ENGINEERING STRUCTURES AND TECHNOLOGIES
ISSN 2029-882X print / ISSN 2029-8838 online
2013 5(3): 123–133
doi:10.3846/2029882X.2013.869415

ACCURACY ANALYSIS OF DESIGN METHODS FOR CONCRETE BEAMS REINFORCED WITH FIBER REINFORCED POLYMER BARS

Edgaras Timinskas, Rūta Jakštaitė, Viktor Gribniak, Vytautas Tamulėnas, Gintaris Kaklauskas

Vilnius Gediminas Technical University, Saulėtekio al. 11, 10223 Vilnius, Lithuania

Received 03 June 2013; accepted 05 November 2013

Abstract. Traditional steel reinforcement does not resist corrosion and its resources are limited; therefore, carbon, glass, aramid and basalt fibre reinforced polymer bars were developed. The composite reinforcement has a high tensile strength and resistance to electromagnetic fields. Different kinds of materials and application of various surface coatings are used in the production of the composite bars. This results in different adhesion to concrete and mechanical properties of composite bars. In comparison with steel reinforcement, glass, aramid and basalt fibre reinforced polymer bars have a lower modulus of elasticity. Thus, structural rigidity provided by these bars is smaller in respect to reinforced concrete elements. Current reinforced concrete design codes and recommendations are based on empirical and simplified methods of strain evaluation, which may be inadequate for design of structures with composite bars. In this paper, an adequacy of the empirical models was checked against the experimental data of concrete beams reinforced with composite bars. The moment-curvature data of 52 beams reported in the literature and conducted by the authors were used for assessment of accuracy of design methods. In order to perform the analysis, different methods from design codes (European (LST 2007), American (ACI Committee 318 2011) and Russian (NIIZhB 2006)) and recommendations (Italian (CNR 2007) and American (ACI Committee 440 2006)) have been selected. The results of the investigation will provide engineers with more information on design of concrete beams with fibre reinforced polymer bars. This will encourage an extensive use of these innovative materials in different types of structures.

Keywords: fibre reinforced polymer bars, FRP, design codes, experimental data, deflection, reinforced concrete, accuracy analysis, short-term loading.

Reference to this paper should be made as follows: Timinskas, E.; Jakštaitė, R.; Gribniak, V.; Tamulėnas, V.; Kaklauskas, G. 2013. Accuracy analysis of design methods for concrete beams reinforced with fiber reinforced polymer bars, *Engineering Structures and Technologies* 5(3): 123–133. http://dx.doi.org/10.3846/2029882X.2013.869415

Introduction

Concrete is a composite material highly resistant to compression; however, it is brittle and has low tensile strength. Therefore, reinforcement bars are used to take over the tensile stresses. Generally, steel reinforcement bars are used for the production of structural elements. Properly designed and built concrete structures could be used for centuries. Unfortunately, due to the low cracking resistance of concrete, unacceptable cracks often appear in the structures. This leads to intensive corrosion of steel reinforcement while the resultant products of the corrosion process continue to erode the concrete. Without the protection and strengthening actions, the structure quickly loses its operational characteristics. Currently, almost half of the budget of the construction industry is spent on the reconstruction and repair of already existing buildings (Cigna *et al.* 2003). It is not surprising that huge finan-

Corresponding author:
E. Timinskas E-mail: *edgaras.timinskas@vgtu.lt*

cial investments and efforts of scientists and engineers from all over the world are made for improvement of structural and technical solutions of concrete structures and creation of new and efficient materials. In order to prevent the corrosion of steel reinforcement, fibre reinforced polymer (FRP) were developed. Unlike steel reinforcement, these bars are more resistant to the effects of cyclic load and electromagnetic fields. FRP bars consist of resin and fibre. Environmental resistance of these bars depends on resin properties. FRP bars are made from glass, carbon, aramid or basalt fibres (GFRP, CFRP, AFRP and BFRP, respectively). Fibres are responsible for mechanical properties of FRP bars, on which the deformational behaviour of reinforced concrete structures depends.

Production of FRP bars involves formation of a cross-section and surface treatment (Fig. 1a). The surface treatment determines the quality of bond between the bars and the concrete matrix. Complex, uneven and rough shape of bars ensures good bond properties; however, such surface treatment results in significant price increase of reinforcement. There is still no global consensus on the most effective shape of FRP bars. Standardisation of the shape would allow a more extensive use of such type of reinforcement in the construction industry. In a similar manner, the development of the shape of steel reinforcement was carried out. However, it took over 100 years to be completed (Fig. 1b).

According to ACI 440.1R-06 design recommendations, in order to ensure the serviceability limit state of existing structures, a characteristic value of tensile strength of GFRP and BFRP, AFRP, CFRP has to be reduced by 20%, 30%, 55%, respectively. As shown in

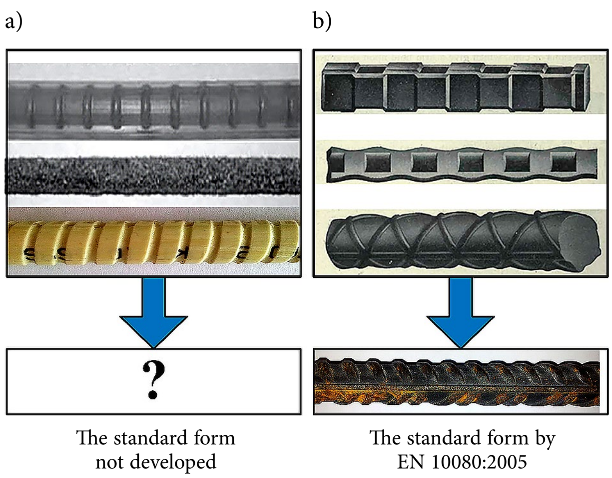

Fig. 1. Shapes of rebar surface: (a) composite, (b) steel

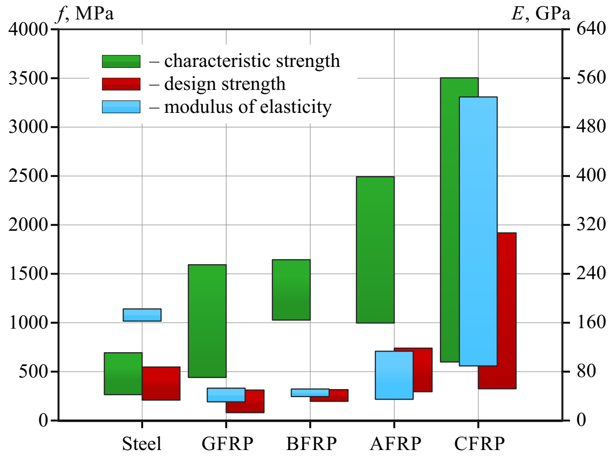

Fig. 2. Comparison of characteristic and design tensile strength and elasticity modulus of reinforcement. Glass, basalt, aramid and carbon fibre reinforced polymer GFRP, BFRP, AFRP, CFRP, respectively

Fig. 2, the CFRP bars have the best mechanical properties, but materials for its production are hardly accessible. Meanwhile, GFRP production uses widely available sand and manufacturing technology is rather simple; therefore, GFRP bars are the most popular among the other types of FRP. In terms of mechanical properties and production complexity, BFRP and AFRP bars are somewhere the middle, but they are rarely used in practice.

A low modulus of elasticity is the main drawback of GFRP bars (Fig. 2). This leads to a smaller structural rigidity provided by these bars in respect to reinforced concrete elements. The bond between composite bars and concrete depends on the surface pattern of these bars (Fig. 1a) and is not always guaranteed. Structural elements with such reinforcement may not meet a serviceability (a limitation of strain and deflection) requirements. Current design codes and recommendations of structural concrete are based on empirical and simplified methods of deformation evaluation, which may be inadequate for design of structures with FRP bars. Therefore, additional and more comprehensive studies of structural elements reinforced with FRP bars have to be performed. In this paper, adequacy of empirical models for deflection evaluation was checked against the experimental data of concrete beams reinforced with composite bars. The moment-curvature data of 52 beams reported in the literature and conducted by the authors were used for assessment of accuracy of deflection calculation.

1. Deflection determination

Behaviour of reinforced concrete is a complex issue. Determination of the behaviour of structural elements is aggravated by different physical and mechanical properties of concrete and reinforcement, non-linear behaviour, cracking, shrinkage and creep, absence of bond between reinforcement and concrete, distribution of rebars in the cross-section and the scale factor make. Modelling of the cracking of concrete and bond between reinforcement and concrete have the greatest influence on the results of deformation evaluation.

Design recommendations for FRP reinforced concrete elements exist in USA (ACI Committee 440 2006), Canada (CSA 2012; CSA 2010), Japan (JSCE 1997) and Italy (CNR 2007), though, there are no design codes for such type of reinforcement. Current European (LST 2007), American (ACI Committee 318 2011) and Russian (NIIZhB 2006) design codes of structural concrete are adapted to the elements reinforced with steel bars; therefore, may be inadequate for design of structures with composite bars.

In this paper, a comparative analysis of the accuracy of deflection determination based on European (LST 2007), American (ACI Committee 318 2011), and Russian (NIIZhB 2006) design codes of structural concrete as well as Italian (CNR 2007) and American (ACI Committee 440 2006) design recommendations for FRP reinforced concrete elements is performed. Element deflection δ for all analysed methods is calculated as follows:

$$\delta = k \cdot \kappa \cdot l_0^2, \quad (1)$$

where: k is the coefficient depending on the loading scheme k = 23/216 for the beams loaded by two concentrated forces); κ represents the curvature corresponding to the maximum bending moment; and l_0 indicates the span length of the beam. All of the methods use the same k and l_0 parameters and only differ in the technique of curvature determination.

2. Accuracy analysis

The authors have shown (Gribniak *et al.* 2013) that the tension-stiffening is very important for the deformation assessment of concrete elements reinforced with composite bars. Still, there is no single approach to determine the effect of tension-stiffening for such type of reinforcement. As mentioned before, the deformations due to the low modulus of elasticity of concrete elements reinforced with FRP bars are different from the ones of concrete elements with steel reinforcement. Therefore, the application of design codes of structural concrete for design of concrete structures reinforced with FRP bars may be inadequate.

2.1. Experimental data

The accuracy analysis of deflection assessment methods of design codes and recommendations was performed in two stages. The data of 46 beams reinforced with FRP bars (466 experimental measurements) collected from 14 various literature sources (Benmokrane *et al.* 1996a, b; Al-Musallam 1997; Aiello, Ombres 2000; Pecce *et al.* 2000; Abdalla 2002; Toutanji, Deng 2003; Leung, Balendran 2003; Belarbi, Wang 2005; Al-Sunna 2006; Rafi *et al.* 2008; Barris Peña *et al.* 2009; Soric *et al.* 2010; Ascione *et al.* 2010) was used in the first stage of the statistical analysis. In order to verify the adequacy of the results of performed statistical analysis, the test data of six beams reinforced with GFRP bars (66 experimental measurements) tested by the authors were used in the second stage. The analysed beams were divided into groups according to the type of reinforcement, the reinforcement ratio, the strength of concrete and the ratio of reinforcement and concrete modulus of elasticity as shown in Figure 3. All beams had a rectangular shape and were tested under four-point bending scheme.

The beams were reinforced with GFRP, CFRP or AFRP bars. The main characteristics of the beams are given in Table 1 (No. 1–46). The reinforcement ratio (ρ) varies from 0.2% to 3.6%, the average compressive strength of concrete (f_{cm}) — from 24.1 MPa to 61.7 MPa, the ratio of reinforcement and concrete modulus of elasticity (n_f) — from 1.00 to 4.39. Other parameters presented in the table are the height (h) and the width (b) of the section, the effective depth (d) and the length of the beam (L).

The operating loading value and the deflection of mid-span point have been recorded during the experimental testing of the beams. In accordance with the Equation (1) and deflection values obtained from the experiments, the moment-curvature diagrams were derived.

The beams tested by the authors were reinforced with GFRP bars (ComBAR, Schöck Bauteile GmbH). The main characteristics of the beams are given in Table 1 (No. 47–52). The reinforcement ratio (ρ) varies from 0.2% to 1.1%, the average compressive strength of concrete (f_{cm}) — from 44.6 MPa to 56.0 MPa.

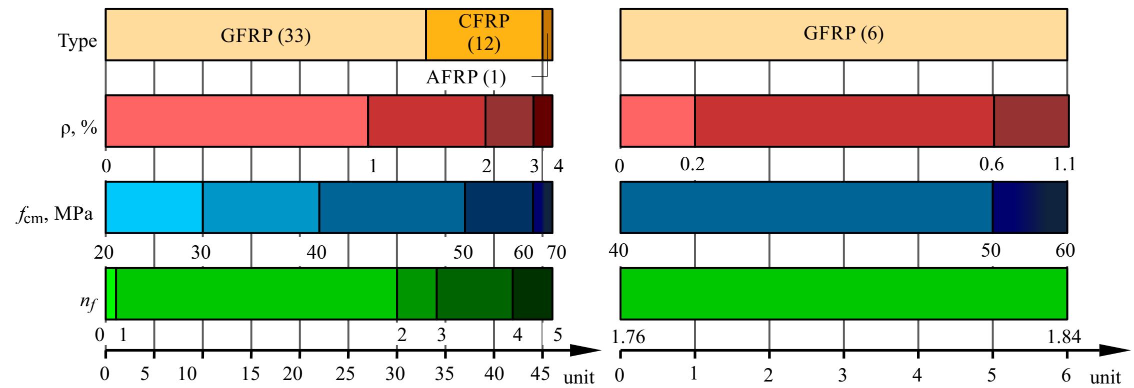

Fig. 3. Test data of 46 beams from the literature and 6 beams from experiments of the authors

Table 1. Geometry and material properties of 46 beams from literature

Programme	No.	FRP type	h, mm	b, mm	d, mm	L, mm	ρ, %	f_{cm}, MPa	n_f
1	1, 2, 3, 4	GFRP	190	140...160	150...170	1800	1.68...2.53	55.2...61.7	2.35...2.57
2	5, 6, 7, 8, 9, 10	GFRP, CFRP	250	500	212	2300	0.200...1.52	28.0	1.26...4.39
3	11, 12, 13, 14, 15, 16	GFRP, CFRP	250	150	225	2300	0.281...3.38	46.5...55.4	1.32...4.22
4	17	CFRP	200	120	180	1750	0.656	42.6	3.80
5	18, 19, 20	GFRP	300	180	255...270	2800	0.521... 1.10	35.0	1.14
6	21, 22	GFRP	200	150	165	2200	0.576	28.5, 48.8	1.26, 1.10
7	23, 24	GFRP	185	500	145	3400	1.22, 0.699	30.0	1.28
8	25, 26, 27, 28	GFRP	210...300	200	158...248	2700	1.14...3.59	31.3...40.7	1.01...1.31
9	29, 30, 31, 32, 33	AFRP, CFRP	200...150	150...250	165...130	2610...2700	0.330...1.15	30.5...46.2	1.37...3.91
10	34, 35, 36, 37	GFRP	300	200	240...270	3000	0.645...2.18	45.0...52.0	1.00...1.25
11	38, 39	GFRP	300, 550	200	260, 510	3000	1.10, 0.562	43.0	1.36
12	40, 41	GFRP	200	150	170	2000	0.616	24.1, 32.0	1.70, 1.56
13	42, 43	GFRP, CFRP	280	200	250	2900	0.760	34.8	1.22...3.93
14	44, 45, 46	CFRP, GFRP	229	178	191	1829	0.780, 1.95, 2.89	48.0	1.11...3.36
15*	47, 48, 49, 50, 51, 52	GFRP	302...305	271...287	243...277	3000...3280	0.198...1.06	44.6...56.0	1.76...1.84

* Geometry and material properties of 6 beams from experiments of the authors

2.2. Analytical method

Accuracy of curvature (deflection) assessment methods of reinforced concrete elements is analysed examining ratio Δ as follows:

$$\Delta = \frac{\kappa_{calc}}{\kappa_{obs}}, \quad (2)$$

where κ_{calc} and κ_{obs} are the calculated and experimentally obtained curvature of the beams, respectively (Timinskas 2014). A logarithmic normalisation:

$$\Theta = \ln(\Delta), \quad (3)$$

was introduced to ensure equal contribution to the accuracy of underestimated ($\Delta < 1$) and overestimated ($\Delta \geq 1$) predictions. The authors kept to the view that a prediction is safe if $\Delta \geq 1$ ($\Theta \geq 0$), meaning that the code overestimates the deflection rather than underestimating it.

Considering the relative deflection Δ as a random variable, statistical methods can be used to assess the accuracy represented by the central tendency and variability. The central tendency is regarded as a precision parameter of the calculation method. The postulate of minimum variability is used to evaluate consistency of the model. Basic statistics, such as means m_Θ and m_Δ (estimator of the central tendency) and standard deviations s_Θ and s_Δ (measure of variability), are calculated for each method. Taking into consideration transformation (3), the deflection Δ is statistically assessed using the following relationships:

$$m_\Delta = exp\left(m_\Theta + 0.5 s_\Theta^2\right);$$
$$s_\Delta^2 = m_\Delta^2\left[exp\left(s_\Theta^2\right) - 1\right]. \quad (4)$$

All of the descriptive statistical data is given in Table 2. The most striking feature is high values of the coefficient of variation υ_Δ obtained for all methods under consideration. To investigate the reasons for that, a regression analysis was carried out.

Table 2. Main descriptive statistical data

Calculation technique	m_Θ	s_Θ	m_Δ	s_Δ	$\upsilon_\Delta = s_\Delta / m_\Delta$
LST EN 1992-1-1:2005	0.548	0.755	2.300	2.016	87.7 %
CNR-DT 203/2006	0.690	0.854	2.870	2.972	103.6 %
ACI 318M-11	–0.118	0.604	1.067	0.707	66.3 %
ACI 440.1R-06	0.242	0.671	1.595	1.202	75.4 %
SP 52-101-2003	0.563	0.802	2.420	2.298	94.9 %

The influence of variation in parameters, such as loading intensity K, reinforcement ratio ρ, average compressive strength of concrete f_{cm}, and ratio of reinforcement and concrete modulus of elasticity n_f on experimental moment-curvature diagrams has been observed. The loading intensity (loading levels) is considered as the ratio K:

$$K = \frac{M - M_{cr}}{M_{u,p} - M_{cr}};\ K = \left(0;0,1;0,2;\ldots;0,9;1\right), \quad (5)$$

where M_{cr} and $M_{u,p}$ are the theoretically calculated cracking and pseudo-cracking moments, respectively. The cracking moment was determined by the expression from the Eurocode 2 (LST 2007):

$$M_{cr} = f_{ct} \cdot \frac{I_{el}}{y_t}, \quad (6)$$

where: I_{el} is the moment of inertia for uncracked cross-section; y_t represents the distance from the neutral axis to the sectional layer most in tension; and f_{ct} indicates the tensile strength of concrete determined by the values of the compressive strength of concrete f_{cm} (Table 1):

$$f_{ct} = \begin{cases} 0,3\left[f_{cm} - 8\right]^{2/3}, when\ f_{cm} \le 58\,MPa; \\ 2,12\ln\left(1 + \left(f_{cm}/10\right)\right), when\ f_{cm} > 58\,MPa. \end{cases} \quad (7)$$

Using Equation (5), the analysis is carried out in 11 load levels (K = 0 corresponding to the cracking moment, and K = 1 refers to the failure of an element). It is important to note that during the experiment, the failure mostly occurs in the compressive zone or due to the reached ultimate strength in the shear zone. Therefore, when the collapse of element is governed by the failure of reinforcement (the relative strength of FRP bars is considered to be 500 MPa), it is necessary to calculate the limit value of pseudo-cracking moment. Having in mind that the effect of tension-stiffening practically disappears with increasing loading, such limitation of the moment allows to assess the influence of constitutive model of tension-stiffening concrete on the results of deflection calculation.

The ratios Δ (2) of all the analysed deflection calculation methods were determined for each loading level. The regression analysis was performed to investigate the influence of variation of a model parameter X (ρ, n_f, $n_f\rho$, f_{cm} or K) on scatter of deflection predictions. This analysis was performed using the linear regression model. Taking into consideration the transformation (3), the logarithmic scale for the ordinate axis is used in Figure 4. Then, the regression model:

$$\Delta = exp\left(a + b \cdot X\right), \quad (8)$$

shown in Figure 4, becomes linear ($\Theta = a + b \cdot X$). The values of the coefficients a and b as well as the coefficient of correlation r are also given in this figure.

It is important to note that, in the ideal case, no correlation between Δ and X should be obtained. The high absolute value of r indicates the presence of such correlation. The coefficient b (slope) characterises the influence of the variation of parameter X on the precision of the method expressed in terms of Δ. As indicated, the accuracy of a deflection prediction method should be independent of the variation of the model parameters, i.e. b should be close to 0. The systematic error of the method is characterised by the constant a (intercept) that ideally should approach 0. It indicates that the predicted values are equal to the experimental deflection, i.e. the method does not have a systematic error.

In this study, the analysed parameters (loading intensity, reinforcement ratio, average compressive strength of concrete, and a ratio of reinforcement and concrete modulus of elasticity) have different units of measurement, which makes it difficult to compare the influence on precision of deflection calculation methods. Therefore, the influence of variation of X on Δ can be compared using the dimensionless factors:

$$B_{\Delta/X} = b\Delta m_X / m_\Delta;\ S_{\Delta/X} = r \cdot b \cdot s_X / s_\Delta\,, \qquad (9)$$

where: m_X and m_Δ are the mean values of the parameters X and Δ, respectively; and s_X and s_Δ are the respective standard deviations. The factor $B_{\Delta/X}$ indicates a relative increase of Δ with a unit increment of X, whereas $S_{\Delta/X}$ shows a relative part of variation of Δ due to scatter of parameter X. These factors are given in Table 3 with extreme values shown in bold.

It should be pointed out that the regression model is developed using sample data and, therefore, it is influenced by sampling variation. Assessing the variation, widths of confidence and prediction intervals constructed for a regression model can be analysed. The confidence interval describes the area where the mean value μ_Θ of ratio Θ with the $(100-\alpha)$ % probability would be inside the interval. In accordance with the expressions by Draper and Smith (1998), the lower $\Theta_{conf,1}$ and the upper $\Theta_{conf,2}$ bounds of the (100 – α) % confidence interval are obtained.

Unlike the confidence interval that assesses the mean prediction μ_Θ, the prediction interval estimates the likely value of the deflection prediction Θ^*, meaning that (100 – α) % of forecast predictions Θ would be inside the interval. As these predictions are associated with errors from the future observation, naturally, the prediction interval becomes wider than the confidence interval.

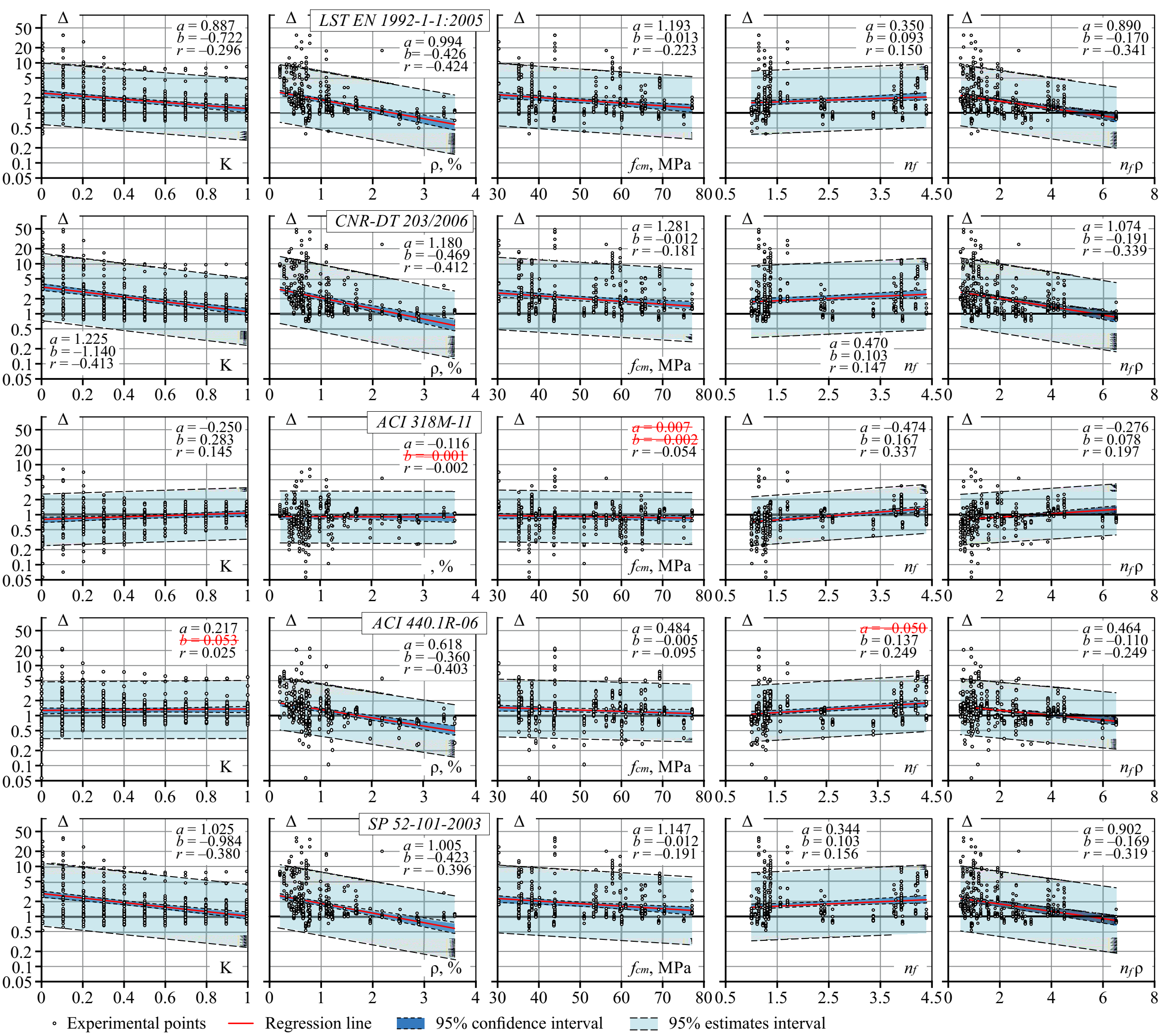

Fig. 4. Regression analysis of the data of 46 experimental beams from literature

Table 3. Coefficients $B_{\Delta/X} | S_{\Delta/X}$ expressed as percentage (extreme values in bold)

Calculation technique	K	ρ	f_{cm}	n_f	n_fρ
LST EN 1992-1-1:2005	–14.8 | 3.3	–19.4 | 6.7	**–28.0** | 1.9	8.6 | 0.8	–14.9 | 4.4
CNR-DT 203/2006	–18.7 | 4.9	–17.1 | 4.9	–20.6 | 0.9	7.7 | 0.6	–13.4 | 3.3
ACI 318M-11	12.4 | 1.8	–0.1 | 0.0	–11.7 | 0.2	33.4 | 9.7	14.8 | 3.3
ACI 440.1R-06	1.6 | 0.0	–23.6 | 9.1	–15.2 | 0.5	18.3 | 3.5	–13.9 | 3.5
SP 52-101-2003	–19.1 | 5.0	–18.3 | 5.5	**–24.2** | 1.3	9.0 | 0.9	–14.0 | 3.5

Table 4. Characteristics of the prediction intervals

Calculation technique	Width, %		Lower (unsafe) bound, %
	min	max	
LST EN 1992-1-1:2005	1483.8	1912.5	–568.7
CNR-DT 203/2006	2150.4	2816.6	–679.0
ACI 318M-11	939.5	1109.7	–317.7
ACI 440.1R-06	1125.6	1414.0	–567.1
SP 52-101-2003	1822.1	2287.0	–625.1

As shown in Figure 4, the confidence and prediction intervals for all the regression models were calculated under the assumption that α = 5%. The numerical parameters of the prediction intervals are also given in Table 4 .

Using the same analytical method as for the analysis of the data from the first stage, the test data from the second stage was analysed. The obtained results are presented in Figure 5.

2.3. The analysis results

In this section, results of statistical analysis are discussed. Design codes and recommended methods for determining curvature were analysed during the stages. During the first stage, 46 concrete beams reinforced with FRP bars were analysed.

Location of confidence and prediction intervals (Fig. 4) with respect to the accurate forecast (Δ = 1) serves as a key factor of the analysis. A calculation method is regarded as precise (with the 95% probability) if its confidence intervals include the value Δ = 1. In the authors' view, the results on the safe side (overestimated predictions, (Δ > 1) are preferred considering the underestimation (Δ < 1) as unsafe.

Authors consider that the design method is reliable if $m_\Delta \geq 1$. This means that the calculation by the design codes are derived element stiffness margin (calculated curvature by the design codes are systematically above the experimental values of the curvature).

Analysing the influence of the reinforcement ratio on the results presented in Figure 4, it is important to note that the expression of linear regression may not be fully adequate. For instance, the regression expression of the methods from European and Russian design codes as well as Italian and American design recommendations is valid only when $\rho \leq 2{,}5\ \%$ (Fig. 4). When $\rho > 2.5\%$, regression becomes a non-linear and $\rho = 2.5\%$ could be considered as a limit value.

Examining the FRP reinforcement, instead of the reinforcement ratio ρ, Torres *et al.* (2012) recommend to analyse a regression relationship between the ratio Δ and $n_f\rho$ (Fig. 4, Table 3). It should be noted that the relative stiffness $n_f\rho$ of reinforced concrete beams is usually higher than six. According to the parameters of the collected sample data, it can be seen that the relative stiffness $n_f\rho$ of the majority of concrete ele-

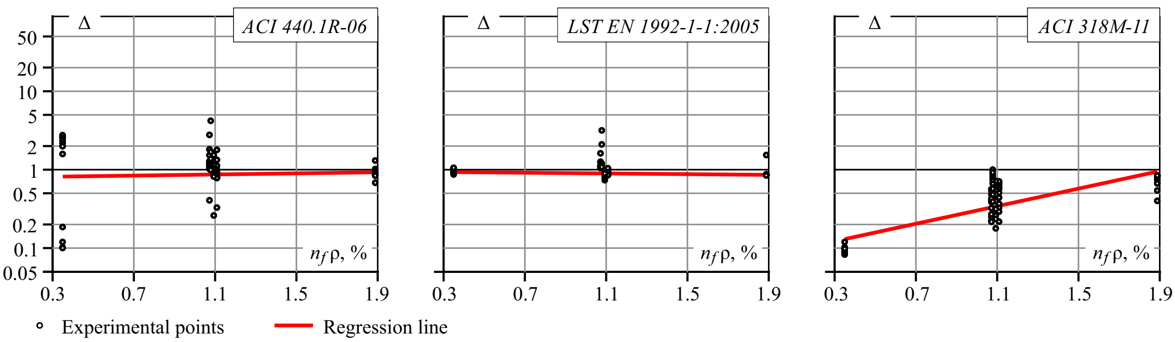

Fig. 5. Analysis of the data of 6 beams from experiments of the authors

ments reinforced with FRP bars is less than 2. Therefore, the authors recommend to perform further studies, when $2 \leq n_f\rho < 6$. Parameter $n_f\rho$ has a similar effect on the precision of each method: the scatter of ratio Δ is approx. 3.3 to 3.5%, only for European design code approx. 4.4% of Δ scatter can be attributed to the variation of $n_f\rho$ ($S_{\Delta/X}$, Table 3).

Summarising the results, it can be stated that the curvature assessed with methods from the American design code ACI 318M-11 and the one obtained from the experimental measurements differs the least; although, on average, it gives the deflection predictions underestimated by 11%. Theoretically, it could be considered as the most accurate method; however, it does not ensure the required structural rigidity and is not adequate for the design of concrete elements reinforced with FRP bars. The Italian design recommendations give the largest margin (overestimation up to 123%) leading to the unreasonable increase of cost of the structural elements. The average standard deviation of the curvature calculated with the methods from European and Russian design codes as well as Italian and American design recommendations are 51%, 52%, 63% and 24%, respectively. Moreover, changing the strength of concrete, the precision of the methods from European and Russian design codes as well as Italian and American design recommendations is improving. With increasing loading level *K*, the precision of the methods from European and Russian design codes as well as Italian design recommendations is improving as well. At the limit loading level, it gives the deflection predictions overestimated by 16%, 4% and 9%, respectively. This can be explained by the fact that with increasing loading, the influence of the tensile concrete on element deformations is decreasing and finally fully disappears (when only reinforcement takes over all tensile stresses).

The precision of the method from the American design recommendations practically does not depend on the loading level *K*. Such method provides the deflection predictions overestimated by 20%, which is the lowest of all analysed methods. Therefore, in the authors' view, the ACI 440.1R-06 method is the most suitable for the design of concrete structures reinforced with FRP bars.

In order to verify the adequacy of the results of performed statistical analysis, the test data from the second stage (six beams reinforced with GFRP bars, 66 experimental measurements in total) was analysed using the same analytical method. The obtained results are presented in Table 5. As already mentioned, the reinforcement ratio ρ is not considered while creating a regression model of the ratio Δ for the concrete elements reinforced with FRP bars (Fig. 5).

Due to a small experimental sample the analysis results are of a qualitative nature (type), i.e. it is not correct to consider statistical parameters such as mean average, standard deviation, etc. Following the data from Figure 5, it can be seen that the method from ACI 318M-11 design code is very sensitive to the change of $n_f\rho$. When $n_f\rho \approx 0.3$, the values of calculated curvature are 7 times lower than the experimental ones. The authors believe that this phenomenon observed at low load values and calculating the cracking moment is due to the overestimation of tensile strength of concrete.

Unlike in the first stage of analysis, curvature values calculated by the method from the European design code LST EN 1992-1-1:2005 were very close to the experimentally obtained results. Moreover, a scatter of the results was the lowest of all analysed methods (Fig. 5). In the authors' view, the main reason for this is ensured bond properties between "Schöck ComBAR" rebars and concrete (Gudonis *et al.* 2012).

As in the first stage, the curvature results calculated by the Russian design code SP 52-101-2003 are very similar to the European ones.

The character of the results of calculations by the American (ACI Committee 440 2006) and Italian (CNR 2007) design recommendations is similar to the one from the first stage. Calculation results of American design recommendations approximately correspond to the experimental data (the regression line is horizontal and close to the unit, see the comments on Equation (8)), although the distribution of the results obviously depends on the change of $n_f\rho$. The Italian design recommendations ensure the margin of deflection calculations (it gives deflection predictions overestimated by 150%). This can be explained by the coefficient β_1 = 0.5 applied in the formula (8-12b) from ACI 440.1R-06 design recommendations. The obtained results show that the application of such value of β_1 regardless of the bond quality between FRP bars and concrete, is too rough for deflection calculations of concrete elements reinforced with various types of FRP bars. In case of the use of "Schöck ComBAR" reinforcement, the coefficient β_1 can be taken equal to 1.0.

Conclusions

According to the European (LST 2007), American (ACI Committee 318 2011), and Russian (NIIZhB 2006) design codes of structural concrete as well as Italian (CNR 2007) and American (ACI Committee 440 2006) design recommendations for FRP reinforced concrete elements, a comparative analysis of the accuracy of deflection assessment methods was performed. The statistical analysis was carried out using the data of 52 concrete beams reinforced with FRP bars (totally 532 experimental measurements) collected from 15 experimental programs. All of the beams were tested by a four-point bending scheme. Most of them were reinforced with GFRP bars (39 beams), the rest – with CFRP bars (12 beams) and one – with AFRP bars. Six beams tested by the authors were reinforced with "Schöck ComBAR" GFRP bars.

A design of concrete elements reinforced with FRP bars should be based on the experimental results of the structural stiffness and the bond properties between FRP bars and concrete. The authors suggest applying the American ACI 440.1R-06 design recommendations for the design of concrete elements reinforced with FRP bars. In case the bond properties between FRP bars and concrete are ensured (e.g. the use of "Schöck ComBAR" reinforcement), the European LST EN 1992-1-1:2005 design code is adequate for the deflection assessment of concrete elements reinforced with FRP bars.

Acknowledgements

The authors gratefully acknowledge the financial support provided by the Research Council of Lithuania (Research Project MIP–083/2012) and the framework of the EU Structural Funds (Research Project VP1-3.1-ŠMM-08-K-01-020). V. Tamulenas gratefully acknowledges the financial support by the project "Promotion of Student Scientific Activities" (VP1-3.1-ŠMM-01-V-02-003) from the Research Council of Lithuania. This project is funded by the Republic of Lithuania and European Social Fund under the 2007-2013 Human Resources Development Operational Programme's priority 3.

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KOMPOZITAIS ARMUOTŲ BETONINIŲ ELEMENTŲ PROJEKTAVIMO METODŲ TIKSLUMO ANALIZĖ

E. Timinskas, R. Jakštaitė, V. Gribniak, V. Tamulėnas, G. Kaklauskas

Santrauka. Tradicinė plieninė armatūra nėra atspari korozijai, jos ištekliai yra riboti, todėl buvo sukurti polimeriniai strypai, armuoti anglies, stiklo, bazalto arba aramido pluoštu. Ši kompozitinė armatūra pasižymi dideliu tempiamuoju stipriu ir atsparumu elektromagnetiniam laukui. Kompozitinių strypų gamyboje naudojamos skirtingos medžiagos ir taikomi įvairūs paviršiaus dengimo būdai, skiriasi jų mechaninės bei sukibimo su betonu savybės. Lyginant su plienine armatūra, stiklo, aramido ir bazalto kompozitiniai strypų tamprumo modulis yra mažesnis, todėl tokiais strypais armuotų konstrukcijų standumas taip pat yra mažesnis nei gelžbetoninių konstrukcijų. Dabartiniuose gelžbetoninių konstrukcijų projektavimo reglamentuose taikomi empiriniai supaprastinti deformacijų nustatymo metodai gali būti netinkami konstrukcijoms, armuotoms polimerine armatūra, projektuoti. Šiame darbe, naudojant mokslinėse publikacijose surinktų 46 eksperimentinių tyrimų ir autorių atliktų 6 sijų bandymų duomenis, buvo įvertintas kompozitais armuoto betono elementų įlinkių skaičiavimo metodų tikslumas. Analizei atlikti buvo pasirinkti Europos (LST EN 1992-1-1:2005), JAV (ACI 318M-11) ir Rusijos (SP 52-101-2003) armuotojo betono konstrukcijų projektavimo normų bei Italijos (CNR-DT 203/2006) ir JAV (ACI 440.1R-06) projektavimo rekomendacijų metodai. Gauti analizės rezultatai suteiks projektuotojams išsamesnę informaciją apie kompozitais armuotų betoninių elementų projektavimą, skatins didins šių inovatyvių medžiagų naudojimo apimtį įvairiose statybinėse konstrukcijose.

Reikšminiai žodžiai: nemetalinė strypinė armatūra, projektavimo normos, eksperimentų duomenys, įlinkiai, armuotasis betonas, tikslumo analizė, trumpalaikė apkrova.

Edgaras TIMINSKAS. PhD student at the Department of Bridges and Special Structures of Vilnius Gediminas Technical University, Lithuania. BSc (2007, Construction Engineering) and MSc (2009, Construction Engineering) from VGTU. Research interests include composite reinforcement.

Rūta JAKŠTAITĖ. MA student at the Department of Bridges and Special Structures of Vilnius Gediminas Technical University. Research interests include composite reinforcement.

Viktor GRIBNIAK. Researcher at the Department of Bridges and Special Structures, VGTU, Lithuania. He received his engineering degree, PhD (2009) from VGTU. A study visit to Polytechnico di Torino (2003). Authored a number of papers. Research interests include mathematical modelling, statistical analysis and numerical simulation of concrete structures.

Vytautas TAMULĖNAS. MA student at the Department of Bridges and Special Structures of Vilnius Gediminas Technical University. Research interests include analysis of reinforced concrete structures under cyclic loading.

Gintaris KAKLAUSKAS. Professor and Head of the Department of Bridges and Special Structures at VGTU, Lithuania. He received his engineering degree, PhD and Dr Habil (DrSc) from VGTU. He was the recipient of Fulbright Fellowship (for research work at the University of Illinois, Urbana-Champaign, 1996) and Marie Curie Fellowship (experienced researchers category, 2002–2003). He is a member of FIB Task Group 4.1 "Serviceability Models". Research interests include various topics on reinforced concrete, particularly constitutive modelling and numerical simulation of reinforced concrete structures.