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A COMPARISON OF MULTI-LOOP PI/PID CONTROLLER DESIGN WITH REDUCTION AND WITHOUT REDUCTION TECHNIQUE

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Abstract — Proportional, Integral and Derivative (PID) controller are most widely used controller in chemical process industries because of their simplicity, robustness and successful practical application. Many methods have been proposed for design of Multi-loop PI/PID controller for Multi-Input Multi-Output (MIMO) process. In this paper we have compared two methods for two by two processes with time delays. One is model order reduction and other is without reduction. Performance index and robustness has been used as the criterion for comparison. Several commonly used simulation examples are included for demonstrating effectiveness of the proposed methods and the results obtained are comparatively same.

Keywords - Multi-loop PID controller tuning; Effective open-loop transfer function (EOTF); Model reduction; Internal model control (IMC); Static decoupling.

I. INTRODUCTION

Most chemical processes are basically multiple input/ multiple output (MIMO) systems. Despite considerable work on advanced multivariable controllers for MIMO systems, multi-loop proportional-integral-derivative (PID) controllers remain the standard for many industries because of their adequate performance with most simple, failure tolerant, and easy to understand structure. In a multi-loop system, once a control structure is fixed, control performance is then determined mainly by tuning each multiple single-loop PID controller. However, because the interactions that exist between the control loops make the proper tuning of the multi-loop PID controllers quite difficult, only a relatively few tuning methods are available to the multi-loop PID controllers and most of them require non-analytical form with complex iterative steps.

Much research has been focused on how to efficiently take loop interactions into account in the multi-loop controller design. Many design methods have been proposed [1], they are:

1. Detuning or Biggest Log Modulus (BLT) methods.
2. Sequential loop closing (SLC) methods.
3. Iterative or Trial-and-error methods.
4. Independent loop methods.
5. Relay auto-tuning methods

The interactions between input/output variables are a common phenomenon and the main obstacle encountered in the design of multi-loop controllers for interacting multi-variable processes. There are two techniques used in this paper to design a multi-loop PID controller.

Method-I: Several researchers have introduced the concept of Effective open-loop transfer function (EOTF). Using this concept the design of multi-loop controller can be reasonably converted to the design of

single-loop controller. On the basis of structure decomposition, the multi-loop control system is completely separated into equivalent individual SISO loops, and thus the effect of process and controller on the loop interactions and subsequent system properties, such as right half plane (RHP) zeros and poles, integrity, and stability, are elucidated. The control performance of the multi-loop system is also closely related to the control loop pairing. The well-known RGA has been widely used for the multi-loop structure design, such as the ratio of open-loop gain to a closed-loop gain. The definition of RGA was extended to dynamic RGA (DRGA), with frequency-dependent terms, by replacing the steady-state gains with the corresponding transfer functions. A multi-loop control system is then decomposed into a set of independent SISO loops represented by corresponding EOTF's, the tuning of the multi-loop PI/PID controller is thus converted to the design of independent single-loop PI/PID controllers.

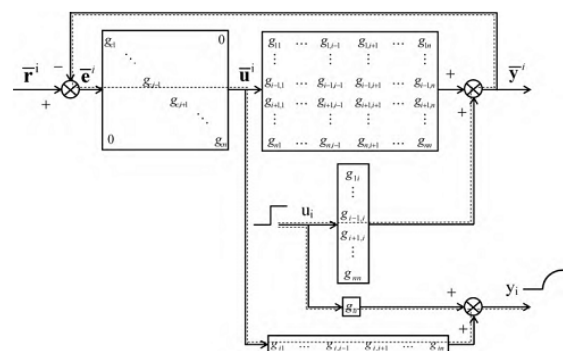


Figure1. Block diagram for the concept of the EOTF in a $n \times n$ multi-loop system: loop i is open while all other loops are closed.

A. Effective open-loop Transfer Function and DRGA

Consider the open-loop stable multi-loop system in Fig.1, where r^i , u^i , and y^i are the set-point, manipulated, and controlled variable vectors, where r_i ,

u_i , and y_i are discarded from r , u , and y , respectively. Let the EOTF of loop i be defined as the transfer function relating u_i with y_i where loop i is open while all other loops are closed. It shows the block diagram for the concept of the EOTF of loops i . The EOTF differs from the original open-loop transfer function (OTF) by transmission interaction through a path including other loops. It is clear that the EOTF corresponds to the actual open-loop transfer function under multi-loop situations and thus, tuning of the controller of loop i should be done based on the EOTF ([2]-[3]), rather than the original OTF, g_{ii} . From the block diagram of Fig.1 with $r^i=0$, u^i is obtained

$$\bar{u}^i = -\bar{G}_c^i \bar{y}^i = -\bar{G}_c^i (\bar{g}^{ic} u_i + \bar{G}^i \bar{u}^i) \quad (1)$$

Where \bar{G}_c^i denoted a multi-loop controller matrix. Therefore, the relation between y_i and u_i is written as

$$y_i = g_{ii} u_i + \bar{g}^{ir} \bar{u}^i = [g_{ii} - \bar{g}^{ir} \bar{G}_c^i (I + \bar{G}^i \bar{G}_c^i)^{-1} \bar{g}^{ic}] u_i \quad (2)$$

Furthermore, the EOTF can be compactly expressed in terms of DRGA, as follows

$$g_{ii}^{\text{eff}} = g_{ii} / A_{ii} \quad (3)$$

Where A_{ii} denotes the i th diagonal element of DRGA and is calculated by

$$A_{ii} = [G \otimes (G^{-1})^T]_{ii} \quad (4)$$

Where the symbol \otimes denotes the element by element multiplication and the subscript T denotes the transpose of a matrix.

B. Reduced EOTF for Controller Design

A simple model reduction technique is applied to approximate the EOTF to a reduced-order model, such as the first-order plus dead time (FOPDT) and the second-order plus dead time (SOPDT) models. One of the most common approaches for controller design is use a reduced-order model that simplifies the process dynamics.

A two-input, two-output (TITO) multi-delay process is one of the most commonly encountered multivariable processes in the process industry. For 2×2 system, the general stable square transfer function matrix is represented as

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \quad (5)$$

The DRGA obtained from (4) is

$$A_{11}(s) = A_{22}(s) = \frac{g_{11}(s)g_{22}(s)}{g_{11}(s)g_{22}(s) - g_{12}(s)g_{21}(s)} \quad (6)$$

Therefore, the EOTF's for the first and second loops are found by using (4) respectively:

$$g_{11}^{\text{eff}}(s) = g_{11}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{22}(s)} \quad (7)$$

$$g_{22}^{\text{eff}}(s) = g_{22}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)} \quad (8)$$

As seen from the above equations, the resulting EOTF's are usually too complicate to be directly

utilized for controller design. To overcome this difficulty, the EOTF's have to be simplified to low-order models, such as FOPDT and SOPDT. To evaluate the proposed EOTF [3], a simple model reduction technique was proposed based on the coefficient matching method.

Expanding in a Maclaurin series in s gives

$$g_{ii}^{\text{eff}} = a_{ii} + b_{ii}s + c_{ii}s^2 + d_{ii}s^3 + e_{ii}s^4 + \dots \quad (9)$$

Where the coefficients of the polynomial are

$$a_{ii} = g_{ii}^{\text{eff}}(0) \quad (10a)$$

$$b_{ii} = \left. \frac{d g_{ii}^{\text{eff}}(s)}{ds} \right|_{s=0} \quad (10b)$$

$$c_{ii} = \left. \frac{1}{2} \frac{d^2 g_{ii}^{\text{eff}}}{ds^2} \right|_{s=0} \quad (10c)$$

$$d_{ii} = \left. \frac{1}{6} \frac{d^3 g_{ii}^{\text{eff}}}{ds^3} \right|_{s=0} \quad (10d)$$

$$e_{ii} = \left. \frac{1}{24} \frac{d^4 g_{ii}^{\text{eff}}}{ds^4} \right|_{s=0} \quad (10e)$$

The FOPDT dynamics as a reduced-order model must be considered first.

$$g_{ii}^{\text{r-eff}} = \frac{ke^{-\theta s}}{\tau s + 1} \quad (11)$$

Expanding the reduced EOTF given by (11) in a Maclaurin series in s gives

$$g_{ii}^{\text{r-eff}}(s) = K - K(\theta + \tau)s + K[\frac{\theta^2}{2} + (\theta + \tau)\tau]s^2 + 0(s^3) \quad (12)$$

TABLE1. Relation between process parameters and polynomial coefficients for typical process models

Model	Relations
$\frac{Ke^{-\theta s}}{\tau s + 1}$	$K = a_{ii}; (\theta + \tau) = \left(-\frac{b_{ii}}{a_{ii}}\right);$ $\left[\frac{\theta^2}{2} + (\theta + \tau)\tau\right] = \left(\frac{c_{ii}}{a_{ii}}\right)$
$\frac{(\pm \tau_a s + 1)Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$K = a_{ii}; (X_0 \mp \tau_a) = \left(-\frac{b_{ii}}{a_{ii}}\right);$ $[X_1 + X_0(\tau_2 \mp \tau_a)] = \left(\frac{c_{ii}}{a_{ii}}\right);$ $[X_2 + X_3(\tau_2 \mp \tau_a)] = \left(-\frac{d_{ii}}{a_{ii}}\right);$ $[X_4 + X_5(\tau_2 \mp \tau_a)] = \left(\frac{e_{ii}}{a_{ii}}\right);$

Where $X_0 = \theta + \tau_1 + \tau_2$, $X_1 = (\theta^2/2) + (\theta + \tau_1)\tau_1$, $X_2 = (\theta^2/6) + ((\theta^2/2) + (\theta + \tau_1)\tau_1)\tau_1$, $X_3 = (\theta^2/2) + (\theta + \tau_1)\tau_1 + (\theta + \tau_2)\tau_2 + \tau_1\tau_2$, $X_4 = (\theta^4/24) + ((\theta^3/6) + (\tau_1\theta^2/2) + \tau_1^2\theta + \tau_1^3)\tau_1$, $X_5 = ((\theta^3/6) + (\tau_1\theta^2/2) + \tau_1^2\theta + \tau_1^3) + (\tau_2\theta^2/2) + \tau_2^2\theta + \tau_2^3 + \tau_2$, $\tau_1\theta + \tau_2\tau_1^2 + \tau_2^2\tau_1$.

Where K , τ , and θ should be identified to approximate g_{ii}^{eff} , comparing the (12) and (9) leads to the following explicit equations for K , τ , and θ .

$$K = a_{ii} \quad (13a)$$

$$\tau = \sqrt{\frac{2c_{ii}}{a_{ii}} - \left(\frac{b_{ii}}{a_{ii}}\right)^2} \quad (13b)$$

$$\theta = \frac{b_{ii}}{a_{ii}} - \sqrt{\frac{2c_{ii}}{a_{ii}} - \left(\frac{b_{ii}}{a_{ii}}\right)^2} \quad (13c)$$

In order for the resulting FOPDT model to be feasible, τ and θ should be real and positive. It is clear from (13) that the following condition should be satisfied for feasible τ and θ values.

$$\sqrt{\frac{2c_{ii}}{a_{ii}}} > \left(-\frac{b_{ii}}{c_{ii}}\right) > \sqrt{\frac{2c_{ij}}{a_{ij}} - \left(\frac{b_{ij}}{a_{ij}}\right)^2} \quad (14)$$

Method-II: In this, the desired transfer functions for individual loops in combination with the dynamic detuning factors, and this, the ideally desired multi-loop controllers can be inversely figured out. Then, by using maclaurin series expansion, the practicable PI/PID controllers are conveniently obtained. Moreover, improved tuning capacities of individual loops are obtained; that is, each loop can be tuned on-line by a single adjustable parameter to cope with the process unmodeled dynamics, which will surely bring much convenience to the system operation in practice.

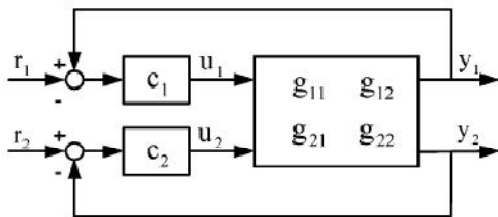


Figure2. General TITO multi-loop control structure

C. Multi-loop Structure Controllability

Consider the general transfer matrix form of two-by-two process with time delays,

From (5), Where $g_{ij}(s) = g_{oij}(s) e^{-\theta_{ij}s}$, and $i, j = 1, 2, \dots$, of which $g_{oij}(s)$ is the delay-free part and a physically proper and stable transfer function. According to the commonly used multi-loop control structure (Fig.1),

$$H = GC(I+GC)^{-1} \quad (15)$$

Where C represents diagonal controller matrix, i.e., $C = \text{diag}\{c_1, c_2\}$. It implies the absolute decoupling regulation of the binary outputs. The multi-loop control structure shown in Fig.1 can be rearranged for analysis as the block diagonal closed-loop structure shown in Fig.2.

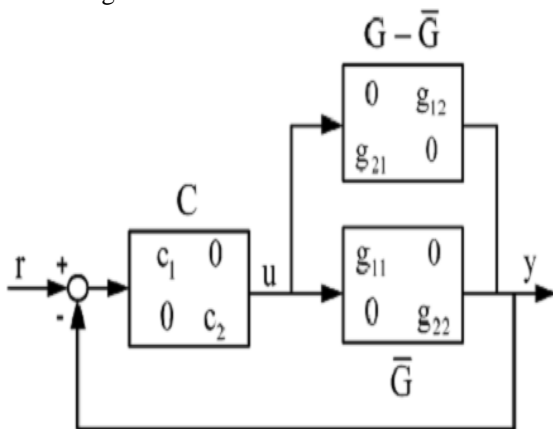


Figure3. Block diagram representation of additive uncertainty

Where G is composed of the diagonal transfer matrix of the process transfer matrix G, i.e., $G = \text{diag}\{g_{11}, g_{22}\}$, which connects the desired pairings between the binary inputs and outputs. Meanwhile, $G - \bar{G}$ is regarded as the additive uncertainty of the diagonal transfer matrix G.

g_{22} , which connects the desired pairings between the binary inputs and outputs. Meanwhile, $G - \bar{G}$ is regarded as the additive uncertainty of the diagonal transfer matrix G.

D. The Desired Closed-Loop Diagonal Transfer Functions

From Fig.3 it can be easily seen that the nominal transfer function matrix of the block diagonal closed-loop system without the additive uncertainty is in the form,

$$\bar{H} = \bar{G}C(I + \bar{G}C)^{-1} \quad (16)$$

Following some linear algebra, the diagonal controller matrix can be derived as

$$C = \bar{G}^{-1}(\bar{H}^{-1} - I)^{-1} \quad (17)$$

Therefore, the multi-loop controllers are obtained in the form of

$$c_i = \frac{1}{g_{ii}} \frac{h_i}{1-h_i} \quad i = 1, 2 \quad (18)$$

Note that g_{ii} contains time delay θ_{ii} . In addition, if g_{ii} has any right-half-plane (RHP) zeros, h_i is required to include them so that the resulting controller c_i will not include them as unstable poles. Hence, according to the H_2 optimal performance objective of the IMC theory, the desired closed-loop diagonal transfer functions are proposed as

$$h_i = \frac{e^{-\theta_{ii}s}}{(\lambda_i s + 1) U_i} \prod_{k=1}^{V_i} \frac{(-z_k s + 1)}{(z_k s + 1)} \quad i = 1, 2 \quad (19)$$

Where λ_i is an adjustable parameter for obtaining the desirable i th system output response, U_i is the relative degree of g_{oii} , $s = z_k^{-1}$ is the RHP zero of g_{ii} , and V_i is the number of these RHP zeros.

However, substituting eq(17) in to eq(15), the actual multi-loop control system transfer matrix, i.e., the transfer matrix of the perturbed block diagonal closed-loop system with the additive uncertainty $G - \bar{G}$ shown in fig.3, in the form of

$$H = G(\bar{G} + \bar{H}(G - \bar{G}))^{-1} \bar{H} \quad (20)$$

The diagonal transfer functions connecting the system inputs and outputs will not be in the form of eq(19) if the multi-loop controllers are to be directly derived from eq(18). To implement the desired closed-loop diagonal transfer functions shown in eq(19) for desired pairings between the system inputs and outputs, a diagonal dynamic detuning matrix $D = \text{diag}\{d_1, d_2\}$ is proposed to modify the diagonal system transfer function matrix shown in eq(16). It follows that

$$D\bar{H} = \bar{G}C(I + \bar{G}C)^{-1} \quad (21)$$

Hence, by using some linear algebra, is yielded the multi-loop controller matrix,

$$C = \bar{G}^{-1}(\bar{H}^{-1}D^{-1} - I)^{-1} \quad (22)$$

Then following a similar calculation as above, one obtains the actual multi-loop control system transfer matrix in the form of

$$H = G(D^{-1}\bar{G} + \bar{H}(G - \bar{G}))^{-1}\bar{H} \quad (23)$$

Therefore if one lets

$$\text{diag}\{G(D^{-1}\bar{G} + \bar{H}(G - \bar{G}))^{-1}\} = I \quad (24)$$

The diagonal dynamic detuning matrix D can be ascertained. Substituting eq(5) and eq(19) into eq(24) and solving it yields the dynamic detuning factors

$$d_1 = 2g_{11}g_{22} / [(h_1 - h_2)g_{12}g_{21} + g_{11}g_{22} + (-1)^m \sqrt{[(h_1 - h_2)g_{12}g_{21} - g_{11}g_{22}]^2 - 4g_{12}g_{21}g_{11}g_{22}(1 - h_1)h_2}] \quad (25a)$$

$$d_2 = 2g_{11}g_{22} / [(h_2 - h_1)g_{12}g_{21} + g_{11}g_{22} + (-1)^m \sqrt{[(h_1 - h_2)g_{12}g_{21} - g_{11}g_{22}]^2 - 4g_{12}g_{21}g_{11}g_{22}(1 - h_1)h_2}] \quad (25b)$$

Where

$$m = \begin{cases} 0, & g_{11}(0)g_{22}(0) > 0 \\ 1, & g_{11}(0)g_{22}(0) < 0 \end{cases} \quad (26)$$

Note that the choice of m in eq(26) is to guarantee $d_1(0) = d_2(0) = 1$ so that the actual multi-loop control system transfer function matrix will be led to identity matrix, i.e., $H(0) = I$. as for $d_1(0) = d_2(0) = 1$, it can be easily identified in the view of that $h_1(0) = h_2(0) = 1$ (see eq 6).

Combining the eq(19) and eq(21) with eq(25a) and eq(25b), the diagonal transfer matrix for deriving the desired multi-loop controllers so as to implement the H_2 optimal closed-loop diagonal transfer functions shown in eq(19) that actually connect the desired pairings between the system inputs and outputs can be in the form of

$$\hat{H} = D\bar{H} = \text{diag} \left\{ \frac{d_i e^{-\theta_{ii}s}}{(\lambda_i s + 1)U_i} \prod_{k=1}^{V_i} \frac{(-z_k s + 1)}{(z_k s + 1)} \right\} \quad (27)$$

II. MULTI-LOOP PI/PID CONTROLLER DESIGN

Method-I: Once a reduced EOTF is obtained, any PID tuning method for SISO system can be applied for the design of each individual PID controller. The IMC-PID design approach is commonly used for the PID controller tuning in the process industry.

First, the reduced EOTF, \hat{H} , is decomposed to

$\hat{H} = p_{Ai}/p_{Mi}$, where p_{Ai} and p_{Mi} are the non-minimum portion with an all-pass form and the minimum phase portion respectively. The conventional IMC filter, f_i , is selected as $f_i = 1/(\lambda_i s + 1)^{m_i}$, in which λ_i is design parameter, the filter order m_i is selected as positive integer.

Then, the ideal feedback controller to yield the desired closed-loop response perfectly is given by

$$g_{ci} = \frac{1}{f_i} \quad (28)$$

Where q_i is the IMC controller and is designed by $q_i = f_i$.

Since the above resulting controller does not have standard PID controller form, it is required to approximate the ideal feedback controller g_{ci} to the equivalent PID controller form.

Expanding g_{ci} in a Maclaurin series in s yields

$$g_{ci} = \frac{f_i(s)}{s} = \frac{1}{s} [f_i(0) + f_i'(0)s + f_i''(0)s^2 + f_i'''(0)s^3 + \dots] \quad (29)$$

The controller given by (28) is interpreted as the standard PID controller by using the first three terms and truncating the higher order terms, given by

$$g_{ci}(s) = K_{ci} \left(1 + \frac{1}{\tau_{ii}s} + \tau_{Di}s \right) \quad (30)$$

Where

$$K_{ci} = f_i'(0) \quad (31a)$$

$$\tau_{ii} = \frac{f_i''(0)}{f_i'(0)} \quad (31b)$$

$$\tau_{Di} = \frac{f_i'''(0)}{2f_i''(0)} \quad (31c)$$

Method-II: According to proposed diagonal transfer matrix eq(27), the ideally optimal multi-loop controllers can be derived by substituting eq(27) in eq(22), it follows that

$$C_{i_ideal} = \frac{1}{g_{ii}} \frac{d_i h_i}{1 - d_i h_i} \quad i = 1, 2 \quad (32)$$

Which implies that the ideally optimal multi-loop controllers proposed in eq(32) have a property of integrating to eliminate the steady deviation of system outputs. Therefore, let

$$M_i(s) = s C_{i_ideal}(s) \quad i = 1, 2 \quad (33)$$

Using the mathematical Maclaurin series expansion, the rational approximation form of eq(32) can be obtained as

$$C_{i_PI} = k_{ci} \left(1 + \frac{1}{\tau_{ii}s} \right) \quad i = 1, 2 \quad (35)$$

Where $k_{ci} = M_i'(0)$ and $\tau_{ii} = M_i''(0)/M_i'(0)$, $i = 1, 2$

Furthermore, the first three terms of eq(34) constitute a standard PID controller, i.e.,

$$C_{i_PID} = k_{ci} \left(1 + \frac{1}{\tau_{ii}s} + \tau_{Di}s \right) \quad i = 1, 2 \quad (36)$$

Where k_{ci} and τ_{ii} are as above, and $\tau_{Di} = M_i'''(0)/2M_i''(0)$, $i = 1, 2$. Obviously the PID controller formula is capable of obtaining better system performance than the PI formula due to a better approximation for the ideally optimal multi-loop controller as shown in eq(32).

In addition, it should be noted that each of the multi-loop PI/PID controllers proposed in eq(35-36) is actually turned by a single adjustable parameter λ_i , which is utilized to obtain the desirable i th system output response, as shown in eq(19).

III. MULTI-LOOP SYSTEM STABILITY ANALYSIS

In this section, to ensure a fair comparison, the performance and robustness of the control systems are measured by the following evaluation criteria.

E. Performance Index

To evaluate closed-loop performance, the integral absolute error (IAE) criterion is considered, which is defined as

$$IAE = \int_0^{\infty} |e(t)| dt \tag{37}$$

Where $e(t) = r(t) - y(t)$

F. Robustness Index

The robust stability is utilized for a fair comparison with other comparative methods. The multiple resources of uncertainty are lumped into a single complex perturbation. The robust stability of multi-loop control system is examined under output multiplicative uncertainty. For a process with an output uncertainty of $[I + \Delta_0(s)]G(s)$, the upper bound of the robust stability is given as

$$\gamma = \bar{\sigma}(\Delta_0) < 1/\bar{\sigma}[(I + G(j\omega)\bar{G}_c(j\omega))^{-1}G(j\omega)\bar{G}_c(j\omega)] < \underline{\sigma}[I + (G(j\omega)\bar{G}_c(j\omega))^{-1}], \forall \omega \geq 0 \tag{38}$$

Where γ represents the degree of robust stability, perturbation as a multiplicative output, and $\bar{\sigma}$ and $\underline{\sigma}$ maximum and minimum singular values, respectively. For a fair comparison, all of the controllers being compared were designed to have the same degree of robust stability in terms of the γ value.

It is necessary to analyze the multi-loop system stability (Method-II) so that the tuning constraints for the adjustable parameter λ_i of the proposed multi-loop PI/PID controllers can be ascertained. The generalized Nyquist stability theorem is represented for the multi-loop system stability.

IV. SIMULATION EXAMPLES

Example1. (Vinante and Luyben (VL) column). A 24-tray tower separating a mixture of methanol and water, examined by Luyben, has the following transfer function matrix.

$$G(s) = \begin{bmatrix} \frac{-2.2 e^{-s}}{7s+1} & \frac{1.3 e^{-0.3s}}{7s+1} \\ \frac{-2.8 e^{-1.8s}}{9.5s+1} & \frac{4.3 e^{-0.35s}}{9.2s+1} \end{bmatrix}$$

Method-I: For this TITO system, it follows from (7) and (8) that the EOTF's for the first and second loops are obtained as

$$g_{11}^{eff}(s) = \frac{-2.2e^{-s}}{7s+1} + \frac{0.85(9.2s+1)e^{-1.75s}}{(7s+1)(9.5s+1)};$$

$$g_{22}^{eff}(s) = \frac{4.3e^{-0.35s}}{9.2s+1} - \frac{16.5e^{-1.1s}}{9.5s+1}$$

The reduced EOTF's for the corresponding EOTF's are constituted as using (13a) – (13c) as follows

$$g_{11}^{r_eff} = \frac{-1.354e^{-0.682s}}{6.661s+1}; g_{22}^{r_eff} = \frac{2.646e^{-0.052s}}{8.841s+1}$$

Method-II: It can be easily seen that the first column is the process transfer matrix is of slightly off-diagonal dominance. A static decoupler $D(0) = G^{-1}(0)$ in front of the binary process inputs and then designed the multi-loop PI controller for the augmented system. Take $\lambda_1 = 2$ and $\lambda_2 = 0.7$ so as to obtain a similar set point response rising speed with the Lee method the first diagonal transfer function of the augmented system transfer matrix; hence, according to the H_2 optimal form of the desired closed-loop diagonal transfer functions shown in eq(19), the first diagonal transfer function of closed-loop control system should be

$$h_1 = \frac{(-0.7329s+1)e^{-0.3s}}{(0.7329s+1)(\lambda_1s+1)}$$

Fig.4 shows the closed-loop responses by several tuning methods. In the simulation study, the unit step set-point changes were sequentially introduced into the individual loops. The controller parameters are shown in Table 2.

TABLE 2. Controller parameters for the VL column

Tuning Method	Loop	K_{cl}	τ_{I1}	τ_{D1}	λ_i
Proposed (Method-I)	1	-1.83	6.71	0.090	1.98
	2	5.54	8.84	0.002	0.55
Proposed (Method-II)	1	-1.54	6.25	-	2
	2	4.35	7.48	-	0.3
Lee	1	-1.31	2.36	-	2
	2	3.97	2.42	-	0.7

Example2. (Wood and Berry (WB) column). Wood and Berry introduce the following model of a pilot-scale distillation column of a eight-tray plus reboiler separating methanol and water.

$$G(s) = \begin{bmatrix} \frac{12.8 e^{-s}}{16.7s+1} & \frac{-18.9 e^{-3s}}{21s+1} \\ \frac{6.6 e^{-7s}}{10.9s+1} & \frac{-19.4 e^{-3s}}{14.4s+1} \end{bmatrix}$$

Method-I: For TITO system, the EOTF's of first and second loop are found as

$$g_{11}^{eff}(s) = \frac{12.8e^{-s}}{16.7s+1} + \frac{6.36(14.4s+1)e^{-7s}}{(21s+1)(10.9s+1)}$$

$$g_{22}^{eff}(s) = \frac{-19.4e^{-3s}}{14.4s+1} - \frac{9.75(16.7s+1)e^{-9s}}{(21s+1)(10.9s+1)}$$

The EOTF's are approximated to the reduced EOTF's by using the proposed model reduction method as follows

$$g_{11}^{r,eff} = \frac{6.370e^{-0.308s}}{10.529s+1}; g_{22}^{r,eff} = \frac{9.655e^{-4.265s}}{6.275s+1}$$

The multi-loop PI controller design methods were employed for the comparison as they have demonstrated effectiveness over other existing methods.

Method-II: In proposed method, take $\lambda_1 = 2.5$ and $\lambda_2 = 6$ in order to obtain the similar set point response rising speed. The controller parameters used are listed in Table 3. For fair comparison, the λ_i values for both the proposed method and Lee et al. [5] were adjusted. The simulation results are shown in Fig.5.

TABLE 3. Controller parameters for the WB column

Tuning Method	Loop	K_{ci}	τ_{ii}	τ_{Di}	λ_i
Proposed (Method-I)	1	0.66	10.55	0.02	2.20
	2	-0.11	7.54	1.04	2.87
Proposed (Method-II)	1	0.244	5.45	0.255	2.5
	2	-0.072	6.27	1.079	6
Lee et al.	1	0.24	8.36	-	4.55
	2	-0.10	7.46	-	4.55

V. CONCLUSION

In this study both the methods gave similar results and performed well. When compare to without

reduction method and model order reduction method, model order reduction method gives slightly better performance than without reduction method. So, these two methods can be used for achieving better results in various TITO process in industry.

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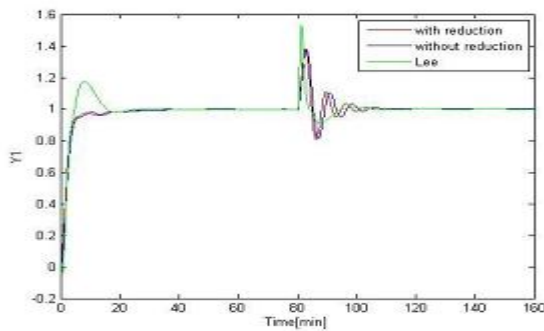


Figure 4. Closed loop responses to the sequential step changes in the set point for the VL columnn

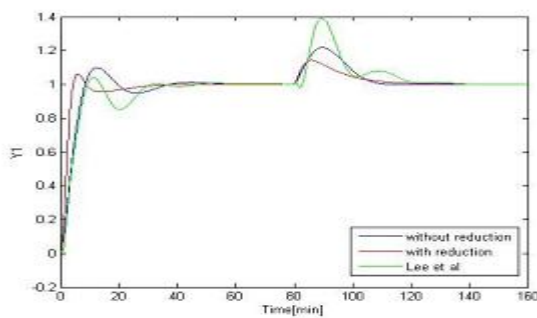
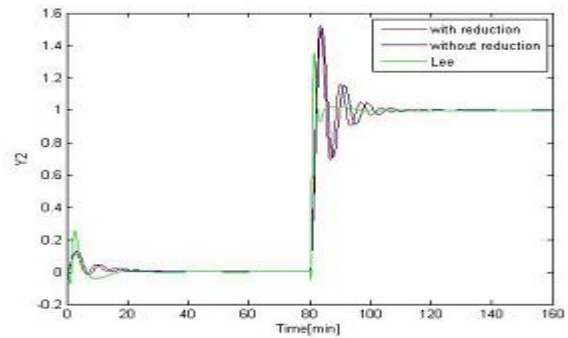


Figure 5. Closed loop responses to the sequential step changes in the set point for the WB columnn.

