Only the raisins or all the pie?

The implementation of the Varga-Neményi¹ method in Finnish schools

Anni Lampinen - Kirsi Puumalainen

Matikkamaa/ Kirstin koulou, Espoo – Turengin koulou, Espoo

At the beginning of a course held at Helsinki University in 2000, Julianna Szendrei asked us, Finnish teachers the question: Who do we want to get to the finish line with – the few best pupils, "the average" or every single pupil, even if there will be some who will have to wear roller skates to get there. According to Eszter C. Neményi, who introduced all her courses with this, the teachers are driven by two kinds of love in their work: the love of children and the love of mathematics. Now, these thoughts are important and familiar to us, Finnish teachers as well. It is exactly the similar values, the similar view on teaching that aroused our interest: How do you teach here in Hungary? What novelties could we learn from you? This was the situation in 2000. Now we are here to tell you about Finnish mathematics teaching and about what we learnt from you, apart from the fact that our school culture is somewhat different from yours, how we are proceeding in the direction you set for us, and how we have formed the seven principles of Varga-Neményi to "our image".

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1. Experience based on reality

"Playing is the work of a child" – this popular Finnish saying is especially valid when it comes to learning mathematics. In order to make our young pupils understand the basics of mathematics, we have to make abstract mathematics accessible for their lives through their personal experience. The purpose of teaching is to give "mathematical googles" to the children. The Finnish Core Curriculum has long emphasized concretisation and activity as the starting points of teaching, yet there are multiple opinions on what it means in practice.

The central issue of the Varga-Neményi method is the personal and direct experience of children. It is not enough for them to hear or read that somebody somewhere once experienced this or that; or that this or that happened. If the pre-school personal experience is missing, it needs to be made up for in the school. We now have plenty of experience of the primal importance of personal experience in two learning groups. When teaching immigrant children, it is essential to be specific and let them experi-

Developing the primary school curriculum and teaching methods, Eszter C. Neményi was one of the closest associates of Tamás Varga. Many Hungarian approaches of mathematics teaching allude to Tamás Varga in their publications, yet many of these are quite far from his original intellectual heritage. That is why, as Julianna recommended, in the title of the authentic curriculum and teaching method presented in Finland we include Neményi.

ence, for many of them do not speak Finnish or speak poorly. We construct our common language learning together in the course of the schooldays. They have to learn to express their lives and activities using words of a language new to them. On the other hand, in the Ruskeasuo-school, which develops teaching methods for physically handicapped children, we experience continually and palpably how indispensable experience is in the learning process; due to their disability, these children could not extract enough experience and knowledge from and about their surroundings. There are some teachers in our postgraduate courses who are working with mentally handicapped children as well.

During our own postgraduate courses, primary school teachers have the opportunity to experience playing and activities from a child's perspective. It partly helps them to reconsider and reform their own work, and partly to perceive what children experience through this kind of teaching method. The experience gained during the course can be transferred into their own classroom almost without change. Oftentimes, a sigh can be heard coming from many participants of our courses: "I wish mathematics had been taught to me this way!"

2. Using tools widely

The usage of tools is traditionally restricted to the first two classes in Finland, and even in those classes tools rather mean aids, helping mechanical calculation. These, of course, have to be left behind as soon as possible, therefore students who need them longer can soon find themselves in the "weak student" category. Which student would want this? For the average Finnish primary school teacher it is an aha-experience to discover that the tools of various activities are for everyone! Even for the brightest prodigy! And that these tools are instruments of conceptualization and thinking development. That we can differentiate with tools – even upwards!

It is not a secret that this method requires a lot of tools, and so the teacher has to prepare many objects and written materials. In order to help their work, we comprised various ready-made materials apart from the coursebook and workbook (for example flesh cards), which can be purchased. Moreover, we organize DIY gatherings, where teachers can prepare tools and teaching materials together. The same can be a pleasant and useful evening program during seminars. Teachers' imagination, creativity and dexterity is fascinating. Time flies when they work together and discuss relevant issues. The students' parents can also be asked to help with this or that, there are always some

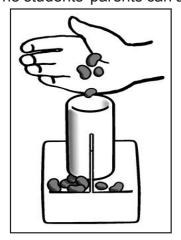


Figure 1.: Number splitter

who volunteer. With all these we want to ensure that being skillful would not be a requirement, neither would the lack of technique be a hindrance in the application of the method.

Learning in nature: Finland is the country of woods and lakes, where we find it important to educate children in relation to nature.

Arts, architecture and culture draws on nature and its proximity. In the latest years, we have developed our pedagogy in general following the "out of the classroom" principle, and within this we developed Varga-Neményi method as well: we took it to the schoolyard and the nearby woods, where tools and materials are literally on hand.

Number splitter: We can make children practice splitting numbers 0-10 into two with the help of this game.

For example, take 7, we drop 7 beans into the funnel, and before that, every player has to jot down what they think will come out of the machine. Those who are right, get a point each. We can also write the sums into a bingo-table and play splitter bingo this way.

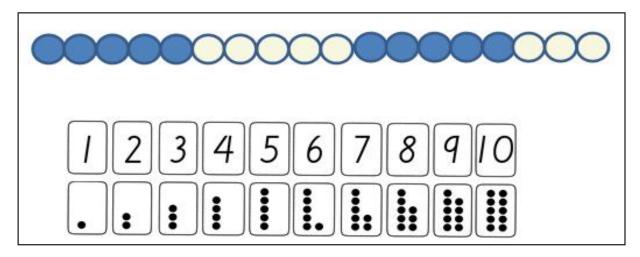


Figure 2.: The power of fives

The power of fives: The improvement of visual recognition of quantities, as well as benefiting from this ability has an important role in the Varga-Neményi method. As it is difficult to estimate the number of more than five things at one glance, we may use the power of fives as a possible visual model: we group quantities in fives, for example, we build up ten from two fives.

We can estimate even larger numbers with fives, for example 18 beads will be three groups of fives and three individual beads.

Children learn to name and show the amount of beads without actually counting them by one. This cannot happen without a sort of visual structuring.

Nuts and bolts boxes and snakes of balls: We make children practice number sequences 0-10 and the quantity a number expresses with various tools. Children experience by touching when they have to line boxes up in order of their weight or they are asked to count the balls in a snake of balls; the speciality of these tools is that they cannot be opened, therefore, children cannot estimate quantities by looking.

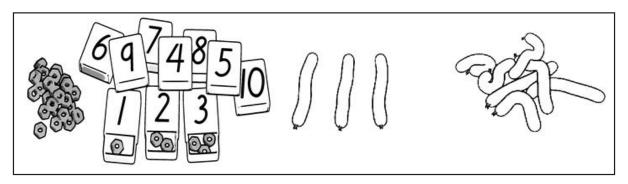


Figure 3.: Boxes and snakes

Story telling: Stories include long processes. Learning to exchange tens to ones by number splitting requires a variety of skills and knowledge. To make these skills and knowledge build up in a way that supports memory, we tell tales and play. Egg boxes

for ten eggs are used in Finland as well. The game is set in the hen house: the farmer collects fresh eggs every morning and evening and puts them into the boxes always the same way – left to right and up from down. This game helps pupils visualize and memorize what happens. In the bookkeeping section they learn the notations of the calculations with the help of arrows showing the exchange.

New and ready-made games: Hally-Gally, Super Farmer and Chocolate fix.

3. Considering personality and age

Pre-school curriculum is only briefly and roughly worked out in Finland. It does not guide pre-school teaching well enough and it does not support teachers in awakening interest in children whose skills have to be improved, who did not receive important inputs of mathematical experience in early childhood. The Varga-Neményi method is a complete system which supports the multiple improvement of children. This method requires a new way of thinking from us, teachers. Therefore, we in the Varga-Neményi Company put the greatest emphasis on improving pre-school teaching. Our colleague, Minna Salminen is developing a new, extensive and structured pre-school curriculum. Pre-school teachers attending our courses are very enthusiastic about these new thoughts as they extend their knowledge and improve the quality of their work. In their opinion, they did not receive enough and good enough training to ensure successful mathematical education of young children.

MAVALKA: Finnish children begin school in the year they become 7, which means that the age difference between the youngest and eldest students in a class can be almost a year. Most children attend pre-school, but up to this year we have paid little attention to the profound differences in knowledge between children beginning school. In the latest few years, we have been trying to help children start school and find the ones who may, sooner or later, have learning difficulties. For this purpose, Anni Lampinen, Hannele Ikäheimo and Marja Dräger developed a method, recommended in pre-school and the first semester of the first class, to realize children's mathematical condition. It consists of number concepts, number line and invariant quantities.

We offer regular help to children who are likely to have learning difficulties in the future, preferably from pre-school, but surely in the first two schoolyears. The form of our help is the so-called Noppa club (noppa=dice), where children can acquire the missing skills and basic knowledge necessary for learning mathematics. This is progress, compared to the recent past, and partly today as well, unfortunately, when we began to deal with mathematics learning difficulties seriously only from the third form, where there are already multiple problems.

AbacoMath: Some modules for pre-school and home practice (numbers 0-20) can be found at the webpages of the Educational Authority by Marja Dräger and Anni Lampinen. http://www.edu.fi/verkko.oppimateriaalit/matematiikan

4. The way to abstraction

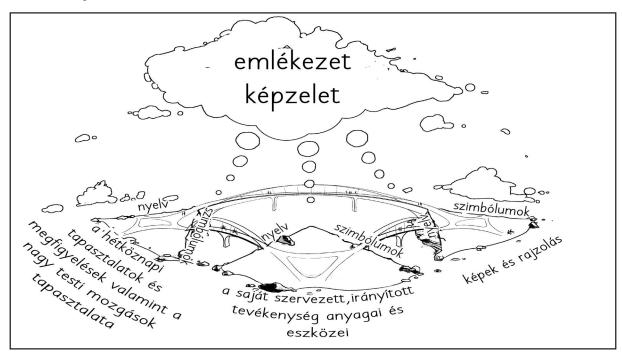


Figure 4. The way to abstraction

We have been thinking what clear, yet rich image of abstraction we shall give primary school teachers, which would help them planning. Finally we dropped anchor by this triangular island (there are three islands in the vertices, and they are interconnected).

- The first island is home to controlled games, like window-shopping, buying
 ice cream, doing everyday housework and so on. Finnish people like baking
 different pies and cakes, which they freeze and take to different charity fairs.
 Playing "birthday game" would certainly excite all children, etc. Gross motor
 skills, moving in space and using their own body as a tool belong to this island.
- On the second island, we put tools in front of each child, and they can explore the new topic by detecting it manually. Among the tools, naturally, there are logic shapes, Cuisenaire rods and many other hand-made or purchased tools.
- On the third island, the children get acquainted with the topics using pictures they get or prepare in advance.

We talk a lot on every island, and we may use the symbols of mathematics everywhere. The islands are interconnected by bridges. The children are enriched with a lot of memories and logical-mathematical experience on each island. These mental images, stored in their imagination together with their verbal representation make a firm base for further mathematical thinking and abstraction.

5. A broad and unified foundation of mathematics

At the first school year, traditional Finnish mathematics teaching begins with writing numbers and some rather short exercises on number concept: quantity, linking numbers and what they represent, relations like more or less, and over all, practising addition and subtraction. Counting strategies are hardly mentioned. Pupils are expected to calculate up to a hundred by the end of the first school year, and up to a thousand by the second. Teaching written addition and subtraction, as well as teaching the multiplication table consumes a lot of time in the second form. This shows that mathematical thinking was thought to be developed by improving computational techniques.

For primary school teachers, who are used to the traditional method, it is new that there is a slow initiation; that they first have to make sure that all children have the basis for learning mathematics on the level of skills; they need to teach children make observations, think diversely, and they have to improve children's concentration span and memory. Instead of quickly turning to a written routine, we first have to establish the number concept of children, we have to make them do a sum in their heads regularly, and we have to deal with other fields of mathematics at the same time. At our teacher training courses, we use a simile: In Finland, we are used to knitting neat stripes of mathematics – we use only one colour at a time and make students only imitate our technique so that they will be able to use it skillfully, and we think they have learnt it. On the contrary, the Varga-Neményi method knits a multi-coloured pattern with various colours at the same time. Teachers have to know colours and patterns in a way that they become able to help children "knit their own mathematics". This way, the method, which first may seem to be confusing, becomes clear and consistent, while it teaches children mathematics and elaborates learning strategies in them.

These are the reasons why Finnish teachers can initially be taken aback by the method. "Why are we doing this? What is it good for? This rather belongs to preschool, doesn't it?" We answer these questions with the question: Do we want to teach mathematics or calculating techniques? If we opt for teaching mathematics, our teaching practice will become varied and richer. Our primary aim at teacher training courses is to introduce teachers to the structure of the curriculum.

The Finnish National Curriculum does not spend a word on combinatorics, logics or sets at all among topics necessary to deal with in the first two years of school. To be honest, logics and sets as words do not occur in the curriculum of upper forms either. In the 70s, sets were part of the curriculum, moreover, learning mathematics began with learning sets. We had to give it up quickly, as teaching methods were too abstract and the teaching material itself was too formal then. Bad teaching, which followed partly from the poor mathematical education of the teachers at the time, made sets discredited in teaching mathematics for years. Apart from this unfortunate failure, teaching division was inconsistent, properties of operations have never been incorporated into the curriculum, not to mention numeral systems. They were invisibly there by the discussed contents, and there were some pupils and teachers who noticed them. But teaching has always been a rush, and there was no time to estimate the significance of the shortcomings of the foundation work or the effect of these shortcomings on building the "Fortress of Mathematics". The motto of this constant hurry was: "We have to finish everything that next year we won't have time to do."

Omission of important mathematical topics from the national curriculum also means that Finnish lower primary teachers do not have personal experience from their childhood about learning these topics, nor have they sufficient professional knowledge to teach them. Therefore, during our teacher training courses besides making teachers acquainted with an innovative pedagogical approach, we broaden and refine their knowledge of mathematics. Teaching the topics above requires more time, greater expertise and more thorough work.

6. The right to be wrong, to dispute and to be delighted

How many mistakes, how much fun can be in a single lesson or a day of a course! Look, learning mathematics is not silent drudgery, not even a job for the bright only. This is where Finnish teachers have the most to learn. But it is exactly this what makes learning enjoyable and fun!

7. The teacher and teaching mathematics

The sixth and the seventh principles are inextricably intertwined.

One of the most important purposes of our teacher training courses is to evoke mutual interest and trust among teachers. It is not only that teachers have the opportunity to talk, it is rather a condition of successful work. This way, even unsure questions and presuppositions, whatever awkwardly they are verbalised, can be set before the whole group to think about them. When someone dares to ask a question, there are probably many who have already been considering asking it.

During the courses, the growth of teachers' courage and confidence in teaching mathematics is gratifying to see, not only for the participants themselves, but for instructors of Varga-Neményi Company as well. We find those moments especially fruitful when a statement of the instructor is challenged; discussing pros and cons shows that teachers are about to be freed from their former wrong concepts that only thinkers who produce one true answer quickly can be valued, even as a person. At this point they are on the way to value a proof or justification of a statement, as well as the diversity of our thinking as our common treasure. These moments make the instructor believe that the participants are becoming better teachers, in the sense of both mathematics and personal wisdom.

Due to the trusting, benevolent and tolerant atmosphere during the meetings, teachers make friends with each other, and keep in touch after the course as well. There are schools all over the country where mathematics is taught exclusively with the Varga-Neményi method in pre-school and the first two grades. Thus, having participated in our courses is an advantage for the teacher when applying to these schools. This is the nicest appreciation of our work!

By increasing teacher's professional knowledge, their self-confidence also increases, as they realize how important it is to make the foundations in pre-school and the first two years to be able to build mathematics on it later. It is no longer their greatest concern what mark they should give for a test, according to the table of marks. When finding an error, they don't just say "it is inattention"; they want to know exactly what and to what extent children know at that moment, what deficiencies hinder them in learning the next topics and how to make each and every student capable of progress. They ponder how much time, resource and professional knowledge is available which can be rearranged to solve learning difficulties even in the present difficult economical situation. We are obliged to do it by the 2013 SEN legislation, which introduced a three-step model to arrange helping students according to their special needs. The theoretical background of the Varga-Neményi method is great help for us in understanding lesson management and the structure of teaching mathematics.

The new syllabi are to be implemented on 1st August 2016. This March, professional groups prepared their plans which are going to be revised and modified as recommended. There was special attention paid to the syllabus of the first two forms. We think these changes are leading us to the right direction. Many teachers applying the Varga-Neményi method will take part in these developments in local groups. At the moment, we are excited to see the preparations of the basis of the national curriculum. What makes our work increasingly interesting is the fact that according to the international agreement of children's rights, their opinion will have to be consulted in each phase of the preparation of the syllabus.

The description of the method, bibliography, reports on the Company's activities and events, as well as information on seminars and courses can be found at our web-

page: www.varganemenyi.fi. There we also provide teachers with ideas and photocopiable materials for free.

After finishing the schoolyear we are going to hold the traditional two-day seminar of the Company in Kuopio – it is going to be the eighth this year! There are going to be lectures and free discussion, opportunities for teachers to meet other teachers teaching the method, brainstorm, exchange ideas and experience, take part in DIY, swimming and sauna – the best beginning for the well-deserved summer holiday!

Finland is the country of great distances: the greatest distance between two teachers of the Varga-Neményi method can be up to 1200 km. We not always find an appropriate teaching partner in our schools, but social media, especially Facebook have been a very suitable forum where we can share the joy and difficulties of teaching, load up photos, ask and give advice freely. Our Facebook group already counts about 800 members.

Educational issues attract the attention of the media as well, especially when the results of PISA assessments are released. In recent years, mainly in local newspapers, many articles have been published on schools and teachers using the Varga-Neményi method. Radio and TV interviews with the President of the Company have created a nationwide audience for this important issue. From the very beginning we enjoy an enthusiastic and positive attitude from the media.

One of the greatest tasks for the future is to stimulate research work which analyses the method. Pirjo Tikkanen wrote her dissertation on this topic in 2008 - "It is easier and more entertaining than I have thought. Finnish and Hungarian fourth graders about mathematics". Some theses appear every year as well, but still there are many exciting topics to discover. And, let's not forget about this, the third and fourth grade teaching materials will have to be reformed to our Finnish way. The work has begun, and there's an atmosphere of great expectations.

We would like to express our sincerest gratitude to the Finnish General Education Authority for the moral and financial help which have made the dissemination and teaching of the Varga-Neményi method possible for years. Without training teachers, we would not get anywhere. Here in Hungary, we would like to thank you all, individuals and institutions alike, what we have got from you; that you enrich our thinking and that you have introduced us to this magical world of teaching and learning mathematics. We could visit your schools and observe your lessons where we have always been most welcomed. You came to visit us in Finland to teach us, putting off your other important jobs. Apart from sharing your knowledge and experience with us, you also became true friends with us. You are always in our hearts and minds, when we teach, talk and prepare teaching materials and tools. You can't even imagine what celebrities you are in our Finland!

With respect and gratitude, in memoriam Julianna Szendrei. Varga-Neményi Company Anni Lampinen and Kirsi Puumalainen