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Provide a multi-purpose fuzzy model for stock portfolio optimization

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ABSTRACT

Researchers in the field of portfolio optimization made efforts to decrease uncertainty in future returns. Any disturbance in the parameter values causes the solution to be non-optimal or impossible. This study designs a strong fuzzy-multipurpose model for stock portfolio optimization based on Tehran Stock Exchange market data. At the end of the paper, the created model is compared with the results of the multi-objective model. The results show that the fuzzy multi-objective optimization model has relative stability and model compared to the multi-purpose optimization model is strong.

1-Introduction

Choosing the optimal investment portfolio is one of the most important issues in the field of financial issues in which an attempt is made to distribute a certain amount of capital among assets in order to achieve a specific goal or goals. In the traditional portfolio selection approach, the investor estimated the yield of the various securities in order to obtain the highest expected return, and then invested in the securities that had the highest expected return. But this view was challenged in 1952 by Markowitz. According to Markowitz, in addition to maximizing the return, the investor should be as careful as possible about the safe realization of this return.

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In the financial world, uncertainty is one of the determinants. By ignoring the cause of financial market fluctuations, these uncertainties create classical approaches to mathematical modeling challenges. This issue is one of the most important problems in modelling and math decision making regarding portfolio selection. The main assumption in math programming is as input data is exactly clear and is equal to nominal value (Sajadi et al, 2010). The data with uncertainty can be in constraints or objective function. If the input data in constraints have value except their nominal value, the constraint is violated or it is not feasible and if the input data of objective function are deviated from their nominal value, the optimization is ignored or optimal solution of nominal problem is not justified (shirazi,2021). Therefore, to make the model more realistic, it can be assumed that some parameters have fuzzy uncertainty (cognitive uncertainty). The approach that has been used in recent years to deal with cognitive uncertainty is solid optimization, which deals with optimization in the worst case. A robust approach to solving optimization problems has been proposed since early 1791 and has recently been extensively studied and proposed.

As optimization issue of portfolio consists of values as stock price, profit rate, risk and etc., these values are not defined exactly and only can be predicted and these problems can be used in robust optimization technique. Various researchers have conducted some studies in this regard.

Non-robust methods consider certain values for the parameters and achieve optimal solutions. Strong methods offer a solution close to the optimal solution and show high costs (low efficiency) but the solution is very reliable. By changing the parameters in an interval, the solution values are very reliable. The first step for robust modeling was presented by Soyster as linear programming model to produce a solution justified for data belonging to ellipsoidal uncertainty sets. The mentioned model presented the solutions as conservative against optimization of nominal problem to be sure of robustness. Then, The optimization community, on the issue of robustness until the work of Ben-Tal and Nemirovski, Ghaoui and Lebret, Ghaoui et al. and Bertsimas, D. & Sim (Bertsimas, D. & Sim, 2004). In robust optimization models like Bertsimas, D. & Sim (2004), the middle value of these intervals is called nominal value. In some cases of real problem for decision maker, exact determination of interval length in which nominal value volatilizes is not easy and determines the interval length is ambiguous. If the decision maker considers the interval length as high, conservatism and costs are increased. If the interval is low, decision making risk is increased. In addition to the balance between risk and cost (return), the decision maker states the interval length not clearly. To solve this problem, an approach is stated in which the decision maker can state the length of interval as fuzzy number and balanced risk is required (Aliahmadi et al, 2015).

2- The theoretical review of robust optimization

Consider the following linear math model:

$$\begin{aligned} &Max\ cx \\ &A(p)x \le b \end{aligned} \tag{1}$$

There are different definitions of strength and required models. One of the important concepts is Constraint Robust Solutions and refers to solutions that are justified for all values of unknown parameters. The mathematical model for achieving robust constraint solutions is as follows:

$$Max\ cx \qquad (2)$$
$$A(p)x \le b \quad \forall p \in U$$

In this model, p is vector of uncertain parameters and U the set of uncertain states.

Another common concept in robust optimization literature is Objective Robust Solutions. This concept is raised when the objective function consists of uncertain parameters. The robust solutions to objective function for all feasible values of uncertain parameters are close to optimal solution. The linear optimization problem is considered as follows:

$$Max\ c(p)x \qquad (3)$$
$$A\,x \le b$$

Here, U is the uncertain set and p uncertain parameters and uncertainty parameters only exist in objective function. Objective Robust Solutions is achieved as follows:

$$Max\ (Max_{p\in U}\, c(p)x) \qquad (4)$$
$$Ax \le b$$

Robustness to objective function is special state of robustness to constraints. In other words by definition of a new variable z and by adding constraint $c(p)x \ge t$, we reach an equivalent model (1). With this approach, robust optimal solution is the robust solution with the best objective function value.

There are various approaches to solve robust problems. Bertsimas and Sim approach (budget uncertainty) maintains the linear model and this method has full control on conservatism degree of each constraint.

Let ith constraint of nominal problem (primary) is as $a_i x \le b_i$ and U_i is the set of constraint i^{th} uncertainty parameters. Based on data uncertainty model $\tilde{a}_{ij} \in U_i$, according to a symmetrical distribution with a mean equal to nominal value a_{ij} is in interval $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$.

It is assumed that in constraint ith, the defined value of Γ_i parameter is changed to the nominal value and by this assumption against constraint violation ith, control is done certainly. One guaranty is given that even if more than Γ_i parameter is changed compared to nominal value, the solution is justified probably. Bertsimas & Sim approach to present robust counterpart of a certain model is called budget uncertainty. It is impossible that all uncertain parameters a_{ij} are changed compared to nominal value. In the presented model, the aim is to be careful against all states in which maximum $|\Gamma_i|$ uncertain coefficient is changed and an a_{it} coefficient is changed as $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it}$ and this is done to justify the solution. In other words, it is assumed that nature is finite in its own behavior and only a subset of coefficients is changed to worsen the objective function. For each i, Γ_i parameter as not an integer has a value in $[0,|U_i|]$. Γ_i is required to regulate the robustness degree of model against its conservatism degree. In this method, if the nature behavior is as the mentioned form, then the robust solution is justified as 100%. In addition, if more than Γ_i parameter is changed, then the robust solution is justified probably. One of the advantages of this method is that it is generalized easily to discrete space optimization problems.

The robust counterpart is the following non-linear model:

$$
\begin{aligned}
& Max\ cx && (5)\\
s.t.\ & \sum_j a_{ij} x_j + \max_{\{s_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \{\sum_{j \in S_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} y_{t_i}\} \le b_i \quad \forall i \\
& -y_j \le x_j \le y_j \\
& l \le x \le u \\
& y_i \ge 0
\end{aligned}
$$

If Γ_i integer is selected, i^{th} constraint is protected as the followings.

$$
\beta_i(x, \Gamma_i) = \max_{\{S_i | S_i \subseteq J_i, |S_i| = \Gamma_i\}} \{\sum_{j \in S_i} \hat{a}_{ij} |x_j|\} \qquad (6)
$$

The above non-linear model is turned into a linear model with some changes. The robust counterpart as 6-2 is equal to Bertsimas and Sim approach with linear structure.

$$
\begin{aligned}
& Max\ cx && (7)\\
s.t.\ & \sum_j a_{ij} x_j + z_i + \sum_{j \in U_i} p_{ij} \le b_i \quad \forall i \\
& z_i + p_{ij} \ge \hat{a}_{ij} y_j \quad \forall i, j \in U_i \\
& -y_j \le x_j \le y_j \quad \forall j \\
& l_j \le x_j \le u_j \qquad \forall j \\
& y_j \ge 0 \quad \forall j \\
& z_i \ge 0 \quad \forall i \\
& p_{ij} \ge 0 \quad \forall i, j \in U_i
\end{aligned}
$$

By changing Γ_i we have flexibility of controlling robustness of model against solution conservatism level.

3- Goal programming model for portfolio optimization

Lee, S., Chesser (1980) introduced goal programming model for portfolio selection. Let $J = \{1.2. \ldots . n\}$ is the set of securities for investment as capital return rate for each of securities $j \in J$ is equal to random variable R_j with the mean $\mu_j = E\{R_j\}$ and . $j = \{1.2. \ldots . n\}$ is invested price in portfolio (decision variables).

Lee, S., Chesser (1980) model is as follows:

(8) $$\min W_1 d_1^+ + W_2(d_2^- + d_3^-) + W_3 \sum_{i=4}^{n+3} d_i^+ + W_4 \sum_{i=n+4}^{2n+3} d_i^- + W_5 d_{2n+4}^-$$

st

(9) $$\sum_{j=1}^{n} x_j + d_1^- - d_1^+ = BC$$

(10) $$\sum_{j=1}^{n} R_j x_j + d_2^- - d_2^+ = DR$$

(11) $$\sum_{j=1}^{n} B_j x_j + d_3^- - d_3^+ = B(BC)$$

(12) $$x_j + d_{j+3}^- - d_{j+3}^+ = V_j$$

(13) $$x_j + d_{2n+2+j}^- - d_{2n+2+j}^+ = D_j$$

(14) $$BC + \sum_{j=1}^{n} R_j x_j + d_{2n+4}^- - d_{2n+4}^+ = M$$

The term (9) considers the budget constraint. The term (10) focuses on portfolio return rate more than DR (Total portfolio income determined based on the view of investor). The term (11) considers portfolio systematic risk. If the investor predicts the market is improved in future, he should select his portfolio close to the market beta. It is assumed that future condition of market is like this and in the term (11), portfolio beta is maximized based on the opinion of investor. The objectives (12), (13) consider investment constraint in each of securities (the max and min investment) and finally the term (14) focuses on maximization of sum of budget and portfolio return.

W_1 to W_5 show the priorities to objectives (constraints) as determined according to the investor.

x_j j: Decision variable indicating the invested money in j^{th} stock.

BC: The allocated budget for investment

R_j : The expected return based on $R_j = \frac{P_{t+1} - P_t + D_t}{P_t}$

P_t: Price at t th period

D_t : Dividend

DR : Total expected income of investment

B_j : The expected beta per share of portfolio

B : The expected systematic risk

V_j: The maximum expected investment of investor in jth stock

D_j : The expected value of investment at ith stock based on beta of stock jth

M: A big value

4-The robust goal programming model for portfolio optimization

In goal programming model of Lee , S. , Chesser(1980), R_j and B_j have uncertainty. Thus, Ghahtarani, A., Najafi (2013) developed the above model and turned it into a multi-objective robust model:

(15) $\min W_1 d_1^+ + W_2(d_2^+ + d_3^+) + W_3 \sum_{i=4}^{n+3} d_i^+ + W_4 \sum_{i=n+4}^{2n+3} d_i^- + W_5 d_{2n+4}^+$

st

(16) $\sum_{j=1}^{n} x_j + d_1^- - d_1^+ = BC$

(17) $-\sum_{j=1}^{n} R_j x_j + d_2^- - d_2^+ + \max_{\{s_1 \cup \{t_1\} | s_1 \subseteq J_1, |s_1| = \Gamma_1, t_1 \in J_1, s_1\}} \times \{\sum_{j \in S_1} \widehat{R}_j y_j + (\Gamma_1 - \lfloor \Gamma_1 \rfloor) \widehat{R}_{t_1} y_t\} = -DR$

(18) $-\sum_{j=1}^{n} B_j x_j + d_3^- - d_3^+ + \max_{\{s_2 \cup \{t_2\} | s_2 \subseteq J_2, |s_2| = \Gamma_2, t_2 \in J_2, s_2\}} \{\sum_{j \in S_2} \widehat{R}_j y_j + (\Gamma_2 - \lfloor \Gamma_2 \rfloor) \widehat{B}_{t_2} y_t\} = -B(BC)$

(19) $x_j + d_{j+3}^- - d_{j+3}^+ = V_j$

(13) $x_j + d_{2n+2+j}^- - d_{2n+2+j}^+ = D_j$

(20) $-BC - \sum_{j=1}^{n} R_j x_j + d_6^- - d_6^+ + \max_{\{s_1 \cup \{t_1\} | s_1 \subseteq J_1, |s_1| = \Gamma_1, t_1 \in J_1, s_1\}} \{\sum_{j \in S_1} \widehat{R}_j y_j + (\Gamma_1 - \lfloor \Gamma_1 \rfloor) \widehat{R}_{t_1} y_t\} = -M$

(21) $-y_j \le x_j \le y_j$

(22) $y_j \ge 0$

5-Fuzzy robust multi-objective programming model

Ghahtarani, A., Najafi (2013) were not informed of the form of distribution of uncertain parameters and considered these parameters as stochastic value fluctuating in a symmetric interval. In their model, the middle value of interval is called as nominal value. In real problems for decision maker, exact determination of interval in which nominal value is fluctuating is not easy and the determination of interval length is not clear. If the decision maker considers the interval length high, conservatism level is increased and high cost is imposed. If the interval length is low, decision making risk taking is increased. In addition to balance between risk taking and cost, in some cases, the decision maker states the interval length not clearly. To eliminate this problem, an approach is presented in which the decision maker can state the interval length as fuzzy numerical and have balanced risk taking.

In linear programing, if the right side coefficients are fuzzy, the model is as follows:

$$\max \sum_{j=1}^{n} C_j x_J \qquad 23$$

s.t.

$$\sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i \quad i = 1, \dots, m \qquad 24$$

$$x_j \ge 0 \qquad 25$$

Generally, fuzzy linear programming problem should be turned into certain equivalent problems and be solved by common methods and the results of these equivalent certain problems are achieved by considering fuzzy conditions in turning fuzzy problems to crisp in crisp model. Let the right side fuzzy values are as follows:

$$M_{\tilde{b}_i} = \begin{cases} 1 & 1 \le b_i \\ \frac{b_i + p_i + y}{p_i} & b_i \le y \le b_i + p_i \\ 0 & y \ge b_i + p_i \end{cases} \qquad 26$$

For each solution as a vector $X = (x_1, x_2, \dots, x_n)$, at first membership degree $D_i(x)$ denoting the membership degree of constraint i^{th} by X vector is defined as follows:

$$D_i(x) = \mu_{\tilde{b}_i}\left(\sum_{j=1}^{n} a_{ij} x_j\right) \qquad 27$$

Each$D_i(x)$ of a set makes a fuzzy set and their commonality $(\bigcap_{i=1}^{n} D_i)$ makes the justified region. As the solution space is fuzzy, the objective function is also fuzzy. Thus, fuzzy objective function should be computed. The upper and lower limit of objective function, Z_L , Z_U are computed as follows:

$$\max Z_L = CX \qquad 28$$

s.t.

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad i = 1, \dots, m \qquad 29$$

$$x_j \ge 0 \qquad 30$$

,

$$\max Z_u = CX \qquad 31$$

s.t.

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i + P_i \quad i = 1, \dots, m \tag{32}$$

$$x_j \geq 0 \tag{33}$$

Membership function of objective function is defined as follows:

$$G(x) = \begin{cases} 0 & Cx \leq Z_L \\ \frac{Cx - Z_L}{Z_U - Z_L} & Z_L \leq Cx \leq Z_U \\ 1 & Cx \geq Z_U \end{cases} \tag{34}$$

The solution of maximum commonality of objective function and region is justified as achieved:

$$\max \min[\cap_{i=1}^{x} D_i(x), G(x)] \tag{35}$$

The variable is changed as:

$$\max \min[\cap_{i=1}^{m} D_i(x), G(x)] \tag{36}$$

Thus:

$$\max \lambda \tag{37}$$

$$\text{s.t.} \tag{38}$$

$$\lambda \leq G(x) \tag{39}$$

$$\lambda \leq D_i(x) \quad \forall i \tag{40}$$

$$\lambda, x \geq 0 \tag{41}$$

Instead of G(x), we can use $\frac{Cx - Z_L}{Z_U - Z_L}$ and instead of $D_i(x)$, we use $\frac{b_i + p_i - \sum a_{ij}x_j}{p_i}$. Thus, we have:

$$\max \lambda \tag{42}$$

$$\text{s.t.} \tag{43}$$

$$\lambda \leq \frac{Cx - Z_L}{Z_U - Z_L} \tag{44}$$

$$\lambda \leq \frac{b_i + p_i - \sum a_{ij}x_j}{p_i} \quad \forall i \tag{45}$$

$$\lambda, x_j \geq 0 \quad \forall j \tag{46}$$

In the above model, rewriting is as follows (Lai , K. K. & Wang, 2006).

$$\max \lambda \quad (47)$$

s.t.

$$\lambda(Z_U - Z_L) - Cx \leq -Z_L \quad (48)$$

$$\lambda p_i + \sum a_{ij} x_j \leq b_i + p_i \qquad \forall i \quad (49)$$

$$\lambda, x_j \geq 0 \quad , \quad \forall j \quad (50)$$

5-1 Fuzzy-robust model of portfolio by goal programming

In the problem space of this model, risk parameters and return are not reliable from the view of decision maker. As the exact distribution of these data is not defined, the data fluctuation is considered in the form of a symmetrical interval. The important point regarding intervals is determining the length of interval as the decision maker is unsure of their exact value. In this thesis, this issue is occurred for the length of risk parameters and return on asset. If we denote the interval length of these parameters with $\hat{R}_j$, $\hat{B}_j$ and based on the previous explanations, this term is consider as triangular fuzzy value and is denoted by $\breve{\hat{R}}_j$, $\breve{\hat{B}}_j$. Thus, fuzzy-robust counterpart is written as follows:

$$\min W_1 d_1^+ + W_2(d_2^+ + d_3^+) + W_3 \sum_{i=4}^{n+3} d_i^+ + W_4 \sum_{i=n+4}^{2n+3} d_i^- + W_5 d_{2n+4}^+ \quad (51)$$

s.t

$$\sum_{j=1}^{n} x_j + d_1^- - d_1^+ = BC \quad (52)$$

$$-\sum_{j=1}^{n} R_j x_j + d_2^- - d_2^+ + Z_1\Gamma_1 + \sum_{j=1}^{n} P_{1j} = -DR \quad (53)$$

$$-\sum_{j=1}^{n} B_j x_j + d_3^- - d_3^+ + Z_2\Gamma_2 + \sum_{j=1}^{n} P_{2j} = -B(BC) \quad (54)$$

$$x_j + d_4^- - d_4^+ = V_j \quad (55)$$

$$x_j + d_5^- - d_5^+ = D_j \quad (56)$$

$$-BC - \sum_{j=1}^{n} R_j x_j + d_6^- - d_6^+ + Z_1\Gamma_1 + \sum_{j=1}^{n} P_{1j} = -M \quad (57)$$

$$Z_1 + P_{1j} \geq \breve{\hat{R}}_j y_j \quad (58)$$

$$Z_2 + P_{2j} \geq \breve{\hat{B}}_j y_j \quad (59)$$

$$-y_j \leq x_j \leq y_j \quad (60)$$

$$0 \leq P_j \, , \quad 0 \leq y_i , \quad 0 \leq Z_1 \, , \quad 0 \leq Z_2 \quad (61)$$

The mentioned model is a linear programing model with fuzzy sources (asymmetrical model) and can be turned into symmetrical model. Thus, deterministic fuzzy-robust counterpart model is as follows:

$$\text{Max } Z = \lambda \quad (62)$$

s.t

$$W_1 d_1^+ + W_2(d_2^+ + d_3^+) + W_3 \sum_{i=4}^{N+4} d_i^+ + W_4 \sum_{i=n+5}^{2n+5} d_i^- + W_5 d_{46}^+ + \lambda Z_L \leq Z_U \quad (63)$$

$$\sum_{j=1}^{n} x_j + d_1^- - d_1^+ = BC \quad (64)$$

$$-\sum_{j=1}^{n} R_j x_j + d_2^- - d_2^+ + Z_1 \Gamma_1 + \sum_{j=1}^{n} P_{1j} = -DR \quad (65)$$

$$-\sum_{j=1}^{n} B_j x_j + d_3^- - d_3^+ + Z_2 \Gamma_2 + \sum_{j=1}^{n} P_{2j} = -B(BC) \quad (66)$$

$$x_j + d_4^- - d_4^+ = V_j \quad (67)$$

$$x_j + d_5^- - d_5^+ = D_j \quad (68)$$

$$-BC - \sum_{j=1}^{n} R_j x_j + d_6^- - d_6^+ + Z_1 \Gamma_1 + \sum_{j=1}^{n} P_{1j} = -M \quad (69)$$

$$Z_1 + P_{1j} + \lambda\left(\overline{R}_{(\max i)} - \overline{R}_{(\min i)}\right) y_j \geq \overline{R}_{(\text{mini})} y_j \quad (70)$$

$$Z_2 + P_{2j} + \lambda\left(\overline{B}_{(\max i)} - \overline{B}_{(\min i)}\right) y_j \geq \overline{B}_{(\text{mini})} y_j \quad (71)$$

$$-y_j \leq x_j \leq y_j \quad (72)$$

$$0 \leq P_j, \quad 0 \leq y_i, \quad 0 \leq Z_1, \quad 0 \leq Z_2, \quad 0 \leq \lambda \leq 1 \quad (73)$$

In this model, λ is the satisfaction degree of constraints.

6-The results of solution of goal programing robust model

In this section the results of robust multi-objective model (Ghahtarani, A., Najafi, 2013) and proposed model. These models are investigated based on data of 20 stocks of TSE.

In this model, based on the opinion of investor, the weights of goals and other variables are determined as follows:

Budget value (BC) in the problem is equal to 1 currency, expected income of investment (DR) as 0.25 currency and Beta risk (B) is 0.9 and high limit of investment in each share is 0.2 currency and lower limit in each share based on its systematic risk is zero.

Table 1-The outputs of multi-objective robust model of portfolio selection and portfolio return against robustness cost

$\Gamma_i(\Gamma_1,\Gamma_2)$	(0,0)	(1,1)	(2,2)	(3,3)	(4,4)	(5,5)	(6,6)	(7,7)	(8,8)	(9,9)	(10,10)	(15,15)	(20,20)
x_1	0.091037	0	0.065924	0.065924	0	0	0	0	0	0	0	0	0
x_2	0	0	0	0	0.111791	0.1117911	0	0	0	0.0897043	0.0897043	0	0
x_3	0	0	0	0	0.0798300	0.0798300	0	0	0	0.0640579	0.0640579	0	0.06979567
x_4	0	0	0	0	0.1279122	0.1279122	0	0	0	0.1026403	0.1026403	0	0.04829599
x_5	0.161068	0.1938868	0.116636	0.116636	0	0	0	0	0	0	0	0	0
x_6	0	0	0	0	0	0	0	0	0	0	0	0	0
x_7	0	0	0	0	0.1430154	0.1430154	0	0	0	0.1147595	0	0.06069747	0
x_8	0	0	0	0	0	0	0.1510569	0	0	0.0925185	0.0925185	0	0
x_9	0	0	0	0	0.0493656	0.0493656	0.0493656	0.0493656	0.0493656	0.0396123	0.0396123	0.0493656	0
x_{10}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1604856	0.1604856	0.2	0.2
x_{11}	0	0	0	0	0	0	0.0932885	0.2	0.2	0	0	0.0932885	0.2
x_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0
x_{13}	0	0	0	0	0	0	0	0.1438294	0.1438294	0	0	0.2	0.100082
x_{14}	0	0	0	0	0.0628439	0.0628439	0.0962151	0.0962151	0.0962151	0.0772057	0.0772057	0.0962151	0.08412118
x_{15}	0	0	0.104341	0.104341	0	0	0.1032754	0.1032754	0.1032574	0	0	0	0.0902941
x_{16}	0	0	0	0	0.0916843	0	0.0916843	0.0916843	0.0091683	0.07357	0.07357	0.0916842	0.0801599
x_{17}	0	0	0	0	0.0819464	0.0819464	0.0819464	0.0819464	0.0819464	0.065756	0.065756	0.0819464	0.0716460

x_{18}	0.2	0.2	0.2	0.2	0	0	0	0	0	0	0	0	0.00901
x_{19}	0.1297113	0.1378822	0.1506988	0.1506988	0.0516113	0.051612	0.079874	0.033684	0.033684	0.119690	0.119690	0.0735086	0
x_{20}	0	0.049265	0	0	0	0	0.0532942	0	0	0	0	0.0532942	0.0465953
Budget (Million Rial)	0.7818163	0.781034	0.8376	0.8376	1.0000001	0.9083166	1.0000004	1.0000002	0.9174662	1.0000001	0.8852406	1.0000001	1.0000001
Absolute value of budget deviation	0.2181837	0.218966	0.1624	0.1624	0.0000001	0.0916834	0.0000004	0.0000002	0.025338	.00000001	0.1147594	.00000001	0.0000001
Real return	-0.028526	-0.033615	-0.018355	-0.018355	1.1755345	1.1773699	-0.032233	-0.041444	-0.039793	0.9433888	-0.035038	0.45802371	-0.0522391

Table 2- The output of multi-objective fuzzy-robust optimization model of portfolio and portfolio return against robustness costs

$\Gamma_i(\Gamma_1, \Gamma_2)$	(0,0)	(1,1)	(2,2)	(3,3)	(4,4)	(5,5)	(6,6)	(7,7)	(8,8)	(9,9)	(10,10)	(15,15)	(20,20)
X1	0.0402248	0	0	0	0	0.0012016	.000995	.00084859	0.000731	0.000656	0.0005892	0	0.0040633
X2	0	0	0	0	0	0	0	0	0	0	0	0	0.064296
X3	0.0492123	0.0522102	0.0522102	0.0522102	0.0462087	0.0463670	0.046379	0.0463875	0.0463938	0.0463987	0.0464026	0.052007	0.0459141
X4	0.0788524	0.0836559	0.0836559	0.0836559	0.0740397	0.0742934	0.0743126	0.0743262	0.0743363	0.0743441	0.0743503	0.0833304	0.0735677
X5	0.1292982	0.1371748	0.1371748	0.1371748	0.06682351	0.0670525	0.0670698	0.0670821	0.0670912	0.0670983	0.0671039	0.0752088	0.1206325
X6	0.0438787	0.0465517	0.0465517	0.0465517	0.0412006	0.0413418	0.0413525	0.041361	0.0413657	0.0413701	0.0413735	0.0463705	0.040938
X7	0	0	0	0	0.0827817	0.0830654	0.083087	0.0831021	0.0831134	0.0831221	0.0831291	0.09317	0
X8	0	0	0	0	0	0	0	0	0	0	0	0	0
X9	0	0	0	0	0.0285752	0.0286731	0.0286805	0.0286858	0.0286897	0.0286927	0.0286951	0	0

X10	0.1232908	0.1308013	0.1308013	0.1308013	0.1157656	0.1161624	0.1161925	0.1162137	0.11623	0.1162417	0.1162514	0.1302924	0.115028
X11	0.0575088	0.061012	0.061012	0.061012	0.0539988	0.0541838	0.0541978	0.0542077	0.0542151	0.0542208	0.0542253	0.6077465	0.0536545
X12	0	0	0	0	0	0	0	0	0	0	0	0	0.0061008
X13	0	0	0	0	0.0662594	0.0664864	0.0665036	0.0665158	0.0665249	0.0665318	0.0665374	0.0745738	0
X14	0	0	0	0	0.0556927	0.0558835	0.0558980	0.0559082	0.0559159	0.0559217	0.0559264	0	0
X15	0	0	0	0	0	0	0	0	0	0	0	0	0.0593983
X16	0	0	0	0	0	0	0	0	0	0	0	0	0.0527319
X17	0	0	0	0	0.0474337	0.0475962	0.0476085	0.0476172	0.0476237	0.0476287	0.0476327	0	0.0471313
X18	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039	0.2001039
X19	0.0919505	0.0975519	0.0975519	0.0975519	0.0863383	0.0866342	0.0866566	0.0866724	0.0866842	0.080067	0.0867006	0.0971724	0.0857879
X20	0.0328544	0.0348557	0.0348557	0.0348558	0.0308492	0.0309549	0.0309629	0.0309686	0.0309728	0.030976	0.0309786	0.0347201	0.0306525
Budget	0.8471748	0.8439174	0.8439174	0.8439176	0.996071	1.0000001	1.0000002	1.0000008	0.9999916	0.9933736	1	1.4946958	1.0000007
Budget deviation	0.1528252	0.1560826	0.1560826	0.1560824	0.003929	0.0000001	0.0000002	0.0000008	0.0000084	0.0066264	0	0.4946958	0.0000007
Return	-0.033806	-0.032633	-0.032633	-.0326325	0.6735647	0.6757137	0.6759057	0.6760399	0.6761412	0.6760547	0.6762799	0.7547609	-.03364277

As shown in Figures 1 2, by increasing conservatism, portfolio return is reduced. As some different objectives are satisfied simultaneously, in some levels of conservatism, there are some distributions. Fuzzy-robust multi-objective programming model compared to multi-objective robust model has stable condition in terms of deviation from objective and disturbance in reduction of return and this is one of the advantages of the mentioned model.

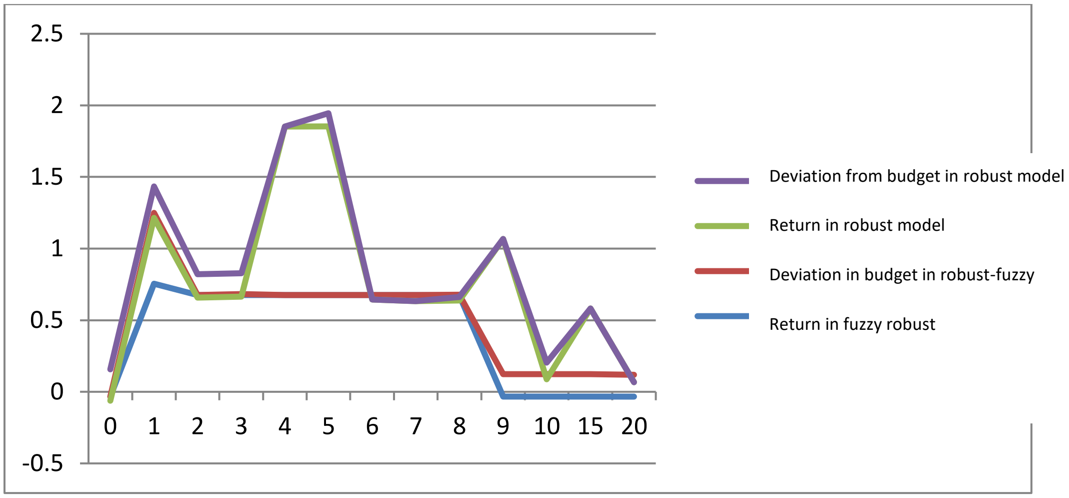

Figure 1- The relationship between deviation from budget in robust multi-objective models and robust-fuzzy against return to robustness value

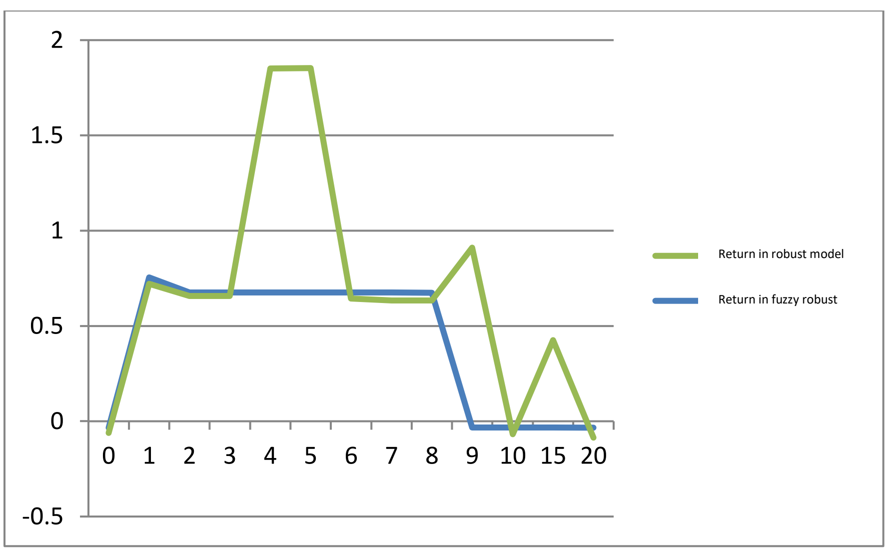

Figure 2- The relationship between return in robust multi-objective models and fuzzy-robust compared to robust value

7- Conclusion

This paper is used to optimize portfolio and explains Lee, S., Chesser model and then to consider uncertainty of data of return and risk, multi-objective robust model of Ghahtarani, A., Najafi is defined. Then, to remove the problems in Ghahtarani, A., Najafi model, their model is developed as fuzzy robust and the results are compared after implementation in TSE. The results showed high stability of fuzzy robust model in various levels of conservatism compared to robust model.

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