Supplemental Material: Universal properties of boundary and interface charges in multichannel one-dimensional models without symmetry constraints

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In the following, we list the Hamiltonians used in the numerical examples of section V in the main text.

The $N_c = Z = 3$ Hamiltonian used in the calculation of the contour (Fig. 2) is given by

$$v_1 = \begin{pmatrix} -0.4944582 & 0.57434879 + 0.76768438i & 0.38010216 - 0.3227788i \\ 0.57434879 - 0.76768438i & 0.17028087 & -0.12163262 + 0.08683064i \\ 0.38010216 + 0.3227788i & -0.12163262 - 0.08683064i & -0.1872981 \end{pmatrix}, \tag{1}$$

$$v_2 = \begin{pmatrix} 0.54582815 & -0.36852351 - 0.04013338i & 0.17708711 - 0.15488239i \\ -0.36852351 + 0.04013338i & 0.98698514 & 0.23707274 + 0.03260454i \\ 0.17708711 + 0.15488239i & 0.23707274 - 0.03260454i & -0.89519627 \end{pmatrix}, \tag{2}$$

$$v_3 = \begin{pmatrix} 0.09034543 & -0.0529429 + 0.66493465i & -0.40929606 + 0.50975004i \\ -0.0529429 - 0.66493465i & 0.62717073 & -0.70806207 - 0.46843796i \\ -0.40929606 - 0.50975004i & -0.70806207 + 0.46843796i & -0.84868633 \end{pmatrix}, \tag{3}$$

$$t_1 = \begin{pmatrix} 0.24106425 + 0.75954503i & 0.39951955 - 0.83987445i & -0.70939688 + 0.59471041i \\ 0.08062616 - 0.62072405i & -0.64056866 + 0.46943078i & 0.69665083 + 0.56323945i \\ -0.15140389 + 0.62351802i & -0.4158642 + 0.61600451i & -0.47909182 - 0.468285i \end{pmatrix}, \tag{4}$$

$$t_2 = \begin{pmatrix} 0.95472693 + 0.27556248i & 0.79839864 + 0.0917834i & 0.71196756 + 0.51063241i \\ 0.3859219 - 0.06135333i & 0.49234291 - 0.03195245i & -0.8642979 - 0.31707168i \\ 0.35721307 - 0.61547973i & 0.68259592 + 0.97858545i & -0.45682217 - 0.87495712i \end{pmatrix}, \tag{5}$$

$$t_3 = \begin{pmatrix} 0.53283463 - 0.01128521i & -0.15628193 + 0.98906666i & 0.03970061 - 0.72966195i \\ 0.36798589 + 0.38419227i & 0.42645765 + 0.71823889i & 0.70707806 - 0.69956256i \\ 0.84265784 - 0.19864427i & -0.25684613 + 0.07105722i & -0.49867149 - 0.33685095i \end{pmatrix}. \tag{6}$$

The $N_c = 3, Z = 4$ Hamiltonian used in the calculation of the shifted boundary charge invariant example (Fig. 3) is given by

$$v_0 = \begin{pmatrix} 0.90918084 & -0.24572285 - 0.66887345i & 0.07488023 + 0.43240801i \\ -0.24572285 + 0.66887345i & 0.56819934 & -0.39061585 - 0.22247052i \\ 0.07488023 - 0.43240801i & -0.39061585 + 0.22247052i & -0.17051496 \end{pmatrix}, \tag{7}$$

$$\delta_v = \begin{pmatrix} 0.02774156 & 0.2859403 - 0.01955917i & -0.28670403 - 0.09992215i \\ 0.2859403 + 0.01955917i & -0.07877183 & -0.64531684 + 0.08964048i \\ -0.28670403 + 0.09992215i & -0.64531684 - 0.08964048i & -0.93476734 \end{pmatrix}, \tag{8}$$

$$t_0 = \begin{pmatrix} -0.88944725 - 0.76770169i & -0.32851701 + 0.4908495i & 0.17681831 + 0.44297607i \\ 0.27956042 - 0.15460026i & 0.69664396 - 0.80789705i & -0.06244329 - 0.36763672i \\ 0.33002784 + 0.57796944i & -0.9874527 + 0.92545997i & 0.27819225 - 0.47315019i \end{pmatrix}, \tag{9}$$

$$\delta_t = \begin{pmatrix} -0.58603661 - 0.1884779i & 0.37426473 - 0.12101691i & 0.34827591 + 0.0989828i \\ -0.66360162 + 0.84043632i & 0.29328977 + 0.06031052i & -0.23946274 + 0.35388921i \\ 0.49854692 + 0.282901i & 0.00510658 - 0.49200114i & 0.9002086 - 0.00963393i \end{pmatrix}. \tag{10}$$

The $N_c = 4, Z = 3$ Hamiltonian used in the calculation of the potential impurity interface charge example (Fig. 4) is given by

$$v_1 = \begin{pmatrix} -0.60687141 & 0.12885404 - 0.28928961i & 0.17126335 + 0.82789126i & -0.01562436 - 0.42902267i \\ 0.12885404 + 0.28928961i & -0.23745729 & -0.02768967 - 0.16012022i & -0.5607436 + 0.09055274i \\ 0.17126335 - 0.82789126i & -0.02768967 + 0.16012022i & 0.99461468 & 0.22963881 - 0.35765889i \\ -0.01562436 + 0.42902267i & -0.5607436 - 0.09055274i & 0.22963881 + 0.35765889i & -0.74220396 \end{pmatrix}, \quad (11)$$

$$v_2 = \begin{pmatrix} -0.71744785 & -0.02397275 - 0.16690444i & 0.21099762 - 0.01700339i & -0.13142301 + 0.38309352i \\ -0.02397275 + 0.16690444i & 0.76671614 & 0.23552629 - 0.32028269i & 0.85170206 + 0.07966246i \\ 0.21099762 + 0.01700339i & 0.23552629 + 0.32028269i & 0.44805615 & 0.76446223 + 0.41996331i \\ -0.13142301 - 0.38309352i & 0.85170206 - 0.07966246i & 0.76446223 - 0.41996331i & -0.28404753 \end{pmatrix}, \quad (12)$$

$$v_3 = \begin{pmatrix} 0.41620817 & -0.36871262 + 0.96755171i & 0.35036057 + 0.25990987i & -0.2083217 + 0.41540971i \\ -0.36871262 - 0.96755171i & 0.05247915 & -0.32631566 + 0.48538255i & -0.21221584 - 0.31134165i \\ 0.35036057 - 0.25990987i & -0.32631566 - 0.48538255i & 0.42443833 & 0.49352163 + 0.31176955i \\ -0.2083217 - 0.41540971i & -0.21221584 + 0.31134165i & 0.49352163 - 0.31176955i & -0.53993851 \end{pmatrix}, \quad (13)$$

$$t_1 = \begin{pmatrix} -0.40522514 + 0.28553213i & -0.65262785 + 0.30669345i & 0.16121097 - 0.51314027i & 0.83699647 + 0.99655913i \\ -0.67139569 + 0.21442743i & 0.17163123 + 0.12591744i & -0.63191814 - 0.13116329i & 0.92153573 - 0.25782209i \\ -0.61992322 + 0.28293355i & -0.35989309 + 0.18055828i & -0.36809224 + 0.64191059i & -0.39694743 + 0.23910722i \\ -0.5400065 - 0.79888228i & 0.30690518 - 0.03598716i & 0.7106456 - 0.08485762i & -0.84147776 - 0.27194106i \end{pmatrix}, \quad (14)$$

$$t_2 = \begin{pmatrix} 0.76176153 - 0.77515945i & 0.80689694 + 0.53367223i & 0.77512597 + 0.81955277i & 0.39171962 + 0.3681039i \\ -0.86282357 + 0.87501381i & 0.84833074 + 0.75070133i & -0.31790326 + 0.44673359i & -0.6401189 + 0.09685302i \\ 0.45167651 - 0.0597095i & 0.85496192 + 0.63632698i & -0.79501046 + 0.57622953i & -0.15800389 - 0.06831002i \\ -0.76541453 - 0.31212747i & -0.23971583 - 0.11841097i & -0.7392837 + 0.60364822i & -0.98611912 - 0.02725172i \end{pmatrix}, \quad (15)$$

$$t_3 = \begin{pmatrix} -0.51495422 - 0.68189032i & -0.07052153 - 0.75208249i & -0.29828693 + 0.3386923i & -0.22381824 - 0.19244218i \\ 0.72135027 - 0.90520099i & -0.12802552 - 0.18513815i & 0.78158383 - 0.76246528i & -0.51494 + 0.65168529i \\ 0.14336053 + 0.64694445i & 0.73489241 + 0.05943474i & 0.35838066 + 0.42942297i & -0.15150548 - 0.4830256i \\ -0.11175007 - 0.20852716i & 0.45727396 - 0.80496684i & 0.91112461 - 0.73402748i & 0.47394746 + 0.12491548i \end{pmatrix}. \quad (16)$$

The $N_c = 5, Z = 5$ Hamiltonian used in the calculation of the link-weakened interface charge example (Fig. 5) is given by

$$v_1 = \begin{pmatrix} -0.84739301 & 0.15192162 - 0.35657652i & 0.39340146 - 0.72879555i & 0.05229739 - 0.32752712i & -0.07163662 - 0.39608318i \\ 0.15192162 + 0.35657652i & 0.17513174 & -0.59635007 + 0.77099924i & -0.55614948 + 0.54347114i & -0.25198908 - 0.09196395i \\ 0.39340146 + 0.72879555i & -0.59635007 - 0.77099924i & -0.30659615 & 0.38055026 + 0.24921779i & 0.57581586 + 0.12289488i \\ 0.05229739 + 0.32752712i & -0.55614948 - 0.54347114i & 0.38055026 - 0.24921779i & -0.48690864 & 0.03716538 - 0.1713455i \\ -0.07163662 + 0.39608318i & -0.25198908 + 0.09196395i & 0.57581586 - 0.12289488i & 0.03716538 + 0.1713455i & 0.40536641 \end{pmatrix}, \quad (17)$$

$$v_2 = \begin{pmatrix} -0.95387331 & 0.01899544 - 0.52384378i & -0.06753597 + 0.23454486i & 0.08562934 + 0.21877701i & 0.39594011 + 0.18401971i \\ 0.01899544 + 0.52384378i & 0.33904945 & -0.12323575 + 0.58328219i & -0.26313186 - 0.16264879i & 0.24225949 + 0.57721432i \\ -0.06753597 - 0.23454486i & -0.12323575 - 0.58328219i & -0.47888902 & 0.02524494 + 0.29215537i & 0.38316456 - 0.06532999i \\ 0.08562934 - 0.21877701i & -0.26313186 + 0.16264879i & 0.02524494 - 0.29215537i & 0.18735596 & 0.25849069 - 0.08107532i \\ 0.39594011 - 0.18401971i & 0.24225949 - 0.57721432i & 0.38316456 + 0.06532999i & 0.25849069 + 0.08107532i & -0.42200492 \end{pmatrix}, \quad (18)$$

$$v_3 = \begin{pmatrix} -0.01520067 & -0.16572919 + 0.2040341i & 0.19093657 + 0.25573464i & -0.09632023 + 0.4730379i & 0.1955689 + 0.06831824i \\ -0.16572919 - 0.2040341i & 0.95158912 & 0.17033616 - 0.64845533i & -0.19121934 + 0.82801133i & -0.5146001 + 0.13980757i \\ 0.19093657 - 0.25573464i & 0.17033616 + 0.64845533i & -0.79367107 & 0.30450466 - 0.79284896i & 0.25800025 + 0.18407783i \\ -0.09632023 - 0.4730379i & -0.19121934 - 0.82801133i & 0.30450466 + 0.79284896i & -0.5404087 & 0.2743869 + 0.22916055i \\ 0.1955689 - 0.06831824i & -0.5146001 - 0.13980757i & 0.25800025 - 0.18407783i & 0.2743869 - 0.22916055i & 0.83056931 \end{pmatrix}, \quad (19)$$

$$v_4 = \begin{pmatrix} -0.56018624 & -0.30849839 + 0.13792053i & -0.09705989 - 0.20756389i & -0.01036744 - 0.45287618i & -0.42858509 - 0.73719311i \\ -0.30849839 - 0.13792053i & -0.70401226 & 0.74535439 - 0.05772667i & -0.00306166 + 0.4168619i & -0.88935293 - 0.14014358i \\ -0.09705989 + 0.20756389i & 0.74535439 + 0.05772667i & 0.41833938 & -0.91932691 - 0.18471939i & 0.72000204 + 0.0246125i \\ -0.01036744 + 0.45287618i & -0.00306166 - 0.4168619i & -0.91932691 + 0.18471939i & 0.49192261 & -0.62016451 + 0.07836037i \\ -0.42858509 + 0.73719311i & -0.88935293 + 0.14014358i & 0.72000204 - 0.0246125i & -0.62016451 - 0.07836037i & 0.96386105 \end{pmatrix}, \quad (20)$$

$$v_5 = \begin{pmatrix} -0.57167868 & -0.13678641 + 0.562379i & -0.76211957 + 0.13162859i & -0.06327828 - 0.023827533i & 0.04930314 - 0.01028572i \\ -0.13678641 - 0.562379i & 0.04751132 & -0.34385889 - 0.69313275i & 0.655804 + 0.85336297i & 0.75116565 - 0.00889398i \\ -0.76211957 - 0.13162859i & -0.34385889 + 0.69313275i & -0.254086 & -0.04476031 - 0.13983655i & -0.1657417 - 0.11132861i \\ -0.06327828 + 0.023827533i & 0.655804 - 0.85336297i & -0.04476031 + 0.13983655i & 0.38239353 & 0.13682864 - 0.11788784i \\ 0.04930314 + 0.01028572i & 0.75116565 + 0.00889398i & -0.1657417 + 0.11132861i & 0.13682864 + 0.11788784i & 0.04561698 \end{pmatrix}, \quad (21)$$

$$t_1 = \begin{pmatrix} 0.80047348 - 0.85591018i & 0.23149352 + 0.44369256i & -0.23002532 + 0.17816297i & -0.39921379 + 0.84223912i & -0.014936 - 0.89550549i \\ 0.90443249 - 0.4340823i & 0.39078169 - 0.26052319i & 0.32689639 + 0.48807573i & 0.3168691 + 0.1521063i & -0.90555087 + 0.87287847i \\ 0.89201688 - 0.27274039i & -0.37060051 + 0.76101001i & -0.37897776 - 0.07010669i & -0.16707652 + 0.63068628i & -0.22471651 - 0.31333483i \\ -0.03780648 - 0.53863946i & 0.21728634 - 0.18827376i & -0.55491977 - 0.93217545i & 0.19125778 - 0.74817493i & -0.02959721 + 0.07231875i \\ -0.53426739 + 0.6217711i & 0.63639065 - 0.14903255i & 0.85241615 + 0.4489814i & -0.14613378 + 0.748662i & 0.2936694 + 0.58832297i \end{pmatrix}, \quad (22)$$

$$t_2 = \begin{pmatrix} 0.46992338 + 0.16509969i & -0.30436618 + 0.47388353i & 0.49098679 + 0.98164082i & -0.54695302 + 0.51214401i & -0.37586555 + 0.17905454i \\ 0.02707689 + 0.10524192i & 0.93233687 + 0.7648932i & -0.58810739 + 0.40664544i & 0.5716518 - 0.83174955i & -0.65173778 + 0.206885i \\ 0.55975019 - 0.53669275i & 0.26586444 + 0.29762377i & -0.49429293 - 0.1435033i & 0.92650894 - 0.84611881i & -0.70455786 - 0.48409096i \\ 0.59037792 - 0.54091199i & 0.85788389 + 0.24560461i & 0.47476284 + 0.49301538i & -0.69627716 - 0.68994854i & 0.55997991 + 0.56031605i \\ 0.63311455 - 0.39776219i & 0.8913214 + 0.8528227i & 0.17073605 - 0.13321926i & -0.44129796 + 0.97878991i & 0.91049575 - 0.51784284i \end{pmatrix}, \quad (23)$$

$$t_3 = \begin{pmatrix} -0.56639054 + 0.95027621i & -0.52393862 + 0.04569261i & -0.09484941 - 0.76356329i & -0.5153158 + 0.49819619i & 0.61664959 - 0.59327422i \\ -0.30669642 + 0.68436901i & 0.56985208 - 0.00448519i & 0.67788516 - 0.30013223i & -0.53975312 - 0.525708i & 0.88646412 - 0.47259366i \\ -0.08189609 + 0.6883885i & -0.67302818 - 0.04206554i & 0.65960854 - 0.27529002i & -0.8154709 + 0.37974371i & 0.66780298 + 0.04558129i \\ 0.37393171 - 0.70087709i & -0.60133584 + 0.6991957i & 0.3181342 - 0.05591992i & 0.56664716 + 0.8622991i & 0.82282491 - 0.5321042i \\ 0.30922302 + 0.37900218i & -0.49970823 + 0.38016214i & 0.1028031 + 0.44109531i & 0.41420559 - 0.97487061i & 0.52013369 + 0.86341996i \end{pmatrix}, \quad (24)$$

$$t_4 = \begin{pmatrix} -0.89886607 + 0.06847595i & -0.9166851 + 0.49653758i & -0.53174656 + 0.89220242i & -0.72362803 + 0.11476783i & 0.74469864 + 0.499776i \\ -0.48299081 - 0.49399424i & -0.27648116 + 0.01925675i & -0.16821297 - 0.89373263i & 0.33594719 - 0.16372056i & 0.2663226 + 0.77684248i \\ 0.28205414 + 0.69985943i & 0.59756142 - 0.11459214i & -0.51297052 + 0.90461684i & -0.10778088 - 0.94646817i & 0.33354676 + 0.32009903i \\ 0.29740809 - 0.66640539i & 0.18472455 + 0.95000883i & -0.21045865 + 0.17550876i & 0.9077019 - 0.77096572i & -0.90006669 + 0.19303063i \\ -0.14091869 + 0.60143292i & 0.2484769 + 0.50801011i & 0.04669818 + 0.85133981i & 0.49710369 - 0.8538442i & 0.89424029 + 0.01437209i \end{pmatrix}, \quad (25)$$

$$t_5 = \begin{pmatrix} -0.92432482 + 0.03243803i & -0.3920592 + 0.52241285i & 0.96245486 + 0.14039386i & 0.30996196 + 0.84366345i & 0.28303399 - 0.22646471i \\ 0.60217582 + 0.84160544i & -0.72616155 - 0.05684327i & 0.01138053 - 0.50204865i & 0.08245395 - 0.85951913i & 0.13246862 - 0.09641901i \\ 0.73965747 + 0.30292888i & 0.75622971 + 0.28772264i & 0.18217603 - 0.57157653i & -0.74675744 - 0.34726472i & 0.39416056 + 0.7220469i \\ 0.95941996 - 0.67000463i & -0.7789017 + 0.20999963i & 0.75104373 - 0.19257495i & -0.97048715 - 0.63885318i & 0.9377695 + 0.83096135i \\ 0.28838462 + 0.43587651i & -0.06389155 - 0.16687494i & -0.54631375 + 0.29305552i & -0.47303696 + 0.519649i & -0.59851634 + 0.94455056i \end{pmatrix}. \quad (26)$$