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The fragility of planetary systems

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ABSTRACT
We specify the range to which perturbations penetrate a planetesimal system. Such perturbations can originate from massive planets or from encounters with other stars. The latter can have an origin in the star cluster in which the planetary system was born, or from random encounters once the planetary system has escaped its parental cluster. The probability of a random encounter, either in a star cluster or in the Galactic field depends on the local stellar density, the velocity dispersion and the time spend in that environment. By adopting order of magnitude estimates, we argue that the majority of planetary systems born in open clusters will have a *Parking zone*, in which planetesimals are affected by encounters in their parental star cluster but remain unperturbed after the star has left the cluster. Objects found in this range of semimajor axis and eccentricity preserve the memory of the encounter that last affected their orbits, and they can therefore be used to reconstruct this encounter. Planetary systems born in a denser environment, such as in a globular cluster are unlikely to have a Parking zone. We further argue that some planetary systems may have a *Frozen zone*, in which orbits are not affected either by the more inner massive planets or by external influences. Objects discovered in this zone will have preserved information about their formation in their orbital parameters.

Key words: minor planets, asteroids: general – planets and satellites: dynamical evolution and stability – planets and satellites: individual: Sedna, 2012VP$_{133}$ – planet–star interactions – stars: individual: WISE J072003.20-084651.2.

1 INTRODUCTION

Planetary systems seem to be composed of one or more stars, orbited by about a dozen planets and many minor bodies (Galilei 1632). The latter can be roughly divided into hundreds of moons and dwarf planets, and many millions of planetesimals. The objects closer to the star seem to be organized in a disc-like structure in which also the planets reside, and which flares to become spherical at larger distance from the stellar host.

This view of planetary systems is heavily based on the Solar system (see the review Adams 2010), but so far its generality cannot be excluded, because each of the several thousands planetary systems known today (Howard 2013) seem to comply to this characteristic. This view is also supported by our limited understanding of the formation of planetary systems (Kokubo & Ida 2002; Bouwman et al. 2008; Ida, Lin & Nagasawa 2013).

The orbits of planets and minor bodies within a few stellar radii are affected by tidal evolution (Zahn 1977). Once the star leaves the main sequence, copious mass-loss starts to affect the entire planetary system (Veras et al. 2011). We refrain from discussing these complexities here, but concentrate on the dynamically affected regime: sufficiently far away and sufficiently early in its evolution to remain unaffected by the stellar host.

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The effect of perturbations on the Solar system either from the local planets, external perturbations from the birth cluster or even from the Galaxy have been studied quite extensively (for a few recent studies see e.g. Kaib & Quinn 2008; Brasser et al. 2012; Schwamb 2014, and references therein). We start with a discussion on the Solar system in Section 2 and generalize in Section 3.

2 PERTURBING THE SOLAR SYSTEM

2.1 The effect of internal perturbations from the planets

Apart from mass-loss and tidal evolution the inner regions of the Solar system are most strongly affected by dynamical interactions with the giant planets. We can calculate the range of dynamical reorganization caused by the widest massive planet in a planetary system by adopting its apocentre distance. For the Solar system, this range is currently determined by the planet Neptune, which affects the orbits of minor bodies to a distance $a \gtrsim 30$ au. Within this distance the planets in the Solar system have caused major changes in the orbital distributions of the minor bodies (Levison et al. 2008). We could argue that within a distance of 30 au/$(1 - e)$, the minor bodies in the Solar system quickly lose memory of the mechanism that brought them in these orbits. Here, e is the eccentricity of the planetesimal.

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2.2 The effect of external perturbations from the parental star cluster

We adopt the view that all stars are born in a clustered environment (Lada & Lada 2003). The densities of these environments vary enormously from $\sim$1 star pc^{-3} to more than 10^6 stars pc^{-3} (Portegies Zwart, McMillan & Gieles 2010). The time that the Solar system with minor bodies remains in the environment determines the degree by which it is dynamically affected by stellar encounters (see the review by Davies et al. 2006). Those encounters play a major role in the evolution of these environments. The degree by which the planetary system is affected by such encounters depends on the duration and the intensity of the exposure. Further internal reorganization of the perturbed planetary system enables the external perturbations to propagate, generally on a much longer time-scale, to the inner parts of the planetary system.

The way in which a planetary system is affected by dynamical encounters depends on the mass of the encountering star, its impact parameter with respect to the host star, the velocity v and the direction with respect of the planetesimal disc (Steinhausen & Pfalzner 2014). This complicated combination of parameters and their mutual relations in terms of the degree of perturbations for the planetary system can be summarized in a cross-section $\langle\sigma\rangle$.

Most important for preserving the integrity of the planetary system is its eccentricity. Here, we adopt a lower limit to the perturbation of the eccentricity of $\delta e = 0.1e$ to be fatal for the particular planetesimal. The cross-section of eccentricity perturbation of a planetary system around a star with mass m, through an encounter with another star of mass M has been estimated by means of integrating small-N systems and averaging over the various encounter angles (Li & Adams 2015):

$$\langle\sigma\rangle = (1 - f_{\rm b})\langle\sigma_{\rm single}\rangle + f_{\rm b}\langle\sigma_{\rm binary}\rangle. \quad (1)$$

Here, $\sigma_{\rm single}$ is the cross-section for encountering a single star and $\sigma_{\rm binary}$ is for binaries. The parameter $f_{\rm b}$ is the binary fraction, which is between 0 and 1. We adopt $f_{\rm b} = 0.5$. In the adiabatic regime where the encounter velocity is comparable to the orbital velocity and which is suitable for encounter in star clusters, both cross-sections have a similar form (Li & Adams 2015, with subscript X = single or X = binary depending on the configuration of the encountering object):

$$\langle\sigma_{\rm X}\rangle = \sigma_0 \frac{a}{[\rm au]} \left(\frac{m}{[{\rm M}_\odot]}\right)^{-1/3} \left(\frac{v}{[{\rm km\,s^{-1}}]}\right)^{-\gamma} \exp\left(b(1 - e_{\rm f})\right). \quad (2)$$

Here, v is the relative velocity for which we adopted the velocity dispersion in the cluster and a is the orbital semimajor axis of the planetary system before the encounter. The post-encounter eccentricity, $e_{\rm f} = e + \delta e$. Equation (2) was calibrated for initially circular orbits, but we apply them here also for eccentric orbits. According to Li & Adams (2015), the cross-sections do not depend much on the pre-encounter eccentricity (but see Heggie & Rasio 1996).

The parameters σ_0, b and γ depend on the binarity of the encountering object. For a single star $\sigma_0 \simeq 1000$ au^2, $b = 8/5$ and $\gamma = 6/5$, whereas for a binary $\sigma_0 \simeq 4050$ au^2, $b = 4/3$ and $\gamma = 7/5$ (see Li & Adams 2015). With this cross-section, the local stellar density n and the velocity dispersion $\langle v\rangle$ we can calculate the encounter rate:

$$\Gamma = n\langle\sigma\rangle\langle v\rangle. \quad (3)$$

In Fig. 1, we present the expected value for the number of encounters in the Sun's parental star cluster. Here, we adopted the cluster parameters derived by Portegies Zwart (2009): a total mass of about $M_{\rm cl} = 2 \times 10^3$ M$_\odot$ and a virial radius of 2 pc, which result in a stellar density of about 100 stars pc^{-3} and a velocity dispersion of $\langle v\rangle \simeq 2$ km s^{-1}. The lifetime of a star cluster in the Galactic disc can be estimated from (Lamers, Gieles & Portegies Zwart 2005):

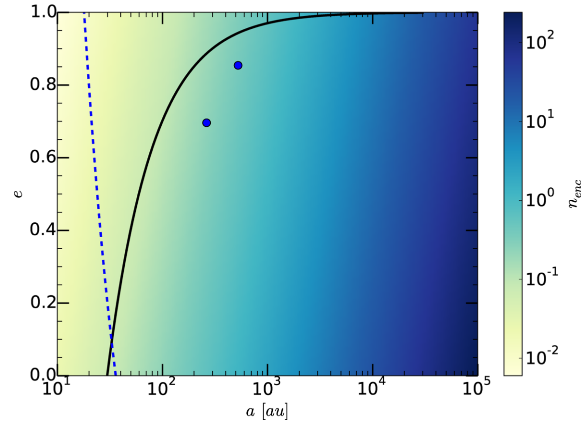

Figure 1. Fragility of the Solar system while still a member of its birth cluster. The shades represent the probability distribution $n_{\rm enc}$, for the number of encounters that perturb the Solar system at a given semimajor axis a and eccentricity e. For the birth cluster, we adopted the mass of 2×10^3 M$_\odot$ and virial radius of 2 pc to remain constant over a time-scale of 200 Myr. The solid black curve gives the semimajor axis up to which Neptune can perturb orbits. The blue dashed curve gives the distance to which star Q (Jilkova et al. 2015), that passed the Sun with an impact parameter of 320 au, has perturbed the Solar system. According to the colour scaling along the right edge of the figure, such a close encounter could have occurred roughly once while the Solar system was a member of the star cluster. The two bullet points give the orbital parameters of Sedna (Brown, Trujillo & Rabinowitz 2004) and 2012VP$_{113}$ (Trujillo & Sheppard 2014).

$$t_{\rm cl} = 2.24\,{\rm Myr} \left(\frac{M_{\rm cl}}{[{\rm M}_\odot]}\right)^{0.60}. \quad (4)$$

For the star cluster in which the Sun was born this results in a lifetime of about 200 Myr.

Our adopted cluster lifetime and the assumption of a constant density within this period are very approximate. However, our intention is to estimate the relative importance of experiencing an encounter in the parental star cluster or in the Milky Way Galaxy, for which this approach suffices. In order to estimate the importance of these assumptions we integrated the cluster mass and radius evolution from the simulations by Portegies Zwart et al. (2001). They aimed their simulations at mimicking Pleiades, Praesepe and Hyades, which have comparable initial conditions as the Solar birth cluster. According to these simulations, clusters with such parameters survive for as long as a Gyr during which the cluster mass drops linearly with time. In this period the cluster expands by about a factor of 3, roughly proportional to the square root of time. With this slightly more elaborate estimate we argue that we overestimate the exposure of encounters in the star cluster by about a factor of 2. This would be consistent with adopting 100 Myr for the cluster exposure calculation in Fig. 1, rather than 200 Myr.

The distance to which these perturbations induced by close encounters penetrate into the Solar system, depends on the distance of closest approach $q_{\rm enc}$, and the eccentricity of the encounter $e_{\rm enc}$, and can be expressed in (Kobayashi & Ida 2001)

$$a(1 + e) \simeq \left(\frac{1}{5}\right)^{2/3} q_{\rm enc} \left((1 + M/{\rm M}_\odot)(1 + e_{\rm enc})\right)^{-1/3}. \quad (5)$$

Recently, Jilkova et al. (2015) argue that the planetesimals Sedna (Brown et al. 2004) and 2012VP$_{113}$ (Trujillo & Sheppard 2014)

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were captured by a close encounter from a $M \simeq 1.8$ M$_\odot$ star that passed the Solar system with a closest approach of $q_{\rm enc} \simeq 227$ au and relative velocity $v \simeq 4.3$ km s^{-1} (which corresponds to an eccentricity $e_{\rm enc} = 2.6$). According to equation (5), such an encounter would have perturbed the Solar system to a distance of about 36 au. In Fig. 1, we present the distance to which such and encountering star perturbs the planetesimal around the Sun. For completeness we include the two objects Sedna and 2012VP$_{113}$ to indicate how dramatically the encounter with star Q perturbed the Solar system.

2.3 The effect of external perturbations from the Galactic encounters

Once the parental star cluster dissolves, the planetary system can only be perturbed by internal reorganization, and by the Galaxy. This latter can be subdivided in a global perturbation from the slowly varying Galactic tidal field, but the occasional close encounters with field stars are more important (see however Fernández 1997).

The probability of an encounter with a Galactic field star is much smaller than of a close encounter in a star cluster, but the lower encounter rate is compensated in part by the longer time spent in the relatively low-density environment of the Galaxy compared to the time spent in the star cluster.

For estimating the effect of an encounter with a field star we cannot simply adopt equation (2) because this is tuned for low-velocity (and low-eccentricity) encounters, whereas Galactic encounters tend to occur with a much higher velocities. We therefore adopt the classic gravitational focused cross-section (Binney & Tremaine 1987)

$$\sigma = \pi a^2 \left(1 + \frac{2G(m+M)}{a v_{\rm enc}^2}\right), \quad (6)$$

for calculating the encounter rate between the Solar system ($m = 1$M$_\odot$) and another Galactic star of mass M to a semi-major axis a.

To provide an upper limit to the effect an encounter has on the orbital parameters of the planets or planetesimals we assume that the perturbed object and the closest approach are aligned. The heliocentric impulse gained by the object at distance r from the Sun is then given by (Rickman 1976)

$$\Delta v = \frac{2GM}{v_{\rm enc}} \frac{r}{q_{\rm enc}(q_{\rm enc} - r)}. \quad (7)$$

Here, we assumed that the relative velocity, $v_{\rm enc} = 30$ km s^{-1}, remains constant during the encounter and the mass of an encountering star $M = 0.5$ M$_\odot$. The perturbation is effective at aphelion (i.e. $r = a(1 + e)$) and we estimate the distance to which the perturbation penetrates the Solar system at the point where the impulse gained by the object is comparable to its velocity at aphelion. This is a rather arbitrary choice, but suffices to indicate to which distance a passing field star may have affected planetesimals in the Solar system.

In Fig. 2, we present the number of encounters that perturbed the Solar system by a passing Galactic disc star, after the parental star cluster has been dissolved. The probability distribution is calculated using equation (3) and adopting $n = 0.20$ stars pc^{-3} and a velocity dispersion in the local standard of rest of $v = 30$ km s^{-1} (Holmberg, Nordström & Andersen 2007), and the time the Solar system spent in the Galaxy, $t_{\rm Gal} = 4.3$ Gyr. For comparison with Fig. 1, we include the curve for planetary perturbations (solid black curve) and the two objects Sedna and 2012VP$_{113}$.

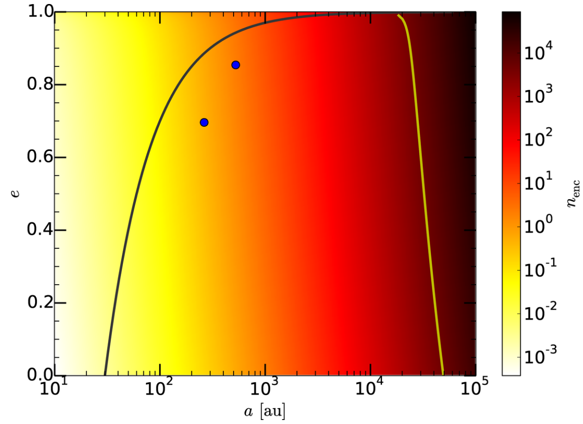

Figure 2. The fragility of the Solar system in the Galactic field. The shades represent $n_{\rm enc}$: the probability density distribution for the number of encounters that affect the orbits in the Solar system to a given semimajor axis a and eccentricity e. Here, we adopted a stellar density of $n = 0.20$ stars pc^{-3}, a velocity dispersion of $v = 30$ km s^{-1} and the typical mass of an encountering star of $M = 0.5$ M$_\odot$. The solid black curve and the two bullet points give the Neptune's perturbing distance, and the orbital parameters for Sedna and 2012VP$_{113}$, see also Fig. 1. The yellow curve gives the distance to which the Solar system was perturbed (by 1 per cent of its velocity at aphelion) due to the recent encounter with the 0.15 M$_\odot$ binary star WISE J072003.20−084651.2, which grazed the Solar system, but clearly did not come in close enough to perturb the inner Oort cloud.

Recently the Solar system had a close encounter with the $M \simeq 0.15$ M$_\odot$ binary star WISE J072003.20−084651.2 (nicknamed 'Scholz's star' after its discoverer Scholz 2014) at a distance of $q = 0.25^{+0.11}_{-0.07}$ pc (Mamajek et al. 2015). We estimate the perturbation by this encounter using the impulse approximation (Heggie 1975; Rickman 1976), in which the duration of the encounter is assumed to be much shorter that the period of the perturbed orbit (opposite to the adiabatic regime in which equation 5 is valid). The impulsive approximation is fulfilled for the encounter with Scholz's star, which had a relative velocity of $v_{\rm enc} = 83.2$ km s^{-1} (Mamajek et al. 2015). We indicate this distance by the yellow curve in Fig. 2.

According to our analysis such an encounter (at $q_{\rm enc} \sim 5.2 \times 10^4$ au) should occur ∼5000 times during the ∼4.3 Gyr sojourn of the Solar system through the Galactic disc. Such encounter is therefore quite a likely event, which occurs roughly once every million years. The close approach of Scholz's star occurred only 70 000 yr ago (Mamajek et al. 2015), which seems amazingly recent. If we naively divide the two time-scales, such an encounter should already have happened ∼60 000 times.

The effect of this particular encounter has hardly perturbed the Oort (1927) cloud down to a distance of 10^5 au from the Sun. But an encounter with an equally low-mass star three orders of magnitude closer in would have affected the outer most planets. With the derived probability distribution, equation (3), using equation (6) for the cross-section and equation (7) to estimate the distance to which such an encounter affects the Solar system, such an encounter would be very unlikely to happen over the lifetime of the Solar system.

Considering the analysis of Jilkova et al. (2015), the encounter that introduced the Sednitos (a family of planetesimals with orbits similar to the 2003VB$_{12}$ Sedna) into the Solar system occurred in the parental cluster, and since then no other stellar encounter has perturbed Edgewordt–Kuiper (Edgeworth 1943; Kuiper 1951) belt. This picture is consistent with the presence of a *Parking zone* (see Section 3.1) in the Solar system between about 100 au and 1000 au;

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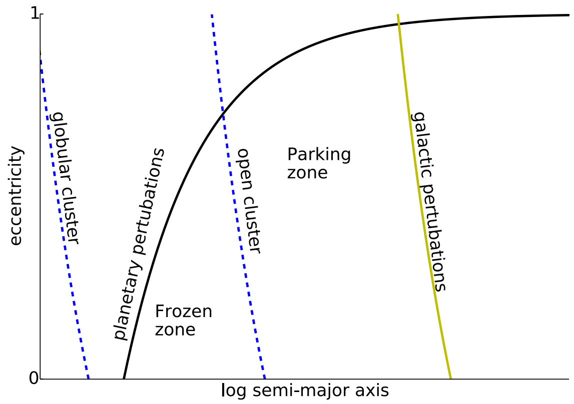

Figure 3. The fragility of a planetary system under the influence of interaction with stars in its parental cluster and a galaxy. The horizontal axis has no scale, because the range to which the effect penetrates depends on the parameters of the planetary system and the environment. The black, yellow and blue curves give the ranges to which various perturbations affect the orbits of the minor bodies. Specific example of the Solar system are presented in Figs 1 and 2. We identify the Frozen zone and the Parking zone.

encounters that affect the Solar system to a distance within 1000 au are extremely rare.

The Parking zone in the Solar system is populated by the Sednitos (Jilkova et al. 2015); planetesimals that share a common argument of pericentre, inclination and perihelion distance. We argue that the Parking zone in the Solar system extends from about 100 au to the about ∼1000 au, near the outer boundary of the inner Oort cloud. The distribution of orbital parameters of planetesimals discovered in this regime bear the information of the last strong encounter the Sun experienced from the time when it was part of its parental cluster.

3 PERTURBING PLANETARY SYSTEMS IN GENERAL

In Fig. 3, we present a generalized view of the fragility of planetary systems. The solid black curve gives the range to which possible massive planets perturb the inner parts of the planetary system. Violent planet scattering can cause massive planets to migrate further outwards, where they can perturb the local planetesimals (Chatterjee et al. 2008). In that case, the solid curve will shift to the right.

The parental star cluster perturbs its planetary systems, but the further evolution of the planetary system determines to what range such a perturbation is preserved over time. For a globular cluster the stellar density is generally higher than for an open cluster, and the probability of spending a prolonged period in a globular cluster is also larger. As a result the range to which the planetary system is perturbed when born in a globular cluster is much closer to the star than for an open cluster. In fact, from the schematic picture (Fig. 3) the influence of random encounters while the member of a globular clusters penetrates all the way to the inner planets. As a consequence, planetary systems in globular clusters, or other massive dense star clusters, are likely to be perturbed by internal as well as external effects. This may explain, in part, the lack of observed planets in globular clusters (Weldrake, Sackett & Bridges 2007).

If born in a low-density cluster with a relatively short lifetime, the range to which random encounters penetrate into the planetary system hardly reaches the influence range of the giant planets. Once the planetary system escapes the star cluster, the perturbations from the Galaxy start to affect the orbits of the objects. This influence prolongs for the remainder of the main-sequence lifetime of the parent star, after which stellar evolution starts to play a major role in the redistribution of the orbits.

3.1 The Frozen and Parking zones

We define the Parking zone as a range in semimajor axis and eccentricity in which the orbits of objects have only been affected by encounters in the parental star cluster, and not by the local planets or by the Galaxy.

Objects that orbit in the Parking zone have therefore not been affected directly by the planets and remain unaffected by close Galactic encounters. The Parking zone is likely to shrink with time, and young stars tend to have a more extended Parking zone than older stars, due to the less prolonged exposure to Galactic encounters.

Objects with orbital parameters in the Parking zone preserve information about the last event that affected their orbits in the planetary system. Once in the Parking zone, orbital parameters are unlikely to be affected either by the planets, because they only affect orbits closer to the star, or by random Galactic encounters, because they tend to affect the outer most regions of the planetary system. Objects found in the Parking zone can therefore be used as tracers to reconstruct the event that introduced them in their current orbits.

To the left of the Parking zone, and to the right of the range to which planets perturb the orbits we recognize the *Frozen zone*.

We define the Frozen zone as a range in semimajor axis and eccentricity in which the orbits of objects have not been affected by the local planets and not by any encounters, in the parental star cluster or the Galaxy.

In this zone minor bodies remain unaffected by either internal influences or external perturbations. Planetesimals found in this regime will preserve information about the formation of the planetary system.

In the Solar system, the Frozen zone is probably very small or completely absent (see also Fig. 1). But other planetary system may have a populated Frozen zone, which can be used to study their origin.

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