Granular Jets

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Worthington jets[1] are a familiar sight in light rain upon puddles and ponds. These narrow vertical jets are formed by the radial collapse of the liquid *"craters"* produced by the impacting rain drops[2–6]. Such jets can also be generated by super-critically forcing the standing Faraday-waves[7–8] on a liquid surface and have recently been cast in the formalism of physical singularities[2,3] to investigate the role of the inertial focusing and the influence of surface tension on their strength. Zeff *et al.*[2] propose that during the collapse of the free-surface cavity the surface develops a curvature singularity which is dominated by inertia and surface tension, hence, generating high-energy vertical jets. We have *discovered* that similar narrow jets occur even for granular materials, where surface tension is absent. This new phenomenon suggests that a singularity in the surface tension

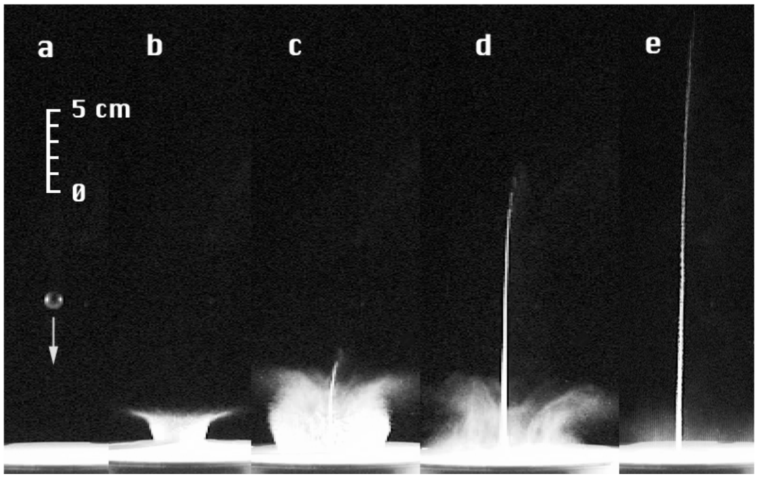

Figure 1. Granular jet generated by the impact of a lead sphere ($U = 5.5$ m/s, $D_b = 1.34\ cm$, $\rho_b = 11.5\ g/cm^3$) onto a deep layer of granular medium, consisting of glass spherical beads 0.079 mm in diameter. The medium sits in a circular glass container 12.2 cm deep and 18.8 cm in diameter. **a**, $t = 18$ ms; **b**, $t = 25$ ms; **c**, $t = 92$ ms; **d**, $t = 160$ ms; **e**, $t = 260$ ms. The upwards speed of the granular jet is about one third of the sphere impact velocity.

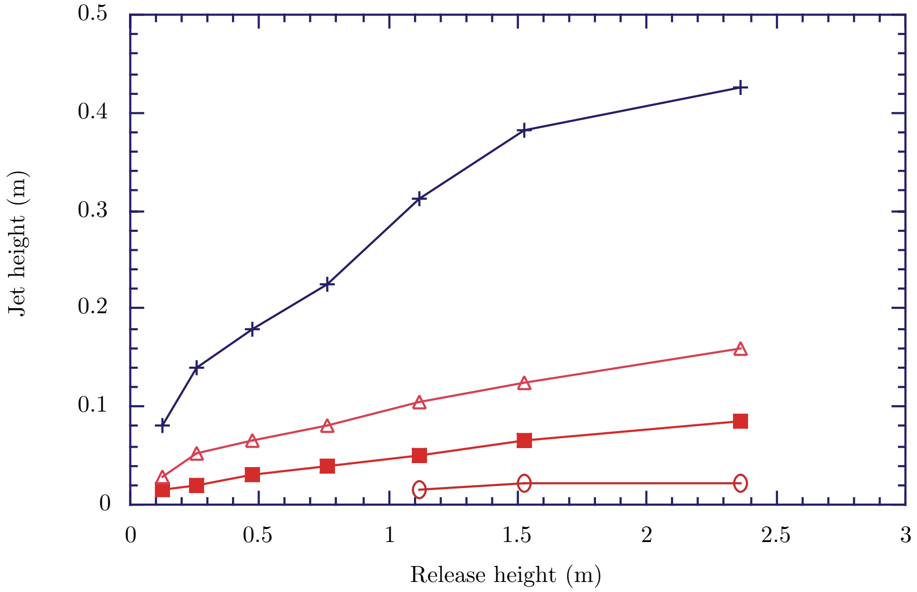

Figure 2. The maximum height attained by the granular jet versus release height of lead sphere. Granular media consist spherical glass beads with four different mean diameters: d_s = 0.080 mm (+), 0.118 mm (△), 0.176 mm (■) and 0.275 mm (◯). The glass has density of 2.48 ± 0.05 g/cm^3. The material was poured slowly between two identical glass containers and leveled using a metallic ruler dragged over the surface. This was done before every impact, to avoid any effects of compaction, which can greatly alter the results for the finest grains.

force is not needed to produce such jets and raises the question whether the inertial focusing is the sole mechanism. The sequence in Figure 1 shows the generation of a granular jet resulting from the impact of a solid lead sphere onto a deep flat layer of granular medium consisting of spherical glass beads. Similar approach was used by Hogrefe *et al.*[4] to generate jets in the liquid case[9]. The impacting sphere produces a deep cylindrical cavity in the sand, which subsequently collapses radially under the gravity-induced "hydrostatic" pressure. The sand converges axisymmetrically towards the center of the cavity. Due to the relatively small compressibility of the granular medium, the radial velocity diverges as $1/r$ when the cavity closes up. This inertial focusing produces a large dynamic pressure-spike driving up the sand in a narrow jet along the axis of symmetry. The granular jets are quite narrow, being comparable in shape to the energetic liquid jets. The jet in Figure 1.d is however about 40 grain-diameters wide, where the grain size clearly would provide an ultraviolet cutoff in this process, thus supporting a continuum viewpoint of the flow. The porosity of the sand-medium may result in an even more pronounced inertial focusing than in the liquid case, as the gas caught on the axis of symmetry can escape between the grains. The tuning of the granular jet does not depend on the layer depth[5] as in the fluids case, however, the granular jet height is strongly dependent on the grain size of the granular medium (see Figure 2). We get the

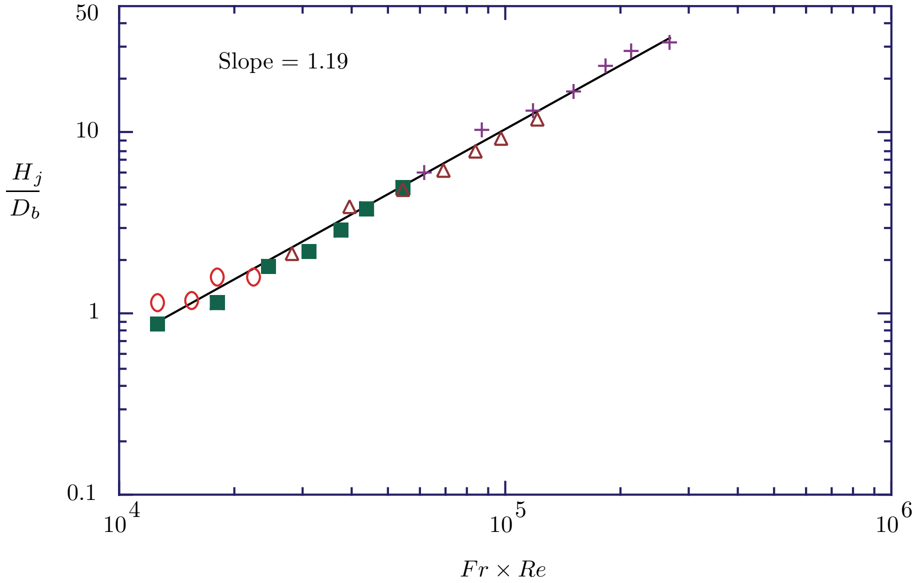

Figure 3. Dimensionless plot of the maximum height attained by the granular jet, versus jetting parameter $Fr \times Re$. Granular media consist spherical glass beads with four different mean diameters: d_s = 0.080 mm (+), 0.118 mm (△), 0.176 mm (■) and 0.275 mm (◯).

highest jets using the finest granular media for the same impact velocity.

We should mention that vertically vibrated thin granular layers have been shown to develop *"oscillons"* which are heaps emanating from the surface after the closing of craters[10], but these are much less energetic than the jets studied here.

Our experiments will be useful in separating the effects of surface tension and inertial focusing[11] on the tuning of singular fluid jets. More importantly, they give insights into the constitutive properties of flowing granular media at high shear-rates. The quantity we use to characterize the "singularity" or jetting event is the maximum height attained by the jet H_j. The absence of surface tension makes simple dimensional analysis more pallatable here than in the liquid case. The only other physical quantities of importance are: the sphere diameter D_b, the impact velocity U, gravity g, along with material properties: the density of the sand ρ_s and the sphere material ρ_b, and finally the effective viscosity of the granular media μ_e. Here we assume that the grain-size d_s only enters the problem through the effective viscosity. The dimensional analysis shows that the jet height should follow an unknown function of only 3 dimensionless parameters:

$$H_j/D_b = \Phi(r_\rho, Re, Fr)$$

i.e. a density ratio $r_\rho = \rho_b/\rho_s$, Reynolds number $Re = \rho_s U D_b/\mu_e$ and a Froude number $Fr =$

$U/\sqrt{gD_b}$. We have studied this relation by keeping r_ρ constant while varying U and μ_e independently, as shown in Figure 3. The effective viscosity of flowing granular media remains an active topic of research[12–17] and is far from fully characterized. We use the results of Savage *et al.*[13–15] based on their shear cell experimental results and the kinetic theory of granular materials,

$$\mu_e \simeq 2\rho_s d_s^2 U^2 / D_b^2$$

which produces a good collapse of the data (see Figure 3). This shows clearly that gravity, inertia and viscous forces all play a role in the granular jetting.

The absence of remnants from granular jets in lunar and planetary craters, is probably due to the much higher material strength during those impacts[18].

High-speed video clips showing the granular jet can be viewed at our web site
http://www.tam.uiuc.edu/Faculty/Thoroddsen/GranularJet.html

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