



Biocontrol of the emerald ash borer: an adapted Nicholson-Bailey model

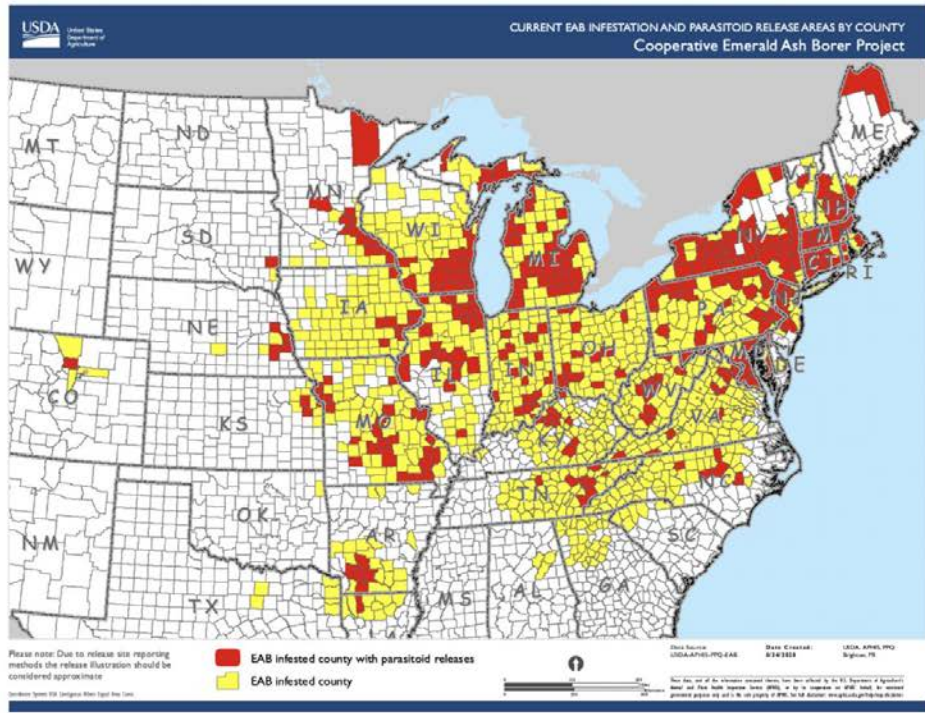
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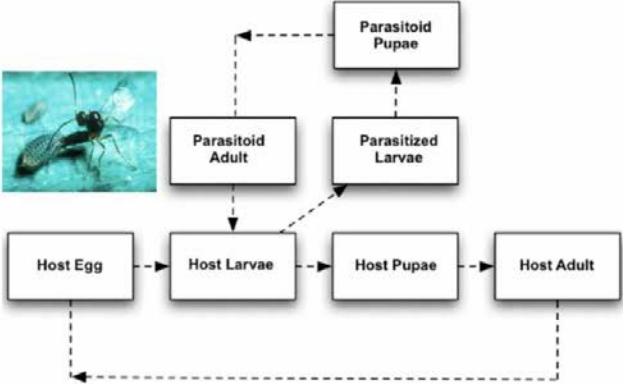
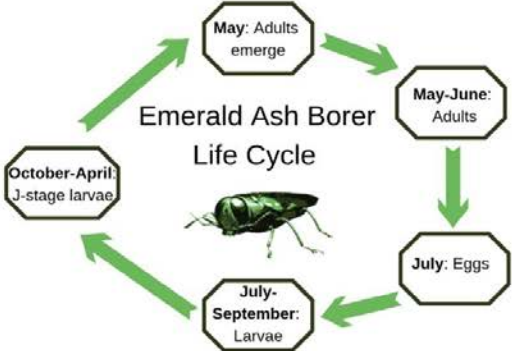
Outline of Talk

- *Agrilus planipennis* (EAB) and the larval parasitoid *Tetrastichus planipennisi* (TP)
- Nicholson-Bailey model
- Partial Refuge Model
 - model parameters
 - equilibria and stability
 - escape probabilities as coordinates
- Biocontrol Questions
- Future Work

Biocontrol Map (2020)

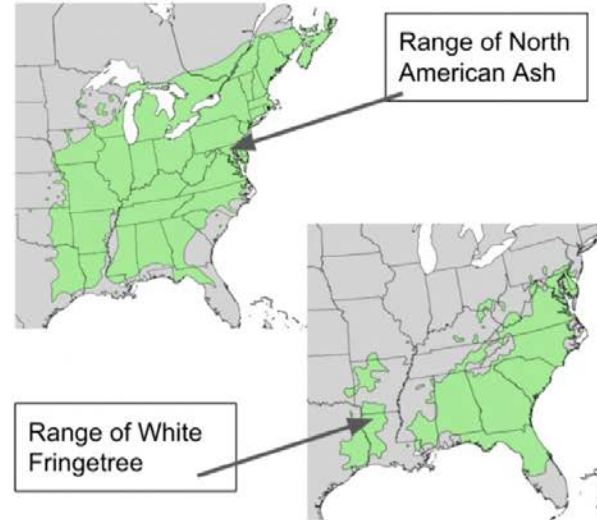


Host and Parasitoid



Ash and Fringetree

- Ash and White fringetree ranges overlap
- Fringetree is suboptimal for EAB
- Fringetree is also suboptimal for TP



Nicholson-Bailey Model (1935)

$$H_{t+1} = R H_t e^{-a P_t}$$

$$P_{t+1} = k R H_t (1 - e^{-a P_t})$$

$e^{-a P_t}$ = escape probability

$1 - e^{-a P_t}$ = parasitism rate

H_t = host density at time t

P_t = parasitoid density

R = host growth rate

a = parasitoid attack rate

k = parasitoids per parasitized host

Unique unstable equilibrium

N-B Suboptimal Refuge

Assume fraction μ of the host population utilizes a suboptimal refuge with refuge growth rate αR . In the absence of parasitoids, the expected growth rate is

$$\gamma R = (\alpha \mu + (1 - \mu)) R$$

Viability assumption : $\gamma R > 1$

Write

$$f(P_t) = \alpha \mu + (1 - \mu) e^{-a P_t}$$

$$H_{t+1} = R H_t f(P_t)$$

$$P_{t+1} = k R H_t (\gamma - f(P_t))$$

Equilibrium and Stability

$$H_{t+1} = R H_t f(P_t)$$

$$P_{t+1} = k R H_t (\gamma - f(P_t))$$

$$f(P_t) = \alpha \mu + (1 - \mu) e^{-a P_t}$$

Equilibrium Conditions

$$1 = R f(P^*)$$

$$P^* = k H^* (\gamma R - 1)$$

Stability: Singh & Emerick (2020)

$$1 + k \gamma R^2 H^* f_p^* > 0$$

$$\text{where } f_p^* = \partial_{P_t} f \Big|_{P_t=P^*}$$

Partial Refuge Model

Refuge is also suboptimal for the parasitoid.

Attack rate: βa , $0 < \beta < 1$.

$$H_{t+1} = R H_t f(P_t)$$

$$P_{t+1} = k R H_t (\gamma - f(P_t))$$

$$f = \alpha \mu e^{-\gamma \beta a P_t} + (1 - \mu) e^{-(1-\gamma) a P_t}$$

$R, kR; a$ ex-refuge growth; attack rates
 α, β discount factors (suboptimal)
 μ, γ % host, parasitoid using refuge

Refuge assumption : $\gamma \beta < 1 - \gamma$

Partial Refuge: Equilibrium and Stability

$$f = \alpha \mu e^{-\nu \beta a P_t} + (1 - \mu) e^{-(1-\nu) a P_t}$$

$$\text{Set } X^* = a P^* \text{ and } \lambda = \alpha e^{(1-(1+\beta)\nu^*) X^*}$$

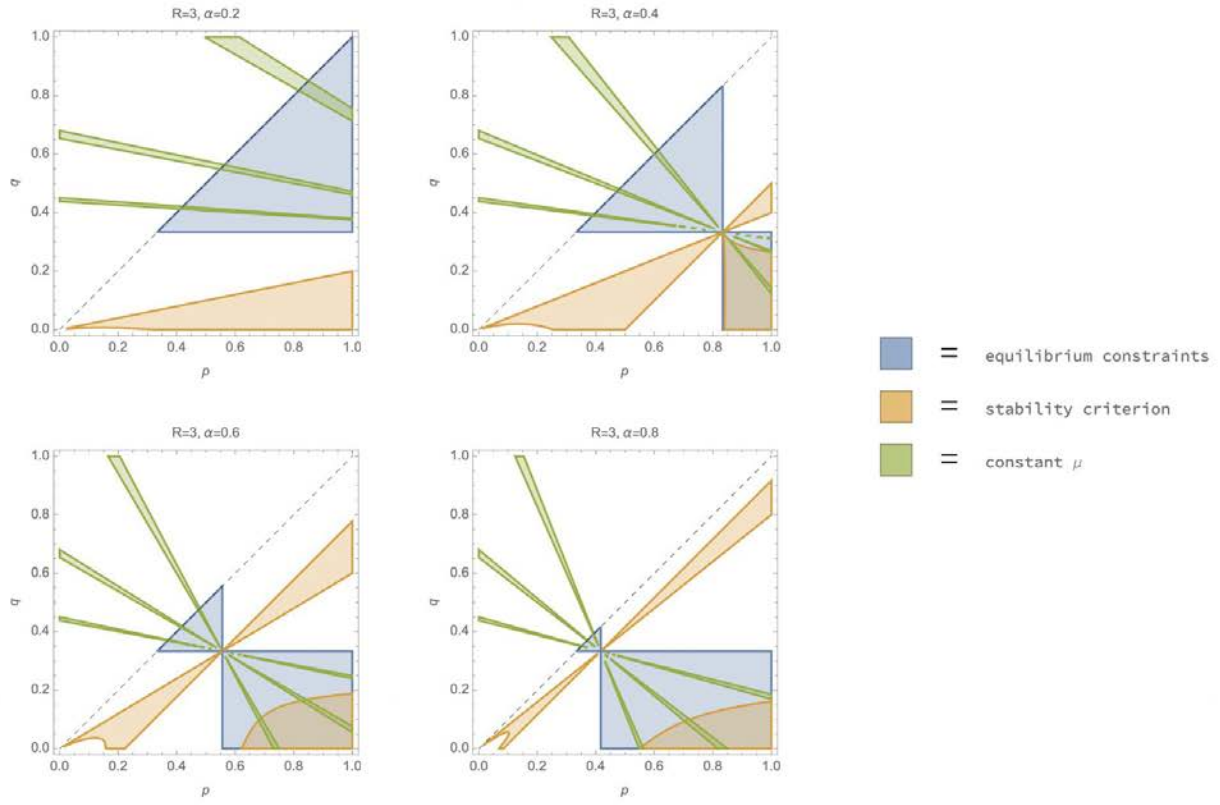
Equilibrium: an α, μ -weighted average of the escape probabilities

$$\frac{\alpha \mu^*}{\alpha \mu^* + (1 - \mu^*)} e^{-\nu^* \beta X^*} + \frac{1 - \mu^*}{\alpha \mu^* + (1 - \mu^*)} e^{-(1-\nu^*) X^*} = \frac{1}{\gamma R}$$

Stability: a λ, μ -weighted average of their logarithms.

$$\frac{\lambda \mu^*}{\lambda \mu^* + (1 - \mu^*)} \ln (e^{-\nu^* \beta X^*}) + \frac{1 - \mu^*}{\lambda \mu^* + (1 - \mu^*)} \ln (e^{-\nu^* \beta X^*}) > \frac{1}{\gamma R} - 1$$

Using the escape probabilities p and q as primary variables clarifies the picture.



Formulas in terms of p, q, β

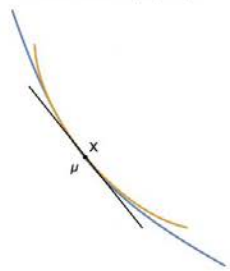
$$\mu^* = \frac{1 - q R}{R (\alpha p - q)} \qquad \nu^* = \frac{\ln (p)}{\ln (p) + \beta \ln (q)}$$

$$X^* = -\frac{1}{\beta} (\ln (p) + \beta \ln (q)) \qquad H^* = \frac{X^*}{a k (\gamma R - 1)}$$

In particular

$$\nabla H^* = \frac{1}{a k (\gamma R - 1)} \left(\nabla X^* - \frac{(1 - \alpha) R}{\gamma R - 1} \nabla \mu^* \right)$$

Level curves of μ^*, H^*, X^*

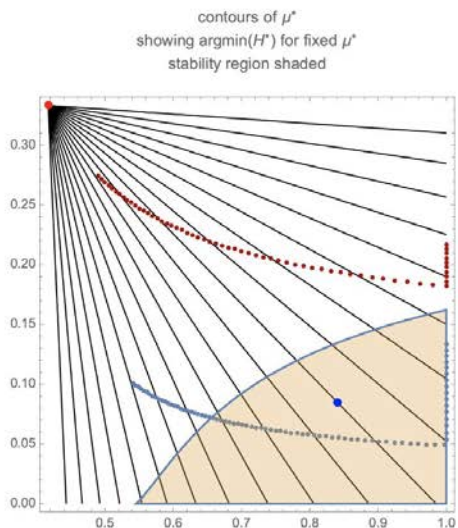


Biocontrol Questions

1. Suppose μ^* is proportional to $\varphi = \frac{\text{fringetree density}}{\text{ash+fringe density}}$. What is the minimum H^* that can be achieved for given β ?
2. For a fixed parasitoid density X^* (again for a given β), what is the worst case maximum density H^* ?
3. How do the answers change as β , i.e. the in-refuge attack rate, increases?

Biocontrol Answers μ^* , H^* , X^* , β

min H^* for given μ and max H^* for given X^* .



• $\beta = 0.7$

• $\beta = 0.1$

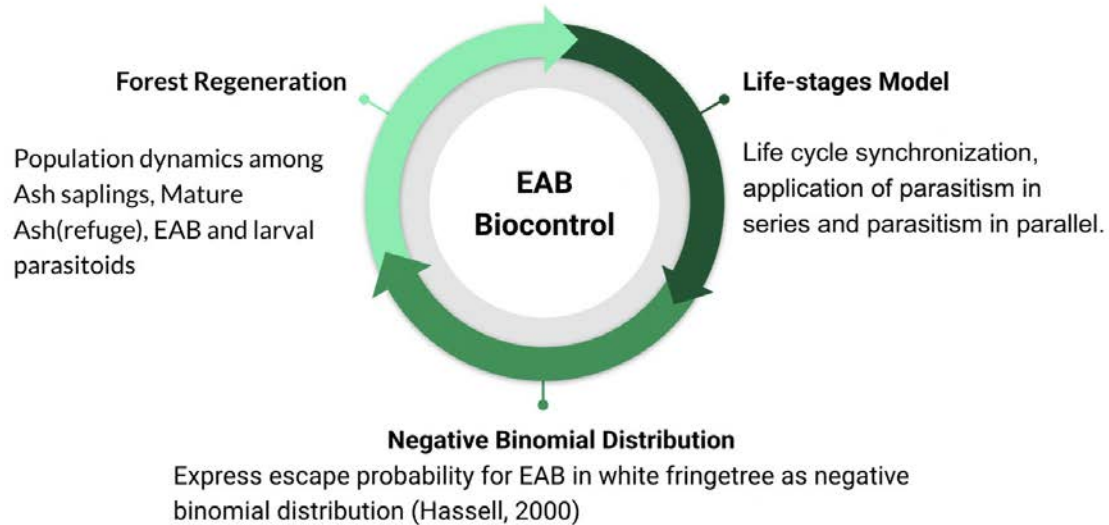
Data at blue point $(p, q) = (.84, .085)$

• $H^* = 1.74$, $X^* = 3.05$, $\mu^* = 0.423$, $v^* = 0.191$

Conclusions

- Suboptimal refuge can stabilize N-B equilibria
 - reasonable in-refuge host growth rate
 - low ex-refuge escape probability
 - high in-refuge escape probability
- Model provides insight for biocontrol questions
 - best- and worst-case scenarios
 - parasitoid response to host range expansion
 - effect of increased efficiency of parasitoids

Future Work



References:

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Thank You!