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Carrier Frequency Offset Estimation Algorithm in the Presence of I/Q Imbalance in OFDM Systems

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Abstract - In this paper, we analyzed the feasibility of using a virtual carrier based carrier frequency offset estimation algorithm in the presence of I/Q imbalance in OFDM systems. Based on the analysis of the signal model with both receiver CFO and I/Q imbalance impairment we conclude that it is feasible to extending the virtual carrier based CFO estimation algorithm to the I/Q imbalance scenario. The CFO estimation performance is evaluated through computer simulation. Impact of parameters on the estimation performance is investigated and it is consistent with our analysis. After CFO correction, a blind based I/Q imbalance estimation and compensation algorithm is applied. The final detection performance demonstrates that the virtual carrier based CFO estimation performance is good enough for subsequent I/Q imbalance estimation and compensation.

Keywords - Carrier frequency offset (CFO), In-phase and quadrature-phase (I/Q) imbalance, orthogonal frequency division multiplexing (OFDM), virtual carrier.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) technique has been adopted as the basic modulation scheme for many modern broadband wireless communication systems, such as IEEE 802.11a/g/n Wireless Local Area Networks (WLAN) [1], IEEE 802.16d/e Wireless Metropolitan Area Networks (WiMAX) [2] and 3GPP Long Term Evolution (LTE) [3] systems, etc.

OFDM is robust against frequency selective fading and timing error. However, it is very sensitive to carrier frequency offset (CFO) between the receiver and transmitter oscillators. CFO will introduce Inter-Carrier-Interference (ICI) among subcarriers and therefore results in significant performance degradation. Many algorithms have been proposed for frequency synchronization in OFDM systems which can be divided into the following main categories: The first group is based on the duplicate structure of the Cyclic Prefix (CP) in each OFDM symbol [4], [5]. The second group is based on specific-designed training symbols for synchronization [6], [7]. Other algorithms use hybrid schemes [8]. Most of these algorithms exploit the duplicate structure in OFDM signals and use correlation-based methods, either in time-domain or frequency-domain, to extract the frequency offset information. However, these frequency synchronization algorithms will suffer from significant performance degradation, if not unusable, in the presence of receiver I/Q imbalance due to the image interference introduced by it.

In the recent years, the zero-IF (or direct-conversion) architecture has been regarded as an attractive alternative to the conventional super heterodyne architecture in low-power, fully integrated receiver design. In direct-conversion receivers, the I/Q demodulation is performed in the analog domain. As a result, the gain and phase imbalance between the I and Q path come into being due to the imperfection of the analog component design. Rather than mitigating the I/Q imbalance by increasing the design time and component cost, I/Q imbalance is usually compensated digitally in base band. Many algorithms are proposed for I/Q estimation and compensation with the assumption that the system is frequency synchronized. [9]-[11] In the presence of both CFO and I/Q imbalance, they must be estimated and compensated either in sequence or jointly [12].

In this paper, we analyzed the feasibility of extending a virtual carrier based CFO estimation algorithm [13] to the situation where I/Q imbalance exists. The impact of algorithm parameter choice on the CFO estimation performance is analyzed. A blind based I/Q compensation algorithm [11] is used after CFO estimation and correction to evaluate the virtual carrier based CFO estimation performance. The paper is organized as follows. Section II describes the receiver signal model in the presence of CFO and I/Q imbalance in OFDM systems. Section III briefly introduces the principle of the virtual carrier based CFO estimation algorithm and analyzes the feasibility of extending it to the I/Q imbalance scenario. The CFO estimation performance in the presence of I/Q imbalance is presented in Section IV. Conclusions are drawn in Section V.

II. SIGNAL MODELING

In OFDM systems, suppose the RF signal before down conversion is *r*(*t*) and

$$r(t) = y(t)e^{j2\pi f_c t} + y^*(t)e^{-j2\pi f_c t} \qquad (1)$$

Where *y*(*t*) is the equivalent low pass complex baseband signal of *r*(*t*).

In direct-conversion receiver, *r*(*t*) is down-converted by a mixer with CFO and I/Q mismatch. This imperfection can be modeled by a complex Local Oscillator (LO) with time function

$$\tilde{x}_{LO}(t) = \cos(2\pi f_{LO}t) - jg\sin(2\pi f_{LO}t + \phi) \qquad (2)$$

Where $f_{LO} = f_c + \Delta f$ is the frequency of LO. *fc* is carrier frequency and Δ*f* denotes CFO between transmitter and receiver. Parameter *g* is the receiver I/Q amplitude imbalance and ϕ is the phase imbalance.

The amplitude imbalance expressed in decibels is defined as

$$\varepsilon = 10\log(1 + \left|\frac{1-g}{1+g}\right|) \qquad (3)$$

Define two parameters *K*1 and *K*2, which related to I/Q imbalance parameters by

$$K_1 = \frac{1 + ge^{-j\phi}}{2}, K_2 = \frac{1 - ge^{j\phi}}{2} \qquad (4)$$

Then $\tilde{x}_{LO}(t)$ can be reformulated as

$$\tilde{x}_{LO}(t) = K_1 e^{-j2\pi f_{LO}t} + K_2 e^{j2\pi f_{LO}t} \qquad (5)$$

Thus in the presence of CFO and I/Q imbalance, the received signal r(t) after down-conversion and low pass filtering will be denoted by *z*(*t*) and

$$\begin{aligned} z(t) &= LP\{r(t)\tilde{x}_{LO}(t)\} \\ &= K_1 y(t)e^{-j2\pi\Delta ft} + K_2 y^*(t)e^{j2\pi\Delta ft} \end{aligned} \qquad (6)$$

Equivalently, the RF imperfection described by (5) can be also described in frequency domain as

$$Z(f) = K_1 Y(f + \Delta f) + K_2 Y^*(-(f - \Delta f)) \qquad (7)$$

Z(*f*) and *Y*(*f*) are the Fourier transform of *z*(*t*) and *y*(*t*) respectively.

In OFDM system, CFO will cause Inter-Carrier-Interference while I/Q imbalance will introduce image interference from mirrored subcarriers. Let the superscript (*n*) denotes the *n*th OFDM symbol and *T* be OFDM symbol duration (excluding the CP duration). If we ignore the noise term for the moment, in the absence of CFO and I/Q imbalance, the demodulated signal at the *m*th subcarrier of the *n*th OFDM symbol will be

$Z_m^{(n)} \equiv Z^{(n)}(f = m/T) = Y_m^{(n)}$ Here, $Y_m^{(n)} = H_m^{(n)} S_m^{(n)}$, $S_m^{(n)}$ is the transmitted data on the mth subcarrier of the nth OFDM symbol and $H_m^{(n)}$ is the frequency-domain channel response at *m*th subcarrier frequency of the *n*th OFDM symbol. When there is I/Q imbalance but no CFO,

$$Z_m^{(n)} = K_1 Y_m^{(n)} + K_2 (Y_{-m}^{(n)})^* \qquad (8)$$

When there is CFO but no I/Q imbalance,

$$Z_m^{(n)} = Y_m^{(n)} + \mathrm{ICI}_1 \qquad (9)$$

When there are both I/Q imbalance and CFO,

$$Z_m^{(n)} = K_1 Y_m^{(n)} + K_2 (Y_{-m}^{(n)})^* + K_1\mathrm{ICI}_1 + K_2\mathrm{ICI}_2 \qquad (10)$$

In (9) and (10), ICI1 is caused by $y(t)e^{-j2\pi\Delta ft}$ ICI2 is caused by $y^*(t)e^{j2\pi\Delta ft}$ in (6). Since in practice |*K1*| >> |*K2*|, thus |K1ICI1| >> | K2ICI2| in (10).

III. ANALYSIS OF THE VIRTUAL CARRIER BASED CFO ESTIMATION ALGORITHM IN THE PRESENCE OF I/Q IMBALANCE

In OFDM systems, virtual carriers mean the un-modulated (or zero modulated) subcarriers in the guard band. In [13] a virtual carrier based CFO estimation algorithm is proposed without considering the I/Q imbalance effect. In the following of the section, we will examine whether it is feasible to extend this CFO estimation algorithm to the situation with I/Q imbalance in OFDM receivers. The basic idea of the virtual carrier based CFO estimation algorithm is that the total energy in the virtual carrier space should be zero, ignoring the noise component for the moment, if no CFO presents. If there is CFO in the receiver, the total energy in the virtual carrier space will be non-zero due to the ICI introduced by CFO to the virtual carrier space. In the proposed CFO estimation algorithm, to estimate the CFO the received signal should be corrected by an estimated CFO δ*f* first. Then the energy in the virtual carrier space is calculated. The optimal CFO estimation will be the one resulting in the minimum energy in virtual carrier space. When there are CFO and I/Q imbalance presenting in the OFDM receiver, the CFO in the received signal described by (5) can not be completely corrected simply by complex multiplication. In this case, the demodulated signal on the *m*th subcarrier of the *n*th OFDM symbol after CFO correction by δf will be

$$Z_m^{(n)}(\delta f) = \sum_{l=0}^{N-1} z^{(n)}(t = kT/N)e^{j2\pi(\delta fT+m)k/N} \qquad (11)$$

$$m = 0,1,2...N-1$$

Where N is the total number of subcarriers. When δf = Δf

$$Z_m^{(n)}(\Delta f) = K_1 Y_m^{(n)} + K_2 (Y_{-m}^{(n)})^* + K_2 \mathrm{ICI}_2 \qquad (12)$$

When $\delta f \neq \Delta f$

$$Z_m^{(n)}(\delta f) = K_1 Y_m^{(n)} + K_2 (Y_{-m}^{(n)})^* + K_1 \mathrm{ICI}_1 + K_2 \mathrm{ICI}_2 \qquad (13)$$

In (13), |ICI1| >> | ICI2| if |K1| >> |K2| and it is true for realistic situation. If *m* and *–m* correspond to the indices of virtual subcarriers, $Y_m^{(n)} = Y_{-m}^{(n)} = 0$. Thus an accurate CFO estimation and correction will result in small energy leakage (≈| *K*2ICI2|2) in the virtual carrier space while false CFO estimation and correction will introduce large energy leakage (≈|*K*1ICI1 + *K*2ICI2|2). Based on the above analysis, CFO can also be estimated by finding the minimum energy leakage in the virtual carrier space in the presence of I/Q imbalance. Therefore, a cost function *J* (δ*f*) can be constructed as the total energy in an observed virtual carrier set MVC over a certain number of OFDM symbols

$$J(\delta f) = \sum_{n}^{N_s} \sum_{m \in M_{VC}} Z_m^{(n)} (Z_m^{(n)})^* \qquad (14)$$

Where Ns is the number of OFDM symbols used in cost function calculation. Thus the optimal CFO estimation could be *opt* δ*f* which minimize the cost function *J* (δ*f*). Hereinafter, we will examine the characteristics of ICI terms in (12) and (13) which are important for virtual carrier selection in CFO estimation. If the normalized CFO is Δ*fT* (normalized with respect to the subcarrier spacing), the demodulated signal on the *m*th subcarrier of the the *n*th OFDM symbol could be represented by (without I/Q imbalance)

$$Z_m^{(n)}(\Delta fT) = \sum_{l=0}^{N-1} c_{m-l} Y_l^{(n)} = c_0 Y_m^{(n)} + \sum_{\substack{l=0 \\ l \neq m}}^{N-1} c_{m-l} Y_l^{(n)} \qquad (15)$$

Where *k c* is the ICI coefficients and defined by [14]

$$c_k = \frac{1}{N} e^{j\frac{\pi(\Delta fT - k)(N-1)}{N}} \cdot \frac{\sin(\pi(\Delta fT - k))}{\sin\frac{\pi(\Delta fT - k)}{N}} \qquad (16)$$

Where *k* is the distance, in terms of number of subcarriers, between the target subcarrier and its interfere subcarriers. When Δ*fT* =0.3 , the magnitude of the ICI coefficients *k c* is plotted in Fig.1 as a function of subcarrier spacing *k*.

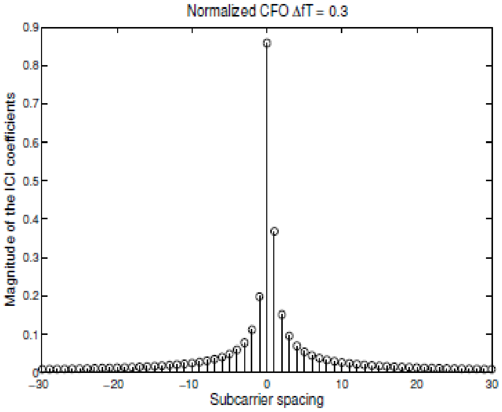

Fig. 1. Magnitude of the Inter-Carrier-Interference coefficients

As can be seen in Fig.1, only the subcarriers in the vicinity of the target subcarrier have significant impact on it. With the distance *k* increasing, the impact of an interfere subcarrier on the target subcarrier gets smaller. In the case depicted in Fig.1 (Δ*fT* = 0.3), only 3~5 subcarriers in the vicinity of the target subcarrier dominate the ICI. In theory, virtual carriers do not cause ICI to other subcarriers. Only the data subcarriers cause ICI to other subcarriers. Therefore, the virtual carriers which are far from the data subcarriers can be regarded as ICI free.

IV. SIMULATION RESULTS

In this section, we will evaluate the feasibility and performance of the CFO estimation algorithm in the presence of I/Q imbalance through computer simulation. In the simulation, the normalized CFO added to the OFDM system is Δ*fT* = 0.3 . The receiver amplitude imbalance ε in (3) is 0.5 dB and phase imbalance ϕ is 4 degree. FFT size for OFDM is N = 1024, the number of data subcarriers Nd = 600. Modulation scheme used on each subcarrier is 64-QAM. The reference channel model used in simulation is 6-ray TU (Typical Urban). Fig.2 is the cost function *J* (δ*f*) depicted as a function of estimated CFO δ*fT* in the TU channel (SNR = 10dB). As can be seen from Fig.2, the optimal CFO estimation can be obtained by searching for the minimum of $J(\delta f)$ in certain frequency range. The searching window may not be very wide after receiver performed coarse timing and frequency synchronization.

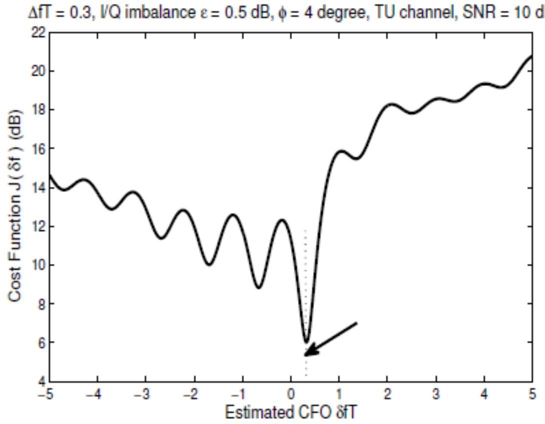

Fig. 2. Cost function for CFO estimation
I/Q imbalance: $\varepsilon = 0.5$ dB, $\phi = 4$ degree

To examine the impact of choice of algorithm parameters on CFO estimation performance, we first investigate the CFO estimation MSE in AWGN channel. Here CFO estimation MSE is defined as

$$MSE = E\{(\Delta\hat{f}\,T - \Delta f T)^2\} = E\{(\delta f_o T - \Delta f T)^2\} \quad (17)$$

Where o δf is the optimal estimation of Δf which minimizes the cost function $J(\delta f)$.The CFO estimation MSE vs different number of virtual carrier and OFDM symbol used is shown in Fig. 3. We can see from the simulation result that the MSE will decrease when more virtual carriers and OFDM symbols are used for CFO estimation. However when the number of virtual carrier used is greater than a threshold (in the simulation case the threshold approximates 10) the decreasing rate of MSE with the increasing number of virtual carrier used will get slow or even can be neglected. This simulation results are consistent with our analysis in Section III. In high SNR, the virtual carriers close to the data subcarriers dominate the performance of CFO estimation. In this case, we just use 5 virtual carriers on either side of the spectrum. From Fig.3 we can see that MSE will decrease when more OFDM symbols are used in CFO estimation. But the performance improvement will get smaller with the increasing of OFDM symbols used in estimation. From the principle of the CFO estimation algorithm, we know that the introducing of more OFDM symbols in estimation will increasing the computation complexity significantly compared with the introducing of more virtual carriers. Thus the number of OFDM symbol used should be a tradeoff between performance and complexity. From the simulation results, we may choose the number of OFDM symbols used to be 6.

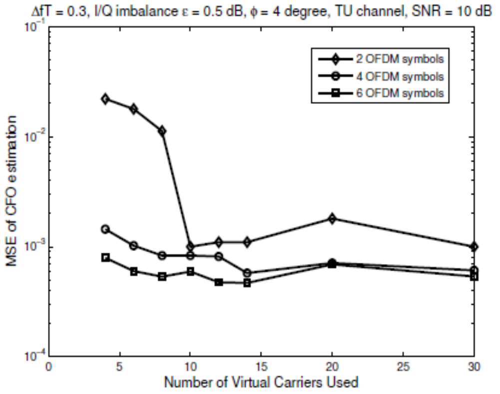

Fig. 3. MSE of CFO estimation vs number of virtual carriers and OFDM symbols used
I/Q imbalance $\varepsilon = 0.5$ dB, $\phi = 4$ degree

The MSE performance is also evaluated with respect to different SNR in TU channel. The simulation result is shown in Fig.4. In this simulation, more virtual carriers are used for cost function calculation to average out the noise effect in low SNR situation. For SNR = 5 dB, the MSE is obtained by using 40 virtual carriers and 6 OFDM symbols in CFO estimation. From the estimation MSE shown in Fig.4, for low SNR the CFO estimation accuracy may not be adequate for fine frequency synchronization. Therefore, after performing CFO estimation in the presence of I/Q imbalance, the CFO in the local oscillator is corrected through feedback loop.Then I/Q imbalance can be estimation and compensated in the presence of small residual CFO. After I/Q imbalance is compensated, fine frequency synchronization can be carried out either in time or frequency domain.

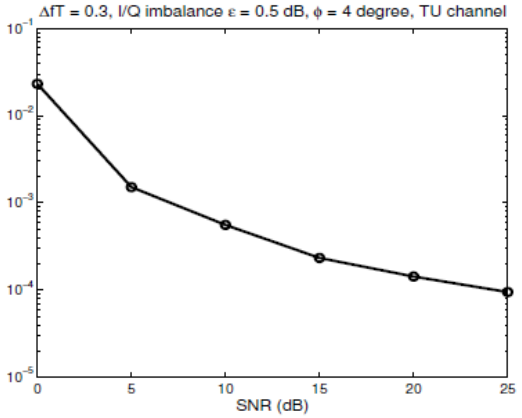

Fig. 4. MSE vs SNR in TU channel
I/Q imbalance: $\varepsilon = 0.5$ dB, $\phi = 4$ degree

The impact of CFO estimation performance on the subsequent I/Q imbalance estimation and compensation stage is also investigated through simulation. In the simulation, CFO is first estimated in the presence of I/Q imbalance and corrected. Then a blind based algorithm [11] is used for I/Q imbalance estimation and compensation. Typical I/Q imbalance parameters are used in the simulation. The amplitude imbalance ε is 0.2 dB and phase imbalance ϕ is 2 degree. The final detected symbol error rate (SER) performance vs SNR for different cases is plotted in Fig.5. From the simulation results we can see that before CFO and I/Q imbalance compensation there will be an error floor which will cause significant performance loss especially in high SNR region. If we use the CFO estimation results to correct the received signal then perform I/Q imbalance estimation and compensation thereafter, the SER performance will almost overlap with the one which is I/Q imbalance compensated but without CFO. Therefore, the CFO estimation performance is good enough for I/Q imbalance estimation and compensation stage to work properly.

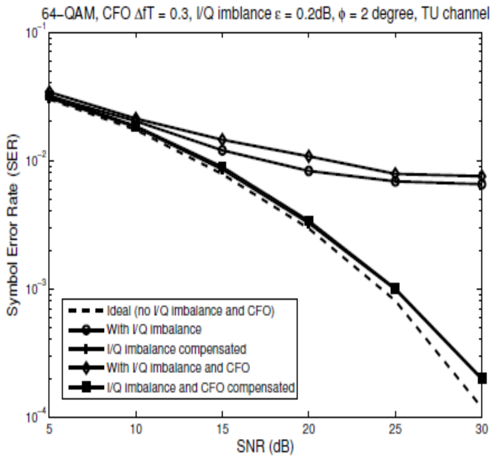

Fig. 5. SER vs SNR before/after CFO, I/Q imbalance compensation in TU channel, I/Q imbalance ε = 0.2 dB, ϕ = 2 degree

The 64-QAM constellation before and after CFO and I/Q imbalance compensation is plotted in Fig.6 the upper-left figure shows the constellation with I/Q imbalance only and the upper-right figure is the constellation after I/Q imbalance compensation. The bottom-left figure shows the constellation in the presence of both CFO and I/Q imbalance. In this scenario, the constellations are getting blurred thus result in poor SER performance. The constellation after CFO and I/Q imbalance compensation is shown in the bottom-right figure. As can be seen that it is similar to the upper-right one which is I/Q imbalance compensated without CFO. That explains why they have similar detection performance.

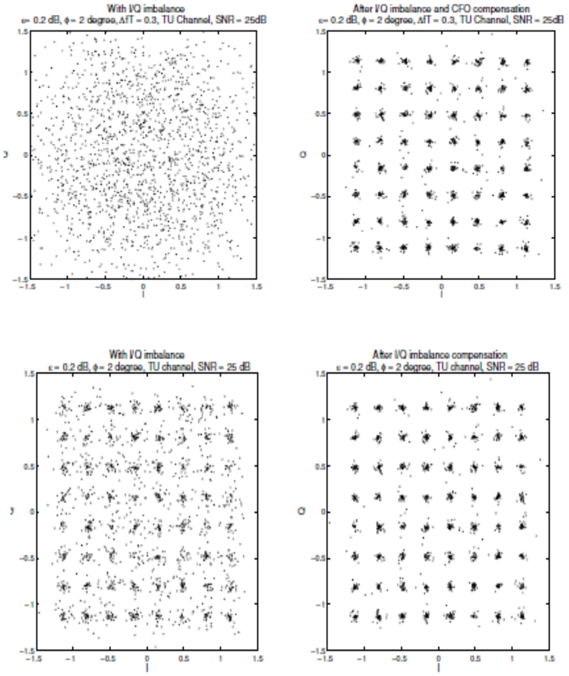

Fig. 6 : 64-QAM constellation before/after CFO, I/Q imbalance compensation in TU channel (SNR = 25dB) I/Q imbalance: ε = 0.2 dB, ϕ = 2 degree.

V. CONCLUSIONS

In this paper, we analyzed the feasibility of using a virtual carrier based CFO estimation algorithm in the presence of I/Q imbalance in OFDM systems. Based on the analysis of the signal model with receiver CFO and I/Q imbalance impairment we conclude that the virtual carrier based CFO estimation algorithm is applicable for the I/Q imbalance scenario. The conclusion is verified through computer simulation. The CFO estimation performance is investigated through computer simulation as well. Impact of algorithm parameters choice on the CFO estimation performance is investigated and it is consistent with our analysis. After correcting CFO with the estimated results, a blind based I/Q imbalance compensation algorithm is applied. The final detection SER performance demonstrates that the virtual carrier based CFO estimation performance is good enough for subsequent I/Q imbalance estimation and compensation.

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