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Synchronization and Anti-Synchronization of Two Identical Hyperchaotic Systems Based on Active Backstepping Design



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Abstract— This paper presents an active backstepping design method for synchronization and anti-synchronization of two identical hyperchaotic Chen systems. The proposed control method, combining backstepping design and active control approach, extends the application of backstepping technique in chaos control. Based on this method, different combinations of controllers can be designed to meet the needs of different applications. Numerical simulations are shown to verify the results.

Keywords: Synchronization, anti-synchronization, hyperchaotic system, active backstepping design.

I. INTRODUCTION

Since the pioneering work of Pecora and Corroll, chaos synchronization has become an active research subject in the nonlinear field for its many potential applications in chemical reactions, biological systems, information processing, power converters, secure communications, etc. The excitement is well comprehended in the academic community as its potential implications and applications are bountiful. Another interesting phenomenon discovered was the anti-synchronization (AS), which is noticeable in periodic oscillators. It is a well-known fact that the first observation of synchronization between two oscillators by Huygens in the seventeenth century was, in fact, an AS between two pendulum clocks. Recent re-investigation of Huygens experiment by Blekhman [1] shows that either synchronization or AS can appear depending on the initial conditions of the coupled pendula. Here, AS can also be interpreted as anti-phase synchronization (APS) [2,3]. In other words, there is no difference between AS and APS for oscillators with identical amplitudes. So far, there exist many types of synchronization such as complete synchronization (CS) [4], phase synchronization (PS) [5,6], anti-synchronization [2,3], partially synchronization [8], generalized synchronization (GS) [9], lag synchronization [10], Q-S synchronization [11], projective synchronization [12,13], etc. In this paper we will consider the synchronization and anti-synchronization of two identical hyperchaotic Chen systems. The active backstepping design [14-19] is employed to realize the synchronization and anti-synchronization of hyperchaotic systems. The organization of this paper is as follows. In Section 2, the hyperchaotic Chen system is described. Section 3,

presents the synchronization between two identical hyperchaotic Chen systems by active backstepping design. In Section 4, the anti-synchronization between two identical hyperchaotic Chen systems is realized by active backstepping design. Finally, conclusion is given in Section 5.

II. SYSTEM DESCRIPTION

The hyperchaotic Chen system is given by

$$\left\{ \begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -x_1 - x_2 - x_3 + x_1^2 x_2 \end{array} \right. \quad (1)$$

where x_1 and x_2 are state variables, and a and b are real constants.

III. Synchronization of two identical hyperchaotic Chen systems via active backstepping

In this section, we develop a design procedure via active backstepping which can realize the synchronization and anti-synchronization of two identical hyperchaotic systems by giving the corresponding update laws. To verify the effectiveness of the proposed method, two identical hyperchaotic Chen systems as the master and slave system for detailed description is used.

Suppose the master system is defined as the following form

$$\begin{cases} \dot{x}_1 = a(y_1 - x_1) + w_1 \\ \dot{y}_1 = dx_1 - x_1z_1 + cy_1 \\ \dot{z}_1 = x_1y_1 - bz_1 \\ \dot{w}_1 = y_1z_1 + rw_1 \end{cases} \quad (2)$$

and the slave system is taken as follows:

$$\begin{cases} \dot{x}_2 = a(y_2 - x_2) + w_2 + u_1 \\ \dot{y}_2 = dx_2 - x_2z_2 + cy_2 + u_2 \\ \dot{z}_2 = x_2y_2 - bz_2 + u_3 \\ \dot{w}_2 = y_2z_2 + rw_2 + u_4 \end{cases} \quad (3)$$

where $u_i(t)(i = 1,2,3,4)$ are controllers to be designed by the following backstepping method.

Define $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$, and $e_4 = w_2 - w_1$.

Subtracting Eq. (3) from Eq. (2), we have

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 &= de_1 - x_2z_2 - x_1z_1 + ce_2 + u_2 \end{aligned} \quad (4)$$

$$\dot{e}_3 = x_2y_2 + x_1y_1 - be_3 + u_3$$

$$\dot{e}_4 = y_2z_2 + y_1z_1 + re_4 + u_4$$

Our goal is to find the controller functions which can make the systems (2) and (3) realize the synchronizatin by active backstepping design, i.e.

$$\lim_{t \rightarrow \infty} \|e_i(t)\| = 0, i = 1, 2, 3, 4.$$

The backstepping design procedure includes four steps as below:

Step 1.

Let $k_1 = e_1$, its derivative is

$$\dot{k}_1 = \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \quad (5)$$

Choose $e_2 = \alpha_1(k_1)$ is regarded as a virtual controller. For the design of $\alpha_1(k_1)$ to stabilize k_1 - subsystem (5), select the first Lyapunov function

$$v_1 = \frac{1}{2}k_1^2 \quad (6)$$

The derivative of v_1 is

$$\dot{v}_1 = k_1(\alpha_1(k_1) - ak_1 + \alpha_3(k_1, k_2, k_3) + u_1) \quad (7)$$

If we set $\alpha_1(k_1) = 0, \alpha_3(k_1, k_2, k_3) = 0,$

and $u_1 = -ak_1$

Then $\dot{v}_1 = -2ak_1^2 < 0$ is negative definite.

This implies that the k_1 - subsystem (5) is asymptotically stable. Since the virtual control function α_1 is estimative, the error between e_2 and α_1 is

$$k_2 = e_2 - \alpha_1(k_1) \quad (8)$$

Study the following (k_1, k_2) -subsystem

$$\dot{k}_1 = -2ak_1 + ak_2 + k_4 \quad (9)$$

$$\dot{k}_2 = -ck_2$$

Step 2.

In order to stabilize the (k_1, k_2) -subsystem (9), choose the second Lyapunov function defined by

$$v_2 = v_1 + \frac{1}{2}k_2^2 \quad (10)$$

Its derivative is

$$\begin{aligned} \dot{v}_2 &= -2ak_1^2 \\ &+ k_2(dk_1 + ck_2 - x_1z_1 - x_2z_2 + u_2) \end{aligned} \quad (11)$$

Let $u_2 = x_1z_1 + x_2z_2 - dk_1 - 2ck_2$ then $\dot{v}_2 = -2ak_1^2 - ck_2^2 < 0$ makes subsystem (9) asymptotically stable. Similarly, assume that $k_3 = e_3 - \alpha_2(k_1, k_2)$ then we will have:

$$\begin{cases} \dot{k}_1 = -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 = -ck_2 \\ \dot{k}_3 = -(b+1)k_3 \end{cases} \quad (12)$$

Step 3.

To stabilize system (12), consider the following Lyapunov function

$$v_3 = v_2 + \frac{1}{2}k_3^2 \quad (13)$$

Taking its derivative, we have

$$\dot{v}_3 = -2ak_1^2 - ck_2^2 + k_3(-be_3 + x_1y_1 + x_2y_2 + u_3) \quad (14)$$

Let $u_3 = -x_1y_1 - x_2y_2 - k_3$ and $\alpha_2(k_1, k_2) = 0$, then $\dot{v}_3 = -2ak_1^2 - ck_2^2 - k_3^2 < 0$ is negative definite and makes the (k_1, k_2, k_3) system (12) asymptotically stable. similarly assume that $k_4 = e_4 - \alpha_3(k_1, k_2, k_3)$, then we obtain

$$\begin{aligned} \dot{k}_1 &= -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 &= -ck_2 \\ \dot{k}_3 &= -(b+1)k_3 \\ \dot{k}_4 &= (r-1)e_4 \end{aligned} \tag{15}$$

Step 4.

In order to stabilize system (15), the following Lyapunov function can be chosen as:

$$v_4 = v_3 + \frac{1}{2}k_4^2 \tag{16}$$

Its derivative is

$$\dot{v}_4 = -2ak_1^2 - ck_2^2 - k_3^2 + k_4(r\alpha_3(k_1, k_2, k_3) + y_1z_1 + y_2z_2 + u_4) \tag{17}$$

Let $u_4 = -y_1z_1 - y_2z_2 - k_4$ and $\alpha_3 = 0$, so $\dot{v}_4 = -2ak_1^2 - ck_2^2 - k_3^2 - k_4^2 < 0$, is negative definite and the (k_1, k_2, k_3, k_4) system (15) is asymptotically stable.

3.1. Simulation and results

In this simulation the parameters are chosen as $a = 35, b = 3, c = 12, d = 7, r = 0.5$, so that, the systems exhibit a hyperchaotic behavior. The initial values of the master and slave systems are $x_1(0) = -20, y_1(0) = 0, z_1(0) = 0, w_1(0) = 15$ and $x_2(0) = 5, y_2(0) = 7, z_2(0) = 9, w_2(0) = 11$. Fig. 1 displays the time response of states x_1, y_1, z_1, w_1 for the master system (2) and the states x_2, y_2, z_2, w_2 for the slave system (3). Fig. 2 displays the synchronization errors of systems (2) and (3). It can be seen from the figures that the synchronization errors converge to zero rapidly.

IV. Anti-synchronization of two identical hyperchaotic Chen systems via active backstepping

In order to determine the control functions and to realize the anti-synchronization via active backstepping between the systems in Eqs. (2) and (3). We add Eq. (2) to Eq. (3) to get:

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + e_4 + u_1 \\ \dot{e}_2 &= de_1 - x_2z_2 + x_1z_1 + ce_2 + u_2 \end{aligned} \tag{18}$$

$$\dot{e}_3 = x_2y_2 - x_1y_1 - be_3 + u_3$$

$$\dot{e}_4 = y_2z_2 - y_1z_1 + re_4 + u_4$$

where $e_1 = x_2 + x_1, e_2 = y_2 + y_1, e_3 = z_2 + z_1$, and $e_4 = w_2 + w_1$. Our goal is to find proper control functions, such that system Eq. (3) globally anti-synchronizes system Eq. (2), i.e.

$$\lim_{t \rightarrow \infty} \|e_i(t)\| = 0, i = 1, 2, 3, 4.$$

Now, as previous part, we design the controllers of Eq. (3) according to active backstepping method.

Step 1.

Let $k_1 = e_1$, its derivative is

$$\dot{k}_1 = \dot{e}_1 = a(e_2 - e_1) + e_4 + u_1 \tag{19}$$

Let $e_2 = \alpha_1(k_1)$ is regarded as a virtual controller. For the design of $\alpha_1(k_1)$ to stabilize k_1 -subsystem (19), choose the first Lyapunov function

$$v_1 = \frac{1}{2}k_1^2 \tag{20}$$

Its derivative is given by

$$\dot{v}_1 = k_1(a\alpha_1(k_1) - ak_1 + \alpha_3(k_1, k_2, k_3) + u_1) \tag{21}$$

If we set $\alpha_1(k_1) = 0, \alpha_3(k_1, k_2, k_3) = 0$,

and $u_1 = -ak_1$.

Then $\dot{v}_1 = -2ak_1^2 < 0$ is negative definite. This implies that the k_1 -subsystem (19) is asymptotically stable. Since the virtual control function α_1 is estimative, choose

$$k_2 = e_2 - \alpha_1(k_1) \tag{22}$$

Study the following (k_1, k_2) -subsystem

$$\begin{cases} \dot{k}_1 = -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 = -ck_2 \end{cases} \tag{23}$$

Step 2.

To stabilize the (k_1, k_2) -subsystem (23), the following Lyapunov function is chosen

$$v_2 = v_1 + \frac{1}{2}k_2^2 \tag{24}$$

Its derivative is

$$\begin{aligned} \dot{v}_2 &= -2ak_1^2 \\ &+ k_2(dk_1 + ck_2 + x_1z_1 - x_2z_2 + u_2) \end{aligned} \tag{25}$$

if $u_2 = -x_1z_1 + x_2z_2 - dk_1 - 2ck_2$ then $\dot{v}_2 = -2ak_1^2 - ck_2^2 < 0$ makes subsystem (23) asymptotically stable. Similarly, assume that $k_3 = e_3 - \alpha_2(k_1, k_2)$ then

$$\begin{cases} \dot{k}_1 = -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 = -ck_2 \\ \dot{k}_3 = -(b+1)k_3 \end{cases} \tag{26}$$

Step 3.

For stabilization of system (26), let the Lyapunov function candidate V_3 be such that:

$$v_3 = v_2 + \frac{1}{2}k_3^2 \quad (27)$$

Taking the time derivative of V_3 , gives

$$\dot{v}_3 = -2ak_1^2 - ck_2^2 + k_3(-be_3 + x_1y_1 + x_2y_2 + u_3) \quad (28)$$

Select $u_3 = +x_1y_1 - x_2y_2 - k_3$ and $\alpha_2(k_1, k_2) = 0$, so $\dot{v}_3 = -2ak_1^2 - ck_2^2 - k_3^2 < 0$ is negative definite and the (k_1, k_2, k_3) system (26) is asymptotically stable. Now, choosing

$k_4 = e_4 - \alpha_3(k_1, k_2, k_3)$, cause the following equation

$$\begin{aligned} \dot{k}_1 &= -2ak_1 + ak_2 + k_4 \\ \dot{k}_2 &= -ck_2 \end{aligned} \quad (29)$$

$$\dot{k}_3 = -(b+1)k_3$$

$$\dot{k}_4 = (r-1)e_4$$

Step 4.

To stabilize system (15), the following Lyapunov is selected:

$$v_4 = v_3 + \frac{1}{2}k_4^2 \quad (30)$$

The time derivative of V_4 is

$$\dot{v}_4 = -2ak_1^2 - ck_2^2 - k_3^2 + k_4(ra_3(k_1, k_2, k_3)y_1z_1 - y_2z_2 + u_4) \quad (31)$$

If $u_4 = +y_1z_1 - y_2z_2 - k_4$ and $\alpha_3 = 0$, so $\dot{v}_4 = -2ak_1^2 - ck_2^2 - k_3^2 - k_4^2 < 0$, is negative definite and makes the (k_1, k_2, k_3, k_4) system (15) asymptotically stable.

4.1. Simulation and results

To verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for the anti-synchronization between two identical hyperchaotic Chen systems. The parameters and initial values are chosen as section (3). Fig. 3 displays the time response of states x_1, y_1, z_1, w_1 for the master system (2) and the states x_2, y_2, z_2, w_2 for the slave system (3). Fig. 4 displays the synchronization errors of systems (2) and (3). the simulation results indicate that the proposed active backstepping controller works well in both phenomenon synchronization and anti-synchronization

V. CONCLUSION :

This paper mainly presents the synchronization and anti-synchronization between two identical hyperchaotic systems based on an active backstepping design. We have proposed this control scheme for hyperchaos synchronization and anti-synchronization by using the Lyapunov stability theory. Finally, numerical simulations were provided to show the effectiveness of our method.

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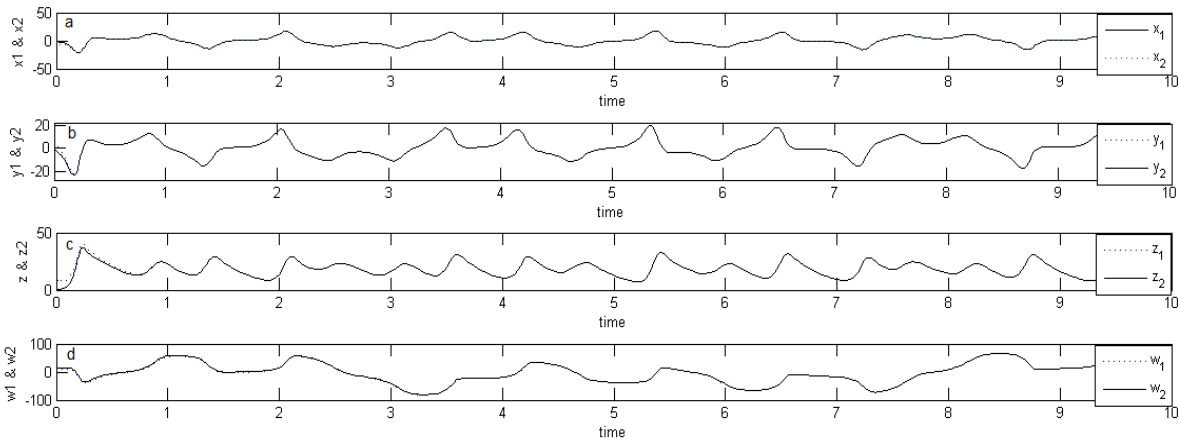


Fig. 1. The time response of states for master system (2) and the slave system (3) for synchronization via active backstepping (a) signals x_1 and x_2 ; (b) signals y_1 and y_2 ; (c) signals z_1 and z_2 ; (d) signals w_1 and w_2 .

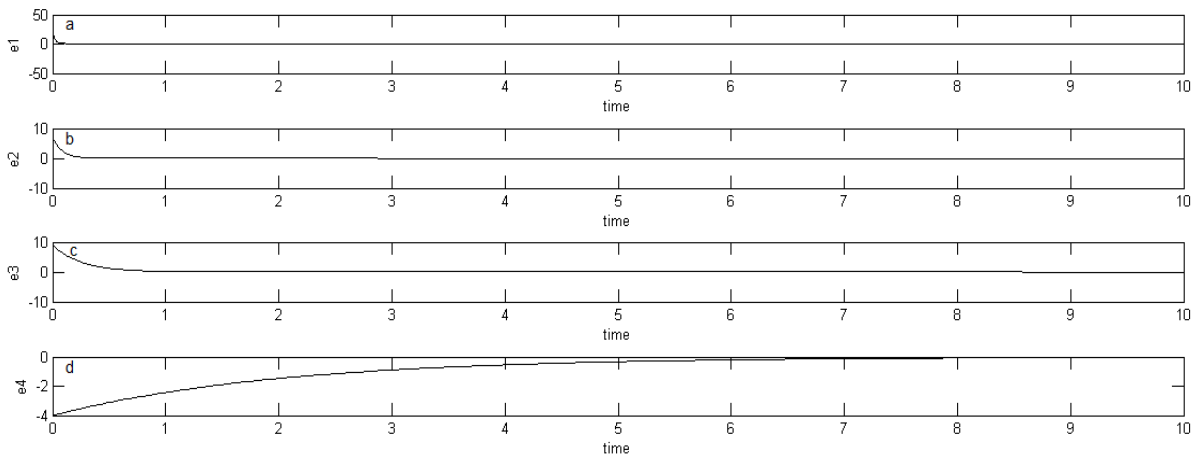


Fig. 2. Dynamics of synchronization errors for two identical hyperchaotic Chen systems (master system (2) and slave system (3)) (a) e_1 ; (b) e_2 ; (c) e_3 ; (d) e_4 .

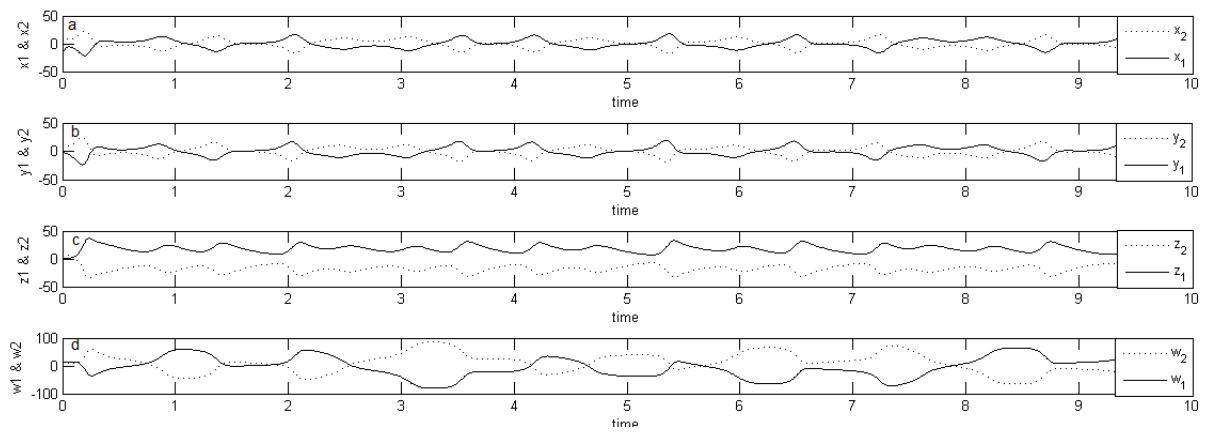


Fig. 3. The time response of states for master system (2) and the slave system (3) for anti-synchronization via active backstepping (a) signals x_1 and x_2 ; (b) signals y_1 and y_2 ; (c) signals z_1 and z_2 ; (d) signals w_1 and w_2 .

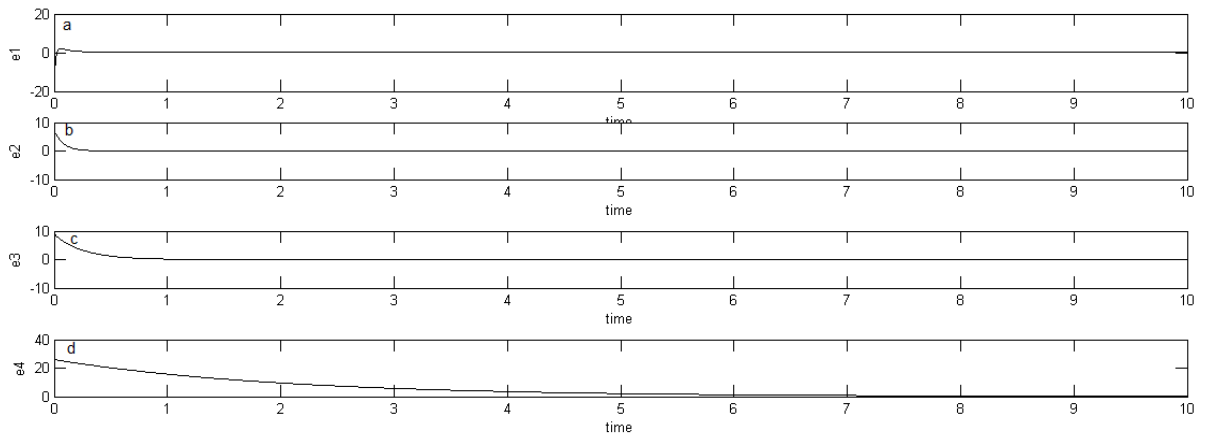


Fig. 4. Dynamics of anti-synchronization errors for two identical hyperchaotic Chen systems (master system (2) and slave system (3)) (a) e_1 ; (b) e_2 ; (c) e_3 ; (d) e_4 .