Assessing Budget Risk with Monte Carlo and Time Series Bootstrap

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Master's Dissertation

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Mestrado Integrado em Engenharia e Gestão Industrial

2021-06-28

Abstract

Budgets are important management tools recognized for the help they provide in planning, communication, monitoring the expense performance, and even as a source of motivation to collaborators. However, recently there has been criticism of the traditional Budgeting Process due to its cumbersomeness, long duration, and eventual diversion of the focus from the day-to-day activities. These factors allied to the recurrent budget deviations that occur as a result of not considering the uncertainty of unpredictable circumstances can derail all the time spent in planning. Therefore, there is the need to improve the Budgeting Process by incorporating the uncertainties that expenses may be subject to. The usage of Simulation is one such technique that can be used.

The main goal of this thesis is to produce a decision support tool for the Expense Budgeting Process of a retail company division to help them assess the risk of a certain budget given as input. Firstly, an extensive review on Simulation related techniques and a preliminary analysis of the Expense components of the division and its Budgeting Process are done. Having that in mind, the next step is to generate alternative historical series of each Expense component with Time Series Bootstrapping Techniques. The techniques implemented are the Moving Block Bootstrap, Model Based techniques (based on Exponential Smoothing, SARIMA and TBATS), the BLD Bootstrap and the recent K-Means Based Bootstrap. From the results obtained, generally, the last one is the method which creates the most similar series to the original ones.

In a next step, the annual distributions of the Expense components are obtained from the most similar generated series and a Monte Carlo Simulation is performed to obtain the Total Expense Distribution. For the simulation two sampling techniques are considered: independent and correlated inputs. The reason for the implementation of both is to assess the impact of correlations as they are generally difficult to obtain, but at the same time yield more realistic results. When correlations are considered, as some of their coefficients based on historical data are negative, the Total Expense distribution has less spread and the consequent budget less risk.

Finally, it is crucial to understand the relative importance that each Expense component has on the Total Expense distribution. For that purpose, the Sensitivity Importance Measures of Variance explained, R-squared decomposition and the Moment Independent index of Borgonovo are implemented. The results show that the order of importance changes when correlations are considered, which underlines the importance of including them in the Simulation to obtain more realistic results.

Resumo

Os orçamentos são importantes ferramentas de gestão amplamente usadas para auxiliar o planeamento, comunicação, monitorização da despesa e motivação dos colaboradores. No entanto, o Processo tradicional de Orçamentação tem sido alvo de críticas por, eventualmente, desviar do foco das tarefas diárias e de maior valor agregado, ao exigir uma grande intensidade de todos os envolvidos na sua elaboração e ter uma grande duração. Além disso, os recorrentes desvios orçamentários que ocorrem como resultado de não se considerar incertezas nos orçamentos das despesas podem inviabilizar todo o tempo inicialmente gasto no planeamento. É, então, imperativo melhorar o processo de orçamentação incluindo estas incertezas, sendo a Simulação uma técnica que possibilita a sua realização.

Esta tese pretende produzir uma ferramenta de apoio à decisão para o Processo de Orçamentação de uma divisão de uma empresa de retalho, de forma a avaliar o risco de um orçamento dado como *input*. Primeiramente foram realizadas uma extensa revisão teórica sobre tópicos relacionados com Simulação e uma análise preliminar às componentes da Despesa. Tendo isso em conta, o passo seguinte foi a geração de séries alternativas com base no histórico para cada componente da Despesa, recorrendo a técnicas de *Bootstrap* para séries temporais. As técnicas implementadas foram o *Moving Block Bootstrap*, técnicas de *Bootstrapping* baseadas em modelos estatísticos (como os modelos de Amortecimento Exponencial, *SARIMA* e *TBATS*), o *BLD Bootstrap* e o recente *Bootstrap* com base no algoritmo de *clustering K-Means*. Os resultados obtidos demonstraram que, geralmente, o último foi o método que criou as séries mais similares às originais e, portanto, mais plausíveis de terem ocorrido no passado.

Seguidamente, foram obtidas as distribuições anuais de cada componente da Despesa por agregação anual dos valores mensais das suas séries mais similares selecionadas. Estas distribuições foram o ponto de partida para a realização da Simulação de Monte Carlo, que originou a distribuição da Despesa Total. Para a simulação, duas técnicas na geração das realizações foram implementadas: *inputs* independentes e correlacionados. A razão da implementação de ambas é conseguir avaliar o impacto das correlações, que são normalmente difíceis de obter, mas originam resultados mais realistas. A partir dos resultados, verificou-se que na presença de correlações, e uma vez que alguns dos seus coeficientes a partir do histórico são negativos, a distribuição da Despesa Total tem uma menor amplitude, apresentando um mesmo orçamento menor risco.

Por último, foi realizado um estudo da importância relativa que cada componente da Despesa tem em impactar a distribuição da Despesa Total. Para isso, foram implementadas as medidas de Importância Variância explicada, Decomposição do R-Quadrado e o índice independente do momento de Borgonovo. Os resultados sugerem que na presença de correlações, a ordem de importância das diferentes componentes muda, o que realça a importância da sua inclusão na Simulação de forma a obter resultados mais realistas.

Acknowledgements

This thesis marks the end of my Master's in Industrial Engineering and Management at FEUP, a degree that I am proud of having been a student for these past five years. The vast scientific areas it covers, the high-quality staff and apprentices are what make this degree so special and attractive.

Firstly, I would like to thank Sonae MC for having welcomed me so well, even at this time of the pandemic. It was a pleasure to be part of such a big company that still thrives to be constantly better. Thank you, André Moutinho, my advisor at the company, for all the availability, suggestions, and patience to deal with my requests. The Analytics team of ABS (Ana, Carolina, Diana, and Pedro) has also a special place in my heart as considered me as one of its members. Lastly, I'd also like to praise Carlos for his willingness to help and proactivity.

My work was substantially improved with the careful revision and suggestions of my FEUP advisor, Eng. Miguel Gomes, whom I thank for all his availability, quickness to respond, and attention to detail.

To my friends and colleagues who accompanied me on this journey and because of their excellence motivated me to continue to aspire for more, but also calmed me down during the most difficult periods, a massive thank you.

Finally, I would like to praise my family members that made all of this possible, by giving me valuable advice from day one, spending their time to contribute to my success, and also being an excellent example.

"I've always believed that if you put in the work, the results will come."

Michael Jordan

Contents

Acronyms and Symbols

ABS	Accounting Business Solutions division of Sonae
AICc	Akaike Information Criterion
ANN	Artificial Neural Network
ARMA	Autoregressive Moving Average model
BLD	Box-Cox and Loess-based decomposition bootstrap
CVaR	Conditional Value at Risk
ERP	Enterprise Resource Planning
ETS	Exponential Smoothing family models
IID	Independent and Identically Distributed
KS	Kolmogorov-Smirnov hypothesis test
LMG	R-Squared decomposition Importance Measure
MBB	Moving Block Bootstrap
MCS	Monte Carlo Simulation
NBB	Non-Overlapping Block Bootstrap
NRMSE	Normalized Root Mean Square Error
OAT	One-at-a-time Local Sensitivity Methods
SARIMA	Seasonal Autoregressive Integrated Moving Average model
S_i	Variance Explained Importance Measure
STL	Seasonal and Trend decomposition using Loess
TBATS	Trigonometric seasonality, Box-Cox transformation, ARMA errors, Trend and Seasonal components model
VaR	Value at Risk
δ_i	Moment Independent Importance Measure of Borgonovo

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Chapter 1

Introduction

Budgets are management tools widely used to support planning and help managing resources and activities of organizations (Lidia, 2014). They are appreciated for their control purposes and according to Libby and Lindsay (2010) organizations intend to continue using them in the future.

There are several studies that point the advantages of Budgeting in an organization. Lidia (2014) underlines the positive impact they have on managers by giving them a sense of safety and certainty when operating in an uncertain environment. Libby and Lindsay (2010) refer the support that they offer in translating the strategy into more objective actions, whilst interacting with the different divisions of the company. Other advantages expressed are its positive role in planning, resource allocation, performance evaluation, and even in motivating collaborators.

However, recently some authors have also criticized the usage of these tools and have urged its improvement. Common drawbacks of its implementation are related to its high cost and the fact that it is very time-consuming, which can hinder the focus on the core activities and innovation (Lidia, 2014). Moreover, the fact that the majority of budgets are deterministic and not consider uncertainties in the Budgeting Process can be viewed as a limiting view that often results in inadequate plans (Lord, 1977; Rubin and Patel, 2017; Hager et al., 2015; Crum and Rayhorn, 2019). Some even recognize that they can be easy to manipulate through controlling the budgetary slack. Budgetary Slack is defined as “the intentional overestimation of expenses and/or underestimation of revenue during budget setting” (Eaton, 2005, 7). This is where the application of risk management techniques have an important role in helping improve budgets by considering the uncertainties in the Budgeting Process and helping reduce the budgetary slack existent by defining appropriate contingencies, leading to more accurate and effective budgets (Elmassri and Harris, 2011; Robinson et al., 2018).

For this matter, Simulation can be a useful tool in helping to consider different scenarios and support the decision making regarding budget feasibility. This thesis arises in this context, where a division of a major Portuguese company (Accounting Business Solutions of Sonae) wants to develop an application that helps estimating the probability of cost overrun for a certain budget level and the general risk associated with its expenses. A detailed description of the division and its budgeting process is provided in Chapter 3.

The remainder of this chapter continues introducing the topics explored in this dissertation. Section 1.1 provides the scope and context of the project and its underlying motivation. Then, the goals for this thesis are presented in Section 1.2, followed by the approach used to assess the risk of expenses surpassing a given budget level and the relative importance of input variables (Section 1.3). This chapter finalizes with the structure outline of the rest of the dissertation (Section 1.4).

1.1 Project Context and Motivation

This dissertation is based on a project that takes place at the Accounting Business Solutions (ABS) of Sonae Group, a division that has more than 300 collaborators.

The Budgeting Process of this division is subject to several factors that are difficult to predict and that influence the evolution of expenses over the years. For instance, the variation in the number of people in the teams or the change in the way of allocating costs makes the construction of the budget not necessarily a trivial task. Additionally, there is a greater burden on the teams in the budget construction process and its monitoring, as the division lacks a default risk assessment tool to support it. This can make the budget creation process very demanding and time consuming.

In this sense, the incorporation of risk management, and in particular Simulation, to help assess budgets risk before they are approved or assist in its monitoring compliance is seen as an asset for the division, allowing it to make decisions based on data and consider the uncertainty that may impact its performance. There are two main advantages of creating a program designed specifically for this purpose instead of using other solutions available on the market:

1. Possibility of having a tailored solution to manage risk according to the specific needs of the team;

2. Possibility of integration with data from databases to manage risk without having to set up the environment and manually run simulations (more dynamic solution).

1.2 Goals

This thesis is expected to produce a decision support tool for the budgeting process of the expenses of ABS, allowing to assess the risk of exceeding a certain budget level and the expected value of expenses based on historical data. Moreover, a detailed comprehension of the relative importance and contribution of each input variable of the Expense is also necessary through a Sensitivity Analysis.

As secondary objectives, this work also aims to compare different Time Series Bootstrap methods on real-world data and more generally propose a methodology that companies can use to assess risks from historical data.

1.3 Approach

To assess the risk of a budget, firstly the total expense distribution needs to be estimated. In the present dissertation, this estimation was based on historical data since it allows the analysis to be backed up with data and minimize subjective inputs that may bias the results (Paté-Cornell and Dillon, 2006). The period included in the analysis refer to the expenses from 2016-2020 for each component of the Expense. To be able to obtain each of the component's distributions, similar plausible series for this period were generated through time series bootstrapping techniques. Once the distributions of the components were formed a Monte Carlo Simulation was performed to obtain the final total expense distribution, which was compared to a certain budgeted level to estimate its risk.

Afterwards, a sensitivity analysis was conducted in order to assess the relative importance of the input expense components in changing the total expense distribution.

Figure 1.1 details how these phases were spanned in the timeline of the project.

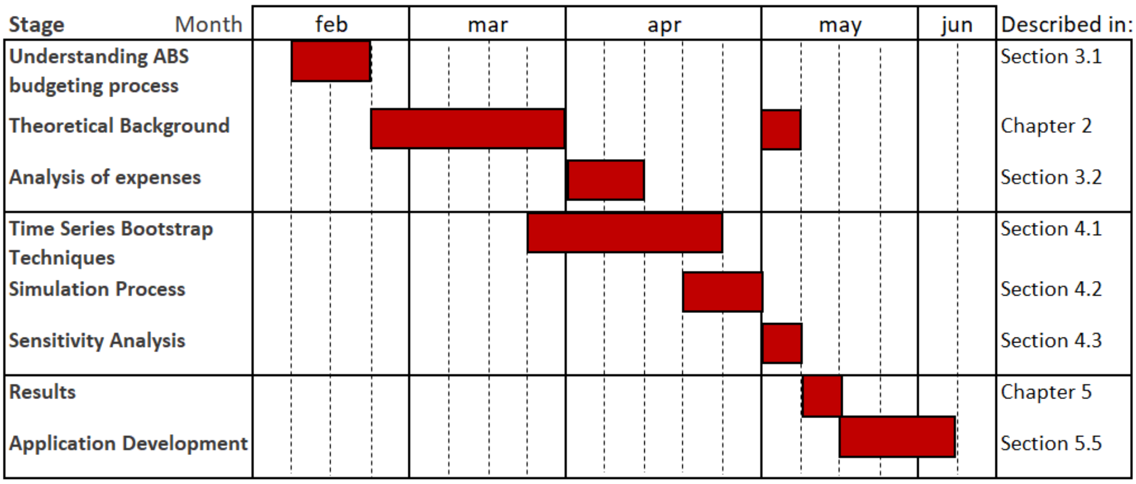


Figure 1.1: Timeline of the project

1.4 Thesis Outline

In addition to this introductory chapter, the thesis features five more chapters.

Chapter 2 consists of a theoretical background on risk management topics and analyzes previous works that are related to the techniques used in the dissertation.

Chapter 3 presents the budgeting process of the Accounting Business Solutions of Sonae, identify its limitations and finalizes with an analysis of the components of the Expense.

Chapter 4 presents the methodology followed to create the Expense components' distributions, from the initial time series bootstrap procedure, passing to the Monte Carlo Simulation process and correlations consideration. Lastly, some sensitivity importance measures are described.

Chapter 5 begins to show the performance and validation of the time series bootstrapping techniques. Then it characterizes the obtained distributions and the resultant Total Expense distribution and evaluates the effect of considering correlations in the sampling. The results regarding the relative importance of the Expense components are also described. Lastly, this chapter describes how the previous analysis can be extended to monitor budget risk throughout the year and shows the developed interactive application that enables its execution.

Chapter 6 summarizes the main findings of this thesis and also presents the opportunities for further improvement.

Chapter 2

Theoretical Background

Managing risk is extremely important to the decision-making process, allowing leaders to make careful and reasoned decisions. For this matter, many techniques have been proposed with gradually increasing complexity. The present chapter presents the main concepts used throughout the dissertation from a theoretical perspective with reference to relevant academic literature. Its objective is not to be a through literature review on the topic of risk, measures and applications, but to be regarded as an introduction by comparing from a rather macroscopic view the principal techniques. The chapter is divided into two sections: Section 2.1 that introduces the concept of Risk from a theoretical point of view; and Section 2.2 that is more focused towards Risk Assessment Techniques and the application of Simulation to Budgets.

2.1 Theory on Risk Management

Although Risk is a multidisciplinary field, the base foundation knowledge is common. This section serves as an introduction to the topic of risk, presenting core concepts related to Risk and the Risk Management process. It finalizes by underling the importance that managing risk has on Decision Making.

2.1.1 An overview on the concept of Risk

The concept of risk is not new to mankind, having been used by the Ancient Greeks to help them make decisions (Bernstein, 1996). However, the risk as a scientific field is relatively young, with around 40 years old (Aven, 2016). For more information about the origins of risk see Althaus (2005) and Aven (2012).

There are a number of different contrasting definitions of risk that may create tension, partly due to its multidisciplinary application or the difficulty of some authors to distinguish between the concept of risk with the way it is perceived or measured. Some definitions are based on probabilities, others on consequences (sometimes referred as hazards), or expected values. Moreover, some studies consider risk as subjective and, thus, dependent on the level of prior knowledge, while others consider it as being constant. According to Aven (2012), who studied the evolution of the concept, there has been a shift from narrow perspectives based on probabilities and expected values to the inclusion of consequences and uncertainties.

Aven (2012) classified existing definitions present in scientific studies into nine categories. Table A.1 of Appendix A summarizes the main aspects of each one. The definition that considers risk as Consequence and Uncertainty ($R = C\&U$) is viewed by several authors to be the most appropriate and complete as it considers two dimensions (the potential loss of an event and the chance of it occurring) and allows the separation of risk with the way it is measured.

2.1.2 Risk Management Process

The International Organization for Standardization in its 2018 Risk Management Guidelines defines risk management as the "coordinated activities used to direct and control an organization with regard to risk" (ISO, 2018). The process of risk management consists of the following steps, as shown in Figure 2.1: defining the Scope, Context and Risk criteria; Risk Assessment and Risk Control. Additionally, there are two continually acting stages: the Communication and Consultation with internal and external stakeholders and the Monitoring and Review. The output of the process is the Recording and Reporting stage that aims to communicate the risk management activities across the organization and provide information for decision making.

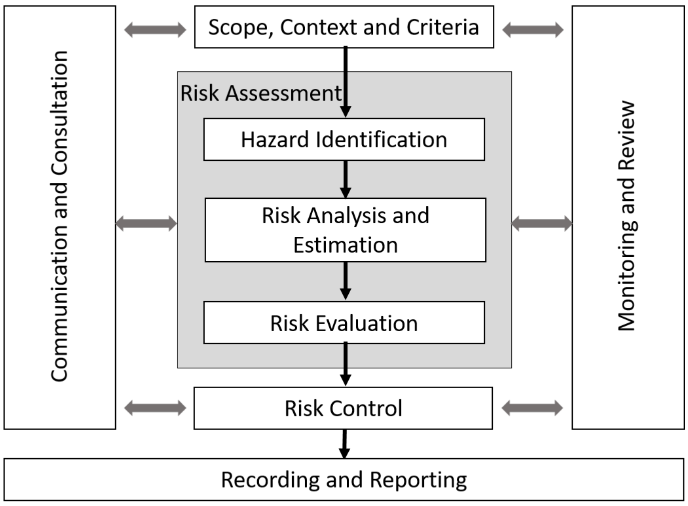


Figure 2.1: The Risk Management Process (ISO, 2018; Purdy, 2010)

The risk assessment stage comprises the Hazard Identification, which consists of identifying relevant sources of risk for the operation; the risk analysis that focus on characterizing risk (likelihood and consequences) in a qualitative, semiquantitative or quantitative manner; and the risk evaluation that by comparing the level of risk analyzed with the one initially defined as acceptable (risk criteria) decides if risk control/treatment is needed (Roberts and Graves, 2020; ISO, 2018). This stage will be the focus of the dissertation, with particular relevance of the first two tasks (Hazard Identification and Risk Analysis).

2.1.3 Risk Management and Decision Making

Risk Management and Decision Making are viewed as complementary fields, with the former being an input to the latter (ISO, 2018). Some authors (Paté-Cornell and Dillon, 2006; Howard, 2007; Borgonovo et al., 2018) even consider them as synonyms and opt for accounting only the risk analysis stage as an input for decision analysis (both forming the decision making process). This is a more restricted view when compared with the process presented in subsection 2.1.2, as they disregard the possibility of the risk analyst to perform a pre-evaluation (risk evaluation) by comparing with the risk criteria of the company. Paté-Cornell (2007) clearly defines the distinct roles of the risk analyst and of the decision maker. The risk analyst is intended to present as exactly as possible the state of knowledge (assumptions of the model, sources of information) and all conceivable alternatives, whilst the decision maker uses this information to evaluate and choose between the alternatives.

Apostolakis (2004) underlined the importance of responsible decisions not just to be risk based, but rather risk informed. This idea is in tune with the model proposed by Hansson and Aven (2014) that characterizes the steps involved in a risk informed decision (Figure 2.2).

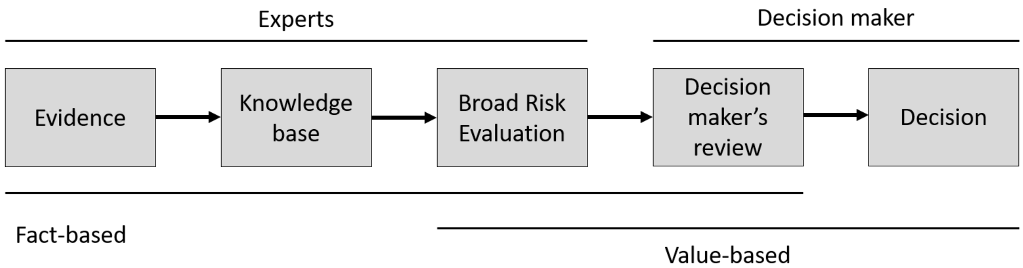


Figure 2.2: Model representing the stages of a risk informed decision-making (Hansson and Aven, 2014; Aven, 2016)

These authors first consider that there is a value-free (fact-based) stage where an objective analysis is performed (forming the evidence and knowledge base). This is followed by an evaluation stage performed by risk experts who make a summary judgment on the risks and uncertainties involved. This point of view is contrasting to the one of Paté-Cornell (2007) as in the Broad Risk Evaluation step facts and values are both considered.

For the actual decision making, the risk assessment report is combined with other non-scientific topics or decision making tools such as policy-related considerations, cost-benefit analysis, cost-effectiveness analysis and multi-attribute analysis (Aven, 2016).

The link between risk analysis and decision theory was formally established and further explored by Borgonovo et al. (2018), who showed that for every risk analysis there is a corresponding model of decision theory. According to these authors, this study is viewed as a starting point for risk and decision making being used more as a joint methodology.

2.2 Measuring Risk and Applications

This section comprises an overview of Risk Assessment techniques and Measures of Risk, introducing Simulation as an alternative to the popular Scenario Analysis techniques. Sensitivity Analysis techniques are also presented as relevant complementary techniques to better understand and validate the models created. Furthermore, literature on Bootstrapping methods for time series is also explored, due to its usefulness to perform statistical inference regarding population distributions and confident intervals in the case of dependent data. The section ends with an analysis on the application of Simulation to Budgeting.

2.2.1 Risk Assessment Techniques and Measures of Risk

Since risk is applied in many different areas, it is clear that the different disciplines have different methods for assessing and managing risks (Aven, 2012). Ahmed et al. (2007) reviews some of the techniques used in Project Management (which have application in budgets) and McNeil et al. (2015) in the case of Finance applications.

Grimaldi et al. (2012) proposed the following three dimensions for classifying risk assessment techniques:

- The application in the phase of the risk management process (see Subsection 2.1.2). The techniques applied in the risk analysis stage can be divided as qualitative, semi-quantitative, quantitative and hybrid. Hybrid techniques are similar to semi-quantitative, but have more precise and realistic quantitative results (Khan et al., 2015);

- The phase of the life cycle/scope that the problem to be analyzed is in (i.e. conceptualization, planning, execution or termination). Depending on the stage that the problem is in, the knowledge of probabilities and consequences vary. For instance, in the planning phase the probabilities of occurrence of hazards are difficult to be defined, whereas in the following phases they become more certain and, thus, techniques for characterizing the consequences of the decisions made become more relevant;

- The corporate maturity towards risk. This relates to the context of the organization performing the risk assessment, by considering its commitment to high quality data and the seriousness with which it faces the risk. The more its commitment to risk assessment, generally the more quantitative techniques it employs.

This classification was further extended in the ISO 31010 Risk Assessment Techniques Standard (2019) by considering more techniques and other criteria such as the cost of implementation, effort needed and time horizon (short term, medium or long term).

For risk analysis and estimation (which will be the focus of this dissertation), popular techniques include the usage of Scenario Analysis and Simulation. Scenario Analysis consists of defining possible and/or extreme scenarios (such as best and worst cases) and estimating a probability and consequence. Probabilities can be estimated based on expert judgment, historical data (for which Bootstrap Methods can be considered) or prediction models. The first is subjective and can incorporate bias; the second reflects what happened in the past (which may not happen in the future) and the third requires the devise of an expression or data to train the model, which implies a greater effort. The outcome of the Scenario Analysis can be represented in a Risk Matrix, allowing to perform risk evaluation.

Simulation based techniques allow to consider more scenarios than Scenario Analysis by exploring different combinations of the input's probability space. Monte Carlo Simulation (MCS) is the most common simulation method used to propagate input uncertainty to a model output. This simulation method is classified as a planning quantitative risk analysis technique that enables to assess the likelihood, consequences and risks (Grimaldi et al., 2012; ISO, 2019). It also helps in the risk evaluation stage by allowing to see the entire outcome distribution. Other useful measures to estimate risk besides the distribution are the expected outcome value, the Value at Risk (VaR) that corresponds to the tail X percent (usually 1% or 5%) extreme value, or the Conditional Value at Risk (CVaR) that corresponds to the expected loss in the worst X % of cases (ISO, 2019).

Several studies refer that for high dimensions, Monte Carlo Simulation does not converge easily, being necessary an exponential number of iterations as the input variables increase. This occurs because the sampling executed is random which may lead to clusters around the mean and extreme values not being sampled. Alternatives to Monte Carlo Simulation are based on different sampling methods and include the variance reduction method Latin Hypercube Sampling (LHS) that by first dividing each cumulative probability input function in equally distanced intervals ensures that a more uniform sampling occurs; or the Quasi Monte Carlo Simulation (QMC) that instead of random sequences uses quasi random low discrepancy sequences, which are deterministic and ensures a stricter uniform coverage. We refer to Singhee and Rutenbar (2010) and Kucherenko et al. (2015) for comparisons between these different sampling methods.

Other common technique is Sensitivity Analysis which can be viewed as a complementary quantitative technique to Scenario Analysis and Simulation, used to better understand and validate the models created (ISO, 2019).

2.2.2 Sensitivity Analysis and Importance Measures

Saltelli (2002) defines Sensitivity Analysis (SA) as the study of how the uncertainty in the output can be allocated to the different sources of uncertainty in the model input. Sensitivity analysis helps managers understand the developed models, being a bridge between the analysts and decision makers (Borgonovo and Plischke, 2016). Importance measures are the measures given by sensitivity analysis that allow to rank inputs based on the influence they have on the model output.

The literature distinguishes between value and decision sensitivity (Borgonovo and Plischke, 2016). The first quantifies the change in model output due to changes in the model inputs, whilst the second involves determining ranges of input values so that the optimal decision does not change. In this study, we will focus on Value sensitivity.

Value sensitivity methods are classified as local and global methods. Local methods are used when one is interested in determining the output of a model as a result of changes around a point of interest in the input space. Examples of local sensitivity methods are: one-at-a-time (OAT), that are commonly represented in Tornado Charts, in which changes in the output are calculated by varying each input variable by a certain amount whilst maintaining the others constant; Scenario analysis that allows to combine changes of various inputs; and Differentiation methods that consist of calculating partial derivatives.

Global methods take into consideration the entire input distribution, as opposed to only specific points and they allow for probabilistic input distributions. Borgonovo and Plischke (2016) classifies the different global sensitivity methods:

- Screening Methods that are used to identify the least important model inputs, allowing the analyst to fix its value (factor fixing) or eliminate them from the model (model simplification). A well-known technique is the Morris Method that extends the OAT approach to a set of discrete input ranges calculating the average and variance of finite changes on the output.

- Regression-based methods (also called non-parametric) that are used when linearity between the inputs and output can be assumed. Measures of sensitivity include the Standardized Regression Coefficient (SRC) and the R-squared contribution of each input (LMG) that can be used in the presence of correlated inputs (Grömping, 2015).

- Variance based methods that assess the importance of inputs based on the expected reduction in the variance of the output provided that the value of input is known. A widely used variance-based sensitivity measure are the Sobol indices (Sobol, 1993). These methods are useful when the analyst wants to identify the parameters that reduce the variance the most. To extend the analysis for correlated variables, Xu and Gertner (2008) proposed using regression to decompose the total variance explained in its uncorrelated and correlated parts. Hao et al. (2012) generalized the analysis for nonlinear additive models by using Artificial Neural Networks (ANNs).

- Density based methods are based on the difference between the probability distributions of the output and of the conditional output known the value of a certain input. An example of an input importance metric is the δ_i (Borgonovo, 2007). This class of methods have the advantage of being moment free, meaning that they look at the entire output distribution without referring to any of its moments. This is particularly relevant when there are correlations between inputs as in this case variance-based methods fail to determine the most influential set of inputs, making Density Based the preferred methods (Borgonovo, 2006).

- Regionalized Sensitivity Analysis methods first divide the output in classes and then assess the differences between the classes depending on each input.

2.2.3 Bootstrap methods for Dependent Data (Time Series)

The Bootstrap is a technique proposed by Efron (1979) as an extension of the Jacknife method. It consists of sampling with replacement from a sample (resampling) so as to obtain an empirical distribution that by maintaining the original relation between the "population" and the sample will approximate an unknown theoretical population distribution, as the number of resamples gets bigger (the law of large numbers). This method has the advantage of not being necessary to know the population parameters (which happens often) and still being able to perform statistical inference regarding their distribution and confidence intervals (Lahiri, 2003).

For dependent data (as it is the case of time series) the Bootstrap of Efron (1979) (called IID Bootstrap) is inadequate as it considers the data IID (independent and identically distributed) and so it completely ignores its dependence structure (Lahiri, 2003).

There are several techniques that try to capture the dependence of the data. One of the first ones implemented was the class of methods denominated Model Based which consisted on fitting a model to the time series data and then resampling the residuals with the IID Bootstrap. The bootstrap series were then formed by adding the model fitted values with the residuals bootstrapped. This class of Bootstrap methods had the limitation of being dependent on a model and assuming the residuals were independent. To overcome the first limitation the class of Block Bootstrap Methods was introduced which try to create bootstrap samples by resampling blocks of data instead of single observations. There are many variations of the Block bootstrap methods such as the Non-Overlapping Block Bootstrap (NBB) or the Moving Block Bootstrap (MBB). The latter has the advantage over the former of considering more blocks of data, which is particularly useful when the initial sample size is small (Bergström, 2018). Politis and Romano (1994) introduced the Stationary Bootstrap (SB) which by resampling blocks with different lengths according to a geometric distribution ensured that the bootstrapped series was stationary, a property which is not guaranteed for the NBB or the MBB even if the initial series is. However, this method converges slower.

For the block bootstrap methods, the choice of the optimal block length is a relevant ongoing discussion, with Politis (2003) dividing the approaches in two categories: cross validation (selecting based on criteria such as Mean Square error) or plug-in methods (more analytical).

Another type of time series bootstrap is the Sieve Bootstrap, which can be considered an extension of the Model Based methods by not considering the model residuals directly IID. It first consists on fitting a model to the data and then applying typically an Autoregressive (AR) model to the residuals. The residuals of this Autoregressive model are bootstrapped with the IID Bootstrap.

More recently, the literature is more focused on capturing series not stationary, which is an assumption of the Block Bootstrap methods referred. For this matter, we refer to the study of Cordeiro and Neves (2009) who use the Sieve Bootstrap after decomposing the time series with an ETS model and lately the study of Bergmeir et al. (2016) who developed a novel bootstrapping procedure involving a Box-Cox transformation, STL decomposition and the MBB. This last method has the advantage of not being model dependent and not assuming that the residuals of the STL decomposition are IID. Furthermore, for particularly noisy time series Laurinec et al. (2019) proposed the K-Means based Bootstrap, which does not create bootstrap series with a model, but samples from similar points of the original time series.

2.2.4 Budgeting under uncertainty and application of Monte Carlo Simulation

Collier and Berry (2002) define the budgeting process as a "formal method by which plans are established for future time periods". The authors consider that whilst the uncertainty may be set aside in the budget (i.e. the final document), this should be considered in the process of budgeting. Several authors agree with this statement and even state that the main critics referred to budgets (being easy to manipulate or difficult to prepare (Lidia, 2014)) are founded on the fact that the process is deterministic and only considers point estimates (Lord, 1977; Rubin and Patel, 2017; Hager et al., 2015; Crum and Rayhorn, 2019). Thus, the traditional plans tend to ignore uncertainty or significantly reduce the potential risk that the organization faces (Lord, 1979).

Scott (1998) referred that risk can be included in the process of budgeting through three ways: excluding risk, comparing risk or modelling risk. The first does not really include risk in the budgeting process; the second uses simple analysis such as sensitivity analysis and probability; and the last has an explicit formal use of probability models.

Modelling risk in budgets is also known as Probabilistic Budgeting and it can be done with a Monte Carlo Simulation that allows for a robust analysis of the potential risk associated with each input variable of the budget. Stochastic Simulation gives useful insights about the main drivers of the outcome of analysis (e.g. profit, costs, Net Present Value etc.). This technique is widely present in the literature as a tool to help creating budgets in the field of Project Management applied to construction projects (Urgilés et al., 2019) or as a capital budgeting tool (Platon and Constantinescu, 2014).

Nevertheless, the literature about the application of Monte Carlo Simulation in modelling the primal financial budget of a company is scarce (Hager et al., 2015). Hager et al. (2015) pointed the large widespread obtained for the outcome distribution (i.e. total profit or expense) as a result of considering a vast number of variables and the presence of interactions in the model as the main reason for the lack of application of stochastic modelling to the budgeting process of firms. Moreover, the subjective uncertain inputs which are usually included in the models are also a cause of great variability. These critics were already pointed out in the initial studies of Lord (1977). Additionally, Shim and Siegel (2005) refer the difficulty of creating the model as another possible reason.

To overcome these problems, Hager et al. (2015) recommends considering the model only as "accurate enough" and solely include primary drivers of the output variable. For this matter, Moro Visconti et al. (2018), outlined the importance of big data on the predictive ability of stochastic modelling, especially for short term budgets.

Chapter 3

Problem Description and Data Exploration

In this chapter, it is presented the main characteristics of the retail company Sonae MC, particularly the division of Accounting Business Solutions (ABS) where this dissertation's budget study simulation will be implemented. Moreover, the Budgeting Process of the ABS division is described and its limitations referred, building a foundational knowledge for the application of the simulation later introduced in Chapter 4. With the aim of increasing the knowledge of the data that will be subject to research, an initial data exploration analysis is also performed. This analysis is more of a preparing step for the methodology introduced in Chapter 4.

Section 3.1 introduces Sonae MC, the budgeting process of the ABS division and its limitations. This section is followed by the preprocessing and initial data exploration in Section 3.2, with focus on a trend and seasonality study and a comparison of the different components of the Expense. Section 3.3 presents a synthesis of the most relevant ideas and conclusions of the chapter.

3.1 Description of the company and the Budgeting Process of ABS

Sonae is a multinational company managing a diversified portfolio of businesses in retail, fashion, financial services, technology, shopping centers and telecommunications. Sonae businesses' units include Sonae MC, Sonae Fashion, Worten, Sonae FS, ISRG, Sonae IM, Sonae Sierra and NOS.

Sonae MC is a leading food retailer in Portugal. This company offers many options to consumers, counting with more than 1200 stores in the Iberian Region and a diversified portfolio ranging from big supermarkets to small convenience stores. Sonae Fashion is responsible for Sonae's specialized retail in fashion and manages brands such as MO, Zippy and Deeply. Worten is the company's bet on retail electronics.

Managing the accounting of these three main business units is a challenge and it is the scope of the Accounting Business Solutions (ABS) division. This team has more than 300 collaborators and it is responsible for the development of solutions that allow the control of the company's accounts, including the provision of shared accounting services, management of the purchase and sale processes and the relationship with suppliers and client companies.

The ABS division is constituted by six teams:

- **Accounting:** Responsible for controlling income and expense accounts in order to present the information succinctly and in accordance with accounting standards;
- **Accounts Payable:** Responsible for managing the accounts payable, namely payments and terms agreed with Sonae's suppliers and partner entities;
- **Accounts Receivable:** Responsible for managing accounts receivable from Sonae customers;
- **Business Partners & Innovation:** This team has two main functions, the Business Partners and the Innovation. The first has the function of serving as a communication bridge between ABS and the other Sonae companies; and the second is responsible for the development of new technologies in order to improve the process efficiency;
- **Consolidation:** Team that is responsible for preparing the financial statements of the various Sonae companies;
- **Document Solutions:** Manages the scanning, archiving and shipping of documents relevant to the company.

The Budget for the Business Partners & Innovation is divided in different cost centers: Budget for Business Partners Sonae MC, Budget for Business Partners Worten and Sonae Fashion and Budget for Innovation. Throughout the years some changes have occurred with how these cost centers are divided, for instance, before the year 2018 Business Partners for Worten and Sonae Fashion were split in different cost centers.

Each team prepares its budget and the budget of ABS is the result of the sum of the various budgets of the member teams.

The Budgeting process starts in September of the previous year and each team is responsible to devise the budget for the following components:

- **Personnel Expenses:** Done in collaboration with the Human Resources department, each team must plan the number of employees that will leave and the ones needed for the following year. Moreover, wages need to be considered as well as holidays, performance bonuses and insurance;
- **Operating Expenses:** This component includes mainly the Selling, General & Administrative expenses (SG&A) such as displacement costs and telecommunication expenses;
- **Other activity charges** include specialized and more punctual expenses such as specialized work expenses, software licenses, advertising expenses and mainly revenues from charged services to clients;

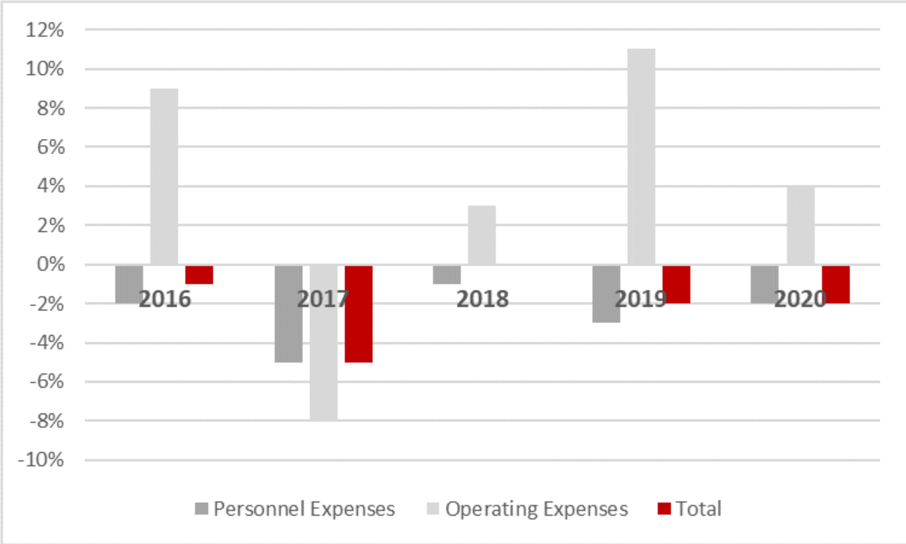


Figure 3.1: Percentage deviations of Expense from Budget in the ABS division

- **Office material expenses**;
- **Amortizations and Depreciations**;
- **Financial expenses** (interests receivable and payable).

Having the budget been duly justified, it is then approved and after each month of execution, a careful variance analysis is conducted to assess the reasons for deviations of the budget. Figure 3.1, depicts the percentage deviation from the budget for the years 2016-2020.

From the current Budgeting Process described above, there are some **limitations** that can be pointed out. The first, is the fact that there is no formal data driven decision-making support application for the creation of budgets, being the process based more on intuition and experience. Besides, the current process is deterministic, not considering the uncertainty in the estimation of the budget. This can be a problem as alerted by Moro Visconti et al. (2018), which may lead to biased or slack in budgets. Thirdly, the teams are unaware of the impact that the components have on the overall budget and how changing a component budgeted value may change the probability of budget cost overrun.

The estimation of the level of risk (probability to surpass the budget input and the consequent expected surplus) is a fundamental step in helping to consider uncertainties in the Budgeting Process, offering more confidence in the approval of budgets.

3.2 Data Exploration

Analyzing the information present in databases is crucial for companies to get valuable insights from the data collected. In this section, in order to do an initial exploration of the data used in this dissertation, the steps of a Data Mining process were implemented. Data Mining refers

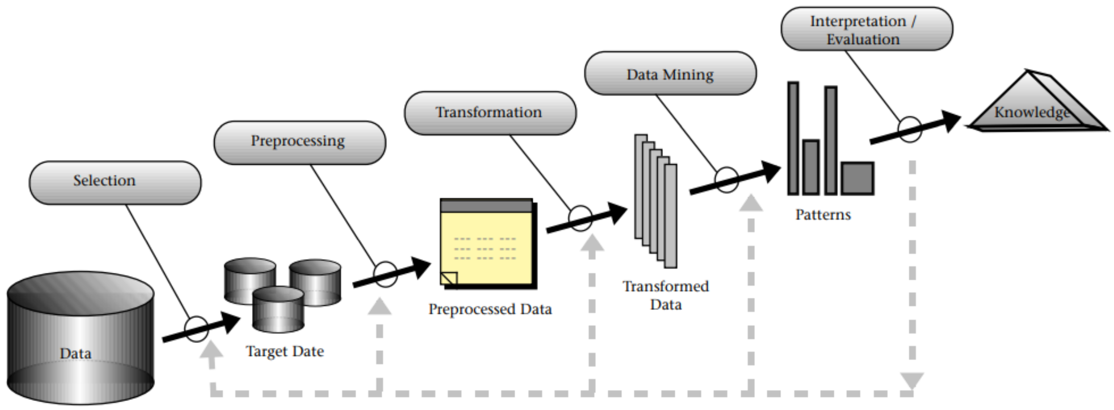


Figure 3.2: Knowledge Discovery in Databases through Data Mining (Fayyad et al., 1996)

to discovering interesting patterns and to get valuable knowledge from a big amount of data (Han et al., 2012). According to Fayyad et al. (1996), the Data Mining process has the following phases: 1) Selection; 2) Preprocessing; 3) Transformation; 4) Data Mining; 5) Interpretation/Evaluation. Figure 3.2 illustrates these steps.

Selection – The data used for this dissertation refers to the historic expenses of each cost center of the ABS division. This data was extracted from the database using SQL and has a time horizon ranging from 2016-2020. It was chosen not to include previous years as before this period there were major differences in the structure of the ABS division. In total, there were 209498 expenses in the dataset.

Preprocessing – Before the analysis of the data, a preprocessing stage occurred in which missing values were assumed to be equal to zero (meaning that no expenses occurred in that period). Outlier values were identified by visually analyzing the Expense components. They were subject to a careful inspection, having been conducted several interviews with the Accounting team to assess the reasons for its occurrence (errors in registration or simply abnormal values). The values corresponding to registration errors were corrected. For instance, there were cases when compensations were wrongly considered in a certain component or the mistake was corrected, but in the following month after having been occurred. In these last cases, the time series of the Expense components had unnecessary variability (as they were corrected, but in a posterior moment), and so we balanced them by eliminating the error cause right from the start. Fifteen values were corrected with this procedure.

Transformation – For confidentiality reasons the data was scaled in order not to be perceptible the magnitude of expenses, however, the proportions and the sign are the same.

The database contains thousands of records, needing a high level of attention and time for the study. In order to reduce the amount of data to be dealt with, expenses were monthly aggregated according to the main components of the budget already referred in Section 3.1. This made the dataset to be reduced to 960 records. Furthermore, the cost centers of Business Partners Sonae MC, Business Partners Worten & Fashion and Innovation were aggregated (forming the BPI_Innovation team) so as to remove the effects of changes that occurred as a result of separations and fusions between them during this period. The reasoning for this procedure was based on the fact that by including these cost centers separately, unnecessary great variability in the data would be created, which is not desirable when doing the simulation.

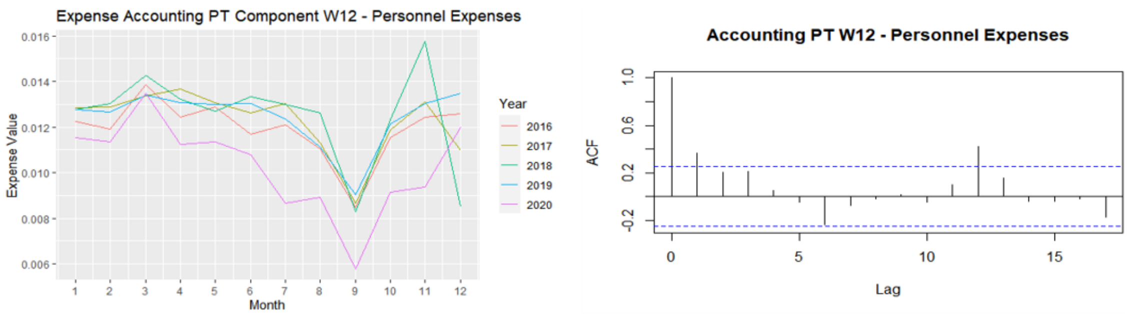
With the purpose of considering the effect of inflation and the time value of money, the monthly expenses were also capitalized to the year when the simulation is taking place (2021). This is an important step particularly if more data into the past is considered or the inflation rate in the period of analysis is not stable, which if disregarded can reduce the viability of the results of a simulation that is based on historical data. The annual inflation rates were collected from the public database PORDATA (2021).

Data Mining and Interpretation/Evaluation – Figure 3.3 shows the magnitude of the Expense components for each team, by displaying its average value in a bar and the minimum and maximum values registered during the period in analysis. It can be seen that Accounting PT, Accounts Payable and BPI_Innovation are the teams with the highest expenses. Besides, the class components having the highest expenses and variability are the *W12 – Personnel Expenses* and *W13 – Operating Expenses*. Therefore, the data used for the simulation study will be referent to these two classes. Note that despite, the variability registered in components *WOEA – Other Activity Charges* for two teams, it refers to more specialized expenses, thus being more deterministic. Besides, these components are mainly composed of revenues (which explains the negative sign), being out of scope for this project that intends to analyze the budget of expenses of ABS. The remaining classes have a small impact on total expenses and can be considered deterministic, as they are a result of planning (Office Material expenses) or calculations obtained from the ERP software (Amortizations and Depreciations).

The starting point for the seasonality study of Expense components was a visual inspection of its time series. Figure B.1 and B.2 of Appendix B represents the time series for the two class components selected for further study. From these graphs it can be concluded that there are some components, especially in the *W12 – Personnel Expenses* that seem to have a yearly seasonality. This can be justified as there occurs some repeatability of events throughout the years, such as every March there are increased personnel expenses as a result of the attribution of performance bonuses related to the previous year; and a decrease in September due to holidays. This analysis can be corroborated with the Figure 3.4 that depicts that the Personnel expenses of the Accounting team have yearly seasonality across the years.

Nonetheless, some components (especially in the Operating Expenses Category) can be interpreted as more random as there is no clear dependence with the previous months. Figure

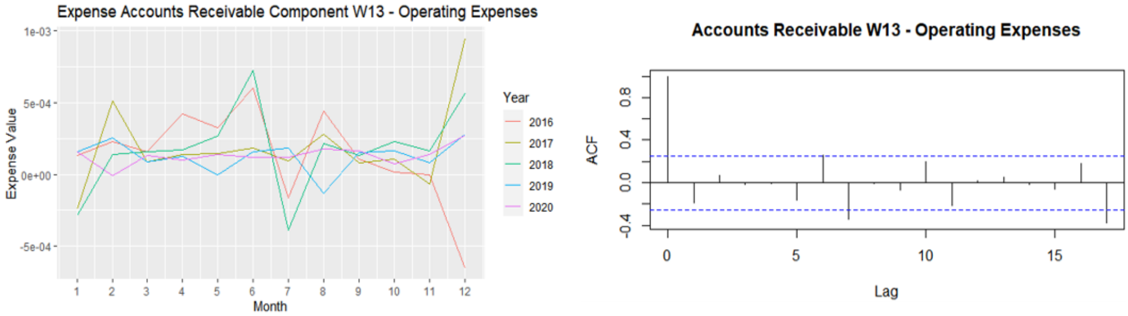
Figure 3.3: Comparison of magnitude and variability of Expense Components



(a) Seasonal plot of monthly expenses

(b) Autocorrelation Plot

Figure 3.4: Seasonality plots for *W12 Personnel Expenses* of the Accounting PT team



(a) Seasonal plot of monthly expenses

(b) Autocorrelation Plot

Figure 3.5: Seasonality plots for *W13 Operating Expenses* of the Accounts Receivable team

3.5 illustrates this fact, for the component *W13 – Operating Expenses* of team Accounts Receivable.

Table 3.1 extends this analysis and summarizes the existence of evidence of trend and yearly seasonality for each of the components analyzed. The p-values to assess the evidence of trend and seasonality were computed with the *seasplot()* function from the *R* package *tsutils*. A FALSE evidence conclusion means that the null hypothesis of the tests (i.e. series does not have trend/seasonality) failed to be rejected.

3.3 Synthesis

In this chapter, it was described the Budgeting Process of the ABS division. This division is constituted by 6 teams that have to include in its budget 6 components. The resultant budget for the ABS is the sum of the budgets of each team. For the data analysis, a time horizon from 2016-2020 is considered.

It was verified that Accounting PT, Accounts Payable and BPI_Innovation are the teams with the highest expenses. Moreover, as the components of class *W12 – Personnel Expenses* and *W13 – Operating Expenses* are the ones that have higher expenses and variance, they were chosen to be subject to the simulation study. The other four class components were disregarded as they can be considered deterministic and so will not influence the uncertainty of the budget.

Based on the sales pattern of the data and with regard to the trend and seasonality study, it was concluded that all the components referred to Personnel Expenses had yearly seasonality, whilst the Operating Expense components were more random with not so much dependence present in its time series. This conclusion is important as it underlines the necessity of considering time dependent methods to recreate the expenses as it will be discussed in Chapter 4.

The following chapter will expose the methodology used in order to obtain, from the historical data, the distributions of each Expense component. This next chapter is an extension to gain further insights from the data and to be able to assess the risk of a certain budget.

Table 3.1: Summary of trend and seasonality study

Component	Team	Trend		Yearly Seasonality	
		p-value[a]	Evidence of trend	p-value[b]	Evidence of Seasonality[c]
W12 – Personnel Expenses	Accounting PT	0.181	FALSE	≈0	TRUE
	Accounts Payable	≈0	TRUE	≈0	TRUE
	Accounts Receivable	≈0	TRUE	0.002	TRUE
	Consolidation	≈0	TRUE	≈0	TRUE
	Document Solutions	≈0	TRUE	0.009	TRUE
	BPI_Innovation	0.5	FALSE	0.038	TRUE
W13 – Operating Expenses	Accounting PT	≈0	TRUE	0.073	FALSE
	Accounts Payable	0.001	TRUE	0.515	FALSE
	Accounts Receivable	0.292	FALSE	0.27	FALSE
	Consolidation	≈0	TRUE	0.137	FALSE
	Document Solutions	≈0	TRUE	0.454	FALSE
	BPI_Innovation	≈0	TRUE	0.015	TRUE

[a] p-value to test evidence of trend, obtained according to the Cox-Stuart test
[b] p-value to test evidence of yearly seasonality, obtained according to Friedman test
[c] A FALSE evidence conclusion means that the null hypothesis of the tests (i.e. series does not have trend/seasonality) failed to be rejected

Chapter 4

Methodology

This chapter describes the methodology used to obtain the Expense components' distributions and the Monte Carlo Simulation that enabled to evaluate the risk of a budget inputted by the ABS Division of Sonae. Sensitivity Analysis with Importance Measures will also be presented. First, an overview of the methodology will be addressed in Section 4.1. Then, Section 4.2 will start with the Time series Bootstrapping Techniques implemented and their comparison and further validation. Section 4.3 will present the specificities of the simulation performed and the process used to generate correlated random variables in the sampling. Section 4.4 will introduce the theory behind the sensitivity importance measures implemented and, finally, Section 4.5 presents a synthesis of the most relevant ideas and conclusions of the chapter.

4.1 Methodology Overview

The goal of this project is to develop a decision support tool that enables to assess the risk of a certain budget level based on historical data expenses. This tool aims to give the decision maker the necessary information concerning the risk of the budget, by presenting the output according to the most appropriate risk definition described in Subsection 2.1.1. Throughout this dissertation, the risk of a certain budget is characterized by the probability of occurring a cost overrun (as a way of quantifying the uncertainty that the performance will deviate from a certain budget level) and by the consequence of this event occurring (expressed by both the expected and maximum deviation of the total expenses from the budgeted level if the budget is surpassed).

For this purpose, Simulation will be used because it makes it possible to consider the variability of the input variables of the expense and assess more quantitatively its impact on the output (total expenses) even in the absence of equations that are complex to formulate as a result of several mathematical operations between distributions (Bakhshi and Touran, 2014). Besides, in the simulation model implemented, historical data will serve as a basis for the creation of the distribution of each component's expense as it offers a more expedite way of considering the variability of components than defining mathematic relationships between causal variables. This approach makes it faster to evaluate the risk of a budget and not consider many necessary inputs to be given

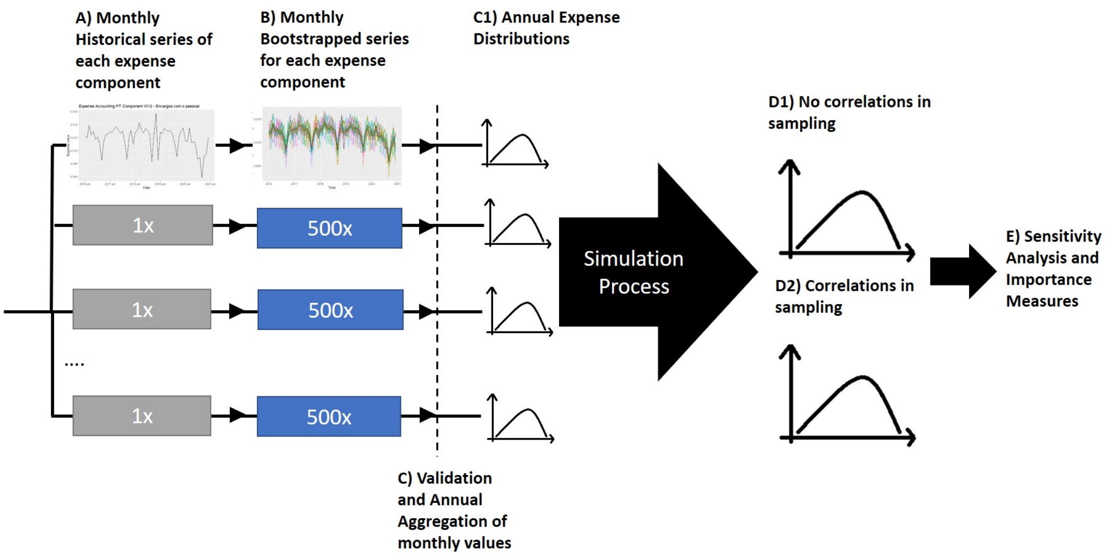


Figure 4.1: Scheme of the steps implemented in this dissertation

by the decision maker, which can be a source of bias in many risk analyses (Paté-Cornell and Dillon, 2006). Additionally, by considering the historical aggregated expenses of each Expense component, many relationships between its inner variables are already regarded and so more realistic results can be obtained.

Figure 4.1 summarizes the approach taken in this dissertation. The starting point was an analysis of the monthly historical series of each Expense component which was already presented in Section 3.2. From these series and in order to expand the data necessary to conduct the simulation several possible series were created from the original ones through time series bootstrapping techniques. This enabled to introduce variability in the input data and simulate different plausible expenses occurring for each component. Afterwards and as the expense budget to be analyzed is assessed annually, an annual aggregation of the monthly values of each series created was performed resulting in the annual expense distribution for each Expense component.

Then with these input distributions a Monte Carlo Simulation was conducted, for which two approaches were adopted related to the sampling procedure, or in other words, the way the realizations were sampled from each of these distributions. The first one (D1) was the standard procedure in which each sampling was assumed to be independent, meaning that there was total freedom to choose a value from the distribution. The second (D2) was considering the correlations existent between each of the annual expenses. This second approach is motivated because by reality correlations between variables may exist and consider those will yield more accurate results and a lower forecast confidence interval if negative correlations prevail. (Mun, 2012).

Finally, to assess the importance and contribution that each input expense variable has on the total expense distribution generated, a sensitivity analysis with some state-of-the-art importance measures introduced in Subsection 2.2.2 was performed.

All of the conducted analyses were done using the *R* programming language.

Figure 4.2: Block Formation in Block Bootstrap Methods

4.2 Time Series Bootstrapping

As it was concluded from Section 3.2, several Expense components' series have trend and seasonality, implying that the normal bootstrap procedure of Efron (1979) is not recommended as it does not consider the time dependence present in the data. Therefore, to create additional series for the simulation, Time Series Bootstrapping Techniques were implemented. The implemented techniques can be divided in: the Moving Block Bootstrap that considers series stationary, Model Based and Box-Cox and Loess-based decomposition (BLD) Bootstrapping that can be used for non-stationary series (ideally seasonal) and the K-means Based Bootstrap procedure which adapts better to noisy series. For each Expense component five hundred bootstrapped series were generated by each of these techniques. Subsection 4.2.1 will present these techniques and Subsection 4.2.2 the procedure followed to compare the performance of the enunciated methods and its validation.

4.2.1 Time Series Bootstrapping Techniques

Moving Block Bootstrap – Considering a time series with n observations, the series created through the Moving Block Bootstrap result from resampling with replacement from blocks of data until a new series with the same dimension is created. Each block has the same length defined as L and as they overlap, for a series of n observations there are $n-L+1$ blocks. Figure 4.2 resumes the process of block formation when $L=3$. The indices $1:L$ define the first block of data, $2:L+1$ define the second block and $n-L+1:n$ the last block. In *R*, this procedure can be implemented using the function *tsbootstrap* from the *tseries* package.

Model Based Bootstrap – Another class of methods implemented were Model Based techniques that can adapt better for non-stationary series with seasonality, which is the case of the majority of the expense series as analyzed in Section 3.2. For these techniques it was considered a statistical model that could incorporate trend and seasonality present in the data and then the moving block bootstrap was applied to the residuals of the model. This way it was not assumed that the residuals were IID. The series created resulted from adding the fitted values from the statistical model with the bootstrapped residuals. Three different models were implemented: SARIMA, ETS and TBATS.

- SARIMA is a model that is very similar to the Autoregressive Integrated Moving Average (ARIMA) model, but it has the advantage of considering seasonality (Hyndman and Athanasopoulos, 2018). An ARIMA model is a combination of an autoregressive and a moving average model. In the first, the variable of interest is explained as a linear combination of its past values and in the moving average model it results from adding past fitted errors in a regression similar formulation. The integrated part means that firstly the series is differenced so that it becomes stationary. In *R* this model can be implemented using the *auto.arima* function from the *forecast* package, that automatically returns the best SARIMA model according to the AICc criteria.
- ETS represents the Exponential Smoothing family models. The simplest one is the Simple Exponential Smoothing (SES) that models the series' values as a result of the average of past observations with weights decaying exponentially as the observations get older. The most advanced is the Holt Winters that considers the level (Lt), the trend (Tt) and the seasonality (St) of a time series. The best ETS model for a time series according to the AICc criteria can be returned in *R* using the *ETS* function from the *forecast* package.
- TBATS is a model proposed by de Livera et al. (2011) that consists on performing Box-Cox Transformation, Autoregressive Moving Average (ARMA) errors correction, and smoothed Trend and Seasonal Components. This model can handle multiple seasonality, being the seasonal components described by trigonometric functions based on Fourier Series. It is an extension of the BATS model, which generalizes the ETS models for multiple seasonality. In R it can be implemented with the *TBATS* function from the *forecast* package.

BLD Bootstrap – The BLD Bootstrap was proposed by Bergmeir et al. (2016) and consists on first doing a Box Cox Transformation to stabilize the variance of the series, and then obtain its trend, seasonal and remainder components with the Seasonal and Trend decomposition using Loess (STL). If the series is not seasonal only the trend and residuals are obtained through Loess decomposition. Then, the remainder components are bootstrapped with the Moving Block Bootstrap and are summed to the trend and seasonality parts previously obtained. This technique is model free as it does not use statistical models to fit the series. In *R* it can be implemented with the function *bld.mbb.bootstrap* of the *forecast* package.

K-Means Based Bootstrap – The K-Means based Bootstrap is a Time Series Bootstrap procedure proposed by Laurinec et al. (2019), so that the created series could have a better adaptation when series were noisy. This method does not create new points, but rather samples from existent points in the series that can be considered similar according to the K-Means clustering result. In other words, after the application of the K-Means clustering algorithm to the points of the series there occurs n samples with replacement in which each value sampled belongs to the cluster assigned to the ith point.

K-Means is a clustering algorithm that partitions the data in K clusters, by minimizing the distances inside each cluster and maximizing the ones between clusters. The optimal number of clusters was defined according to the minimum value of the Davies-Bouldin Index. The maximum number of clusters was defined to be 6 to avoid overfitting the data. As the K-Means clustering may not converge depending on the initial centroids randomly chosen, the algorithm was performed 10 times for each series and the final optimal number of clusters chosen was the one with the smallest Davies-Bouldin index of the best of each of these 10 iterations. The K-means was implemented in *R* using the function *kmeans* of the *stats* package and the Davies-Bouldin Index was calculated using the *intCriteria* function of the package *clusterCrit*.

4.2.2 Comparison and Validation of the Time Series Bootstrapping Techniques

In order to pick, for each expense series, the time series bootstrapping method that produced the most similar and plausible series, the Normalized Root Mean Square Error (NRMSE) was used, as, similarly to the commonly used RMSE, it penalizes values that are more distant to the original ones.

The NRSME has the advantage over the RMSE of allowing to compare the quality of the fitness between the different Expense components even if their order of magnitude is different, as it is normalized by the order of their values. This is particularly useful for interpretability since the data was scaled for confidentiality reasons. For the normalization, the difference between the maximum and minimum values was chosen as the series can have negative values and so the mean is not a viable option. Note that this criterion compares points in the same time period, which enables to maintain the time dependence of the original series. Equations 4.1 and 4.2 show the RMSE and NRMSE formulas used respectively, where y_i represents each monthly value observed in the original expense series and $\hat{y}_i$ represents its correspondent bootstrap expense series value.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} \tag{4.1}$$

$$NRMSE = \frac{RMSE}{y_{max} - y_{min}} \tag{4.2}$$

After selecting the best method for each Expense component and to validate if the values from the five hundred series created belonged to the same distribution as the original ones, a Kolmogorov-Smirnov test (KS) was performed to each series created. The minimum p-value was evaluated and compared to the threshold of 5%. If above, the null hypothesis that the two series follow the same distribution failed to be rejected and so the method successfully created similar series. If not, then the similarity of the series is questionable, and it will be necessary to perform a careful graphical analysis of each series to make further conclusions about the selected bootstrap method.

4.3 Simulation Process

After having the most suitable monthly series for each Expense component, an annual aggregation of the monthly values was performed so that the end result would be its annual distribution based on the historical and generated similar data related to the period 2016-2020.

A Monte Carlo Simulation could now be performed from sampling from each of these input distributions and summing the values of each realization of the simulation so that the total expense distribution could be obtained. One hundred thousand iterations were done to guarantee the convergence of the output distribution and the sampling of the extreme values from the input distributions. For the sampling procedure two options were tested for further comparison: considering each expense component as independent from each other and considering its historical annual correlations in the sampling.

Generating correlated random variables – The procedure implemented to generate correlated random variables was adapted from Neine and Curran (2021). It uses the Cholesky Decomposition of the correlation matrix to transform independent normal random variables into correlated normal random variables. These variables are then transformed into the desired distributions whilst having correlations approximately equal to the original correlation matrix. The algorithm has the following steps:

1. From the annual expense component's correlation matrix (C) use the Cholesky Decomposition to decompose C in two triangular matrices (L e L^t). $LL^t = C$
2. Simulate X uncorrelated standard normal random variables, where X has the same dimension as the number of expense components to be included in the simulation. $X \sim N(0,1)$
3. Obtain the Y correlated standard normal random variables by $Y = LX$. Because of the Probability Integral Transform theorem computing the cumulative of each distribution of Y (which are continuous distributions) yields the standard uniform distributions Z (Angus, 1994). $Z = Cdf(Y),\ Z \sim U(0,1)$
4. Compute the inverse transform of the cumulative distributions of the annual expenses, using the uniform values of Z. This enables to sample values from each of the Expense component's annual distribution with an expected correlation of C between expenses.

To obtain the correlation matrix (C), Spearman's rank correlation for the annual expenses was used as it does not assume that the correlated variables have an underlying normal distribution or its relationship is linear. This rank coefficient is according to Mun (2012) the most commonly used and most appropriate in the context of Monte Carlo Simulation.

To characterize the output total expense distribution it was used, besides the graphical representation, its statistics (mean, mode, median, variance, skewness and kurtosis) as well as its percentiles to get a sense of the values of the distribution. For the assessment of the risk of a given budget, the total annual expense distribution obtained was compared with the total budgeted value and the following metrics were devised: probability of the expenses exceeding the budget; the expected and the maximum percentage deviations if the expenses exceeded the budget and the cost of uncertainty. This last one is defined as the expected expense value occurring if the expenses pass the budgeted level.

4.4 Sensitivity Analysis and Importance Measures

With the intention of exploring the importance and contribution that each input expense distribution has on the total expense distribution, three importance measures were implemented: R-Squared contribution of each input (LMG), estimation of variance explained by each input with regression (S_i) and the moment independent importance measure of Borgonovo (2007) δ_i.

LMG is a technique that allocates a share of the R-Squared to each input variable (Grömping, 2015). It is also known as Shapley Value Regression and can be used in the presence of correlated input variables as it considers the sequence that the predictors appear in the linear regression model. In other words, the method does all the possible sequences of including an input in a linear regression model and computes the mean marginal contribution to the R-Squared. The sum of all the values from each variable equals the total R-Squared of the regression model (which in this case is 100% as the relationship between the total expense and its inputs is perfectly linear) and the variables with the highest contribution have a higher relative importance. In *R* it can be implemented using the *LMG* function of the *sensitivity* package.

Variance explained by each input (S_i) and decomposition in its correlated and uncorrelated parts with regression was initially proposed by Xu and Gertner (2008). The methodology adopted was the improved one suggested by Hao et al. (2012), but instead of using ANNs a linear regression was performed as, for this problem, the relationship between output and inputs is known to be linear. Being $\widehat{V}$ the total variance of output y (i.e. total annual expense simulated values) defined as,

$$\widehat{V} = Var(y) = \frac{1}{N-1}\sum_{j=1}^{N}(y_j - \bar{y})^2 \tag{4.3}$$

The Variance contribution by x_i to y is given by:

$$\widehat{V_i} = \frac{1}{N-1}\sum_{j=1}^{N}(\widehat{y_j{}^{(i)}} - \bar{y})^2 \tag{4.4}$$

Where $\widehat{y_j{}^{(i)}}$ denotes the fitted values from regression, $\widehat{y_j{}^{(i)}} = f^i(x_i)$, being $x_i \in X$. X represents the realizations input matrix obtained through Monte Carlo and x_i its input expense variable i.

The uncorrelated variance contribution of input i, $\widehat{V_i^U}$, can be obtained by $\widehat{V_i^U} = \widehat{V} - \widehat{V_{(-i)}}$, where $\widehat{V_{(-i)}}$ denotes the variance contribution by all variables except x_i. $\widehat{V_{(-i)}}$ can be calculated the same way as $\widehat{V_i}$, but instead of doing the regression with x_i it uses all variables except x_i, $x_{(-i)}$. The correlated contribution of variable i can now be obtained by $\widehat{V_i^C} = \widehat{V_i} - \widehat{V_i^U}$.

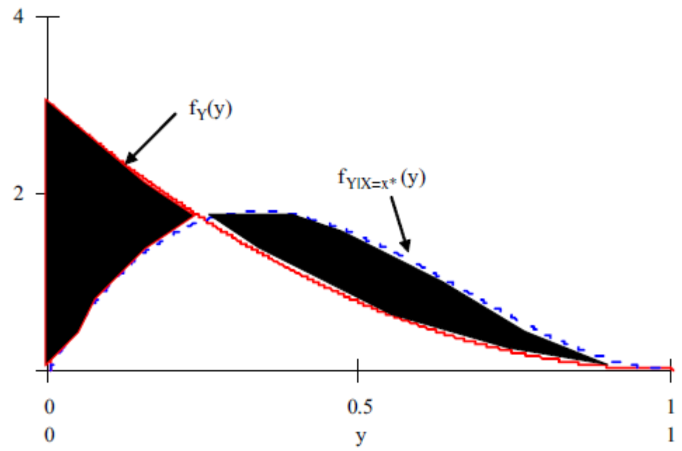


Figure 4.3: Graphical illustration of the term inside the integral of Equation 4.6 (Borgonovo, 2007)

The sensitivity indices are the ratios of the corresponding variances:

$$Si = \frac{\widehat{V_i}}{\widehat{V}},\ Si^U = \frac{\widehat{V_i^U}}{\widehat{V}},\ Si^C = \frac{\widehat{V_i^C}}{\widehat{V}} \tag{4.5}$$

$\boldsymbol{\delta_i}$ is a Moment independent importance measure proposed by Borgonovo (2007), meaning that it does not evaluate the relative importance of an input variable based on the change of a moment of the output's distribution (e.g. variance), but on its whole distribution itself. The index is defined in Equation 4.6 and can be interpreted as the expected shift between the output distribution fY and the conditional output distribution $fY|X_i$. The quantity ½ is introduced for normalization. The term inside the integral graphically represent the shaded area in Figure 4.3, where the conditional distribution is defined for a specific value of X_i, x_i*.

$$\delta_i = \frac{1}{2}\mathbf{E}_{X_i}\left[\int |f_Y(y) - f_{Y|X_i}(y)|dy\right] \tag{4.6}$$

Table 4.1 summarizes the properties of the δ_i sensitivity index. This index is well defined regardless of the dependence structure of the input. The higher the value of δ_i, the higher the relative importance of X_i. In *R*, it can be implemented with the *sensiFdiv* function of the *sensitivity* package by setting the parameter Total Variation "TV".

Table 4.1: Properties of δ_i (Borgonovo, 2007)

No.	Property	Meaning
1	$0 \leq \delta_i \leq 1$	Bounds of δ_i
2	$\delta_i = 0$	In case Y is independent of X_i, $\delta_i = 0$
3	$\delta_{1,2,\ldots n} = 1$	The value of the index when all parameters are considered simultaneously is 1
4	$\delta_{ij} = \delta_i$	Applies in case Y is independent of X_j
5	$\delta_i \leq \delta_{ij} \leq \delta_i + \delta_{i,j}$	Bounds of δ_{ij}

4.5 Synthesis

This chapter intends to describe the methodology used in this dissertation to obtain the total distribution of the annual Expense which will be used to compare with a given budget input.

Firstly, Time Series Bootstrapping methods were implemented so that additional valid series could be generated and be used to obtain the distributions for each expense component. The methods implemented were: the Moving Block Bootstrap which has better results when the series are stationary; Model Based Bootstrap procedures (based on ETS, SARIMA, or TBATS) and the BLD Bootstrap which can be used for non-stationary series (preferably seasonal); and the K-Means based bootstrap that allows creating series even if they are noisier. For each Expense component, five hundred bootstrapped series were created with each of the techniques. These different generated series were compared using the NRMSE criteria and the method with the lowest value was selected. A further validation using the KS hypothesis test was performed to check if there was statistical evidence that the selected generated series were different than the original ones.

Secondly, an annual aggregation of the monthly values for each expense components' series was performed, yielding its historical annual distributions. A Monte Carlo Simulation with one hundred thousand iterations followed, having been considered two different sampling procedures for further comparison: 1) independent random sampling; 2) correlated random sampling. The annual total expense distribution results from summing all the realization values of the expenses simulated.

Thirdly, the obtained output distribution was compared with a given certain budget level and the risk of the budget and the characterization of the distribution estimated.

Finally, a sensitivity analysis through importance measures was implemented using the R-Squared contribution of each input (LMG), the estimation of variance explained by each input (S_i) and the moment independent importance measure of Borgonovo (2007) δ_i.

The following chapter 5 will present the results obtained with this methodology.

Chapter 5

Results

In this chapter, it will be presented and discussed the results obtained from applying the algorithms and methodology described in Chapter 4. The annual total expense distribution will be analyzed, and the impact of correlations assessed. The obtained distributions will be compared with a certain budget level chosen in order to estimate its risk and then the relative importance of each expense component assessed through the application of the sensitivity importance measures enunciated in Section 4.4. Finally, a functionality extension that allows monitoring budget risk throughout the year and the interactive application developed that performs these analyses will also be described.

In Section 5.1 are presented and analyzed the performance of the time series bootstrapping methods implemented. Section 5.2. shows the distributions of the components obtained by annually aggregating the monthly values of each best generated series. The total output expense distribution considering independent and correlated random sampling are also showed. Then, in Section 5.3. the results from the sensitivity importance measures to assess the variables that have the most influence on the total expense are showed and the effect of correlations is evaluated. Section 5.4 presents how the principles enunciated before can be used to extend the functionality and monitor the budget risk throughout the year. Section 5.5 describes the application developed that executes interactively the features introduced in the previous sections to help monitor the risk of a certain budget and assess the annual expense distributions. Lastly, Section 5.6 synthetizes the main takeaways from this results chapter.

5.1 Performance of Time Series Bootstrapping techniques

As it was previously explained in Section 4.2, several Time Series Bootstrapping techniques were implemented in order to create similar monthly expense series for each expense component. For the techniques that rely on the definition of the block length (MBB, Model Based techniques and BLD), different lengths were tested to evaluate the impact of this parameter on the results. The lengths of the blocks tested were 1; 3; 6; 12 and 24. The choice of length 1 can be justified if the residuals to be bootstrapped for the model are independent and do not have any remaining time dependence. For the case of the MBB it corresponds to the normal Bootstrap of Efron (1979). The

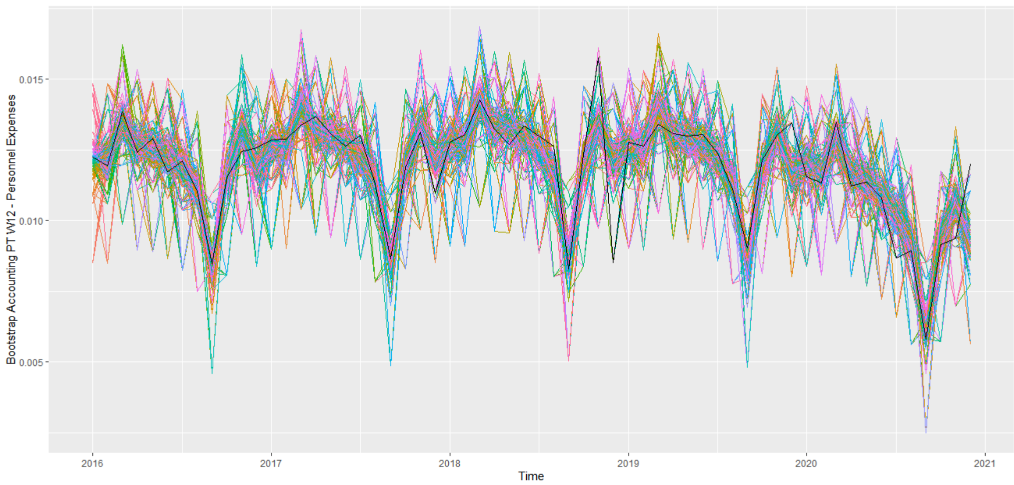


Figure 5.1: Bootstrap generated series with BLD for *Accounting PT W12 Personnel Expenses*

other values were chosen due to the general fact that the expenses can be analyzed in quarters so there can be seasonality of multiple of this time period. The maximum length of 24 derives from the recommendation of Bergmeir et al. (2016) to ensure that any remaining seasonality is captured. As for the K-Means Based bootstrap the maximum number of clusters allowed to be formed was 6 to allow some variability in the data, but at the same time isolate more distinct values. For all the methods five hundred series were created.

The performance of these techniques was evaluated with the NRMSE for each Expense component. Table 5.1 shows the results of the Expense component *Accounting PT W12 Personnel Expenses*. The technique that delivered the best results was the BLD with a block length of 3 and a corresponding NRMSE of 0.1142. The five hundred generated series with the BLD bootstrap for this expense component and the original monthly expense series can be seen in Figure 5.1 in colors and in black respectively. The complete results are presented in Appendix C for the remaining components.

A summary of the results concerning the best techniques and its parameter values is presented in Table 5.2. Additionally, it can be seen the minimum p-value obtained from doing the KS test to each of the five hundred generated series as described in Subsection 4.2.2.

The results demonstrate that, generally, the MBB technique yields the worst results. This can be justified as the majority of the expense series in analysis is not stationary, with the exception of *Accounts Receivable W13 Operating Expenses* that did not present trend and seasonality from the study conducted in Section 3.2. For this component, the obtained results inside the block dependent techniques are very competent. Moreover, as of the model-based techniques implemented TBATS performs better, but BLD can create even more similar series. This may be due to the fact that BLD can also account for non-seasonality with the Loess Decomposition, contrarily to TBATS that works better for seasonal components.

Another relevant insight is that the choice of the model is more important and has more impact on the results than the block length that only changes slightly the NRSME values. This suggests

Table 5.1: NRMSE values of the different Time Series Bootstrapping Techniques for *Accounting PT W12 Personnel Expenses*

Method	Block Based Methods					K-Means	
	Block Length					Number of	
	1	3	6	12	24	Clusters	NRMSE
MBB	0.2540	0.2537	0.2522	**0.2423**	0.2488	–	–
ETS	0.1444	**0.1425**	0.1453	0.1483	0.1587	–	–
SARIMA	0.1599	**0.1589**	0.1607	0.1640	0.1604	–	–
TBATS	0.1372	0.1338	**0.1337**	0.1362	0.1396	–	-
BLD	0.1176	**0.1142**	0.1152	0.1168	0.1205	–	–
KMEANS	–	–	–	–	-	2	**0.1321**

Table 5.2: NRMSE of the best selected Time Series Bootstrapping Technique for each Expense component

Expense Component	Time Series Method	Block Length or Number of clusters	NRMSE	Minimum p-value of KS test
Accounting PT W12	BLD	3	0.114	0.120
Accounting PT W13	KM	4	0.095	0.375
Accounts Payable W12	KM	6	0.057	0.809
Accounts Payable W13	KM	5	0.045	0.181
Accounts Receivable W12	KM	5	0.072	0.375
Accounts Receivable W13	KM	3	0.094	0.181
Consolidation W12	KM	4	0.069	0.509
Consolidation W13	KM	6	0.041	0.509
Document Solutions W12	KM	3	0.106	0.267
Document Solutions W13	BLD	3	0.176	0.120
BPI_Innovation W12	KM	5	0.062	0.375
BPI_Innovation W13	KM	6	0.057	0.660

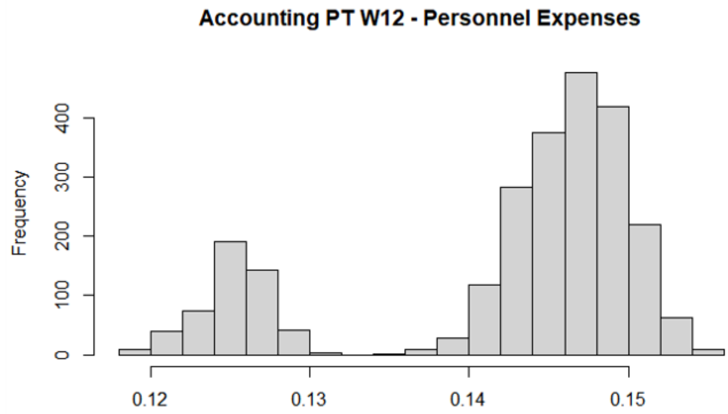


Figure 5.2: Annual Distribution obtained for *Accounting PT W12 Personnel Expenses*

that the residuals tend to have the same magnitude and do not present much time dependence between them. This result goes in accordance Radovanov and Marcikić (2014).

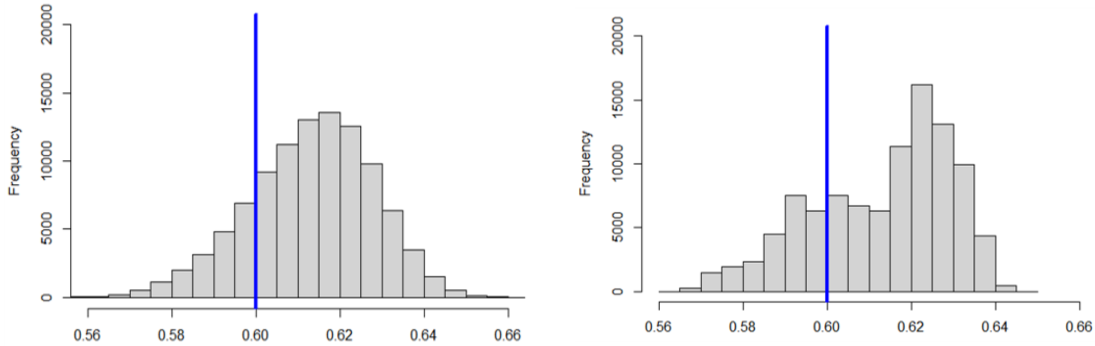
The technique that generally had the best results was the K-Means Based Bootstrap. This has to do with its capacity to isolate more extreme values in separate clusters and choose randomly from the rest of the points. The end result is that generated series are more similar to the original one, as suggested by the higher p-values from the KS test.

Moreover, it can be concluded that the components that had the highest NRMSE were the *Document Solutions W13*, *Accounting PT W12* and *Document Solutions W12*. It can also be verified that mostly there is a higher gap of NRMSE values between block length-based techniques and the K-Means for the *W13 Operating Expenses* class components. This fact can be justified by its more random nature as explained in Section 3.2, which results in increased modelling difficulty with statistical models. This conclusion can be corroborated by the fact that more components belonging to the *W13 Operating Expenses* class needed the highest number of clusters allowed to be formed.

To finalize, note that for all the optimal series created none rejected the KS test, which gives assurance that they can be considered similar to the original series. In other words, they could be a plausible expense alternative series for the period 2016-2020. The graphs in Appendix D show the generated optimal series for all Expense components.

5.2 Simulation Process

After the generation of the additional expense series, the next step was to perform for each component an annual aggregation of the monthly values of each series. The resulting distribution is shown in Figure 5.2 for *Accounting PT W12 – Personnel Expenses*. As it can be observed the distribution is centered around two pick values due to the fact that the year 2020 was very atypical in terms of personal costs that decreased considerably. The distributions for the remaining components are presented in Appendix E.



(a) Independent Random Sampling

(b) Correlated Random Sampling

Figure 5.3: Total Expense Annual Distributions

From these distributions a Monte Carlo Simulation with one hundred thousand iterations was performed resulting in the distribution of the total annual expense shown in Figure 5.3. Subfigure 5.3a corresponds to when the independent random sampling between the components is considered, whilst Subfigure 5.3b refers to the correlated one. The blue vertical line depicts the total budgeted value, that for this example is considered to be 0.60. Table 5.3 summarizes the differences between the two distributions and at the bottom presents the risk metrics for the budget level of 0.60. In Figure 5.4 it is depicted the historical annual Spearman's rank correlations between the components.

As it can be observed, the presence of correlations in the sampling does little change to the expected value of the expense. Nonetheless, it affects considerably the spread of the distribution (characterized by the range) that decreased by 26%. This reduction in the tail values can also be seen because of a lower value of kurtosis in the correlated distribution indicating the presence of less extreme values compared to its mean. The decrease in spread can be justified by the presence of negative correlations between some components, corroborating the conclusion of Mun (2012) with respect to the effect of correlations. Besides, as the components influence others, the total expense distribution does not center nicely in one value, presenting more uncertainty and variability when correlations are considered (higher standard deviation). Another impact of

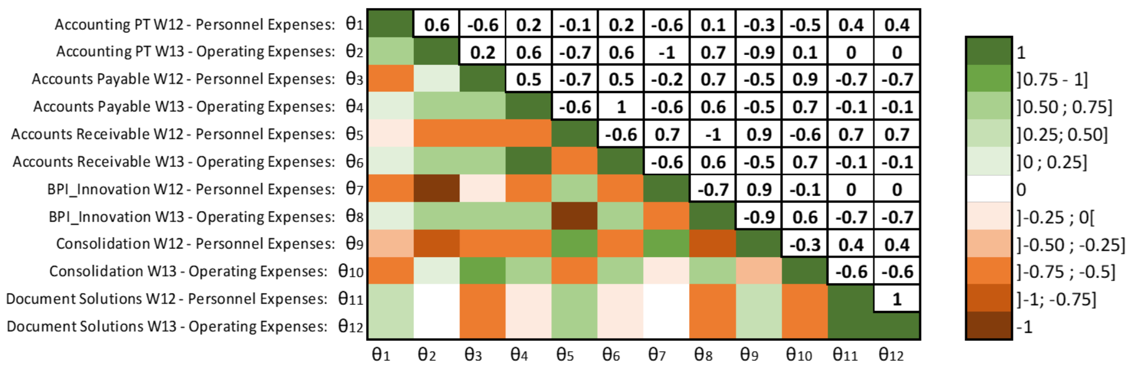


Figure 5.4: Spearman's Rank Correlation Coefficients

Table 5.3: Comparison of Total Expense Distribution Statistics with Independent and Correlated Random Sampling

Statistics	Random Sampling Independent	Random Sampling Correlated	Change (%)
Mean	0.6131	0.6131	–0.001
Median	0.6142	0.6177	0.5762
Mode	0.6080	0.6298	3.5969
Standard Deviation	0.0147	0.0165	11.9565
Skewness	–0.3221	–0.5656	75.593
Kurtosis	2.9086	2.3941	–17.6896
Range = Max - Min	0.1132	0.0831	–26.622
95% prediction interval	[0.5818; 0.6392]	[0.5770; 0.6369]	–
Value at Risk (5%)	0.6357	0.6348	–0.1323
Probability of expense exceed budget	81.2410	75.7410	–5.5pp
Cost of uncertainty	0.6183	0.6207	0.3967
% Expected Deviation if budget exceeded	3.0472	3.4560	0.4088pp
% Maximum Deviation if budget exceeded	11.1254	7.5280	–3.5974pp

the correlations, in this case, is the shift of the expenses to the right which is suggested by the skewness difference between the two distributions.

Because of the presence of negative correlations present in some of the components, the risk of the budget is lower when correlations are considered as the reduction in the probability of the total expense exceeding the budget indicates. However, the consequence of the budget being surpassed is slighter higher due to the shift of the distribution to the right, which can make the conclusion not that straightforward and necessary to consider the whole risk profile as discussed in Subsection 2.1.1.

Note that this same analysis presented in this subsection can be applied to any input distribution or combination of expense distributions.

5.3 Sensitivity Importance Measures

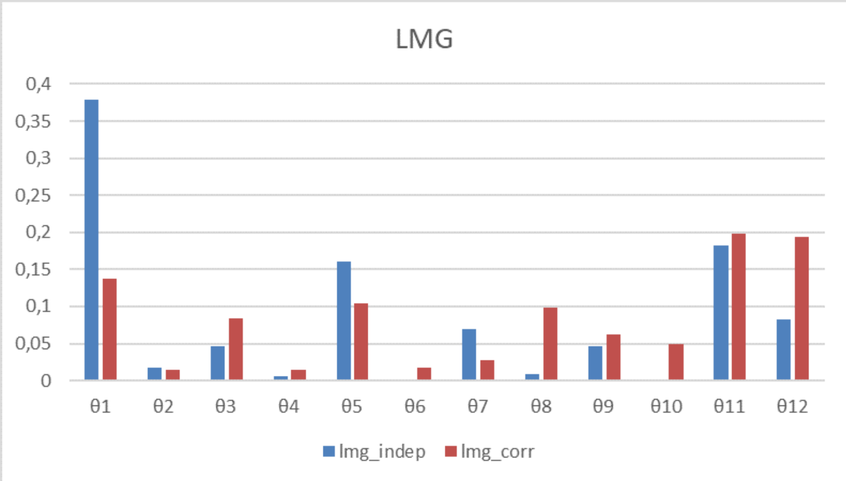
Now that the distribution of the total expense is fully characterized, it is useful to know what input expense components influence this output distribution the most. For that, the sensitivity importance measures LMG, total variance explained estimation ratio (S_i) and the moment independent index δ_i were implemented as described in Section 4.4. Figure 5.5 shows the values obtained for each Expense component by the different measures tested and Table 5.4 presents the respective importance rankings when considering independent or correlated random sampling.

From the results it can be observed that when independent random sampling is considered, the rankings given by the measures to the inputs converge in defining the most and least important input expense variables. Additionally, the values of the LMG and variance ratio are very similar, meaning that in the presence of independent expenses, its marginal contribution to the R-Squared corresponds to the ratio of explained variance, as expected. Besides, as the rankings coincide with the δ_i, variance can be used to describe the effect on the outcome distribution. This happens because it is a direct reflection of model function decomposition and structure (Borgonovo, 2006). Variance based measures also provide guidance in determining the best components that can reduce total variance and, thus, are particularly useful as single-handedly characterize variance and distribution effect when independence between inputs can be assumed. The three most influential components belong to the *W12 Personnel Expenses* class and concern the teams Accounting PT, Document Solutions and Accounts Receivable. Contrarily, the three least important ones belong to the *W13 Operating Expenses* class and refer to the teams Accounts Payable, Consolidation and Accounts Receivable.

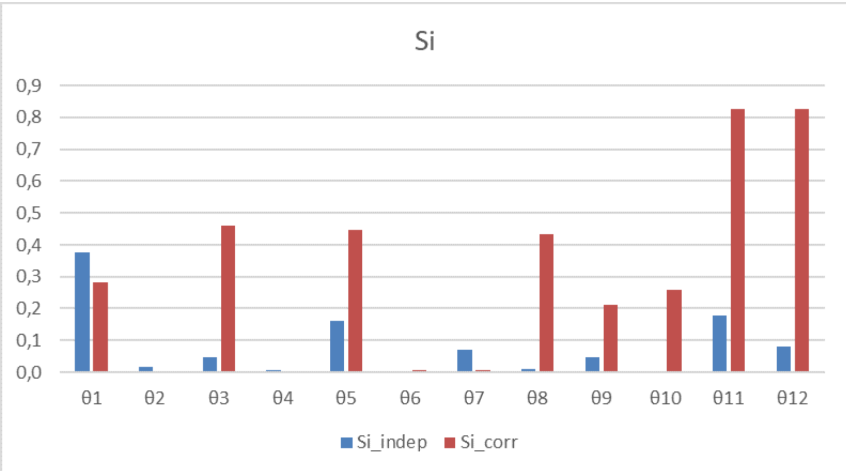
In comparison, when correlations are considered (and as they differ considerably from zero) the values from the computed measures are distinct. LMG values suggest that the marginal contribution to the R-Squared of each input variable becomes more faded and similar as its effect is partially explained by other variables. Differently, the variance explained ratios (S_i) increase because each input can now influence more of the output by changing other input variables. In fact, the explained variance of each input is mainly due to the presence of correlations as it is corroborated by the correlated ratios shown in Table F.1 of Appendix F. Still, the rankings given by the LMG and S_i are, in general, similar since they are essentially related to the variance of the output. As far as the δ_i is concerned, its values are increased in the correlated analysis being in tune with the results from the correlation study of Borgonovo and Tarantola (2008). This fact means that each input causes a bigger expected shift by influencing other variables. The three most important components given by δ_i are the Document Solutions team expenses and the Consolidation *W12 Personnel Expenses*. The ones with the least importance are the *Accounting PT W12 Personnel Expenses*, *Accounts Payable W13 Operating Expenses* and *Accounts Receivable W13 Operating Expenses*.

Note from Table 5.4 that the model inputs that influence variance the most are not necessarily the ones that impact more the distribution. However, the two most and least importance variables are the same in both measures.

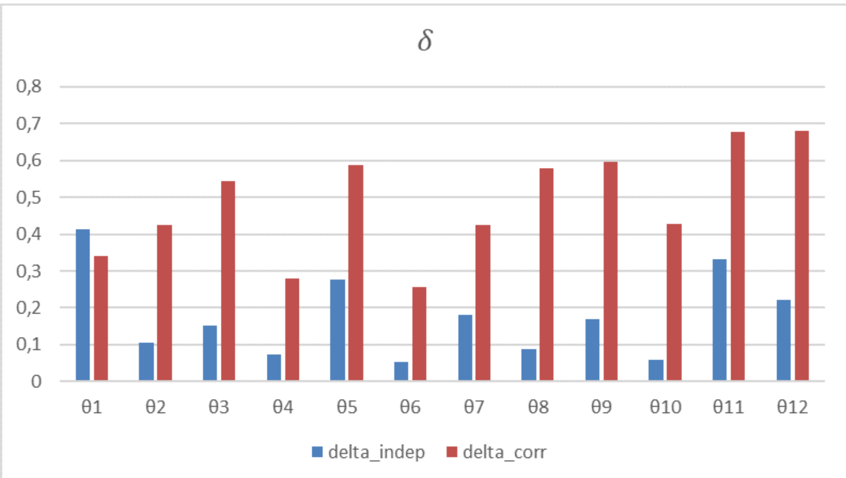
From these results, it means that addressing ways to control more the expenses especially in the Document Solutions team may have potential in reducing the total expense variance and positively influence the total expense distribution. Therefore, a joint utilization of variance-based techniques and the δ_i allows to better characterize input importance and comprehend its uncertainty propagation.



(a) LMG values



(b) S_i values



(c) δ_i values

Figure 5.5: Importance Measures implemented for Independent (in blue) and Correlated Random Sampling (in red)

Table 5.4: Expense component rankings according to the different Importance Measures implemented

Component	Random Sampling					
	Independent			Correlated		
	LMG	S_i	δ_i	LMG	S_i	δ_i
θ_{12}	4	4	4	2	1	1
θ_{11}	2	2	2	1	2	2
θ_9	7	7	6	7	8	3
θ_5	3	3	3	4	4	4
θ_8	9	9	9	5	5	5
θ_3	6	6	7	6	3	6
θ_{10}	11	11	11	8	7	7
θ_2	8	8	8	12	12	8
θ_7	5	5	5	9	9	9
θ_1	1	1	1	3	6	10
θ_4	10	10	10	11	11	11
θ_6	12	12	12	10	10	12

*Components ordered by the δ_i rank obtained in the Correlated Random Sampling.

5.4 Functionality extension: Monitoring the budget risk throughout the year

By changing the way the monthly aggregation of each generated series by the time series bootstrap is done, one can extend the previous analysis and monitor the risk of a budget throughout the year of its execution. The key idea is in considering that in a certain present month t, there occurred already $1 : t-1$ months of expenses that are known and so the remaining budget portion (i.e. total annual budgeted value minus the sum of already incurred expenses) can be compared with the historical simulated values of the rest of the year (t : 12). These simulated values result from considering only the aggregation of the expenses of the series from month t until December. The result is the historical distribution of the expenses of the rest of the year.

Then, one can collect the probability of the expenses from the historical data surpass the remaining budget for the rest of the year and the expected and maximum percentage deviation from this budget level if the budget is surpassed (as shown in Section 5.2) to characterize the risk of the budget in that precise moment in time. Figure 5.6 shows the monitor graphs that can be used to characterize the risk of the budget for the rest of the year based on the probability and consequence. The values are derived assuming that the present month is June, the total budgeted value in the beginning of the year was 0.60 and the already incurred expenses were the ones present in Table 5.5. For this example, in the simulation correlated random sampling was performed. For reference Figure 5.7 presents the distribution of the rest of year total expenses from June until December (with correlated sampling) alongside the remaining budget available ($0.35 = 0.60 - 5 \times 0.05$).

Table 5.5: Example of Incurred Costs to be used in Monitoring the budget risk throughout the year

Month	Total incurred expenses in month i
January	0.05
February	0.05
March	0.05
April	0.05
May	0.05

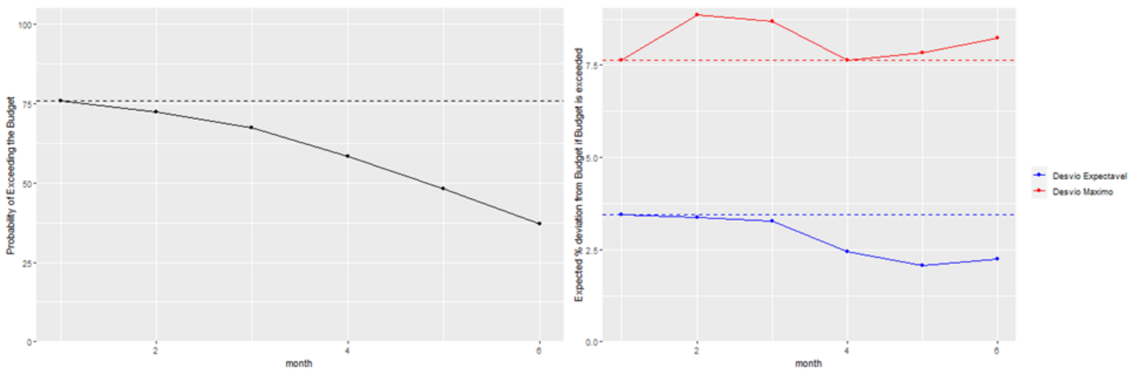


Figure 5.6: Monitoring budget risk throughout the year (Example)

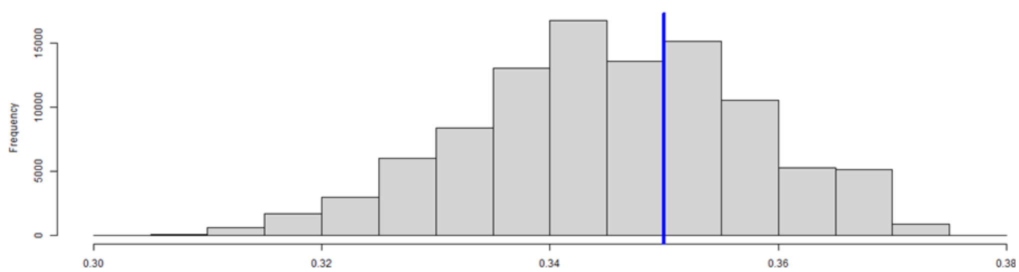


Figure 5.7: June-December Total Expense Distribution (Example)

5.5 Application Developed: Visualize and Monitor risk interactively

With the purpose of helping monitor the risk interactively, showing the expense distributions and risk profile of a certain input budget, a web application was developed in Shinny[1].

The application consists of several input fields that enable to select a specific or group of expense components to be analyzed and compared to its budgeted input so that the risk profile can be assessed through Monte Carlo historical simulation. This analysis can be done considering independent or correlated random sampling and be performed at any moment in time as explained in Section 5.4. This way, a manager can easily benefit from the application and ease its budgeting process by having a clearer picture of the risk of its team and of the total budget. The analyses possible to be conducted with the application are:

- Visualize the distribution of a specific, group of or total expenses;
- Calculate the risk of a budget at any moment in time by giving the incurred expenses until that moment;
- Visualize the risk evolution of budget compliance (as in Figure 5.6);
- Obtain the statistics that describe the simulated expense distribution;
- Calculate the probability of achieving at least a certain percentage of savings compared to a budgeted level.

Figure 5.8 shows the application interface developed.

5.6 Synthesis

This chapter's purpose was to describe the results obtained from applying the methodology enunciated in Chapter 4. It began by presenting the results of the time series bootstrapping techniques implemented (MBB, ETS, SARIMA, TBATS, BLD and K-Means based bootstraps), which were compared according with the NRMSE criteria. From the results, K-Means Based Bootstrap was the one that, generally, created more similar series (i.e. had the lowest NRMSE). This has to do with its capacity to isolate more extreme expenses and sample from other similar values to create new series. Contrarily, MBB was the worst because the majority of expense series was not stationary (i.e. presented trend and/or seasonality) as already stated in Section 3.2. All the best generated series can be considered sufficiently similar as it was corroborated by the minimal p-values of the KS hypothesis tests.

Afterwards, the total expense distribution was obtained through Monte Carlo Simulation from the best generated bootstrap series and for that two versions were considered when generating the realizations: independent random sampling or correlated random sampling. It was verified that when correlations between expenses were considered, the output distribution had a less spread

[1] Shiny is an open-source R programming language package that allows to build interactive web applications.

and consequently less risk for a given same budget level. This fact was explained by the existence of significant negative correlations between some expense components.

From the sensitivity analysis performed, it is important to remember that when correlations between variables are considered in the simulation, variance based measures (LMG and S_i) do not describe the model structure and the effect on the total expense distribution and, thus, δ_i is the preferred importance measure to assess the relative importance of input expense variables. Nonetheless, a joint usage of distribution-based measures (δ_i) and variance-based ones is advisable, as the last gives useful insights in estimating which input variables most influence the variance of the total expense distribution and, consequently, can be used to reduce it if addressed. When considering the independence of the expense components the three most important variables were *Accounting PT W12 Personnel Expenses*, *Document Solutions W12 Personnel Expenses* and *Accounts Receivable W12 Personnel Expenses*; and the least ones were *Accounts Payable W13 Operating Expenses*, *Consolidation W13 Operating Expenses* and *Accounts Receivable W13 Operating Expenses*. Contrarily, when the effect of correlations is accounted in the simulations the three most important variables were the Document Solutions team expenses and the *Consolidation W12 Personnel Expenses*. The ones with the least importance were the *Accounting PT W12 Personnel Expenses*, *Accounts Payable W13 Operating Expenses* and *Accounts Receivable W13 Operating Expenses*.

The last two Sections of this Chapter were destinated to generalize and extend the analysis presented in the previous sections and for that a method to monitor the risk of a certain budget throughout the year was presented. The main idea is in aggregating the monthly values from the generated series from the present moment until December to get the distributions referred to the rest of the year. After Monte Carlo Simulation is performed, the result is the total expense distribution for the rest of the year, which by being compared to the remaining budget can assess its risk based on historical expenses. Finally, to ease the execution of these analyses an interactive application in Shiny was developed and described in Section 5.5.

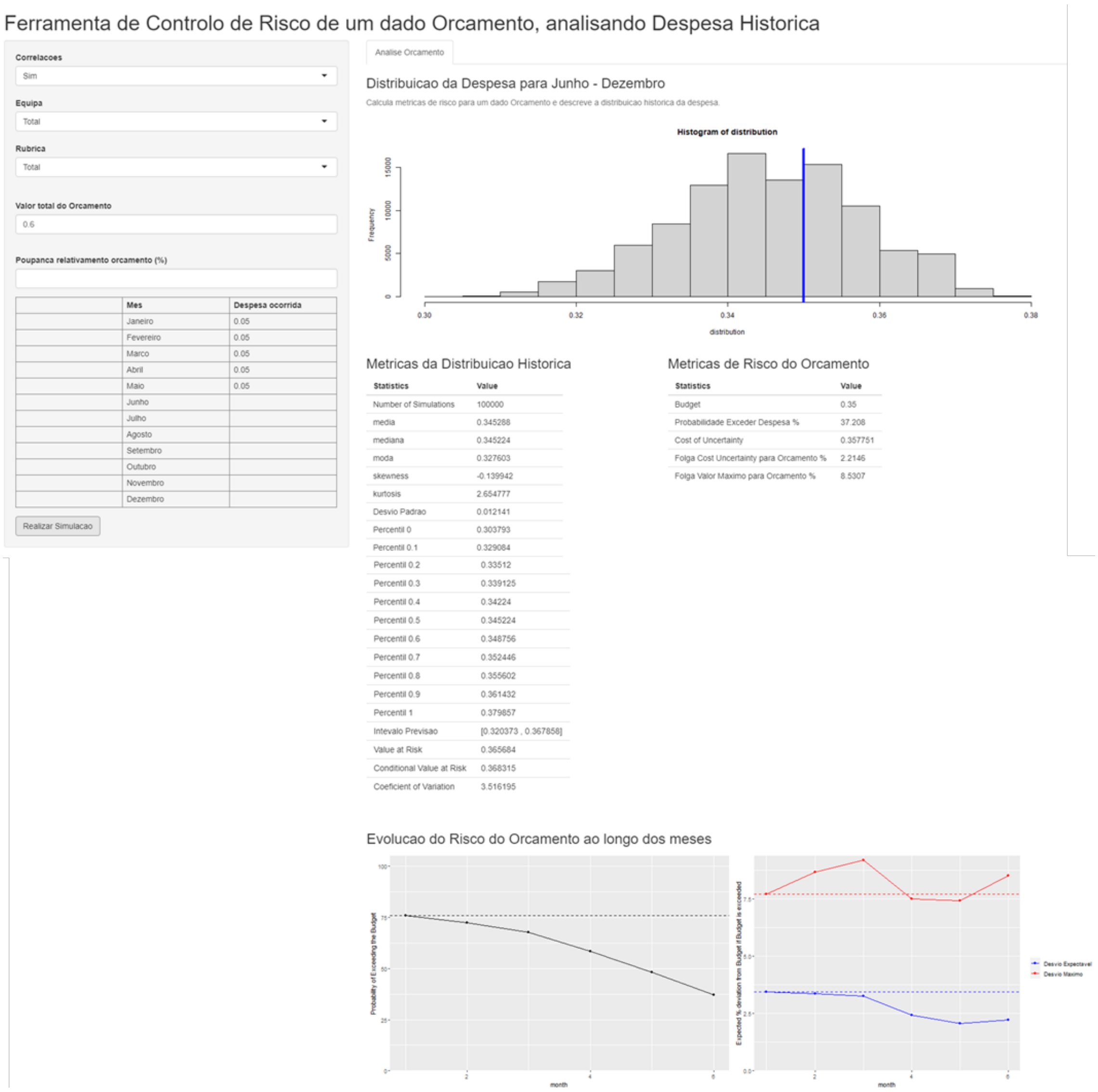


Figure 5.8: Interface of the Application developed

Chapter 6

Conclusion

In order to create accurate budgets, companies need to consider uncertainty in the process of budgeting. This means that they have to acknowledge that the budget compliance will be subject to uncertain factors that may influence the company's performance during the year and create a budget based on that. For this matter, Simulation is a valuable tool that enables managers to propagate the uncertainty and variability of factors that form the expense components and create a distribution that can be used to assess the risk of a budget. This way managers have in mind the contingency applied to the budget and the risk of occurring a cost overrun before its execution. This has several advantages such as approving the budget based on evidence and eventually turn the budgeting process and validation faster. The present dissertation proposes a method to assess the risk of a budget using historical data expenses. It has the main advantages of needing low maintenance (i.e. needs to be updated annually with historical values of the recent monthly expenses) and reduced input from the manager, not being subject to bias.

Firstly, an analysis to the components of the expense was conducted. The main takeaway from this analysis is that the Personal expenses have a more seasonal behavior and are, generally, easier to model in comparison with the Operating expenses. These last ones can be considered more random, despite its lower order of magnitude. Therefore, by considering the variability of Operating expenses in simulation one can see its distribution and help in setting a budget value for it.

For the creation of similar expense series through time series bootstrapping techniques, it can be concluded that there is not a universal best technique that creates the most similar series, despite K-Means Based Bootstrap being very competent for the majority of the cases. Moreover, it could be verified that techniques that relied on the bootstrap of the residuals adapt better to only bootstrapping blocks of data (as it happens in the MBB). Additionally, one can conclude that the internal parameters of bootstrapping techniques such as the block length do not have much influence as the initial primal choice of the method.

From this study, it can also be underlined the importance of considering correlations existent between the Expense components in the Monte Carlo Simulation as it allows the simulated results to be closer to reality. Depending on the magnitude of correlations, the output total expense distribution can be more or less spread when correlations are positive or when there are some negative correlations respectively (Mun, 2012). This change in the distribution has an impact on the risk of a budget, which in the present analysis was lowered because of the presence of negative correlations between some expenses. Besides, the relative importance of input expense components changes when correlations are regarded as it was verified by the *Accounting PT W12 Personnel Expenses* that was the most impactful in the independent random sampling, but was one of the least in the correlated study. This order of importance provides valuable information to the manager so he can focus on initiatives to better control the most important expense components and eventually reduce its impact on total expenses. For that δ_i is the preferred measure to identify the most important input variables since it considers the shift in the whole output distribution when fixing the input variable (Borgonovo, 2007). The rankings given by the sensitivity analysis suggested that focusing on the Document Solutions' expenses can have a positive impact in reducing the uncertainty of total expenses. When the goal is to identify the most relevant expense components in reducing total expense variability, variance-based measures (such as S_i) should be used (Section 5.3).

Regarding the evaluation of the risk of a budget it is important to do the assessment by comparing the total budgeted value with the total expense distribution as performed in Section 5.2. This way we are allowing possible compensations between the budgeted components. In other words, the ultimate risk of the total budget is lower than the risk of its components as there could be compensations between the different teams resulting from gaps between incurred expenses and budgeted values. However, the analysis of the risk of a certain component is still useful specially in understanding if the budgeted value is under or over dimensioned according to the historical expense data.

Finally, Simulation has recently registered a wider stream adoption due to its increased ease of execution with proper software. Having that in mind, an interactive application was developed tailored to the needs of the ABS division, allowing the manager to monitor the risk of a certain budget throughout the year and visualize its evolution.

Future Research

Despite the methodology proposed being useful for the company to start considering risk in the budgeting process and use minimal input subjective values, the contents of the thesis are not without its limitations and possibility of further improvements.

Firstly, there are some limitations to the analysis since the expense distributions are built from historical values and consequently may not be necessarily adapted to the future strategy of the division. Therefore, it is advisable to use the budgeting risk assessment tool developed as a reality check and one more useful information when creating budgets and not as a unique tool to decide if the budget is well dimensioned or not. This decision has to be made according to other criteria as well such as being critical with the inclusion of some atypical years and most importantly the strategy of the division for the future. So, the proposed methodology gives more meaningful risk evaluations when the teams are in a more balanced expense plateau. For example, if a team had an exponential growth year after year, the risk evaluation of its new year's budget would always be very high as the past expenses were in a lower level than the new year's level.

The reason behind using historical values in the simulation was so that the initial consideration of risk would be the most objective as possible and backed up with data. An improved second version would be to try to generate the distributions according to the new year's plans. For that there could be three options:

1. Adapt the presented methodology by filtering irrelevant years where its expenses were very atypical; apply a growth factor to each year or obtain the historical series normalized by some important change variables (e.g. number of employees) that could then be given according to the new year's values and multiplied to these normalized series;

2. Construct a mathematical model with ranges from input values for each expense component related to the new years' strategy and do a Monte Carlo Simulation assuming specific distributions for each component expense (e.g. triangular or beta) as it occurs in several Project Management studies (Elkjaer, 2000; Tamošiūnienė and Petravičius, 2006; Rubin and Patel, 2017). The reason for not doing this in this initial stage is because attributing input ranges to the expenses and assuming specific distributions is a more subjective approach and can eventually lead to a huge spread in distributions with inconclusive results (Hager et al., 2015);

3. Train Machine Learning models with the historical data considering explanatory variables that could be given according to the new year's strategy. This way and by considering possible values from expense components, it would be possible to create plausible series for the expense into the future. This would be an alternative to the time series bootstrapping stage and the rest of the methodology would be maintained.

The selection of the maximum number of clusters to be six could be changed. The choice of this value was based on the fact that the generated series were similar both from visual inspection and low NRMSE values. However, the maximum number defined can be considered conservative. This is a disadvantage of the K-Means Based Bootstrap, which makes the decision more subjective and more subject to overfitting. Therefore, it would be interesting to test besides statistical models, machine learning models with explanatory variables to see if the generated series would pass the tests of generating similar series.

Lastly, the results from the correlated study despite showing the importance regarding correlations, should be taken with a grain of salt. This is because the correlations were estimated based only on 5 available points (i.e. the annual expenses from 2016-2020), which makes the coefficient values very sensitive to new points. A possible solution would be the managers, based on their experience and rationale, try to estimate possible coefficients between the most relevant expenses or considering the independent random sampling for now as it could be viewed as a more conservative approach (since it gave higher risk for the same budgeted level). As more data is gathered, using correlations in sampling would become a more viable option.

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Appendix A

Comparison of Risk Definitions

Table A.1: Summary of Risk Definitions (Aven, 2012)

Classification	**Meaning**	**Comments/Critics**
1) Risk = Expected value ($R = E$)	The well-known definition that "risk is probability times consequence"	It does not consider the decision maker's risk aversion or risk-seeking attitude. In multiple scenarios, this definition is misleading (Kaplan and Garrick, 1981).
2) Risk = Probability of an (undesirable) event ($R = P$)	Risk defined as a probability (measure of uncertainty)	Restricts the measurement of uncertainty through probabilities, which is limited. Furthermore, the concept of probability is not very precise as it can depend on the perspective considered (frequentist vs subjective/Bayesian). Additionally, it ignores the potential loss.
3) Risk = Objective Uncertainty ($R = OU$)	It assumes that the distribution of the outcome is known (from past experience or statistics)	Few situations in life have objective distributions assumed to be known, thus corresponding to a narrow view on what risk is. Besides, it disregards the usage of subjective probabilities (Bayesian perspective).
4) Risk = Uncertainty ($R = U$)	Similar to 3) but it assumes that the underlying distribution is unknown. Risk is seen as a deviation from a reference (usually historical average value).	It captures both types of uncertainty: aleatory (through variation) and epistemic (due to lack of knowledge). However, it disregards the potential consequences.
5) Risk = Potential/-possibility of a loss ($R = PO$)	Acknowledgment that something negative can occur (loss), without quantifying it.	The same critics as 4), but it restricts that uncertainty is measured according to possibility theory.

Classification	**Meaning**	**Comments/Critics**
6) Risk = Probability and scenarios/Consequences ($R = P\&C$)	Risk is the the triplet (si, pi, ci), where si is the ith scenario, pi its probability and ci its consequence (Kaplan and Garrick, 1981).	Provides the answers to the questions What can happen? How likely is that to happen? If it happen, what are the consequences? It differs from 1) as it considers that "risk is probability and consequence", thus being more general and allowing risk to be represented not by a single value but by a risk curve (Kaplan and Garrick, 1981).
7) Risk = Event or consequence ($R = C$)	Loss of something that human's value	This definition does not go along with how risk is viewed in its everyday use as it ignores the chance of the event occurring.
8) Risk = Consequences + Uncertainty ($R = C\&U$)	Risk can be described in the form (C',Q,U), where C' specifies the event/consequence; Q is a measure of uncertainty (using probability, interval probabilities or belief functions) and K is the knowledge/assumptions that Q is based on.	Aven (2012) argue that this is the most appropriate type of risk definition as it allows uncertainty to be measured through different tools (and not just probability), and it considers the consequences of events. Definitions 2), 4), 6), 7) and 9) can be considered special cases.
9) Risk as effect of uncertainty on objectives ($R = ISO$)	Relates risk with uncertainty and the impact that it has on something that human's value (objectives)	Too general. It can be considered as a special case of 8) or 7) (Aven, 2012).

Appendix B

Expenses of the different teams in the ABS Division

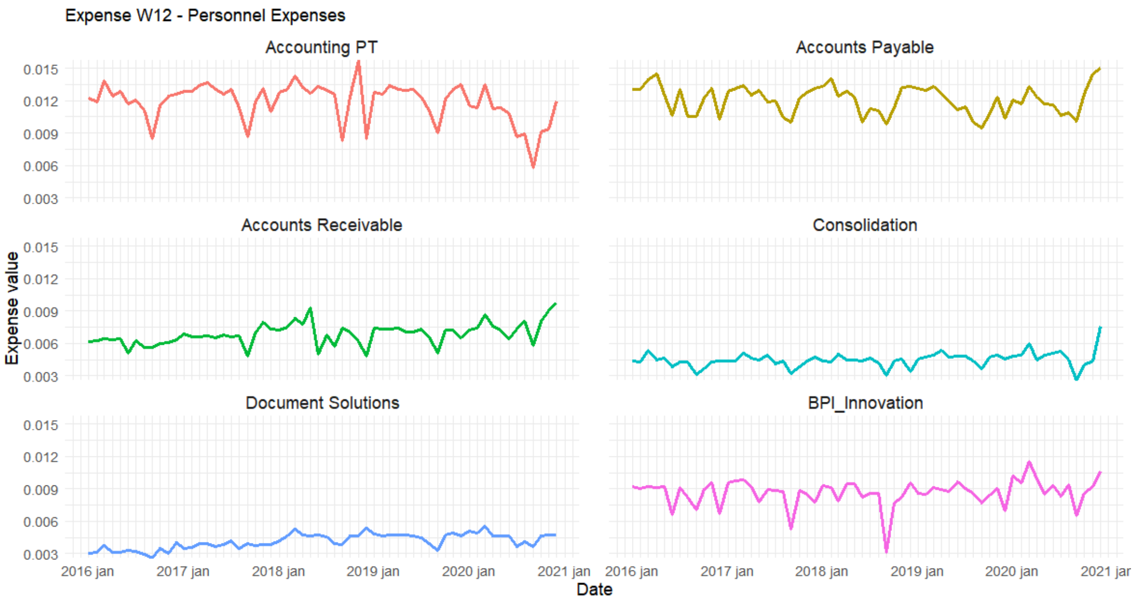


Figure B.1: Expense of *W12 Personnel Expenses* for the different teams

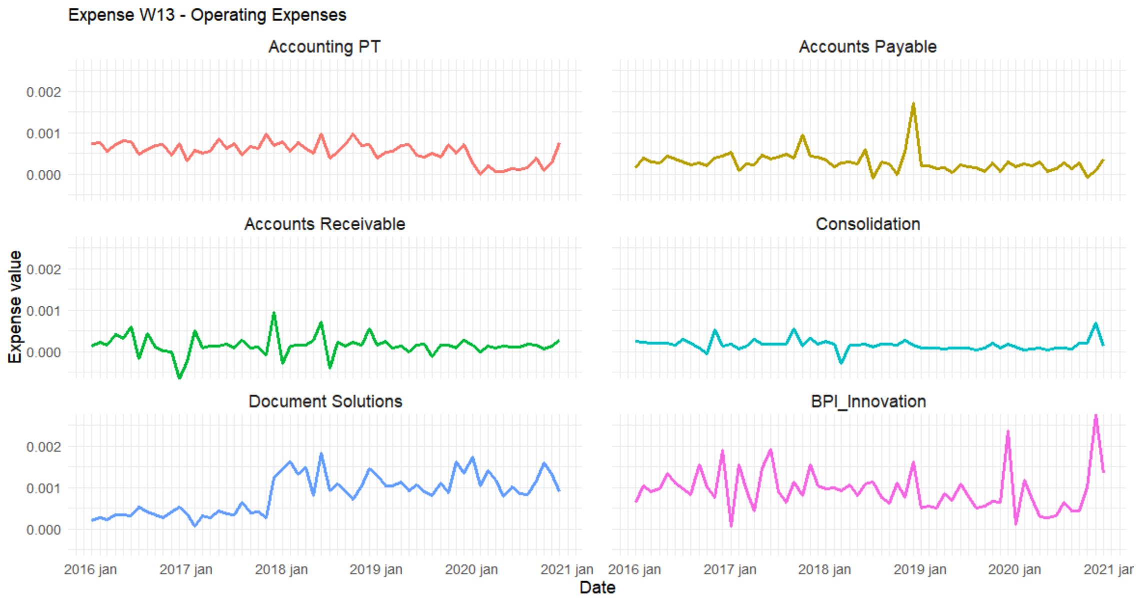


Figure B.2: Expense of *W13 Operating Expenses* for the different teams

Appendix C

Performance of Time Series Bootstrapping Techniques

Accounting PT W13							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.3384	0.3390	0.3387	**0.3261**	0.3272	-	-
ETS	0.2763	0.2703	**0.2692**	0.2750	0.2756	-	-
SARIMA	0.2737	**0.2691**	0.2707	0.2733	0.2709	-	-
TBATS	0.2337	**0.2267**	0.2274	0.2305	0.2332	-	-
BLD	0.1920	**0.1896**	0.1902	0.1916	0.1947	-	-
KMEANS	-	-	-	-	-	4	**0.0949**

Accounts Payable W12							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.3292	0.3272	0.3226	0.3185	**0.3145**	-	-
ETS	0.1874	0.1841	0.1837	0.1841	**0.1817**	-	-
SARIMA	0.2044	0.1987	**0.1967**	0.1996	0.2038	-	-
TBATS	0.1898	0.1894	0.1869	0.1892	**0.1843**	-	-
BLD	0.1484	0.1488	0.1487	0.1471	**0.1453**	-	-
KMEANS	-	-	-	-	-	6	**0.0571**

Accounts Payable W13							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	**0.1916**	0.1964	0.1947	0.2035	0.2176	-	-
ETS	**0.1920**	0.1935	0.1967	0.2006	0.2190	-	-
SARIMA	0.1948	**0.1947**	0.1975	0.2007	0.2154	-	-
TBATS	**0.1875**	0.1905	0.1938	0.1978	0.2168	-	-
BLD	**0.2042**	0.2093	0.2117	0.2084	0.2139	-	-
KMEANS	-	-	-	-	-	5	**0.0454**

Accounts Receivable W12							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.2903	0.2841	0.2786	**0.2786**	0.2846	-	-
ETS	0.2688	**0.2665**	0.2680	0.2680	0.2765	-	-
SARIMA	0.2676	0.2647	0.2669	**0.2637**	0.2746	-	-
TBATS	0.2196	0.2177	**0.2160**	0.2200	0.2289	-	-
BLD	**0.1748**	0.1768	0.1768	0.1783	0.1898	-	-
KMEANS	-	-	-	-	-	5	**0.0721**

Accounts Receivable W13							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.2099	**0.2095**	0.2149	0.2130	0.2241	-	-
ETS	**0.2082**	0.2094	0.2144	0.2159	0.2203	-	-
SARIMA	**0.2084**	0.2107	0.2135	0.2154	0.2197	-	-
TBATS	**0.2080**	0.2112	0.2121	0.2151	0.2223	-	-
BLD	0.3980	**0.3928**	0.4135	0.4301	0.4628	-	-
KMEANS	-	-	-	-	-	3	**0.0942**

Consolidation W12							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.1974	0.1877	0.1844	0.1805	**0.1786**	-	-
ETS	0.1324	0.1240	0.1222	**0.1222**	0.1241	-	-
SARIMA	0.1703	0.1598	0.1570	0.1528	**0.1494**	-	-
TBATS	0.1250	0.1143	0.1128	**0.1114**	0.1133	-	-
BLD	0.1180	0.1090	0.1075	**0.1057**	0.1075	-	-
KMEANS	-	-	-	-	-	4	**0.0691**

Consolidation W13							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.1936	0.1896	0.1895	**0.1863**	0.1892	-	-
ETS	0.1916	0.1887	0.1868	**0.1861**	0.1881	-	-
SARIMA	0.1926	0.1903	**0.1864**	0.1879	0.1895	-	-
TBATS	0.1547	0.1532	**0.1516**	0.1520	0.1518	-	-
BLD	**0.1517**	0.1521	0.1526	0.1529	0.1543	-	-
KMEANS	-	-	-	-	-	6	**0.0411**

Document Solutions W12							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	**0.3311**	0.3267	0.3242	0.3227	0.3157	-	-
ETS	**0.1475**	0.1476	0.1480	0.1504	0.1489	-	-
SARIMA	0.1622	**0.1617**	0.1661	0.1654	0.1663	-	-
TBATS	0.1401	**0.1398**	0.1437	0.1424	0.1438	-	-
BLD	0.1212	**0.1210**	0.1216	0.1241	0.1236	-	-
KMEANS	-	-	-	-	-	3	**0.1064**

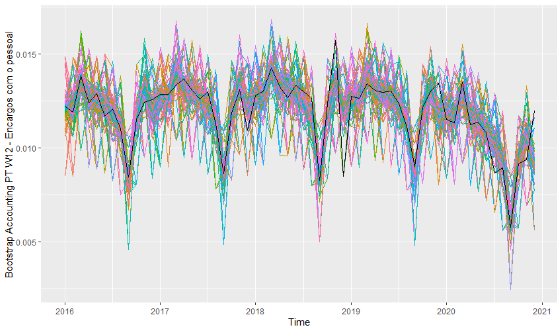
Document Solutions W13							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	**0.3637**	0.3640	0.3685	0.3652	0.3764	-	-
ETS	0.2388	**0.2364**	0.2392	0.2438	0.2503	-	-
SARIMA	**0.2363**	0.2367	0.2376	0.2413	0.2479	-	-
TBATS	0.2424	**0.2410**	0.2461	0.2453	0.2561	-	-
BLD	0.1755	**0.1737**	0.1739	0.1745	0.1772	-	-
KMEANS	-	-	-	-	-	2	**0.1871**

BPI_Innovation W12							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.2080	**0.2078**	0.2094	0.2136	0.2165	-	-
ETS	0.1479	**0.1461**	0.1476	0.1474	0.1491	-	-
SARIMA	0.1932	0.1930	0.1927	**0.1940**	0.1967	-	-
TBATS	**0.1404**	0.1406	0.1427	0.1426	0.1454	-	-
BLD	0.1315	**0.1312**	0.1332	0.1327	0.1336	-	-
KMEANS	-	-	-	-	-	5	**0.0621**

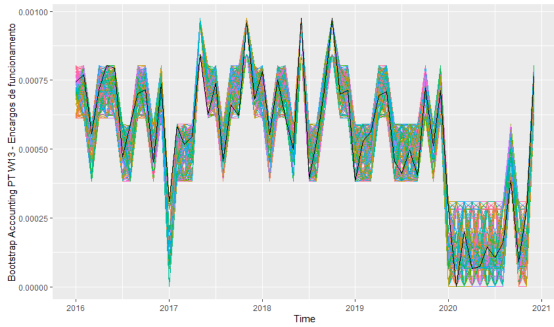
BPI_Innovation W13							
Method	Block Length					K-Means	
	1	3	6	12	24	Number of clusters	NRMSE
MBB	0.2603	0.2542	0.2518	0.2559	**0.2442**	-	-
ETS	0.2222	0.2211	0.2227	0.2229	**0.2174**	-	-
SARIMA	0.2489	0.2428	0.2382	0.2402	**0.2351**	-	-
TBATS	0.2606	0.2553	0.2509	0.2540	**0.2449**	-	-
BLD	0.1906	0.1892	0.1847	0.1820	**0.1799**	-	-
KMEANS	-	-	-	-	-	6	**0.0565**

Appendix D

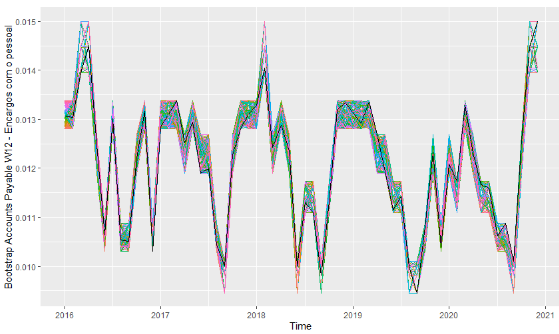
Time Series Bootstrapping optimal selected Series for each Expense Component



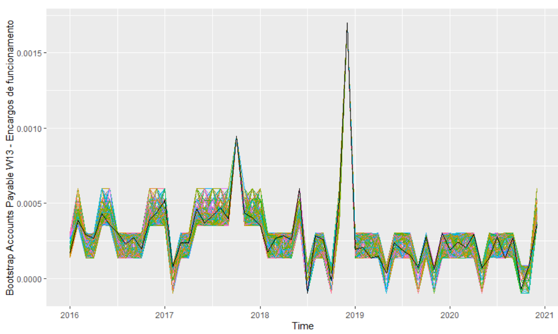
(a) Accounting PT W12



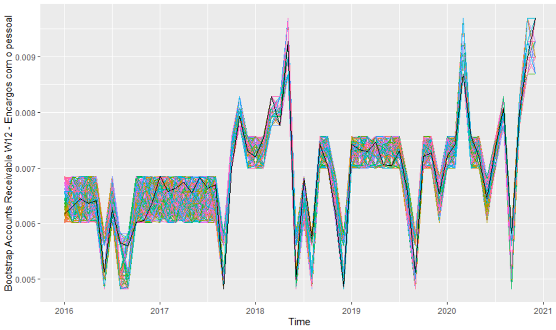
(b) Accounting PT W13



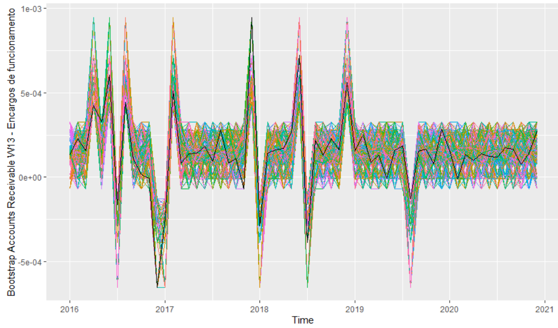
(c) Accounts Payable W12



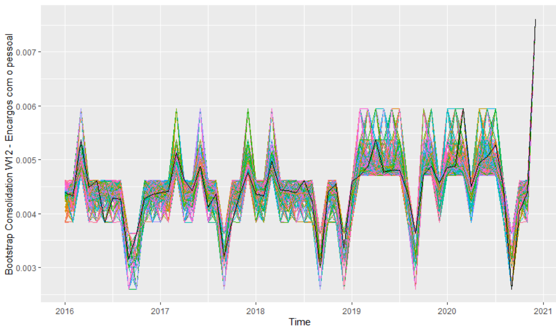
(d) Accounts Payable W13



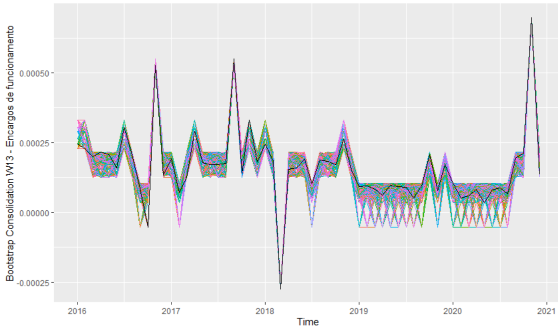
(e) Accounts Receivable W12



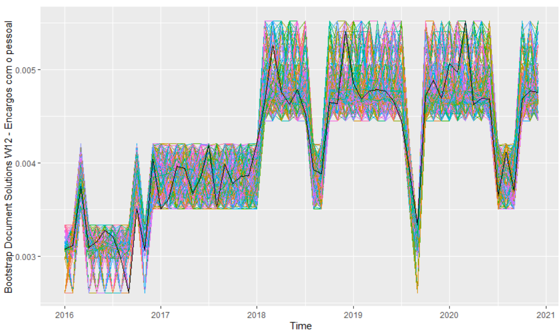
(f) Accounts Receivable W13



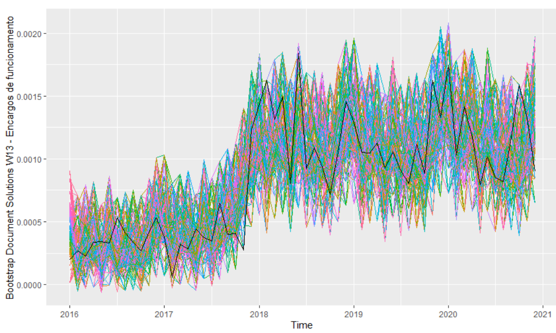
(g) Consolidation W12



(h) Consolidation W13

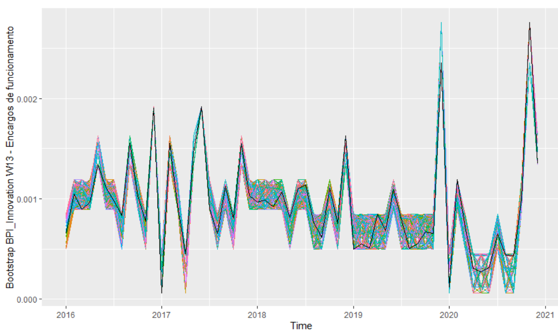


(i) Document Solutions W12



(j) Document Solutions W13

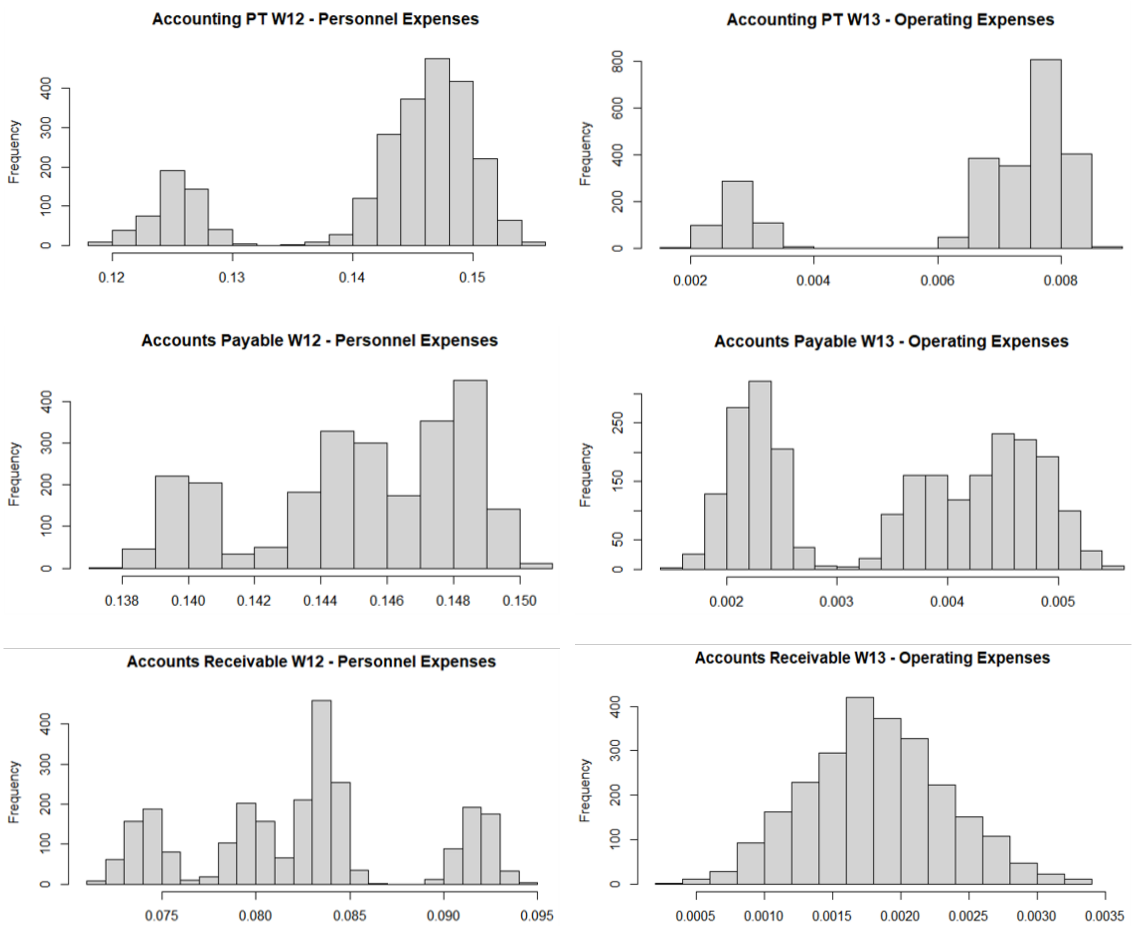
(k) BPI_Innovation W12

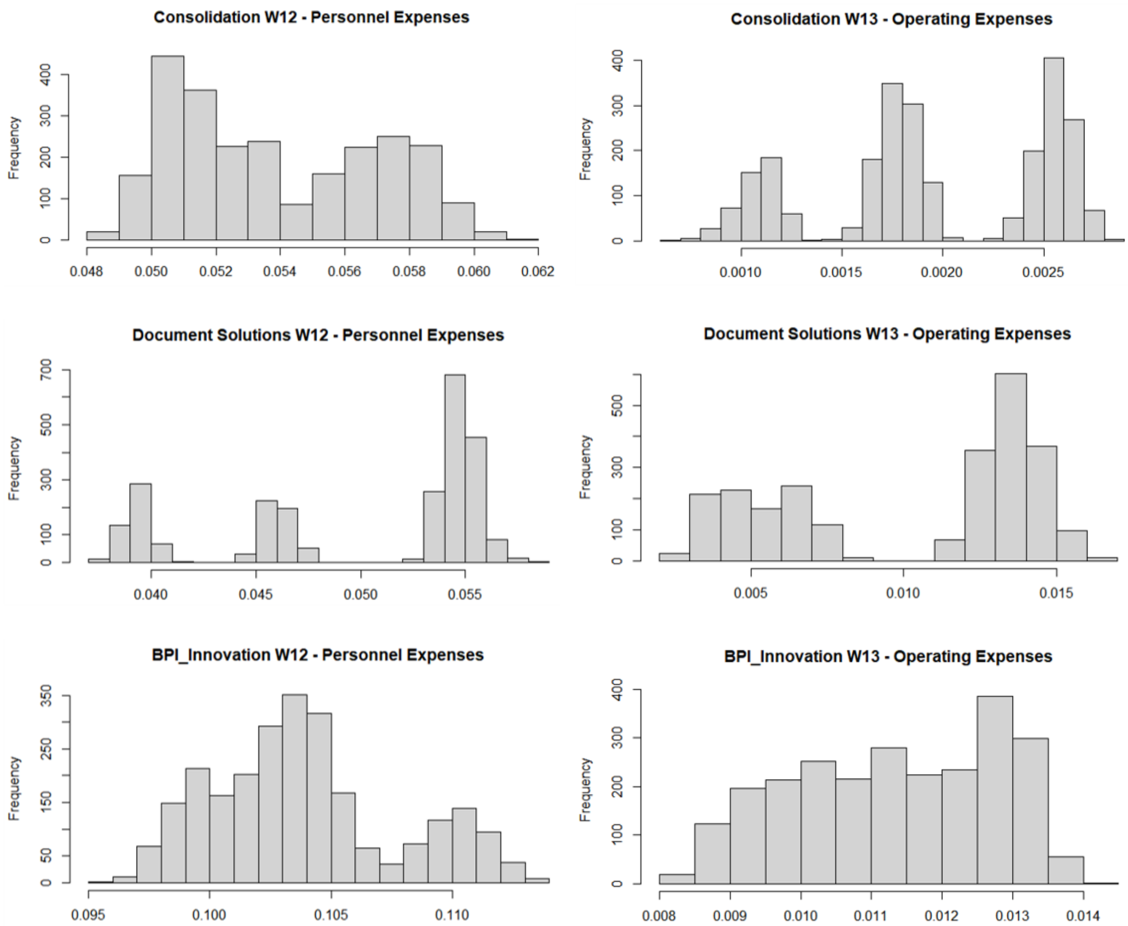


(l) BPI_Innovation W13

Appendix E

Annual Distribution of each Expense component




Consolidation W12 - Personnel Expenses
Frequency
0 100 200 300 400
0.048 0.050 0.052 0.054 0.056 0.058 0.060 0.062
Consolidation W13 - Operating Expenses
Frequency
0 100 200 300 400
0.0010 0.0015 0.0020 0.0025
Document Solutions W12 - Personnel Expenses
Frequency
0 100 300 500 700
0.040 0.045 0.050 0.055
Document Solutions W13 - Operating Expenses
Frequency
0 100 300 500
0.005 0.010 0.015
BPI_Innovation W12 - Personnel Expenses
Frequency
0 50 150 250 350
0.095 0.100 0.105 0.110
BPI_Innovation W13 - Operating Expenses
Frequency
0 100 200 300 400
0.008 0.009 0.010 0.011 0.012 0.013 0.014

Appendix F

Si Sensitivity Measure decomposition

Table F.1: Variance explained (S_i) of each component decomposed in its uncorrelated and correlated parts for the independent and correlated random sampling cases

Component	Independent Random Sampling			Correlated Random Sampling			
	S_i	Si^U	Si^C	S_i	Si^U	Si^C	Rank_Si
Accounting PT W12	0.377	0.380	-0.004	0.281	0.088	0.193	6
Accounting PT W13	0.017	0.018	-0.001	0.002	0.002	0.001	12
Accounts Payable W12	0.048	0.046	0.001	0.460	0.004	0.457	3
Accounts Payable W13	0.006	0.006	0.000	0.004	0.000	0.003	11
Accounts Receivable W12	0.161	0.159	0.002	0.446	0.007	0.439	4
Accounts Receivable W13	0.001	0.001	0.000	0.005	0.000	0.005	10
BPI_Innovation W12	0.070	0.068	0.001	0.006	0.002	0.004	9
BPI_Innovation W13	0.009	0.009	0.000	0.435	0.000	0.434	5
Consolidation W12	0.047	0.046	0.001	0.212	0.002	0.211	8
Consolidation W13	0.002	0.001	0.000	0.259	0.000	0.259	7
Document Solutions W12	0.179	0.184	-0.005	0.825	0.004	0.821	2
Document Solutions W13	0.082	0.083	-0.001	0.828	0.001	0.826	1