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PROBABILITY-BASED DESIGN OF AN OPTIMAL ELASTIC–PLASTIC TRUSS

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Abstract. According to Eurocode EN 1990 and Lithuanian Technical Regulation of Construction STR 2.05.03:2003, structures should be designed to satisfy reliability requirements. The reliability of a structure can be achieved using one of 3 methods: partial factor (PF), PF assisted by testing and direct probability-based method. When PF methods are used, the determined reliability of a structure is often greater than required; therefore the direct probability methods allow a more cost-efficient design. The reviewed literature suggests that even greater economical effect can be achieved by combining probability-based design methods with optimization. Unfortunately, the literature presents very few such methodologies. This article focuses on an optimal design of a truss under variable repeated loading at shakedown. The authors propose a model of a truss volume minimization problem with direct probabilistic evaluation of safety margin. The developed technique allows finding minimum volume of desirable reliability structure when loading, provided stochastic parameters are known in advance. The finite element method is applied for the discretisation of a structure. Mathematical programming is used to resolve the optimization problem.

Keywords: direct probability design, truss, volume minimization, mathematical programming.

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Introduction

Modern national and international design standards allow using partial factor (PF) and direct probability-based (PB) methods for ensuring the required reliability of a structure (Holický *et al.* 2004, Vrowenvelder 2002). However, only PF method is used in practice, because of its disposition of definite calculation methods, characteristic values of material properties, determined combinations of loading etc. Calculations based on PF method are rather simple and comprehensible. On the other hand, they produce poorer accuracy. In the mathematical sense, the goal of partial factors is to ensure required (standardised) reliability of an element or an entire structure (Köhler *et al.* 2007; Užpolevičius 2006; Kudzys, Kliukas 2010). This means that a very small (admissible) possibility of structural failure should be permitted, the value of which would be based on economic and social expenses.

As PB method determines the structural collapse probability more accurately than PF or permissible stress methods, more cost-efficient structures can be designed. This method is presented in the Eurocode standards (LST EN 1990), although it does not offer definite calculation methodologies. Scientists are interested in creating such methodologies (Simões 2012; Užpolevičius, Amšiejus 2007; Mrázik, Križma 1997). In direct PB methods, the safety reserve of a structure is described by stochastic variables (Holický *et al.* 2005, Kudzys 2005), such as the failure (collapse) probability, structural reliability and the index of reliability. The index of reliability is described in the standard EN 1990. National standards indicate values for the index

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of reliability in terms of certain structures, depending on their significance (i.e. the significance of damage in case of collapse).

Some authors indicate that compared with PF or other methods, the economic effect of 20% can be achieved by using the probability-based design methods (Užpolevičius 2006). Considering these facts, it is rational to use PB design for structural optimization problems (Schuëller, Jensen 2008). Thus, we can ensure the optimal required reliability and design of a structure (by chosen criteria) at the same time. It is a complex problem and, therefore, not many solutions are proposed in the literature.

In order to ensure a more effective design, plastic properties of materials can be taken into account in optimization problems. European Eurocode standards (LST EN 1993) allow for designing steel structures with plastic hinges (plastic deformations). It is rational to apply the shakedown theory for the design of statically indeterminate structures (Marti 2008; Kaliszki, Lógó 1998; Giambanco *et al.* 1994). The shakedown theory exploits plastic properties of elements in order to reduce the required cross-sections in an optimal project of a structure. Residual internal forces of the structure in the state of shakedown ensure that after the complete loading cycle, the structure will not collapse and stay in pseudo elastic state (plastic deformations will stop). The loading cycle is a time period when all signs of plastic deformation development in the structure can be observed (Atkočiūnas 2011).

Unfortunately, examples of such design techniques (methods) are rarely discussed in literature. Therefore, the purpose of this paper is to demonstrate the method for the effective design of a more economical structure with required standardised reliability, using simple mathematical calculations and an optimization algorithm.

This article focuses on a truss volume optimisation problem with the implemented probability-based design methodology. A truss is subjected to variable repeated loading, i.e. time varying, independent forces or their combinations. The assumptions and mathematical formulations of the direct PB design method are presented in detail. A mathematical model of a truss optimization problem, which allows finding minimum volume of desirable reliability structure is proposed. The finite element method is applied for discretisation of a structure and mathematical programming is used for the numerical solution to the problem. The numerical example of a truss volume minimization problem is presented. Experiments were performed under the assumption of small displacements.

1. Discrete model of an elastic–plastic truss

A discrete model of a truss is composed of n finite elements (members) connected to the nodes. The state of stress is generated only by axial forces, therefore, it is uniaxial. Statically admissible pseudo elastic axial forces $\boldsymbol{N} = [N_1, N_2, ..., N_k, ..., N_n]^T$ and the residual forces $\boldsymbol{N}_r$ constitute the total internal forces of an elastic–plastic truss: $\boldsymbol{N} = \boldsymbol{N}_e + \boldsymbol{N}_r$. Acting loading is described by the vector $\boldsymbol{F} = [F_1, F_2, ..., F_m]^T$, where m is a degree of freedom of a structure. Thereby equilibrium equations of an entire discrete structure read as follow:

$$\boldsymbol{AN} = \boldsymbol{F}, \tag{1}$$

where $\boldsymbol{A}(m \times n)$ is the coefficient matrix of equilibrium equations. It is also used for determination of residual axial forces $\boldsymbol{N}_r = [N_{r1}, N_{r2}, ..., N_{rk}, ..., N_{rn}]^T$:

$$\boldsymbol{AN_r} = \boldsymbol{0}. \tag{2}$$

Elastic internal forces are calculated using an influence matrix of internal forces $\boldsymbol{\alpha}$:

$$\boldsymbol{N_r} = \alpha \mathrm{F},\ \alpha = \boldsymbol{KA}^T(\boldsymbol{AKA}^T)^{-1}, \tag{3}$$

where $\boldsymbol{K}$ is the stiffness matrix composed of individual stiffness of discrete elements $k_k = (EA_{pk})/l_k$ (l_k is the length of k-th element, A_{pk} is cross-sectional area, E is the modulus of elasticity). The axial force of an individual discrete element is calculated using the sub matrix (line) of α, which is related to k-th element: $N_{ek} = \alpha_k \boldsymbol{F}$. Thus, without loss of generality, the axial force can be expressed as a function of cross-sectional areas $\boldsymbol{A}_p$, lengths $\boldsymbol{l}$, modulus of elasticity $\boldsymbol{E}$ and loading vector $\boldsymbol{F}$:

$$N_{ek} = f(\boldsymbol{A}_p, \boldsymbol{E}, \boldsymbol{l}, \boldsymbol{F}), \tag{4}$$

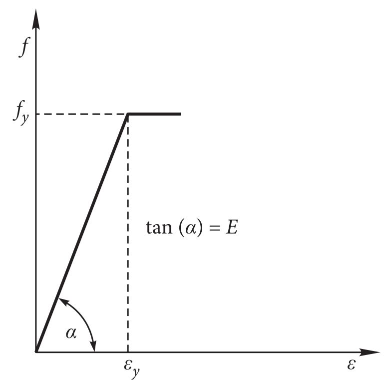

Fig. 1. Physical model of ideal elastic–plastic material

The limit axial force $N_{0k} = A_{pk} f_{yk}$ (k = 1, 2, ..., n) is assumed to be constant over the entire finite element; f_{yk} is the material yield stress. An ideal elastic–plastic stress–strain state model is applied for the structure (Fig. 1).

2. Probability-based truss design

When PB method is applied to a truss design, all variables X that describe load effect and structural resistance are random with the normal distribution of probability density: $X \in N(\mu_X, \sigma^2_X)$. For the sake of simplicity, element buckling is ignored is this study, therefore, a form of cross-section has no influence on the calculations. Thus, the structural resistance function of k-th element reads as follows:

$$R_k = r(f_{yk}, A_{pk}, \Delta R), \tag{5}$$

the function of load effect is:

$$E_k = e(N_{ek}, N_{rk}, \Delta E). \tag{6}$$

Consequently, the safety margin function is:

$$\begin{aligned} Z_k &= R_k - E_k = r(f_{yk}, A_{pk}, \Delta R) - e(N_{ek}, N_{rk}, \Delta E) = \\ &z(f_{yk}, A_{pk}, N_{ek}, N_{rk}, \Delta R, \Delta E) = \\ &N_{0k} - N_{ek} - N_{rk} + \Delta R + \Delta E = \\ &A_{pk} \cdot f_{yk} - \boldsymbol{\alpha}_k \boldsymbol{F} - N_{rk} + \Delta R + \Delta E, \end{aligned} \tag{7}$$

where ΔR is the error of the structural resistance calculation model and ΔE is the error of the load effect calculation model.

Variables of structural resistance and load effect functions are random and have normal distributions; therefore, the function of safety reserve has the normal distribution too: $Z_k \in N(\mu_{Zk}, \sigma^2_{Zk})$. The reliability index of the safety margin function β_k is the main criteria of the probability-based design method. A graphical interpretation of the probability density function $h(Z_k)$ is shown in Fig. 2.

Eurocode standards require that the reliability index of all structural elements should be higher or equal to the indicated value β_{nk}. This way, sufficiently small failure probability α_k of elements is ensured (Fig. 2).

The reliability index β_k of the element k is calculated according to the formula:

$$\beta_k = \frac{\mu_{Zk}}{\sigma_{Zk}}, \tag{8}$$

where μ_{Zk} is the value of the safety margin in mean points:

$$\mu_{Zk} = z(\boldsymbol{\mu_x}) = \mu_{Apk} \cdot \mu_{fyk} - \mu_{Nek} - \mu_{Nrk} + \mu_{\Delta R} + \mu_{\Delta E}, \tag{9}$$

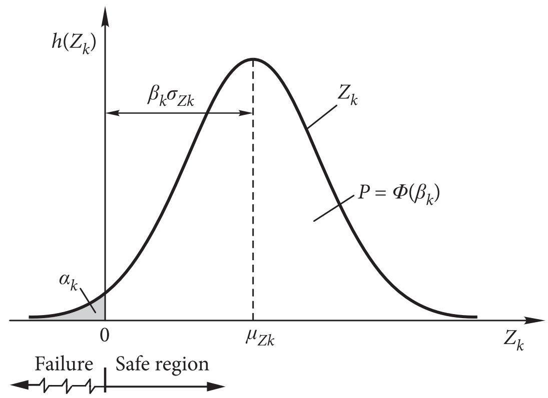

Fig. 2. The probability density function of the safety margin Z_k

σ_{Zk} is the mean square deviation of the safety margin function:

$$\sigma_{Zk} = \left[\left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial f_{yk}} \sigma_{fyk} \right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial A_k} \sigma_{Apk} \right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial N_{ek}} \sigma_{Nek} \right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial N_{rk}} \sigma_{Nrk} \right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial \Delta R} \sigma_{\Delta R} \right)^2 + \left(\frac{\partial z(\boldsymbol{\mu_x})}{\partial \Delta E} \sigma_{\Delta E} \right)^2 + 2\sigma_{RE}\sigma^2_{\Delta E}\sigma^2_{\Delta R} \right]^{1/2}. \tag{10}$$

After differentiation operations we get:

$$\sigma_{Zk} = [(A_{pk} \cdot \sigma_{fyk})^2 + (f_{yk} \cdot \sigma_{Apk})^2 + (\sigma_{Nek})^2 + (\sigma_{Nrk})^2 + (\sigma_{\Delta \mathrm{R}})^2 + (\sigma_{\Delta \mathrm{E}})^2 + 2\sigma_{RE}\sigma^2_{\Delta \mathrm{E}}\sigma^2_{\Delta \mathrm{R}}]^{1/2}, \tag{11}$$

where σ_{Xk} is the square deviation of random variable X, $\sigma_{\Delta \mathrm{R}}$, $\sigma_{\Delta \mathrm{E}}$, σ_{RE} are the square variations of resistance and effect functions and their correlation (Ditlevsen, Madsen 2007). The square deviation of the axial force N_{ek} function (4) is calculated in the same manner:

$$\sigma_{Nek} = \left[\left(\frac{\partial N_{ek}}{\partial A_p} \sigma_{Apk} \right)^2 + \left(\frac{\partial N_{ek}}{\partial E} \sigma_E \right)^2 + \left(\frac{\partial N_{ek}}{\partial l} \sigma_l \right)^2 + \left(\frac{\partial N_{ek}}{\partial F} \sigma_F \right)^2 \right]^{1/2}. \tag{12}$$

Calculations of such partial derivatives are rather complicated, thus simplified methods, such as Richardson extrapolation, are applied (Jankovski, Atkočiūnas 2008).

3. The mathematical model of a truss volume optimization

The mathematical model of a truss volume minimization problem with constraints for reliability of elements, when the truss is subjected to variable repeated loading, reads as follows:

find

$$\min \boldsymbol{L}^T \boldsymbol{A}_p, \tag{13}$$

subject to

$$\boldsymbol{A}\boldsymbol{N}_r = \boldsymbol{0}, \tag{14}$$

$$|\boldsymbol{\beta}| \geq \boldsymbol{\beta}_n, \tag{15}$$

$$\boldsymbol{A}_p \geq \boldsymbol{A}_{\min}. \tag{16}$$

It is a linear mathematical programming problem. The objective function (13) express volume of truss elements: $\boldsymbol{L}^T$ is the vector of truss elements and $\boldsymbol{A}_p = [A_p, A_{p2}, ..., A_{pn}]^T$ is the vector of corresponding cross-sectional areas. Conditions (14) describe the equilibrium of the residual axial forces $\boldsymbol{N}_r$. Conditions (15) ensure required reliability indices of structural elements; $\boldsymbol{\beta} = [\beta_1, \beta_2, ..., \beta_k, ..., \beta_n]^T$ is the vector of truss element reliability indices and β_n is the vector of standardized reliability indices. Conditions (16) are construction regulation constraints (implied by the designer or indicated in standards); $\boldsymbol{A}_{\min}$ is the vector of minimal allowed cross-sectional areas. The unknowns of the problem (13)–(26) are the vector of the element's cross-sectional areas $\boldsymbol{A}_p$ and the vector of residual axial forces $\boldsymbol{N}_r$.

The components of the vector $\boldsymbol{\beta}$ in conditions (15) are written in modulus to satisfy the validity of safety margin function both for positive and negative axial forces, i.e. for tension and compression. Thus, the expression for the k-th element of a truss is:

$$|\beta_k| = \frac{|\mu_{Zk}|}{\sigma_{Zk}} = \frac{\dfrac{\mu_{Apk} \cdot \mu_{fyk} + \mu_{Nek,\max} + \mu_{Nrk} + \mu_{\Delta R} + \mu_{\Delta E}}{\sigma_{Zk}}}{\dfrac{\mu_{Apk} \cdot \mu_{fyk} - \mu_{Nek,\min} - \mu_{Nrk} + \mu_{\Delta R} + \mu_{\Delta E}}{\sigma_{Zk}}}, \tag{17}$$

where $\mu_{Nek,\max}$ and $\mu_{Nek,\min}$ are the extreme pseudo elastic axial forces of k-th element. A time function of variable repeated loading $\boldsymbol{F}(t)$ is often substituted by the combinations $\boldsymbol{F}_j$, which describe all vertexes of loading locus. Thereby $\mu_{Nek,\max}$ and $\mu_{Nek,\min}$ are extracted from all possible internal forces caused by loading combinations $\boldsymbol{F}_j$ ($j = 1, 2, ..., p$, $p = 2^m$) in element k (Merkevičiūtė, Atkočiūnas 2005). Thus, the conditions (17) are essentially yield conditions with stochastic variables.

4. Numerical example of a truss optimization

A bridge type truss subjected to a pair of forces that can take any of five positions (discretised moving load) is analysed (Fig. 3). The main task is to find the minimum volume of the truss by solving the problem (13)–(16) and ensure that reliability indices of all members are higher or equal to $\beta_n = 3.8$.

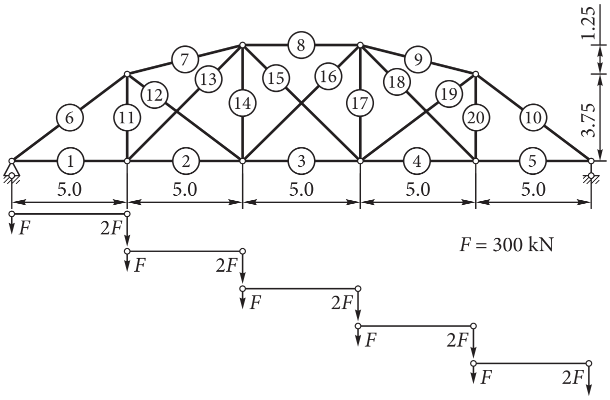

Fig. 3. Truss subjected to moving load

The following presumptions are considered: the length of elements are invariable (determined values); variations of elastic internal forces are assessed all at once, without considering the influence of variations of cross-sectional areas A_p, nor the modulus of elasticity E, or external forces $\boldsymbol{F}$; the structural resistance and the load effect functions are independent, i.e. $\sigma_{RE} = 0$; the error of the load effect calculation model is neglected – $\sigma_{\Delta \mathrm{E}} = 0$. The effect of the variable repeated loading is evaluated through vectors of extreme internal forces and applied to the corresponding equation of the reliability index calculation (17).

The following stochastic parameters were known in advance: the steel yield stress (μ_{fy}, σ_{fy}) = (530 MPa, 58.3 MPa); the cross-sectional areas $\upsilon_{Ap} = 0.05$; elastic axial forces $\sigma_{Ne} = 40$ kN; errors of the structural resistance calculation model ($\mu_{\Delta \mathrm{R}}$, $\sigma_{\Delta \mathrm{R}}$) = (21 kN, 6 kN). The predefined minimal cross-sectional area for all elements is $A_{\min} = 5$ cm^2.

This is a continuous optimization problem, discrete values of real steel cross-sections are neglected, thus calculation results are mathematically ideal values, which satisfy the model (13)–(16) conditions without reserve. Real cross-sections can be ascribed to members of the truss subsequent to analysis of continuous optimisation results. Members could also be grouped before the optimization (e.g. all web elements could be the same), but in this case we assumed that all elements could be different. Unification of elements is more rational when stress (or internal forces) distribution is available, i.e. if optimization results are considered.

The constraints of the model (13)–(16) are dependent on the unknowns (cross-sectional areas); therefore, the solution algorithm is performed in the following iterative manner:

- initial cross-sectional areas $A_{pk,init}$ are prescribed for all elements $k \in K$;
- the global stiffness matrix $\boldsymbol{K}$ is composed with the initial cross-sectional areas and mean values of the elastic axial forces $\mu_{Nek,\max}$, $\mu_{Nek,\min}$ and a deviation σ_{Zk} of structural resistance function distribution for all $k \in K$ are calculated;
- new cross-sectional areas $A_{pk,new}$ are received by solving the linear mathematical programing problem (13)–(16);
- new cross-sectional areas are prescribed as initial ones $A_{pk,init} = A_{pk,new}$ and the cycle of problem (13)–(16) solution is repeated while a change of truss volume value in adjacent iterations is as small as desired.

Such iterative calculation is reasonable in case of the analysed example (convergence achieved after 6 iterations, see Fig. 4). It allows resolving simpler formulation of the optimization model compared with possible nonlinear solution if variables as functions in the constraints are introduced. Calculations were performed using *Matlab* programming environment. The results are presented in Table 1. The optimal solution consists of cross-sectional areas $\boldsymbol{A}^*$ and residual axial forces $\boldsymbol{N}_r^*$. Extreme axial forces in every element k are also calculated: $N_{max,k} = \mu_{Nek,\max} + N_{rk}$ and $N_{min,k} = \mu_{Nek,\min} + N_{rk}$. These forces do not exist in the structure at the same time, but arise in separate elements on different stages of the shakedown process. The optimal distribution of cross-sectional areas is relatively shown in Fig. 5. The optimal volume of the truss, calculated with respect of above stated conditions, is $V_{\text{opt}} = 0.43926$ m^3.

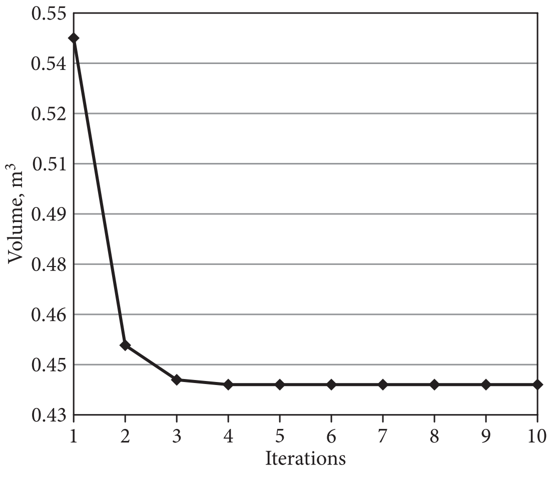

Fig. 4. Convergence of the iterative problem solution

Table 1. Results of the problem solution

Element No	A^*, cm^2	N_r^*, kN	N_{max}, kN	N_{min}, kN	β_n
1	52.61	0.00	800.00	0.00	3.80
2	53.73	27.98	818.99	27.98	3.80
3	56.79	19.28	870.92	19.28	3.80
4	52.23	17.21	793.62	17.21	3.80
5	57.33	0.00	880.00	0.00	3.80
6	64.46	0.00	0.00	–1000.00	3.80
7	59.00	28.85	28.85	–908.13	3.80
8	60.24	19.28	19.28	–929.09	3.80
9	58.55	17.74	17.74	–900.52	3.80
10	70.43	0.00	0.00	–1100.00	3.80
11	34.21	27.98	482.82	27.98	3.80
12	18.64	–34.98	203.53	–203.53	3.80
13	16.59	–39.58	165.71	–165.71	3.80
14	26.90	40.27	353.37	40.27	3.80
15	16.36	–27.27	161.41	–161.41	3.80
16	21.88	–27.27	262.86	–178.01	3.80
17	25.91	32.19	335.70	22.20	3.80
18	23.49	–21.51	292.03	–117.74	3.80
19	14.26	–24.33	122.16	–122.16	3.80
20	35.96	17.21	513.62	17.21	3.80

The influence of individual stochastic variables to the results of calculations is further analysed. The square products in the formula (11) have a meaning of different variable errors in the safety margin calculation function. For example, an actual function with numerical values for the 8-th element reads as follows:

$$\sigma_{Z8} = [(A_{p8} \cdot \sigma_{fy8})^2 + (f_{y8} \cdot \sigma_{Ap8})^2 + (\sigma_{Ne8})^2 + (\sigma_{Nr8})^2 + (\sigma_{\Delta \mathrm{R}})^2]^{1/2} =$$
$$[(0.006 \cdot 58300)^2 + (530000 \cdot 0.05 \cdot 0.006)^2 + 40^2 + 6^2]^{1/2} =$$
$$[12.24 \cdot 10^4 + 2.53 \cdot 10^4 + 0.16 \cdot 10^4 + 36]^{1/2}.$$

From these results we can easily determine, that the greatest influence for this variable is achieved by the first component, i.e. the square product of the cross-sectional area and the square deviation of yield stress.

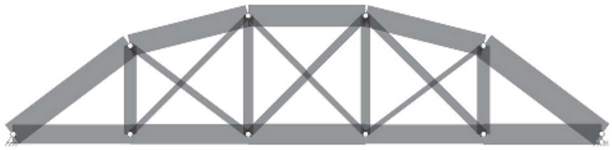

Fig. 5. Comparative scheme of optimal truss elements cross-sections (according to Table 1)

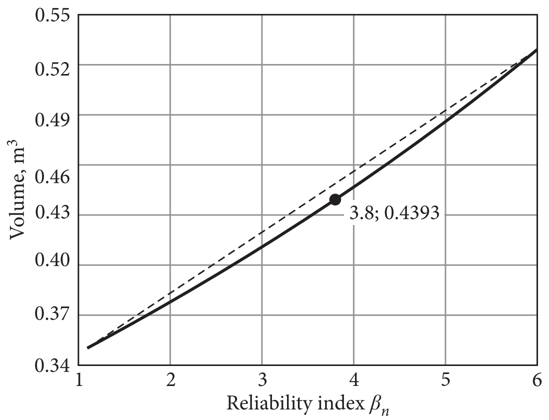

Fig. 6. Truss volume dependence on the reliability index β_n

The cross-sectional area of optimization problem is unknown; therefore, the most effective way to improve the reliability of structures is to reduce the square deviation of yield stress. The latter value is mostly influenced by steel manufacturing technology and the quality control during manufacturing. It is worth mentioning, that the deviation of the production of a particular steel manufacturer can be smaller than the one indicated in the standards, where the worst case scenario is always implicated. Thus, a possibility of more economical design emerges with the knowledge of particular steel product characteristics.

The relation between the reliability index β_n and the optimal truss volume according to the problem (13)–(16) is further analysed. After solving a set of problems, nonlinear distribution of these values was found (Fig. 6). It is evident that the volume of truss increases with higher values of desired reliability. A solution to the abovementioned detailed example (when $\beta_n = 3.8$) is also in the graph.

Conclusions

1. A proposed methodology allows effective and economical designing of structures with direct probabilistic evaluation of the safety margin using a moderate amount of mathematical calculations and an optimization algorithm.
2. The considered method is based on implication of the probability-based design in the mathematical programming optimization problem of a discrete structure.
3. A numerical example showed that the greatest influence to the optimal volume of structure has the variation of the steel yield stress; therefore, the reduction of its value is the most effective way to a more economical design. The latter value is mostly influenced by steel manufacturing technology and its quality control, thus it is recommended to consider controlling this parameter.
4. Further analysis of the developed optimization model showed a nonlinear dependence of the optimal truss volume on the reliability index β_n. That validates the necessity to apply optimization techniques in each particular case and does not allow any prejudgement of possible structural reliability.

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OPTIMALIOS TAMPRIAI PLASTINĖS SANTVAROS PROJEKTAVIMAS TIKIMYBINIU STATISTINIU METODU

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Santrauka. Europos projektavimo normos EN 1990 ir Lietuvoje galiojantys statybos techniniai reglamentai STR 2.05.03:2003 reglamentuoja konstrukcijų projektavimą užtikrinant normuotąjį patikimumą. Konstrukcijos patikimumas gali būti užtikrintas trimis metodais: dalinių koeficientų (DK), DK su bandymais ir tikimybiniu informaciniu statistiniu (TIS) metodu. Taikant DK ar DK su bandymais metodus suprojektuotos konstrukcijos patikimumas dažnai viršija normuotojo patikimumo reikšmę, o tiesioginis tikimybinis projektavimas sudaro prielaidas projektuoti ekonomiškiau. Kaip rodo literatūros šaltinių analizė, pasiekti dar didesnį konstrukcijos ekonomiškumą galima jungiant TIS metodą su konstrukcijos optimizavimu. Straipsnyje pademonstruota, kad, pasitelkus nesudėtingus matematinius skaičiavimus ir pritaikius optimalaus sprendinio paieškos algoritmą, galima efektyviai projektuoti statybines konstrukcijas užtikrinant reikiamą normuotąjį patikimumą. Straipsnyje nagrinėjama kintamosios kartotinės apkrovos veikiama prisitaikanti santvara. Sudarytas tūrio minimizavimo uždavinio modelis, tiesiogiai tikimybiškai įvertinant konstrukcijos atsparumo atsargą. Iš pateikto skaitinio pavyzdžio rezultatų nustatyta, kad didžiausią įtaką optimaliam santvaros tūriui iš visų atsitiktinių dydžių turi plieno takumo įtempių kvadratinė nuokrypa. Todėl jos mažinimas yra efektyviausias būdas siekiant didesnio konstrukcijos ekonomiškumo. Konstrukcijos diskretizacijai taikomas baigtinių elementų metodas. Skaitiniam optimizavimo uždavinio sprendimui pasitelktas matematinis programavimas.

Reikšminiai žodžiai: tiesioginis informacinis statistinis projektavimas, santvara, optimizacija, matematinis programavimas.

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