ABSTRACT

A COMPUTATIONAL MODEL FOR SWITCH SURFACES

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Computation of contact points between the wheel and rail is a fundamental problem in dynamic simulation of trains. For this purpose, an accurate model of a switch surface is needed. The objective of this study is to develop a mathematical model for a switch. The model for the switch will be based on skin surface scheme whereby a number of profiles will be placed along a track curve. Each profile will be divided into segments of similar shapes. Then, each set will be used in an interpolation scheme to generate the switch surface.

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A COMPUTATIONAL MODEL FOR SWITCH SURFACES

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DEDICATION

To my parents, Alireza and Shahrzad, and my brother Ali

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CHAPTER 1

INTRODUCTION

1.1. Background

Intersecting of surfaces has been studied from the time of ancient mathematicians to the present, and still lots of important issues in vast variety of problems related to surfaces are left unsolved. Finding contact points between two surfaces, which is called common or collinear normals between the intersecting surfaces, is an important issue for detecting loops and singularities of surface intersections. For finding common normal it is necessary to have complete information about the two surfaces. Accuracy in modeling of surface has the most impact in accuracy of results in finding common normal. In this study, a mathematical model of wheelset and switch will be shown.

1.2. Literature Review

A general review of the subject can be found in Pratt and Geisow [1]. For marching and subdivision methods, the most efforts have been put on improving the accuracy and efficiency; however, some recent developments have been focused on techniques dealing with singularities of intersection curves [2-10]. One of the most important problems regarding computation of common normals is multibody modelling of railroad because of the forces generated at the contact area. For finding the common normal, four nonlinear equations should be performed and solved. These four equations are demonstrated by parameters such as normals of both wheel and rail surface and also the line segment connecting end points of common normals in order to be parallel to normals of the rail. Embedded constraints and elastic are two prerequisite basic approaches implemented to develop wheel and rail interaction (Shabana et al. [11-15]). In embedded constraint approach, the constraint equations governing the location of common normal are augmented with a second set of equations enforcing the distance between the end points of common normal in order to become zero [15]. In this case, the wheel has four degrees of freedom respecting to the rail, and wheel penetration and lift are not permitted. In elastic approach, locations of common normals are computed by considering penetration of wheel and rail surface [15].

Generating an estimate for each common normal followed by an iterative nonlinear algebraic equation solver to refine the estimates according to a desired accuracy is another problem in computation of common normals. Fallahi and Sunil [16] used Cauchy index for gradient of signed distance function to identify the grids in space parameter of wheel surface where a common normal exists. Center of grids is used as an estimate for location of common normal and it is shown that this approach is not computationally efficient. Shabana et al. [12] used nodal investigation to identify an approximate location of common normals.

Auciello et al. [17] developed an optimized version of four-equation approach for a straight track laid along the x-axis. This development led them to a very efficient computational algorithm applied only to a straight track. Stationary property of signed distance function at common normal locations is used to locate the common normals [6] and two equations are needed to solve. Signed distance is the projection of the vector that connects two end points of the line segment-connected end point on the wheel surface to the intersection of the line emanating from that point and parallel to a fixed transverse direction (not tangent) and rail surface, and it is minimum. Malvezzi et al. [18] developed a specific signed-distance study for a straight track and they used vertical direction as transverse direction and stated a computational efficient algorithm for locating common normals.

Based on this literature review, it is clear that a mathematical model for a wheelset and a switch, a set of algebraic equations to locate the contact points and a computational approach to solve for coordinates, tangents, and normals to the switch and rail surface are needed. Finding the switch surface is the objective of this thesis.

1.3. Problem Definition

In this study surface of wheelset and switch will be generated by given data from the Federal Railroad Administration (FRA). An input file contains three sets of data including center track, right track and left track curve. Each track curve is represented by a set of nodes and each node can be identified by eight factors. Factors are S (arc length along the track), C_H (horizontal curvature), X, Y, Z (coordinates in absolute coordinate system) and $\emptyset, \theta$ and

ψ (Euler angels) as rotation angels around X, Y, Z respectively (Figure 1). Another input file contains different set of nodes and each set represents Y and Z coordinates of them in local coordinate system. By fitting spline through these nodes, profile of rail or switch, depending on value of arc length (S), can be shown (Figure 2).

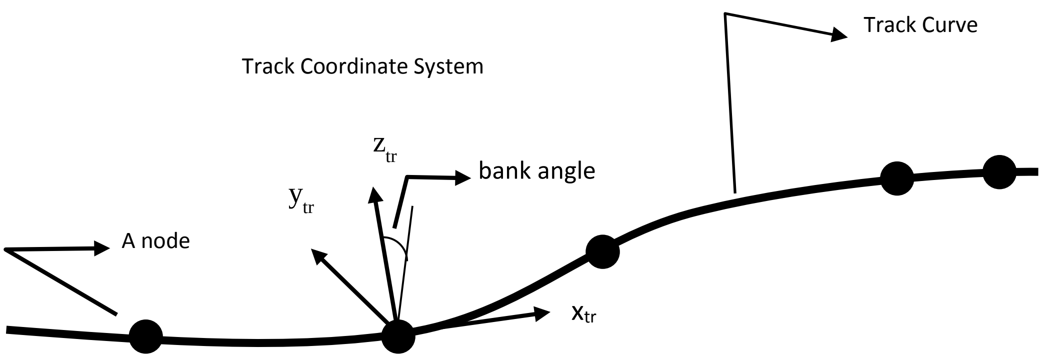

Figure 1: A Track Curve with Track Coordinate System

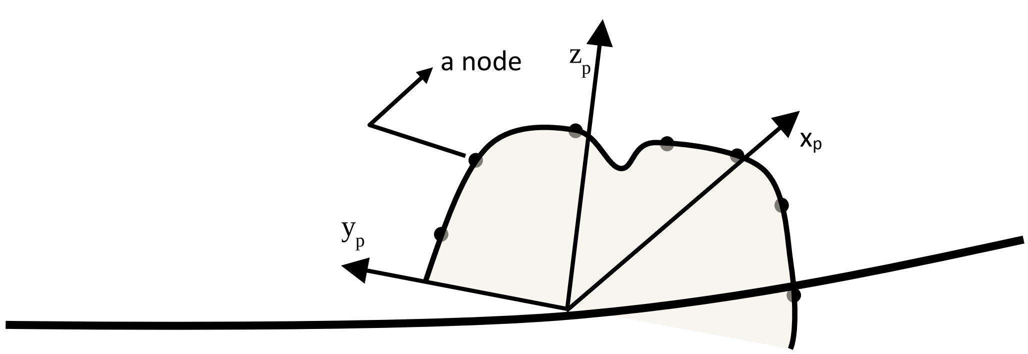

Figure 2: Switch Profile – Generated by Fitting Spline

By using aforementioned data and fitting splines into the nodes, switch profiles will be generated in different section of track curve (S). Then switch surface can be demonstrated by interpolating new nodes on switch profiles and fitting splines to them. Figure 3 shows two switches which are tongue rail and crossing rail.

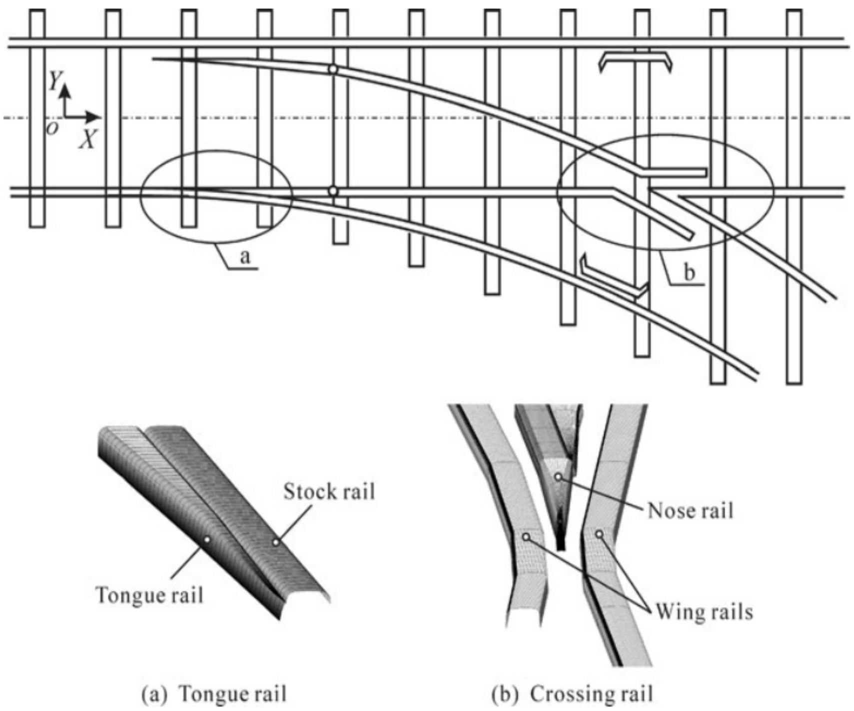

Figure 3: Two Different Switches (Tongue Rail and Crossing Rail)

CHAPTER 2

SWITCH PROFILES AND PARAMETRIC EQUATION

2.1. Overview

In this chapter, mathematical preliminaries for switch surface modeling and basic equations and concepts are shown. This includes the definition of coordinate systems, orientation matrixes, spline, and transformation of node in different coordinate systems. Figures 4 and 5 show rail profile and switch profile at the very beginning of switch.

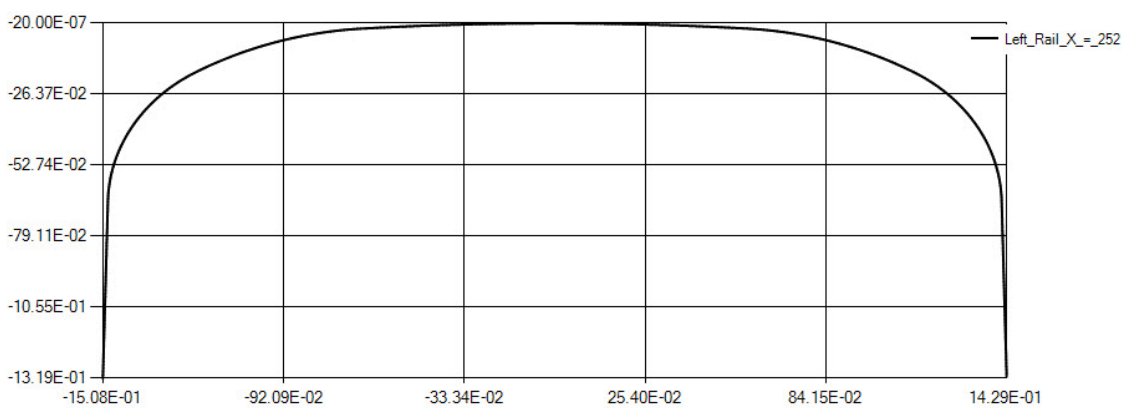

Figure 4: Rail Profile

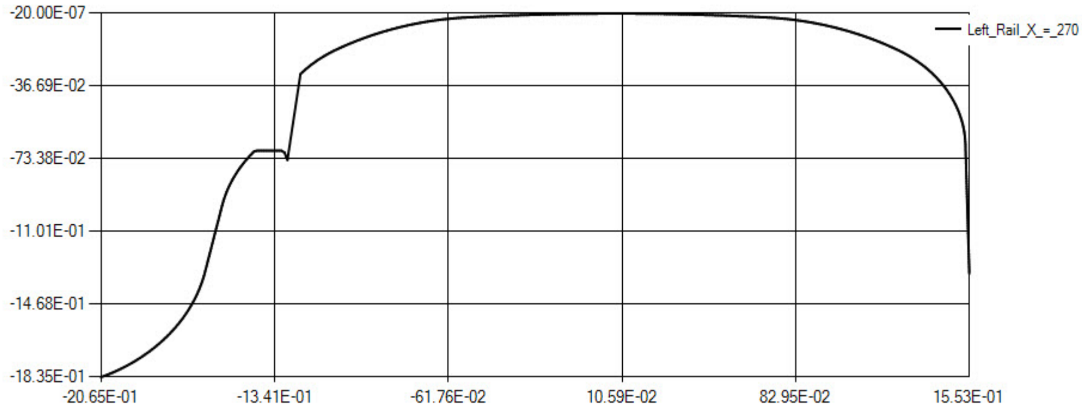

Figure 5: Switch Profile

Figure 6 shows left, right, and center track, wheelset, global coordinate system, and local coordinate system. Global coordinate system is located at an arbitrary location and it is fixed. The track coordinate system is set up in a way that its X axis is tangent to the track curve and its origin is located at the distant S from the beginning of the track curve. Orientation of the track coordinate system is relative to the global system and it is defined by a set of Euler angles ($\emptyset, \theta$ and ψ) which are rotation angels around X, Y, Z respectively. They are all functions of the track arc length (S).

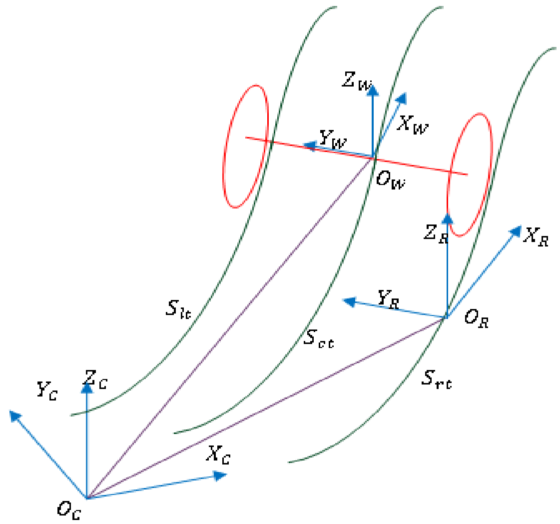

Figure 6: Important Definitions Symbols

2.2. Basic Rotation Matrix and Coordinate Transformation

Rotation matrix is defined as matrix multiplier for a point which has rotation on a coordinate system. This rotation can be over X, Y and Z axis and the rotation angles are ($\emptyset, \theta$ and ψ) respectively. In railroad studies, the negative sign is in a different place compared with rotation matrixes definition in classical mathematics. The y-axis is set up using the right-hand rule.

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\phi) & -sin(\phi) \\ 0 & s(\phi) & c(\phi) \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta)0 & 0 & cos(\theta) \end{bmatrix}$$

$$R_z(\psi) = \begin{bmatrix} c(\psi) & -sin(\psi) & 0 \\ sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Position of a node in local coordinate system can be defined in global coordinate system if there is a relation between global coordinate system and local coordinate system. Mathematical expression for the transformation of a node from local coordinate system to a global coordinate system is shown by assuming two different coordinate systems and an arbitrary point on space. Figure 7 shows two coordinate systems, namely $O_1X_1Y_1Z_1$ and $O_2X_2Y_2Z_2$. The position vector of a node (P) in coordinate system $O_1X_1Y_1Z_1$ and $O_2X_2Y_2Z_2$ are denoted by r_1^P and r_2^P.

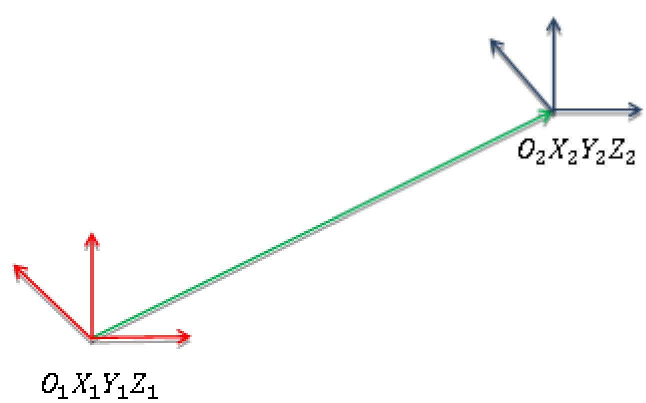

Figure 7: Relation Between the Two Coordinates

The orientation of the coordinate system $O_2X_2Y_2Z_2$ relative to coordinate $O_1X_1Y_1Z_1$ is described by Euler angles ϕ_2^1, θ_2^1 and ψ_2^1. ϕ_2^1, θ_2^1 and ψ_2^1 are body fix rotations about X, Y, Z axis of the coordinate system $O_2X_2Y_2Z_2$ relative to coordinate system $O_1X_1Y_1Z_1$. The orientation matrix of coordinate system $O_2X_2Y_2Z_2$ relative to coordinate $O_1X_1Y_1Z_1$ is:

$$A_2^1 = R_Z(\psi_2^1)R_y(\theta_2^1)R_x(\phi_2^1)$$

The coordinates of point P in coordinate system $O_1X_1Y_1Z_1$ and $O_2X_2Y_2Z_2$ are related by:

$$r_1^P = R_2^{O_2} + A_2^1 r_2^P$$

$R_2^{O_2}$ is the position vector of the origin of coordinate system $O_2X_2Y_2Z_2$ relative to $O_1X_1Y_1Z_1$ coordinate system.

2.3. Coordinate of a Node on Rail or Switch Profile

A rail surface can be made by swiping a rail profile along a track curve which can be left or right track curve. Rail or switch profile can be sketched by fitting spline through the nodes data. For the nodes on both switch and rail profiles, Y and Z values of noedes are used as variable to fit spline. Y is independent variable and Z is dependent variable to Y. Therefore, for both switch and profile as shown in figure 8:

$$z_{rp}^P = z_{rp}^P\left(y_{rp}^P\right)$$

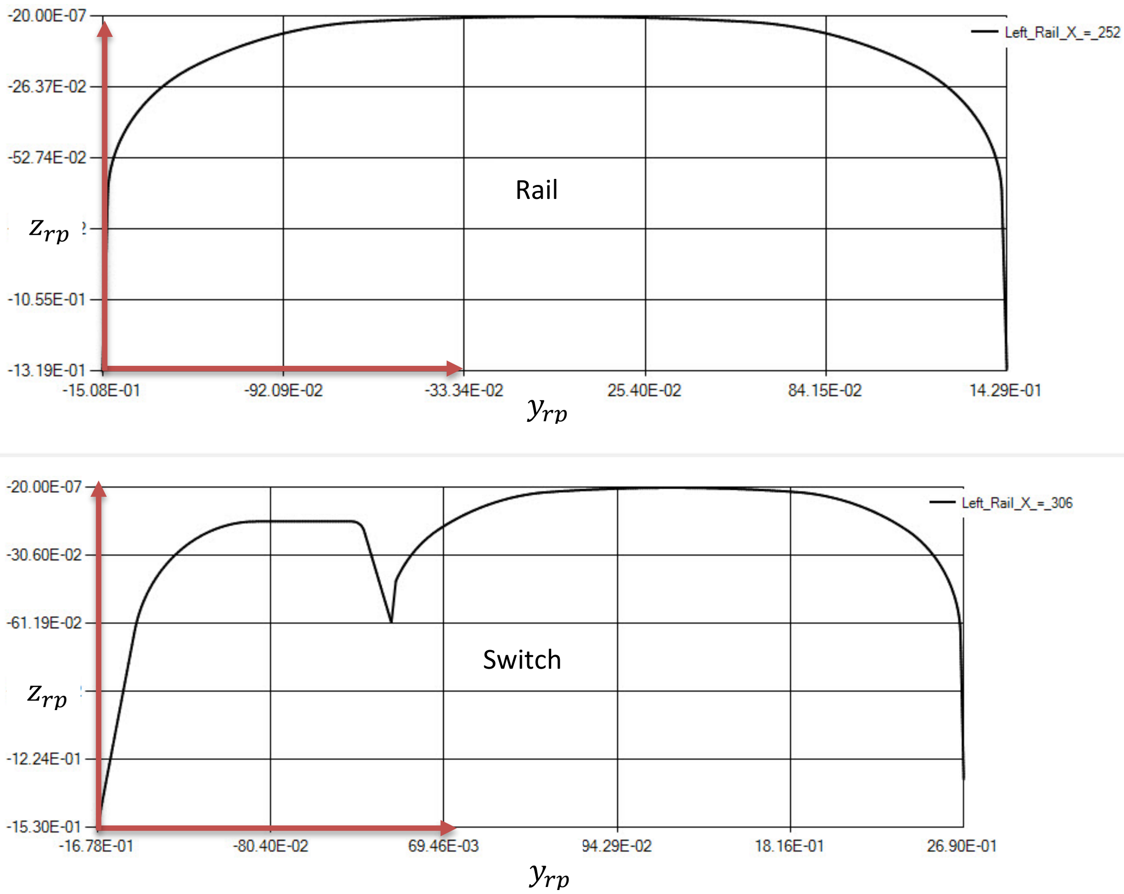

Figure 8: Rail and Switch Profile Coordinates

2.4. Different Switch Profiles

By fitting splines through the set of nodes in different places on track curve (for different values of S), different switch profile can be generated. Figure 9 and 10 shows switch profiles in different location on track curve.

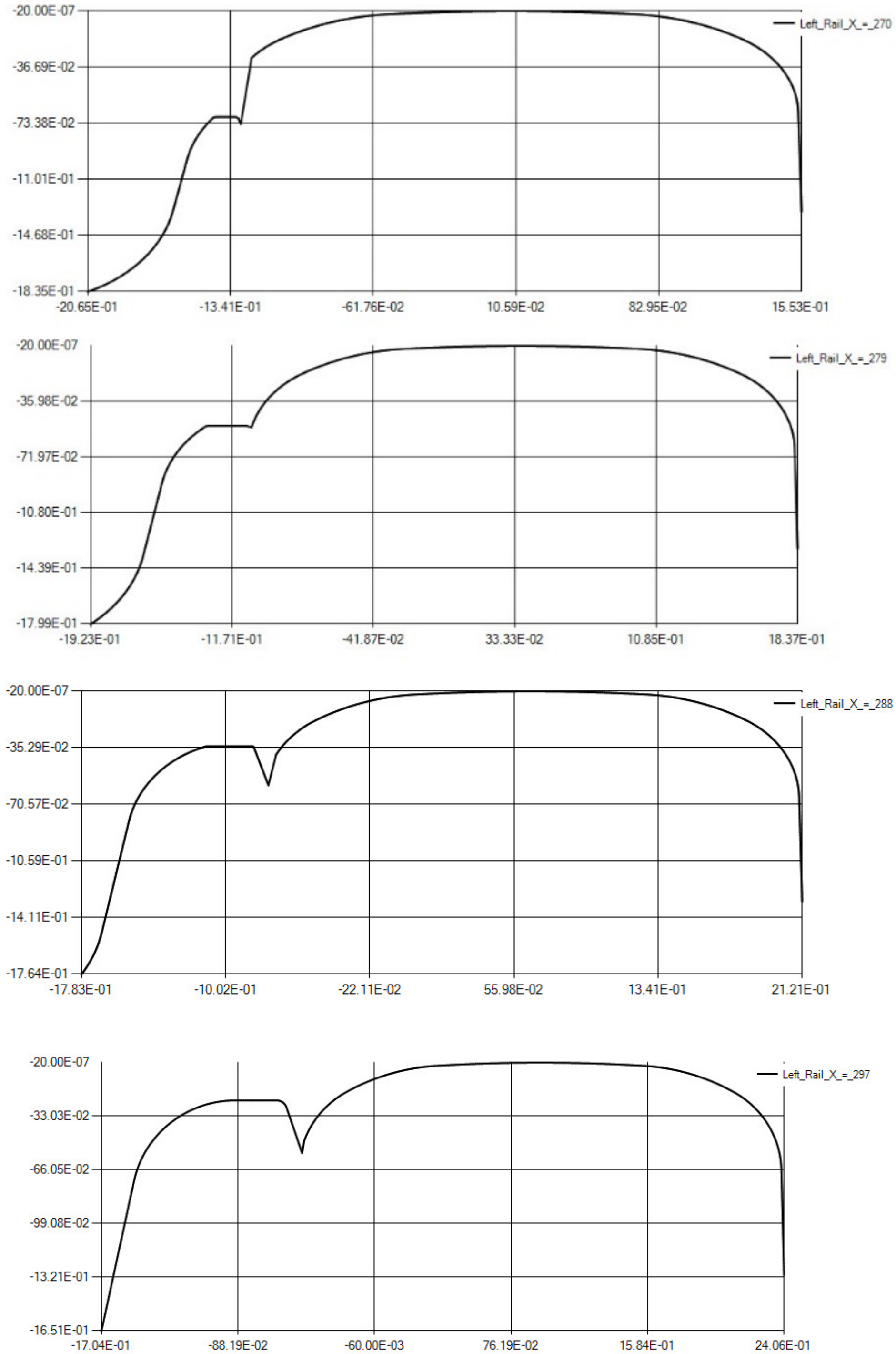

Figure 9: Switch Profiles (First Four Profiles)

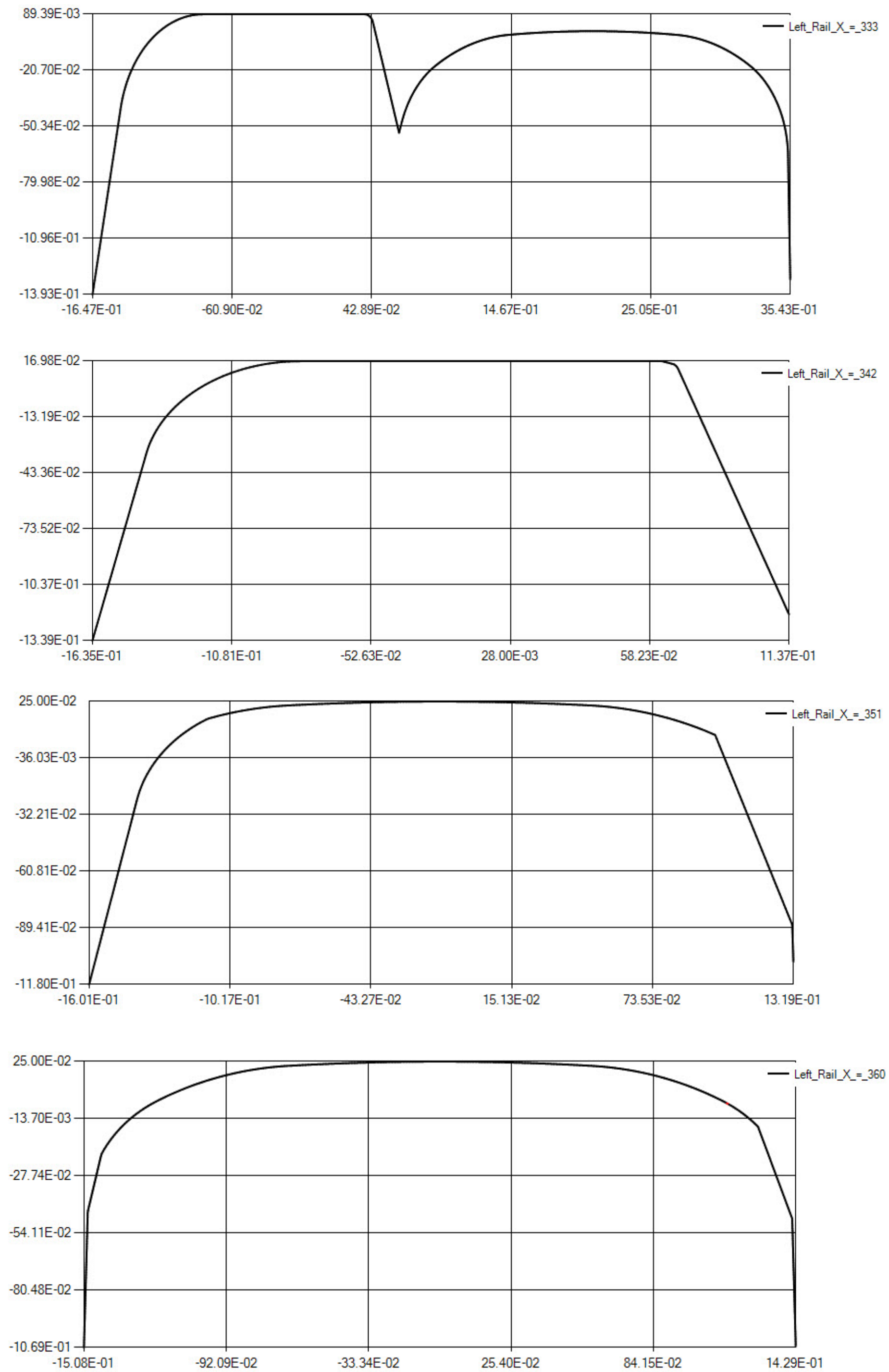

Figure 10: Switch Profiles (Second Four Profiles)

2.5. Segments on Switch Profiles

Most of the switch profiles can be divided to three different segments. The first and third segments' property is that the second derivation of each node on them is a negative value and property of the second segment is that the second derivative of each node on it is positive value. Figure 11 is showing how a switch profile is divided to three segments. These segment allows to keep track of switch profile change in particular S values.

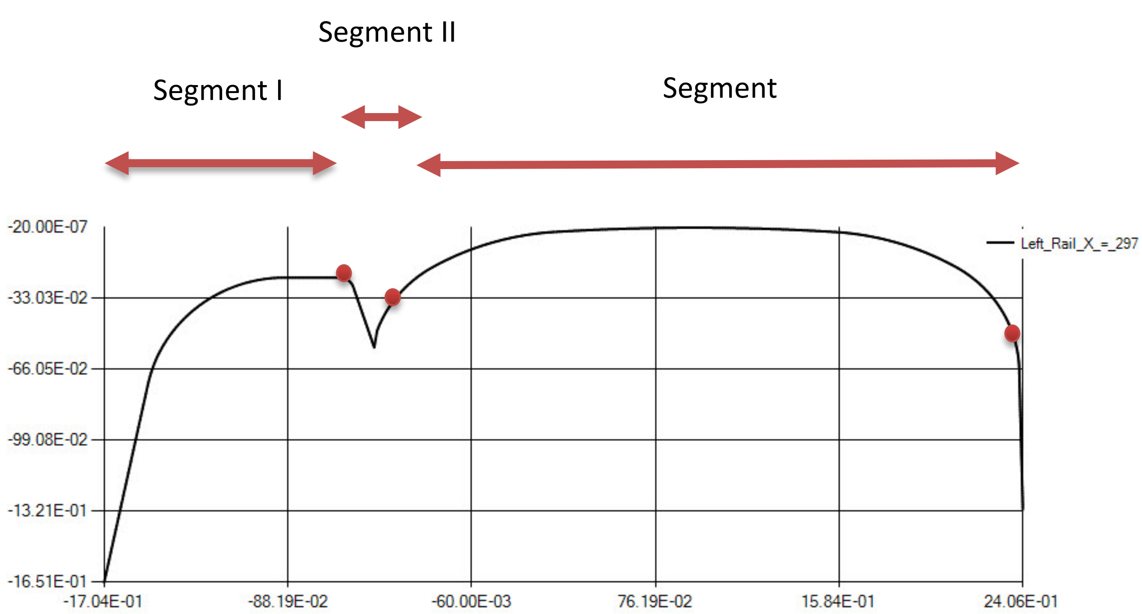

Figure 11: Segments on Switch Profiles

CHAPTER 3

SWITCH SURFACE MODEL

3.1. Surface

There are several methods to generate a surface from a profile. One characteristic of the switch surface is that its cross section is varying along the track. To capture this characteristic in the surface model, it is proposed to develop a surface constructor for variable section surfaces. To accomplish this goal, two tasks should be accomplished. The first task is the computational scheme to compute a variable section surface (the surface constructor). The second task is the input data model for the surface constructor. The surface constructor will use a space curve (track curve) and a number of cross sections. The approach for constructing the switch surface will be to place the cross section along the track curve and construct a surface that follows the track curve and pass through the cross sections. In the following, the data model to support this operation and the scheme to generate the switch surface are discussed.

3.2. Scheme of Switch Surface Model on Track Curve

The switch cross section data model consists of y- and z-coordinates of a number of points on each cross section. To differentiate each cross section a number should be assigned to them. Pictorially, the data model is shown in Chapter 1, Figure 2. These nodes reside in yz-plane of the profile coordinate system. A spline will be fitted into each set of nodes of a cross section; this spline will be used to interpolate arbitrary points on the profile. The data model for the track curve consists of a set of nodes and coordinate systems (track coordinate system). The track coordinate system at each node is defined as follows:

- x-axis of the track coordinate system is tangent to track
- z-axis of the track coordinate system is perpendicular to the x-axis
- yz-plane of the track coordinate system makes a specified angle called bank angle from the vertical plane passing through the x-axis of the track coordinate system.

The track data consist of the position of the nodes on the track curve as measured by the arc length to that node, the x-, y-, and z-coordinates of the nodes, and the Euler angles of the track coordinate system relative to the absolute coordinate system. The arc length will be used as the independent parameter and the rest of the data will be used as dependent parameter; a number of splines will be fitted. These splines will be used to interpolate the coordinates and Euler parameters of the track coordinate system at an arbitrary point on the track (see Figure 1). To implement the switch surface constructor, each cross section is divided to three segments and their end points are recorded (see Figure 11). The rationale for dividing the cross sections into three segments is to skin similar shapes to achieve better representation (avoid warping) for the switch surface. Next, the cross sections are placed along the track curve at their specified locations and orientation, which is demonstrated in Figure 12.

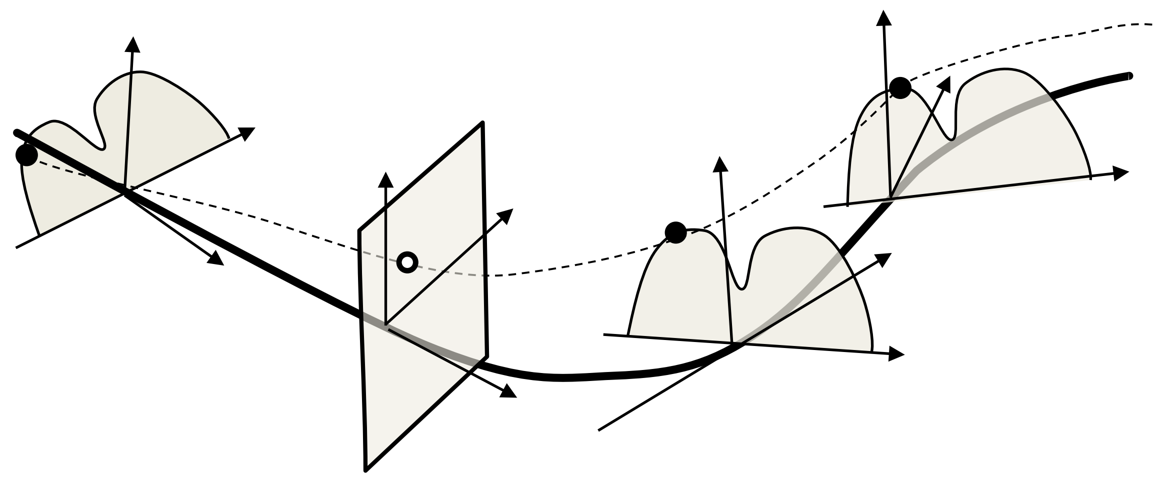

Figure 12: The Procedure for Construction of a Skinned Surface

3.3. Interpolation of Nodes

The mathematical representation of the switch surface means, for a given switch surface parameters, to compute the x-, y-, and z-coordinates of the corresponding point in

3D and tangents and normal vectors at that point. The steps to accomplish this task are as defining y_{min} and y_{max} to be the y-coordinate of the end points of a segment. Then,

$$y_p = (1 - t)y_{min} + ty_{max}$$

Varying parameter t between zero and one ensures that y_p stays within a segment. For a given value of t, the above will be used to compute the y-coordinate of the point (in profile coordinate system) on the segments with the same segment number. Then the z-coordinate can be computed using the spline fit. These points are shown as solid black circles in Figure 13. The y- and z-coordinates of the profile points (solid black) are used as dependent parameters and the arc lengths that locate the cross sections along the track curve will be used as the independent parameter for a spline fit. This spline function will be used to interpolate a point on a cross section which is arbitrarily located along the track curve. The interpolated point is shown as hollow circle in Figure 13. The position and orientation of the track coordinate system at this location can be interpolated using the track data and should be used to compute the absolute coordinate of the interpolated profile point. Same procedure will be implemented for the derivatives of the position vector of the interpolated point. This completes the mathematical model (switch surface constructor) of the switch surface. This surface will be blended to the surface of the incoming and outgoing rail to the switch surface. Figure 13 shows a set of nodes obtained by interpolation, then for each set a spline is fitted.

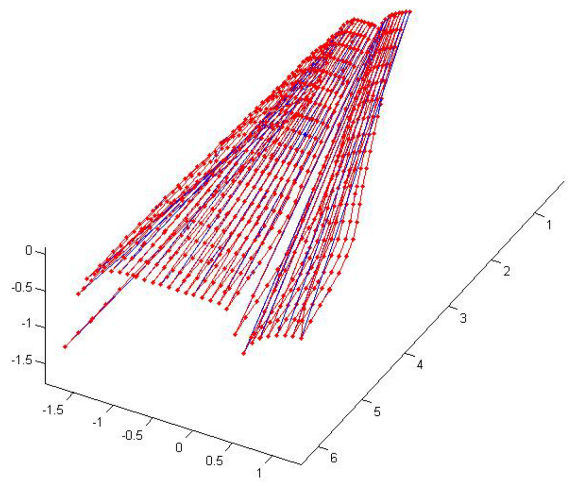

Figure 13: Interpolated Nodes and Fitted Splines

3.4. Switch Surface for Segment I, II and III

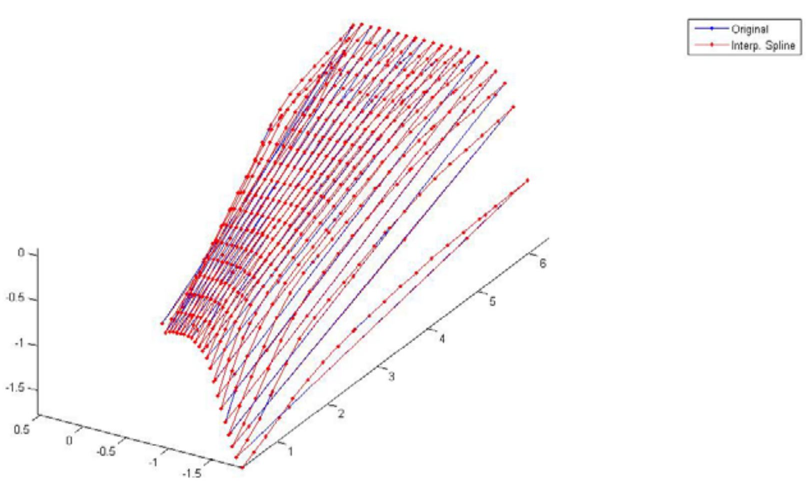

Figure 14: Switch Surface for First Segment

All the switch profiles are divided into three segments and the aforementioned method for modeling is applied to the segments. Figures 15, 16 and 17 show the switch surface for the first, second and third segments. Finally, complete switch surface can be generated by combining the interpolated nodes and splines of all three segments. Figure 17 is the complete model of switch surface.

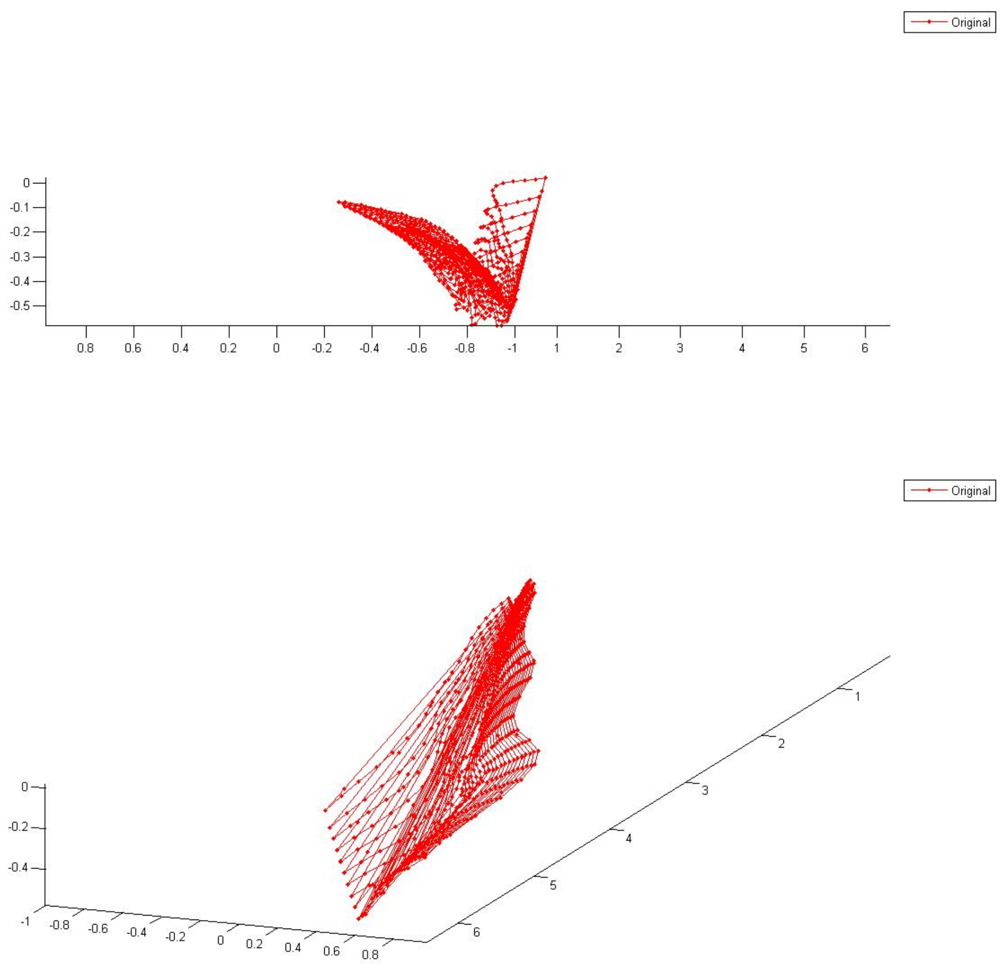

Figure 15: Switch Surface for Second Segment from Two Different Views

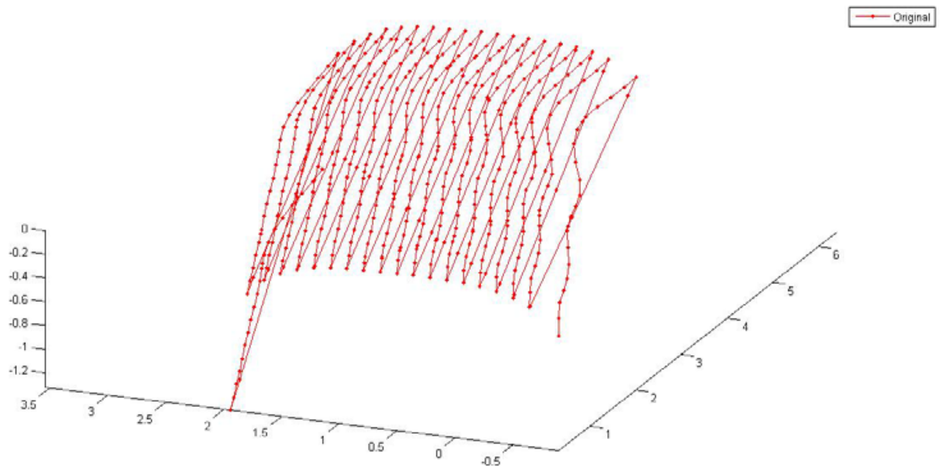

Figure 16: Switch Surface for Third Segment

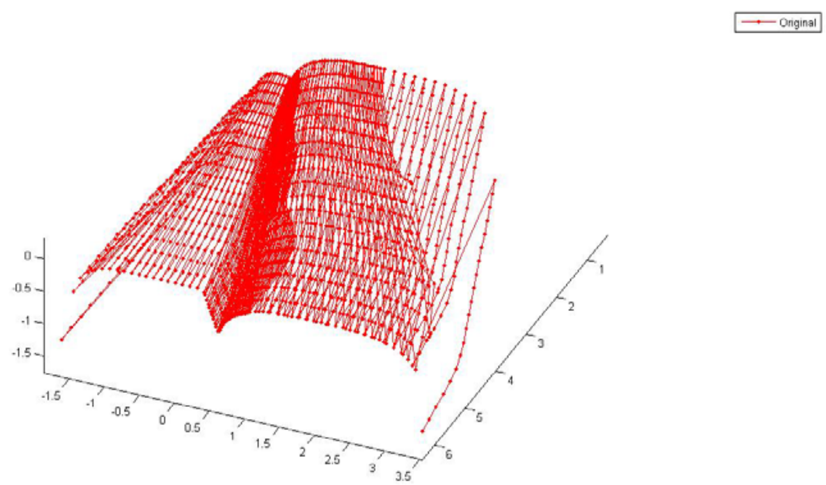

Figure 17: Complete Switch Surface

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