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No 784

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

THE UNIVERSITY OF WARWICK

Testing for seasonal unit roots in heterogeneous panels in the presence of cross section dependence*

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January 2007

Abstract

This paper presents two alternative methods for modifying the HEGY-IPS test in the presence of cross-sectional dependency. In general, the bootstrap method (BHEGY-IPS) has greater power than the method suggested by Pesaran (2007) (CHEGY-IPS), although for large T and high degree of cross-sectional dependency the CHEGY-IPS test dominates the BHEGY-IPS test.

JEL Classification: C12; C15; C22; C23.

Keywords: Heterogeneous dynamic panels, Monte Carlo, seasonal unit roots, cross sectional dependence.

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*This paper was started while Jesús Otero was a Visiting Fellow in the Department of Economics at the University of Warwick. He would like to thank the Universidad del Rosario for its financial assistance.

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1 Introduction

In a recent paper, Otero *et al.* (2005) use the approach of Im, Pesaran and Shin (2003) (IPS) to propose seasonal unit root tests for dynamic heterogeneous panels. The test statistics are based on standardised $t-bar$ and $F-bar$ statistics, which are simply averages of the Hylleberg *et al.* (1990) (HEGY) tests across groups and are referred to as the HEGY-IPS tests. Based on Monte Carlo simulations, these standardised statistics are found to follow a standard normal distribution even for a relatively small number of data points. An important assumption underlying the HEGY-IPS tests is that of cross section independence among the individual time series in the panel.[1] However, these HEGY-IPS tests suffer from size distortions in the presence of cross section dependence. This paper considers two alternative procedures to correct for these distortions: the first uses a generalisation of the cross sectionally augmented IPS (CIPS) test put forward by Pesaran (2007), and the second applies a bootstrap methodology.

The plan of the paper is as follows. Section 2 briefly reviews the HEGY-IPS approach to seasonal unit root testing in panels and its generalisation in the presence of cross section dependence. Section 3 presents the main results.

2 HEGY-IPS panel seasonal unit root test

Generalising the HEGY test for seasonal unit roots to a panel in which there is a sample of $i = 1, \ldots, N$ cross sections observed over $t = 1, \ldots, T$ time periods, we have:

$$y_{4it} = \mu_{it} + \pi_{1i}y_{1it-1} + \pi_{2i}y_{2it-1} + \pi_{3i}y_{3it-2} + \pi_{4i}y_{3it-1} + \sum_{j=1}^{p_i} \varphi_{ij}y_{4i,t-j} + \varepsilon_{it}, \qquad (1)$$

where $\mu_{it} = \alpha_i + \beta_i t + \sum_{s=1}^{3} \gamma_{is}D_{st}$, D_{st} is a seasonal dummy variable that takes the value of 1 in quarter s (and zero otherwise) and $\varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon_i})$. Also, $y_{1it} = y_{it} + y_{it-1} + y_{it-2} + y_{it-3}$, $y_{2it} = -y_{it} + y_{it-1} - y_{it-2} + y_{it-3}$, $y_{3it} = -y_{it} + y_{it-2}$ and $y_{4it} = \Delta_4 y_{it} = y_{it} - y_{it-4}$.

In a univariate context, HEGY test for the existence of a unit root by testing $H_0 : \pi_1 = 0$ against $H_1 : \pi_1 < 0$, and for the existence of a seasonal unit root by testing $H_0 : \pi_2 = 0$ against $H_1 : \pi_2 < 0$ and simultaneously testing $H_0 : \pi_3 = \pi_4 = 0$ against $H_1 : \pi_3 < 0, \pi_4 \neq 0$. A null hypothesis of a seasonal unit root is only rejected when both the $t-$test for π_2 and the joint $F-$test for π_3 and π_4 are rejected. Subsequently, Ghysels *et al.* (1994) suggest using a test of $H_0 : \pi_2 = \pi_3 = \pi_4 = 0$ against $H_1 : \pi_2 < 0, \pi_3 < 0, \pi_4 \neq 0$.

In a panel context, the null hypothesis to test for the presence of a unit root, for example, becomes $H_0 : \pi_{1i} = 0 \quad \forall i$ against $H_1 : \pi_{1i} < 0$ for $i = 1, 2 \ldots, N_1$, $\pi_{1i} = 0$, for $i = N_1 + 1, N_1 + 2, \ldots, N$.

[1] In an independent piece of research, Dreger and Reimers (2005) generalise the HEGY test to cover a heterogeneous panel of data, assuming cross sectional independence.

This allows some, but not all, of the individual series to have a unit root, but assumes that a non-zero fraction of the processes are stationary.

The HEGY statistics from estimating equation (1) for the i^{th} group are given by the $t-$ratios on π_{ji}, $j = 1, 2$ and the $F-$tests of the joint significance of π_{3i}, π_{4i} and $\pi_{2i}, \pi_{3i}, \pi_{4i}$. Denote the estimated $t-$ratio as $\tilde{t}_{jiT}$ $(j = 1, 2)$ and the $F-$test as $\tilde{F}_{jiT}$ $(j = 2, 3)$. For a fixed T define the average statistics:

$$\tilde{t}_j bar_{NT} = \frac{1}{N}\sum_{i=1}^{N}\tilde{t}_{jiT} \;\; j = 1, 2,$$

and

$$\tilde{F}_j bar_{NT} = \frac{1}{N}\sum_{i=1}^{N}\tilde{F}_{jiT}, \;\; j = 2, 3.$$

After a suitable standardisation, using mean and variance obtained by Monte Carlo simulation (as tabulated in Otero *et al.* (2005)), the resulting standardised statistics, denoted $W_{\tilde{t}_1 bar}$, $W_{\tilde{t}_2 bar}$, $W_{\tilde{F}_2 bar}$ and $W_{\tilde{F}_3 bar}$, have a standard normal distribution. However, these tests suffer from increasing severe size distortions as the degree of cross-sectional dependency, $E(\varepsilon_{it}\varepsilon_{jt}) = \omega$ for $i \neq j$ in equation (1), increases (these results are not reported to save space).

A number of procedures have been suggested to allow for cross-sectional dependence in panel unit root tests that focus on the zero or long run frequency. In this paper we consider two such approaches. First, we follow Pesaran (2007), who augments the standard ADF regressions with the cross section averages of lagged levels and first-differences of the individual series in the panel. The corresponding cross-sectionally augmented HEGY regression is given by:

$$\begin{aligned} y_{4it} &= \mu_{it} + \pi_{1i}y_{1it-1} + \pi_{2i}y_{2it-1} + \pi_{3i}y_{3it-2} + \pi_{4i}y_{3it-1} \\ &\quad + c_i\bar{y}_{1t-1} + d_i\bar{y}_{2t-1} + e_i\bar{y}_{3t-2} + f_i\bar{y}_{3t-1} \\ &\quad + \sum_{j=0}^{p_i}\delta_{ij}\bar{y}_{4t-j} + \sum_{j=1}^{p_i}\varphi_{ij}y_{4i,t-j} + \varepsilon_{it}, \end{aligned} \tag{2}$$

where $\bar{y}_{1t}$ is the cross section mean of y_{1it}, defined as $\bar{y}_{1t} = (N)^{-1}\sum_{i=1}^{N} y_{1it}$, and similarly for $\bar{y}_{2t}$, $\bar{y}_{3t}$ and $\bar{y}_{4t}$. The cross-sectionally augmented versions of the HEGY-IPS tests, denoted as the CHEGY-IPS, are then:

$$\text{CHEGY-IPS}_{t_j} = N^{-1}\sum_{i=1}^{N} t_{\pi_{ji}}, \qquad j = 1, 2,$$

where $t_{\pi_{ji}}$ denotes the $t-$ratio on π_{ji} in equation (2), for $j = 1, 2$, and

$$\text{CHEGY-IPS}_{F_j} = N^{-1}\sum_{i=1}^{N} F_{\pi_{ji}}, \qquad j = 2, 3,$$

where $F_{\pi_{ji}}$ denotes the $F-$test of the joint significance of π_{3i}, π_{4i} and $\pi_{2i}, \pi_{3i}, \pi_{4i}$, also in equation (2), for $j = 2, 3$ respectively.

Critical values of the CHEGY-IPS$_{t_j}$ and CHEGY-IPS$_{F_j}$ are reported in Table 1, for different combinations of deterministic components, based on a Monte Carlo simulation (with 20,000 replications) of the model:

$$\Delta_4 y_{it} = y_{it} - y_{it-4} = \mu_{it} + \rho y_{it-4} + \sum_{j=1}^{p_i} \varphi_{ji} \Delta_4 y_{i,t-j} + \varepsilon_{it}, \tag{3}$$

where $\varepsilon_{it} \sim N(0,1)$, $p_i = 0$, $N = (5, 15, 25, 40)$, $T = (20, 40, 60, 100)$, and $\rho = 0$, with the first 100 time observations for each cross-sectional unit being discarded.

As an alternative procedure to test the presence of unit roots in panels that exhibit cross-sectional dependency, Maddala and Wu (1999) and more recently Chang (2004) have considered bootstrapping unit root tests which, in the context of the HEGY-IPS, we denote as BHEGY-IPS. In order to implement this procedure, we start off by resampling the restricted residuals $\Delta_4 y_{it} = y_{it} - y_{i,t-4} = \varepsilon_{it}$ after centering, since y_{it} is assumed to be a seasonally integrated series under the null hypothesis; this is what Li and Maddala (1996) refer to as the sampling scheme S_3 which is appropriate in the unit root case. To preserve the cross-correlation structure of the error term within each cross section i, and following Maddala and Wu (1999), we resample the restricted residuals with the cross-section index fixed. Also, in order to ensure that initialisation of ε^*_{it}, i.e. the bootstrap samples of ε_{it}, becomes unimportant, we follow Chang (2004) who advocates generating a large number of ε^*_{it}, say $T+Q$ values and discard the first Q values of ε^*_{it} (in our simulations we choose Q equal to 100). Lastly, the bootstrap samples of y^*_{it} are calculated by taken partial sums of ε^*_{it}. These Monte Carlo simulation results are based on 2,000 replications each of which uses 100 bootstrap repetitions.

3 Main results

The empirical size results for both the CHEGY–IPS and BHEGY-IPS tests based on equation (3) with $E(\varepsilon_{it}\varepsilon_{jt}) = \omega = (0.3, 0.5, 0.7, 0.9)$ for $i \neq j$, are approximately correct and hence are not reported to save space. Table 2 reports the power of these tests at the 5% significance level for the linear trend case, when in equation (3) $\rho = -0.1$, for differing degrees of cross-sectional dependency. In general, we observe that the BHEGY-IPS test out performs the CHEGY-IPS test. However, the extent of this dominance falls as the degree of cross-sectional correlation increases and as N increases. For large N and high ω there are cases in which the CHEGY-IPS test dominates. Similar results are observed when other deterministic components are included in the test regressions.

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Table 1. Critical values of the CHEGY-IPS test statistic

Statistic	Level		Constant				Constant, trend				Constant, seasonal				Constant, seasonal, trend			
		$N \setminus T$	20	40	60	100	20	40	60	100	20	40	60	100	20	40	60	100
CHEGY-IPS$_{t1}$	1%	5	-2.54	-2.65	-2.68	-2.77	-3.01	-3.09	-3.17	-3.22	-2.57	-2.60	-2.66	-2.76	-3.14	-3.06	-3.15	-3.21
	5%		-2.16	-2.33	-2.40	-2.46	-2.60	-2.79	-2.86	-2.94	-2.14	-2.31	-2.39	-2.45	-2.61	-2.77	-2.86	-2.93
	10%		-1.97	-2.16	-2.24	-2.29	-2.39	-2.63	-2.72	-2.79	-1.93	-2.14	-2.23	-2.28	-2.36	-2.61	-2.71	-2.78
	1%	15	-2.06	-2.21	-2.28	-2.36	-2.49	-2.67	-2.74	-2.82	-2.07	-2.19	-2.27	-2.35	-2.53	-2.66	-2.74	-2.81
	5%		-1.83	-2.03	-2.11	-2.17	-2.26	-2.50	-2.57	-2.66	-1.81	-2.02	-2.10	-2.16	-2.24	-2.48	-2.57	-2.65
	10%		-1.73	-1.94	-2.01	-2.07	-2.14	-2.40	-2.49	-2.57	-1.69	-1.92	-2.00	-2.06	-2.10	-2.39	-2.48	-2.56
	1%	25	-1.92	-2.11	-2.17	-2.25	-2.37	-2.56	-2.64	-2.70	-1.93	-2.09	-2.16	-2.23	-2.40	-2.55	-2.63	-2.70
	5%		-1.76	-1.97	-2.03	-2.10	-2.18	-2.43	-2.51	-2.57	-1.72	-1.96	-2.02	-2.10	-2.15	-2.41	-2.50	-2.57
	10%		-1.67	-1.89	-1.95	-2.02	-2.09	-2.35	-2.43	-2.50	-1.62	-1.87	-1.94	-2.02	-2.03	-2.33	-2.42	-2.50
	1%	40	-1.85	-2.05	-2.12	-2.16	-2.28	-2.48	-2.57	-2.64	-1.85	-2.04	-2.11	-2.16	-2.30	-2.47	-2.57	-2.63
	5%		-1.71	-1.92	-1.99	-2.05	-2.13	-2.37	-2.46	-2.53	-1.67	-1.91	-1.99	-2.04	-2.10	-2.35	-2.45	-2.52
	10%		-1.63	-1.85	-1.92	-1.98	-2.04	-2.31	-2.40	-2.47	-1.58	-1.84	-1.92	-1.97	-1.99	-2.29	-2.39	-2.46
CHEGY-IPS$_{t2}$	1%	5	-1.96	-2.06	-2.09	-2.15	-1.93	-2.05	-2.10	-2.15	-2.60	-2.63	-2.67	-2.73	-2.62	-2.62	-2.68	-2.73
	5%		-1.59	-1.72	-1.79	-1.83	-1.57	-1.71	-1.79	-1.83	-2.14	-2.30	-2.38	-2.44	-2.15	-2.30	-2.38	-2.44
	10%		-1.40	-1.55	-1.62	-1.65	-1.38	-1.54	-1.61	-1.65	-1.93	-2.14	-2.22	-2.28	-1.92	-2.14	-2.22	-2.28
	1%	15	-1.51	-1.64	-1.70	-1.75	-1.50	-1.64	-1.69	-1.75	-2.07	-2.19	-2.27	-2.33	-2.09	-2.20	-2.27	-2.34
	5%		-1.30	-1.46	-1.52	-1.56	-1.29	-1.46	-1.52	-1.56	-1.81	-2.02	-2.09	-2.16	-1.82	-2.03	-2.10	-2.16
	10%		-1.19	-1.36	-1.41	-1.45	-1.18	-1.36	-1.41	-1.46	-1.68	-1.92	-2.00	-2.07	-1.68	-1.93	-2.00	-2.07
	1%	25	-1.39	-1.54	-1.61	-1.65	-1.39	-1.54	-1.61	-1.65	-1.92	-2.11	-2.16	-2.24	-1.94	-2.11	-2.16	-2.24
	5%		-1.23	-1.40	-1.46	-1.50	-1.23	-1.40	-1.46	-1.50	-1.72	-1.95	-2.03	-2.09	-1.72	-1.96	-2.03	-2.09
	10%		-1.15	-1.31	-1.37	-1.41	-1.14	-1.31	-1.37	-1.41	-1.62	-1.87	-1.95	-2.01	-1.61	-1.87	-1.95	-2.01
	1%	40	-1.32	-1.48	-1.54	-1.60	-1.31	-1.48	-1.54	-1.60	-1.84	-2.03	-2.11	-2.17	-1.85	-2.03	-2.11	-2.17
	5%		-1.18	-1.35	-1.42	-1.47	-1.17	-1.36	-1.42	-1.47	-1.67	-1.90	-1.98	-2.05	-1.67	-1.91	-1.99	-2.05
	10%		-1.11	-1.28	-1.34	-1.39	-1.10	-1.28	-1.35	-1.39	-1.59	-1.84	-1.92	-1.98	-1.58	-1.84	-1.92	-1.98
CHEGY-IPS$_{F2}$	99%	5	4.94	4.29	4.39	4.44	5.02	4.25	4.34	4.40	8.72	6.78	6.93	7.00	10.11	6.76	6.93	7.00
	95%		3.57	3.35	3.49	3.53	3.57	3.30	3.45	3.51	6.11	5.57	5.74	5.90	6.45	5.55	5.72	5.90
	90%		2.98	2.93	3.06	3.10	2.98	2.89	3.02	3.09	5.12	4.99	5.18	5.38	5.27	4.95	5.15	5.37
	99%	15	3.43	3.30	3.37	3.45	3.52	3.24	3.34	3.43	6.26	5.38	5.49	5.59	7.02	5.37	5.48	5.61
	95%		2.82	2.82	2.88	2.97	2.82	2.77	2.86	2.94	4.91	4.75	4.87	5.04	5.20	4.72	4.86	5.03
	90%		2.51	2.57	2.65	2.73	2.52	2.53	2.62	2.72	4.36	4.43	4.57	4.73	4.51	4.40	4.55	4.73
	99%	25	3.10	3.02	3.12	3.20	3.18	3.00	3.08	3.18	5.65	5.04	5.17	5.34	6.19	5.01	5.17	5.34
	95%		2.63	2.65	2.75	2.83	2.64	2.62	2.72	2.82	4.57	4.52	4.68	4.86	4.84	4.51	4.66	4.85
	90%		2.40	2.46	2.56	2.64	2.40	2.43	2.53	2.63	4.13	4.26	4.44	4.60	4.27	4.23	4.42	4.59
	99%	40	2.89	2.89	2.98	3.07	2.97	2.85	2.95	3.04	5.12	4.80	4.98	5.14	5.57	4.79	4.97	5.13
	95%		2.51	2.57	2.66	2.76	2.52	2.54	2.64	2.74	4.36	4.37	4.56	4.74	4.58	4.35	4.55	4.74
	90%		2.32	2.41	2.50	2.59	2.31	2.37	2.48	2.58	4.00	4.15	4.36	4.53	4.15	4.13	4.34	4.52
CHEGY-IPS$_{F3}$	99%	5	4.54	3.90	3.98	3.89	4.70	3.89	3.94	3.87	8.79	6.48	6.49	6.49	9.93	6.48	6.48	6.47
	95%		3.38	3.15	3.21	3.24	3.39	3.10	3.17	3.22	6.18	5.42	5.48	5.55	6.53	5.39	5.46	5.54
	90%		2.87	2.79	2.87	2.89	2.86	2.75	2.83	2.88	5.23	4.88	5.01	5.13	5.39	4.86	4.99	5.12
	99%	15	3.26	3.09	3.14	3.15	3.30	3.05	3.12	3.14	6.29	5.22	5.26	5.36	7.08	5.20	5.26	5.34
	95%		2.73	2.69	2.73	2.78	2.73	2.65	2.71	2.77	5.00	4.65	4.74	4.87	5.31	4.64	4.74	4.86
	90%		2.46	2.47	2.53	2.59	2.45	2.44	2.51	2.58	4.46	4.37	4.48	4.61	4.65	4.35	4.47	4.60
	99%	25	3.01	2.86	2.92	2.97	3.06	2.82	2.89	2.95	5.69	4.92	5.00	5.09	6.34	4.91	4.97	5.09
	95%		2.57	2.55	2.61	2.68	2.57	2.52	2.59	2.66	4.68	4.47	4.57	4.70	4.95	4.44	4.56	4.70
	90%		2.36	2.38	2.45	2.51	2.36	2.35	2.43	2.50	4.26	4.23	4.36	4.49	4.42	4.22	4.35	4.49
	99%	40	2.81	2.76	2.82	2.87	2.86	2.73	2.80	2.86	5.21	4.72	4.83	4.94	5.73	4.70	4.82	4.93
	95%		2.45	2.48	2.55	2.61	2.45	2.45	2.53	2.60	4.48	4.33	4.48	4.61	4.70	4.32	4.47	4.60
	90%		2.28	2.33	2.40	2.47	2.27	2.30	2.38	2.46	4.12	4.14	4.29	4.42	4.29	4.12	4.29	4.42

Table 2. Power of the CHEGY-IPS and BHEGY tests. Linear trend case

Statistic	ω	T=20				T=40				T=60				T=100			
		N=5	N=15	N=25	N=40	N=5	N=15	N=25	N=40	N=5	N=15	N=25	N=40	N=5	N=15	N=25	N=40
CHEGY-IPS$_{t1}$	0.3	5.60	5.70	5.65	6.05	5.85	5.25	6.30	7.55	6.10	7.65	8.25	9.35	7.70	9.45	11.65	13.60
BHEGY-IPS$_{t1}$		8.00	7.85	8.40	8.70	7.45	8.40	9.15	9.95	7.50	9.05	9.55	10.65	9.75	14.65	15.00	17.15
	0.5	5.50	5.40	6.00	6.65	5.55	5.45	6.95	7.40	6.05	8.10	8.50	9.50	7.05	9.65	11.70	13.15
		7.45	7.70	7.60	7.00	7.05	8.45	8.50	8.05	7.35	7.55	9.05	9.05	9.15	12.90	11.95	12.10
	0.7	5.30	6.20	5.80	7.15	5.70	5.85	7.65	7.00	5.85	7.90	7.85	9.10	6.90	9.45	11.80	13.55
		6.90	7.40	6.60	6.40	7.45	7.35	7.50	7.40	6.60	6.75	8.30	7.55	8.65	9.90	9.60	9.85
	0.9	5.75	6.30	6.35	7.10	6.15	6.25	7.80	7.15	6.15	7.80	8.00	10.00	6.75	9.60	12.30	14.00
		7.45	6.60	6.50	6.20	7.30	6.75	7.85	7.00	5.25	6.20	7.50	7.55	7.80	7.65	8.40	8.20
CHEGY-IPS$_{t2}$	0.3	6.75	7.40	8.80	9.40	9.55	11.45	15.15	15.85	11.60	20.95	25.20	27.35	20.10	43.05	55.60	61.45
BHEGY-IPS$_{t2}$		27.25	48.35	60.10	66.45	40.25	73.85	82.80	87.50	54.10	86.00	92.60	94.75	76.45	97.00	98.65	99.15
	0.5	7.00	7.00	9.10	9.75	10.00	11.70	15.70	16.30	12.05	21.30	25.25	28.05	19.25	43.05	55.65	61.25
		22.30	35.00	40.70	42.70	34.40	57.40	60.65	65.20	46.30	69.50	75.40	79.20	67.15	87.50	89.70	91.90
	0.7	7.00	7.75	9.25	9.85	10.75	12.40	15.55	15.30	12.35	21.35	25.75	29.25	19.25	43.30	56.25	62.15
		18.55	24.05	25.85	25.35	28.35	38.75	40.55	41.35	36.95	47.45	52.70	53.75	54.20	68.40	69.35	72.05
	0.9	6.75	8.45	8.90	10.20	10.45	12.65	15.15	14.95	12.65	21.40	25.40	28.50	19.40	42.45	54.40	64.25
		13.65	16.20	16.60	16.25	20.95	23.10	23.55	23.15	23.10	26.55	27.55	29.75	35.60	40.25	40.60	41.05
CHEGY-IPS$_{F2}$	0.3	6.35	7.15	8.45	7.70	10.35	14.25	14.30	15.70	12.15	20.15	23.95	28.70	23.55	44.40	55.05	65.15
BHEGY-IPS$_{F2}$		10.55	13.20	12.00	12.70	21.80	36.40	46.75	49.90	36.95	61.50	75.55	82.50	68.40	93.25	97.25	98.45
	0.5	6.95	6.50	9.05	7.85	10.20	14.80	14.40	16.70	12.30	19.45	24.40	28.30	23.55	44.95	54.05	63.50
		9.40	10.20	8.40	8.95	19.25	25.95	29.75	29.90	33.95	45.05	52.70	55.20	60.80	79.50	82.60	85.40
	0.7	6.15	6.80	8.50	8.05	10.15	13.35	14.30	16.35	13.20	18.85	24.25	28.60	23.85	45.10	53.60	62.80
		8.85	8.75	8.05	8.15	17.05	18.75	20.55	20.85	26.80	31.90	35.65	35.65	50.25	59.45	61.70	63.50
	0.9	6.10	7.75	8.20	8.10	10.15	13.35	14.90	15.95	12.70	19.20	23.40	27.95	24.15	44.95	54.55	63.15
		8.65	8.70	7.95	8.05	15.70	15.10	15.65	16.75	20.90	21.80	23.95	22.65	35.55	38.25	38.10	40.10
CHEGY-IPS$_{F3}$	0.3	6.85	7.85	9.20	8.95	11.80	15.80	18.25	20.60	15.40	27.25	34.55	38.75	33.05	62.70	74.60	84.35
BHEGY-IPS$_{F3}$		13.35	19.05	19.05	21.85	28.70	47.05	58.20	65.60	47.30	74.80	86.90	91.75	81.15	97.80	99.50	99.65
	0.5	6.50	7.80	9.55	9.05	11.15	15.65	18.10	20.70	15.90	27.30	33.95	38.40	32.35	61.75	74.50	84.15
		11.80	13.30	13.55	12.90	24.70	34.80	39.60	40.05	40.80	57.30	64.50	69.20	74.15	89.20	92.30	94.20
	0.7	6.35	8.10	9.80	9.55	11.05	14.80	17.75	19.85	17.15	26.70	33.45	38.00	32.90	62.30	74.30	83.75
		10.65	11.15	10.90	9.75	22.10	24.85	25.05	26.60	33.00	40.55	43.20	45.25	61.95	72.55	73.70	75.85
	0.9	5.95	8.50	8.85	9.80	11.00	14.85	17.45	19.60	17.80	26.25	32.95	37.85	32.95	63.45	74.45	82.80
		10.75	10.20	9.65	9.90	17.80	19.25	18.20	20.45	23.60	27.00	28.25	28.25	44.30	48.20	47.90	48.60