45.

Helms, B.P., F.R. Kaen, and R.E. Rosenman. 1984. “Memory in Commodity Futures Contracts.” *The Journal of Futures Markets*. 10:559-567.

Herner, R.A. 1983. “The Origin of Predictable Behavior.” *American Economic Review*. 73:560-595.

Hoffman, L.A., S. Schwartz, and G.V. Chomo. 1996. *Wheat: Background for 1995 Farm Legislation*. Agricultural Economic Report Number 712. Economic Research Services, United States Department of Agriculture. Washington, D.C.

Hosking, J.R.M. 1981. “Fractal Differencing.” *Biometrika*. 68:165-176.

Hurst, H.E. 1951. "Long-Term Storage of Reservoirs". *Transactions of American Society of Civil Engineers*. 116:770-799.

Jarque, C.M. and A.K. Bera. 1980. “Efficient Tests for Normality, Homoscedasticity and Serial Independence of Regression Residuals.” *Economic Letters*. 6:255-259.

Kang, T. and B.W. Brorsen. 1995. “Conditional Heteroskedasticity, Asymmetry, and Option Pricing.” *The Journal of Futures Markets*. 15:901-928.

Leuthold, R.M., J.C. Junkus, and J.E. Cordier. 1989. *The Theory and Practice of Futures Markets*. Lexington Books. D.C. Health and Co., Lexington, MA.

Lin, W., P. Riley, and S. Evans. 1996. *Feed Grains: Background for 1995 Farm Legislation*. Agricultural Economic Report Number 714. Economic Research Services, United States Department of Agriculture. Washington, D. C.

Lo, A. 1991. “Long-Term Memory in Stock Market Prices.” *Econometrica*. 59:1279-1313.

Lord, R. 1996. *Sugar: Background for 1995 Farm Legislation.* Agricultural Economic Report Number 711. Economic Research Services, United States Department of Agriculture. Washington, D.C.

Mandelbrot, B. 1969. "The Robustness of the Rescaled Range R/S in the Measurement of Noncyclic Long-Run Statistical Dependence." *Water Resources Research*. 5:967-988.

Mandelbrot, B. 1972. "Statistical Methodology for Non-Periodic Cycles: From the Covariance to R/S Analysis." *Annals of Economic and Statistical Measurements*. 1:259-290.

Mandelbrot, B. and J.R. Wallis. 1969. "Computer Experiments with Fractional Gaussian Noises." *Water Resources Research*. 5:228-267.

Milonas, N.T., P.E. Koveos, and G.G. Booth. 1985. "Memory in Commodity Futures Contract: A Comment." *The Journal of Futures Markets.* 5:113-114.

Nelson, D.B. 1990. "ARCH Models as Diffusion Approximations." *Journal of Econometrics*. 45:7-38.

Newey, W. and K. West. 1987. "A Simple Positive Definite, Heteroscedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica*. 55:703-705.

Peters, E.E. 1989. "Fractal Structure in the Capital Markets." *Financial Analysts Journal.* July-August:32-37.

Peters, E.E. 1991. *Chaos and Order in the Capital Markets.* John Wiley & Sons, Inc. New York.

Peters, E.E. 1994. *Fractal Market Analysis*. John Wiley & Sons, Inc. New York.

Phillips, P.C.B. and P. Perron. 1988. "Testing Unit Roots in Time Series Regression." *Biometrika.* 75:335-346.

Scacciavillani, F. 1994. "Long Memory Processes and Chronic Inflation." *International Monetary Staff Papers*. 41:488-501.

Smillie, K.R. 1966. *An Introduction to Regression and Correlation*. Ryerson Press. New York.

Taylor, S. 1986. *Modeling Financial Time Series*. John Wiley & Sons, Inc. New York.

Tomek, W.G. 1994. "Dependence in Commodity Prices: A Comment." *The Journal of Futures Markets.* 14:103-109.

Wei, A. 1997. *Long Agricultural Futures Price Series: ARCH, Long Memory, or Chaos Processes?* Ph.D. dissertation. University of Illinois at Urbana-Champaign, 1997.

Wei, A. and R.M. Leuthold. 1998. *Long Agricultural Futures Prices: ARCH, Long Memory, or Chaos Processes?* Office for Futures and Options Research Working Paper 98-03. University of Illinois at Urbana-Champaign.

Yang S.R. and B.W. Brorsen. 1992. "Nonlinear Dynamics of Daily Cash Prices." *American Journal of Agricultural Economics*. 74:706-715.

Yang S.R. and B.W. Brorsen. 1993. "Nonlinear Dynamics of Daily Futures Prices: Conditional Heteroscedasticity or Chaos?" *The Journal of Futures Markets*. 13:175-191.