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# FREE INTERACTOR MATRIX METHOD FOR CONTROL PERFORMANCE ASSESSMENT OF MULTI-VARIATE SYSTEMS

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**Abstract**—In this paper, an alternative method for the assessment of multi-vitiate control loop performance with consider twocircumstances. First, known time delays between each pair of inputs and outputs, and second, without relying on any a priori knowledge about the process model or timedelays. The performance of the control loop is calculated from data driven autoregressive moving average (ARMA) and prediction error model. It is clear that the limited data in scalar measure used for performance assessment results tends to steady-state as time tends to infinity, but large number of samples gives risen in scalar measures and tends to infinity as time samples tends to infinity and therefore it becomes difficult to calculate the performance index. In this paper, the later problem is solved by considering initial part of scalar measures with steady value for next-to-next time samples to calculate the control-loop performance index which would be utilized to decide healthy working of the control loop. Simulation example is included to show the performance index of multi-variate control loop.

***Keywords***- *Interactor matrices; Performance assessment; Performance Index; Multi-variate systems; Moving average; prediction error*

## I. INTRODUCTION

Control performance assessment (CPA) techniques provide an indication of how current controller performance compares with what would be considered to be ideal. The ideal performance is typically referred to as a 'benchmark'. There are two fundamental requirements for any CPA algorithm. The first is that it should be able to detect any change in the performance of a control system and the second is that it should be able to identify the potential improvement that can be made to the performance of the control system if it were to be re-tuned or re-designed [1]. The performance of a control system relates to its ability to deal with the deviations between controlled variables and their set-points (or desired values). These deviations can be quantified by a single number, the performance index (indicator/potential/metric). Traditional performance measures (such as rise time, settling time, overshoot, offset from set-point, integral error criteria, etc.) have been used by [2,3,4,5]. In the case of frequentdeterministicdisturbances. The most widespread criterion considered for CPA is the variance (or, equivalently, the standard deviation), particularly for regulatory control. The performance of a control loop might be deemed unacceptable if the variance of the controlled variable exceeds some critical values, because of its direct relationship to process performance, product quality, and profit. In line with [6,7,8] the control performance indices (CPIs) should be scaled to lie within [0,1], where values close to 1 mean better/tighter control:

$$\eta = \frac{J_{des}}{J_{act}}. \quad (1)$$

Where is any ideal, optimal or desired/expected valuefor a given performance criterion (typically the variance), and the actual value extracted from measured data. This definition is chosen for its practical acceptance and is not in line with[33], who use the reverse index. However, all indices (of the same category) are equivalent, i.e., they can easily be transformed into each other.

Several method exposures for CPA like Linear Quadratic Gaussian (LQG)[9], Model Prediction Control (MPC)[10,11]user-specified(US)[12,13,14]Minimum Variance Control (MVC) [15,16,17,18,19].Among a number of approaches for control performance monitoring, minimum variance control (MVC) - benchmark remains the most popular benchmark. One of the reasons for the suitability of MVC benchmark to assess performance of control loops in the industry is that it is non-intrusive and routine closed-loop operating data are sufficient for the calculation of this benchmark [20,21,22]. However, this convenience holds only in the univariate case where the time delay is the only a priori knowledge that needs to be available. For multi-variate processes, this simplicity is lost and the time delay is no longer a simple technical concept. An interactor matrix is needed for multi-variate process, and its calculation is beyond the knowledge of the time delay between each pair of inputs and outputs. The earlier work in this area is Huang[30,34] and Harris[29]. Both approaches require an explicit knowledge of the interactor matrix [23].

## II. INTRACTOR MATRIX

Consider the following multi-variate process

$$Y_t = TU_t + Na_t. \quad (2)$$

Where $T$ and $N$ are proper (causal), rational transfer function matrices in the backshift operator $q^{-1}$; $Y_t$, $U_t$ and $a_t$ are output, input and noise vectors of appropriate dimensions. $a_t$ is further assumed to be white noise with zero mean and $Var(a_t) = \sum a$. $N$ is rational realization of disturbance spectrum with the standard assumptions [35] that $N(q^{-1} = 0) = I$ and $N$ is minimum phase, both of which are true through an appropriate realization of the disturbance spectrum. For every $n \times m$ proper, rational polynomial transfer function matrix $T$, there exists non-singular, $n \times n$ (non-unique) polynomial matrix $D$, such that $|D| = q^r$, $D^T D = I$ and

$$lim_{q^{-1} \to 0} DT = lim_{q^{-1} \to 0} \tilde{T} = K. \quad (3)$$

where is a full rank constant matrix, the integer is defined as the number of infinite zeros of , and is the delay-free transfer function (factor) matrix of $T$ which contains only finite zeros. The matrix is known as the unitary interactor matrix, an equivalent form of the conventional lower triangular interactor matrix and can be written as

$$D = D_0 q^d + D_1 q^{d-1} + \cdots + D_{d-1} q. \quad (4)$$

Where is denoted as the order of the interactor matrix and is unique for a given transfer function matrix [24,25,26], and are coefficient matrices. The interactor matrix can be one of the three forms described in the sequel. If is of the form: then the transfer function matrix is regarded as having a simpleinteractor matrix. If is a diagonal matrix, i.e., , then is regarded as having a diagonal interactor matrix. Otherwise, is considered to have a general interactor matrix.

The computation of the interactor matrix needs a complete process model or at least the first few Markov parameters of the process model[21], which is beyond the knowledge of time delays between each pair of the inputs and outputs. This requirement of process model information has been the main difficulty to the application of the multi-variate control performance assessment technique.

If the pair-wise time delays are unknown or the interactor matrix has been determined to be non-diagonal, then it is not possible to estimate minimum variance from closed loop routine operating data. We shall consider an alternative method for the assessment of multi-variate control loop performance without relying on any a priori knowledge of the interactor matrices [23].

## III. ASSESSMENT OF MULTI-VARIATE CONTROL PERFORMANCE WITHKNOWN PAIR-WISE TIME DELAYS

It has been shown in [20,29] that the first d terms of thefollowing moving average expansion of the interactor filteredmulti-variate closed-loop output are feedback controlinvariant, where d is the order of the interactor matrix.

$$\tilde{Y}_t = q^{-d} = \tilde{F}_0 a_t + \tilde{F}_1 a_{t-1} + \cdots + \tilde{F}_{d-1} a_{t-(d-1)} + \tilde{F}_i a_{t-d} + \cdots \quad \ldots\ldots(5)$$

The first terms represent the closed-loop output of ifthe minimum variance feedback control is implemented,where the minimum variance is in the sense of minimizingthe trace of the covariance of . Due to the property of theunitary interactor matrix, the trace of the covariance of is the same as that of . If the interactor matrix is known,then Eq. (5) can be easily obtained through time seriesanalysis of followed by the filtering of and thenthe moving average expansion, and the minimum varianceterm can be calculated, which can be used as a benchmarkfor multi-variable control performance assessment.

The problem in practical application is the interactormatrix as discussed in the last section, calculation of which,except for the diagonal interactor matrix, needs a prioriknowledge of the process model. In particular, an experimentand identification effort has to be undertaken in orderto calculate the interactor matrix.

Unlike univariate control performance assessment, formulti-variate control performance assessment, knowingpair-wise time delays is not sufficient for calculating minimumvariance unless the interactor matrix has a simpleor diagonal structure. However, if the time delays betweeneach pair of inputs and outputs are indeed known, weshould search for a possible simple or diagonal structureof the interactor matrix, which can directly lead to the computationof the multi-variate minimum variance. Both thesimple and the diagonal interactor matrices can be calculatedfrom the time delays between each pair of inputsand outputs of the process. One may surprisingly find thatthe simple and diagonal interactor matrices are not uncommon,particularly in industrial process, where the sparsestructure of the transfer function matrix is often observed.The sparse structure also facilitates the determination ofthe interactor structure.

Consider a multi-variable transfer function matrix of dimension n · m given by

$$T = \begin{bmatrix} T_{11} q^{-d_{11}} & T_{12} q^{-d_{12}} & \cdots & T_{1m} q^{-d_{1m}} \\ T_{21} q^{-d_{21}} & T_{22} q^{-d_{22}} & & T_{1m} q^{-d_{2m}} \\ & \vdots & \ddots & \vdots \\ T_{n1} q^{-d_{n1}} & T_{n2} q^{-d_{n2}} & \cdots & T_{nm} q^{-d_{nm}} \end{bmatrix} (6)$$

where is a scalar transfer function from the th input to the th output. Define a delay matrix

$$\psi = \begin{bmatrix} t_{11}q^{-d_{11}} & t_{12}q^{-d_{12}} & \cdots & t_{1m}q^{-d_{1m}} \\ t_{21}q^{-d_{21}} & t_{22}q^{-d_{22}} & & t_{1m}q^{-d_{2m}} \\ \vdots & & \ddots & \vdots \\ t_{n1}q^{-d_{n1}} & t_{n2}q^{-d_{n2}} & \cdots & t_{nm}q^{-d_{nm}} \end{bmatrix} \quad (7)$$

where $d_{ij}'s$ are time delays that are assumed known; $t_{ij}$ is the first non-zero impulse response coefficient from the jth input to the ith output, which is typically unknown. From W, we can obtain a diagonal matrix

$$\theta = \begin{bmatrix} q^{d_1} & & & \\ & q^{d_2} & & \\ & & \ddots & \\ & & & q^{d_n} \end{bmatrix} \quad (8)$$

where $d_1 = \min\{d_{ij}: j = 1, \ldots, m\}$.

**Example 2:** Consider four processes discussed in [29].The transfer functions matrices are given in Table 1. With sampling interval , the four continuous-timetransfer function matrices can be transferred to discrete-timetransfer function matrices (by assuming zero-order hold).The time delay matrices W are summarized in the first rowof Table 2. The H matrices are obtained and summarizedin the second row. The multiplications are listed in the third row, and their determinants are shownin the fourth row. The fifth row shows the conditions for thedeterminants to be zero. It is not difficult to find out thatWood–Berry and Wardle–Wood both have the diagonalinteract matrices; Ogunnaike and Ray has the simple interactormatrix structure

unless the first non-zero impulseresponses of the four sub-transfer functions satisfy the condition, which is not the case; Vinante–Luyben doesnot have the simple or diagonal interactor matrix.

## IV. ASSESSMENT OF MULTI-VARIAT CONTROL FREE INTRACTOR MATRIX

There are several interactor matrix-free methods in the literature, mainly based on closed-loop impulse response [21,27,28] and variance of multi-step prediction errors [27,29,37]. Earlier work in using interactor-free approach may be traced back to [31,32].

Consider a closed-loop multi-variate process represented by moving average model. It is a time series model of close loop transfer function of process whenorder of auto regressive part of ARMA model iszero. This is obtained by MATLAB2011a software.

The output model should come to constant and for this we can change the order of moving average part and the delay order. So, predilection error comes out from moving average and from that the covariance matrix is obtained.

Table 1Four classical multi-variable processes

| PROCESS | $g_{11}(s)$ | $g_{12}(s)$ | $g_{21}(s)$ | $g_{22}(s)$ |
|---|---|---|---|---|
| Wood and Berry (WB) | $\frac{12.8e^{s}}{16.7s+1}$ | $\frac{-18.9e^{-3s}}{21s+1}$ | $\frac{6.6e^{-7s}}{10.9s+1}$ | $\frac{-19.4e^{-3s}}{14.4s+1}$ |
| Vinante and Luyben (VL) | $\frac{-2.2e^{s}}{7s+1}$ | $\frac{1.3e^{-0.3s}}{7s+1}$ | $\frac{-2.8e^{-1.8s}}{9.5s+1}$ | $\frac{4.3e^{0.35s}}{9.2s+1}$ |
| Wardle and Wood (WW) | $\frac{0.126e^{-6s}}{60s+1}$ | $\frac{-0.101e^{-12s}}{(48s+1)(45s+1)}$ | $\frac{0.094e^{-8s}}{38s+1}$ | $\frac{-0.12e^{-8s}}{35s+1}$ |
| Ogunnaike and Ray (OR) | $\frac{22.89e^{-0.2s}}{4.572s+1}$ | $\frac{-11.64e^{-0.4s}}{1.807s+1}$ | $\frac{4.689e^{-0.2s}}{2.174s+1}$ | $\frac{5.8e^{-0.4s}}{1.801s+1}$ |

Table 2Determination of interactor structure for four classical multivariable processes

| | $WB$ | $VL$ | $WW$ | $OR$ |
|---|---|---|---|---|
| $\psi$ | $\begin{bmatrix} t_{11}q^{-2} & t_{12}q^{-4} \\ t_{21}q^{-8} & t_{22}q^{-4} \end{bmatrix}$ | $\begin{bmatrix} t_{11}q^{-2} & t_{12}q^{-1} \\ t_{21}q^{-2} & t_{22}q^{-1} \end{bmatrix}$ | $\begin{bmatrix} t_{11}q^{-7} & t_{12}q^{-14} \\ t_{21}q^{-9} & t_{22}q^{-9} \end{bmatrix}$ | $\begin{bmatrix} t_{11}q^{-1} & t_{12}q^{-1} \\ t_{21}q^{-1} & t_{22}q^{-1} \end{bmatrix}$ |
| $\theta$ | $\begin{bmatrix} q^{2} & \\ & q^{4} \end{bmatrix}$ | $\begin{bmatrix} q & \\ & q \end{bmatrix}$ | $\begin{bmatrix} q^{7} & \\ & q^{9} \end{bmatrix}$ | $\begin{bmatrix} q & \\ & q \end{bmatrix}$ |
| $K = \lim_{q^{-1}\to 0} \theta\psi$ | $\begin{bmatrix} t_{11} & 0 \\ 0 & t_{22} \end{bmatrix}$ | $\begin{bmatrix} 0 & t_{12} \\ 0 & t_{22} \end{bmatrix}$ | $\begin{bmatrix} t_{11} & 0 \\ t_{21} & t_{22} \end{bmatrix}$ | $\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$ |
| $\det(K)$ | $t_{11}t_{22}$ | $0$ | $t_{11}t_{22}$ | $t_{11}t_{22} = t_{12}t_{21}$ |
| $Cond.$ | $None$ | $Any t_{ij}$ | $None$ | $\frac{t_{11}}{t_{12}} = \frac{t_{21}}{t_{22}}$ |

The size of covariance matrix depends on how much sample is going to be studied, for each point we bring out smaller covariance matrix for example in the fourth point we take first term of general matrix and scalar measure of *i.e.*the sum of original diameter terms of covariance matrix, this value after some limited number of samples tends to be fixed so we assume this as the final value to calculate that is obtained from (9).

$$p_i = \frac{s_\infty - s_i}{s_\infty} \qquad (9)$$

In the flowing shows an algorithm of this intractor matrix-free method.

First find the closed-loop multi-variate process; define the data in iddata form, (input, delay),thendefine the $T$, $N$ and $Q$ which are proper (causal), rational transfer function matrices in the backshift operator$q^{-1}$.

Find the closed-loop multi-variate process represented by;

$$CLTF = \frac{N}{(I+TQ)} \qquad (10)$$

Find the output regarding to input by LSIM syntax

The closed-loop multi-variate process represented by a moving average form;

$$Y_t = F_0 a_t + F_1 a_{t-1} + \cdots + F_{i-1} a_{t-(i-1)} + F_i a_{t-i} + \cdots \qquad (11)$$

This is obtain in Matlab-software by syntax as well as IDENT-Toolbox, In IDENT toolbox after dataimport, in time domain we should select the linear parametric models and change the order which the part of auto regressive be zero by select *an=0* and estimate the model.

Since at is white noise, the optimal *i*th step prediction isgiven by;

$$y_{t|t-i} = F_i a_{t-i} + F_{i+1} a_{t-i-1} + \cdots \quad (12)$$

The prediction error$e_{t|t-i} = Y_t - Y_{t|t-i}$ is given by;

$$e_{t|t-i} = F_0 a_t + F_1 a_{t-1} + \cdots + F_{i-1} a_{t-(i-1)} \quad (13)$$

Prediction error represented by flowing syntax;

(14)

When

*E* = prediction model

*Pe* =syntax for finding prediction of MA model and DATA

*ARX* =moving average model

*DATA* = two input and output data, in IDDATA form.

The covariance of the prediction error can be calculated as;

$$COV(e_{t|t-i}) = F_0 F_0^T + F_1 F_1^T + \cdots + F_{i-1} F_{i-1}^T \qquad (15)$$

and its scalar measure$(s_i)$

$$s_i \triangleq tr[COV(e_{t|t-i})] = tr(F_0 F_0^T + F_1 F_1^T + \cdots + F_{i-1} F_{i-1}^T) \quad (16)$$

## V. SIMULATIONEXAMPLE

**Example 2:**Consider a multi-variable process with the open-loop transfer function matrix T and disturbance transfer function matrix N given by;

$$T = \begin{bmatrix} \frac{q^{-1}}{1-0.4q^{-1}} & \frac{0.5q^{-2}}{1-0.1q^{-1}} \\ \frac{0.3q^{-1}}{1-0.4q^{-1}} & \frac{q^{-2}}{1-0.8q^{-1}} \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{1}{1-0.5q^{-1}} & \frac{-1}{1-0.6q^{-1}} \\ \frac{q^{-1}}{1-0.7q^{-1}} & \frac{1}{1-0.8q^{-1}} \end{bmatrix}$$

The white noise excitation is a two-dimensional normally distributed white noise sequence with

.

Consider that the following multi-loop controller is implemented in the process:

$$Q = \begin{bmatrix} k\frac{0.5-0.20q^{-1}}{1-0.5q^{-1}} & 0 \\ 0 & \frac{0.25-0.200q^{-1}}{(1-0.5q^{-1})(1+0.5q^{-1})} \end{bmatrix}$$

In this example, three controller gains are considered, respectively.As per the study on 2000 samples the output data and ARX model are plotted in the below figure (Fig. 1).

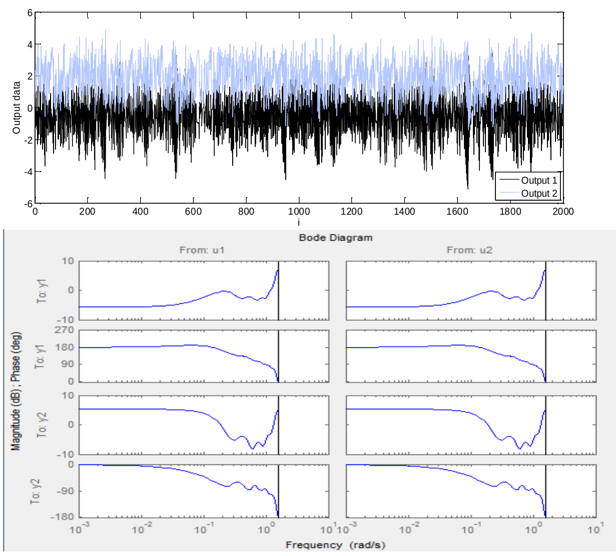


**Fig. 1.output data and ARX model**

In Fig.2. iscalculated and plotted for three different range of samples. It indicates that the closed-loop settling time increases withthe increase in controller gain. Also it shows, for first few samples the value is constant, but for large number of samples final value ( ) increases gradually with increase in .

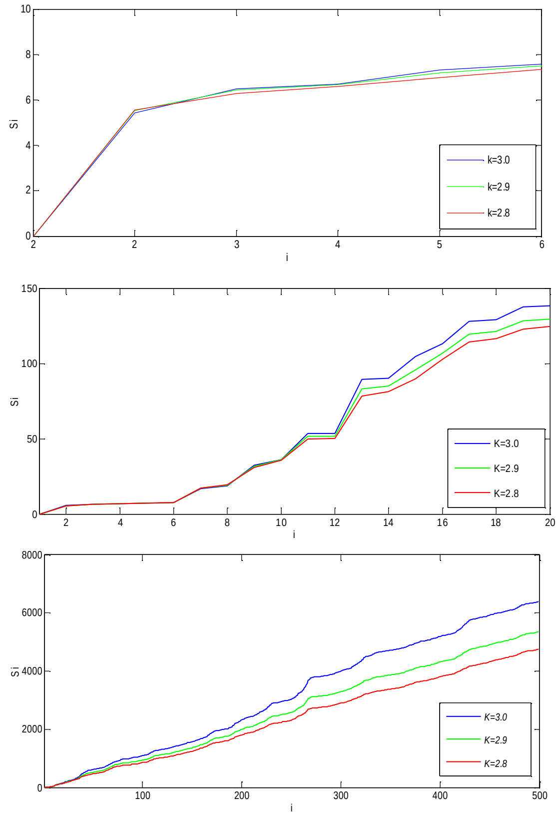


Fig. 2. **Plot for first 6,20 and 500samples**

The closed-loop potential $p_i$ define as

$$P_i = \frac{S_\infty - S_i}{S_\infty} \qquad (17)$$

Since $S_i$ is monotonically increasing with an increase in $i, P_i$ is monotonically decreasing.$S_0 = tr[cov(Y_t - Y_{t|t})] = 0$, $P_0 = 1$. Therefore, $P_i$ starts from 1 at $i = 0$ and monotonically decreases to 0 and $0 \ll P_i \ll 1$. Unlike the impulse response or variance of prediction error, $P_i$ is dimensionless and facilitate the comparison of control performance. Due to the monotonically decreasing nature of the potentials and fixed starting and ending values of the potentials, the area below the potential plot well reflects the rate of its decaying. Therefore, it is possible to define a scalar index to monitor the change of the closed-loop potential. This index is called relative closed-loop potential index and can be calculated as

$$\eta_p \triangleq \frac{\sum P_i^{(2)}}{P_i^{(1)}} - 1 \qquad (18)$$

Where is a reference potential calculated, for example, from the data sampled before control tuning, and is calculated from data sampled after the tuning.

However for calculating the final scalar measure( )must be constant.This alternative solutions method is checked for different delay and different spread times ( ),the plot reflects for a limited number of samples when tends to infinity the value of tends to be constant value, but for a large number of samples tends to increase incrementally as shown inFig. 2.Here we assume is constant subsequently and plot as shown in second and third graph in Fig. 3.

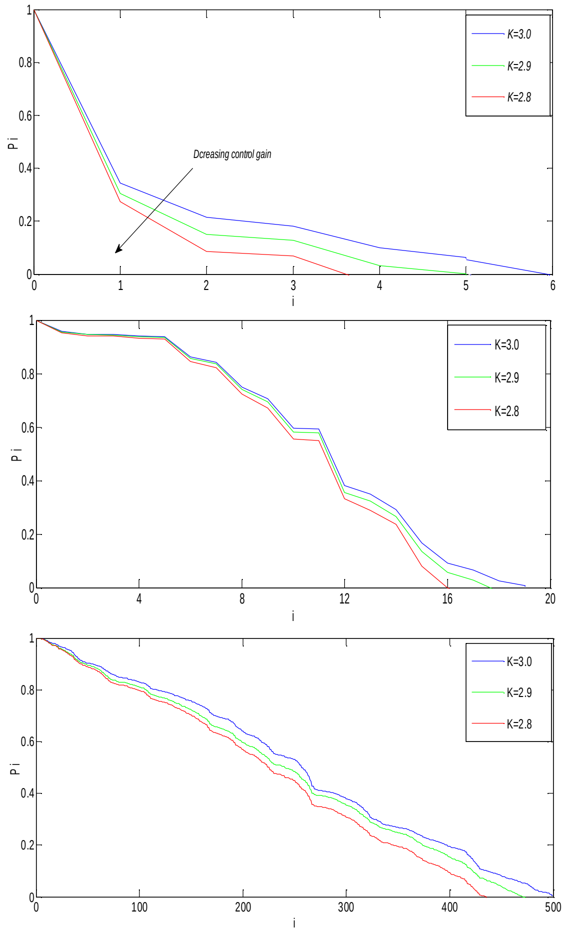


**Fig. 3. Plot for first 6,20 and 500 samples**

## VI. CONCLUTION

In this paper, the discussion of alternative and simplesolutions to multi-variate feedback control performanceassessment with prior knowledge and without any prior knowledge of theinteractor matrices. The proposed an algorithm to obtained performancemeasure based on closed-loop potential and the solution isbased on the multi-step optimal prediction error.This alternative method has been mentioned which is acceptable for limit of samples and it should improve to find proper control assessment index. The simulation examples have shown the features of the proposedalgorithms.

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