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CAR ROLLOVER COLLISION WITH PIT CORNER

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Abstract. The paper presents a theory of body collision with pit corner. The theory can be successfully applied in approaching tasks coping with the orientation of details in technological processes and control systems, the motion of elements in mechanisms having a gap, collisions in motion of different sport objects, the analysis of car accidents etc. The applied theory deals with car motion after collision with a specific obstacle i.e. pit corner. The study is based on linear and angular momentum theorems and two restitution coefficients of normal impulse from both sides of the obstacle. The obtained results can be used for assessing road accidents in case of car collision with one or more obstacles. The object of contact may involve hitting a plane foundation, the ditch and edging of a highway or a rock in the field.

Keywords: collision, impact, pit corner, car rollover, kinetic energy, symbolic calculation.

1. Introduction

A car accident, which is sometimes called a car crash or wreck, can be accepted as an incident in which an automobile collides with anything causing damage to the car including other types of vehicles, telephone poles, buildings, trees or the situation in which a driver loses control over the vehicle and damages it in some other way such as driving into a ditch or rolling over (Lama *et al.* 2007; Kinderytė-Poškienė and Sokolovskij 2008; Sokolovskij 2007a, b; Sokolovskij *et al.* 2007; Tautkus and Bazaras 2007; Prentkovskis *et al.* 2007; Pelenytė-Vyšniauskienė and Jurkauskas 2007; Vaidogas 2007). A car accident sometimes may refer to an automobile striking a man or an animal. 1.2 million people are killed in car crashes worldwide each year, whereas the number of the injured is about forty times higher.

Accidents are divided into a few categories: rear-end collisions, head-on collisions, rollovers, side impact collisions, truck under-ride accidents, backup accidents and suicides. 280 000 rollover accidents occur in the US each year and result in about 10 000 fatal accidents.

The article examines several cases of rollover motion of a car using the theory of impact on a body against an obstacle. The results may be used for assessing road accidents in cases of car collision with one or more obstacles and car rollover (Fig. 1–4). The obstacle may include plane fundament, the ditch and edging of a highway or a rock in the field.

Fig. 1. The start of right side car rollover (frame from video)

Fig. 2. Left side car rollover (frame from video)

Fig. 3. Left side car rollover with one collision

Fig. 4. More than one collision in car rollover

2. Collision Equation

Suppose that body 1 collides pit corner with two rectangular sides 2 and 3 (Fig. 5). The impact impulse has two components $\bar{S}_x$, $\bar{S}_y$ (Виба 1988; Viba *et al.* 2000; Kepe and Viba 1999). To calculate the parameters of collision, the theorems of change in linear and angular momentum and two restitution coefficients of normal impulse R_1 and R_2 by sides 1 and 2 (1), (2) can be used (Lavendelis *et al.* 1997; Плявниекс *и др*. 1969).

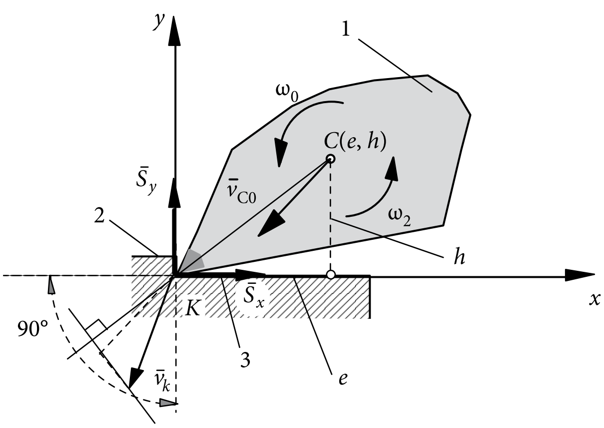


Fig. 5. A scheme of collision from two sides

$$
\begin{aligned}
& m \cdot \dot{x}_{c1} - m \cdot \dot{x}_{c0} = S_{x1}; \\
& m \cdot \dot{y}_{c1} - m \cdot \dot{y}_{c0} = S_{y1}; \\
& J_c \cdot \omega_1 - J_c \cdot \omega_0 = S_{x1} \cdot h - S_{y1} \cdot e; \\
& 0 = \dot{x}_{c1} + \omega_1 \cdot h; \\
& 0 = \dot{y}_{c1} - \omega_1 \cdot e;
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
& m \cdot \dot{x}_{c2} - m \cdot \dot{x}_{c0} = S_{x2} \cdot \left(1 + R\right)_1; \\
& m \cdot \dot{y}_{c2} - m \cdot \dot{y}_{c0} = S_{y2} \cdot \left(1 + R_2\right); \\
& J_c \cdot \omega_2 - J_c \cdot \omega_0 = \\
& S_{x2} \cdot \left(1 + R_1\right) \cdot h - S_{y2} \cdot \left(1 + R_2\right) \cdot e,
\end{aligned} \tag{2}
$$

where: m, e, h – the mass of the body and the coordinates of mass center C; J_c – the moment of inertia against CSV axis; $\dot{x}_{c0}, \dot{y}_{c0}$ – the projection of the side sliding velocity of mass centre C before collision; $\dot{x}_{c1}, \dot{y}_{c1}$ – the projection of the side sliding velocity of mass center at the end of the first phase of collision; $\dot{x}_{c2}, \dot{y}_{c2}$ – the projection of the side sliding velocity of mass center at the end of the second phase of collision; $\omega_0, \omega_1, \omega_2$ – the angular velocity of the body before collision, at the end of the first phase of collision and at the end of full collision; $\bar{S}_{x1}, \bar{S}_{y1}$ – impulses at the end of the first phase of collision when contact point K stops that is:

$$\bar{v}_{K1} = \bar{\omega}_1 \times \overline{CK} = 0,$$

where: $\bar{v}_{K1}$ – the velocity of contact point at the end of the first phase of collision (Fig. 5).

A solution of eight equations (1), (2) by symbolic calculation is (3)–(6):

Given:

$$
\begin{aligned}
& m \cdot v_{x1} - m \cdot v_{x0} = S_{x1}; \\
& m \cdot v_{y1} - m \cdot v_{y0} = S_{y1}; \\
& J_c \cdot \omega_1 - J_c \cdot \omega_0 = S_{x1} \cdot h - S_{y1} \cdot e; \\
& 0 = v_{x1} + \omega_1 \cdot h; \\
& 0 = v_{y1} + \omega_1 \cdot e.
\end{aligned} \tag{3}
$$

Find:

$$
\left(S_{x1}, S_{y1}, v_{x1}, v_{y1}, \omega_1\right) \to
\begin{bmatrix}
-m \cdot \dfrac{h \cdot J_c \cdot \omega_0 + h \cdot e \cdot m \cdot v_{y0} + v_{x0} \cdot J_c + v_{x0} \cdot m \cdot e^2}{J_c + m \cdot h^2 + m \cdot e^2} \\
-m \cdot \dfrac{-e \cdot J_c \cdot \omega_0 + e \cdot h \cdot m \cdot v_{x0} + v_{y0} \cdot J_c + v_{y0} \cdot m \cdot h^2}{J_c + m \cdot h^2 + m \cdot e^2} \\
\dfrac{-J_c \cdot \omega_0 + h \cdot m \cdot v_{x0} - e \cdot m \cdot v_{y0}}{J_c + m \cdot h^2 + m \cdot e^2} \cdot h \\
\dfrac{-\left(-J_c \cdot \omega_0 + h \cdot m \cdot v_{x0} - e \cdot m \cdot v_{y0}\right)}{J_c + m \cdot h^2 + m \cdot e^2} \cdot e \\
\dfrac{-\left(-J_c \cdot \omega_0 + h \cdot m \cdot v_{x0} - e \cdot m \cdot v_{y0}\right)}{J_c + m \cdot h^2 + m \cdot e^2}
\end{bmatrix}. \tag{4}
$$

Given:

$$m \cdot v_{x2} - m \cdot v_{x0} = S_{x1} \cdot \left(1 + R_1\right);$$
$$m \cdot v_{y2} - m \cdot v_{y0} = S_{y1} \cdot \left(1 + R_2\right);$$
$$J_c \cdot \omega_2 - J_c \cdot \omega_0 =$$
$$S_{x1} \cdot h \cdot \left(1 + R_1\right) - S_{y1} \cdot e \cdot \left(1 + R_2\right). \quad (5)$$

Find:

$$\left(v_{x2}, v_{y2}, \omega_2\right) \rightarrow$$
$$\begin{bmatrix} \dfrac{m \cdot v_{x0} + S_{x1} + S_{x1} \cdot R_1}{m} \\ \dfrac{m \cdot v_{y0} + S_{y1} + S_{y1} \cdot R_2}{m} \\ -\dfrac{S_{y1} \cdot e - J_c \cdot \omega_0 - S_{x1} \cdot h - S_{x1} \cdot h \cdot R_1 + S_{y1} \cdot e \cdot R_2}{J_c} \end{bmatrix}. \quad (6)$$

Formulas (3)–(6) are useful for calculating collisions if all parameters of the system and the initial conditions of motion are given.

In addition, existence conditions must be satisfied as follows:

1. Before collision, the velocity direction of contact point K must be inside 90° sector of pit corner (7) (Fig. 5):

$$v_{x0} + \omega_0 \cdot h < 0; \; v_{y0} - \omega_0 \cdot e < 0. \quad (7)$$

2. After collision, the velocity direction of contact point K must be outside 90° sector of pit corner (8), (9) (Fig. 5):

$$v_{x2} + \omega_2 \cdot h > 0; \quad (8)$$

$$v_{y2} - \omega_2 \cdot e > 0. \quad (9)$$

3. Calculation of Remaining Kinetic Energy

Remaining kinetic energy in collision, expressed as a percentage of initial kinetic energy f is:

$$f = \frac{E_2}{E_0} \cdot 100\%,$$

where: E_0 – initial kinetic energy:

$$E_0 = \frac{m \cdot \left(v_{x0}^2 + v_{y0}^2\right)}{2} + \frac{J_c \cdot \omega_0^2}{2};$$

E_2 – kinetic energy after collision:

$$E_2 = \frac{m \cdot \left(v_{x2}^2 + v_{y2}^2\right)}{2} + \frac{J_c \cdot \omega_2^2}{2}.$$

Then:

$$f = \frac{\dfrac{m \cdot \left(v_{x2}^2 + v_{y2}^2\right)}{2} + \dfrac{J_c \cdot \omega_2^2}{2}}{\dfrac{m \cdot \left(v_{x0}^2 + v_{y0}^2\right)}{2} + \dfrac{J_c \cdot \omega_0^2}{2}} \cdot 100\%. \quad (10)$$

For modeling collision, existence conditions (7)–(9) must be satisfied using special switch operators in form (11):

$$f = \frac{\dfrac{m \cdot \left(v_{x2}^2 + v_{y2}^2\right)}{2} + \dfrac{J_c \cdot \omega_2^2}{2}}{\dfrac{m \cdot \left(v_{x0}^2 + v_{y0}^2\right)}{2} + \dfrac{J_c \cdot \omega_0^2}{2}} \cdot 100\% \times$$
$$\frac{1 + \dfrac{v_{x2} + \omega_2 \cdot h}{\left|v_{x2} + \omega_2 \cdot h\right|}}{2} \cdot \frac{1 + \dfrac{v_{y2} - \omega_2 \cdot e}{\left|v_{y2} - \omega_2 \cdot e\right|}}{2}. \quad (11)$$

Using equations (4), (6) and (11), different computer calculations of car rollover with one, two or more collisions may be investigated.

Consequently, to examine car crashes and insurance, initial velocity v_0 may be found out.

4. Modeling

The impact of different coefficients R_1 and R_2 on remaining kinetic energy was investigated in modeling tasks. The values of both coefficients in regions $0 \le R_1 \le 1$, $0 \le R_2 \le 1$ varied with step value 0.01 (12). 3-D graphics were designed for visualizing results (see next tasks below), see Figs 6–12.

$$m := 1500; \; B := 2; \; H := 1.5;$$
$$e := \frac{B}{2}; \; J_c := m \cdot \frac{B^2 + H^2}{12};$$
$$h := 0.75; \; v_{x0} := -50 \cdot \frac{1}{3.6};$$
$$v_{y0} := 0; \; \omega_0 := 0.$$

Fig. 6. Car sliding with collision against a road border or rock

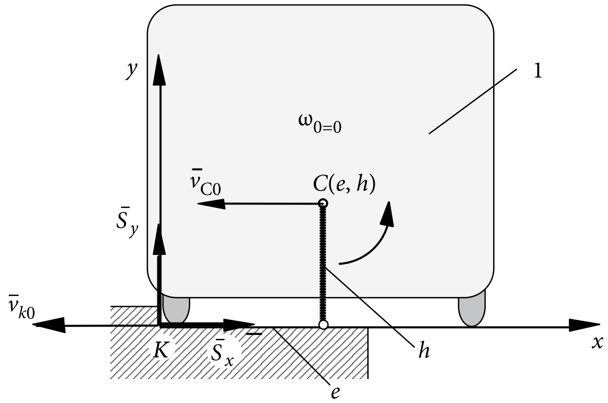
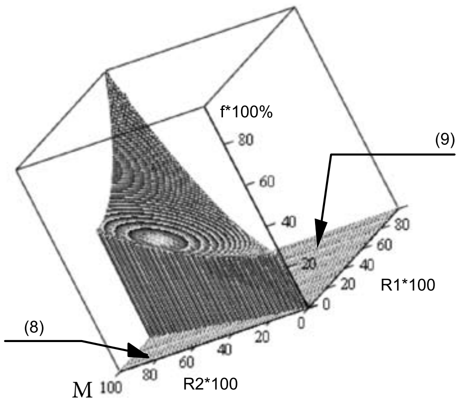


Fig. 7. Vector scheme for collision with rotation



$$M_{99,100} = 98.551;$$
$$\max(M) = 98.551;$$
$$M_{35,100} = 51.931;$$
$$M_{49,50} = 44.531;$$
$$\min(M) + M_{4,5} = 27.121.$$

Fig. 8. A graph of remaining kinetic energy M (in %) as a function of R_1, R_2

$$m := 1500;\ H := 1.5;$$
$$v_{x0} := -50 \cdot \frac{1}{3.6};\ e := \frac{B}{2};$$
$$J_c := m \cdot \frac{B^2 + H^2}{12};\ h := 0.75;$$
$$B := 2;\ \omega_0 := \frac{-v_{x0}}{3 \cdot h};\ v_{y0} := 0.$$

Fig. 9. Car sliding motion and rotation

$$N := 100;$$
$$i := 0...N;\ j := 0...N;$$
$$R_{1i} := 0 + 0.01 \cdot i;$$
$$R_{2j} := 0 + 0.01 \cdot j;$$
$$M_{i,j} := f\left(R_{1i}, R_{2j}\right). \quad (12)$$

4.1. Side Sliding Without Rotation

In the first example, a task of modeling a car was solved: mass m = 1500 kg; width B = 2 m; height H = 1,5 m; coordinate h = 0.75 m; velocity v_{x0} = 50 km/h; angular velocity before collision $\omega_0 = 0$.

The arrows in Fig. 8 show conditions (8) and (9). The extreme values of kinetic energy are shown by M % (for example: $M_{35,100}$ denote that $R_1 = 0.35$, $R_2 = 1.00$ and M % = 51.931%).

The carried out analysis concludes that in regions $0 \le R_1 \le 0.5$ and $0 \le R_2 \le 0.5$, remaining kinetic energy makes about 27–45%.

4.2. Side Sliding and Rotation

Sliding collision and rotation was calculated applying parameters shown in Figures 9 and 10:

In regions $0 \le R_1 \le 0.5$ and $0 \le R_2 \le 0.5$, remaining kinetic energy-makes about 42–56%.

4.3. Collision with Rotation Around the Axis

This type of collision exists under special initial conditions, for example, when a car (after first collision against road border) collides with the roof against the ground second time (Fig. 11–12).

In this case, in regions $0 \le R_1 \le 0.5$ and $0 \le R_2 \le 0.5$, remaining kinetic energy makes about 21–41%.

5. Calculating Collision Series

Traffic accidents with rollover motion provide the possibility of calculating the number of collisions a car has made. Afterwards, an expert inspects a character of car hull deformations within collisions and may approximately evaluate coefficients R_1 and R_2 (in very small region $R_{1,2} = 0 \div 1.$). At a later stage, initial car kinetic energy and initial driving velocity v_0 may be calculated.

Fig. 13 indicates a full overturn cycle with five rollover collisions. For example, an expert decided that during the first and following collisions, remaining energy made 51%, a serious deformation of the roof border on the right side of a car – 27%, roof deformation – 41%, a minor deformation of the roof border on the left side of a car – 70% and insufficient damping in tires – 90%. When the *SSF* (*Static Safety Factor*) is given, for example *SSF* = 1.25, the value of maximal v_0 and minimal v_1 initial velocity can be calculated and is equal to: v_1 = 47.86 km/h; v_0 = 50.45 km/h (see the following calculation).

Remaining kinetic energy when a car stops moving after four collisions and takes an overturned position on the left is:

$$0.51 \cdot 0.27 \cdot 0.41 \cdot 0.7 = 0.04.$$

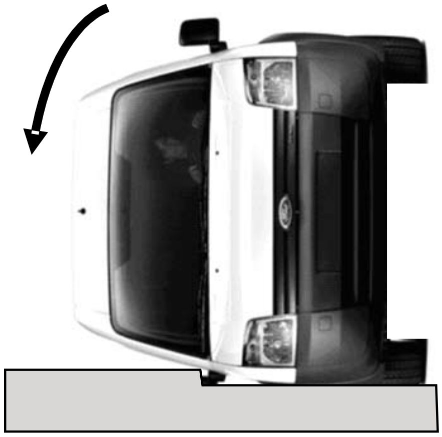


Fig. 10. The model of car rotation

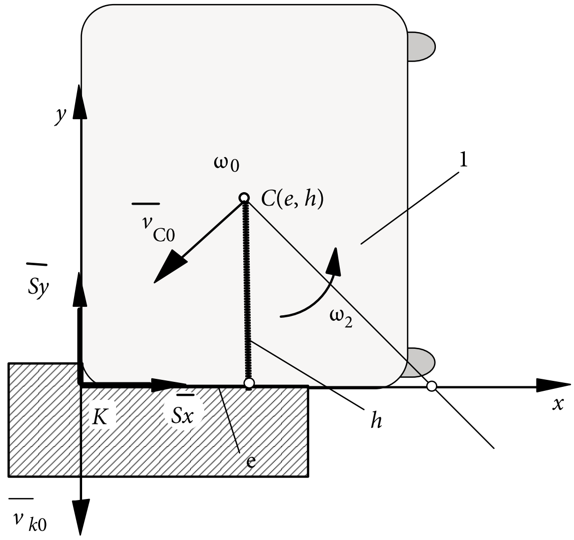


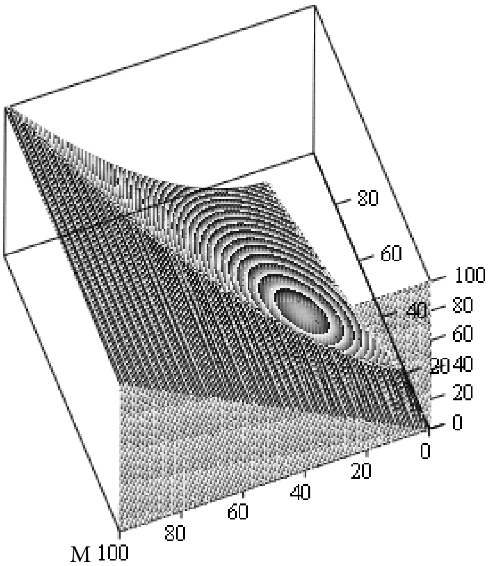
Fig. 11. The scheme of calculating rotation

$m := 1500;\ B := 1.5;\ H := 2;$

$h := 1;\ v_{x0} := -50 \cdot \frac{1}{3.6};$

$\omega_0 := \frac{-v_{x0}}{3 \cdot h};\ e := \frac{B}{2};$

$J_c := m \cdot \frac{B^2 + H^2}{12};\ v_{y0} := -\omega_0 \cdot e.$



$M_{100,99} = 98.435;$
$\max(M) = 98.435;$
$M_{100,35} = 48.086;$
$M_{49,50} = 44.531;$
$M_{50,50} = 40.870;$
$\min(M) + M_{2,1} = 21.172.$

Fig. 12. Calculation results

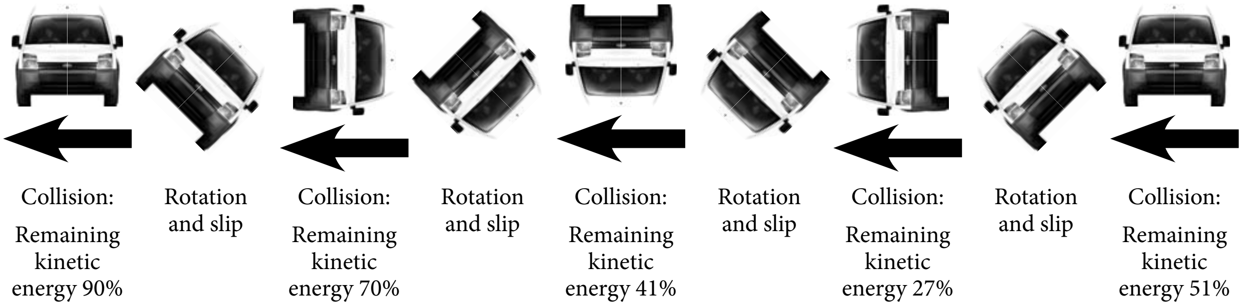


Fig. 13. Full overturn cycle with five rollover collisions (pictures from the right side to the left ←)

Remaining kinetic energy when a car stops moving after five collisions and takes a normal position on tires is:

$$0.51 \cdot 0.27 \cdot 0.41 \cdot 0.7 \cdot 0.9 = 0.04.$$

Symbolic calculation (Fig. 14) gives velocities v_1 and v_0 as follows (t – distance between tires; Δ – a maximal rise of mass center at the moment of rollover):

Given:

$$t = 1.5;$$

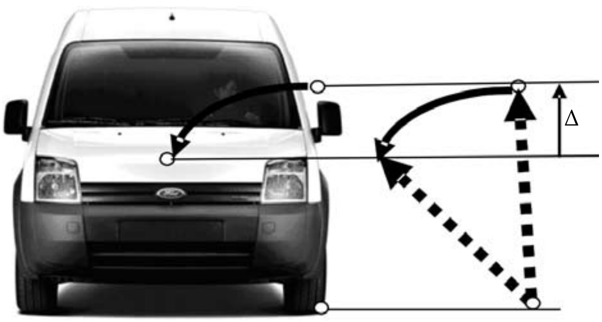
$$SSF = 1.25;$$

$$SSF = \frac{t}{2 \cdot h};$$

$$\Delta = \sqrt{h^2 + \left(\frac{t}{2}\right)^2} - h;$$

$$m \cdot 9.81 \cdot \Delta = \frac{m \cdot \left(\frac{v_1}{3.6}\right)^2}{2} \cdot 0.04;$$

$$m \cdot 9.81 \cdot \Delta = \frac{m \cdot \left(\frac{v_0}{3.6}\right)^2}{2} \cdot 0.036.$$



Find:

$$(v_0, v_1, t, h, SSF, \Delta) \rightarrow \begin{pmatrix} 50.458518280616946058 \\ 47.869153537197825341 \\ 1.5000000000000000000 \\ 0.6000000000000000000 \\ 1.2500000000000000000 \\ 0.36046863561492730297 \end{pmatrix}$$

Fig. 14. Symbolic calculation in which Δ – a maximal increase of mass center within last two rollovers

6. Conclusion

The theory of impact on a body against an obstacle in a pit corner allows calculating collisions in technological processes and control systems, the impact of the motion of elements in mechanisms having a gap, the collisions of elements in car crashes etc. More specifically, several cases of car rollover motion can be investigated using this theory (adding braking and sliding energy must be taken into account).

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