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A New Hammerstein Model for Non-Linear System Identification

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Abstract— In the present work a newer type of black box non-linear model in Hammerstein structure is proposed. The model has Wavelet Network coupled with Orthonormal Basis Functions which is capable of modeling a class of non-linear systems with acceptable accuracy. Wavelet basis functions have the property of localization in both the time and frequency domains which enables wavelet networks to approximate severe non-linearities using few number of parameters. Orthonormal Basis functions possess the ability to approximate any linear time invariant system using appropriate basis functions. The efficacy of the model in modeling is demonstrated using numerical examples.

I. INTRODUCTION

System identification is an important field of science with its applications spanning several industries such as aerospace, power, civil engineering, and naval engineering. It involves the use of statistical methods to build mathematical models of dynamical systems from measured data. Black box modeling is an elegant system identification technique that is employed when there is no a priori knowledge or insight about the system except the input output data of the system. In this type of modeling, using the input-output data a model is developed such that the model maps the input space to the output space of the system with the best possible accuracy. Linear systems can be modeled perfectly with the help of existing models. However, modeling of non-linear systems is an open research problem.

Extensive study in the domain of non linear system identification has produced several models, the most important among them are the polynomial NARMAX (Non Linear Auto Regressive Moving Average with eXogenous inputs) model [1],[2] neural networks [3], fuzzy logic based models [4], neuro-fuzzy networks [5], wavelet multiresolution decompositions [6] and wavelet networks [7]. The common criteria for judging the efficacy of the model include model parsimony, ease of development of the model and the accuracy of the model.[8] All of these techniques are found lacking, when it comes to displaying all these traits simultaneously. The NARMAX model can gather global information of the system dynamics efficiently but fails to approximate local dynamics parsimoniously[1],[2]. Neural networks can identify the process dynamics efficiently, but

there is no rigorous way of determining the number of hidden layers and neurons in the network model[3]. Fuzzy logic based models can represent very complex non-linearities like discontinuities and jumps and saturations [4]. However, none of these models possess all the merits stated above.

Wavelet decomposition is gaining in popularity as a formidable tool in the field of signal processing.[9] Wavelet decomposition is essentially a decomposition of any function $f \in L^2(R^n)$ in terms of dilated and translated versions of a basis function, called the "mother wavelet". Due to the property of time and frequency localization of wavelet functions, severe non-linearities can be approximated in an efficient manner. However, this property is not effective for approximating low-order non-linearity in a function.

Zhang and Benvesite [7] discovered the similarity between wavelet decomposition and single-hidden layer neural network and wavelet network was developed. In this wavelet network, non-orthogonal redundant wavelets, called wavelet frames are used in discrete wavelet transformation to approximate functions with lesser number of terms. Although wavelet network can approximate any static non-linearity in an effective manner, the linear dynamic part of the system also has to be represented by appropriate means. The dynamic part can be integrated into the model in two ways - either the Hammerstein Model structure or the Weiner type structure. This paper advocates the use of a Hammerstein type model. Hammerstein Model structure consists of a static non-linearity followed by a linear dynamic portion. It can be used to model only a specific class of non-linear systems whose dynamic part can be considered as a Linear Time Invariant (LTI) system.

There are several orthonormal basis functions used for linear system identification such as the Laguerre filters, the Kautz filters and finite impulse response (FIR) filters. All of these fall in the category of generalized orthonormal basis filters (GOBF). [10] A transfer function of an LTI system can be represented as a weighted sum of these basis functions. [11] The poles and the weights of the basis functions can be estimated using iterative optimization methods which use either steepest descent or Newton's algorithms.[11],[13].

Orthonormal basis functions(OBF) can represent any linear dynamic system efficiently [10] ; likewise wavelet

networks can represent any static non-linear function with considerable accuracy. OBF can be used piecewise linear models for mildly non-linear system, but they fail when they are tried to model severe non-linear systems. On the other hand, wavelet network fails miserably for mildly non-linear systems. These demerits of OBF and wavelet networks are removed when they are used together in a Hammerstein Model. [8]

A variety of algorithms exist for identification of the Hammerstein model such as the cross-correlation algorithm developed by Hunter and Korenberg [16] and the separable least squares (SLS) optimization algorithm developed by Westwick and Kerney. [17] However all these methods use polynomials to approximate the static non-linearity. Highly non-linear systems require the use of high order polynomials which include oscillations. Accuracy of such polynomial approximations tends to degenerate at the edge of the data set, rendering the model to be unreliable. [18]

In this paper, a combination of wavelet network and Kautz filters in the Hammerstein model structure is used for black box modeling of non-linear systems. The model development is executed through learning algorithms based on gradient based optimization.

The paper is organized as follows. Section II presents a brief overview of the different kinds orthonormal basis functions (OBF) along with methods to select poles of OBF. Section III describes the wavelet network and the structure of the Hammerstein model. In section IV, the algorithm used for training of the wavelet networks is discussed. In Section V, an illustrative example is presented to compare the efficiency of the proposed model with models that exist in literature. It is followed by application to the model to a real physical system, namely the DC to DC boost converter. Finally, section VI addresses the conclusions.

II. ORTHONORMAL BASIS FILTERS

Orthonormal Basis Functions (OBF) [14] are a very general category of filters that admit a variety of real or conjugate poles. The commonly known FIR, Laguerre and Kautz filters are restrictive special cases of Orthonormal Basis Filters. [10] These bases are capable of representing almost all type of linear, causal and stable systems. The generalized OBF can be described as:

$$B_n(z) = \frac{z\sqrt{1-|\xi_n|^2}}{z-\xi_n} \prod_{k=0}^{n-1} \left(\frac{1-\bar{\xi}_k z}{z-\xi_k} \right) \quad (1)$$

where ξ represents the vector containing the poles of the filter. An elementary transfer function $G(z)$ for a stable and causal LTI system can be decomposed as:

$$G(z) = \sum_{n=0}^N g_n B_n(z, \xi) \quad (2)$$

where $\{g_n\}$ represents the set of Fourier coefficients, N is the truncating order. Such a system can also be described in discrete state space representation of the form: [14]

$$\begin{aligned} X(k+1) &= AX(k) + Bu(k) \\ y(k) &= \theta^T X(k) \end{aligned} \quad (3)$$

with X being the state vector of dimension $(N+1)$:

$$X(k) = [x_0(k) \ x_1(k) \ \dots \ x_N(k)]^T \quad (4)$$

where:

$$x_n(k) = Z^{-1} \{B_n(z, \xi)\} u(k) \quad (5)$$

A is a matrix of dimension $(N+1) \times (N+1)$ defined by:

$$A(p, q) = \begin{cases} \xi_{p-1} & \text{if } p = q \\ a(p, q) & \text{if } p > q \\ 0 & \text{if } p < q \end{cases} \quad (6)$$

where:

$$a(p, q) = (-1)^{p+q+1} \alpha_{p-1} (1 - \xi_{q-1} \bar{\xi}_{q-1}) \prod_{l=q+1}^{p-1} \alpha_{l-1} \bar{\xi}_{l-1} \quad (7)$$

B and θ are vectors of dimension $(N+1)$:

$$B(p) = (-1)^{p+1} \alpha_{p-1} \prod_{l=q-1}^{p-1} \alpha_{l-1} \bar{\xi}_{l-1} \quad (8)$$

$$\theta = [g_0 \ g_1 \ \dots \ g_N]^T \quad (9)$$

where:

$$\alpha_l = \sqrt{\frac{1-|\xi_l|^2}{1-|\xi_{l-1}|^2}} \quad (l > 0) \quad \text{and} \quad \alpha_0 = \sqrt{1-|\xi_0|^2}$$

$u(k)$ is the discrete input signal

$y(k)$ is the discrete output signal

A. Kautz filters

Kautz filters are a form of orthonormal basis filter realization obtained by choosing a pair of complex conjugate poles $(\beta, \bar{\beta})$ for the orthonormal basis functions. The result is the well known Kautz basis, whose z-domain representation is given as:

$$\begin{aligned} \Psi_{2m}(z) &= \frac{z\sqrt{(1-c^2)(1-b^2)}}{z^2 + b(c-1)z - c} \times \left[\frac{-cz^2 + b(c-1)z + 1}{z^2 + b(c-1)z - c} \right]^{m-1} \\ \Psi_{2m-1}(z) &= \frac{z(z-b)\sqrt{(1-c^2)}}{z^2 + b(c-1)z - c} \times \left[\frac{-cz^2 + b(c-1)z + 1}{z^2 + b(c-1)z - c} \right]^{m-1} \end{aligned} \quad (10)$$

with $\Psi_{2m}(z)$ and $\Psi_{2m-1}(z)$ denoting the even and odd Kautz functions respectively. The scalars b and c are real-valued parameters satisfying $|b| < 1$ and $|c| < 1$. These parameters are related to the pair of Kautz poles $(\beta, \bar{\beta})$ as

$$\begin{aligned} b &= (\beta + \bar{\beta}) / (1 + \beta \bar{\beta}) \\ c &= -\beta \bar{\beta} \end{aligned} \quad (11)$$

Kautz filters are a good choice for modeling LTI systems having oscillatory response for given step input, i.e. there exist at least one pair of complex conjugate poles.

B. Selection of Poles for Kautz filters

Let us suppose Kautz filters are used to model a LTI system. To calculate the optimal poles of the system it is necessary to solve the optimization problem:

$$\min_{\theta} J = \frac{1}{2} \sum_{k=1}^{Nd} [y(k) - \hat{y}(k)]^2 \quad (12)$$

where $\theta, J, y(k), \hat{y}(k)$ are, respectively, the vector of parameters containing the poles of the Kautz filters and the weights of the basis functions, the prediction error, the measured output of the system and the predicted output using the model. A gradient-based optimization algorithm is used to solve this problem, wherein the gradient of the prediction error is computed with respect to the vector of parameters, which is updated at every iteration in a direction opposite to the gradient of the error. A detailed description of this method is given in [10].

III. WAVELET NETWORKS

A. Wavelet Transforms

Wavelet transforms are used to represent a function as a sum of wavelets functions. Wavelet transforms, unlike Fourier transforms, can provide a good time-frequency localization of the function simultaneously. Continuous Wavelet Transform (CWT) of any function at a scale $a > 0$ and translation $b \in \mathbb{R}$, $f \in L2(\mathbb{R})$ is given by ([8], [9])

$$X(a, b) = \int_{\mathbb{R}^n} f(x) |a|^{-1/2} \varphi\left(\frac{x-b}{a}\right) dx \quad (13)$$

where $\varphi(x)$ is a continuous function called the mother wavelet. The purpose of the dilation and translation parameters is to generate daughter wavelets which are simple dilated and translated versions of the mother wavelet. Owing to the time - frequency localization properties of the wavelet transform, if the dilation and translation parameters are chosen intelligently, any function can be approximated with the help of only a few daughter wavelets. The function $f(x)$ can be reconstructed back by using the inverse wavelet transform:

$$f(x) = C_{\varphi}^{-1} \int_0^{\infty} a^{-(n+1)} \int_{\mathbb{R}^n} w_{a,b} a^{-n/2} \Psi\left(\frac{x-b}{a}\right) da db \quad (14)$$

The inverse wavelet transform is not implementable in computers without discretization. So the inverse wavelet transform when discretized, takes the following form:

$$f(x) = \sum_i w_{a,b}^i a_i^{-n/2} \Psi\left(\frac{x-b_i}{a_i}\right) \quad (15)$$

B. Structure of Wavelet Network

It was first proposed by Zhang and Benveniste [7] that the discrete form of the inverse wavelet transform can be viewed as a one-hidden-layer Artificial Neural Network (ANN). By optimally choosing the parameters w_i, a_i, b_i with $\varphi(\cdot)$ as the hidden layer activation function and a linear function in the output layer, the construction becomes similar to that of single hidden layer neural networks, called Wavelet Network. Wavelet functions satisfy the "universal approximation" property. The structure of the wavelet network is as follows:

$$g(x) = \sum_{i=1}^N w_i \Psi[D_i R_i(x - t_i)] + \bar{g} \quad (16)$$

where x is the input vector, t_i are translation vectors, D_i are diagonal matrices built from dilation vectors, while R_i are rotation matrices. The parameter \bar{g} is introduced to account for functions with non-zero average, since the multi-dimensional wavelet function $\varphi(x)$ is with zero mean. This structure is illustrated in Fig.1.

C. Training of the Wavelet Network Model

This section details the training methodology of the parameters of the wavelet network. The learning is based on a sample of random input/output pairs $\{x, f(x)\}$ where $f(\cdot)$ is the function to be approximated. A stochastic gradient type of algorithm is used in the present work, as described in detail by Zhang and Benveniste [7]. A short summary of the algorithm is as follows:

i. Network Initialization

The efficiency of the learning algorithm and the accuracy of the final model are heavily dependent on the initialization of the dilation and translation parameters of the wavelet network. For the one-dimensional case, Zhang and Benveniste have proposed a method that estimates the "centre of gravity" of a function $f(x)$ in an interval $[a, b]$ and uses it to initialize the translation parameter. The dilation parameter is taken to be proportional to the length of the interval. The interval is then split about the centre of gravity and the next two translation and dilation parameters are estimated from the sub-intervals. The same method can be adapted to the multi-dimensional case. However, the approach used by Oussar and Dreyfus [15] is found to be more expedient. This approach uses a family of wavelets described by the relation:

$$\Omega_d = \{2^{m/2} \phi(2^m x - n), (m, n) \in \mathbb{Z}^2\} \quad (17)$$

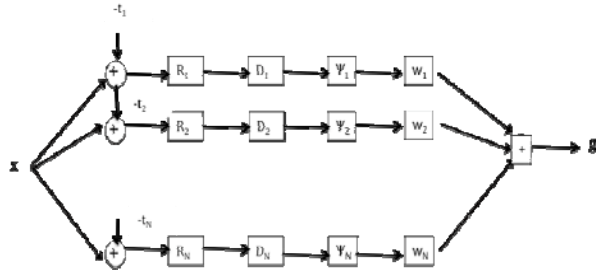


Figure 1. General Hammerstein Model

from which appropriate wavelet functions are chosen. Depending upon the input domain $\{ a_k, b_k \}$ of the k 'th input, the values of the translation parameters are chosen such that:

$$a_k \leq 2^{-m} n \leq b_k \quad (18)$$

This results in a library of wavelet functions that can be used for approximation. These wavelet functions are ranked in order of decreasing relevance using the Gram Schmidt Method, [15] from which the most relevant wavelets functions are used in the wavelet network. This approach of initialization of wavelet network parameters is adopted in the present work.

ii. Learning algorithm

Using the above approach, all the parameter w_i, t_i, D_i and R_i are collected in a vector θ . Representing the network with the parameters θ as $g_\theta(x)$, the following objective function is minimized :

$$C(\theta) = \frac{1}{2} E \left\{ [g_\theta(x) - y]^2 \right\} \quad (19)$$

The stochastic gradient algorithm modifies the parameter vector θ at each iteration in the opposite direction of the gradient of the functional

$$c(\theta, x_k, y_k) = \frac{1}{2} [g_\theta(x) - y]^2 \quad (20)$$

For calculation of the stochastic gradient and setting constraints on the Adjustable Parameters, refer to [7].

STRUCTURE OF THE HAMMERSTEIN MODEL

The Hammerstein model consists of a static non-linear part followed with a linear dynamic part. Note that Hammerstein model are applicable only to a specific class of non-linear systems whose dynamic portion can be modeled as an LTI system and the non-linearity is at the input side of the system. Fig.2 illustrates the schematic diagram of a general Hammerstein model, where $y(t)$ is the measured output $u(t)$ and $\tilde{u}(t)$ are input and output of the non-linear box respectively, $A(z)$ and $B(z)$ are polynomials of degrees

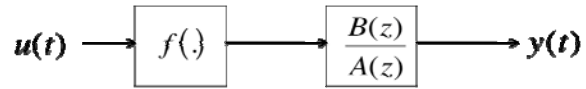


Figure 2. General Hammerstein Model

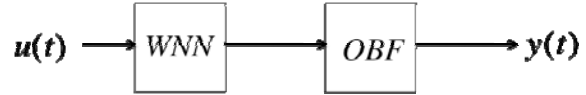


Figure 3. Hammerstein Model with static non-linear Wavelet Network and dynamic linear Orthonormal Basis Filters

n_a and n_b in the unit backward shift operator $z^{-1}[z^{-1}y(t) = y(t-1)]$ with

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_{n_a} z^{-n_a}$$

$$B(z) = b_1 z^{-1} + b_2 z^{-2} + \dots + b_{n_b} z^{-n_b}$$

Fig.3 shows the schematic diagram of the Hammerstein model proposed by us, which consists of Wavelet Network for approximating the static non-linearity and orthonormal basis filters for modeling the linear dynamic portion.

V. ILLUSTRATIVE EXAMPLE

To illustrate the efficacy of our model, it is applied to an artificial system with complicated static non-linearity. It is supplemented by application of the model to a DC-DC Boost Converter. The simulations presented in this section have been carried out on Simulink tool in Matlab 2008a on a 2.26 GHz Intel Core 2 Duo processor with an associated 2GB 1067 MHz DDRAM .

A. Numerical System

Consider a system whose static non-linearity is described by the function :

$$f(u) = \begin{cases} -1 + \cos u & 0 \leq x \leq \frac{\pi}{2} \\ -1 + \frac{2}{\pi} \left(x - \frac{\pi}{2} \right) & \frac{\pi}{2} < x \leq \frac{3\pi}{2} \\ 1 - \sin \left(x - \frac{3\pi}{2} \right) & x \leq 2\pi \end{cases}$$

and the linear dynamic part by the transfer function :

$$G(s) = \frac{5}{s^2 + 2s + 5}$$

The system is excited with a random sequence of pulses with sufficient pulse period to allow the system to stabilize to a steady value, before it is excited by another pulse. The input-output data extracted from this experiment is used to train the wavelet network and Kautz filter functions, to form the Hammerstein model.

One dimensional wavelet network consisting of 5 wavelons is used for approximating the static non-linearity. Each

wavelon consists of a dilation and translation parameter and has a weight associated with it. These, together with the offset g make for 16 parameters for the wavelet network. A data set of 300 sample points is used for training the wavelet network, according to the algorithm described in section III.C . 2000 learning iterations are performed.

Kautz filter functions of the second order are used for modeling the linear dynamics. An input data set of 100 points is used for selection of optimal poles for the Kautz filters, using the iterative optimization method described in section II.B . The unknown parameters for the Kautz model are the real and imaginary parts of the poles and the weights respectively. Therefore the entire Hammerstein model has a total of just 20 unknown parameters, thus forming a parsimonious model.

The comparison is performed by modeling the same system with the aid of a Hammerstein model that uses polynomial functions for the static non-linearity and a second order ARX model for modeling the linear subsystem. The coefficients of the polynomial can be computed using the least squares approximation and the ARX model is identified using the system identification tool available in MATLAB. This model has 10 unknown parameters, 6 for the polynomial non-linearity and 4 for the second order ARX model. The performance of the two models is compared in Fig. 4.

Mean absolute error of conventional model is 0.1732 whereas that of the proposed model is 0.0619

It should be mentioned that there is no significant change in the performance of the conventional Hammerstein model by increasing the order of the ARX model, or by increasing the order of the polynomial used for approximating the static non-linearity.

Evidently, for complex non-linearities involving sharp changes and jumps, the conventional Hammerstein model is going to fail. However the proposed model consisting of wavelet network for identifying the non-linearity will provide a good performance for any type of non-linear system.

B. DC-DC Boost Converter

A DC-to-DC boost converter is a power converter used to step up DC voltage. A simple DC-to-DC converter consists of an inductor as an energy storing device and two switches - a transistor and a diode. Filters made of capacitors are added at the output to reduce the output voltage ripple. The key principle that drives a boost converter is the tendency of an inductor to resist changes in current. When the switch is closed, the inductor acts like a load and gets charged. When the switch is open, the only path offered to the current through the inductor is through the diode, the capacitor and the load. This results in transferring the energy accumulated during the ON - state into the capacitor. The voltage produced by the inductor during the discharging phase is

related to the rate of change of current and not the original charging voltage, thus allowing the output voltage to be different.

By operating the switch at a high frequency, it is possible to generate an output DC voltage greater than the input voltage. The switch can be made using a MOSFET and it can be regulated using a PWM(Pulse Width Modulated) signal. The output DC voltage is related to the duty cycle of the pulse wave. The approximate relation between the input and output voltage can be given as:

$$\frac{V_o}{V_i} = \frac{1}{1-D} \tag{21}$$

From the above expression it can be seen that the output DC voltage is always higher than the input voltage (as the duty cycle always goes from 0 to 1) and it increases non linearly with increase in the duty cycle. The DC-DC Boost Converter can be considered to be a non-linear system with input $u(k)$ as the duty cycle of the pulse wave and the output $y(k)$ as the output DC voltage. The configuration of the DC/DC converter used in the present study employs an inductor of inductance 69 micro henries ; capacitor of 550 micro farads ; Diode of resistance 0.001 ohms and forward voltage 0.8 Volts ; MOSFET with FET resistance of 0.1ohms and internal diode resistance of 0.01 ohms .The input DC voltage is 10V and pulse period of the PWM signal is 10 micro seconds. Simulink Model of the System (Matlab 2008a) is given in Figure 5. The dynamic characteristics of the DC-DC Boost Converter are illustrated in Figure 6 . From the dynamics of the DC-DC converter, it is clear that for large negative changes in the duty cycle, the fall in output DC voltage is very slow. The reason for this is that when the duty cycle decreases, the average charge on the capacitor has to decrease, so the average current through the inductor has to decrease. However, as the inductor current can never be negative, with the presence of the diode, discharging of average charge of the capacitor is very slow. This effect can be seen in Figure 6

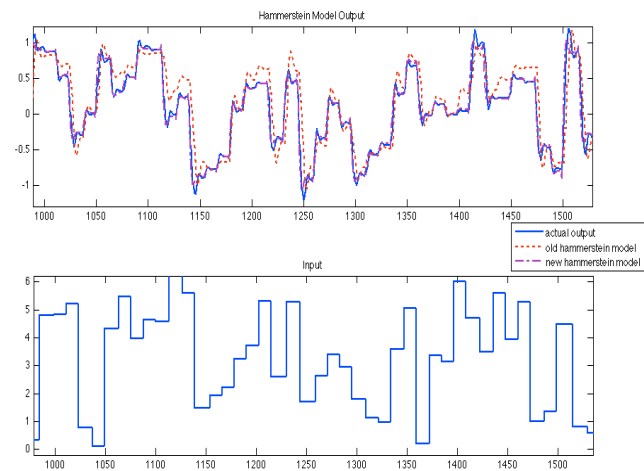


Figure 4. Hammerstein Model Output for Numerical System

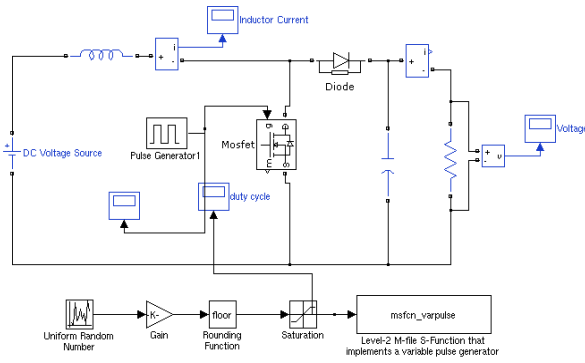


Figure 5. Simulink Model of Boost Converter

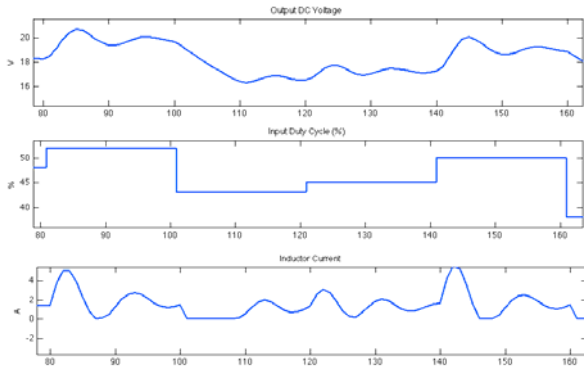


Figure 6. Dynamic Characteristics of DC-DC Boost Converter

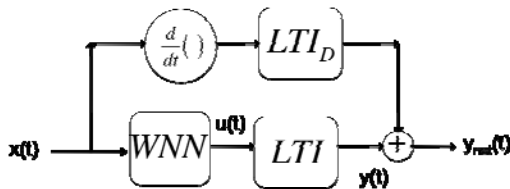


Figure 7. Hammerstein Model with Disturbance Filter

To overcome this problem, we have proposed a modified model with an additional disturbance filter.

Figure 7 illustrates the new model. LTI_D is the "disturbance filter". The filter can be trained using data corresponding to the response of the system to a sequence of negative steps in the input duty cycle. Once the disturbance filter has been integrated into the model, the output of the modified Hammerstein model can be estimated for an input duty cycle in a wide range.

Figure 8 compares the actual output with the output of the Hammerstein model and the Modified Hammerstein Model. Mean absolute error of approximation of Hammerstein model is 0.3872 and Mean absolute error of approximation of Hammerstein model with disturbance is 0.2678. Evidently, the Hammerstein Model with a disturbance filter provides a better approximation of the non-linear dynamics

of the DC-DC Boost Converter. However, it can be seen that the dynamic characteristics of the Boost converter change with the region of operation and the range of the duty cycles.

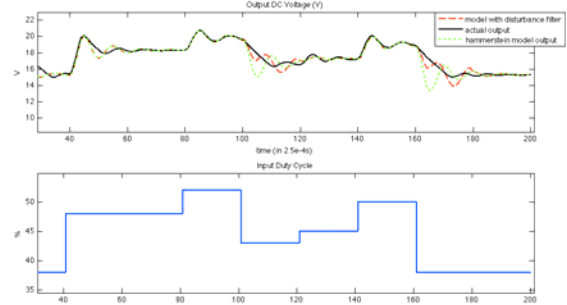


Figure 8. Hammerstein Model Prediction for DC-DC Boost Converter

VI. CONCLUSIONS

In this paper, a new method has been proposed for black box modeling of non-linear systems that combines the virtues of wavelet networks and orthonormal basis functions. The model possesses the ability to model complicated non-linear systems parsimoniously and can be trained using simple newton-based optimization algorithms. Simulation results reveal the robustness and effectiveness of the proposed method.

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