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Master thesis

in Physics

submitted by

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born in Valencia

2021

A Phenomenological Analysis

of the $\triangle$EFT:

Type-2 Seesaw Effective Field Theory

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at the

Max-Planck-Institut für Kernphysik

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Eine phänomenologische Analyse der $\triangle$EFT: Type-2 Seesaw Effective Field Theory:

In dieser Arbeit wird eine phänomenologische Analyse der Erweiterung der type-2 seesaw mechanism effective field theory durchgeführt. Hierzu wird eine vollständige nicht-redundante Basis erzeugt und mit der Warschauer Basis verglichen. Anschließend werden drei unterschiedliche Ansätze genutzt, um die die verschiedenen Operatoren der effective field theory zu untersuchen und einzuschränken.

Erstens wird das Modell unter der Annahme untersucht, dass die Energien viel niedriger sind als die Skala der type-2 seesaw Masse sind, indem man ihren Beitrag zu den Wilson-Koeffizienten der Warschau-Basis betrachtet. Diese Analyse wird um die dimension-5 Operatoren der type-2 effective field theory erweitert. Der zweite Ansatz besteht darin, den Einfluss dieser Operatoren auf die Standardmodell-Kopplungen zu untersuchen. Zudem wurde die Möglichkeit untersucht, diese Operatoren zusammen mit der Warschauer Basis zu beschränken. Der letzte Ansatz beleuchtet den Einfluss einiger Operatoren auf die LHC-Suche nach neuen Teilchen. Durch diese Analysen werden einige neue, interessante Wechselwirkungen dieses Modells aufgezeigt und eine Analyse anhand von CMS-Daten durchgeführt.

Zum Abschluss dieser Arbeit wird ein kurzer Überblick über mögliche ultraviolette Ergänzungen dieses Modells diskutiert.

A Phenomenological Analysis of the $\triangle$EFT: Type-2 Seesaw Effective Field Theory:

In this thesis a phenomenological analysis of the type-2 seesaw mechanism effective field theory expansion is performed. A complete non-redundant basis is obtained and compared to the Warsaw basis. Then, three different approaches are used to study and constrain the different operators of the effective field theory.

Firstly, the model is studied assuming much lower energies than the scale of the type-2 seesaw mass by looking at its contribution to the Wilson coefficients of the Warsaw basis. This analysis is extended to include dimension-5 operators of the type-2 effective field theory. The second approach consists in probing the contribution of these operators to standard model couplings. We also study the possibility of constraining these operators along with the Warsaw basis. The last approach consists of studying the impact of some operators to LHC searches for new particles. Some new interesting interactions of this model are pointed out, and an analysis is carried out by using CMS data.

Finally, a brief overlook into possible ultraviolet completions of this model is discussed at the end of this thesis.

Contents

1 Introduction

Even though, we are in the era where more data is available for particle physics, and in where larger energies in colliders are achieved, we have little clue of what the new physics actually is. Only few a deviations here and there are popping up [1, 2], although not with enough significance yet to claim anything. In this landscape of great data and small deviations physicists have developed a set of tools that allow for indirect searches and a big independence of the possible new physics, that is Effective Field Theories (EFT).

On the other side, it is widespread to assume new physics must exist. This is due to different open questions of the Standard model. One of them being: why are neutrino masses so tiny compared to the rest of standard model particles? Since the discovery of neutrino oscillations, first pointed out by the Homestake experiment [3], and finally unambiguously observed by the SNO experiment [4], we know that neutrinos must have a mass, but it has always been an issue why these particles have a smaller mass with respect to the rest of SM particles. This has led to several standard model extensions that introduce new particles which suppress the mass of the neutrinos, a very famous one the seesaw mechanisms.

It is then an interesting idea to study these models in the framework of effective field theories. The difference between a "normal" quantum field theory and an effective one, is that we allow for terms in the Lagrangian that exceed the typical requirement of having dimension 4 (or d in a d-dimensional space-time). This is why they are called non-renormalizable theories, a bad name since they are at least "approximately renormalizable". These larger dimensional terms, called operators $\mathcal{O}$, found in the Lagrangian of equation 1.1 can be used for parameterizing new physics through its coefficients C, called Wilson coefficients.

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{D=4} + \sum_{i\geq 4} \mathcal{L}_i = \mathcal{L}_{D=4} + \sum_i \frac{C_i^{(D=5)}}{\Lambda} \mathcal{O}_i^{(D=5)} + \frac{C_i^{(D=6)}}{\Lambda^2} \mathcal{O}_i^{(D=6)} + \mathcal{O}(\Lambda^{-3}) \quad (1.1)$$

A typical way of understanding how new physics is parameterized by these higher-dimensional coefficients is by thinking of Fermi theory, which we will talk more in the following sections. In this theory of the weak interaction there was no gauge boson and science could still be performed, and measurements and predictions were made with these theories. This is because one does not need a heavy gauge boson at low momentum since, these particle would never be produced on-shell in for example the decay of the muon. So, processes such as this were thought as a 4-point fermion interaction, that is a dimension-6 operator. So, in that same way new physics could be creating sets of higher dimensional operators with the standard model particles, and, if we are able to constrain them, we could point to where to look. One classic example of dimension-5 operator is the Weinberg operator, eq. (1.2), which would produce neutrino masses.

$$\mathcal{L}_5 = \frac{C_5}{\Lambda}\mathcal{O}_5 = \frac{C_5}{\Lambda}(\tilde{\varphi}^\dagger l_p^c)^T(\tilde{\varphi}^\dagger l_r) \tag{1.2}$$

This operator is related to the mass of the neutrinos and could be produced, for example by a triplet $SU(2)_L$ scalar particle, which is known as type-2 seesaw mechanism. This is the model we will work with in this thesis, but we will also allow for higher-dimensional operators in our Lagrangian. In this case part of the usefulness of the EFTs will be lost, since as we will see, it is a very complex model, and simplifications are required if one wants to study its operators. On the other hand, we will also see that new couplings and phenomenology, potentially useful in collider searches also appears. That is why we will make a phenomenological study of the effective field theory extension of the type-2 seesaw mechanism.

This thesis is structured as follows: in section 1 we will review the standard model, as well as the neutrino current landscape. We will then briefly talk about the seesaw mechanisms and, in particular, about the type-2 seesaw mechanism, the model we will work with. We will then change topics to the discussion of effective field theories, sec. 1.3.1. Next we will start by writing down all the possible terms of this extension, 2 and move to make a phenomenological analysis of this basis 3, using three different approaches. Then in the final section 4 we will briefly discuss different models that could generate the operators of the type-2 effective field theory extension of this work.

1.1 The Standard Model

The Standard Model (SM) of particle physics is so far the best theory that describes nature in its most fundamental blocks. It is difficult to say when was the starting point, since several classical developments such as Maxwell's electromagnetic theory in 1865 [5] or the quantum mechanical theory of the electron by Dirac in 1928 [6] were crucial for the creation of quantum field theory. But certainly, we can point towards 1947-1949 as the breakthrough of the SM, when Feynman, Tomonaga, Schwinger and Dyson solved in a clear way the problem of divergences introducing renormalization. This led to many other theories that completed the standard model as we know nowadays, such as: the theory of Quantum Chromodynamics (QCD) [7, 8], the theory of the Electroweak (EW) interaction [9–11] and the most recent discovery the Higgs boson [12].

The SM is based on the Lagrangian formalism. Thus, we describe our theory in terms of the symmetry group respected in the Lagrangian and the matter content, that

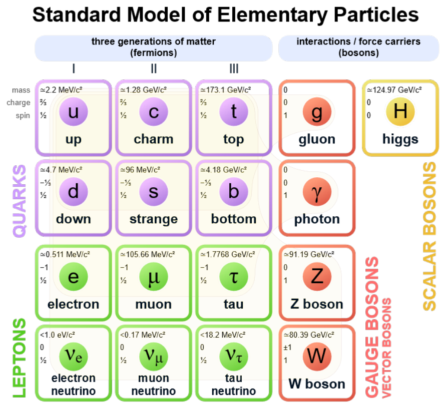


Figure 1.1: The matter content of the Standard Model. Source: Wikipedia.

is, what fields or particles are seen in nature. In the case of the SM the symmetry group is $SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\text{broken}} SU(3)_c \times U(1)_{\text{EM}}$. The first group represents the colour symmetry of QCD, the second one the $SU(2)$ group applied to left-handed particles and the global phase invariance $U(1)$ represented by the hypercharge Y. This symmetry group is broken, by the Spontaneous Symmetry Breaking (SSB) produced by the Higgs potential (we will see this later), giving mass to the SM fermions and gauge bosons of $SU(2)_L$, leaving invariant only one gauge boson: the photon.

The matter content, seen in fig (1.1) is organized in 2 blocks, bosons and fermions. Bosons are particles with an integer spin, following the Bose-Einstein distribution, and are the interaction carriers. Fermions have a half-integer spin, following the Fermi-Dirac, distribution. In the fermionic sector we find leptons and quarks, quarks have a colour charge and they interact with the strong force as well as with the electroweak force, however leptons only interact with the electroweak gauge bosons. Both families of fermions come in 3 generations, of increasing mass but equal in the other lepton numbers (except neutrinos, which have no mass in the traditional SM).

There are 8 gluons, due to the representation of the $SU(3)_c$ group and 4 EW gauge bosons: 3 massive bosons and a massless one, the photon. The last boson is the Higgs which by SSB gives mass to the SM.

1.1.1 Symmetries in the Standard Model

In order to set the notation that we will use throughout this thesis and define many of the useful operators, we will get into some of the details of the SM. First following [13]. The symmetries of the SM are given by the Lie Groups, represented in a general way as $SU(N)$. Apart from being a group, Lie groups must also be differentiable manifolds, and all the elements of this group are continuously connected to the identity, so they can be written as:

$$U = \exp(i\theta^a T^a)\mathbb{1} \tag{1.3}$$

T^a are the generators of the group and θ^a are parameters that set the position of the element of the group in the space of the generators. The generators are operators, normally matrices, that given U, one can find by expanding U close to $\mathbb{1}$. These generators form a Lie algebra, defined by the commutator:

$$\left[T^a, T^b\right] = if^{abc}T^c \tag{1.4}$$

Where f^{abc} is called the antisymmetric structure constant, and defines the algebra of the group. In $U(1)$, this constant is 0, and the gorup is Abelian, that is, it commutes. In $SU(2)$ this constant is the levi-civita symbol ε^{abc} and $T^a = \sigma^a/2$ where σ^a are the Pauli matrices. For $SU(3)$ the generators are the so-called Gell-Mann matrices $T^a = \lambda^a/2$ and the structure constant can be also obtained following eq. (1.4). One can choose different representations for T^a, the lower dimensional representations are called fundamental, such as the ones mentioned before, however, one can choose a different one, for example the adjoint representation $(T^a)_{bc} = if^{abc}$ (one can check, this representation fulfills the Lie algebra as well).

Fields in the SM transform following different representations, for example quarks transform under the fundamental representation of $SU(3)_c$ and the doublets of $SU(2)_L$ under the fundamental representation of this group. However, gauge fields transform under the adjoint representation of their respective symmetry. Furthermore, the laws of nature, for example Maxwell's theory of electromagnetism, are not globally invariant but locally invariant, which takes $\theta^a \to \theta^a(x)$, and makes it space-dependent. This is an important fact, since terms that contain derivatives will break the symmetry, so one introduces the covariant derivative:

$$D_\mu = \partial_\mu - igG^a_\mu T^a \tag{1.5}$$

Where g is just a constant and G^a_μ fields that will ensure that when D_μ is applied to a field ψ, if ψ transforms as $\psi \to U\psi$, then $D_\mu\psi \to UD_\mu\psi$ by transforming:

$$G^a \to G^a_\mu + \frac{1}{g}\partial_\mu\theta^a(x) - f^{abc}\theta^b(x)G^c_\mu \tag{1.6}$$

So this is the important point that we want to emphasize after this interlude into group theory: if we want to have local symmetries of some Lie group in our Lagrangian, we are forced to have some bosonic fields that transform as eq. (1.6). Thus, it is fair to say that symmetries impose the bosons that we need in our Lagrangian.

Just one more remark: there exists another important object called the strength field defined as:

$$F_{\mu\nu} \equiv \frac{i}{g}[D_\mu, D_\nu] = \partial_\mu G_\nu - \partial_\nu G_\mu + g f^{abc} G^b_\mu G^c_\nu T^a \tag{1.7}$$

With $G_\mu = G^a_\mu T^a$. This is not $SU(N)$ invariant, it transforms in the adjoint representation as $F^a_{\mu\nu} \to F^a_{\mu\nu} - \theta^b f^{abc} F^c_{\mu\nu}$. So, if one wants to write an invariant kinetic term for the gauge bosons in the Lagrangian, one can use the following invariant object $F^a_{\mu\nu} F^{a,\mu\nu}$.

1.1.2 The Standard Model Lagrangian

After the previous section we have all the tools to write a Lagrangian under the symmetry group of the SM. First we will start by introducing the fermion sector.

Fermion Sector

One symmetry that we have not talked about is Lorentz invariance. It is a requirement of all relativistic Lagrangians to be Lorentz invariant, in this way Dirac proposed [6] an equation that fulfilled this requirement and that managed to describe fermions (electrons in the original paper). This Lagrangian reads as follows:

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\mathbb{1}\right)\psi \tag{1.8}$$

ψ is the spinor field, a 4-dimensional vector that lives in the spinor space, and $\bar{\psi} = \psi^\dagger \gamma^0$. γ_μ are the Dirac matrices, which are 4-dimensional objects defined by the anticommuting property of $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ where $\eta^{\mu\nu}\mathbb{1} = \text{diag}\{1, -1, -1, -1\}$. And m is the mass of the Fermion. All these objects and definitions are imposed by the requirement of having Lorentz invariance and also reproducing the Klein-Gordon equation.

The spinor field ψ and γ_μ have different representations: Dirac representation, Weyl representation or Majorana representation. For the first one we have:

$$\gamma^0 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{pmatrix}, \gamma^2 = \begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & \sigma^3 \\ -\sigma^3 & \end{pmatrix} \tag{1.9}$$

One can define a fifth matrix, which anticommutes with the other four $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ $\{\gamma^5, \gamma^\mu\} = 0$ and this allows us to define the projector:

$$P_L = \frac{1}{2}\left(\mathbb{1} - \gamma^5\right) \quad P_R = \frac{1}{2}\left(\mathbb{1} + \gamma^5\right) \tag{1.10}$$

These projectors satisfy the properties: $P^2_{L/R} = P_{P/R}$, $P_L P_R = 0$ and $P_R + P_L = \mathbb{1}$ and the important thing about these operators is that they allow us to write the spinor field ψ into two different chirality fields: $\psi_{L/R} = P_{L/R}$ with $\gamma^5\psi_{L/R} = \mp\psi_{L/R}$. So if we apply $\mathbb{1}\psi = (P_L + P_R)\psi = \psi_L + \psi_R$ we can rewrite the Lagrangian (1.8) into:

$$\mathcal{L}_{\text{Dirac}} = i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R - m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) \tag{1.11}$$

Experiments, such as Wu's experiment [14], tell us that nature does not treat in the same way left-handed and right-handed particles particles, this is seen in the symmetry group only applied to the left-handed particles: $SU(2)_L$.

Now we want to focus on making the Lagrangian invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y$, under which fermions would transform in the fundamental representation:

$$\psi \longrightarrow U\psi = e^{iY\alpha(x)}e^{i\beta^a(x)\sigma^a/2}e^{i\theta^a(x)T^a}\psi \tag{1.12}$$

In order to do this we replace the derivative with:

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - ig'YB_\mu - igW^i_\mu\frac{\sigma^i}{2}P_L - ig_sG^a_\mu T^a \tag{1.13}$$

Where Y is the hypercharge operator, since different fields may have different hypercharge. After making this replacement kinetic terms will preserve the symmetry, left-handed quarks will interact with the three fields, left-handed leptons only with B_μ and W^i_μ, and so forth and so on.

So the Lagrangian can be written as:

$$\mathcal{L} = i\bar{\psi}^a_L\gamma^\mu\partial_\mu\psi^a_L + i\bar{\psi}^a_R\gamma^\mu\partial_\mu\psi^a_R - m_a\left(\bar{\psi}^a_L\psi^a_R + \bar{\psi}^a_R\psi^a_L\right) \tag{1.14}$$

Where we include an index a to ψ^a to account for all the particles in the SM, that we can find described by their quantum numbers in Table 1.1.

Before finishing this section, the mass term of eq. (1.14) is not invariant. One can see this very cearly, since ψ_L transforms under $SU(2)_L$ while ψ_R does not. So, this term is not invariant and one could think that under this symmetry group fermions could not have mass. We will later see that this is solved by introducing a scalar doublet, the Higgs boson. So, right now we will define the fermionic Lagrangian as:

$$\mathcal{L}_{\text{Fermionic}} = i\bar{\psi}^a_L\gamma^\mu\partial_\mu\psi^a_L + i\bar{\psi}^a_R\gamma^\mu\partial_\mu\psi^a_R \tag{1.15}$$

Gauge Sector

With the tools of the previous section about symmetries, we can formulate an invariant Lagrangian for the gauge bosons:

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}\left(W^i_{\mu\nu}\right)^2 - \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 - \frac{1}{4}\left(B_{\mu\nu}\right)^2 \tag{1.16}$$

	Fields	$SU(3)_c$	$SU(2)_L$ Isospin	$U(1)_Y$ Hypercharge
Quarks	$q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	Triplet $q = \begin{pmatrix} q_r \\ q_g \\ q_b \end{pmatrix}$	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	1/6
	$u = u_R$	Triplet $u = \begin{pmatrix} u_r \\ u_g \\ u_b \end{pmatrix}$	Singlet	2/3
	$d = d_R$	Triplet $d = \begin{pmatrix} d_r \\ d_g \\ d_b \end{pmatrix}$	Singlet	-1/3
Leptons	$l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	Singlet	$\begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$	-1/2
	$e = e_R$	Singlet	Singlet	-1

Table 1.1: Fields of the SM and their representation under the different symmetry groups. One can also find in this table the values of the eigenvalues of the isospin I_3 and hypercharge Y operators. The subindices L and R describe the chirality state of the fermion $\psi_{L/R} = P_{L/R}\Psi$.

Where the $B_{\mu\nu}$, $W^i_{\mu\nu}$ and $G^a_{\mu\nu}$ are the respective field strength tensors for the electroweak and gluon fields, which can be written more concretely like:

$$B^2_{\mu\nu} = (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \tag{1.17}$$

$$\left(W^i_{\mu\nu}\right)^2 = \left(\partial_\mu W^i_\nu - \partial_\nu W^i + g\varepsilon^{ijk} W^j_\mu W^k_\nu\right)^2 \tag{1.18}$$

$$\left(G^a_{\mu\nu}\right)^2 = \left(\partial_\mu G^a_\nu - \partial_\nu G^a + g f^{abc} G^b_\mu G^c_\nu\right)^2 \tag{1.19}$$

Two important remarks: the difference between an Abelian group such as $U(1)_Y$ and non-Abelian such $SU(2)$ or $SU(3)$ in terms of physics is that these two last groups, will have bosons interacting among themselves. This is due to the noncommutation of the fields, which produces the last terms of the previous two expressions resulting in 3- and 4-particle bosonic vertices.

The second point is that mass terms like $m_W W^i_{\mu\nu} W^{i,\mu\nu}$ are forbidden by the $SU(2)_L$ symmetry. However, in nature we observe 3 massive gauge bosons. So, this problem along with the problem with the masses of the fermions bring us to the next point, the Higgs boson.

Higgs sector

The two problems mentioned above can be solved by introducing a scalar field which is a doublet under $SU(2)_L$, singlet under colour and hypercharge 1/2. The Lagrangian of this field would be written as:

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \varphi)^\dagger D^\mu \varphi - V(\varphi) \tag{1.20}$$

φ is the Higgs doublet and $V(\varphi)$ is the Higgs potential:

$$V(\varphi) = -m_\varphi^2 |\varphi|^2 + \lambda |\varphi|^4 \tag{1.21}$$

This potential has the famous Mexican hat shape. This potential has a degeneracy around the minimum, but the field configuration of the minimum is different at each point. Furthermore, the minimum does not respect the symmetry of $SU(2)_L$. We call vacuum expectation value $v_d = \sqrt{m_\varphi^2/\lambda} \approx 246$ GeV to the value of the field at the minimum and we choose it to be along the direction of the neutral component of the field (this is not arbitrary, it will ensure that the photon field remains massless).

$$\varphi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v_d + h) \end{pmatrix} \tag{1.22}$$

$$\begin{aligned} \mathcal{L}_{Higgs} = \frac{1}{2}\frac{(v_d+h)^2}{4}\left(g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-gW_\mu^3 + g' B_\mu)^2\right) + (\partial_\mu h)^2 \\ -\frac{m_\varphi^2}{2} h^2 - \lambda v_d h^3 - \frac{\lambda}{4} h^4 \end{aligned} \tag{1.23}$$

Two remarkable things have happened, first, the quadratic term on h now has the proper sign to be a mass term $m_\varphi \approx 125$ GeV (before, it was the opposite) and now the gauge fields have also acquired a mass, but we must make the following redefinitions:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \tag{1.24}$$

$$\begin{pmatrix} Z_\mu^0 \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \tag{1.25}$$

Where $\cos\theta_w = g/(g^2 + g'^2)$ and we also define $e = g \sin\theta_w$. Now we see that the terms in equation eq. (1.23) are masses of the gauge bosons $m_W = v_d\, g/2$ and $m_Z = v_d \sqrt{g^2 + g'^2}/2$. Thus, we have obtained a mass term for the gauge bosons in a $SU(2)_L$ invariant way through the SSB mechanism. We also observe that all except one generators of $SU(2)_L \times U(1)_Y$ have been broken:

$$T^1 \langle\varphi\rangle \neq 0,\ T^2 \langle\varphi\rangle \neq 0,\ (T^3 - Y) \langle\varphi\rangle \neq 0,\ (T^3 + Y) \langle\varphi\rangle = 0 \tag{1.26}$$

So we define a new operator, associated to the gauge boson A_μ called charge: $Q \equiv T^3 + Y$ that is a conserved quantity, since the vacuum expectation value of the Higgs doublet field $\langle\varphi\rangle$ conserves this quantity. We can now make the identification of the field A_μ with the photon field. One last remark, the ratio of the mass of the gauge bosons is a well measured parameter in the standard model [15]:

$$\rho = \frac{M_w^2}{M_Z^2 \sin\theta_w} = 1.00038 \pm 0.00020 \tag{1.27}$$

This ratio is modified if other particles contribute to the masses of the gauge bosons in a different way, being a really helpful parameter to constrain possible new physics. The Higgs field also solves the problem of the fermionic masses, by introducing what is called a Yukawa coupling:

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= -Y_u \bar{q}\,\tilde{\varphi} u_R - Y_d \bar{q}\,\varphi d - Y_L \bar{l}\,\varphi e_R + \text{h.c} \\ &\xrightarrow{SSB} -\frac{v_d}{2}\bar{u}_L Y_u u_R - \frac{v_d}{2}\bar{d}_L Y_d d_R - \frac{v_d}{2}\bar{e}_L Y_E e_R + \text{h.c.} \end{aligned} \tag{1.28}$$

Here we have defined $\tilde{\varphi} = i\sigma^2\varphi^*$. These terms would be normal mass terms if Y_u were diagonal in flavour space, but actually that is not generally the case. If we think of u, for instance, as $u = (u, c, t)$ we have to diagonalize the matrix Y_u by the unitary transformation $u_{L/R} \to V_{u_{L/R}} u_{L/R}$ and similarly for the other terms. This diagonalization has a physical consequence: the vertices of the weak interaction mix u and d-type quarks of different generations through the terms:

$$\mathcal{L}_{Weak} \supset \frac{g}{\sqrt{2}}\bar{u}_L V_{u_L}^\dagger V_{d_L} W_\mu^+ \gamma^\mu d_L + \frac{g}{\sqrt{2}}\bar{d}_L V_{d_L}^\dagger V_{u_L} W_\mu^- \gamma^\mu u_L \tag{1.29}$$

Where we define the CKM matrix (named after Cabbibo, Kobayashi and Maskawa [16, 17]) $V_{CKM} \equiv V_{u_L}^\dagger V_{d_L}$. This matrix is not diagonal, so it implies that the weak interaction mixes the different quark families. Since the Z-boson does not couple up-type quarks and down-type quarks, the rotations are canceled due to unitarity. We also observe that if we do not have massive neutrinos, we could make the necessary rotation to diagonalize the electron Yukawa matrix, without any impact in the weak sector, since the neutrinos could be rotated freely as they would not have a Yukawa term. However, this is not the case, neutrinos do have a mass, so the same phenomenon happens in the lepton sector, where there is also mixing between generations. This is the topic of the following section.

1.2 Neutrino Physics

1.2.1 Neutrino Oscillations

The observation of neutrino oscillations makes it clear, neutrinos have masses and they must be different. There is a simple, but 'wrong', derivation that we will follow. Using quantum mechanics it is simple to describe how an oscillating state evolves in time. Let us call $|\nu_\alpha\rangle$, with $\alpha = e, \mu, \tau$ the neutrino flavour state and $|\nu_i\rangle$, with $i = 1, 2, 3$ the neutrino mass state. Assuming, the neutrino starts in a definite flavour state, the evolution is given by:

$$|\nu_\alpha\rangle(t) = U^*_{\alpha,k} e^{-iE_k t} |\nu_k\rangle \tag{1.30}$$

Where we have projected the flavour state onto the mass basis, through the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) matrix U_{PMNS} (Which we will write U). To calculate the probability of finding a flavour β we use Born's rule:

$$P_{\alpha\to\beta}(t) = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = |U_{\beta k} U^*_{\alpha k} e^{-iE_k t}|^2 = U^*_{\beta j} U_{\alpha j} U_{\beta k} U^*_{\alpha k} e^{-i(E_k - E_j)t} \tag{1.31}$$

The energy of a relativistic particle is given by $E_j = \sqrt{p_j^2 + m_j^2}$, and by using that $m_j^2 \ll p_j^2$ and $p_j \approx p_k \equiv p$ we can simplify the result of the argument of the exponential:

$$-(E_k - E_j) \approx -(p^2 + \frac{m_k^2}{2p^2} - p^2 - \frac{m_j^2}{2p^2}) \approx \frac{m_k^2 - m_j^2}{2E} \equiv \frac{\Delta m_{kj}^2}{2E} \tag{1.32}$$

We make now two last replacements: the trivial $t \sim L$ where L is the distance traveled by the neutrino and the second replacement $L_{kj}^{\text{osc}} = \frac{4\pi E}{m_{kj}^2}$, which is called oscillation length. This finishes our derivation of the formula for the probability of oscillation of a neutrino α into β:

$$P_{\alpha\to\beta}(L) = U^*_{\beta j} U_{\alpha j} U_{\beta k} U^*_{\alpha k} e^{-i2\pi L/L_{kj}^{\text{osc}}} \tag{1.33}$$

If the traveled length is small compared to the oscillation length of the neutrino, then we do not observe any oscillation, since the state did not have time to oscillate. However, if it is large we may not be able to resolve the frequency of oscillation and we will only observe an average decrease (or increase) on the number of neutrinos. But, since there are different oscillation lengths it may happen that one of them is larger than the other two. In this case we could neglect the oscillation to one of the other neutrino species. Let us write in the 2-neutrino case:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{1.34}$$

And then simplify eq. (1.33):

$$P_{\alpha\to\beta}(L) = \sin^2 2\theta \sin^2\left(\pi \frac{L}{L^{\mathrm{osc}}}\right) = \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2[\mathrm{eV}^2]L[\mathrm{km}]}{E[\mathrm{GeV}]}\right) \tag{1.35}$$

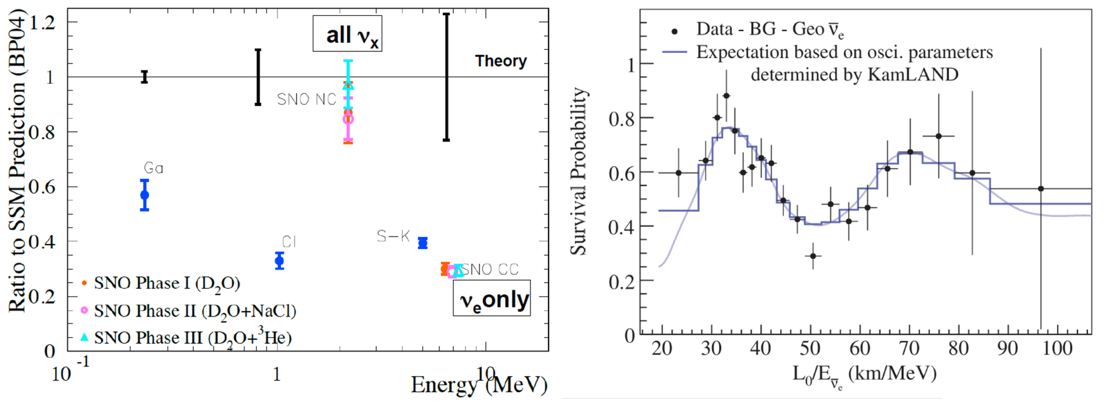
For $\beta \neq \alpha$. This formula works in the 2-neutrino case, which is valid when the oscillation lengths are widely separated. We can see now, that if the masses are the same we would have no oscillations of this kind. Besides the mass, we need the mixing angle θ to be different than zero. The second equality is written in the typical scale of neutrino experiments.

These type of oscillations were first pointed out by the Homestake experiment [3] which measured a decrease in the electron neutrino flux coming from the sun with respect to the predictions, initiating what was called the solar neutrino problem. This experiment could not tell whether the neutrinos measured came from the sun or not, so other experiments were designed for that, e.g. Super-Kamiokande [18], DayaBay [19], Sudbury Neutrino Observatory [4], being this last experiment the one to measure not only a decrease in the flux of electron neutrinos, but also to measure all three flavours. This showed that the total number of neutrinos was conserved, but the number of electron neutrinos was lower than expected. This experiment established the three-neutrino oscillation theory and other neutrino experiments took enough data to determine the values of the parameters of the PMNS matrix. For, the three-neutrinos oscillation we can parametrize the PMNS matrix as:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.36}$$

In Table 1.2 we can find the values of the fit done with a large collection of data performed in Ref. [20]. Δm_{21} is measured from solar neutrinos, that is why it is also called Δm_{sol}. On the other hand, $\Delta m_{32} \approx \Delta m_{31}$ is also called Δm_{atm} since it is measured from atmospheric neutrinos, which have the appropriate energies and length to be measured. The last parameter δ is the CP -hase, similar to the CP phase in the complex CKM matrix.

However, we find that in Table 1.2, there are things that these measurements cannot tell us. One of them is the order of the mass state, this is known as the neutrino hierarchy problem. Experiments can tell the value of the mass difference but not the sign, so it is still an open question whether m_3 is larger than m_1, Normal Hierarchy (NH), or whether m_1 is larger m_3, Inverted Hierarchy (IH). The second one is the mass of the states themselves, we just measure their differences, so the actual value of the mass is yet unknown and other experiments that do not rely on oscillations must determine their value, e.g. the Karlsruhe Tritium Neutrino Experiment (KATRIN) [25].



(a) (b)

Figure 1.2: In (a), we have a summary of neutrino oscillation experiments comparing the ratio of observed events and predicted events. Here Neutral Current (NC) processes are in good agreement whereas Charged Current (CC) processes are fewer than expected. In (b) we observe typical oscillation pattern, in this case from KamLand experiment [22], where antineutrinos are detected 180 km away from nuclear reactors.

The mass of the neutrino has not been measured yet, but some upper bounds have been stablished. For some time the best upper bound on the sum of the mass of the three neutrinos has come from cosmology [21], on $\sum_i m_i < 0.2$ eV at 95% CL, but now KATRIN [26] experiment has been able to set a sub-eV upper bound of $\sum_i m_i < 0.8$ eV by direct measurement and aims to set it lower in the future.

Parameter	best fit $\pm 1\sigma$
$\Delta m_{21} [10^{-5}\, eV^2]$	$7.50^{+0.22}_{-0.20}$
$\vert\Delta m_{31}\vert [10^{-3}\, eV^2]$	$2.55^{+0.02}_{-0.03}$
$\vert\Delta m_{31}\vert [10^{-3}\, eV^2]$	$2.45^{+0.02}_{-0.03}$
$\sin^2\theta_{12}/10{-}1$	3.18 ± 0.16
$\sin^2\theta_{23}/10^{-1}$ (NH)	5.74 ± 0.14
$\sin^2\theta_{23}/10^{-1}$ (IH)	$5.78^{+0.10}_{-0.17}$
$\sin^2\theta_{13}/10^{-2}$ (NH)	$2.200^{+0.069}_{-0.062}$
$\sin^2\theta_{13}/10^{-2}$ (IH)	$2.225^{+0.064}_{-0.070}$
δ/π (NH)	$1.08^{+0.13}_{-0.12}$
δ/π (IH)	$1.58^{+0.15}_{-0.16}$

Table 1.2: Best fit values and 1σ deviations of the experimental measurements of neutrino oscillations of Ref. [20].

In Figure 1.2a we find a summary of the measured number of events divided by the expected number of neutrinos performed by different experiments, Gallex, Super-Kamiokande or SNO. We observe that measurements performed on only one flavour (through CC) are fewer than expected but, when we observe processes that are sensitive to all flavours of neutrinos, such as NC processes, we find the expected amount of them, as shown in the plot by the SNO experiment. The second one, Fig. 1.2b is a typical oscillation pattern in the flux of electron antineutrinos. In this case the Kamioka Liquid Scintillator Argon Antineutrino Detector (KamLAND), detected antineutrinos produced in nuclear power plants at 180 Km, being sensitive to the θ_{13} angle.

As a final remark, we want to point out that even if the equation (1.33) is 'correct', the derivation is not. There are several issues on this derivation, all of them coming from the approximations. This rises several problems as the conservation of energy, momentum and other apparent paradoxes (well explained in Ref. [23]). However, this is just a problem of the simple 'proof' we have used, more fine derivations use wave packets and allow neutrinos to travel with different momenta and take into account the uncertainty in the production. These considerations modify the previous eq. (1.33), by adding some suppression consisting in what is called coherence. This coherence is represented by the coherence length: neutrinos with different masses travel at different velocities, thus, different mass states will propagate a different distance. When the distance the neutrinos have traveled is larger than the coherence length, the neutrinos do not overlap anymore and detectors cannot resolve these oscillations.

$$P_{\alpha\to\beta}(L) = U^*_{\beta j} U_{\alpha j} U_{\beta k} U^*_{\alpha k} \exp\left\{ -i2\pi \frac{L}{L^{\text{osc}}_{kj}} - \left(\frac{L}{L^{coh}_{kj}}\right)^2 - c\left(\frac{\sigma_x}{L^{\text{osc}}_{kj}}\right)^2 \right\} \quad (1.37)$$

Equation (1.37) contains all the corrections due to the correct wave-packet treatment. In this equation σ_x is the spatial uncertainty of the neutrino production and c is an unimportant constant for us. The second term is called localization term, and suppresses the oscillations in which the spatial uncertainty is larger than the oscillation lengths, which never happens in real experiments.

1.2.2 Type-I Seesaw Mechanism

Although we will work with the type-2 seesaw mechanism, it is interesting for pedagogical reasons to cover the key points of the type-1 seesaw mechanism. These mechanisms try to explain why the neutrino masses are much smaller than the rest of the particles of the SM. Both of this models attempt to do it by introducing a new kind of particle, a right-handed neutrino for the type-1 mechanism and a complex scalar particle, triplet under $SU(2)_L$, for the type-2 mechanism.

As we have pointed out, neutrinos have only been observed to be left-handed, but since we observe all fermions to have left and right components, we could try to extend the SM by adding a right-handed neutrino and a coupling with the SM Higgs particle. For one neutrino we could introduce a term such as:

$$\mathcal{L}^{\text{SM+RH}\nu} = -Y\bar{L}_L\varphi l_R - Y\bar{L}_L\tilde{\varphi}N_R + \text{h.c.} \tag{1.38}$$

This is a perfect valid extension and would lead to a neutrino mass term $-m_D\bar{\nu_L}\nu_R$, albeit it does not explain why neutrino masses are so tiny, since the value Y would need to be much smaller than the rest of the yukawa couplings in the SM so that $m_D = v_d Y/\sqrt{2} < 0.2$ eV, $Y < 1.2 \cdot 10^{-12} \ll y_{\text{top}} \sim 1$. However, if neutrinos were Majorana particles there would be an extra term in the Lagrangian that could explain this fact. We define the charge conjugate operator as: $\psi^c = \xi_c C\bar{\psi}^T$, and then we place Dirac equation in terms of the chiral components of the spinors:

$$i\gamma^\mu\partial_\mu\psi_R = m\,\psi_L \tag{1.39}$$

We can now take this expression and try to leave it as the definition of the charge conjugated spinor given above. Then we apply the charge conjugate and obtain:

$$i\gamma^\mu\partial_\mu C\bar{\psi}_R^T = m\,C\bar{\psi}_L^T \tag{1.40}$$

It is important to note that $C\bar{\psi}_L^T$ transforms as a right-handed component, so this whole expression can be thought as $i\gamma^\mu\partial_\mu\psi_L = m\,\psi_R$ which motivates the Majorana condition $\psi_R = \xi_c C\bar{\psi_L}^T$. This condition also implies that $\psi = \psi_L + \psi_R = \psi_L + \xi_c C\bar{\psi_L}^T$ and by using the definitions of the charge conjugate operator $\psi = \psi^c$, where ξ_c is a complex phase $|\xi_c|^2 = 1$. A Majorana neutrino is then its own charge conjugate, and it can only be true if the fermion is neutral, so neutrinos are the only particles that can be Majorana particles. As pointed out before, in this case we can add an extra term in the lagangian

$$\begin{aligned}\mathcal{L}^{\text{type-1}} &= -Y\bar{L}_L\varphi l_R - Y\bar{L}_L\tilde{\varphi}N_R - \frac{m_R}{2}\bar{N}_R C^\dagger N_R + \text{h.c.}\\ &\overset{SSB}{\supset} m_D\bar{\nu}_L N_R + \frac{m_R}{2}N_R^T C^\dagger N_R + \text{h.c.}\end{aligned} \tag{1.41}$$

And by assuming the neutrino is a majorana particle we can use the property: $\bar{\nu}_L N_R = \overline{N_R^C}\nu_L^C$ to write the previous equation as:

$$\mathcal{L}_\nu^{\text{type-1}} = -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \overline{N_R^C}\end{pmatrix}\begin{pmatrix}0 & m_D\\ m_D & m_R\end{pmatrix}\begin{pmatrix}\nu_L^C\\ N_R\end{pmatrix} \tag{1.42}$$

We could have added a diagonal term corresponding to a Majorana left-handed mass term, however this would be only possible by introducing a complex triplet scalar particle (see the next section). Now, by introducing a unitary matrix to make the mass matrix diagonal:

$$\mathcal{L}_{\nu}^{\text{type-1}} = -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \overline{N_R^C}\end{pmatrix} U^\dagger U \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} U U^\dagger \begin{pmatrix}\nu_L^C \\ N_R\end{pmatrix}$$
$$= -\frac{1}{2}\begin{pmatrix}\bar{\nu}_1 & \overline{\nu_2^C}\end{pmatrix} \text{diag}\{m_\nu, M_R\} \begin{pmatrix}\nu_1^C \\ \nu_2\end{pmatrix} \text{ with } U = \begin{pmatrix}\cos\theta_\nu & \sin\theta_\nu \\ -\sin\theta_\nu & \cos\theta_\nu\end{pmatrix} \quad (1.43)$$

With the eigenvalues: $\frac{1}{2}(m_R \mp \sqrt{m_R^2 + 4m_D^2})$ and $\tan\theta_\mu = 2m_D/m_R$. Now, if we use the following assumption $m_R \gg m_D$ we could write the masses as $m_\nu \approx m_D^2/m_R$ and $M_R \approx m_R$. Furthermore, since $m_D = v_d Y/\sqrt{2}$ we could rewrite the eigenvalue of the mass $m_\nu = \frac{v_d}{\sqrt{2}} Y \frac{m_D}{m_R}$ and $Y\frac{m_D}{m_R}$ would act as an effective Yukawa coupling suppressed by m_R. The states $\nu_1^C = \cos\theta_\nu \nu_L^C - \sin\theta_\nu N_R \approx \nu_L^C$ and similarly $\nu_2 \approx N_R$, so the eigenstates would be composed mainly by a left-handed and a right-handed neutrino respectively. This kind of mechanism can be extended to 3-neutrino models, see for example [24].

This extension of the standard model offers a way of creating light neutrino masses through a very massive right-handed neutrino that suppresses the mass of the neutrinos, this effect is described by the typical image of the seesaw, Fig. 1.3, where the heavy neutrino sits on one side rising the lighter neutrino on the other side.

Figure 1.3: Typical image representing the seesaw mechanism, where the heavy right-handed neutrino lifts the lighter left-handed neutrino. Source: Symmetry Magazine.

1.2.3 Type-II Seesaw Mechanism

Even though the type-1 seesaw mechanism gives a reasonable explanation to the smallness of the neutrino masses, it is not the only model available. There are many, but one that only requires adding one more extra particle is the so-called type-2 seesaw mechanism. In this case we add a complex triplet scalar, with hypercharge $Y = 1$. This allows us to write a Majorana mass term of the type $\sim \bar{\nu}_L C^\dagger \nu_L$ in a $SU(2)_L$ invariant way, which is not possible with the SM matter content since it would not respect the $U(1)_Y$ symmetry.

This new field could be written as Δ^i for $i = 1, 2, 3$, transforming under the adjoint representation of $SU(2)_L$. However, this is not the most useful form of the triplet, since it is not diagonal to the weak isospin operators. The triplet could be written in a matrix form by summing its components with the generators of the symmetry, in this case the Pauli matrices.

$$\Delta(1, 3, +1) = \frac{\sigma^i}{\sqrt{2}}\Delta^i = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & \Delta^+/\sqrt{2} \end{pmatrix} \tag{1.44}$$

With the redefinition of the fields $\Delta^+ = \Delta^3$, $\Delta^{++} = \frac{1}{\sqrt{2}}(\Delta^1 - i\Delta^2)$ and $\Delta^0 = \frac{1}{\sqrt{2}}(\Delta^1 + i\Delta^2)$.

In this matrix form we can check that the triplet transforms as $\Delta \overset{SU(2)_L}{\rightarrow} U_L \Delta U_L^\dagger$ where U_L corresponds to the doublet transformation of the gauge group $SU(2)_L$. In this way it is simple to find the possible terms that respect the gauge group of the standard model $SU(2)_L \times U(1)_Y$ (note that this field has no colour charge).

To be able to make singlet combinations with this field, one will need to either multiply it with two doublet fields or to use the trace. For example the kinetic term can be written like this:

$$\mathcal{L}_{kin} \supset \text{Tr}\left[(D^\mu \Delta)^\dagger (D_\mu \Delta)\right] \tag{1.45}$$

With the covariant derivative given as:

$$D_\mu \Delta = \partial_\mu \Delta - i\frac{g}{2}\left[W_\mu^a \sigma^a, \Delta\right] - i\frac{g'}{2} B_\mu \Delta \tag{1.46}$$

It will be important for the following section to check some properties of the this derivative. As a derivative this should follow the Leibniz rule $\partial_\mu(\Delta^\dagger \Delta) = (D_\mu \Delta)^\dagger \Delta + \Delta^\dagger D_\mu \Delta$. However this is not true. This property is not true, since by inserting eq. (1.46) we obtain:

$$(D_\mu \Delta)^\dagger \Delta + \Delta^\dagger D^\mu \Delta = \partial_\mu(\Delta^\dagger \Delta) - i\frac{g}{2}\left[W_\mu^a \sigma^a, \Delta^\dagger \Delta\right] \tag{1.47}$$

Only combinations of fields that form a singlet fulfill the Leibniz rule, so the correct statement of the Leibniz rule for the triplet would be $\partial_\mu \operatorname{Tr}\big(\Delta^\dagger\Delta\big) = \operatorname{Tr}\big((D_\mu\Delta)^\dagger\Delta\big) + \operatorname{Tr}\big(\Delta^\dagger D_\mu\Delta\big)$. This is true as well in for example combinations with the Higgs field:

$$\partial_\mu(\tilde{\varphi}^\dagger\Delta^\dagger\varphi) = (D_\mu\tilde{\varphi})^\dagger\Delta^\dagger\varphi + \tilde{\varphi}^\dagger(D_\mu\Delta)^\dagger\varphi + \tilde{\varphi}^\dagger\Delta^\dagger D_\mu\varphi \tag{1.48}$$

As said, this property will be needed in the following sections. By now let us continue by writing the potential of the Lagrangian, combination of triplet ad doublet scalar fields:

$$\begin{aligned} V(\varphi,\Delta) = &-m_\varphi^2\varphi^\dagger\varphi + \mathbf{M}^2\mathrm{Tr}\Delta^\dagger\Delta + (\mu\varphi^T i\sigma^2\Delta^\dagger\varphi + \text{h.c.}) + \frac{\lambda}{4}(\varphi^\dagger\varphi)^2 \\ &+\lambda_1\varphi^\dagger\varphi\mathrm{Tr}\Delta^\dagger\Delta + \lambda_2(\mathrm{Tr}\Delta^\dagger\Delta)^2 + \lambda_3\mathrm{Tr}(\Delta^\dagger\Delta)^2 + \lambda_4\varphi^\dagger\Delta\Delta^\dagger\varphi \end{aligned} \tag{1.49}$$

All these couplings are in principle free parameters of the theory, however one can require different constraints in order to reduce the possible parameter space. Usually we demand these couplings to respect perturbative unitarity and to be bounded from below [27, 28], this last requirement is called vacuum stability [28]. A review of the model and a list of this constraints can be found in Ref. [29]. Notice that the μ parameter can be complex, however, its phase can be absorbed in Δ.

This potential, eq. (1.49), modifies some parameters of the SM, since it introduces a new vacuum expectation value (vev): $\langle\Delta^0\rangle = \frac{v_t}{\sqrt{2}}$ different from the Higgs vev $\langle\varphi^0\rangle = \frac{v_d}{\sqrt{2}}$. For instance, the first modified value would be the Higgs mass, since after SSB we would obtain the following masses:

$$m_\varphi^2 = \frac{\lambda}{4}v_d^2 + \frac{\lambda_1+\lambda_4}{2}v_t^2 - \sqrt{2}\mu v_t \tag{1.50}$$

$$M^2 = -\frac{\lambda_1+\lambda_4}{2}v_d^2 - (\lambda_2+\lambda_3)v_t^2 + \frac{\mu v_d^2}{\sqrt{2}v_t} \tag{1.51}$$

As we will see later, the vacuum expectation value of the triplet will be required to be smaller than v_d so the modification to the Higgs mass is not necessarily large. On the other hand, we see that the smaller v_t is, the larger its mass could be. The next modifications with respect to the SM come from the kinetic term in eq. (1.45), introducing a contribution to the mass of the weak bosons: $m_W^2 = g^2(v_d^2 + 2v_t^2)/4$ and $m_Z^2 = (g^2 + g'^2)(v_d^2 + 4v_t^2)/4$, this modification sets the upper bound for the vev of the triplet through the measurement of the ρ parameter, eq. (1.27):

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_w} = 1 - \frac{2v_t^2}{v_d^2 + 4v_t} \tag{1.52}$$

Which sets the upper bound on $v_t \leq 4.8$ GeV [29], this makes the vacuum expectation value $v^2 \equiv v_d^2 + 2v_t^2 \sim (246)^2$ GeV^2 to come mainly from the Higgs doublet.

Another important modification is the mixing of the scalar particles. There are 10 degrees of freedom, 3 of which go to the $W^{\pm}$- and Z-bosons, the other 7 are real particles. To make this explicitly we write φ and Δ after SSB:

$$\varphi = \begin{pmatrix} \varphi^+ \\ \frac{v_d + h + iz_d}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad \Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \frac{v_t + H + iz_t}{\sqrt{2}} & \frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \tag{1.53}$$

Since there is no doubly-charged scalar in the Higgs doublet, the $H^{\pm\pm} \equiv \Delta^{\pm\pm}$ does not mix with SM particles and they are purely coming from the triplet, with the following mass:

$$m_{H^{++}}^2 = \frac{\mu v_d^2}{\sqrt{2} v_t} - \frac{\lambda_4}{2} v_d^2 - \lambda_3 v_t^2 \tag{1.54}$$

The other scalar particles mix among each other and due to the diagonalization of their respective mass matrices, one obtains the following combinations

$$\begin{pmatrix} H^+ \\ G^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Delta^+ \\ \varphi^+ \end{pmatrix} \tag{1.55}$$

$$\begin{pmatrix} A \\ G^0 \end{pmatrix} = \begin{pmatrix} \cos\beta' & -\sin\beta' \\ \sin\beta' & \cos\beta' \end{pmatrix} \begin{pmatrix} z_t \\ z_d \end{pmatrix} \tag{1.56}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_t \\ h_d \end{pmatrix} \tag{1.57}$$

We have denoted by G the Goldstone bosons, H^+ the singly-charged scalar particle, A is a CP-odd neutral scalar, H is a new CP-even neutral Higgs particle and finally h is the Higgs particle that we know at $\sim 125\,\text{GeV}$. Since the triplet vev is so small in comparison $\tan\beta = \frac{\sqrt{2}v_t}{v_d} \approx 0$ and $\tan\beta' = \frac{2v_t}{v_d} \approx 0$ the mixing between the charged and CP-odd Higgs is negligible. This is something we should have expected: the triplet vev is constrained by the ρ parameter, thus the mass of the EW bosons comes mainly from the doublet. This is obviously given by the Goldstone bosons, which give the longitudinal component to these bosons. Hence it is expected that if the triplet vev is small, the Goldstone bosons should be mainly coming from the doublet, as it is the most SM-like configuration, and thus closer to the experimental results.

The masses of these particles are:

$$m_{H^+}^2 = \frac{2\sqrt{2}\mu - \lambda_4 v_t}{4 v_t} (v_d^2 + 2v_t^2) \tag{1.58}$$

$$m_A^2 = \frac{\mu}{\sqrt{2} v_t} (v_d^2 + 4v_t^2) \tag{1.59}$$

The case of the CP-even Higgs is different since its mixing angle depends on the interplay of many parameters:

$$\tan 2\alpha = \frac{-2\sqrt{2}\,\mu\, v_d + 2(\lambda_1 + \lambda_4)\, v_t\, v_d}{\frac{\lambda}{2} v_d^2 - \frac{\mu v_d^2}{\sqrt{2} v_t} - 2(\lambda_2 + \lambda_3) v_t^2} \tag{1.60}$$

It can be the case that both masses sit close to each other, this is known as maximal mixing. This can occur if the denominator cancels, and then the masses are degenerate. This scenario is highly disfavored, see [29] where they set an upper bound of $|\sin\alpha| < 0.3$ at 95% C.L. A final remark about this sector, is that under the approximation $v_t \ll v_d$ we can find some interesting relations:

$$m_{H^+}^2 - m_{H^{++}}^2 \simeq m_{H^+}^2 - m_{H^{++}}^2 \simeq \lambda_4 v^2/4 \tag{1.61}$$

$$m_H^2 \simeq (2m_{H^+}^2 - m_{H^{++}}^2)(1 + t_\alpha^2) - m_h^2 t_\alpha^2 \tag{1.62}$$

Where $t_\alpha \equiv \tan\alpha$. And for example in [29] one can find the results given in terms of the mass difference $\Delta m \equiv m_{H^+} - m_{H^{++}}$

The importance for neutrino physics is that this model can create a Majorana mass term without introducing a right-handed neutrino, which has not been observed, and gives also an explanation to the smallness of the neutrino masses. As we mentioned, with SM particles one cannot write a term producing $m_L \overline{\nu^c}_L \nu_L$. However, the triplet does this job, and allows one to introduce a gauge invariant term that after SSB produces this kind of mass:

$$\mathcal{L}_{\text{Y}}^{\text{type-2}} = f_{ab} \overline{l_a^c} i\sigma^2 \Delta l_b + \text{h.c.} \tag{1.63}$$

Where $a, b = e, \mu, \tau$ and σ^2 is the second Pauli matrix. Multiplying each term we obtain:

$$\mathcal{L}_Y = -f_{ab} \bar{l}_a^c l_b \Delta^{++} - \sqrt{2} f_{ab} \overline{\nu_a^c} l_b \Delta^+ + f_{ab} \overline{\nu_a^c} \nu_b \Delta^0 + \text{h.c.} \tag{1.64}$$

After SSB this term gives $f_{ab} \frac{v_t}{\sqrt{2}} \overline{\nu_a^c} \nu_b$ which except by the necessary diagonalization it is a proper mass term. There is nothing new to the mass matrix diagonalization with respect to the other sections so we will skip it, however, it is important to remark that for Majorana neutrinos 2 extra complex phases appear, and even though they do not modify neutrino oscillations measurements (not even in matter, see [24]) they can modify experiments relying on the neutrinoless double β decay ($0\nu\beta\beta$) [30].

The important fact is that v_t can be small, by inverting eq. (1.51) and using $v_t \ll v_d$ we find that $v_t \simeq \frac{\mu v_d^2}{\sqrt{2} M^2}$, giving a neutrino mass:

$$m_{ab}^\nu = 2 f_{ab} \frac{v_t}{\sqrt{2}} = f_{ab} \frac{\sqrt{2}\mu v_d}{M^2} v_d = Y_{ab}^{\text{eff}} v_d \tag{1.65}$$

With $Y_{ab}^{eff} \equiv \sqrt{2} f_{ab} \frac{\mu v_d}{M^2} \ll 1$ (notice that μ has energy dimensions) if $M \gg v_d$ which would be the case assuming and $v_t \ll v_d$. This is again similar to the idea of a seesaw mechanism, a large mass of the triplet would suppress the mass of the neutrinos making them tiny. Besides, $f_{\alpha\beta}$ would be related to the PMNS matrix by the diagonalization by the equation (1.66):

$$f = \frac{v_t}{\sqrt{2}} U^*_{\text{PMNS}} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger_{\text{PMNS}} \tag{1.66}$$

Thus, once the mass of the neutrinos is known, by multiplying the PMNS matrix the values of the Yukawa-like couplings are known.

We will finish up this section by summarizing some bounds of this model set by different experiments. Among all the possible decays there are two regimes that are particularly interesting, and that play an important rôle in the phenomenology of this model. The coupling f_{ab} which gives mass to the neutrinos, also gives the decay strenght of H^+ and H^{++} into $l^+\nu_l$ and l^+l^+ respectively. Furthermore, if we fix the product $\sqrt{2} f_{ab} v_t \overset{!}{=} m^\nu_{ab}$ we see that if v_t is really small the coupling f_{ab} will have to be larger in order to give the neutrino mass. In this regime the decay of these two particles will be dominated by this coupling f_{ab}.

On the other hand v_t could still be larger so that f_{ab} would have to compensate to reproduce the neutrino mass. In this scenario the important processes for the decays (and also interesting production channels for collider searches) are $H^+ \to W^+Z$ and $H^{++} \to W^+W^+$. This scenario is particularly interesting for production in colliders since through Vector Boson Fusion (VBF) charged bosons could be created. Other channels independent of v_t are possible, e.g. $W^+W^- \to H^{++}H^{--}$.

Firstly, the EW S and T parameters bound the mass difference to be $-40\,\text{GeV} < \Delta m < 50\,\text{GeV}$. In the case of $v_t = 10^{-9}$ the mass limit is $m_{H^{++}} \gtrsim 750$ GeV and comes from lepton flavour violation processes and NH for the neutrino masses (we will see this in more detail in section 3.1.1). Both ATLAS [31, 32] and CMS [33] have looked for different channels of this kind of model, looking for doubly and singly-charged Higgs particles. In [31] the ATLAS collaboration sets a bound of $m_{H^{++}} \gtrsim 667$ GeV by looking at the $\mu^+\mu^+$ channel for $v_t = 10^{-9}$ GeV and $v_t = 10^{-7}$ GeV, or $m_{H^{++}} \gtrsim 450$ GeV for $v_t = 10^{-4}$. By using ATLAS data in Ref. [29] were able to set a larger lower bound of $m_{H^{++}} \gtrsim 740$ GeV looking at $H^{++}H^-$ and leptonic decay. In the regime where v_t is large, the ATLAS collaboration [32] sets a mass bound of $m_{H^{++}} \gtrsim 220$ GeV.

As one can see from this section, it is difficult to establish a bound that is independent of v_t. This is because different values of v_t make different processes dominant, both for production and decay channels. It is then only possible to study and set bounds of the particle's mass by assuming the most important decay channel and setting v_t accordingly to this assumption. Hence, regions where v_t is neither large nor tiny are not properly covered, since many decays or production channels could be relevant.

1.3 Effective Field Theories

1.3.1 General Aspects of the Effective Field Theories

There are many reasons why Effective Field Theories (EFTs) have become really popular during the last few decades. First, EFTs describe theories up to a certain energy-scale Λ, at this point we do not know what happens, new particles or effects may arise. However, even if our knowledge of this new physics is unknown we can still formulate theories, test them and make predictions. This is the heart of EFTs. One can put it in a more mathematical framework called the decoupling theorem [34]. This theorem states [35]: when considering a one particle irreducible (1PI) Feynman diagram, if the momenta of the external particles p^2 is smaller than M^2 (the mass of an internal heavier state), then apart from coupling constants and renormalization field strengths the graph will be suppressed by some power of M relative to a graph with the same number of external particles but no internal particle. In the original paper they apply it to a simple case, but one can write it for a n-point Green function as:

$$\prod_i^n Z_i G^n_{\text{full}}(p_1, p_2, ..., p_n; \mu) = \prod_i^m Z_i G^n_{EFT}(p_1, p_2, ..., p_n; \mu) + \frac{1}{M^2} \prod_i^k Z_i G'^n_{EFT}(p_1, p_2, ..., p_n; \mu) + ... \tag{1.67}$$

The first term contains operators (we call operator to each term composed by fields in a Lagrangian) with dimension $d \leq 4$ and the field strength Z_i so that the theory is finite, the second one contains $d = 6$ and needs of new field strengths to make finite the new operators of dimension 6 of the theory. Removing a field whose mass is larger than the energy scale of the experiment is sometimes referred to as "integrating out" a particle. And it is part of the utility of EFTs: we do not need to know the UV (high energy) physics in order to build a IR (low energy) Lagrangian.

An equivalent way to think about this theorem is as a momentum expansion of a propagator in a Feynman diagram: if the external momentum is smaller than the mass of the mediating particle the propagator term can be expanded as $\frac{1}{p^2-M^2} \approx -\frac{1}{M^2}(1 - \frac{p^2}{M^2})$ and to leading order we recover the result of the decoupling theorem, a Feynman diagram with no internal particle, suppressed by some power of M.

Arguably, the most famous effective quantum theory is Fermi's theory (1933) of the β-decay [36]. Nowadays, we know that the β-decay is mediated via a W boson, $d \rightarrow u\, e\, \bar{\nu}_e$, but back then we did not know about this boson, but this knowledge was not needed to make experiments and predictions. The process only involves a tiny portion of energy compared to the mass M_W of the mediator. So, we can describe this theory as a 4-fermion interaction rather than a interaction mediated via a W-boson:

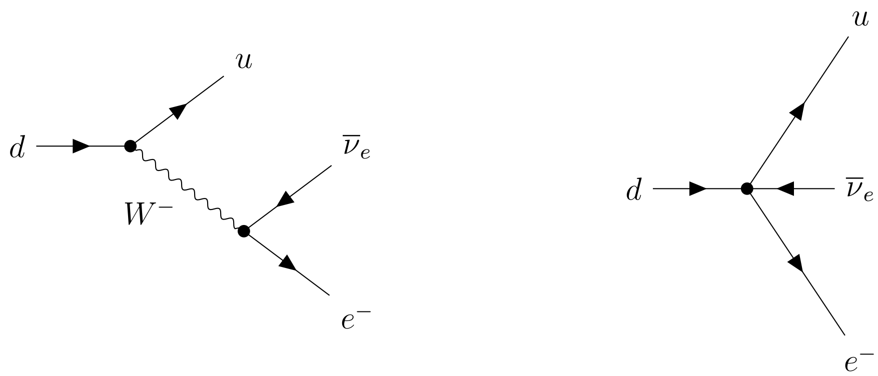


Figure 1.4: (left) The Feynman diagram for a β-decay mediated through a W-boson, (right) the external momentum is much smaller than the mass of the W-boson, so we can describe the theory as an effective 4-fermion interaction.

Both theories are different since they have different matter content, but we can set them so that we obtain the same result, this is achieved by the matching condition. The couplings of the IR theory, called Wilson coefficients, will be fixed by the condition of obtaining the same S matrix element in both theories, when taking the limit $p^2 \ll M^2$ [35]:

$$\langle k_1, k_2, ..., k_a | \, S_{1...n} \, |p_1, p_2, ..., p_n\rangle \, |^{UV}_{p^2 \ll M^2} = \langle k_1, k_2, ..., k_{n'} | \, S_{1...n} \, |p_1, p_2, ..., p_b\rangle^{EFT} \tag{1.68}$$

This is again very similar to the idea of the decoupling theorem, eq. (1.67) but it gives a way of obtaining the Wilson coefficient in terms of the UV theory parameters, e.g. for the case of Figure 1.4, eq. (1.68) gives:

$$-\left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{p^2 - M^2} \approx \frac{g^2}{2M^2} + \mathcal{O}(\frac{p^2}{M^4}) = \frac{4G_f}{\sqrt{2}} \tag{1.69}$$

The matching condition becomes a bit more involved when we introduce quantum corrections, but EFTs have a very elegant way of solving it. Since heavy states decouple from the IR theory, UV divergences of these states will not appear in the EFT. On the other hand, both theories will have the same IR divergences, since the lower energy states are present in both theories. So, when we want to calculate the matching condition at the quantum level, there exists a recipe one may follow (from Ref. [37]):

Calculate the graphs of the full theory, and expand in all IR scales. Then, from the possible UV and IR divergences terms with $1/\epsilon$ will appear. These contributions will be canceled by the counter-terms of the Lagrangian, so they will not play a rôle in the matching. This process will give a mismatch between these theories, this mismatch is a finite part which will be the matching contribution at a scale $\mu \sim M$, with M the UV scale. If we want then to go to a lower scale we then apply the renormalization group equations (RGE) to obtain the matching contribution at another scale.

The reason behind this set of rules is the following: The matching contribution is $I_M = (I_{full} + I_{full}^{c.t.}) - (I_{EFT} + I_{EFT}^{c.t.})$. The counter-terms cancel the divergences, and as we have argued, the EFT and full theory have the same divergences in the IR. Finally, the reason why expanding in the IR scales in the full theory graphs results in the matching contribution is because by expanding in this scale we are breaking the non-analytic IR integral, leaving only the non-analytic UV integral. This will give a divergence plus a finite contribution of the UV scale, this finite term cannot be produced in the EFT, so it is the mismatch between the two theories. There is a second requirement, if one calculates a loop with regularization scheme that is mass dependent there will be problems, namely, the momentum expansion will break, see for example [37, 38].

After this arguments, we can start building a theory with the quantum states that are relevant for the energies that we want to study. We organize this Lagrangian in powers of momentum, since we are making a momentum expansion (remember eq. (1.67)):

$$\mathcal{L} = \sum_{D\geq 0, i} \frac{C_i^{(D)}(\mu)}{\Lambda^{D-d}} \mathcal{O}_i^{(D)} \tag{1.70}$$

In this Lagrangian $C_i^{(D)}$ are the so called Wilson coefficients, and $\mathcal{O}_i^{(D)}$ the operators which contain the fields, the D indicates the energy dimension of the operator and d the space-time dimension. We are used to work with Lagrangians of $D \leq d$ operators, but one could continue the expansion, the only problem is that this theory would be non-renormalizable. To understand what we mean as renormalizable we will use the power counting formula. Calculate an amplitude produced by an operator $\mathcal{O}_i^{(D)}$. This will be proportional to $\mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^{(D-d)}$. If we insert several operators the formula becomes:

$$\mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^n \quad \text{with } n = \sum_i (D_i - d) \tag{1.71}$$

The important fact is that this formula holds as well at the quantum level.

So, we can see what happens if we only introduce terms with $D \geq d$. With $d = 4$ the amplitude with two $D = 5$ operators, will be of order $\mathcal{O}\left(\frac{p^2}{\Lambda^2}\right)$ and we can start seeing the problem: if we have quantum loops with divergences in this amplitude, we will need a counter-term of dimension $D = 6$ to cancel them. If an amplitude with two dimension 6 operators has divergences we will need a counter-term with dimension 8, and so forth and so on. This is why theories containing operators of dimension $D \geq d$ are called non-renormalizable. However, these theories are 'approximately renormalizable'. In this momentum expansion the higher the p/Λ ratio the more suppressed it is, so if we formulate our Lagrangian to a certain order, and we are consistent not including graphs that go further than the chosen order, EFTs have the same power than the renormalizable theories.

We briefly mentioned above that in order to go from one energy scale to another one must use the RGE. This part is really important since if one does not do it with care, the momentum expansion might break. For instance, if one calculates a loop with the $\overline{MS}$ renormalization scheme, one will obtain graphs proportional to $\log \overline{\mu}^2/M^2$. If we set $\overline{\mu} \sim M$ there is no problem, however if we set the energy scale away from M the logarithm increases, being possible to break the expansion. That is why the correct way to go from one scale to another is through the RGE (1.72):

$$\mu \frac{\mathrm{d}c_i}{\mathrm{d}\mu} = \gamma_{ij} c_j \tag{1.72}$$

Where $\gamma = Z^{-1}\mu\frac{\mathrm{d}Z}{\mathrm{d}\mu}$ is the anomalous dimension matrix and Z are the field strength renormalization matrix, which is given by the loop calculations. In this case Z is a matrix because it can happen that a loop transforms an operator into another, for example at tree level after integrating out the W-boson, we would have operators like[1] $(\bar{c}^\alpha \gamma^\mu b_\alpha)(\bar{d}^\beta \gamma_\mu P_L u_\beta)$ but never $(\bar{c}^\alpha \gamma^\mu b_\beta)(\bar{d}^\beta \gamma_\mu P_L u_\alpha)$, where the colour indices α and β have been exchanged, since W-bosons do not carry colour. However, when considering loops a gluon could couple to two of the fermions producing the second operator. So, the Z field strength would take the first operator to the second in the renormalization, making Z a matrix. So, the whole correct process can be summarized in Fig. 1.5 (taken from [38]) where the heavier state Φ is integrated out, the Lagrangians are matched at the scale of this particle and at to gro from one scale to another the RGE is used.

A last important point that becomes important in the model-building aspect of the EFTs is the field redefinition. Quantum field theories are in general invariant under field redefinitions, but in renormalizable lagrangian the only ones allowed are linear transformations of the fields, for example the redefinition of the W-bosons of eq. (1.24), however in EFTs other field transformations other field transformations are allowed [37]. It is not our purpose to study this in detail but there is a set of transformations that are used to reduce terms with multiple derivatives by using the equations of motion. If we redefine a field by $\phi' = F[\phi]E[\phi]$ where $E[\phi]$ is the equations of motion operator and $F[\phi]$ a general term containing any combination of ϕ we can proof that any S-matrix with an insertion of this transformation will be 0. That's why this operators can be dropped, let us see an example of this application (from [37]), take the Lagrangian of a scalar field ϕ:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{C_1}{\Lambda}\phi^3\partial^2\phi + \frac{C_2}{\Lambda^2}\phi^6 \tag{1.73}$$

[1] Example taken from [37].

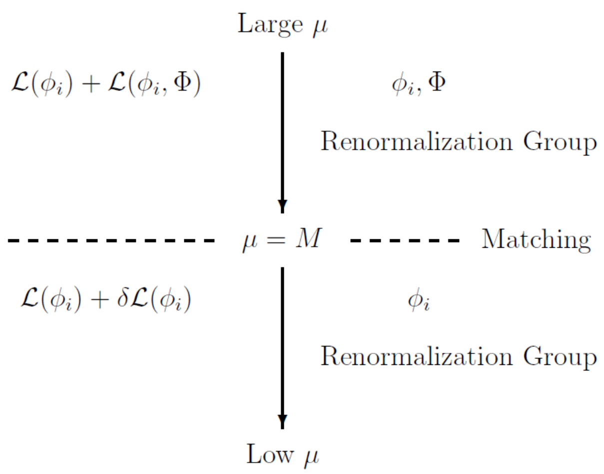


Figure 1.5: Scheme of the correct procedure to integrate out a heavy state Φ with a mass M, and go from an energy scale to a different one. Source: Ref. [38].

We see that, by addin the term $\frac{C_1}{\Lambda^2}\phi^3[m^2\phi + \frac{\lambda}{3!}\phi^3]$ to eq. (1.73) we will have the equations of motion in the term proportional to C_1:

$$\frac{C_1}{\Lambda^2}\varphi^3\left(\partial^2\phi + m^2\phi + \frac{\lambda}{3!}\phi^3\right) \tag{1.74}$$

As we have discussed, operators proportional to to the EoM can be dropped, so in order to obtain this expression we add and subtract these necessary terms to obtain the previous operator and then redefine the Wilson coefficients. By adding and subtract these terms we obtain:

$$\begin{aligned}\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \left[\frac{1}{4!}\lambda + \frac{C_1}{\Lambda^2}m^2\right]\phi^4 + \frac{C_1}{\Lambda^2}\phi^3\left[\partial^2\phi + m^2\phi + \frac{\lambda}{3!}\phi^3\right] \\ + \left[\frac{C_2}{\Lambda^2} - \frac{C_1}{\Lambda^2}\frac{\lambda}{3!}\right]\phi^6\end{aligned} \tag{1.75}$$

By dropping the operators proportional to the equations of motion we eliminate the operator $\phi^3\partial^2\phi$ and by redefining the other coefficients the Lagrangian is reduced to just:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda'\phi^4 + \frac{C_6}{\Lambda^2}\phi^6 \tag{1.76}$$

Where we redefined: $\lambda' = \frac{1}{4!}\lambda + \frac{C_1}{\Lambda^2}m^2$ and $C_6 = C_2 - C_1\frac{\lambda}{3!}$, reducing the number of coefficients, thus making it simpler and eliminating what was a redundancy. This is the same as making the field redefinition $\phi \to \phi + \frac{C_1}{\Lambda^2}\phi^3$, which means that using EoM is the same as making a particular field redefinition. The use in EFTs of this property is typically to reduce the number operators with derivatives, since it is a way of systematically eliminate redundancies and avoid making mistakes.

1.3.2 The Standard Model as an EFT

If we look into the past of particle physics we see that discoveries of new fundamental particles have been decreasing in time, even though the energy scale, luminosity and amount of data has never been larger. Furthermore, the scientific community is quite certain that new states must exist, e.g. due to the proofs for Dark Matter (DM). Then the question arises: what is the best way to look for these new states without important deviations from the SM predictions? One possible good answer is EFTs. These theories allow for a systematic search of these physics within the global picture of whole data set available. This is the downside, if we want EFTs to be powerful, we need large amounts of data to be able to determine and constrain the new operators that may be causing deviations, without breaking the very good predictions of the SM.

To make clearer why large amounts of data are required we will start by writing down the basis of the Standard Model Effective Field Theory. It is very important that this basis is non-redundant and complete, in order to make systematic and unbiased constraints. A basis is complete when we have written down all possible operators (i.e. combinations of fields of a particular dimension) that respect the symmetries of the theory, in this case SMEFT. This basis must be non-redundant, which means that the operators must be independent from each other, in other words they cannot be related by some property, e.g. Fierz identities.

Finding a non-redundant complete basis took some years, from one of the first complete basis [42], in fact with redundancies, to the first complete non-redundant basis [44] took 20 years, known as the Warsaw basis. It is unique and there are other popular choices of basis, such as the SILH basis (Strongly-Interacting Little Higgs) [43] that can be found in the literature. To show some of the technicalities of the EFTs and in particular the SMEFT we will write down the Warsaw basis. First, following the ordering given by the momentum expansion of eq. (1.70), we only have one operator, the so-called Weinberg operator:

$$\mathcal{L}_5 = \frac{C_5}{\Lambda}\mathcal{O}_5 = \frac{C_5}{\Lambda}(\tilde{\varphi}^\dagger l_p^c)^T(\tilde{\varphi}^\dagger l_r) \tag{1.77}$$

Where $r, p = 1, 2, 3$ indicate the generation number. This operator became really famous, since after SSB neutrinos get a Majorana mass term $C_5\frac{v_d^2}{\Lambda}\overline{\nu^c}\nu$. Also producing lepton number violation processes such as the $0\nu\beta\beta$. When we increase the dimension of the operators the simplicity breaks and we find many more possible combinations, listed in tables 1.3 and 1.4). In this table capital indices $A, B, C = 1, ..., 8$ denote

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
$\mathcal{O}_G$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_\varphi$	$(\varphi^\dagger\varphi)^3$	$\mathcal{O}_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$\mathcal{O}_{\tilde{G}}$	$f^{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$\mathcal{O}_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \tilde{\varphi})$
$\mathcal{O}_W$	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{\varphi D}$	$(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D_\mu\varphi)$	$\mathcal{O}_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$\mathcal{O}_{\tilde{W}}$	$f^{IJK}\tilde{W}_\mu^{I\nu}G_\nu^{J\rho}G_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$\mathcal{O}_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$\mathcal{O}_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\varphi\tilde{G}}$	$\varphi^\dagger\varphi\, \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$\mathcal{O}_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)\,(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)T^A\tilde{\varphi}G_{\mu\nu}^A$	$\mathcal{O}_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)\,(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\varphi\tilde{W}}$	$\varphi^\dagger\varphi\, \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\tilde{\varphi}W_{\mu\nu}^I$	$\mathcal{O}_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)\,(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tilde{\varphi}B_{\mu\nu}$	$\mathcal{O}_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)\,(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\varphi\tilde{B}}$	$\varphi^\dagger\varphi\, \tilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)T^A\varphi G_{\mu\nu}^A$	$\mathcal{O}_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)\,(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi W_{\mu\nu}^I$	$\mathcal{O}_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)\,(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\varphi\tilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi$	$\mathcal{O}_{\varphi ud}$	$(\varphi^\dagger D_\mu\varphi)\,(\bar{u}_p\gamma^\mu d_r)$

Table 1.3: Dimension six operators containing Higgs fields, strength tensors of the gauge bosons fermions and their combination, obtained in Ref. [44].

the number of the Gell-Mann matrices, $i, j, k = 1, 2, 3$ indices of the group $SU(2)_L$, r, p, s, t indicate the generation and $\alpha,\ \beta,\ \gamma\ = 1, ..., 8$ colour indices. In the following sections we will use τ^I as the Pauli matrices.

In total there are 2499 operators, taking into account all possible indices, however, making a global fit to these operators becomes impossible. That is why normally we pick a subset of operators, and in many cases assume some kind of symmetry, just to quote two: $U(3)^5$ making all operators flavour conserving (for instance [47]), in total 59 operators (excluding the barion number-violating operators), and the one typically used in top-quark physics assuming $U(2)^3$ in the quark sector and flavour invariance in the lepton sector (e.g. [46]), which requires the study of 85 independent parameters (17 of them complex phases). So, even in the simplified cases scientists tend to do Global fits with a reduced set of operators.

Different operators can be constrained by different types of measurements. An example of this is Figure 1.6 from Ref. [50], where we find a scheme of the different data sets that constrain different Wilson coefficients, in this case with specific top, di-boson and Higgs observables. For example, the Electroweak Precision Observable (EWPO) are typically constrained by LEP measurements since some of these operators modify EW parameters of the SM, e.g. the masses of the gauge bosons, the Fermi constant G_F or the fermionic couplings to the Z boson.

The SMEFT is not the only effective theory used nowadays, there exists a large list

$(\bar{R}R)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma_\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma_\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma_\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma_\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma_\mu u_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma_\mu d_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma_\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma_\mu d_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma_\mu u_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma_\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma_\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma_\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma_\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma_\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma_\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma_\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma_\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma_\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma_\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma_\mu T^A d_t)$
$(\bar{R}L)(\bar{L}R)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}[(d_p^\alpha)^T C u_r^\beta][(q_s^{\gamma j})^T C l^k_t]$		
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}[(q_p^\alpha)^T C q_r^\beta][(u_s^{\gamma j})^T C l^k e_t]$		
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}[(q_p^\alpha)^T C q_r^\beta][(q_s^{\gamma j})^T C l^k l_t]$		
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}[(d_p^\alpha)^T C u_r^\beta][(u_s^{\gamma j})^T C l^k e_t]$		
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma_{\mu\nu} u_t)$				

Table 1.4: Operators containing only fermionic fields, obtained in Ref. [44].

of EFTs that are helpful in different energy scale of experiments. An old EFT is the Chiral. For instance, at low energy-scales the running of the QCD coupling makes really difficult to study all the interactions, in fact it is impossible to do in a perturbative way. That is why an EFT of the asymptotic states π and K was developed in the 60s-70s called Chiral Perturbation Theory (ChPT), which allows the study of these mesons without the need of knowing all the difficult processes of the QCD interactions at low energies. A more modern theory is the Low-energy Effective Field Theory (LEFT), referred to as the EFT that generalizes the Fermi interaction mentioned above. This kind of theory has been used for a long time to study flavour physics and recently due to the so-called flavour anomalies increasing in popularity.

In this introduction we pointed out the most important concepts of Effective Field Theories and we have given a short overview of the current landscape of the SMEFT. We have talked about the validity of these theories up to a certain scale and discussed that they are approximately renormalizable. We mentioned as well several technical aspects that are important for the correct construction of these theories that will be useful in the next sections. Furthermore,in the absence of large deviations from the SM model and the large amount of possible extensions, EFTs are the best way to explore the rich amounts of data and constrain by indirect measurements the space of parameters.

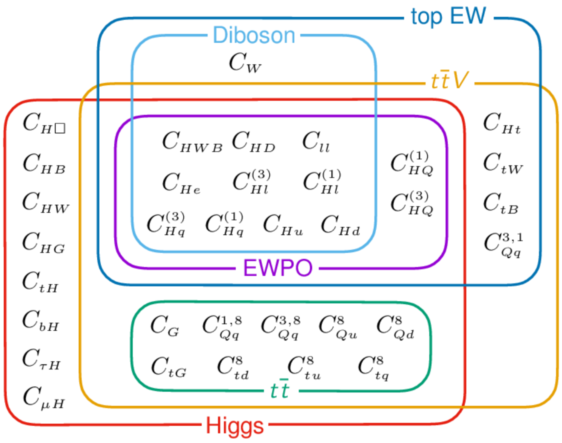


Figure 1.6: Scheme indicating the different data sets used in Ref. [50] to constraint Higgs, top and di-boson Wilson coefficients, which exemplifies the use of different data sets to constrain different directions of the SMEFT-operators space.

2 Extension of the Type-II Seesaw Mechanism

We have mentioned that EFTs are the best method to study the global picture of data, but there is another interesting use in phenomenology. The extensions in different Beyond the Standard Model (BSM) theories can produce new interesting couplings that lift the constrains of the renormalizable Lagrangians or give new production channels that would increase the possible ways of observing these models. These extensions could be justified by arguing that most of these theories do not explain all the questions of the SM (or at least they would have a difficult time doing so). For instance the type-2 seesaw mechanism, in which we will focus now, would not make a good DM candidate, since the neutral particles could decay to neutrinos, and, even though this coupling could be really suppressed it is certain that in order to explain neutrino masses it would not be zero. Even if a model could explain all of the open question there still exists a much higher scale that could generate new operators, that is gravity, which comes into play at energies of the order of the Planck scale $\Lambda_{\text{Planck}} \sim 10^{19}$ GeV.

The disadvantage of doing this is twofold, first the large amount of operators generated for every model adds complexity to the already large amount of operators of the SMEFT. Secondly, these new states have not been observed, therefore many of these new operators can not be constrained. So in this case these extensions are more interesting in the phenomenological aspect of new production channels and/or modification to suppressed processes, e.g. Lepton Flavour Violation (LFV) processes or Flavour Changing Neutral Currents (FCNC).

A motivation for expanding the type-2 seesaw mechanism is that as one can see from section 1.2.3 there is no coupling to quarks, so in colliders this model can only be produced in processes where vector bosons are emitted. Hence, an extension in which direct couplings of quarks or gluons to the triplet field are generated would have a large impact in these channels. So, in the next section we discuss the dimension-6 Lagrangian of the type-2 seesaw mechanism EFT expansion (ΔEFT).

2.1 Complete Dimension Dimension-6 Basis

The type-1 seesaw mechanism has already been extended to a dimension 6 basis [48], so we continue with this idea and we extend the type-2 seesaw mechanism, the ΔEFT. This basis has also been worked in [49], however we have reached to a basis with a few differences, and we try to justify the choice of our basis.

2.1.1 Dimension 5 Operators

In this section we are aiming to formulate a Lagrangian with operators up to dimension 6, in the momentum expansion of eq. 2.1:

$$\mathcal{L}_{\Delta\text{EFT}}^{D\leq 6} = \mathcal{L}_{\text{type-2}}^{D\leq 4} + \mathcal{L}_{\Delta\text{EFT}}^{D=5} + \mathcal{L}_{\Delta\text{EFT}}^{D=6} = \sum_i C_i^{(4)}(\mu)\,\mathcal{O}_i^{(4)} + \sum_j \frac{C_j^{(5)}(\mu)}{\Lambda}\,\mathcal{O}_j^{(5)} + \sum_k \frac{C_k^{(6)}(\mu)}{\Lambda^2}\,\mathcal{O}_k^{(6)} \tag{2.1}$$

As we have mentioned in the previous sections the prescription to build an EFT is to follow the symmetries that we want to fulfill, in this case the same as the SMEFT but with an extra field, the complex scalar $SU(2)_L$ triplet Δ. The operators that make the dimension-5 Lagrangian are found in Table 2.1 divided in two groups, operators containing fermions and operators containing only scalar particles.

Fermions		Scalars	
$\mathcal{O}_{\Delta e}$	$(\overline{e_p^c} e_r)\text{Tr}(\Delta\Delta)$	$\mathcal{O}^{(1)}_{\Delta^3\varphi^2}$	$\text{Tr}(\Delta^\dagger\Delta)\varphi^\dagger\Delta\tilde{\varphi}$
$\mathcal{O}_{\Delta qd}$	$\bar{q}_p\Delta d_r\tilde{\varphi}$	$\mathcal{O}^{(3)}_{\Delta^3\varphi^2}$	$\varphi^\dagger\Delta\Delta^\dagger\Delta\tilde{\varphi}$
$\mathcal{O}_{\Delta le}$	$\bar{l}_p\Delta e_r\tilde{\varphi}$	$\mathcal{O}_{\varphi^4\Delta}$	$(\varphi^\dagger\Delta\tilde{\varphi})(\varphi^\dagger\varphi)$
$\mathcal{O}_{\Delta qu}$	$\varphi^\dagger\Delta\bar{u}_r q_p$		

Table 2.1: Dimension 5 operators of the type-2 seesaw mechanism EFT expansion.

In this dimension-5 Lagrangian there are 7 operators. These operators contain terms that couple directly the Δ field to quarks, making it very interesting for collider searches. Other possible phenomenological observables may be affected, like the coupling of the fermions to the Higgs, although this would be proportional to v_t, hence small, and Higgs couplings are poorly measured at the LHC. The scalar part of this Lagrangian is not of such a phenomenological relevance, since in absence of evidence of the Δ particle, the possible modifications induced by these operators in the SM Lagrangian are to the Higgs quartic coupling or the Higgs mass, but again proportional to $\sim v_t$.

In the literature, e.g. [40], the operator $\varphi^\dagger D^\mu\Delta D_\mu\tilde{\varphi}$ also appears in the dimension-5 Lagrangian. As we have seen in section 1.3.1, field redefinitions do not change the S-matrix, in particular, fields can be redefined by using the EoM to eliminate the operators containing derivatives. We prove in the appendix section A.1 that a particular combination of total derivatives (unimportant in the Lagrangian), since can be dropped by integration "by parts", relates this operator with a term proportional to the equations of motion and operators that are already found in the basis, so by renaming the Wilson coefficient we show that this operator is redundant.

2.1.2 Dimension 6 Operators

The dimension 6 Lagrangian offers, as expected, a much richer phenomenology since many more couplings are allowed by the symmetries and the looser restriction on the mass dimension. We find the final basis after all redundancies have been removed in tables (2.2, 2.3).

Δ^6 and $\Delta^4 D^2$		$\Delta^4\varphi^2$ and $\Delta^2\varphi^4$		$\Delta^2\varphi^2 D^2$ and $\tilde{\varphi}^2\varphi^2\Delta^2$	
$\mathcal{O}^1_{\Delta}$	$[\text{Tr}(\Delta^\dagger\Delta)]^3$	$\mathcal{O}^1_{\varphi\Delta}$	$\text{Tr}(\Delta^\dagger\Delta)\,(\varphi^\dagger\varphi)^2$	$\mathcal{O}_{\Delta\Box\varphi}$	$\text{Tr}(\Delta^\dagger\Delta)\,\Box\,(\varphi^\dagger\varphi)$
$\mathcal{O}^2_{\Delta}$	$\text{Tr}(\Delta^\dagger\Delta)^2\,\text{Tr}(\Delta^\dagger\Delta)$	$\mathcal{O}^2_{\varphi\Delta}$	$(\varphi^\dagger\Delta\Delta^\dagger\varphi)\,(\varphi^\dagger\varphi)$	$\mathcal{O}^1_{\Delta D\varphi}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}^\mu\Delta)$
$\mathcal{O}^3_{\Delta}$	$\text{Tr}\big(\Delta^\dagger\Delta\big)^3$	$\mathcal{O}^1_{\Delta\varphi}$	$\text{Tr}(\Delta^\dagger\Delta)\,(\varphi^\dagger\Delta\Delta^\dagger\varphi)$	$\mathcal{O}^2_{\Delta D\varphi}$	$(\overleftrightarrow{D}^\mu\varphi)^\dagger\Delta^\dagger\Delta\overleftrightarrow{D}^\mu\varphi$
$\mathcal{O}_{\Delta\Box}$	$\text{Tr}(\Delta^\dagger\Delta)\,\Box\,\text{Tr}(\Delta^\dagger\Delta)$	$\mathcal{O}^2_{\Delta\varphi}$	$\text{Tr}(\Delta^\dagger\Delta)^2\,(\varphi^\dagger\varphi)$	$\mathcal{O}^3_{\Delta D\varphi}$	$(\varphi^\dagger D^{I\mu}\varphi)\text{Tr}(\Delta^\dagger D^I_\mu\Delta)$
$\mathcal{O}^1_{D\Delta}$	$\text{Tr}(\Delta^\dagger D^\mu\Delta)^*\,\text{Tr}\,(\Delta^\dagger D_\mu\Delta)$	$\mathcal{O}^3_{\Delta\varphi}$	$(\text{Tr}\Delta^\dagger\Delta)^2\,(\varphi^\dagger\varphi)$	$\mathcal{O}^1_{\tilde{\varphi}\Delta}$	$(\varphi^\dagger\Delta\tilde{\varphi})\,(\varphi^\dagger\Delta\tilde{\varphi})$
$\mathcal{O}^2_{D\Delta}$	$\text{Tr}[\Delta^\dagger\Delta\,\Box(\Delta^\dagger\Delta)]$	$\mathcal{O}^4_{\Delta\varphi}$	$\varphi^\dagger\Delta\Delta^\dagger\Delta\Delta^\dagger\varphi$	$\mathcal{O}^2_{\tilde{\varphi}\Delta}$	$(\tilde{\varphi}^\dagger\Delta\varphi)\,(\varphi^\dagger\Delta^\dagger\tilde{\varphi})$
$\Delta^2 X^2$		$\Delta^2\psi^2\varphi$		$\psi^2\Delta^2 D$	
$\mathcal{O}_{\Delta G}$	$\text{Tr}(\Delta^\dagger\Delta)\,G^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}^{(1)}_{le\varphi}$	$\text{Tr}(\Delta^\dagger\Delta)\,(\bar{l}_p e_r\varphi)$	$\mathcal{O}^{(1)}_{\Delta l}$	$\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}_\mu\Delta)\,(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{\Delta\tilde{G}}$	$\text{Tr}(\Delta^\dagger\Delta)\,\tilde{G}^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}^{(1)}_{qd\varphi}$	$\text{Tr}(\Delta^\dagger\Delta)\,(\bar{q}_p d_r\varphi)$	$\mathcal{O}^{(3)}_{\Delta l}$	$\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}^I_\mu\Delta)\,(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{\Delta W}$	$\text{Tr}(\Delta^\dagger\Delta)\,W^I_{\mu\nu}W^{I\mu\nu}$	$\mathcal{O}^{(1)}_{qu\varphi}$	$\text{Tr}(\Delta^\dagger\Delta)\,(\bar{q}_p u_r\tilde{\varphi})$	$\mathcal{O}_{\Delta e}$	$\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}_\mu\Delta)\,(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{\Delta\tilde{W}}$	$\text{Tr}(\Delta^\dagger\Delta)\,\tilde{W}^I_{\mu\nu}W^{I\mu\nu}$	$\mathcal{O}^{(3)}_{le\varphi}$	$\bar{l}_p\Delta\Delta^\dagger e_r\varphi$	$\mathcal{O}^{(1)}_{\Delta q}$	$\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}_\mu\Delta)\,(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{\Delta B}$	$\text{Tr}(\Delta^\dagger\Delta)\,B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(3)}_{qd\varphi}$	$\bar{q}_p\Delta\Delta^\dagger d_r\varphi$	$\mathcal{O}^{(3)}_{\Delta q}$	$\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}^I_\mu\Delta)\,(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{\Delta\tilde{B}}$	$\text{Tr}(\Delta^\dagger\Delta)\,\tilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}^{(3)}_{qu\varphi}$	$\bar{q}_p\Delta\Delta^\dagger u_r\tilde{\varphi}$	$\mathcal{O}_{\Delta u}$	$\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}_\mu\Delta)\,(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\Delta WB}$	$\text{Tr}(\Delta^\dagger\tau^I\Delta)\,W^I_{\mu\nu}B^{\mu\nu}$			$\mathcal{O}_{\Delta d}$	$\text{Tr}(\Delta^\dagger i\overleftrightarrow{D}_\mu\Delta)\,(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{\Delta\tilde{W}B}$	$\text{Tr}(\Delta^\dagger\tau^I\Delta)\,\tilde{W}^I_{\mu\nu}B^{\mu\nu}$				
$\mathcal{O}_{\Delta WW}$	$\text{Tr}(\Delta^\dagger\tau^I\Delta\tau^J)\,W^I_{\mu\nu}W^{J\mu\nu}$				
$\mathcal{O}_{\Delta\tilde{W}W}$	$\text{Tr}(\Delta^\dagger\tau^I\Delta\tau^J)\,\tilde{W}^I_{\mu\nu}W^{J\mu\nu}$				

Table 2.2: Operators of dimension 6 extension of the type-2 seesaw mechanism that are lepton number conserving. We have ordered them in a similar way to the Warsaw basis [44], only-scalar terms on top and in the lower row, combinations with the gauge bosons, leptons and Higgs doublet and lepton respectively.

Many operators that are found in this table are very similar in structure to those of the Warsaw basis, for example the lower row of Table 2.2. In this row there are a few different combinations due to the fact that Δ is a triplet, for example $\mathcal{O}^{(3)}_{le\varphi}$. The upper number indicates that it is a triplet-triplet combination, to see write a property of the Pauli matrices that will be useful in this work:

$$\tau^a_{jk}\tau^a_{mn} = 2\delta_{jn}\delta_{km} - \delta_{jkj}\delta_{mn} \tag{2.2}$$

Terms like $\text{Tr}\big(\Delta^\dagger\Delta\big)$ and $\bar{l}_p e_r\varphi$ are singlets under the gauge group of the SM, but we can for example make a triplet by adding a Pauli matrix: $\bar{l}_p\tau^I e_r\varphi$ and $\text{Tr}\big(\Delta^\dagger\tau^I\Delta\big)$,

L-Violating	
$\mathcal{O}_{\Delta l}^{(1)}$	$\mathrm{Tr}(\Delta^\dagger\Delta)\ (\bar{l}_p^c i\sigma^2\Delta l_r)$
$\mathcal{O}_{\Delta l}^{(3)}$	$(\bar{l}_p^c i\sigma^2\Delta\Delta^\dagger\Delta l_r)$
$\mathcal{O}_{\Delta\varphi l}^{1}$	$(\varphi^\dagger\varphi)\ (\bar{l}_p^c i\sigma^2\Delta l_r)$
$\mathcal{O}_{\Delta\varphi l}^{2}$	$(l_p^T i\sigma^2\varphi)\ C\ (\varphi^\dagger\Delta l_r)$
$\mathcal{O}_{l\Delta D}$	$\bar{l}_r^c i\sigma^2\gamma^\mu e_r\Delta D_\mu\varphi$
$\mathcal{O}_{\Delta lB}$	$(\bar{l}_r^c i\sigma^2\sigma^{\mu\nu}\Delta l_p)\ B_{\mu\nu}$
$\mathcal{O}_{\Delta lW}$	$(\bar{l}_r^c i\sigma^2\sigma^{\mu\nu}\Delta\tau^I l_p)\ W_{\mu\nu}^I$

Table 2.3: Dimension 6 operators of the ΔEFT that are lepton number violating.

so the singlet combination of this $3 \otimes 3$ is just multiplying this terms and summing the index I. Now by using the property of eq. (2.2) we can rewrite this term like:

$$\mathrm{Tr}\big(\Delta^\dagger\tau^I\Delta\big)\bar{l}_p\tau^I e_r\varphi = \Delta_{ij}^\dagger\tau_{jk}^I\Delta_{ki}\bar{l}_{p,m}\tau_{mn}^I e_r\varphi_n =$$
$$2\Delta_{ij}^\dagger\Delta_{ki}\bar{l}_{p,k}e_r\varphi_j - \Delta_{ij}^\dagger\Delta_{ji}\bar{l}_{p,m}e_r\varphi_m = 2\bar{l}_p e_r\Delta^\dagger\Delta\varphi - \mathrm{Tr}\big(\Delta^\dagger\Delta\big)\bar{l}_p e_r\varphi \tag{2.3}$$

By renaming the couplings we have a different way of writing the triplet and singlet combinations both ways are equivalent and our choice of basis is this last one. As we can see, also the operators of the type Δ^2X^2, where X represents any field strength of the gauge bosons, there are new ways of combining the triplet with the W-bosons, $\mathcal{O}_{\Delta WW}$ and $\mathcal{O}_{\Delta\tilde{W}W}$. The row containing field combinations $\psi\Delta^2D$ has the same structure and has one operator less than in the Warsaw basis.

The first row containing combinations of the triplet, the doublet and derivatives is quite different from the SMEFT. Again here there are some operators that can be reduced by using the properties of the Pauli matrices. Since it is different to the Warsaw basis we will explain a couple of example to show the kind of relations between operators, for example operators such as $\mathrm{Tr}(\Delta\Delta)\,\mathrm{Tr}\big(\Delta^\dagger\Delta^\dagger\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big)$ respect all the symmetries of the gauge group, however it is related to other operators through the Pauli matrices property:

$$\{\sigma^i,\sigma^j\} \equiv \sigma^i,\sigma^j + \sigma^j,\sigma^i = 2\delta_{ij}\, I \tag{2.4}$$

Taking operator $\mathrm{Tr}\big(\Delta^\dagger\Delta\Delta^\dagger\Delta\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big)$ and applying this property we have:

$$\mathrm{Tr}\big(\Delta^\dagger\Delta\Delta^\dagger\Delta\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big) = \mathrm{Tr}\big(\Delta^\dagger\Delta\big)^3 - \mathrm{Tr}\big(\Delta^\dagger\Delta^\dagger\Delta\Delta\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big) \tag{2.5}$$

Now we can use eq. (2.2) on the couple of $\Delta^i\sigma^i\sigma^j\Delta^j$ and since the Δ^i fields are symmetric under exchange of indices the second term of eq. 2.2 vanishes, leaving the expression as:

$$\mathrm{Tr}\big(\Delta^\dagger\Delta\Delta^\dagger\Delta\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big) = \mathrm{Tr}\big(\Delta^\dagger\Delta\big)^3 - \mathrm{Tr}\big(\Delta^\dagger\Delta^\dagger\big)\,\mathrm{Tr}(\Delta\Delta)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big) \tag{2.6}$$

So, in the end we can turn around the previous equation and write the last operator in terms of the other two, having proved that this operator is redundant. Similarly, the operator $\mathrm{Tr}\big[(\Delta^\dagger\Delta)^3\big]$, does not seem redundant, however, by using again 2.2:

$$\mathrm{Tr}\big[(\Delta^\dagger\Delta)^3\big] = \mathrm{Tr}\big[(\Delta^\dagger\Delta)^2\big](\Delta^i)^\dagger\Delta^i + \mathrm{Tr}\big[(\Delta^\dagger\Delta)^2\sigma^i\big]i\varepsilon_{ijk}\Delta^{j\dagger}\Delta^k \tag{2.7}$$

The first term is the operator $\mathrm{Tr}\big[(\Delta^\dagger\Delta)^2\big]\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big)$ so it can be eliminated by redefining the Wilson coefficient. If we keep working with the second term and use again eq. (2.2) the term proportional to δ_{ij} will be zero since there will be two Δ fields that are symmetric under the exchange of index. The second term gives:

$$\begin{aligned}\mathrm{Tr}\big[(\Delta^\dagger\Delta)^2\sigma^i\big]i\varepsilon_{ijk}\Delta^{j\dagger}\Delta^k = &-\mathrm{Tr}\big(\Delta^\dagger\Delta\Delta^\dagger\sigma^l\big)\varepsilon_{lmi}\Delta^m\varepsilon_{ijk}\Delta^{j\dagger}\Delta^k) = \\ &-\mathrm{Tr}\big(\Delta^\dagger\Delta\Delta^\dagger\Delta^\dagger\big)\,\mathrm{Tr}(\Delta\Delta) + \mathrm{Tr}\big(\Delta^\dagger\Delta\Delta^\dagger\Delta\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big)\end{aligned} \tag{2.8}$$

Where in the last step we used $\varepsilon_{ilm}\varepsilon_{ijk} = \delta_{lj}\delta_{mn} - \delta_{lk}\delta_{mj}$. Now it is simple since the first term can be rewritten by using $\Delta^\dagger\Delta^\dagger = \frac{1}{2}\{\Delta^\dagger,\Delta^\dagger\} = \mathrm{Tr}\big(\Delta^\dagger\Delta^\dagger\big)\mathbb{1}$. So, we can finally write:

$$\mathrm{Tr}\big[(\Delta^\dagger\Delta)^3\big] = -\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta^\dagger\big)\,\mathrm{Tr}(\Delta\Delta) + 2\,\mathrm{Tr}\big(\Delta^\dagger\Delta\Delta^\dagger\Delta\big)\,\mathrm{Tr}\big(\Delta^\dagger\Delta\big) \tag{2.9}$$

And now the first term we already proved that can be written in terms of the other operators, so this operator is also redundant. There are several more operators that through the same properties can be reduced, but we chose to show in this work two of them as the most representative.

The last set of operators that are reduced in a different way are those which contain derivatives. We will just pick one to show the methodology. When trying to combine 2 derivatives and 4 fields there are in total 6 possible combinations of derivatives acting on different fields, but we can make use of some properties to reduce two of the operators. Our choice is to make it as similar to the Warsaw basis [44] as possible, so by using the Leibniz rule we can relate the two operators Tr($\Delta^\dagger\Delta$) $((D^\mu\varphi)^\dagger D_\mu\varphi)$, Tr($(D^\mu\Delta)^\dagger D_\mu\Delta$) $(\varphi^\dagger\varphi)$ with total derivatives:

$$(\varphi^\dagger\varphi)\partial^\mu\mathrm{Tr}(\Delta^\dagger D_\mu\Delta) = (\varphi^\dagger\varphi)\left[\mathrm{Tr}((D^\mu\Delta)^\dagger D_\mu\Delta) + \mathrm{Tr}(\Delta^\dagger D^\mu D_\mu\Delta)\right] \tag{2.10}$$

$$\partial^\mu(\varphi^\dagger\varphi)\mathrm{Tr}((D_\mu\Delta)^\dagger\Delta) = (\varphi^\dagger\varphi)\left[\mathrm{Tr}((D^\mu\Delta)^\dagger D_\mu\Delta) + \mathrm{Tr}((D^\mu D_\mu\Delta)^\dagger\Delta)\right] \tag{2.11}$$

Now by adding the terms to have the equations of motion, for example in equation (2.10) we obtain:

$$(\varphi^\dagger\varphi)\partial^\mu \mathrm{Tr}(\Delta^\dagger D_\mu\Delta) = (\varphi^\dagger\varphi)\mathrm{Tr}((D^\mu\Delta)^\dagger D_\mu\Delta) + (\varphi^\dagger\varphi)\boxed{\mathrm{EoM}} + M^2\boxed{\varphi^2\Delta^2}$$
$$+\mu\boxed{\varphi^4\Delta} + f\boxed{\varphi^2\Delta\psi^2} + (\lambda_1,\lambda_4)\boxed{\varphi^4\Delta^2} + (\lambda_2,\lambda_3)\boxed{\varphi^2\Delta^4} \quad (2.12)$$

The terms in boxes refer to other operators that are already in the basis, so we can just redefine the couplings and ignore these terms. Then by summing equations (2.10) and (2.11) we obtain:

$$\mathcal{O}_{\Delta\Box} = (\varphi^\dagger\varphi)\Box\mathrm{Tr}(\Delta^\dagger\Delta) = 2(\varphi^\dagger\varphi)\mathrm{Tr}((D_\mu\Delta)^\dagger D^\mu\Delta) + (\varphi^\dagger\varphi)\boxed{\mathrm{EoM}}$$
$$+M^2\boxed{\varphi^2\Delta^2} + \mu\boxed{\varphi^4\Delta} + f\boxed{\varphi^2\Delta\psi^2} + (\lambda_1,\lambda_4)\boxed{\varphi^4\Delta^2} + (\lambda_2,\lambda_3)\boxed{\varphi^2\Delta^4} \quad (2.13)$$

Now we see that $(\varphi^\dagger\varphi)\,\Box\,\mathrm{Tr}(\Delta^\dagger\Delta)$ and $\mathrm{Tr}(\Delta^\dagger\Delta)\Box(\varphi^\dagger\varphi)$ are related by "integration by parts" and dropping the total derivatives. There are 4 more operators: $\mathrm{Tr}(\Delta^\dagger D_\mu\Delta)$ $((D^\mu\varphi)^\dagger\varphi)$, $\mathrm{Tr}((D_\mu\Delta)^\dagger\Delta)$ $((D^\mu\varphi)^\dagger\varphi)$,$\mathrm{Tr}((D_\mu\Delta)^\dagger\Delta)$ $(\varphi^\dagger D^\mu\varphi)$ and $\mathrm{Tr}(\Delta^\dagger D_\mu\Delta)$ $(\varphi^\dagger D^\mu\varphi)$. We will skip the derivation but it can be shown that these 3 of these operators are redundant. This can be shown by writing 4 equations relating the operators to total derivatives using again Leibniz rule. This way we will have 4 equations relating the 6 possible operators, two of them are $\mathcal{O}_{\Delta\Box}$ so the other 3 equations relate the other 4 operators. However, we see that these 4 operators are not self-hermitian, hence we will always have to write the hermitian conjugate even if the number of operators (and Wilson coefficients) necessary is just 1. To do this, the term that we write is $\mathcal{O}_{\Delta\varphi D} = (\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)\mathrm{Tr}(\Delta^\dagger i\overleftrightarrow{D}^\mu\Delta)$, with $\varphi^\dagger i\overleftrightarrow{D}^\mu\varphi = i\varphi^\dagger\left(D_\mu - \overleftarrow{D}_\mu\right)\varphi$ and $\varphi^\dagger\overleftarrow{D}_\mu\varphi = (D_\mu\varphi)^\dagger\varphi$. This operator contains the 4 operators written above, however with just one Wilson coefficient, so we have reduced the number of coefficient, and written it in a self-hermitian way. In these operators we have found a difference with the basis developed in [49].

This procedure is applied to the other possible classes of operators with derivatives $\boxed{\Delta^4 D^2}$ and also to the triplet combinations of these classes of operators, with just a few remarks: for the case of 4 Δ fields and 2 derivatives it is not necessary to write $\overleftrightarrow{D}_\mu$ since we can pick an operator that is self-hermitian, for instance: $\mathrm{Tr}\big(\Delta^\dagger D_\mu\Delta\big)^*\,\mathrm{Tr}\big(\Delta^\dagger D_\mu\Delta\big)$. The choice of the operators $\mathcal{O}^2_{\Delta D\varphi}$ and $\mathcal{O}^3_{\Delta D\varphi}$ is explained in appendix A.2.

A different set of operators appears in this model due to the hypercharge of the triplet field. This field allows configurations in which we have two lepton doublets l and l^c, since they are allowed by invariance under $U(1)_Y$. These operators are listed in Tab. 2.3 and are lepton-violating operators: we find two operators very similar to the Weinberg operator but with dimension-6, $\mathcal{O}^{1,2}_{\Delta\varphi l}$. We also find other combinations which combine the lepton number violating structure $\bar{l}^c\Delta l$ combined with gauge bosons, which could be interesting for LFV processes.

2.1.3 Choice of Basis and Scales

Apart from the choice of basis allowed by total derivatives, rewriting some operators using some properties or using the EoM, there are still some more choices to cover. Fields can be chosen to be on its mass basis or on its flavour basis. This change in fermionic sector is done by applying the CKM or PMNS matrix to the appropriate fields. In the SM normally we transform the fields by a transformation that leave their Yukawa matrix diagonal, for instance $\bar{d}'_L Y_d d'_R = \bar{d}_L V_{d_L} Y_d V^\dagger_{d_R} d_R$ such that $V_{d_L} Y_d V^\dagger_{d_R} = Y_d^{\text{diag}}$, and as we saw in equation (1.29) this makes the CKM matrix $V_{\text{CKM}} = V^\dagger_{d_R} V_{d_L}$ appear in the W-boson interaction. In this case, the ΔEFT extension adds more operators that are in general non-diagonal in flavour which makes that coefficients such as $\mathcal{O}_{\Delta d}$ to be transformed as:

$$(C_{\Delta d})^{pr}\bar{d}'_p\gamma^\mu d'_r = (C_{\Delta d})^{pr}(V^\dagger_{d_L})_{rt}(V_{d_L})_{ps}\bar{d}_s\gamma^\mu d_t \tag{2.14}$$

In the case where it is flavour-diagonal $(C_{\Delta d})^{pr} \sim C_{\Delta d}\delta^{pr}$ we do not need to take care of these matrices, however, in general this is not the case and we would have to deal with all different matrices that are needed to diagonalize the Yukawa matrices. As discussed in [61] we can make U(3) flavour transformations, which leave SM gauge interactions invariant but modify Yukawa interactions and Wilson coefficients, making the specific transformation that leave Y_D and Y_e diagonal and $Y_U = V^\dagger_{CKM} Y_U$, we then only need to transform $u_L \to V_{CKM} u_L$ so we only need to keep track of the CKM matrix in the operators where u_L appears. Results and bounds set for other choices of basis will be different.

During this work we will assume that the scale $\Lambda \gg M$ where M is the mass of the triplet. Although it seems trivial, this hierarchy of scales should never be broken, otherwise the *ansatz* of the whole EFT does not work and this basis would not be useful. During this work we will generally assume the mass of the triplet not to be larger than 1 TeV, thus we will be always assuming $\Lambda \gtrsim 1$ TeV. Different specific assumptions on the masses and scale will be given on the different sections.

3 Phenomenology

There are several strategies to constrain the different parameters of the model, one of them is to consider the mass of the triplet larger than the scale of the experiment, then observe which operators of the SMEFT and with the data that we currently have on this operators infer the bounds on the parameters of our model. This is the first approach we are going to follow in this study.

The second strategy is to obtain information on different parameters through modifications of the SM parameters. This is well known in the SMEFT [62], where well measured quantities such as the Fermi constant G_F or, for instance, the couplings of the fermions to the Z bosons are modified by some of the Wilson coefficients. Some operators of the ΔEFT also modify these couplings, and can be constrained with different assumptions with data from the Large Electron Positron collider (LEP) [64] and LHC.

The last strategy we will use is more straightforward, and consists on looking for the signal (rather absence of it) of the doubly- and singly-charged Higgs to constrain some of the new possible production channels that appear in the ΔEFT.

3.1 Below the Triplet Mass Scale

3.1.1 Lepton Flavour Violating Processes

In this section we assume that the scale of the experiment is much lower than the mass of the triplet Δ, $p^2 \ll M^2$. In this case this heavy state cannot be produced on-shell, so we look for indirect measurements to constrain our model. We will start with the renormalizable Lagrangian and then we will generalize it to a subset of dimension 5 operators.

The best way to test the type-2 seesaw mechanism at a low scale is through the LFV processes. Since the coupling to the fermions f_{ab} of equation (1.63) is related to the PMNS matrix, it will be in general non-diagonal. This coupling also generates the $l_a l_b H^{++}$ vertex, so the charged Higgs will couple leptons of different generations, producing this LFV processes, for instance the $\mu \to 3e$ at tree level and $\mu \to \gamma e$ at loop level as we can see in Figure 3.1. These two processes have an upper bound on the branching ratio of the muon of $BR(\mu \to 3e) < 1 \times 10^{-12}$ at 90% C.L. by the SINDRUM experiment [51] and $BR(\mu \to e\gamma) < 4.2 \times 10^{13}$ at 90% C.L. by the MEG experiment [52]. Assuming that $p^2 \leq m_\mu^2 \ll M_{H^{++}}^2$ we can obtain the branching ratio for both decays [53]:

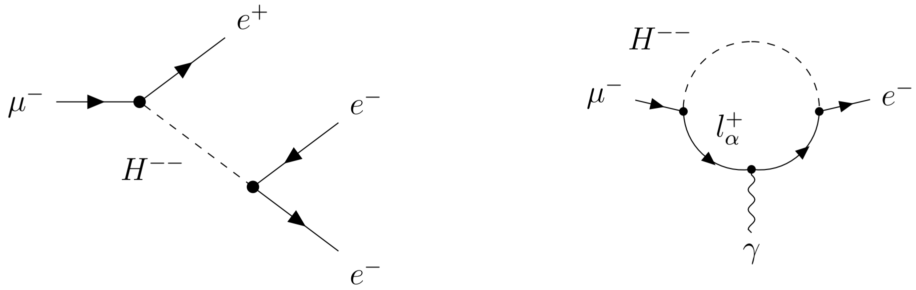


Figure 3.1: Figures of the LFV processes that could happen in the type-2 seesaw mechanism (left) decay of the muon into 3 electrons at tree level through H^{--} and (right) the loop-induced decay of the muon into a photon and an electron.

$$Br(\mu \to 3e) = \frac{|f_{\mu e} f_{ee}^*|^2}{4\, G_f^2\, m_{H^{++}}^4} \tag{3.1}$$

$$Br(\mu \to \gamma e) = \frac{\alpha_{em}}{3\pi} \frac{|f_{\mu\alpha} f_{\alpha e}^*|^2}{G_f^2} \left(\frac{1}{m_{H^{++}}^2} + \frac{1}{8 m_{H^{++}}^2} \right)^2 \tag{3.2}$$

The factors $f_{\alpha\beta}$ are given by the PMNS matrix as we saw in eq. (1.66) if the neutrino masses are known. Since they are not, these couplings can only be tested for different assumption of the mass states. Notice that if the oscillations parameters are known we just need one known mass state to determine the rest of the parameters. Hence, we will obtain upper bounds on the parameters v_t by using 4 different cases a) $m_1 = 0$ eV NH, b) $m_1 = 0.1$ eV NH, c) $m_3 = 0$ eV IH and d) $m_3 = 0.1$ eV IH. We choose $m = 0$ eV as the limit case and $m = 0.1$ to investigate what happens in the nearly degenerate case (all the mass differences are smaller than the absolute mass $\Delta m < (0.1\,\text{eV})^2$). That is why cases b) and d) should be almost the same, since in that scenario the masses of the neutrinos are nearly degenerate, the hierarchy is negligible. However, the CP-phase given by the fits, Tab. 1.2, is different for the IH and NH, so the nearly degenerate case will be different depending on which CP-phase we choose, so we will consider both cases labeled as we mentioned above.

It is also not trivial to determine which process puts the more stringent constrain. The $\mu \to e\gamma$ process is loop suppressed, however the experimental constrain is currently better than for the tree level process $\mu \to 3e$. But that is not the only reason, it has been shown [54] that the muon to electron photon process is independent of the Majorana phases, however the μ to $3e$ process is not. Furthermore, the loop-induced decay of the muon is almost independent of the neutrino mass spectrum:

$$(m^\dagger m)_{e\mu} \sim \frac{\sqrt{6}}{8}\Delta m^2_{12} + \frac{\sqrt{2}}{2}\Delta m^2_{31} s_{13} \quad \text{(NH, and degenerate)} \tag{3.3}$$

$$(m^\dagger m)_{e\mu} \sim \frac{\sqrt{6}}{8}\Delta m^2_{12} - \frac{\sqrt{2}}{2}\Delta m^2_{31} s_{13} \quad \text{(IH)} \tag{3.4}$$

Where we have set all three phases to 0, to test the dependence on the neutrino oscillation parameters. Even if the IH and NH are different, the difference is not as big as one might imagine since the second term is an order of magnitude larger than the first one. In the case of μ to $3e$ it is the opposite, the three cases are different:

$$m^\dagger_{ee} m_{e\mu} \sim \frac{\sqrt{6}}{31}\Delta m^2_{21} + \frac{\sqrt{2}}{8}\sqrt{\Delta m^2_{21}\Delta m^2_{31}}\, s_{13} \quad \text{(NH)} \tag{3.5}$$

$$m^\dagger_{ee} m_{e\mu} \sim \frac{\sqrt{6}}{16}\Delta m^2_{21} + \frac{\sqrt{2}}{4}\Delta m^2_{31} s_{13} \quad \text{(Degenerate)} \tag{3.6}$$

$$m^\dagger_{ee} m_{e\mu} \sim \frac{\sqrt{6}}{16}\Delta m^2_{21} - \frac{\sqrt{2}}{2}\Delta m^2_{31} s_{13} \quad \text{(IH)} \tag{3.7}$$

This will induce that in some cases the constrain of the loop-induced decay becomes better than the tree-level one. To obtain the bounds we assume $m_{H^+} \simeq m_{H^{++}}$ and invert equations (3.1) and (3.2). This gives a relation between $m_{H^{++}}$ and v_t so we plot the curves in this plane to obtain the exclusions 90% C.L. Fig. 3.2. As a general feature of the three plots, the exclusion limit becomes relevant for low values of v_t, this can be seen from eq. (3.1) since $f_{\mu e} \sim m_{\mu e} v_t$ v_t will be in the denominator of the expression, but this comes from the fact that we fix the couplings $f_{\alpha\beta}$ to the neutrino masses and PMNS matrix, this makes that when v_t is small $f_{\alpha\beta}$ must compensate to give the mass matrix, thus, a lower v_t gives a large $f_{\alpha\beta}$ giving a large coupling of $H^{++} \to l^+_\alpha l^+_\beta$.

As we can see from the figures the NH is best constrained by the MEG experiment, however, for the IH and degenerate (for both different δ) scenarios the μ to $3e$ process places more stringent constrains. For $v_t = 10^{-9}$ and NH the bound from MEG is 734 GeV $\gtrsim m_{H^{++}}$, for the IH the bound is given by the μ to 3e process 2.4 TeV $\gtrsim m_{H^{++}}$ and for the degenerate cases the constraints are 3.5 TeV $\gtrsim m_{H^{++}}$ with $\delta = 1.08\pi$ and 7.3 TeV $\gtrsim m_{H^{++}}$ with $\delta = 1.58\pi$, both constrains coming from the μ to $3e$ decay.

For completeness, we can turn around eq. (1.51), and instead of having a exclusion plot in the $v_t - m_{H^{++}}$ we can have exclusion limits of the parameters $\mu - v_t$, valid for the degenerate case, using:

$$M^2_{H^{++}} \simeq \frac{\mu v_d^2}{\sqrt{2} v_t} \tag{3.8}$$

By introducing this approximate relation in equations (3.1) and (3.2) we obtain the exclusion plots shown in Figure 3.3. We again observe that in NH the most stringent

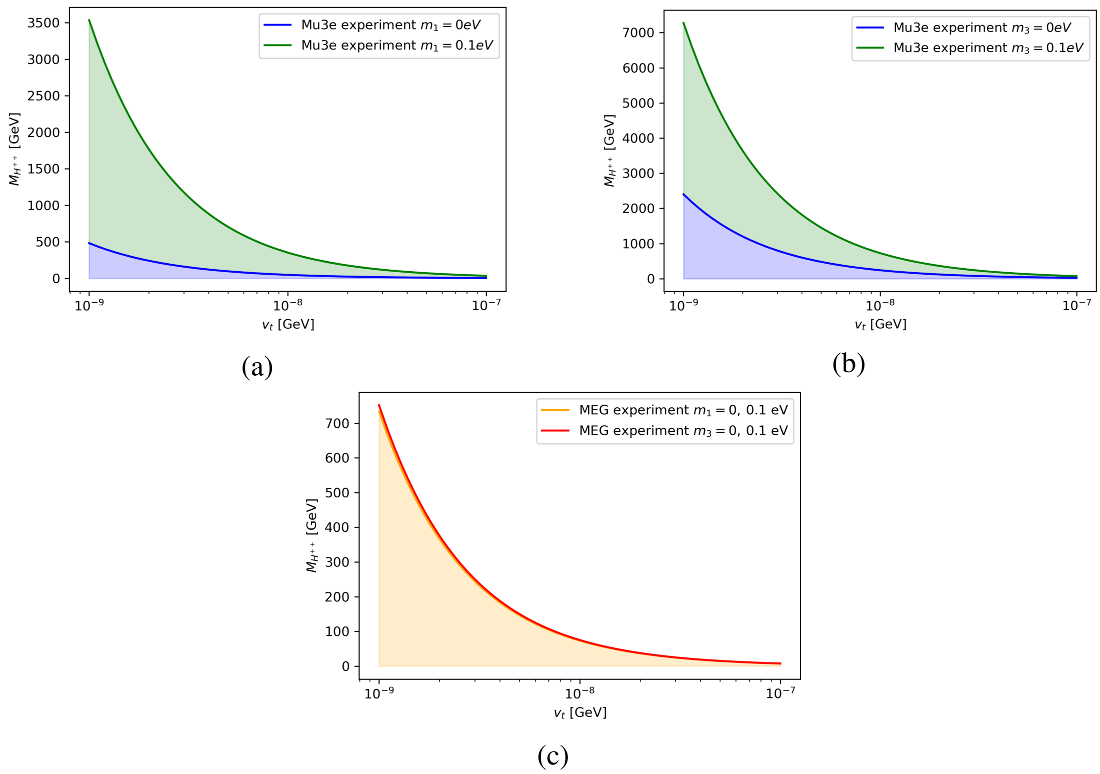


Figure 3.2: In (a) we observe the exclusion plot for the NH for the cases $m_1 = 0$ eV (blue) and $m_1 = 1$ eV (green). In (b) we plot the same assuming IH so this time is the lower state $m_3 = 0$ eV (blue) and $m_3 = 1$ eV (green). In Fig. (c) we have the constrains of the MEG experiment on the loop-induced decay of the muon assuming normal and inverted hierarchy, we also observe that it is almost independent of the choice of mass spectrum.

values come from the MEG experiment while for the rest the μ to $3e$ decay is best for constraining the parameter space available.

In summary, we have observed that both processes are necessary to constrain, in the best possible way, the parameter space available in the type-2 seesaw model, since the $\mu \to e\gamma$ decay currently constrains best the parameters of the model. On the other hand, the IH and degenerate mass spectrum are best constrained by the μ to $3e$ process and in these cases the constrains are much better than for normal hierarchy. As a final remark, experiments such as the Mu3e experiment [55] aim to set a lower upper bound on the branching ratio of 10^{-16} on its final phase, which would set a constrain on the mass of the type-2 seesaw mechanism of ~ 5 TeV at low values of the vev $v_t \sim 10^{-9}$.

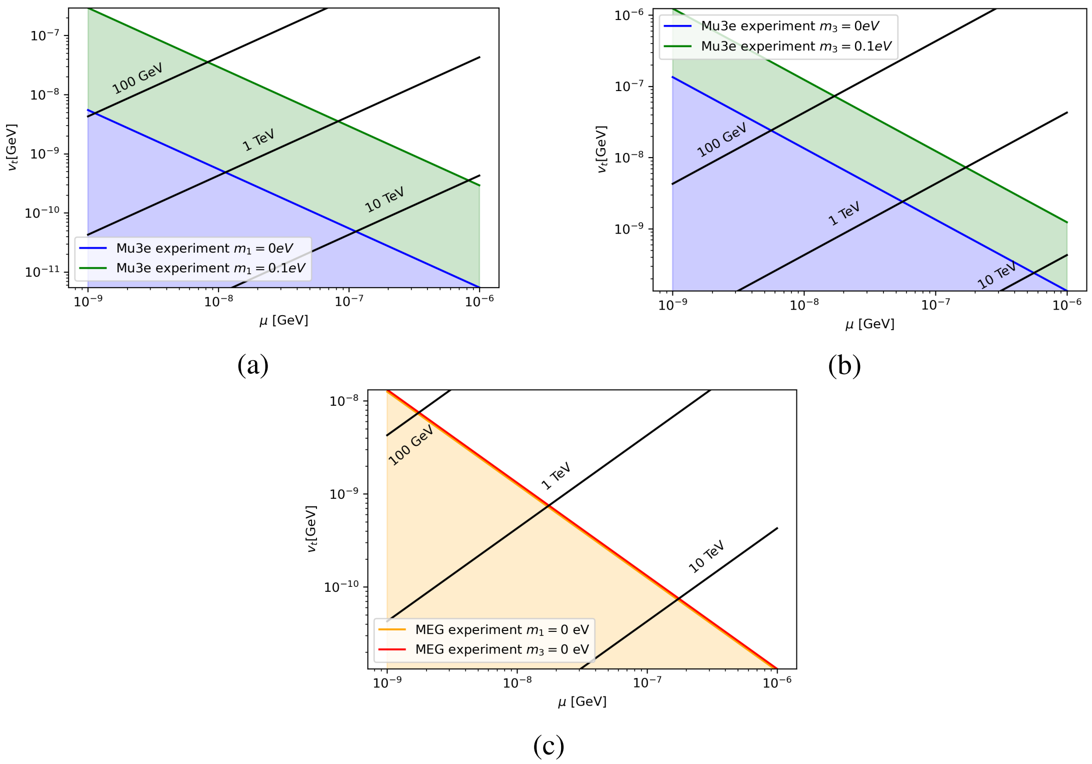


Figure 3.3: Using equation (3.8) we can turn the plots of Figure 3.2 into exclusion plots in the plane $v_t - \mu$, again the plot (a) assumes NH with (b) IH both using the exclusion limit of the SINDRUM experiment, and in (c) NH and IH are plotted with the exclusion limit of the MEG experiment.

3.1.2 Dimension 5 Contributions to the SMEFT

The LFV processes of the last section can be produced by other models, besides since it is commonly thought that BSM particles should have large masses, we could study the decays of the previous section in a model independent way. In the SMEFT basis, the decay of the muon due to a much heavier internal state would be parameterized by the SMEFT operator $\mathcal{O}_{ll}$ for the $\mu \to 3e$ process. What we did in the last section by assuming $M^2 \ll p^2$ was to give a particular expression of $C_{ll} \sim f_{ee} f_{\mu e}^2 / M^2$. In other words, by calculating the Feynman diagram with this approximation we obtained and effective coupling of the SMEFT basis, C_{ll} and then we put a bound on the parameters of the model that generate this coefficient. We can do this approximation in a more general way without the need of calculating any Feynman diagram obtaining all the operators generated by the type-2 seesaw mechanism, and also the some of the dimension 5 coefficients.

Reference [56] offers a general way of obtaining the coefficients of the EFT step by step in perturbation theory. It consists of calculating the saddle point approximation of the Feynman path integral of the action, which gives that the heavy field on the minimum satisfies $\frac{\delta \S}{\delta \Phi} = 0$. This expression leads to the equations of motion which we can solve in an approximate way by expanding it in powers of p^a/M^b, and then substitute the heavy field Φ into the action. In our case we want to integrate out the heavy field Δ^a to do that we obtain the equations of motion upt to linear terms in Δ^a, we cut it here because an operator with more insertions of Δ fields would lead to higher dimension operators. The equation of motion for the type-2 renormalizable Lagrangian (1.49) and (1.63) is:

$$\left[-D_\mu D^\mu - \mathbf{M}^2 - \left(\lambda_1 + \frac{\lambda_4}{2}\right)\varphi^\dagger\varphi\right]\Delta^a = \frac{f_{rp}}{\sqrt{2}}\bar{l^c}_r i\sigma^2\sigma^a l_p + \frac{\mu}{\sqrt{2}}\tilde{\varphi}^\dagger\sigma^a\varphi + \frac{\lambda_4}{2}\varphi^\dagger i\varepsilon_{bac}\sigma^c\varphi\Delta^b + \mathcal{O}(\Delta^2) \tag{3.9}$$

Where we have cut only to linear terms in Δ since going further would generate operators of dimension larger than 6. The last term of this expression can generate some dimension 6 operators, however those operators give a null contribution since the Levi-Civita is antisymmetryc and these operators are symmetric under the $SU(2)_L$ indices. Now, we invert the previous expression and expand it in powers of mass to substitute in the Lagrangian (1.49) and (1.63):

$$\Delta^a \simeq \left[-\frac{1}{M^2} + \frac{D_\mu D^\mu}{M^4} + \left(\lambda_1 + \frac{\lambda_4}{2}\right)\frac{\varphi^\dagger\varphi}{M^4}\right]\left[\frac{f_{rp}}{\sqrt{2}}\bar{l}_r^c i\sigma^2\sigma^a l_p + \frac{\mu}{\sqrt{2}}\tilde{\varphi}^\dagger\sigma^a\varphi + \frac{\lambda_4}{2}\varphi^\dagger i\varepsilon_{bac}\sigma^c\varphi\Delta^b\right] \tag{3.10}$$

In the previous expression we only added operators up to dimension 4, but later on we will include also non-renormalizable operators. Now, we can substitute into the Lagrangian to obtain the SMEFT operators generated by the triplet. We will start by looking to contributions to the operators that only contain Higgs doublets.

$$\mathcal{L}_{int} \supseteq m^2\varphi^\dagger\varphi - \frac{\lambda}{4}(\varphi^\dagger\varphi)^2 - \frac{|\mu|^2}{2}\varphi^\dagger\sigma^a\tilde{\varphi}\left[-\frac{1}{M^2} + \frac{D_\mu D^\mu}{M^4} + \left(\lambda_1 + \frac{\lambda_4}{2}\right)\frac{\varphi^\dagger\varphi}{M^4}\right]\tilde{\varphi}^\dagger\sigma^a\varphi - \frac{|\mu|^2}{2M^4}\left(\lambda_1 + \frac{\lambda_4}{2}\right)\varphi^\dagger\varphi\tilde{\varphi}^\dagger\sigma^a\varphi\varphi^\dagger\sigma^a\tilde{\varphi} + \text{h.c.} \tag{3.11}$$

We can use again the properties of the Pauli matrices (2.2), and skipping some calculations we get:

$$\mathcal{L}_{int} \supseteq m^2\varphi^\dagger\varphi - \frac{1}{4}\left(\lambda - 4\frac{|\mu|^2}{M^2}\right)(\varphi^\dagger\varphi)^2 - \frac{|\mu|^2}{2M^4}\varphi^\dagger\sigma^a\tilde{\varphi}\, D_\mu D^\mu\, \tilde{\varphi}^\dagger\sigma^a\varphi$$
$$-\frac{|\mu|^2}{M^4}\left(\lambda_1 + \frac{\lambda_4}{2}\right)(\varphi^\dagger\varphi)^3 + \text{h.c.} \tag{3.12}$$

Here we already obtained some of the SMEFT operators like $\mathcal{O}_\varphi$ and some modifications to the quartic coupling of the Higgs potential. Now it requires some more calculations that we will skip to work out the term with derivatives of the last expression. This term gives two kind of operators: with two derivatives acting on just a field or the derivatives acting on different fields. The first term can be rewritten in other operators by using the classical equations of motion of the Higgs field as we showed on the section 1.3.1. The other terms with derivatives acting on different fields will generate operators $\mathcal{O}_{\varphi\Box}$ and $\mathcal{O}_{\varphi D}$, obtaining in the end the following contributions:

$$\mathcal{L}_{int} \supseteq m^2\varphi^\dagger\varphi - \frac{1}{4}\left(\lambda - 4\frac{|\mu|^2}{M^2} + 8\frac{|\mu|^2 m^2}{M^4}\right)(\varphi^\dagger\varphi)^2 + 2\frac{|\mu|^2}{M^4}\varphi D^\mu\varphi\left(\varphi^\dagger D_\mu\varphi\right)^*$$
$$+\frac{|\mu|^2}{M^4}\varphi^\dagger\varphi\Box(\varphi^\dagger\varphi) + \frac{|\mu|^2}{M^4}\varphi^\dagger\varphi\left(y^{e*}_{rp}\bar{l}_p e_r\varphi + y^{d*}_{rp}\bar{q}_p d_r\varphi + y^{u*}_{rp}\bar{q}_p u_r\tilde{\varphi}\right)$$
$$-\frac{|\mu|^2}{M^4}(\lambda_1 + \lambda_4 - \lambda)(\varphi^\dagger\varphi)^3 + \text{h.c.} \tag{3.13}$$

After eq. (3.1.2) we omitted the calculation with the leptonic operator on the right-hand side of the equation as well as operators of dimension 5, in order to tackle the problem step by step. Now, we consider the contributions given by this leptonic operator.

$$\mathcal{L}_{int} \supseteq -\frac{\mu f_{rp}}{2M^2}\bar{l}^c_r i\sigma^2\sigma^a l_p\tilde{\varphi}^\dagger\sigma^a\varphi + \text{h.c.} - \frac{\mu f_{rp} f^*_{st}}{2M^2}\bar{l}^c{}_r i\sigma^2\sigma^a l_p\bar{l}_s\sigma^a i\sigma^2 l^c_t \tag{3.14}$$

The first term is simple to rewrite using eq. (2.2), getting the Weinberg operator. The second term, requires also the following Fierz identity, to leave it in terms of the Warsaw basis:

$$l_p\bar{l}_s = -\frac{1}{2}\bar{l}_s\gamma^\mu l_p\gamma_\mu \tag{3.15}$$

Now, we can just apply again the property of the Pauli matrices of eq. (2.2), and find that the operators generated are:

$$\mathcal{L}_{int} \supseteq -\frac{\mu f_{rp}}{M^2}(\tilde{\varphi}^\dagger l_r)^T C(\tilde{\varphi}^\dagger l_p) + \text{h.c.} + \frac{f_{rp} f^*_{st}}{2M^2}(\bar{l}_r\gamma^\mu l_t)(\bar{l}_s\gamma_\mu l_p) \tag{3.16}$$

We have discussed the tree-level contributions to the lagrangian coming from renormalizable terms, now, we start adding the dimension 5 operators that also have a tree-level contribution. Actually, the only terms that we need to use from eq. (3.1.2) are the first order term in the expansion and the term containing only Higgs particles, since all other terms would generate dimension 7 operators (or larger) when combined with dimension 5 operators. Those that contribute at tree level are linear in Δ, and give the following contributions:

$$\mathcal{L}_{int} \supseteq \frac{\mu}{M^2}\varphi^\dagger\varphi\left(\mu C^{rp}_{\Delta le}\bar{l}_p e_r\varphi + \mu C^{rp}_{\Delta qd}\bar{q}_p d_r\varphi + (\mu C^{rp}_{\Delta qu})^*\bar{q}_p u_r\tilde{\varphi}\right) + \frac{2\mathcal{R}\text{e}(\mu C_{\varphi^4\Delta})}{M^2}(\varphi^\dagger\varphi)^3 + \text{h.c.} \tag{3.17}$$

Notice that the dimension 6 operator with only 6 Higgs fields gives the real part of the coefficient since we the coefficients may be complex and then the hermitian conjugate removes the imaginary part. Finally, we can write all the possible contributions to the SMEFT dimension 6 operators from the dimension 5 terms of our basis:

$$\mathcal{L}_{>4} \equiv \sum_i C_i\mathcal{O}_i = -\frac{\lambda'}{4}(\varphi^\dagger\varphi)^2 + C_5\mathcal{O}_5 + C_\varphi\mathcal{O}_\varphi + C_{\varphi D}\mathcal{O}_{\varphi D} + C_{\varphi\Box}\mathcal{O}_{\varphi\Box} + C_{ll}\mathcal{O}_{ll} + C_{e\varphi}\mathcal{O}_{e\varphi} + C_{d\varphi}\mathcal{O}_{d\varphi} + C_{u\varphi}\mathcal{O}_{u\varphi} + \text{h.c.} \tag{3.18}$$

With the following contributions:

$$\lambda' = \left(\lambda - 4\frac{|\mu|^2}{M^2} + 8\frac{|\mu|^2 m^2}{M^4}\right) \tag{3.19}$$

$$(C_5)_{rp} = -\frac{\mu f_{rp}}{M^2} \tag{3.20}$$

$$C_{\varphi\Box} = \frac{|\mu|^2}{M^4} \tag{3.21}$$

$$C_{\varphi D} = 2\frac{|\mu|^2}{M^4} \tag{3.22}$$

$$C_\varphi = -\frac{|\mu|^2}{M^4}(\lambda_1 + \lambda_4 - \lambda) + \frac{2}{\Lambda}\frac{\mathcal{R}\text{e}(\mu C_{\varphi^4\Delta})}{M^2} \tag{3.23}$$

$$(C_{ll})_{rpst} = \frac{f_{rp}f^*_{st}}{2M^2} \tag{3.24}$$

$$(C_{e\varphi})_{rp} = \frac{|\mu|^2}{M^4}y^{e*}_{rp} + \frac{1}{\Lambda}\frac{\mu}{M^2}(C_{\Delta le})_{rp} \tag{3.25}$$

$$(C_{d\varphi})_{rp} = \frac{|\mu|^2}{M^4}y^{d*}_{rp} + \frac{1}{\Lambda}\frac{\mu}{M^2}(C_{\Delta qd})_{rp} \tag{3.26}$$

$$(C_{u\varphi})_{rp} = \frac{|\mu|^2}{M^4}y^{u*}_{rp} + \frac{1}{\Lambda}\frac{\mu^*}{M^2}(C_{\Delta qu})^*_{rp} \tag{3.27}$$

3.1.3 Constrains from Higgs Measurements to Dimension 5 Operators

It is worth it going back a step back and see what we have won. Previous to the last section we had a set of coefficients of dimension 5 containing Δ fields in the Lagrangian, however, not a single particle related to this field has yet been observed, thus studying this operators becomes a challenge. In a similar way to the Fermi theory we have assumed that the heavy state is heavier than the momentum scale we work with and hence it can only be produced virtually as in the case of the W boson in Fermi's theory.

This way we can work with SM fields as external states where we have information on several coefficients. Furthermore, we have written the contributions of the ΔEFT at low energies in terms of the Warsaw basis, which several groups have studied in a model-independent way [39, 57], meaning that we can benefit from their data to put constrains in our operators by using the relations obtained in equations (3.19 - 3.27).

In this analysis we will use the data from [39] since they provide the χ^2 function, constraining many operators generated by the ΔEFT model. This χ^2 function provided uses 159 data points, including LEP measurements on EWPO, Higgs production data of the runs 1 and 2 of the LHC and some more data on production on W boson pair production. This then included in the expression:

$$\chi^2 = (y - \mu(C))^T V^{-1} (y - \mu(C)) \tag{3.28}$$

Where y is the vector of central values, V^{-1} is the inverse of the covariance matrix (which takes into account correlations when possible) and $\mu = \mu_{SM} + \mu_{EFT}(C)$ is the theoretical predictions of the observable, which we separate in a SM contribution plus a correction of the EFT coefficient of the subset of the Warsaw basis chosen. In this case the global analysis done is to linear order on the Wilson coefficients, which makes the likelihood function associated to this χ^2 function is Gaussian distribution. In our case we will drop the generality of the global analysis and focus on the operators generated by our model of equations (3.19 - 3.27), however not all operators can be measured with this set of data, for example λ' has not been yet measured at the LHC, so this operator and C_φ will be dropped. Weinberg operator is also dropped since it is typically measured at $0\nu\beta\beta$ experiments, which are at a much lower scale than LHC experiments, although it is also possible to study this operator in colliders [58]. This analysis also makes the assumption of $U(3)^5$ symmetry on the dimension-6 coefficients of the SMEFT. This is, however not consistent with our model since we need the coefficient $f_{\alpha\beta}$ to be different for each component, otherwise neutrino masses would be degenerate and no oscillations would be seen.

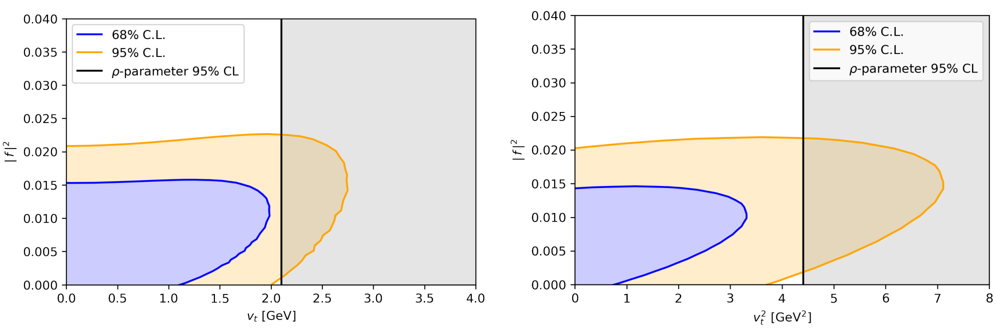
Using eq. (3.8), we can rewrite μ in terms of v_t, setting $M = 1$ TeV, which is a more interesting parameter in the type-2 seesaw mechanism. In any case this is not such a bad approximation since colliders do not constrain $v_t - f$ as well as the LFV processes, so the differences between the different components of f can be neglected.

Neglecting operators of dimension 5 we just get a constrain of the $v_t - f$ plane. Minimizing the χ^2 function we can find the best-fit values and draw the confidence level intervals in which the parameters can lie at different confidence level.

The result of Fig. 3.4 has already been obtain in Ref. [39], however we used expression (3.8) to rewrite the parameter μ as the vev and setting the mass of the triplet to 1 TeV. f is the coupling that through v_t should give mass to the neutrinos, this means that $\sqrt{2} f v_t \lesssim 0.2$ eV, for instance for a value of $v_t \sim 1$ GeV then $f \lesssim 1.4 \times 10^{-10}$. So, looking at Fig. 3.4 we see that we are far from constraining the set of values for $v_t - f$ that can generate the masses of neutrinos that we observe. On the other hand the maximum value of the vev at 95% C.L. is of $v_t \lesssim 2.7$ GeV, a good bound on this value.

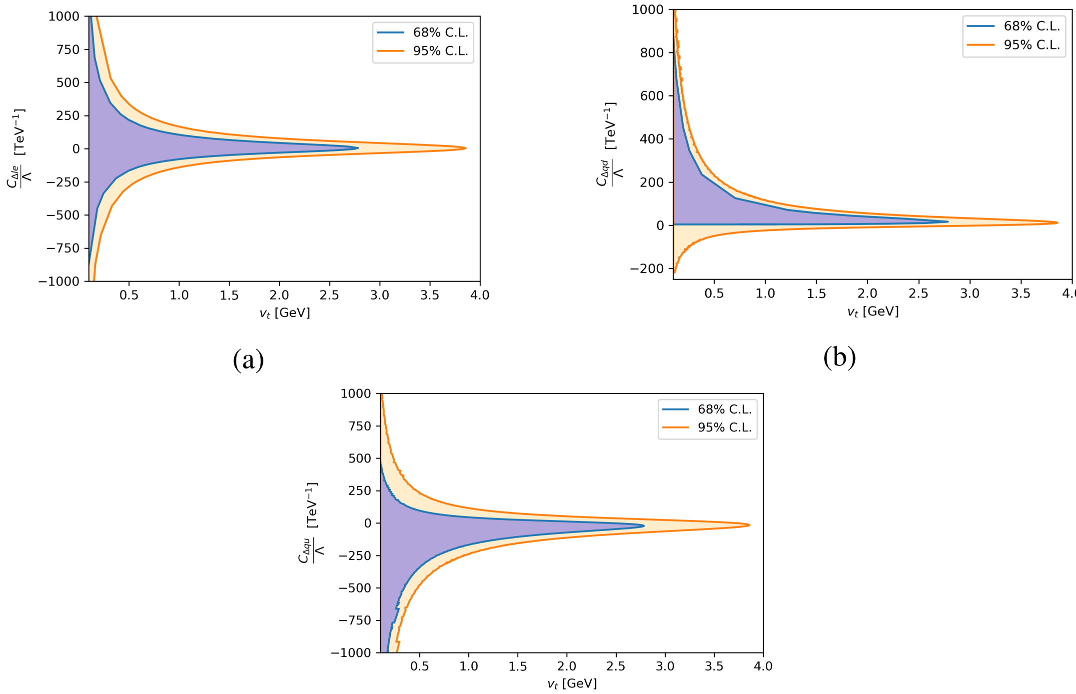
Although the bound on v_t is good, it is clear that LFV processes put more stringent constrains for these set of parameters when we set them to the mass of the neutrinos, however, this procedure is a very useful for constraining $C_{\Delta le}$, $C_{\Delta qu}$ an $C_{\Delta qd}$, since Higgs data is needed to analyze these couplings. Minimizing again the χ^2 function this time we will draw the confidence levels profiling the parameters that are not shown in the plot using MINUIT [59]. The three C.L. can be found in Fig. 3.5

The form of plots 3.5a-3.5c can be understood from equations (3.25)-(3.27), where one must once again apply equation (3.8). The parameter v_t is multiplied to each of the dimension 5 Wilson coefficient so the dependence is clear, if v_t is low the available space for the Wilson coefficient is large, on the other hand if v_t is larger the Wilson coefficients have more stringent constrains. The value of v_t is constrained at 95% C.L.



(a) (b)

Figure 3.4: Confidence level regions of the vacuum expectation value v_t and f at 68% (blue) and 95%(orange) with $\chi^2/\text{dof} = 0.989$, and setting the mass of the triplet to $M = 1$ TeV. (left) for the $f - v_t$ plane and (rigth) in the $f - v_t^2$ plane, which helps comparing this result with that of Ref. [47]. The black line corresponds to the 95% exclusion limit from the ρ-parameter, $v_t \leq 2.1$ GeV obtained in Ref. [60].



(a) (b)

(c)

Figure 3.5: Allowed regions for the different Wilson coefficients of dimension 5 with respect to the vev v_t. In plot (a) $C_{\Delta le}$, (b) $C_{\Delta qd}$ and (c) $C_{\Delta qu}$ for the different confidence levels 68% (blue) and 95% (orange) with χ^2/dof=0.99.

to be lower than $v_t \lesssim 4$ GeV, a bit higher than for the previous plots, since the more parameters we allow to come into the fit the more range of values will have the rest of parameters to vary.

To have better constrains of these parameters, better measurements of Higgs observables are required. Operators $C_{u\varphi}$ and $C_{d\varphi}$ contribute to Higgs production processes such as in the tth channel and also ggF at loop level. However, the strongest contributions of all these 3 operators are to the decays of the Higgs, $h \to \bar{b}b$ for $C_{d\varphi}$, $h \to \bar{c}c$ for $C_{u\varphi}$ and $h \to \bar{\mu}\mu$ for the Wilson coefficient $C_{e\varphi}$. These decays have still some room for improvement, for instance the signal strength, $\mu = \frac{(\sigma \mathrm{BR}^f)_{\mathrm{obs}}}{(\sigma \mathrm{BR}^f)_{\mathrm{SM}}}$ where f indicates the final state and σ is the production cross section, of the process $pp \to h \to \mu\mu$ is $\mu = -0.1 \pm 1.5$. So, to get good constrains on these coefficients these decays also need to be measured with a better precision.

3.2 Modified of Electroweak Parameters

It is a known fact that in the SMEFT some of the dimension-6 operators modify some of the definitions of the SM parameters. For instance the Fermi constant is defined through the muon decay into an electron and two neutrinos. As shown in Fig. 1.4 this interaction is a 4-lepton interaction that in the SMEFT can also be produced by the operator C_{ll}. Another contribution comes from the operator $C_{\varphi l}^{(3)}$, this term contains an extra contribution coupling of the W boson:

$$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar{l}_p\gamma^\mu\tau^I l_r) \supset \frac{g}{2}\frac{1}{G_F^{\mathrm{SM}}}W_\mu^-\bar{e}_L\gamma^\mu\nu_L \tag{3.29}$$

All in all, the definition of the SMEFT in the Warsaw basis of the Fermi constant is:

$$\hat{G}_F = \frac{1}{\sqrt{2}\bar{v}^2} - \frac{\sqrt{2}}{4}((C_{ll})_{\mu ee\mu} + (C_{ll})_{e\mu\mu e}) - \frac{1}{\sqrt{2}}((C_{\varphi l}^{(3)})_{ee} + (C_{\varphi l}^{(3)})_{\mu\mu}) \tag{3.30}$$

Assuming now $U(3)^5$ symmetry, we obtain the expression:

$$\hat{G}_F = \frac{1}{\sqrt{2}\bar{v}^2} - \frac{1}{\sqrt{2}}C_{ll} + \sqrt{2}C_{\varphi l}^{(3)} \tag{3.31}$$

The notation that we will use is the same as in refs. [62, 63] which obtained these expression that we are now reproducing. Constants with a hat are inferred from input measurements and those which have a bar are the expressions obtained from the canonically normalized Lagrangian in the EFT approach. Operators such as $\mathcal{O}_{\varphi W}$ in the Warsaw basis modify the typical form of the kinetic terms for the gauge field as $(1 + C_{\varphi W}v^2)W_{\mu\nu}W^{\mu\nu}$ by absorbing these couplings in the gauge fields we canonically normalize the Lagrangian. However this adds modifications to some couplings, these modified couplings through this process will have a bar on them. In this notation also Wilson coefficients have been redefined so that the factor $1/\Lambda^2$ is implicit in each of them.

The EW sector of the SM model is only given by three constants, $\{g, g', v\}$, the two couplings to the $SU(2)_L \times U(1)_Y$ group and the vacuum expectation value. These three values can be determined experimentally for instance by the set of measurements on the parameters $\{\alpha, G_F, M_Z\}$, where α is the fine structure constant, or on the other hand we can measure $\{M_W, G_F, M_Z\}$. These two different working schemes are called the α and M_W schemes respectively.

In the following sections we apply this logic of the SMEFT and apply it in the α-scheme to the ΔEFT which modifies as well some of the parameters of the SM model. Then these operators will be constrained with LEP [64] data and analyzed using different assumptions.

3.2.1 Modified Parameters by the $\triangle$EFT

The ΔEFT contains several operators that have similar structure as in the SMEFT, so we can imagine the modifications of this model to the SM EW parameters. In this section we will omit the contributions of the Warsaw basis, which will be recovered in the following section. We will analyze those parameters that contribute up to $v_t^2 C/\Lambda^2$ where v_t is the triplet vev and C is any Wilson coefficient, higher powers of v_t will be dropped since $v_t^2 \ll v_d^2$ is assumed. The set of operators used in this section are:

$$\{\mathcal{O}_{\Delta\varphi D}, \mathcal{O}_{\Delta\psi}^{(1)}, \mathcal{O}_{\Delta\psi}^{(3)}, \mathcal{O}_{\Delta WB}\} \tag{3.32}$$

Starting by the definition of the Fermi constant, the result is analog to that of the SMEFT with the exception of the 4-fermion operator which in this case does not appear in the ΔEFT.

$$\hat{G}_F = \frac{1}{\sqrt{2}\bar{v}^2} + \frac{v_t^2}{\bar{v}^2}\sqrt{2}C_{\Delta l}^{(3)} \tag{3.33}$$

Here we already notice the first difference, in this case the deviation from the SM will be also suppressed by powers of v_t/v.

For simplicity we will not probe modifications of the parameters coming from terms which only contain scalar fields, hence there are no modifications to the vev apart from the one induced by the triplet and now on we will drop the bar on v. We note that a consistent analysis must include all these operators. However, one can easily see that the complexity of the problem rises since many operators will modify the definition of the vev. The vev in the SMEFT is only modified by $\mathcal{O}_\varphi$, here many other parameters affect its definition. Besides, these scalar operators are difficult to probe experimentally, for instance the quartic coupling of the Higgs has not been measured yet, so considering more operators will be pointless in the sense that we would not have independent measurements of them. This two-fold difficulty makes us not consider these operators in this work.

Since we consider $\{\hat{\alpha}, \hat{G}_F, \hat{M}_Z\}$ to be fixed by the measurements we should rewrite the rest of the parameters of the EW Lagrangian in terms of these two. Since this has been done in the literature and can be easily found in Refs. [35, 62, 63] we will place these definitions in the appendix B, and refer to these articles for further information. These parameters will take the value:

$$\hat{\alpha}(M_Z) = (127.952)^{-1}, \;\; \hat{G}_F = 1.1663787 \times 10^{-5}\,\text{GeV}, \;\; \hat{M}_Z = 91.1876\,\text{GeV} \tag{3.34}$$

The next modification is to the mass of the Z boson. One can easily see that after SSB not only the kinetic term of the Higgs gives mass to the Z boson, but also all the rest of scalar operators with derivatives. Operators $\mathcal{O}_{\Delta W}$ and $\mathcal{O}_{\Delta B}$ are absorbed in the redefinition so that the kinetic terms have the canonical form $B_{\mu\nu}B^{\mu\nu}$ this introduces a redefinition in the couplings $g' = \bar{g}'(1 + v_t^2 C_{\Delta B})$ and similar for $g = g(1 + v_t^2 C_{\Delta W})$. However the operator $\mathcal{O}_{\Delta WB}$ as shown in [63] modifies the Weinberg angle, introducing a correction to the SM mass of the Z-boson:

$$\bar{M}_Z^2 = \frac{v^2}{4}(\bar{g}^2 + \bar{g}'^2) + \frac{\hat{M}_Z^2}{2} v_t^2 C'_{D\Delta} + \frac{v^2 v_t^2}{2} \bar{g}\bar{g}'\, C_{\Delta WB} \tag{3.35}$$

In this expression we have defined $C'_{D\Delta} \equiv \frac{1}{2} C^1_{\Delta D} + 4\,(C^2_{\Delta D} + C^3_{\Delta D})$, this is done because it is not possible to distinguish these operators in a fit. Since we cannot measure any of the Δ fields, we do not have any observable that constrains a different direction than this particular combination. Following [35] we will denote $\delta k = \bar{k} - \hat{k}$, where k is any parameter, since it will become very useful later on. Using this definition we obtain:

$$\delta G_F = \sqrt{2}\, v_t^2\, C^{(3)}_{\Delta l} \tag{3.36}$$

$$\delta M_Z^2 = \hat{M}_Z^2 \left(\frac{1}{2} v_t^2 C'_{D\Delta} + 2 v_t^2 \cos\hat{\theta}_w \sin\hat{\theta}_w C_{\Delta WB} \right) \tag{3.37}$$

We want to point out that δG_F does not have the same dimensions as G_F, however, to avoid mistakes later on, we will follow analog definitions to those of refs. [35, 62], although in this case it might be misleading. Now on, we will abbreviate $\sin\hat{\theta} \equiv s_{\hat{\theta}}$ and similarly the other trigonometric functions of the Weinberg angle. The rest of the modifications can be written in terms of these two:[1]

$$\delta g = \frac{\hat{g}}{2c_{2\hat{\theta}}} \left[s_{\hat{\theta}}^2 \left(\sqrt{2}\delta G_F + \frac{\delta M_Z^2}{\hat{M}_Z^2} \right) + c_{\hat{\theta}}^2 s_{2\hat{\theta}} v_t^2 C_{\Delta WB} \right] \tag{3.38}$$

$$\delta g' = -\frac{\hat{g}'}{2c_{2\hat{\theta}}} \left[c_{\hat{\theta}}^2 \left(\sqrt{2}\delta G_F + \frac{\delta M_Z^2}{\hat{M}_Z^2} \right) + s_{\hat{\theta}}^2 s_{2\hat{\theta}} v_t^2 C_{\Delta WB} \right] \tag{3.39}$$

$$\delta s_{\theta}^2 = 2 c_{\hat{\theta}}^2 s_{\hat{\theta}}^2 \left(\frac{\delta g}{\hat{g}} - \frac{\delta g'}{\hat{g}'} \right) + \frac{s_{2\hat{\theta}} c_{2\hat{\theta}}}{2}\, v_t^2\, C_{\Delta WB} \tag{3.40}$$

$$\delta M_W^2 = \hat{M}_W^2 \left(\sqrt{2}\delta G_F + 2\frac{\delta g'}{\hat{g}'} \right) \tag{3.41}$$

We start seeing that all these modifications imply that to exclude deviations from the standard model one would need to obtain good measurements on the parameters related to the Z and W bosons. In a similar way to what happens in the SMEFT, e.g. eq. (3.29),

[1] Someone following Refs. [35, 62] should not mistake their $\bar{v}_T$ for our v_t in ur case v_t will always refer to the triplet vev.

the couplings of the fermions to the gauge bosons are also perturbed by the dimension-6 coefficients. We will define the Z-boson coupling to the fermions as:

$$\mathcal{L}_{Z,\text{eff}} = \hat{g}_Z \sum_{\psi=l,\nu,u,d} \bar{\psi}\gamma_\mu \left(\bar{g}_V^\psi - \gamma_5 \bar{g}_A^\psi\right) \psi Z^\mu \tag{3.42}$$

Here, we have omitted any flavour components on the couplings, but they can be recovered by adding indices to the couplings g and fields ψ. We have also used the definition $\hat{g}_Z = -\hat{g}'/c_{\hat{\theta}}$. Now we can write the contribution of the dimension-6 operators to these couplings. Defining $\delta g_{V,A}^\psi$ as:

$$\delta g_{V,A}^\psi = \bar{g}_{V,a}^\psi - g_{V,A}^{\psi,\text{SM}} \tag{3.43}$$

With:

$$g_V^{\psi,\text{SM}} = \frac{T_3}{2} - Q_\psi s_{\hat{\theta}}^2 \quad \text{and} \quad g_A^{\psi,\text{SM}} = \frac{T_3}{2} \tag{3.44}$$

We can easily obtain that the modifications to the fermion-gauge couplings produced by the dimension-6 operators of the ΔEFT are:

$$\delta g_V^\nu = \delta g_A^\nu = \delta g_Z\, g_V^{\nu,\text{SM}} + \frac{v_t^2}{2}\left(C_{\Delta l}^{(3)} - C_{\Delta l}^{(1)}\right) \tag{3.45}$$

$$\delta g_V^l = \delta g_Z\, g_V^{l,\text{SM}} - \frac{v_t^2}{2}\left(C_{\Delta l}^{(3)} + C_{\Delta l}^{(1)} + C_{\Delta e}\right) + \delta s_{\hat{\theta}}^2 \tag{3.46}$$

$$\delta g_A^l = \delta g_Z\, g_A^{l,\text{SM}} + \frac{v_t^2}{2}\left(-C_{\Delta l}^{(3)} - C_{\Delta l}^{(1)} + C_{\Delta e}\right) \tag{3.47}$$

$$\delta g_V^u = \delta g_Z\, g_V^{u,\text{SM}} + \frac{v_t^2}{2}\left(C_{\Delta q}^{(3)} - C_{\Delta q}^{(1)} - C_{\Delta u}\right) - \frac{2}{3}\delta s_{\hat{\theta}}^2 \tag{3.48}$$

$$\delta g_A^u = \delta g_Z\, g_A^{u,\text{SM}} + \frac{v_t^2}{2}\left(C_{\Delta q}^{(3)} - C_{\Delta q}^{(1)} + C_{\Delta u}\right) \tag{3.49}$$

$$\delta g_V^d = \delta g_Z\, g_V^{d,\text{SM}} - \frac{v_t^2}{2}\left(C_{\Delta q}^{(3)} + C_{\Delta q}^{(1)} + C_{\Delta d}\right) + \frac{1}{3}\delta s_{\hat{\theta}}^2 \tag{3.50}$$

$$\delta g_A^d = \delta g_Z\, g_A^{d,\text{SM}} + \frac{v_t^2}{2}\left(-C_{\Delta q}^{(3)} - C_{\Delta q}^{(1)} + C_{\Delta d}\right) \tag{3.51}$$

$$\delta g_Z = -\frac{1}{\sqrt{2}}\delta G_F - \frac{\delta M_Z^2}{2M_Z^2} + s_{\hat{\theta}} c_{\hat{\theta}}\, v_t^2 C_{\Delta WB} \tag{3.52}$$

We have computed the modifications of the EW parameters of the SM, now we can calculate how these variations modify observables such as the decay width of the Z boson, cross-sections, asymmetries... In fact these modifications have already been computed [62], and they are given in terms of these modified parameters δg, δM_Z^2, ... that we have computed. Hence, we will skip how these observables change with respect to these modifications, but for completeness and in order to refer to them in the next section they are listed in the appendix B

For the W-boson the modifications are only dependent on $C_{\Delta}^{(3)}l$ and $C_{\Delta}^{(3)}q$. It can be seen that this is due to the fact that the fermionic part of the operators of the class $\Delta^2\psi^2 D$ is neutral (e.g. only terms like $\bar{u}\gamma_\mu u$ while when we have the triplet combination we can have terms like $\bar{u}\gamma_\mu d$. The contribution from these operators to the W-boson coefficients is:

$$\delta g_V^{W,l,q} = g_A^{W,l,q} = \frac{v_t^2}{2}\left(C_{H(l,q)}^{(3)} + \frac{1}{2}\frac{c_{\hat{\theta}}}{s_{\hat{\theta}}}C_{\Delta WB}\right) - \frac{1}{4}\frac{\delta s_{\hat{\theta}}^2}{s_{\hat{\theta}}^2} \tag{3.53}$$

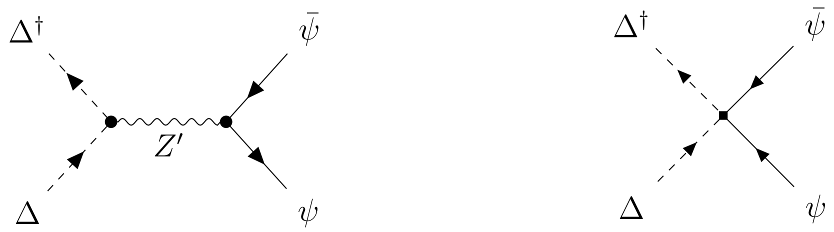
3.2.2 Constraining $\triangle$EFT with LEP

As we have mentioned in the previous section, all these modifications affect EW observables. Clearly, this means looking at observables related to the gauge boson Z. Furthermore, this means using LEP data at the Z-pole resonance [64] in order to put bounds to these modified couplings, as it is done in the SMEFT.

Before starting with the data, we will first discuss the validity of the constrains we are about to obtain. One of the most important, if not the most, aspect of EFTs is that we do not make any judgment about the UV theory generating the effective operators of the model. At the same time, as it has been discussed in previous sections, we are doomed to make some choices, and simplify by assuming symmetries, for instance $U(3)^5$ in some global analyses , or reducing the set of operators due to the complexity of the analysis or even unavailability of measurements, the case of for example $\mathcal{O}_\varphi$. All in all, without simplifications like these, we could not get any constraint, however, we must be clear on what set of operators, which flavour assumptions and with what purpose we are working. It is clear why if we are looking for BSM particles coupling to top quarks we will apply $U(2)$ symmetry to the quark sector, so now we must ask what kind assumptions are we willing to make to reduce the complexity of the work.

We already argued in the previous section that taking into account scalar-only operators was not useful, since measurements would not be available. The subset of operators was also reduced to those which can be measured by the Z-boson, however we still have to make a last decision. It is likely that if the triplet Higgs Δ exists, the way to work in EFTs would be to use SEMFT+ΔEFT operators in the different sections, since not including one of these models would imply making an assumption on the UV completion. However, that is not the case, the type-2 seesaw field Δ has not yet been observed and thus, as we have already said, making a global analysis, if possible, perhaps would not be the most resource-wise option. At the risk of sounding redundant, global analyses are a powerful tools for looking for new physics with a set of external states that are measurable, when this is not the case, a great part of the operators cannot be measured, so other utilities should be found.

After this discussion, we hope to be able to justify the two frameworks in which we will analyze the subset of operators that we have chosen. Firstly we will consider only ΔEFT operators, this is a big assumption that can be justified in some scenarios. We will give an example of one: it has been noted that couplings such as those of the class $\psi^2\varphi^2 D$ could be generated through an exchange of a heavy gauge boson [65]. This is analog in the ΔEFT, to show it we denote this new gauge field as Z'_μ, and write the vertices:



$$g_\Delta \operatorname{Tr}\left(\Delta^\dagger D_\mu \Delta\right) \frac{1}{p^2 - M_{Z'}^2} g_\psi^{\rm SM} \bar{\psi}\gamma^\mu\psi \qquad\qquad -\frac{g_\psi^{\rm SM} g_\Delta}{M_{Z'}^2} \operatorname{Tr}\left(\Delta^\dagger D_\mu \Delta\right) \bar{\psi}\gamma^\mu\psi$$

Figure 3.6: Diagram representing how a dimension-6 operator of the class $\psi^2\Delta^2 D$ as well as the contribution of each coupling.

Figure 3.6 shows a typical diagram that could generate this class of operators. In a general case the couplings to the particles of the SM and the Δ could be different. In fact we could study the case in which $g_\Delta \gg g^{SM}$, this would make $C_{\Delta\psi} \gg C_{\varphi\psi}$. By no means this reduces the magnitude of the assumption, but this exemplifies a scenario in which the bounds of this subset of operators would be valid.

The second framework that we will analyze does not make this assumption and takes into account some of the operators of the SMEFT. As we will discuss later on in more detail, expressions (3.45)-(3.52) are also modified by the coefficients of the Warsaw basis. The problem lies in the fact that one cannot distinguish the SMEFT operators from those of the ΔEFT with measurements of the Z-boson. The operators enter into all the expressions in the same combination, so we need an extra measurement to put constrains on the different directions of the parameter space. We will show how this can be done in section 3.2.3.

So, placing ourselves in the first framework in which we only consider operators containing Δ fields, we are going to use measurements provided by the different experiments of LEP at the resonance of the Z boson. These measurements are listed in Table 3.1. Observables Γ_Z and $\sigma_{\rm had}$ of Tab. 3.1 are the decay width and hadronic cross-

Observable	Measurement	SM prediction
Γ_Z [GeV]	2.4952 ± 0.0023	2.4943 ± 0.0005
σ^0_{had} [nb]	41.540 ± 0.037	41.488 ± 0.006
R^0_l	20.767 ± 0.025	20.752 ± 0.005
$A^{0,l}_{FB}$	0.0171 ± 0.0010	0.0171 ± 0.00009
A_l (P_τ)	0.1465 ± 0.0033	0.1470 ± 0.0004
A_l (SLD)	0.1513 ± 0.0021	0.1470 ± 0.0004
R^0_b	0.21629 ± 0.00066	0.2158 ± 0.00015
R^0_c	0.1721 ± 0.0030	0.17223 ± 0.00005
$A^{0,b}_{FB}$	0.0992 ± 0.0016	0.1031 ± 0.0003
$A^{0,c}_{FB}$	0.0707 ± 0.0035	0.0736 ± 0.0002
A_b	0.923 ± 0.020	0.9347
A_c	0.670 ± 0.027	0.6678 ± 0.0002
M_W [GeV]	80.387 ± 0.016	80.361 ± 0.006
M_W [GeV]	80.370 ± 0.016	80.361 ± 0.006
Γ_W [GeV]	2.085 ± 0.042	2.0896 ± 0.0008
$BR(W \to l\nu)$	0.1086 ± 0.0009	0.10832 ± 0.00005
$BR(W \to$ hadrons$)$	0.6741 ± 0.0027	0.6752 ± 0.0004

Table 3.1: List of measurements used in our analysis. All SM predictions come from [64] except the mass measurements of the W boson which come from [66] and [67]. The SM predictions have been taken from [39] and references therein. Branching ratios, both theoretical predictions and measurements are taken from [15].

section of the Z-boson, similar for the W-boson, where we included the decay width and branching ratios to leptons and hadrons. The observables A correspond to asymmetries in the couplings of the Z-boson:

$$A_{\text{FB}} = \frac{\sigma_{\text{F}} - \sigma_{\text{B}}}{\sigma_{\text{F}} + \sigma_{\text{B}}} \tag{3.54}$$

A_{FB} is the forward-backward asymmetry, in which σ_{F} takes the angles $[0, \pi/2]$ and σ_{B} the angles $[\pi/2, \pi]$. These asymmetries can be rewritten in terms of the couplings, so assuming f is $f = l, c, b$:

$$A_{\text{FB}} = \frac{3}{4} A_l A_f, \quad \text{with} \quad A_f = 2\frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2} \tag{3.55}$$

The observables R are ratios of different partial decay widths: $R^0_l = \Gamma_{Z\to\text{had}}/\Gamma_{Z\to ff}$ and in the case of quarks as the inverse of the previous case $R^0_q = \Gamma_{Z\to qq}/\Gamma_{Z\to\text{had}}$.

We perform a χ^2 fit to the parameters using equation (3.28) but with the observables and measurement of Table 3.1. Again the matrix V^{-1} is the inverse of the covariance matrix, where correlations into different parameters are taken into accounts when available as well as the theoretical errors. The modifications to the SM observables are given in the appendix B, and we keep these modifications to linear order, that is $\mathcal{O}(C/\Lambda^2)$.

It is important to notice that all parameters of the ΔEFT modifying SM observables come with v_t. Although it is theoretically possible, it is technically very difficult to include v_t in the fit. One option would be to include the ρ parameter to constrain the parameter v_t on its own, however the Wilson coefficient $C_{\Delta D}$ would also enter, making the constrain on v_t less powerful. In any case, the real problem becomes the numerical instability of the fit, which is lost when this parameter is introduced freely in the fit.

That is why in this first part we will redefine the Wilson coefficients as:

$$\overline{C} = v_t^2 \frac{C}{\Lambda^2} \tag{3.56}$$

In this first approach the fit is not much different from the one of the Warsaw basis, since many modifications are similar and some are the same. In Figure 3.7 we make a comparison between the values obtained for the ΔEFT basis (Orange), the Warsaw

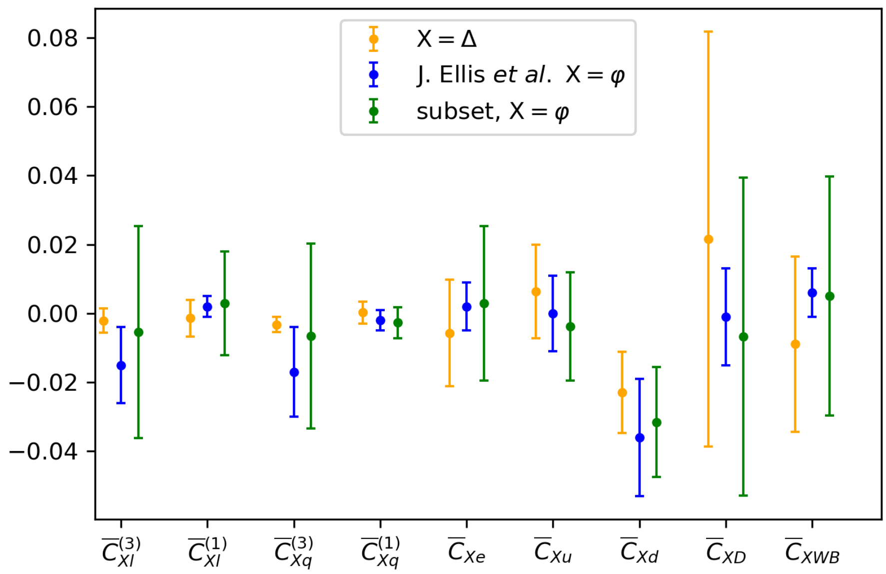


Figure 3.7: Comparison between different basis and analyses. In orange our analysis of the ΔEFT operators that modify EWPO, in green the same thing for the Warsaw basis and in blue the result from a global fit to a larger set of operators. X indicates which set of operators we are plotting. For the ΔEFT we have set $v_t = 5$ GeV.

Operator	ΔEFT	Warsaw Basis	Global Analysis [47]
$\overline{C}^{(3)}_{Xl}$	-0.002 ± 0.003	-0.01 ± 0.03	-0.015 ± 0.011
$\overline{C}^{(1)}_{Xl}$	-0.001 ± 0.005	0.003 ± 0.015	0.002 ± 0.003
$\overline{C}^{(3)}_{Xq}$	-0.003 ± 0.002	-0.01 ± 0.03	-0.017 ± 0.013
$\overline{C}^{(1)}_{Xq}$	0.000 ± 0.003	-0.003 ± 0.005	-0.002 ± 0.003
$\overline{C}_{Xe}$	-0.006 ± 0.016	0.00 ± 0.02	0.002 ± 0.007
$\overline{C}_{Xu}$	0.006 ± 0.014	-0.004 ± 0.016	0.000 ± 0.011
$\overline{C}_{Xd}$	-0.023 ± 0.012	-0.032 ± 0.016	-0.036 ± 0.017
$\overline{C}_{XD}$	0.02 ± 0.06	-0.01 ± 0.05	-0.001 ± 0.014
$\overline{C}_{XWB}$	-0.01 ± 0.03	0.01 ± 0.04	0.006 ± 0.007
$\overline{C}_{ll}$	$\times$	-0.002 ± 0.007	-0.015 ± 0.011

Table 3.2: Values of the Wilson coefficients obtained in different analyses. X is Δ in the first column and φ in the last two. The first column shows the fit of the ΔEFT coefficients, the second row with the subset of the Warsaw basis used in this work and the same data as in the first column. The last column is taken from a global analysis to a large set of observables [47].

Basis (Green) and we also introduced the results from a global fit to a larger set of data and Wilson coefficients (Blue), Ref. [47].

We observe that the fit is consistent with that of the other basis and the global analysis, this is because global analyses include most of the EWPO that we also put into ours, and they are the most constraining to these parameters, so there are only small deviations. We see that the errors of the ΔEFT parameters are lower in general than those of the fit to the subset of operators of the Warsaw basis, this is probably due to the fact that the fit of the ΔEFT has one parameter less than the Warsaw basis, which also contains the coefficient C_{ll}, not shown in Fig. 3.7. Errors of the parameters $C_{\Delta D}$ and $C_{\Delta WB}$ are much larger than the ones in the global fit, similar for the analog operators of the Warsaw basis, this is caused by the fact that these parameters are directly constrained by Higgs measurments of the LHC, such as $h \to \gamma\gamma$, this measurement reduces the fluctuations of these parameters. Values of this plot are found in Table 3.2.

Now we would like to introduce the vacuum expectation value of the triplet into the analysis, however as we said above the fit becomes very unstable, and even though one could introduce some restrictions the fit is still dependent on the initial values. Instead, what we will do is to scan the best fit parameters for different values of v_t, i.e. to minimize the χ^2 function for different values of v_t.

In Fig. 3.8 we show the best fit value of each Wilson coefficient with v_t fixed at a certain value. We scan in the range of $[1\,\mathrm{GeV} - 5\,\mathrm{GeV}]$, since ~ 5 GeV is a sensible upper bound of the vev, and below 1 GeV the constrains become less stringent, besides this range is good enough to show the dependence of these bounds with respect to the vev. We can see by plugging different values of Λ that the bounds of C are above ~ 1. This is due to the range of v_t chosen, clearly for larger v_t we could be setting Λ in the order of $\sim$ TeV, however those values of v_t do not respect the measurements of the ρ parameter. In conclusion these measurements are rather weak when compared to the SMEFT operators, since we cannot constrain in a sensible way values of $\Lambda \sim$ TeV. On the other hand these bounds depend on v_t and even though this parameter is related to the mass, we do not need to make any assumption on the mass triplet.

3.2.3 SMEFT and $\triangle$EFT

As we have mentioned in the previous section, the analysis of just ΔEFT is only valid when making assumptions that relate new physics to be coupled stronger to the type-2 triplet than to the SM particles. However, we introduce a huge assumption by making so, but necessary since studying both basis at the same time would be really difficult, if not impossible.

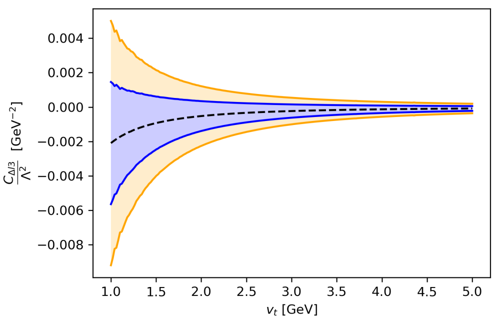
In this section we will study the dependence of the ΔEFT Wilson coefficients studied in the previous section when we introduce different coefficients of the Warsaw basis. In this section we are not aiming to do a fit to all the subset of operators of the ΔEFT and Warsaw basis included in the previous section, since that would require a more careful treatment depending on the goal. What we want to show here is that even if the operators enter in the modification of the EWPO in a similar way, we can add extra data to distinguish them. To be completely specific, we will use now the subset of operators that combines:

$$\{\mathcal{O}_{\Delta u}, \mathcal{O}_{\Delta d}, \mathcal{O}_{\Delta q}^{(1)}, \mathcal{O}_{\Delta q}^{(3)}, \mathcal{O}_{\Delta WB}\} \cup \{\mathcal{O}_{\varphi u}, \mathcal{O}_{\varphi d}, \mathcal{O}_{\varphi q}^{(1)}, \mathcal{O}_{\varphi q}^{(3)}, \mathcal{O}_{\varphi WB}\} \tag{3.57}$$

We will explain the choice of operators later on in this section. First, we can start seeing the problem, these operators are so similar that one cannot constrain them with only EWPO. The reason is that, for instance $\mathcal{O}_{\varphi q}^{(3)}$ and $\mathcal{O}_{\Delta q}^{(3)}$ modify the different parameters always in the same way:

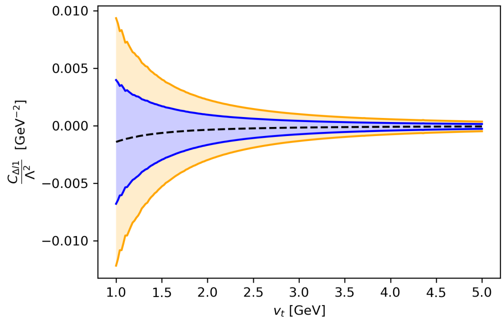
$$\delta g_V^u = \delta g_Z\, g_V^{u,\mathrm{SM}} + \frac{v_t^2}{2}\left(C_{\Delta q}^{(3)} - C_{\Delta q}^{(1)} - C_{\Delta u}\right) + \frac{v_d^2}{4}\left(C_{\varphi q}^{(3)} - C_{\varphi q}^{(1)} - C_{\varphi u}\right) - \frac{2}{3}\delta s_{\hat{\theta}}^2 \tag{3.58}$$

The combination of the two subsets of operators is always the same, hence, with all the measurements of the different observables we will not able to distinguish one subset from the other in the fit. We need some extra measurements that allow us to constrain some different direction. A possible way is to introduce direct measurements of these coefficient, i.e. to look for processes produced or contributions to processes in which only one of these operators participate. For instance, operator $\mathcal{O}_{HWB}$ contributes



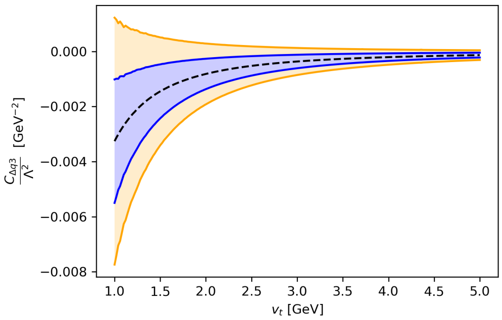
$\frac{C_{\Delta l3}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(a)



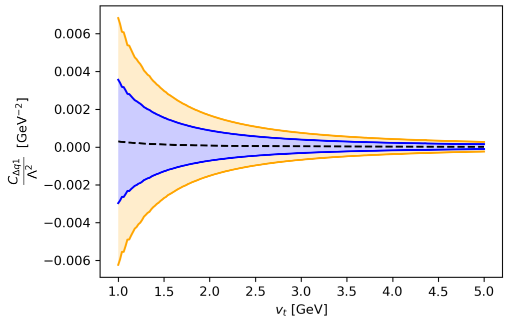
$\frac{C_{\Delta l1}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(b)



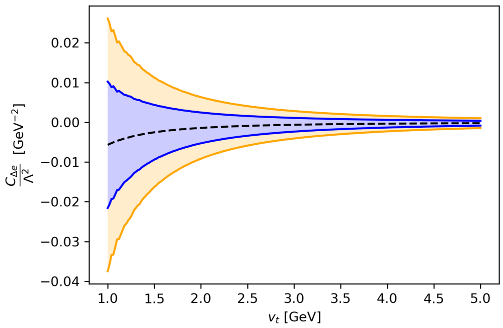
$\frac{C_{\Delta q3}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(c)



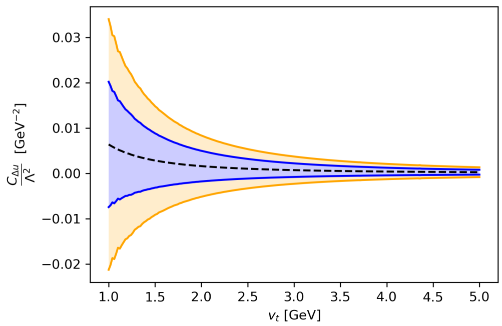
$\frac{C_{\Delta q1}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(d)



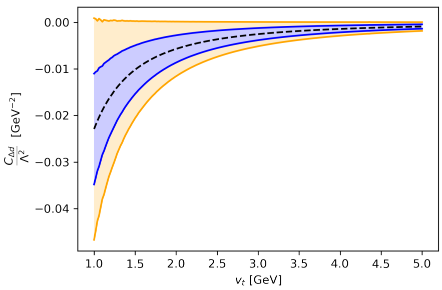
$\frac{C_{\Delta e}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(e)



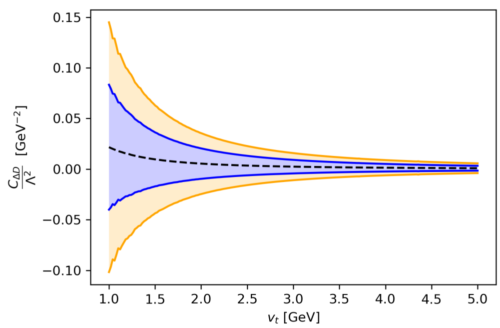
$\frac{C_{\Delta u}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(f)



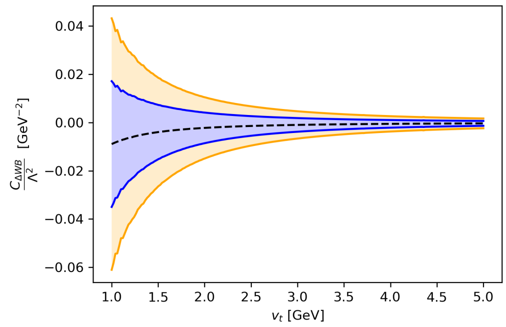
$\frac{C_{\Delta d}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(g)



$\frac{C_{\Delta D}}{\Lambda^2}$ [GeV^{-2}]
v_t [GeV]

(h)



(i)

Figure 3.8: Best fit values (dashed black), $1-\sigma$ (blue) and $2-\sigma$ (orange) values for the different values of the vacuum expectation value of the triplet v_t.

strongly to the Higgs decay $h \to \gamma\gamma$, so if we introduce Higgs measurements in which only operators of the second subset of operators of eq. (3.58) we will be able to constrain the parameter space of these two parameters. Note that operators of the first subset could also contribute, since the Higgs particle in this model is a linear combination of the triplet and the doublet, see eq. (1.57). However this is really supressed when compared to the SMEFT basis:

$$\frac{C_{\Delta WB}}{\Lambda^2}\,\mathrm{Tr}\big(\Delta^\dagger \tau^I \Delta\big) W^I_{\mu\nu} B^{\mu\nu} \sim \frac{C_{\Delta WB}}{\Lambda^2} \sin\alpha\, v_t \cos\theta \sin\theta\,(h A_{\mu\nu} A^{\mu\nu})$$
$$\frac{\delta\sigma^{\Delta\mathrm{EFT}}}{\delta\sigma^{\mathrm{SMEFT}}} = \frac{C_{\Delta WB}}{C_{\varphi WB}} \frac{\sin\alpha\, v_t}{\cos\alpha\, v_d} \lesssim 0.31 \frac{v_t}{v_d} \tag{3.59}$$

Where we used the result from [29] that $\sin\alpha \lesssim 0.3$ at 95% C.L. and that $C_{\Delta WB} \sim C_{\varphi WB}$. If we plug in $v_t \sim 5$ GeV we obtain that the contribution is at least 6.3×10^{-3} times smaller, taking the largest allowed values at 95% C.L. Similarly the other operators behave in the same way, even in the case of maximal mixing the cross section contribution is 0.02 times smaller, hence we will assume only SMEFT operators contributing directly to higgs processes, indirectly through modified couplings they could still enter.

For the Higgs decay into two photons this process is easy to calculate, in fact it has been done and can be found in several references, e.g. [68], $\sigma(h \to \gamma\gamma)/\sigma^{\mathrm{SM}}(h \to \gamma\gamma) = 1 + 26.144\, C_{\varphi WB}$. Typically what one does in these cases is to compute this process through a Monte Carlo simulator giving some values to the Wilson coefficients and then obtain the linear dependence of this process. The most used program is MadGraph [70] where there is a package containing the different operators of the Warsaw basis with different flavour assumptions and input-schemes, this package is called SMEFTsim [69].

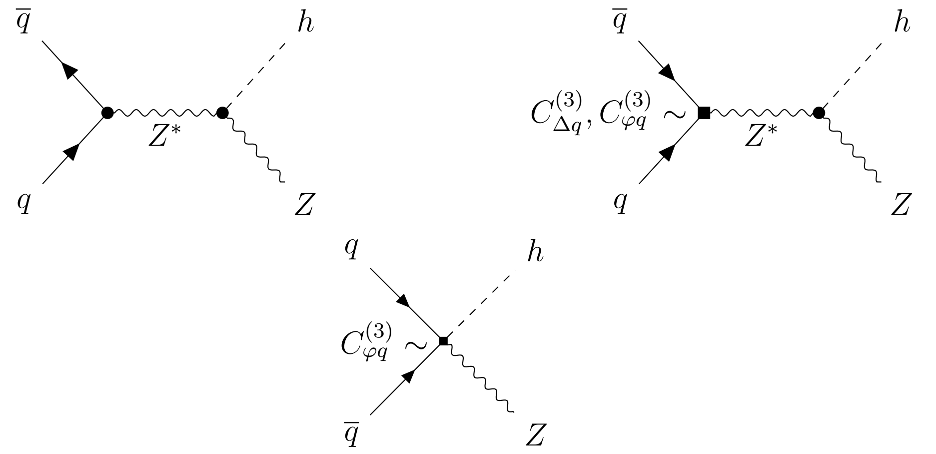


Figure 3.9: Different Feynman diagrams denoting the different contributions from different basis. Dots are SM couplings, squares are couplings of an EFT.

This way of obtaining the different contributions is straightforward in the Warsaw basis, however, in the combination of both basis there exists one problem: for some the processes containing the Z-boson the modifications of the couplings calculated in the previous section also contribute to this process, meaning that to be fully consistent one should take into account also these corrections. This is represented in Figure 3.9 for the production channel $pp \to hZ$. In this case couplings of $\bar{q}qZ$ are modified by the ΔEFT as well as the Warsaw basis, as seen in this figure, however the 4-point interaction is only dependent on $C_{\varphi q}^{(3)}$.

In Figure 3.9 we do not show all contributions, only those given by $C_{Xq}^{(3)}$ but other contributions would also need to be included. That is why we will restrict this analysis to each pair of operators at each time, i.e. setting to 0 the rest of operators not shown in each plot. In order to include correctly both terms we modify slightly the code of SMEFTfit so that we are able to distinguish the contribution of the coupling-modification and contributions that only come from the Warsaw basis, then we make the substitution $C_{\varphi q}^{(3)} \to C_{\varphi q}^{(3)} + 2\frac{v_t^2}{v_d^2} C_{\Delta q}^{(3)}$ on the coupling-correction contributions since, as we have seen in eq. (3.58) it is the way in which these two coefficients are related in this type of contribution.

Since we are not aiming to make a global analysis we will use a reduced data set of measurments of the LHC. We will take the measurements that allow us to reduce the parameter space of the observables subset that we have chosen and that offer the best constrains to the operators. We will show that it is possible to constrain parameters of both basis if we include a larger set of data points which constrain other directions in the parameter space.

We are going to include in our analysis 5 different Higgs signal strengths, listed in

Table 3.3 and performed in the run 2 of the LHC. These signal strength are defined as:

$$\mu(h \to X) = \frac{\sigma^i BR(H \to X)}{(\sigma^i BR(H \to X))_{\text{SM}}} \tag{3.60}$$

Where the numerator of this expression contains also corrections of the EFT up to order $\mathcal{O}(C/\Lambda^2)$. Then by expanding in small corrections of the cross section and branching ratio we obtain:

$$\mu(h \to X) = 1 + \frac{\delta\sigma^i}{\sigma^i_{\text{SM}}} + \frac{\delta BR(h \to X)}{BR(h \to X)_{\text{SM}}} \equiv 1 + \delta\mu_i + \delta\mu_X \tag{3.61}$$

The contributions of the EFT are calculated by the method explained above, and are listed in equations (3.62-3.2.3):

$$\delta\mu_{h\gamma\gamma} = 26.144\, C_{\varphi WB} \tag{3.62}$$

$$\begin{aligned}\delta\mu_{hZ} = {} & 1.717\, C^{(3)}_{\varphi q} + 0.142\left(C^{(3)}_{\varphi q} + 2\frac{v_t^2}{v_d^2} C^{(3)}_{\Delta q}\right) - 0.133\, C^{(1)}_{\varphi q} \\ & -0.010\left(C^{(1)}_{\varphi q} + 2\frac{v_t^2}{v_d^2} C^{(1)}_{\Delta q}\right) + 0.452\, C_{\varphi u} + 0.033\left(C_{\varphi u} + 2\frac{v_t^2}{v_d^2} C_{\Delta u}\right) \\ & -0.155\, C_{\varphi d} - 0.014\left(C_{\varphi d} + 2\frac{v_t^2}{v_d^2} C_{\Delta d}\right) + 0.280\, C_{\varphi WB} - 0.097\left(C_{\varphi WB} + \frac{v_t^2}{v_d^2} C_{\Delta WB}\right)\end{aligned} \tag{3.63}$$

$$\delta\mu_{hW} = 1.838\, C^{(3)}_{\varphi q} + 0.1260\left(C^{(3)}_{\varphi q} + 2\frac{v_t^2}{v_d^2} C^{(3)}_{\Delta q}\right) - 0.187\left(C_{\varphi WB} + \frac{v_t^2}{v_d^2} C_{\Delta WB}\right) \tag{3.64}$$

$$\begin{aligned}\delta\mu_{\text{VBF}} = {} & -0.719\, C^{(3)}_{\varphi q} + 0.260\left(C^{(3)}_{\varphi q} + 2\frac{v_t^2}{v_d^2} C^{(3)}_{\Delta q}\right) - 0.010\, C^{(1)}_{\varphi q} \\ & -0.327\, C_{\varphi u} + 0.017\left(C_{\varphi u} + 2\frac{v_t^2}{v_d^2} C_{\Delta u}\right) - 0.008\, C_{\varphi d} + 0.018\left(C_{\varphi d} + 2\frac{v_t^2}{v_d^2} C_{\Delta d}\right) \\ & +0.020\, C_{\varphi WB} - 0.339\left(C_{\varphi WB} + \frac{v_t^2}{v_d^2} C_{\Delta WB}\right)\end{aligned} \tag{3.65}$$

We introduce these measurements in the previous χ^2 function and we draw minimize each time a pair of Wilson coefficients setting to 0 the rest. We set at $v_t \sim 5$ and $\Lambda \sim 1$ TeV to make this calculation. The results can be seen in Figure 3.10, where again in blue we see the $1{-}\sigma$ region and in orange the $2{-}\sigma$ region. The shape is clearly determined by the combinations of the two different types of measurements, the direction in which the ellipse is best constrained comes from the LEP measurements, while the other direction

Ref.	Process	Observed μ	$\delta\mu$
[71]	$hZ, h \to b\bar{b}$	0.9 ± 0.5	$\delta\mu_{hZ}$
[71]	$hW, h \to b\bar{b}$	1.7 ± 0.7	$\delta\mu_{hW}$
[72]	$gg \to h \to \gamma\gamma$	$1.10^{+0.20}_{-0.18}$	$\delta\mu_{h\gamma\gamma}$
[72]	VBF, $h \to \gamma\gamma$	$0.8^{+0.6}_{-0.5}$	$\delta\mu_{h\gamma\gamma} + \delta\mu_{VBF}$
[73]	VBF, $h \to \bar{\tau}\tau$	$1.11^{+0.34}_{-0.35}$	$\delta\mu_{VBF}$

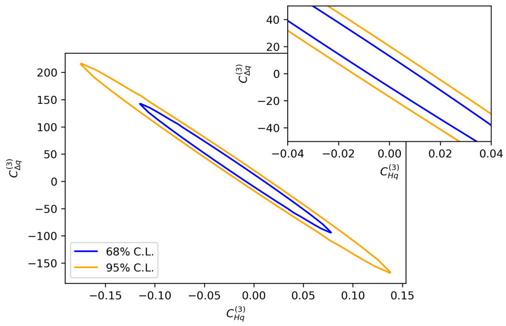
Table 3.3: Higgs signal strength included in the analysis of operators of both basis ΔEFT and Warsaw basis.

comes from Higgs measurements. It is straightforward to notice this if we, for instance, compare the case of the $C_{\Delta WB} - C_{\varphi WB}$ plot to the others. The $C_{\varphi WB}$ is constrained by the measurement of the process $h \to \gamma\gamma$, one of the best measured processes in the Higgs sector. Moreover, the contribution of this coefficient to this process is large, hence the constrain in this direction is more stringent than in the other cases, and that is why in this case the ellipse is not heavily shrunk in one direction as it happens in the other pictures. The coefficient $C_{\varphi q}^{(3)}$ is another similar case, since it has a large coefficient in processes such as the hV production channels, where $V = Z,\, W$.

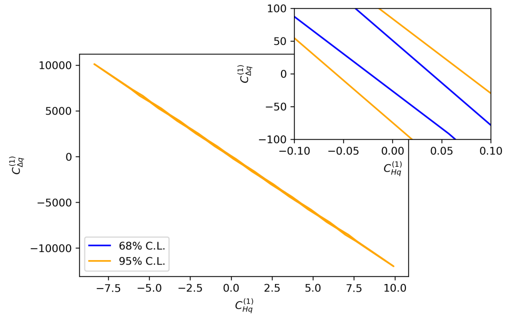
It is clear that a better constrain of these coefficients would be possible if we are able to obtain better measurements on the Higgs signal strengths. However, it is also important to note that many of the couplings contribute very little to the signal strength, so the possibility of constrining them better is limited. Then again, the the constrains on the SMEFT parameters is more stringent since the parameters of the ΔEFT are multiplied by a v_t that is small, however as we can see whn we have a strong dependence and a good measurement of the signal strength, such as in the $C_{\Delta WB} - C_{\varphi WB}$ case, the constraints are more competitive.

Other operators that we have not studied in this section such as those of the class $\Delta^2\varphi\psi^2$ also could be studied in this way, since they modify the couplings of the Higgs to the quarks. However, in this case we do not have the good measurements of LEP, and the method becomes even worse, although some of the couplings would enter strongly in the decay of the Higgs to fermions.

We have shown a possible way of studying both basis at the same time, however, introducing more operators is complicated, since operators such as $C_{Hl}^{(1,3)}$ do not participate in either Higgs decays or production channels (in the LHC), so they only enter through modified couplings of the EW sector. In lepton colliders for instance one could distinguish the two operators since in the production channel it could also enter. It would also be interesting to study the modification of the fit by varying the mixing of the Higgs.



(a)



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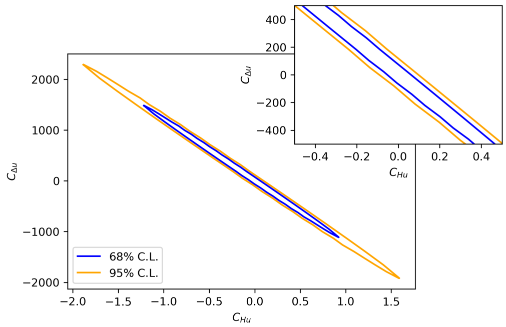

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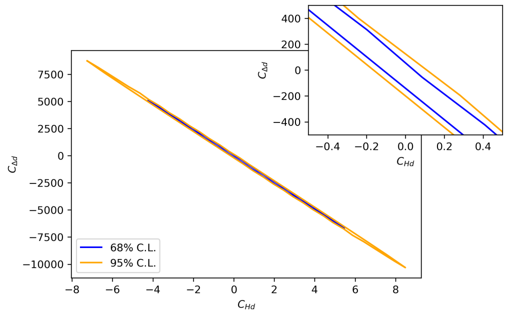

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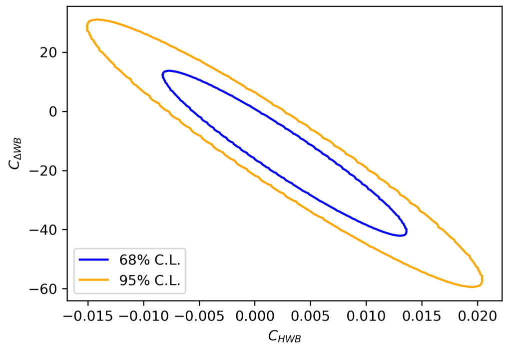

(e)

Figure 3.10: Figures with the 68% (blue) and 95% (orange) confidence levels of the different Wilson coefficients. These fits have been done setting to 0 the rest of the coefficients.

3.3 LHC Searches

Models such as the type-2 seesaw mechanisms, which introduce a new Higgs are very interesting to probe at the LHC, since, for instance, a pair of same-sign leptons has low background and could be well observed in hadron colliders. In fact, models which contain doubly- and singly-charged scalar particles have been studied at the LHC, the

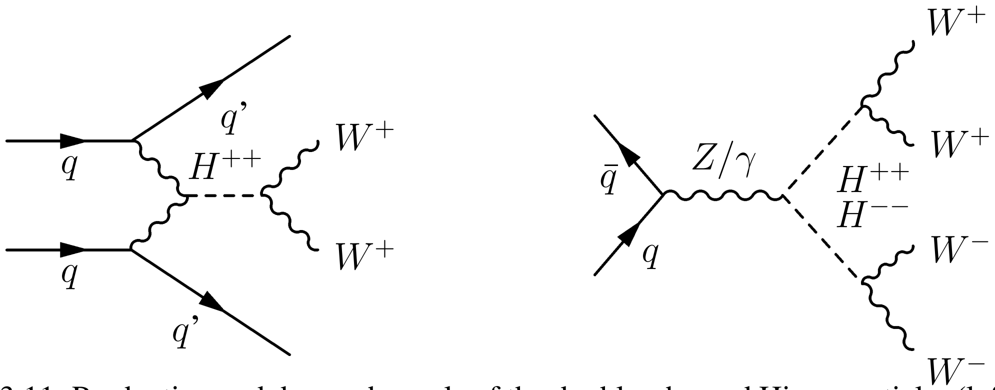

Figure 3.11: Production and decay channels of the doubly-charged Higgs particle. (left) Process analyzed by the CMS collaboration [33] and (right) process studied with the ATLAS data [74].

most recent using the biggest integrated luminosity are the CMS [33] and ATLAS [74]. Since no observation of any of these particles has been made these studies offer a very useful amount of data that can be used for setting bounds on these models.

In Ref. [33] the process studied is the VBF production of $H^{\pm\pm}/H^{\pm}$, decaying into the gauge bosons $W^{\pm}W^{\pm}/W^{\pm}Z$ respectively, while [74] makes an analysis of the production channel of $H^{++}H^{--}/H^{-}H^{++}$ pairs through a Z/W^{+} and decaying into 4 gauge bosons, the two processes are found in Fig. 3.11. Both processes are interesting in the regime where v_t is large, since the vertex $H^{++} \to W^+W^+$ is proportional to v_t. At lower v_t other decay channels start becoming more important, for instance $H^{++} \to W^+H^+$ if masses allow it or $H^{++} \to l^+l^+$.

As we have said many times already, the vacuum expectation value of v_t is not very large, so the VBF production channel would be suppressed, as well as the decay to the gauge bosons. In this point the ΔEFT can come in handy, as several new couplings appear, which do not depend on the vev, particularly operators which couple the charged Higgs to the quarks are of great interest. For instance, operators $\mathcal{O}_{\Delta qu}$ and $\mathcal{O}_{\Delta qd}$ would generate the following couplings:

$$-i\frac{1}{2}v_d\frac{(C_{\Delta qu}-C_{\Delta qd})_{rp}}{\Lambda}(V^{\text{CKM}})^*_{sr}\bar{u}_s d_p H^+ + \text{h.c.} \tag{3.66}$$

This is not the only interesting operator that couples to quarks, in fact other operators such as those of the class $\psi^2\Delta^2 D$ generate couplings to the production channel H^+H^-:

$$i\frac{\left(C^{(1)}_{\Delta q}+C_{\Delta u}\right)_{st}}{\Lambda^2}(V^{\text{CKM}})^*_{rs}(V^{\text{CKM}})_{tp}\bar{u}_r\left(\not{p}_3-\not{p}_4\right)u_p H^-(p_4)H^+(p_3)$$
$$+i\frac{\left(C^{(1)}_{\Delta q}+C_{\Delta d}\right)_{rp}}{\Lambda^2}\bar{d}_r\left(\not{p}_3-\not{p}_4\right)d_p H^-(p_4)H^+(p_3) \tag{3.67}$$

Or to the production channel $H^{++}H^{--}$:

$$i\frac{\left(C^{(1)}_{\Delta q}-C^{(3)}_{\Delta q}+C_{\Delta u}\right)_{st}}{\Lambda^2}(V^{\rm CKM})^*_{rs}(V^{\rm CKM})_{tp}\overline{u}_r\left(\not{p}_3-\not{p}_4\right)u_pH^{--}(p_4)H^{++}(p_3)$$
$$+i\frac{\left(C^{(1)}_{\Delta q}-C^{(3)}_{\Delta q}+C_{\Delta d}\right)_{rp}}{\Lambda^2}\overline{d}_r\left(\not{p}_3-\not{p}_4\right)d_pH^{--}(p_4)H^{++}(p_3) \tag{3.68}$$

We have as well contributions to the production of a single charged Higgs $H^{\pm}$:

$$i\frac{\left(C^{(3)}_{\Delta q}\right)_{rp}}{\Lambda^2}v_t(V^{\rm CKM})^*_{sr}\overline{u}_s\not{p}_3d_pH^{+}+\text{h.c.} \tag{3.69}$$

The pair production processes are independent of the vev, unlike the last expression. However, these couplings are momentum enhanced, so even if v_t is small, larger momentums might compensate making it an interesting vertex. Other couplings are also of interest to produce pairs of singly- and doubly-charged Higgs, for instance it might be interesting the $ggH^{++}H^{--}$ vertex produced by the operator $\mathcal{O}_{\Delta G}$, similarly operators $\mathcal{O}_{\Delta WW}$ $\mathcal{O}_{\Delta WB}$ could produce this pair of charged Higgs in VBF-like processes.

In order to investigate some of these couplings we will use the data of one of the analyses previously mentioned. In our case we will work with the data of the CMS analysis [33], since the analysis from ATLAS [74] uses different cuts for different mass hypotheses, which might induce some bias in our analysis.

These processes are very hard to study, if not impossible, analytically, since they include Parton Density Functions (PDF) of quarks, and we are including many Feynman diagrams that will interfere. That is why the most used resource for this kind of searches is MadGraph [70]. In order to use MadGraph with our model we first need to implement it in FeynRules [75], a program which will take all the information on couplings, particles, and the Lagrangian and write it in what is called UFO (Universal FeynRules Output). This way MadGraph is able to read all the necessary information to perform the necessary computations to set constraints on our model.

For this kind of searches it is also necessary to be able to apply all the cuts performed by the CMS collaboration in Ref. [33], so we need to use different programs in order to get something close to their analysis. We use PYTHIA8 [76] for calculating the hadronic showers in our processes, Delphes3 [77] to simulate the detector response to the final particles, in a faster way than other programs, and MadAnalysis5 [78] to apply all the cuts. It is important to note, that even though great achievements have been made in terms of emulating detector responses, and that many analysis have been recast in MadAnalysis5 with great success, this is an "approximate" way of obtaining similar analyses to those performed with more powerful simulations, but also resource-wise expensive programs such as GEANT4 [79].

Variable	$W^\pm W^\pm$	$W^\pm Z$
Leptons	2 leptons, $p_T > 25/20$ GeV	3 leptons, $p_T > 25/10/20$ GeV
p_T^j	$> 50/30$ GeV	$> 50/30$
$\lvert m_{ll} - m_Z\rvert$	> 15 GeV (ee)	< 15 GeV
m_{ll}	> 20 GeV	$-$
m_{lll}	$-$	> 100 GeV
p_T^{miss}	> 30 GeV	> 30 GeV
b jet veto	Required	Required
τ_h veto	Required	Required
max(z_l^*)	< 0.75	< 1.0
m_{jj}	> 500 GeV	> 500 GeV
$\lvert\Delta\eta_{jj}\rvert$	> 2.5	> 2.5

Table 3.4: Cuts implemented in the CMS analysis of Ref. [33] targeting $W^\pm$ and $W^\pm Z$ pairs.

As we said before, in FeynRules we declare the particles, couplings and the Lagrangian, and then we generate some files that can be read by MadGraph. Different checks have been made in order to make sure that our model works correctly, for instance reproducing the values for the cross section of the processes of Table 1 of Ref. [74] at LO. Also we have checked our program against other models containing a charged Higgs such as the Gerogi-Machacek (GM) model, implemented in FeynRules in Ref. [80] to leading order (LO) and only up to dimension 4 operators.

Once the model has been checked, we must implement the same analysis as CMS. These includes two different set of cuts depending on what we are looking for: a $W^\pm$ pair if we are looking for a doubly-charged Higgs or a $ZW^\pm$ pair for a singly-charged Higgs. These cuts can be found in Tab. 3.4. Some other typical cuts applied for the selection of particles not listed in this table are also applied. We do not aim to explain in depth all these cuts, and we refer to the analysis itself for the details [33], however, we will discuss some of them; for example, the requirement on the invariant mass of the dielectron final state to be larger than 15 GeV so that we can reject some lepton final states that could have come from Z-boson and in which the charge of one of them has been misidentified. The requirement is the other way around for the $W^\pm Z$ cuts, since we want to get all the leptons coming from the Z-boson, so we impose a cut on two leptons with the same flavour and opposite charge to be close to $Z-$boson mass. Other interesting requirement is the Zeppenfeld variable [81], which along with the invariant mass $m_{jj} > 500$ GeV and the pseudorapidity separation cuts are able to target the VBF candidates.

A method employed by many people is to use some fast simulator of detectors such such as DELPHES3 [77] and then use MadAnalysis5 [78]. The detector simulation lies on smearing and efficiency functions implemented in the program, then the files are typically analized by MadAnalysis5, as we can see by the large amount of analysis of CMS and ATLAS reproduced by using these methods [82]. This program has two modes of running, the normal mode, which offers a simple way of implementing cuts with a simple syntax, and the expert mode where the analysis is implemented in a C-file containing all the cuts. To reproduce an analysis of the LHC this last mode is normally necessary since several of the cuts cannot be implemented in the user-friendly version of MadAnalysis5. In this way we write the analysis and try to reproduce the analysis of [33].

In order to check the consistency of the analysis we will generate the background processes of $W^{\pm}W^{\pm}$ and $W^{\pm}Z$ as well as the signal produced by our model. The other backgrounds are taken directly from Ref. [33] since they are either not reproducible with this analysis, such as the non-prompt lepton background which is estimated with the data, or other background processes with not such a big impact. We then try to reproduce figures 3 and 4 from Ref. [33] to check the analysis and simulation of the different processes. The variables used for these plots are the invariant mass of the two jets m_{jj} and the transverse mass of the di-boson pair, defined as:

$$m_T^{WW} = \sqrt{\left(\sum_i E_i\right)^2 - \left(\sum_i p_{z,i}\right)^2} \tag{3.70}$$

Where we sum over the outgoing final leptons and neutrinos, assuming that the longitudinal momentum of the neutrino and its mass are 0. The Signal Regions (SR) chosen for these two variables are for the WW cuts: m_T^{WW}, $[0, 250, 350, 450, 550, 650, 850, 1050, \infty]$ GeV and in m_{jj} $[500, 800, 1200, 1800, \infty]$ GeV. For the WZ cuts the SR is: m_T^{WZ} ($[0, 325, 450, 550, 650, 850, 1350, \infty]$ GeV and m_{jj} $[500, 1500, \infty]$ GeV. As we can see, these are not the bins drawn in Fig. 3.12, in this figure bins are rescaled to reproduce the histograms of Ref. [33], but the histogram used for the fit corresponds to the bi-dimensional histogram corresponding to $m_T^{WW} \times m_{jj}$ (4×8 bins) of the WW SR and $m_T^{WZ} \times m_{jj}$ (2×7 bins) for the WZ SR. This histogram can be found in Fig. 3.13 and it is the plot we will use to obtain the exclusion limits. We have also included in this plots different operators of the ΔEFT that contribute to the VBF processes.

Taking into account the corrections produced by the NNLO corrections used in the signal, and the NNLO also used in the background, which we have only taken to LO, we observe that our analysis is quite in agreement. The most notable differences are in the histograms m_{jj} of the WZ region (bottom figures of Fig. 3.12) where the background is notably larger, but taking into account that the cross-section is reduced by a $\sim 10\%$ when NLO and EW corrections are included, both plots agree. The same thing applies for the diagrams of the WW SR (top figures of Fig. 3.12), where perhaps is less notable, but also taking into account these corrections the cross section is reduced by a $\sim 10 - 15\%$.

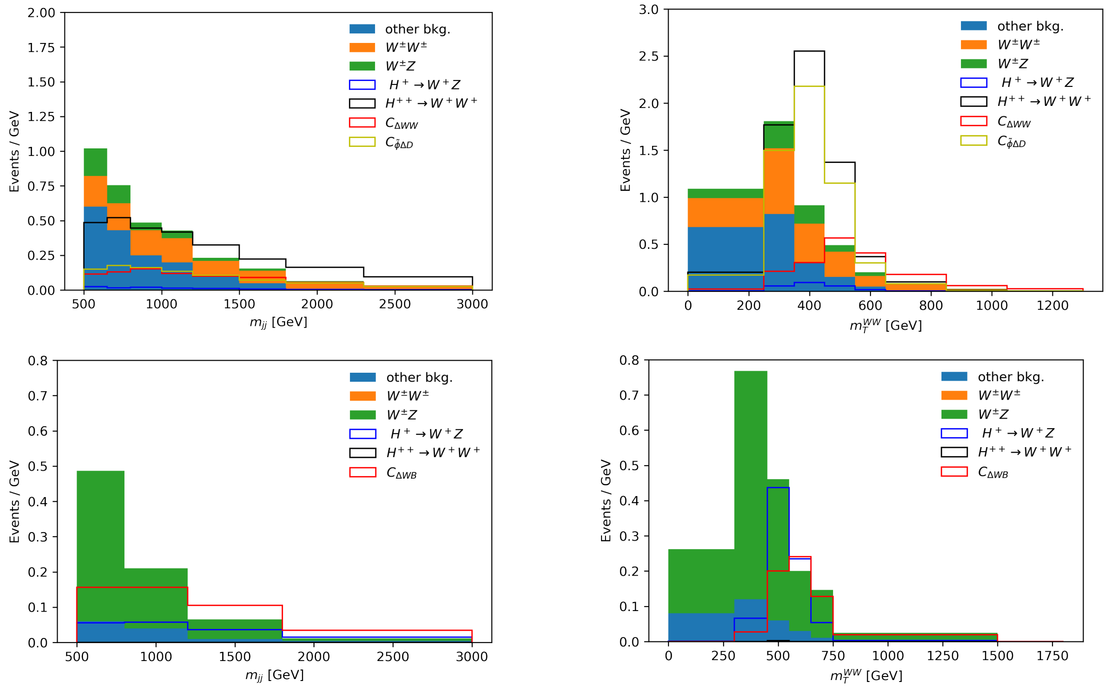

Figure 3.12: Histograms of the $W^{\pm}W^{\pm}$ (top) and $W^{\pm}Z$ (bottom) signal regions, with respect to the di-jet invariant mass m_{jj} (left) and transverse di-boson mass m_T^{WW} (right). In orange we have the background of the $W^{\pm}W^{\pm}$ SM processes and in green the $W^{\pm}Z$ SM background. In blue we observe the background coming from non-prompt leptons and other background that we have not computed. Signals produced by different operators of the ΔEFT basis are also included at arbitrary values.

To generate the signal we use our model implemented in FeynRules to LO and comparing it with the histograms of Ref. [33] we observe a discrepancy in the shape of the function of the histogram m_T^{WW} of the WW SR (top right of Fig. 3.12). We discard, with a reasonable margin of error, a mistake in the analysis since the background and other histograms are in good agreement and most of the cuts are the same for both signal regions. Those cuts which are different should also be correct since the background data is reproduced to a good agreement, taking into account the corrections mentioned above. Besides, we have also used the same PDFs, NNPDF2.3LO for the data of 2016 and NNPDF3.1 NNLO for the 2017-2018 samples. Again, we have also checked the implementation of our model with other models available and they all give the same shape. A probable cause for this deviation might be in the simulation of the detector, a difference between the processes WW and WZ is that for the first one we have 2 neutrinos, while there is only one in the second process, meaning that we probably have

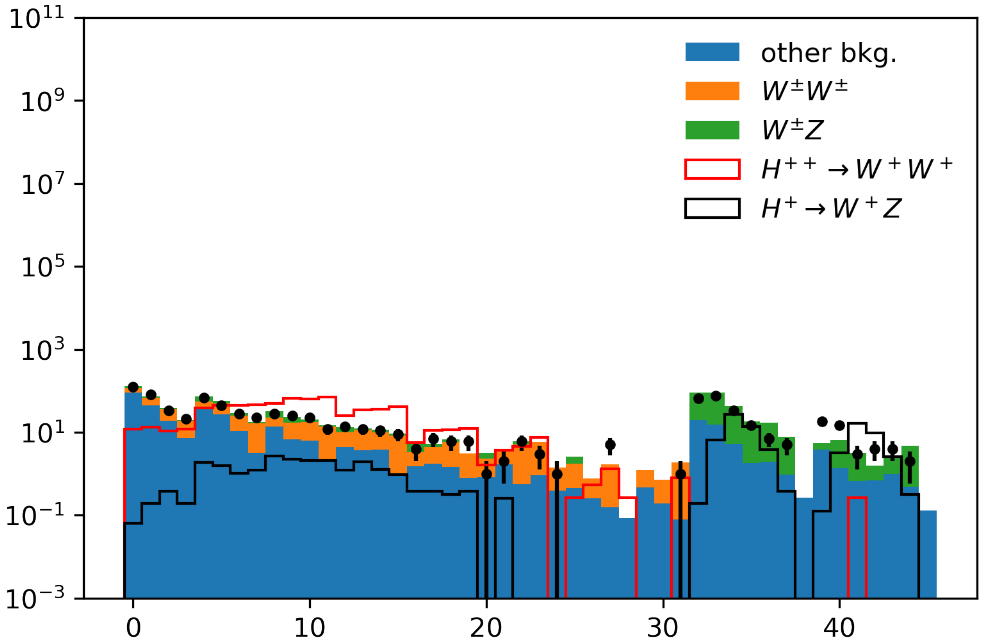

Figure 3.13: Reproduction of Figure 4 of Ref. [33]. Backgrounds of $W^\pm W^\pm$ (orange) and $W^\pm Z$ (green) have been generated at LO, while the other background (blue) has been taken from the CMS analysis.

more missing energy in each process. It is noted that for instance in Delphes [77] they do not apply algorithms to mitigate the pileup in missing energy as they do in CMS [83]. This could explain why the m_T^{WW} is only different in the WW SR and not in the WZ SR, since there would be less missing energy. Even if this is not the direct cause, it is a good point to remember that these fast simulations can not reproduce the detail of the simulations used in LHC analysis, so, we will work with it and take into account these possible deviations in the fit.

We will use the same statistical methods of [33], that is, the modified frequentist approach CL_s criterion of [84] and asymptotic methods of [85] implemented in pyhf [86], a python implementation of HistFactory. Assuming N measurements in a histogram where the expectation value of a certain bin j, has a signal s_j, background, b_j and a number of entries n_j, the Poissonian likelihood can be written as:

$$L(\mu, \theta) = \prod_j^N \frac{(\mu s_j + b_j)^{n_j}}{n_j!} e^{-(\mu s_j + b_j)} \tag{3.71}$$

Our signal and background may depend on parameters in which we are not really interested, called nuisance parameters, represented by θ. The parameter μ is the signal strength and is the parameter of interest on which we will try to derive an upper bound to translate it later into an upper limit on the Wilson coefficients. To test a hypothesized value of μ we use the profile likelihood ratio:

$$\lambda(\mu) = \frac{L(\mu, \hat{\hat{\theta}})}{L(\hat{\mu}, \hat{\theta})} \tag{3.72}$$

Where here $\hat{\hat{\theta}}$ is the maximum-likelihood estimator for a certain μ, and in the denominator we have the $\hat{\mu}$ and $\hat{\theta}$ as the maximum-likelihood estimators. The test statistic that we will use is called q_μ and is defined as:

$$q_\mu = \begin{cases} -2\ln\lambda(\mu) & \hat{\mu} \leq \mu \\ 0 & \hat{\mu} > \mu \end{cases} \tag{3.73}$$

This test statistic is used for deriving upper bounds since one is not interested in values of $\hat{\mu} > \mu$, because these values represent less agreement with μ than with the data obtained. Then, one can calculate the confidence level on the signal as:

$$CL_s = \frac{CL_{s+b}}{CL_b} = \frac{\int_{q_\mu,\text{obs}}^{\infty} p(q_\mu, \mu' = \mu) dq_\mu}{\int_{q_\mu,\text{obs}}^{\infty} p(q_\mu, \mu' = 0) dq_\mu} \tag{3.74}$$

Where in this case $p(q_\mu, \mu')$ are the the distributions of the test statistic q_μ with respect to μ. The formulas for deriving these pdfs in an approximate way are obtained in [85] and implemented in pyhf [86]. The number of background nuisance parameters in this program are one per bin, and by introducing the error of each bin we allow the background parameters to fluctuate along these errors. Once the value of μ at 95% C.L. we obtain the bound on the Wilson coefficients.

Since we chose to use the data from [33] we must look for VBF processes, hence, we will focus on operators $\mathcal{O}_{\Delta WW}$ and $\mathcal{O}_{\Delta WB}$ from our basis. Besides, we will also study the operator $\mathcal{O}_{\tilde{\varphi}\Delta D} = (D_\mu\varphi)^\dagger \Delta D_\mu \tilde{\varphi}$. We argued in section 2 that this operator is redundant, since it can be related to the EoM through integration by "parts" and thus rewrite it in terms in other operators. However, this operator can still be introduced in exchange of some other operator, in fact, the only reason why we discarded this operator was because it is customary to eliminate operators which contain derivatives. This operator can still be produced by some UV completion, for instance at loop level by a heavy gauge boson, while some dimension-5 operators could not be produced in this UV completion. In that case the operators of our basis would have a contribution produced by the redefinition of Wilson coefficients that we use to eliminate this operator, such as it should, since physical results do not depend on the basis. With this discussion we want to argue that it is perhaps more interesting to introduce this operator in exchange of some other operator that is harder to probe experimentally. This operator offers a really interesting coupling for VBF-fusion processes since it is not suppressed by v_t, we can see its contribution in eq. (3.75).

$$- iv_d^2 \frac{C_{\varphi\tilde{\Delta}D}}{\Lambda} \frac{e^2}{2\sin^2\theta_w} H^{++} W^-_\mu W^{-\,\mu} + \text{h.c.} \tag{3.75}$$

The other two operators,$\mathcal{O}_{\Delta WW}$ and $\mathcal{O}_{\Delta WB}$ studied in this section give the following contributions to the VBF-like vertices $H^{++}W^-W^-$ and H^+W^+Z respectively:

$$-i\sqrt{2}\frac{C_{\Delta WB}}{\Lambda^2}\sin\theta_w v_t(p_2^{\mu_1}p_3^{\mu_2} - p_2\cdot p_3\eta^{\mu_1\mu_2})H^+W^-_{\mu_1}(p_2)Z_{\mu_2}(p_3) + \text{h.c.} \tag{3.76}$$

$$-i4\sqrt{2}\frac{C_{\Delta WW}}{\Lambda^2}v_t(p_2^{\mu_1}p_3^{\mu_2} - p_2\cdot p_3\eta^{\mu_1\mu_2})H^{++}W^-_{\mu_1}(p_2)W^-_{\mu_2}(p_3) + \text{h.c.} \tag{3.77}$$

We have included signals of these operators in Figure 3.12. As one can see, the operator $\mathcal{O}_{\tilde{\varphi}\Delta D}$ has the same shape as the signal of the renormalizable type-2 seesaw mechanism. However, for the other 2 operators we a observe a more peaked distribution over the mass of the particle. This difference is produced by the momentum contribution appearing in the new couplings.

All in all, we can now generate different samples of signals and run the analysis to obtain the bounds. To get bounds on the Wilson coefficients we need to make assumptions on the branching ratio and v_t, in this case since some of our operators have a dependence in v_t we will choose 3 and 5 GeV, since they are in the range of maximum allowed values from the mesurement of the ρ-parameter. We will also assume a 100% branching ratio to gauge bosons. With this assumptions we can run our analysis for the three different operators. Notice that v_t and $m_{H^{++}}$ do not fully determine the branching ratio, since if $m_{H^+} < m_{H^{++}}$ the branching ratio can be reduced depending on the difference of the two masses, that is why we have to assume a certain branching ratio.

Results of the 95% C.L. can be found in tables 3.5 and 3.6 for the masses of the particles, $m_{H^{++}} = m_{H^+} = 500$ GeV and $m_{H^{++}} = m_{H^+} = 1$ TeV respectively. Operators $\mathcal{O}_{\tilde{\varphi}\Delta D}$ and $\mathcal{O}_{\Delta WW}$ are tested with the H^{++} channel while $\mathcal{O}_{\Delta WB}$ is tested by producing

	Operator	Observed	Expected	$1-\sigma$ Range
	$C_{\tilde{\varphi}\Delta D}/\Lambda$ [TeV^{-1}]	1.7	1.5	$1.7 - 0.9$
$v_t = 5$ GeV	$C_{\Delta WW}/\Lambda^2$ [TeV^{-2}]	27.8	24.2	$29.1 - 20.1$
	$C_{\Delta WB}/\Lambda^2$ [TeV^{-2}]	75.5	78.8	$94.9 - 65.3$
	$C_{\tilde{\varphi}\Delta D}/\Lambda$ [TeV^{-1}]	2.1	1.9	$2.2 - 0.6$
$v_t = 3$ GeV	$C_{\Delta WW}/\Lambda^2$ [TeV^{-2}]	49.3	43.0	$51.7 - 35.9$
	$C_{\Delta WB}/\Lambda^2$ [TeV^{-2}]	148.9	154.3	$184.3 - 129.8$

Table 3.5: 95% C.L. exclusion limits on the Wilson coefficients assuming 100% decay into gauge bosons and mass of 500 GeV for both particles.

	Operator	Observed	Expected	$1-\sigma$ Range
$v_t = 5$ GeV	$C_{\tilde{\varphi}\Delta D}/\Lambda$ [TeV^{-1}]	2.1	2.0	$2.4 - 1.7$
	$C_{\Delta WW}/\Lambda^2$ [TeV^{-2}]	30.3	30.4	$37.8 - 25.5$
	$C_{\Delta WB}/\Lambda^2$ [TeV^{-2}]	96.9	106.5	$130.2 - 88.2$
$v_t = 3$ GeV	$C_{\tilde{\varphi}\Delta D}/\Lambda$ [TeV^{-1}]	4.1	4.1	$4.9 - 3.5$
	$C_{\Delta WW}/\Lambda^2$ [TeV^{-2}]	42.0	44.4	$54.4 - 36.6$
	$C_{\Delta WB}/\Lambda^2$ [TeV^{-2}]	145.9	167.2	$130.2 - 88.2$

Table 3.6: 95% C.L. upper bounds on the Wilson coefficients assuming 100% decay into gauge bosons, $v_t = 5$ GeV and mass of 1000 GeV for both particles.

a H^+ Higgs, due to this fact the bounds on this last operator are worse in both cases, this is also observed in Ref. [33].

As it is expected, lower values of v_t make the constraints worse, $C_{\Delta WW}$ and $C_{\Delta WB}$ depend directly on v_t, as seen from eq. (3.77) and (3.76). The constraint on $C_{\tilde{\varphi}}\Delta D$ is also slightly worse for the case of v_t, since, even though it does not depend directly on v_t, due to the interference of the Feynman diagram the linear contribution of this coupling gets smaller for lower values of v_t. It is interesting to note that even if the background is lower, the bounds get worse for larger masses. The reason is that for constant couplings the cross section decreases with the mass. So, even if the cross section is more constrained for larger masses (do to less background), since the cross section decreases with the mass, the couplings do not get so constrained. This effect is larger for the operators $\mathcal{O}_{\Delta WW}$ and $\mathcal{O}_{\Delta WB}$, probably due to the binning and the momentum enhancement. The binning affects all of the operators, and as we can see from the signal regions if we have a peak at 1 TeV the signal will be less spread since the width of the bins is larger, so the upper bounds are mainly obtained by large deviations on a smaller number of bins. This is effect is more important in momentum enhanced operators since larger momenta are favored and thus the signal is less spread as we can see from Fig. 3.14.

In previous sections we also discussed that if we expected new physics to be at scales of $\Lambda \sim 1$ TeV, some bounds obtained in section 3.2.1 would be far from probing them. Here, on the other hand, we see we are able to put some more competitive bounds. However, we have had to make assumptions on several parameters of the model, so the constraints are only relevant for this particular scenario. As we have discussed at the beginning of this section, the new amount of couplings linking quarks directly to quarks may be interesting as new ways of producing the type-2 particles, moving out of the restrictions of the model.

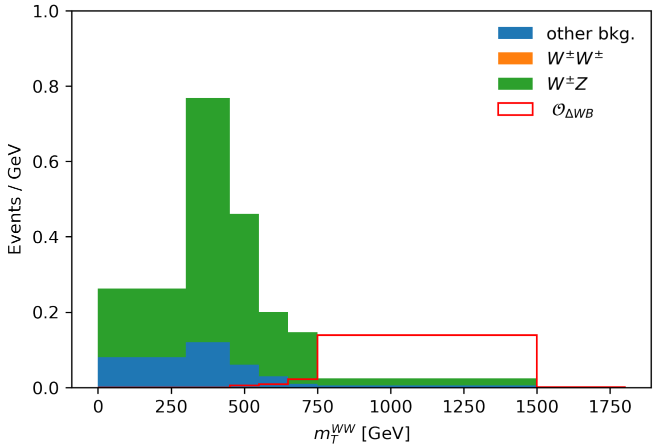

Figure 3.14: WZ SR with a signal produced by operator $\mathcal{O}_{\Delta WB}$ at a mass of 1 TeV. Again the magnitude of the parameter is arbitrary.

4 Ultraviolet Completions

During this thesis we have dealt with great number of operators in a general way, trying to make as few assumptions as possible on the models that could generate the operators. In this last section, we will make the opposite, we will aim to discuss several models that could generate some operators of the type-2 seesaw EFT basis, and discuss the physical implications of these extensions. In particular we will motivate and discuss models that extend the type-2 seesaw mechanism with possible dark matter candidates. First we will give a short review into the classic dark matter evidences, then we will discuss why the type-2 seesaw is not a good candidate and then mention some models and their interplay with the ΔEFT basis.

There are several astrophysical observations that have led us to believe in the existence of a new type of particle in the universe that we have not yet discovered. These observations appear for the first time in the 30s, by the observations of F. Zwicky [88] on the velocity dispersion of the Coma cluster galaxies. Applying the virial theorem, he realized that the speed of the galaxies was not in correspondence to the observed mass, hypothesizing that there was some dark matter that did not interact with light.

It was not until the 70s, that this idea of DM took off, with the observations of V. Rubin on the galaxy rotation curves [89]. In this article it was noticed that the velocity of the stars on the outer parts of the galaxy were not following the expected velocity distribution predicted by the Newtonian mechanics. Several more precise measurements came and solidified the idea that a new kind of particle was necessary.

We know that we need some extra mass, but why do we need a particle? and what do we know about it? Theories of modified gravity were proposed as well to solve this hidden mass problem, however, as more and more observations were made, these theories started facing many difficulties to explain them all. In particular, the Bullet cluster observation [90] , makes it hard to accommodate modified gravity theories. In this cluster, two subclusters are bound gravitationally, and collided leaving a track of electromagnetic radiation produced by this collision. Mapping the distribution of matter by gravitational lensing, we observe that the electromagnetic radiation and the center of mass of the two subclusters are shifted, meaning that a part of the cluster, that does not interact electromagnetically must exist. This observation along with the anisotropies of the cosmic microwave background helped establishing the idea that a new kind of matter was necessary.

Besides of this, several other important assumptions can be made for this kind of matter. We already discussed that this particle should not interact electromagnetically.

Moreover, this particle should be stable. It appears that dark matter does not decay into any SM particle, since the abundance of DM particles does not appear to vary. A final observation is that DM, should be cold. In this case, cold means that the free streaming length of the particle should allow for the creation of small-scale structures in the universe.

With these requirements in hand we can discuss whether any of the particles of the type-2 seesaw mechanism could make a candidate for dark matter. We observe that type-2 seesaw mechanism offers neutral and massive (and hence cold) particles. However, it fails into giving a good stable neutral particle. Two neutral particles appear in the type-2 seesaw mechanism, we called them H and A, referring to a CP-even and CP-odd particles respectively. In both cases these particles are allowed to decay into other particles, for large v_t these particles preferably decay into $H \to ZZ, W^+W^-$ and $A \to hZ$, and for lower v_t the preferred decay is into neutrinos for both particles. So, we can see that stability is a major threat to making this particle a good DM candidate.

This is why, if one wants to make the type-2 seesaw mechanism work for DM, it is quite necessary to introduce an extra particle. This works well with the basis that we have been working with in this thesis. If we introduce a DM along with the triplet and make the new particle to have a larger mass $M_\chi^2 \gg M_\Delta^2$ we inevitably will obtain many effective operators depending on which new χ, DM particle we introduce. Now, we will discuss several extensions that have been used in the literature with the type-2 seesaw mechanism to explain DM and study its interplay with the ΔEFT.

Scalar Gauge-Singlet Dark Matter

The most simplest of the extensions that we could add to the type-2 seesaw mechanism is a gauge-singlet real scalar. This kind of particle could make for a good DM particle, as being a singlet it would not interact electromagnetically. In many models an extra Z_2 parity symmetry is introduce. With this symmetry SM particles transform as $+1$ while DM particles -1, by requiring this symmetry DM particles are made stable. In this case, and following Ref. [91], we also make the type-2 seesaw have a Z_2 value of $+1$. Using D to denote the DM scalar particle the extra terms in the potential are:

$$V_D = \frac{1}{2}m_D^2 D^2 + \lambda_D D^4 + \lambda_\varphi D^2 \varphi^\dagger \varphi + \lambda_\Delta D^2 \,\mathrm{Tr}\big(\Delta^\dagger \Delta\big) \tag{4.1}$$

Due to the requirement of the Z_2 symmetry all the parameters of the ΔEFT basis can only be generated at loop-level. However, as shown in Ref. [91], with this simple extension it is possible to accommodate these two models to fit the measurements of the cosmic ray spectrum made by several different collaborations.

The operators of the ΔEFT basis are listed here:

$$\{\mathcal{O}_\Delta^1,\, \mathcal{O}_{\varphi\Delta}^1,\, \mathcal{O}_{\Delta\varphi}^3\} \tag{4.2}$$

If we break this Z_2 symmetry, other terms in the potential would be possible, for instance $\lambda'_\varphi D\varphi^\dagger\varphi$ or $\lambda'_\Delta D\,\mathrm{Tr}\left(\Delta^\dagger\Delta\right)$ allowing for the previous operators to be generated at tree-level.

Innert Higgs Doublet

Increasing in complexity, Ref. [92] shows that an extra Higgs doublet ϕ, odd with respect to a Z_2 parity symmetry can again produce a sensible candidate for DM. Both of this models benefit from having a combination of the triplet and some dark matter candidate to be able to fit to the electron-positron spectrum data, since a collision of the type $\chi\chi \to \Delta^{++}\Delta^{--}$, where χ is any of the DM candidates of this section or the previous one, would produce an increase in the flux of charged leptons.

Again in this case, the Z_2 makes impossible to generate tree-level operators, since to respect this symmetry and make the DM particle stable each coupling comes with two fields ϕ in the Lagrangian. The operators that can be generated at loop-level also allow for more combinations of $SU(2)_L$:

$$\{\mathcal{O}^1_{\varphi\Delta},\, \mathcal{O}^2_{\varphi\Delta},\, \mathcal{O}^1_{\Delta},\, \mathcal{O}^1_{\Delta\varphi},\, \mathcal{O}^2_{\Delta\varphi},\, \mathcal{O}^3_{\Delta\varphi}, \mathcal{O}^1_{\tilde{\varphi}\Delta}, \mathcal{O}^2_{\tilde{\varphi}\Delta}\} \tag{4.3}$$

Again, by breaking the Z_2 symmetry the previous operators can be generated at tree-level. In this case also operators of the class $\Delta^2\psi^2\varphi$ can be generated, since leptons can be coupled to this Higgs, losing most likely the stability of this inert Higgs used in [92].

$U(1)_{B-L}$ Gauge Boson

Another possible way of DM could come through a new gauge symmetry. The gauge boson of this symmetry, typically called Z', could be by itself a DM particle or a mediator to a DM sector. It is hard to make the Z'-boson a dark matter candidate, since through the mixing with the SM $Z-$boson could decay to a neutrino pair. Hence, it is more interesting to think of it as a mediator between the 2 sectors.

A particular theory containing a Z'-boson would be a theory with $U(1)_{B-L}$ symmetry. This is a global symmetry used in grand unified theories, that would not be broken by chiral anomalies, typically also requires a set of three right-handed neutrinos. In such a theory, the lightest of these right-handed neutrinos could be a dark matter candidate. It can be found in the literature, see for instance [93–95], that one can get in a similar way, a spontaneously broken $U(1)_{B-L}$ symmetry, that solves the neutrino mass question as well as a viable DM candidate, along with a new gauge boson, with a relatively big mass due to experimental constraints.

This is a "better case", for the ΔEFT basis, since even if a Z_2 parity is applied to the DM candidates, the Z' boson acting as a mediator into SM and DM could generate effective couplings in our basis. So, after the SSB of $U(1)_{B-L}$ symmetry, the massive gauge boson would generate the operators of the class $\Delta^2\psi^2 D$ as well as operators of the scalar sector containing derivatives at tree-level:

$$\{\mathcal{O}_{\Delta l}^{(1)}, \mathcal{O}_{\Delta q}^{(1)}, \mathcal{O}_{\Delta e}, \mathcal{O}_{\Delta u}, \mathcal{O}_{\Delta d}, \mathcal{O}_{D\Delta}^{1}, \mathcal{O}_{\Delta\Box}, \mathcal{O}_{\Delta\Box\varphi}, \mathcal{O}_{\Delta D\varphi}^{1}\} \tag{4.4}$$

The Georgi-Machacek Model

We have seen several models that can help solving the dark matter problem, as well as having a triplet to explain neutrino masses. In this section now, even though this model can be accommodated to produce a viable DM candidate [96], we will not consider it in too much detail on this section. The Georgi-Machacek model (GM) [97], is an extension of the SM which contains two scalar triplets, one with hypercharge $Y = 0$, a real triplet, and a complex triplet with hypercharge $Y = 1$. By requiring the two triplets to generate the same vacuum expectation value the ρ-parameter can be kept to be 1. This model also conserves what is called custodial symmetry, which is a global $SU(2)_L \times SU(2)_R$ symmetry, which is tightly related to the ρ-parameter.

To preserve this symmetry, in this model it is customary to write both triplets in a bitriplet, with the same mass term. This is not what we want, since we want the real triplet to generate the operators of the ΔEFT basis. We can see that this symmetry is broken at the loop-level, as pointed out in Ref. [98]. Hence, it is possible to extend the GM model to allow for different mass terms [98], breaking custodial symmetry at tree level and restore it at a higher energy-scale [99]. In that case the Lagrangian can be written in a more general way, which introduces new terms such as[1] $\varphi^\dagger\sigma^a\varphi\xi^a$ and $\chi^\dagger\sigma^a\xi\tilde{\varphi}^\dagger\sigma^a\varphi + \text{h.c.}$. All in all the operators of the ΔEFT produced would be:

$$\{\mathcal{O}_{\tilde{\varphi}\Delta}^{1}, \mathcal{O}_{\tilde{\varphi}\Delta}^{2}, \mathcal{O}_{\varphi\Delta}^{1}, \mathcal{O}_{\varphi\Delta}^{2}\} \tag{4.5}$$

In all these models, also operators off the Warsaw basis can be generated, which can be found in [40]. Hence, studying each different model in the ΔEFT must be complemented also by analyzing the Warsaw basis operators. It is also interesting to note that the GM model can be extended with some other particles as different DM candidates, which would combine and extend this set of operators, e.g. [96].

[1]The whole Lagrangian can be found in Ref. [99],

5 Conclusions

Neutrino oscillations have proved that neutrinos must have a mass, albeit it should be really small compared to the SM model particles. This huge difference in the masses of the SM has led scientist to create models which suppress neutrino masses, typically by including a much heavier state that through its interaction would create this suppression. This is the idea of the seesaw mechanism, that we have shortly reviewed in this work, the type-1 and type-2 seesaw mechanism. Both of these models require the inclusion of a new particle, either a heavier right-handed neutrino or a triplet complex scalar particle.

Since, there are no large fluctuations from the SM predictions, people have come up with a set of tools that allow for indirect searches for these new states, that is EFT. By allowing non-renormalizable terms in the Lagrangian we are able to investigate possible deviations in a UV-completion independent way. Using the tools of EFTs, we have computed a complete non-redundant basis by allowing up to dimension 6 operators in the Lagrangian. We have argued several new ways of reducing the basis that are slightly different from the SMEFT, and discussed several differences and similarities to the Warsaw Basis.

We have used three different methods in order to set bounds, or constrain, the different parameters of this large model. Firstly, we have used LFV processes to study the regime in which $v_t \ll v_d$, by assuming the internal state much larger than the momentum scale of the experiment. This EFT way of thinking has led us to extend this to our dimension-5 operators and obtain bounds on these operators in terms of v_t, by using the data on the SMEFT.

It is known that Warsaw basis operators would modify certain couplings or obseravables of the SM, in particular many couplings related to electroweak observables are modified. This also happens for the ΔEFT and can be used to set bounds on some operators, since these observables are well constrained. We have studied this operators on their own and argued again their dependence on the vacuum expectation value of the triplet. Moreover, we have argued that in a more UV-completion independent framework one should study a combination of the SMEFT+ΔEFT. This is a very large set of operators that cannot be constrained only from EWPO but when including Higgs measurements we we have shown that it is possible to lift the degeneracy of the operators and studied the range of values in which pairs of analog operators can moved within the restrictions imposed by the measurements. However, it is still a very complex framework, so handling this in detail may require a whole dedicated study.

In the third phenomenological study, we have discussed several new ways observing the charged scalar particles of the ΔEFT that are only possible in this extension, for instance, direct couplings with quarks, which does not happen at dimension 4. We have then shown how these operators can be studied in colliders, by recasting the analysis of the CMS [33] and applying it to VBF processes produced by some operators of our basis. This required to make some assumptions on some of the other parameters such as the mass and branching ratios, however these bounds are quite competitive since some of them are able to constrain $C \sim \mathcal{O}(1-100)$ for $\Lambda \sim$ TeV.

Finally, we have given a brief set of models that could produce this basis in the UV, and how the different models would be seen in the ΔEFT through contributions to the Wilson coefficients.

As an overlook, we have studied an EFT extension of a particular model, the type-2 seesaw mechanism. Many more extensions could be done, however it would be interesting, with the knowledge gained, for instance in works such as this, to evaluate the usefulness of these kind of models. As we have argued, global analysis are difficult, if not impossible to make in these models, and the complexity is higher due to the amount of operators. On the other hand, couplings that offer different production channels also appear, albeit sometimes proportional to already experimentally low parameters of the new states. This is why a more general study on these models pointing out the pros and cons, and better directions to study these models could be of great interest.

Appendices

A Reducing Operators of the Lagrangian

A.1 Dimension 5 Operator with Derivatives

The operator $(D_\mu\varphi)^\dagger\Delta D^\mu\tilde{\varphi}$ has been used in the literature [40], however, in a complete non-redundant basis it can be reduced with the EoM and other operators that are already found in the basis. Using Leibniz rule we can get three relations of the operators into a term proportional to the equations of motion and other operators that are already found on the basis.

$$\partial_\mu((D^\mu\varphi)^\dagger\Delta\tilde{\varphi}) = (D_\mu\varphi)^\dagger\Delta D^\mu\tilde{\varphi} + (D_\mu\varphi)^\dagger D^\mu\Delta\tilde{\varphi} + \boxed{\text{EoM}} + ... \tag{A.1}$$

$$\partial_\mu(\varphi^\dagger D^\mu\Delta\tilde{\varphi}) = \varphi^\dagger D^\mu\Delta D^\mu\tilde{\varphi} + (D_\mu\varphi)^\dagger D^\mu\Delta\tilde{\varphi} + \boxed{\text{EoM}} + ... \tag{A.2}$$

$$\partial_\mu(\varphi^\dagger\Delta D^\mu\tilde{\varphi}) = (D_\mu\varphi)^\dagger\Delta D^\mu\tilde{\varphi} + \varphi^\dagger D_\mu\Delta D^\mu\tilde{\varphi} + \boxed{\text{EoM}} + ... \tag{A.3}$$

For instance, combining the previous equations as (A.1) + (A.3) - (A.2) we obtain the following equation:

$$\partial_\mu(\varphi^\dagger\Delta D^\mu\tilde{\varphi}) + \partial_\mu((D^\mu\varphi)^\dagger\Delta\tilde{\varphi}) - \partial_\mu(\varphi^\dagger D^\mu\Delta\tilde{\varphi}) = 2(D_\mu\varphi)^\dagger\Delta D^\mu\tilde{\varphi} + \boxed{\text{EoM}} + ... \tag{A.4}$$

As we can see this operator is completely redundant as we have three relations and three possible operators, so in a complete basis this kind of operator is unnecessary.

A.2 Reducing Dimension-6 Operators with Derivatives

As we did in the previous section of the appendix, for the case of dimension six now we have 4 relations similar as those of the previous section, in this case:

$$\partial_\mu(\varphi^\dagger\Delta\Delta^\dagger D^\mu\varphi) = (D_\mu\varphi)^\dagger\Delta\Delta^\dagger D^\mu\varphi + \varphi^\dagger D_\mu\Delta\Delta^\dagger D^\mu\varphi + \varphi^\dagger\Delta(D_\mu\Delta)^\dagger D^\mu\varphi + \boxed{\text{EoM}} + ... \tag{A.5}$$

$$\partial_\mu(\varphi^\dagger\Delta(D^\mu\Delta)^\dagger\varphi) = (D_\mu\varphi)^\dagger\Delta(D^\mu\Delta)^\dagger\varphi + \varphi^\dagger D_\mu\Delta(D^\mu\Delta)^\dagger\varphi + \varphi^\dagger\Delta(D_\mu\Delta)^\dagger D^\mu\varphi + \boxed{\text{EoM}} + ... \tag{A.6}$$

$$\partial_\mu(\varphi^\dagger D^\mu\Delta\Delta^\dagger\varphi) = (D_\mu\varphi)^\dagger D^\mu\Delta\Delta\varphi + \varphi^\dagger D_\mu\Delta(D^\mu\Delta)^\dagger\varphi + \varphi^\dagger D_\mu\Delta\Delta^\dagger D^\mu\varphi + \boxed{\text{EoM}} + ... \tag{A.7}$$

$$\partial_\mu((D^\mu\varphi)^\dagger\Delta\Delta^\dagger\varphi) = (D_\mu\varphi)^\dagger D^\mu\Delta\Delta\varphi + (D^\mu\varphi)^\dagger\Delta(D^\mu\Delta)^\dagger\varphi + (D_\mu\varphi)^\dagger\Delta\Delta^\dagger D^\mu\varphi + \boxed{\text{EoM}} + ... \tag{A.8}$$

The singlet singlet combination that give rise to operators $\mathcal{O}_{\Delta\Box}$ and $\mathcal{O}_{\Delta D}$ is treated in section 2.1.2. In this case, we have 6 operators and 4 relations that allow for 2 independent operators. In this case we cannot choose the operators at will, for instance the combination: (A.5)-(A.6)+(A.8)-(A.7) gives the relation:

$$\text{Total derivatives} = 2(D_\mu\varphi)^\dagger\Delta\Delta^\dagger D^\mu\varphi - 2\varphi^\dagger D_\mu\Delta(D^\mu\Delta)^\dagger\varphi + \boxed{\text{EoM}} + ... \tag{A.9}$$

Hence, we cannot choose these two operators in our basis since it would be redundant, as it has been shown in the previous equation. Thus we choose one of these operators and the other one we pick a self-hermitian combination of the other 4:

$$\begin{aligned}(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)\,\mathrm{Tr}\big(\Delta^\dagger i\overleftrightarrow{D}^I_\mu\Delta\big) &= \varphi^\dagger\Delta(D_\mu\Delta)^\dagger D^\mu\varphi + (D_\mu\varphi)^\dagger D^\mu\Delta\Delta^\dagger\varphi \\ &\quad - \big[\varphi^\dagger D_\mu\Delta\Delta^\dagger D^\mu\varphi + (D_\mu\varphi)^\dagger\Delta(D^\mu\Delta)^\dagger\varphi\big]\end{aligned} \tag{A.10}$$

Where we skipped an intermediate step in which we used the property (2.2) and the definition of the derivative $\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi = i\varphi^\dagger\tau^I D_\mu\varphi - i(D_\mu\varphi)^\dagger\tau^I\varphi$. This operator is self-hermitian and can be written with just one Wilson coefficient since thanks to equations (A.5)-(A.8) we can reduce these operators to only two, which are non-redundant.

B Modified Electroweak Observables

Since these modifications are known and calculated (see [35, 62, 63]) and we are not adding anything new, we will just mention briefly the most important points and list, for completeness of the work, the different modifications to the EW observables.

In the α-scheme we set $\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ as input parameters, redefining all the rest of the constants in terms of these three (from Ref. [35]):

$$
\begin{aligned}
s_{\hat\theta}^2 &= \frac{1}{2}\left[1-\sqrt{1-\frac{4\pi\hat\alpha}{\sqrt{2}\hat G_F \hat m_Z^2}}\right] & \hat e &= \sqrt{4\pi\alpha} \\
\hat g' &= \frac{\hat e}{c_{\hat\theta}} & \hat g &= \frac{\hat e}{s_{\hat\theta}} \\
\hat v &= \frac{1}{\sqrt{2^{1/4}\hat G_F}} & \hat m_W^2 &= \hat m_Z^2 c_{\hat\theta}^2
\end{aligned} \tag{B.1}
$$

The coupling redefinitions introduced in section 3.2.1, modify as well the observables that we use in our fit in Tab. 3.1. First we use that:

$$\bar\Gamma(Z\to \bar f f) = \frac{\sqrt{2}\hat G_F \hat m_Z^3 N_c}{3\pi}\left(|\bar g_V^f|^2 + |\bar g_A^f|^2\right) \tag{B.2}$$

$$\bar\Gamma(Z\to \text{had}) = 2\bar\Gamma(Z\to \bar u u) + 3\bar\Gamma(Z\to \bar d d) \tag{B.3}$$

Defining the correction of the SMEFT to the SM values as $\bar\Gamma(Z\to \bar f f) = \Gamma^{\text{SM}}_{Z\to\bar f f} + \delta\Gamma_{Z\to\bar f f}$

$$\delta\Gamma^{\text{SM}}_{Z\to\bar l l} = \frac{\sqrt{2}\hat G_F \hat m_Z^3}{6\pi}\left[-\delta g_A^l + \left(-1+4s_{\hat\theta}^2\right)\delta g_V^l\right] \tag{B.4}$$

$$\delta\Gamma^{\text{SM}}_{Z\to\bar \nu \nu} = \frac{\sqrt{2}\hat G_F \hat m_Z^3}{6\pi}\left[\delta g_A^\nu + \delta g_V^\nu\right] \tag{B.5}$$

$$\delta\Gamma^{\text{SM}}_{Z\to\text{had}}\frac{\sqrt{2}\hat G_F \hat m_Z^3}{\pi}\left[-\delta g_A^u - \frac{1}{3}\left(-3+8s_{\hat\theta}^2\right)\delta g_V^u - \frac{3}{2}g_A^d + \frac{1}{2}\left(-3+4s_{\hat\theta}^2\right)\delta g_V^d\right] \tag{B.6}$$

Thus, with these definitions we can write the correction to the decay width of the Z-boson as:

$$\delta\Gamma_Z = 3\delta\Gamma^{\text{SM}}_{Z\to\bar l l} + 3\delta\Gamma^{\text{SM}}_{Z\to\bar\nu\nu} + \delta\Gamma^{\text{SM}}_{Z\to\text{had}} \tag{B.7}$$

Corrections to other observables measured at LEP can be written in terms of these expressions. The ratio of the decay rates, defined as $R_f = \Gamma_{Z\to\text{had}}/\Gamma_{Z\to\bar f f}$ with $f = l,\nu$ the correction according to the definition above is:

$$\delta R_f = \frac{1}{(\Gamma^{\text{SM}}_{Z\to\bar f f})^2}\left[\delta\Gamma^{\text{SM}}_{Z\to\text{had}}\Gamma^{\text{SM}}_{Z\to\bar f f} - \delta\Gamma^{\text{SM}}_{Z\to\bar f f}\Gamma^{\text{SM}}_{Z\to\text{had}}\right] \tag{B.8}$$

When f is a quark the definition is the inverse, so the expression above gets an overall minus sign. The other observables that we use in our fit are the asymmetries. The different asymmetries listed in Tab. 3.1 can be expressed in the SM in terms of the fermion couplings such as:

$$A^f_{\text{FB}} = \frac{3}{4} A_l A_f, \qquad A_f = 2\frac{g^f_V g^f_A}{(g^f_V)^2 + (g^f_A)^2} \tag{B.9}$$

Where $f = l, c, b$, and c and b refer to the c and b quarks. When we introduce the modifications of an EFT we can rewrite them as:

$$\bar{A}_f = \frac{2\bar{r}_f}{1 + \bar{r}^2_f}, \qquad \bar{r}_f = \frac{\bar{g}^f_V}{\bar{g}^f_A} \tag{B.10}$$

Then again we redifine the EW observable as the SM part and a correction, this correction can be written as:

$$\delta A_f = (A_f)^{\text{SM}} \left(1 - \frac{2(r^2_f)^{\text{SM}}}{1 + (r^2_f)^{\text{SM}}} \right) \delta r_f \tag{B.11}$$

with:

$$\bar{r}_f = (r_f)^{(\text{SM})}(1 + \delta r_f), \qquad \delta r_f = \frac{\delta g^f_V}{g^{f,\text{SM}}_V} - \frac{\delta g^f_A}{g^{f,\text{SM}}_A} \tag{B.12}$$

The last observables correspond to the decay width and branching rations of the W-boson. These expressions get modified in a similar way as the Z-boson:

$$\Gamma^{\text{SM}}_{W \to \bar{f}_p f_r} = \frac{N_C |V^f_{pr}|^2 \sqrt{2} \hat{G}_F m^3_W}{12\pi} \tag{B.13}$$

$$\delta\Gamma_{W \to \bar{f}_p f_r} = \frac{N_C |V^f_{pr}|^2 \sqrt{2} \hat{G}_F m^3_W}{12\pi} \left(4\delta g^{W,f}_{V/A} + \frac{1}{2}\frac{\delta m^2_W}{\hat{m}^2_W} \right) \tag{B.14}$$

$$\Gamma^{SM}_W = \frac{3\sqrt{2}\hat{G}_F m^3_W}{4\pi}, \qquad \delta\Gamma_W = \Gamma^{SM}_W \left(\frac{4}{3}\delta g^l_W + \frac{8}{3}\delta g^q_W + \frac{1}{2}\frac{\delta m^2_W}{\hat{m}^2_W} \right) \tag{B.15}$$

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Acknowledgements

It is impossible to thank every single person who has helped me during this work, and during this master. I will also probably forget many names, so this list is not complete.

Firstly, I would like to deeply thank my supervisor Werner Rodejohann for welcoming me in his group. He has also taught me a lot and helped me to get a PhD position in physics. I am also really grateful for how much I have enjoyed discussing this work with him.

I want to thank Tilman Plehn for being the second examiner of this thesis, and also for giving us really useful inputs on the possible ways of studying this model. His help was really appreciated.

I want also to thank all the people working at the MPIK in Heidelberg, that even in COVID-19 times have been always really warm and welcoming. In particular, I thank Ingolf Bischer for helping me at the beginning of this thesis, answering my questions and giving me really good advice on how to tackle the different problems. I also would like to thank all the colleagues in the office that I have talked to, and that indirectly have also helped me with this work and more.

I thank Jorge Terol, who pushed me to apply for a PhD position that I finally obtained and also gave me the idea and guidance to do neutrino physics at the MPIK.

I want to thank my bachelor thesis supervisor, Jordi Vidal, who also helped me to obtain the PhD position, and has written for me several reference letters.

I would like to thank Freddy and Rebecca with whom I have lived two very nice years and who have helped me through the German way of living.

I want to also thank all the friends I made doing physics, and other things, in Heidelberg: Álvaro, Dani, Gloria, Iago, Isa, Jaime, Mireia, Pedro and particularly Paula, whose patience, motivational help and love have been really important to me during these years, I could not have achieved this without her help.

Finally, I would like to thank my parents who have always supported me in whatever I wanted to pursue, spending selflessly their money in my studies and future.

Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 13.09.2021 . . . .