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Anti-synchronization of discrete-time chaotic systems using optimization algorithms

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Abstract - In this paper, anti-synchronization of discrete chaotic system based on optimization algorithms are investigated. Different controllers have been used for anti-synchronization of two identical discrete chaotic systems. A proportional-integral-derivative (PID) control is used and its parameters is tuned by the four optimization algorithms, such as genetic algorithm (GA), particle swarm optimization (PSO), modified particle swarm optimization (MPSO) and improved particle swarm optimization (IPSO). Simulation results of these optimization methods to determine the PID controller parameters to anti-synchronization of two chaotic systems are compared. Numerical results show that the improved particle swarm optimization has the best result.

Keywords: *chaotic system; anti-synchronization; Particle swarm optimization; genetic algorithm*

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I. INTRODUCTION

Many nonlinear dynamical systems have been found to show a kind of behavior known as chaos. A chaotic dynamical system has complex dynamical behaviors that possess some special features such as being extremely sensitive to tiny variations of initial conditions, broad spectra of Fourier transforms, and fractal properties of the motion in phase space [1]. Since synchronization of two coupled chaotic systems with different initial values was demonstrated, there has been an increasing interest in the study of chaos synchronization and its applications in various fields ranging from physics, biology, chemistry, mathematics to engineering [2].

A basic configuration for chaos synchronization is the master–slave (drive-response) pattern, where the response chaotic system must track the drive chaotic trajectory [1]. To synchronize chaotic systems, many kinds of controllers have been presented such as feedback control [3], fuzzy control [4], adaptive control [5] and impulsive control [6]. And there are many optimization methods such as ant colony, harmony search, genetic algorithm, particle swarm optimization, random search, etc.

The concept of synchronization has been extended to the scope, such as generalized synchronization [7, 8], phase synchronization [7], lag synchronization [9], and even anti phase synchronization (APS) [10, 11]. APS can also be interpreted as anti-synchronization (AS), which is a phenomenon that the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the driving system. Therefore, the sum of two signals are expected to converge to zero when either AS or APS appears [12]. Recently, several control method has been applied to anti-synchronize chaotic systems [13–17].

In this paper a proportional–integral–derivative (PID) control as a controller and for tuning the values of the controller's parameters the methods of genetic algorithm (GA), particle swarm optimization (PSO), modified particle swarm optimization (MPSO) and improved particle swarm optimization (IPSO) are used.

The paper is organized as follows:

The problem and control formulation are studied in Section 2. In section 3 optimization algorithms are presented. Numerical results and conclusion are given in section 4 and 5, respectively.

II. PROBLEM DESCRIPTION

In this study, the goal is anti-synchronization of two delayed identical discrete systems by using PID. The master and slave systems are described as follows:

, (1)

, (2)

where is the control.

A. PID Control

PID controllers have been used for decades because they are simple and easy to implement. They are widely applied in industry to solve various control problems. During this time, many modifications have been presented in the literature. In this paper, the transfer function of PID controller is described in the continuous s-domain (Laplace operator) [18] by the following equation

— – , (3)

where U(s) and E(s) are the control and tracking error signals in s-domain, respectively; $K_p$is the proportional gain, K_i is the integral gain, and K_d is the derivative gain. T_i is the integral action time and $T_d$is referred to as the derivative action time or rate time. The output of the PID controller in time domain is given by

$$u(t) = K_p.e(t) + K_i \int_0^t e(\tau)d\tau + K_d.\frac{de(t)}{dt}, \quad (4)$$

where u(t) and e(t) are the control and tracking error signals in time domain, respectively. Using trapezoidal approximations for (4) to obtain the discrete control law, we have

$$\begin{aligned} u(k) &= u(k-1) + K_p.[e(k) - e(k-1)] \\ &+K_i.\frac{T_s}{2}.[e(k) - e(k-1)] \\ &+K_d.\frac{T_s}{2}.[e(k) - 2e(k-1) + e(k-2)], \quad (5) \end{aligned}$$

where T_s is the sampling period. The proportional part of the PID controller reduces error responses to disturbances. The integral term of the error eliminates steady state error and the derivative term of error dampens the dynamic response and thereby improves stability of the system. How to solve these three gains to meet the required performance is the most key in the PID control system. However, it is difficult to find the optimal set of PID gains for nonlinear dynamical systems.

B. *Nonlinear Discrete Chaotic System*

In this section, we illustrate the anti-synchronization by the proposed methods for chaotic systems. We employ Lozi's model as an example of discrete chaotic systems that are considered to be anti-synchronized using the proposed PID control. The master system is given by

$$x_1(k+1) = 1 - a.|x_1(k)| + x(k), \quad (6)$$

$$x(k+1) = b.x_1(k), \quad (7)$$

where $a = 1.7$, $b = 0.5$, and x is the master state. The corresponding slave system is described by

$$y_1(k+1) = 1 - a.|y_1(k)| + y(k) + u(k), \quad (8)$$

$$y_1(k+1) = 1 - a.|y_1(k)| + y(k) + u(k), \quad (9)$$

where y is the slave state and u is the external control force that adopts the PID control of (5). For the identical discrete chaotic systems (7) and (9) without control u, the state trajectories of these chaotic systems will separate each other if their initial conditions are not the same. However, the state trajectories can approach anti-synchronization for any initial condition if an appropriate controller is utilized. Hence the purpose of this paper is to apply the discussed optimization methods approaches to find out the optimal PID control gains such that chaos anti-synchronization for two Lozi's chaotic systems is achieved.

III. REVIEW OF OPTIMIZATION ALGORITHMS

A. *Genetic Algorithms*

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution. The genetic algorithm repeatedly modifies a population of individual solutions. Each individual of population is called chromosome. At each step, the genetic algorithm selects individuals at random from the current population to be parents and uses them produce the children for the next generation. Over successive generations, the population "evolves" toward an optimal solution. You can apply the genetic algorithm to solve a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear.

The genetic algorithm uses three main types of rules at each step to create the next generation from the current population:

- *Selection rules* select the individuals, called *parents*, that contribute to the population at the next generation.
- *Crossover rules* combine two parents to form children for the next generation.
- *Mutation rules* apply random changes to individual parents to form children.

Evaluation of each chromosome is based on a fitness function that is problem-dependent. Given an initial population of elements, GAs use the feedback from the evaluation process to select fitter solution, eventually converging to a population of high-performance solutions. It is necessary to know that GAs do not guarantee a global optimum solution [19].

B. *Partial Swarm Optimization*

Particle Swarm Optimization (PSO) is a population-based optimization method which is inspired by life of natural swarms such as birds and fishes [20, 21]. PSO is basically developed through simulation of bird flocking in two-dimensional space [22]. The position of each agent is represented by XY axis position and also the velocity is expressed by V_x (velocity of X axis) and V_y (velocity of Y axis). Modification of the agent (particle) position is realized by the position and velocity information. In PSO algorithm first an initial population of particles with random positions and velocities is created. In subsequent iterations every particle adjusts its position and velocity by its own experience and other particles' information. A

fitness function determines how good the position of each particle which is a potential solution of the problem is. Based on this information the motion of every particle is a combination of the following terms:

- Current velocity of the particle (inertia term).
- Motion toward the best position of the particle obtained until current iteration (p_{best}) (cognitive term).
- Motion toward the best position of the group obtained until current iteration (g_{best}) (social learning term).

Therefore, if the position and velocity of particle *i* are denoted, respectively, by $X_i = (x_{i,1}, x_{i,2}, \dots, x_{i,d})$ and $V_i = (v_{i,1}, v_{i,2}, \dots, v_{i,d})$, where *d* is the dimension of the search space, the velocity and position of that particle at iteration $t + 1$ will be as follows [12]

$$V_{i,t+1} = w \times V_{i,t} + c_1 \times r_{1,t} \times \left(p_{best,i,t} - X_{i,t}\right) + c_2 \times r_{2,t} \times \left(g_{best,i,t} - X_{i,t}\right), \quad (10)$$

$$X_{i,t+1} = X_{i,t} + k_c \times V_{i,t+1}, \quad i = 1,2, \dots, n, \quad (11)$$

where '*n*' is the population size, '*t*' is the current iteration, *X* is the inertia weight, $r_{1,t}$ and $r_{2,t}$ are random numbers between 0 and 1, the $w \in [0,1]$. c_1 , c_2 and k_c are constants and is the constriction coefficient. Suitable values for X, c_1, c_2, and k_c may be chosen depending on problem.

Using the below equation,

$$w = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times iter, \quad (12)$$

where $w_{max} = 0.9, w_{min} = 0.4$, a certain velocity, which gradually gets close to p_{best} and g_{best}, can be calculated. This method is called Modified PSO (MPSO).

As the development of PSO method, we propose Improved PSO (IPSO). The main differences between proposed IPSO and conventional PSO are:

1) Velocity strategy equation employed in conventional PSO is not suitable for large systems. Also in MPSO the range of minimum and maximum velocity limit is quite large which makes the approach slow rate of convergence, and takes more computational time and local convergence. In order to overcome above difficulty, a new velocity strategy equation is formulated suitably for any number of systems in the proposed IPSO method.

2) The basic system equation of PSO can be considered as a kind of difference equations. Therefore, the system dynamics, namely, search procedure, can be analyzed by the eigen value analysis. The Improved PSO (IPSO) utilizes the eigen value analysis and controls system behaviour so that the system behaviour has the following features:

- The system does not diverge in a real value region and finally converge.
- The system can search different regions efficiently.

The velocity of IPSO can be expressed as given below:

$$V_{i,t+1} = K \times V_{i,t} + c_1 \times r_{1,t} \times \left(p_{best,i,t} - X_{i,t}\right) + c_2 \times r_{2,t} \times \left(g_{best,i,t} - X_{i,t}\right), \quad (13)$$

where

$$K = \frac{2}{\left|2 - \phi - \sqrt{\phi^2 - 4 \times \phi}\right|}, \quad (14)$$

such that $\phi = c_1 + c_2$; $\phi > 4$; $i = 1,2, \dots, n$; '*n*' is the population size. The convergence characteristics of the system are controlled by ϕ. IPSO examines the convergence of the search procedure based on the mathematical theory. The amplitude of the each agent's oscillation decreases as it focuses on a previous best point. The IPSO could generate higher quality solutions than the conventional PSO method. So the proposed approach has stable convergence characteristics, avoids premature convergence and takes less computational time.

C. *Fitness Function*

In each optimization process there is a fitness function that should reach its optimum value. In this paper the fitness function F is defined as [18],

$$F = \sum_{k=1}^{N} |x(k) + y(k)| = \sum_{k=1}^{N} |e(k)|, \quad (15)$$

where *e(k)* is the error signal between the master and slave states and *N* is the total number of sampling. The optimization problem involves finding $[K_p^*, K_i^*, K_d^*]$ in PID control such that the F fitness function of the system is minimized.

IV. NUMERICAL SIMULATION

Here we will illustrate the anti-synchronization PID controller design for the above two Lozi chaotic systems given by equations (7) and (9) with different initial value conditions, $x_1(0) = x(0) = 0.1$, and $y_1(0) = y(0) = 0.6$. We solved the optimization problem with N = 25 and $T_s = 0.01$ sec.

The GA optimization characteristics are as following:

Binary representation with an individual length of 16 bits for each design variable, population size of 150 chromosomes, mutation probability of 0.2, crossover probability of 0.6, keep percent of 0.2, roulette wheel selection with elitism, maximum iterations 1000 and number of initialization 10.

The PSO characteristics are as follows:

Population size of 150, maximum iterations 500, $(K_p, K_i, K_d) \in (0,4)$, w=0.3, $c_1 = c_2 = 2$ and number of initialization 10.

The MPSO characteristics are chosen as:

Population size of 150, maximum iterations 500, $(K_p, K_i, K_d) \in (0,4)$, $w \in [0.4, 0.9]$, $c_1 = c_2 = 2$ and number of initialization 10.

The IPSO characteristics are chosen as:

Population size of 150, maximum iterations 500, $(K_p, K_i, K_d) \in (0,4)$, $c_1 = 2, c_2 = 3$, w=K=0.38 and number of initialization 10.

Convergence results for anti-synchronization of Lozi map via PID control based on fitness function are given in table (1) and table (2) is determining the optimal values of the parameters of PID control of the discussed system via GA, PSO, MPSO and IPSO.

Fig. 1 shows the state responses of the master and slave systems and the error signals using IPSO for tuning the parameters of the controller for Lozi chaotic system. As it can be seen in table (1) the IPSO optimization method gives the best results, because the values of minimum, maximum and mean of fitness function are less than other methods.

TABLE II. CONVERGENCE RESULTS

Optimization method	**Minimum F**	**Mean F**	**maximum F**
GA	8.9642	8.9817	8.9710
PSO	8.9638	8.9734	8.9782
MPSO	8.9638	8.9737	8.9780
IPSO	8.9638	8.9684	8.9782

TABLE III. PID PARAMETRS

Parameter	**GA**	**PSO**	**MPSO**	**IPSO**
K_p	0.0172	0.0185	0.0236	0.0277
K_i	2.4396	2.2302	1.2102	0.3869
K_d	0.0396	0	0.0016	1.0000e-004
Minimum F	8.9642	8.9638	8.9638	8.9638

V. CONCLUSION

In this paper, the methods IPSO, MPSO, PSO and GA are proposed to tune PID controller gains in anti-synchronization application of two chaotic systems. The numerical simulation shows that the well-known Lozi mapping with different initial conditions for the master and slave are chosen to illustrate the proposed scheme, and simulations are also given to verify the effectiveness of the proposed improved particle swarm optimization algorithm.

According to our results the proposed method based on IPSO can be successfully applied to control and anti-synchronization problems of discussed systems. Finally, the optimal values of PID control are achieved and it is compared with other optimization algorithms such as PSO, MPSO and GA. The results show that IPSO has better performance than the other methods.

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