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INVESTIGATION OF CAPACITY GAINS IN MIMO CORRELATED RICIAN FADING CHANNELS SYSTEMS

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Abstract- This paper investigate the effect of Rician fading and *correlation* on the capacity and diversity of *MIMO* channels. The use of antenna arrays at both sides of the wireless communication link (*MIMO* systems) can increase channel capacity provided the propagation medium is rich scattering or Rayleigh fading and the antenna arrays at both sides are uncorrelated. However, the presence of line-of-sight (LOS) component and *correlation* of real world wireless channels may affect the system performance. Along with that we also investigate power distribution methods for higher *capacity gains* and effect of CSI at the transmitter on the capacity for range of SNR. Our investigation follows capacity gain as function of number of antennas and signal-to-noise (SNR) power ratio Block and frequency nonselective Rician fading channel is assumed, and the effect of Rician factor (L) and the *correlation* parameter (ρ) on the capacity and diversity gains of *MIMO* channels are found. *Index*

***Keywords** - Correlation, fading, multiple-input multiple output (MIMO), diversity, Rician channel, spatial multiplexing.*

I. INTRODUCTION

There interest in wireless communication systems is continues substantially that employ antenna arrays at transmitter and receiver side, due to their promise for dramatically increasing the performance and capacity to up to 40 bit/s/Hz when the system design is optimal [1].Two types of gains can be provided by Multiple-input multiple-output (*MIMO*) system are: *diversity* gain and *spatial multiplexing* gain or capacity gain. With T transmit and R receives antennas, the system can provide *spatial multiplexing* gain of g = min (T, R) or *diversity* gain (D) of order T.R [2].Substantial degradation of the *MIMO* architecture performance may results due to *correlation* of a real-world wireless channel [3]–[6].Also, there is a possibility that the line-of-sight (LOS) component may exist in addition to scattered components. So, the *fading* follows the *Rician* distribution, degrading the performance of *MIMO*, compared to Rayleigh *fading* [7-8]. Previously, Sun and Reed [9] presented *diversity* analysis for MPSK transmitted over uncorrelated *Rician fading* channels. Combined effect of *Rician fading*; correlated channels and availability of channel information were not discussed, in most of the published work.

In this paper, we find the effect of *Rician* factor L, and *correlation* parameter (r) on the capacity and effect of *Rician* factor on *diversity* gains of *MIMO* channels. Using hybrid simulations Capacity and *diversity* gains of Block and frequency nonselective *fading MIMO* systems are obtained. AWGN effect is calculated analytically, whereas the effects of *Rician* factor and correlated *fading* are obtained using Monte Carlo simulation. Also, CSI effect at the transmitter on the *capacity gains* is obtained using Monte Carlo simulation.

II. ORGANIZATION OF PAPER

The paper is organized as follows. Section III describes the model of signal and the channel respectively. The *capacity gains* of *Rician fading* channel are discussed in section IV. Section V presents the simulation results and discussion followed by conclusions in section VI.

III. MIMO CHANNEL AND SIGNAL MODEL

Consider a single user *MIMO* system with T antennas at the transmitter and R antennas at the receiver.

1. *MIMO Signal Model*

The system can be represented by the matrix equation [10]–[12].

$$y = \sqrt{\frac{E_t}{T}} Hx + w \qquad (1)$$

where E_t is the total energy available at the transmitter, yisthe R × 1 vector of signals received on the R antennas,xis theT × 1 vector of signals transmitted on the T transmit antennas,wis the R × 1 noise vector consisting of independent complex Gaussian distributed elements with zero mean and variance σ2,and **H** is the R × T channel matrix.

2. *Correlated Rician fading Channel Model*

In *Rician fading* the elements of **H** are non-zero mean complex Gaussians. Hence we can express **H** in matrix notation as [11]–[12]

$$H=pH^{sp}+ qH^{sc} \qquad (2)$$

Where the specular and scattered components of **H** are denoted by superscripts, p>0, q>0 and $p^2 + q^2 = 1$.H^{sp} is a matrix of unit entries denoted as $_1$.If there is no *correlation* at the transmitter or at the receiver side then the entries of Hscareindependent and identically distributed (i.i.d) complexGaussian random variables with zero mean and unitmagnitude variance, usually denoted by . If there is correlated *fading* then the matrix can be modeled as [3], [4].

$$H_{sc}=R_r^{1/2} H_\omega R_t^{1/2} \quad (3)$$

Where and are the *correlation* matrix at the transmitter and at the receiver side, respectively. The *correlation* matrix **R** is defined as [5]

$$\rho_{ij} = \begin{cases} \rho^{j-i}, & i \le 1 \\ \rho^{*}_{ji}, & i > 1 \end{cases} \quad .|\rho| \le 1 \quad (4)$$

Where "*" denotes the complex conjugate. The *Rician* factor, K is defined asa^2/b^2. Thus, the above H matrix can be written as [11]–[12].

$$H = \sqrt{\frac{K}{K+1}} H_1 + \sqrt{\frac{1}{K+1}} R_r^{1/2} H_\omega R_t^{1/2} \quad (5)$$

IV. MIMO CAPACITY

In the following, we assume that the channel is perfectly known to the receiver .Channel knowledge at the receiver can be sustain by constant training and tracking at reciever.Where as the channel knowledge at the transmitter may be available or not. Also, we assume an ergodic block *fading* channel model where the channel remains constant over a block of consecutive symbols, and changes in an independent fashion across blocks. In *fading* channels there are essentially two notions of capacity: ergodic capacity and outage capacity [1] which describes relative behavior of mean and the tail region of capacity, respectively.

Ergodic Capacity: This is the time-averaged capacity of a stochastic channel. It is found by taking the mean of the capacity values obtained from a number of independent channel realizations.

Outage Capacity: The r % outage capacity Cout, r, is defined as the capacity that is guaranteed for (100 – r) % of the channel realizations [1], i.e.,

$$P(C \le c_{out,r}) - r\,\% \quad (6)$$

1. Channel Unknown at the Transmitter

Acquiring channel knowledge at the transmitter is in general very difficult in practical systems. When the transmitter has no channel state information, it is optimal to evenly distribute the available power Γ among the transmit antennas. The *MIMO* channel capacity with Γ=E_t/σ^2 can be written as [13],

$$C = E_H \left\{ \log_2 \det \left(I_m + \frac{\Gamma}{T} W \right) \right\} \quad (7)$$

Where $E_H\{.\}$ denote the expectation over H, m = min (T, R), Imis the m x m identity matrix, Γ is the average signal-to-noise ratio (SNR) per receive antenna, and the m x m matrix W is given by

$$W = \begin{cases} HH^H, R \le T \\ H^H, T < R \end{cases} \quad (8)$$

Using singular value decomposition (SVD), (7) can be decomposed as

$$C = E_H \left\{ \sum_{i=1}^{k} \log_2 \left(1 + \frac{\Gamma}{T} \lambda_i \right) \right\} \quad (9)$$

Where k, (k ≤ g) is the rank of H, and λi (i = 1, 2, ….., k) denotes the positive eigen values of W.

2. Channel Known at the Transmitter

When the channel parameters are known at the transmitter (the channel state information (CSI) is available at the transmitter), the capacity given by (7), or (9), can be increased by assigning the transmitted power to various antennas according to the "water-filling" algorithm [14].

$$C = E_H \{ \sum_{i=1}^{k} \log_2 (1 \mid \Gamma_i \lambda_i) \} \quad (10)$$

WhereΓ_i, is the power assigned to the ith transmitter and µ is chosen to satisfy:

$$\Gamma = \left(\mu - \lambda_i^{-1} \right)^+ \quad (11)$$

$$\Gamma = \sum_{i=1}^{k} \Gamma_i \quad (12)$$

and "+" denotes taking only those terms which are positive.

V. SIMULATION RESULTS

In this section results illustrate the effect of correlated *fading* and *Rician* factor on the *capacity gains* of *MIMO fading* channels. In view of the fact that **H** matrix represents the channel, the elements of **H** matrix are random and depend upon the type of *fading* encountered by the channel. For *Rician fading* channels, the elements of **H** matrix consist of two parts (2). The first part, is fixed while the second part consists of , which is random. The elements of are i.i.d. zero mean circularly symmetric complex Gaussian (ZMCSCG) random numbers with unit variance, generated using software. Here, in all cases we have considered *correlation* at both ends.

Capacity gains of block and frequency nonselective *fading MIMO* systems are obtained using hybrid simulations. The effect of AWGN is calculated analytically, whereas the effects of *Rician* factor, correlated *fading*, and CSI at the transmitter are obtained using Monte Carlo simulation. Then using (6) and the capacity expressions ((9)–(10)), capacity is calculated. The results are then averaged over 10^3channel realizations. In all the cases discussed here, it is assumed that the CSI is available at the receiver whereas CSI at the transmitter may be available or not.

i) *Effect of the signal-to-noise (SNR) power ratio on the capacity:*

We choose a 6x6 system with exponential co-relation at both sides (T_x. and R_x.) to illustrate the effect of SNR on the capacity of *MIMO* system under different assumptions about the underlying channel. Whereas Fig: 1.10 and Fig: 2.14 show results for uniform c0-relation. Fig: 1.1-1.9 show the ergodic and 1 % outage capacities as functions of signal-to-noise power ratio for various values of *Rician fading* parameter L, with or without *correlation* (ρ) at either the transmitter (ρ_t) or at the receiver side (ρ_r) or at both sides. Availability or lack of channel state information (CSI) is also considered. Here in results legends from Top to bottom are:

- *Cwf.ergodic= ergodic capacity with water-filling power distribution among antennas at transmission side*
- *Cep.ergodic= ergodic capacity with equal power distribution among antennas at transmission side*
- *Cwf.outage= outage capacity with water-filling power distribution among antennas at transmission side*
- *Cep.outage= outage capacity with equal power distribution among antennas at transmission side*
- *Cep.outage= outage capacity with equal power distribution among antennas at transmission side*
- *From the figures 1.1-1.10, it is clear that:*

- Ergodic and 1 % outage capacities show the results in same fashion; however, ergodic capacity is higher.
- For low and initial values of SNR, CSI improves capacity for *Rician fading*, where improvement is more for the outage capacity than the ergodic capacity. The effect of CSI is negligible at high SNR's.

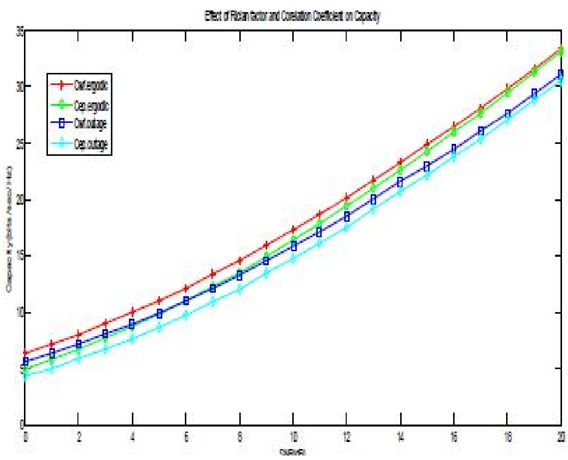

Fig: 1.1: MIMOCapacity vs. SNR, with L=0, ρ=0

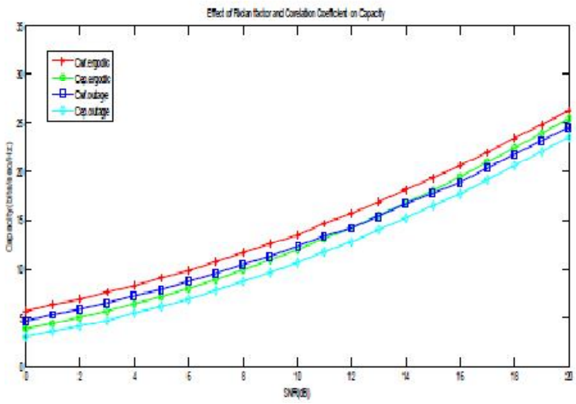

Fig: 1.2: MIMOCapacity vs. SNR, with L=0, ρ=0.7

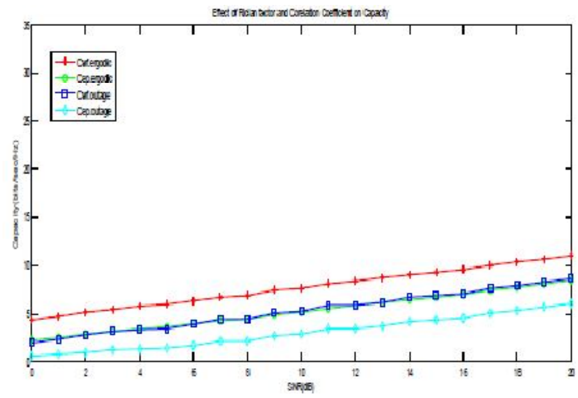

Fig: 1.3: MIMOCapacity vs. SNR, with L=0, ρ=1

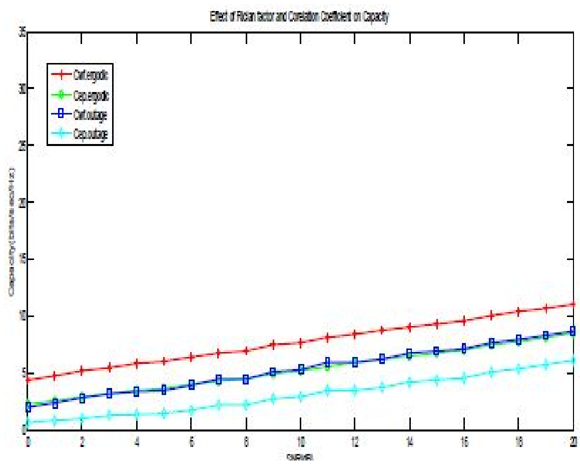

Fig: 1.4: MIMOCapacity vs. SNR, with L=10, ρ=0

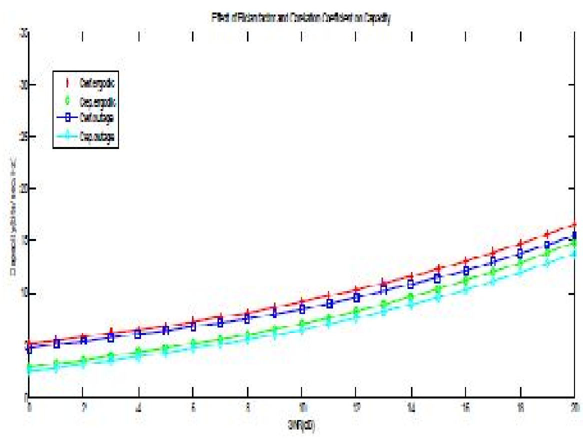

Fig: 1.5: MIMOCapacity vs. SNR, with L=10, ρ=0.7

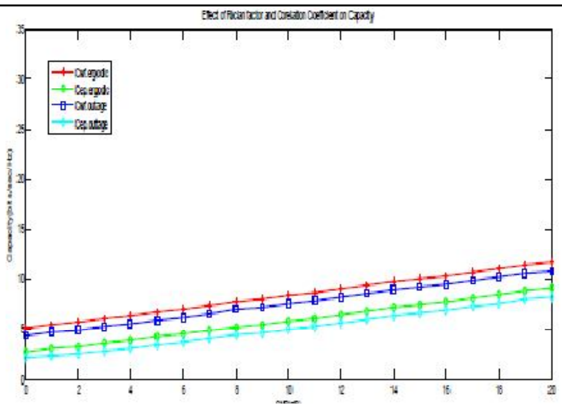

Fig: 1.6: MIMOCapacity vs. SNR, with L=10, ρ=1

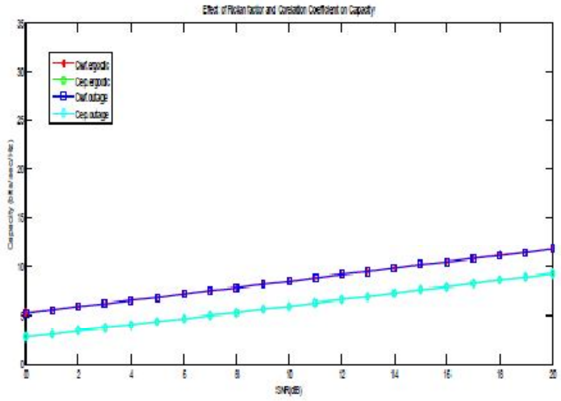

Fig: 1.7: MIMOCapacity vs. SNR, with L=1000, ρ=0

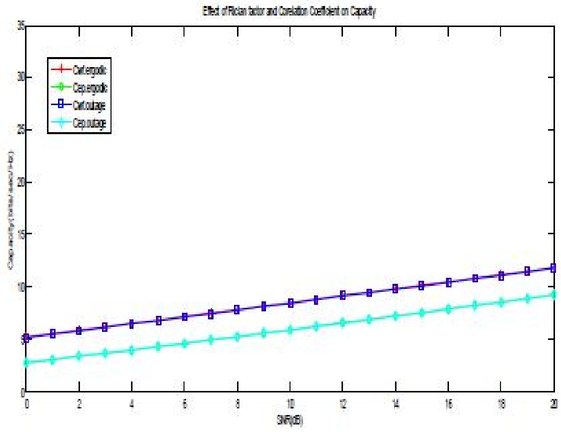

Fig: 1.8: MIMOCapacity vs. SNR, with L=1000, ρ=0.7

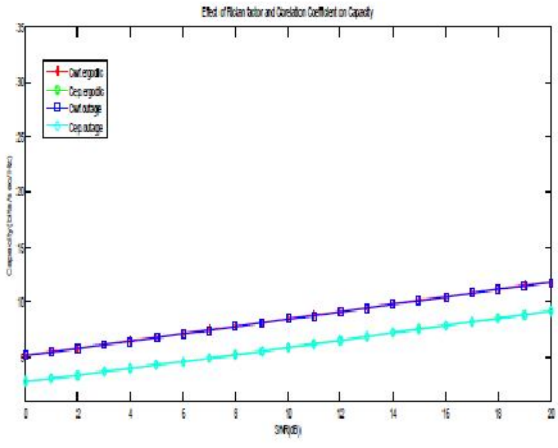

Fig: 1.9: MIMOCapacity vs. SNR, with L=1000, ρ=1

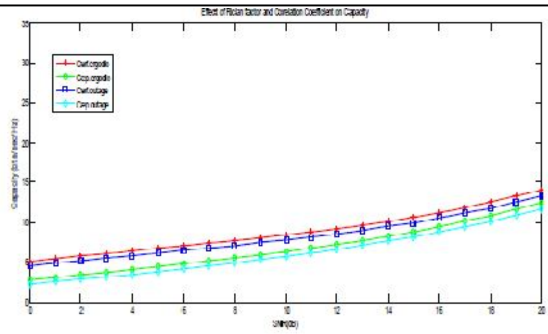

Fig: 1.10: MIMOCapacity vs. SNR, with L=1000, ρ=1 (Uniform co-relation)

- The increase in L factor emphasizes the deterministic part of the channel, results in rank 1 channel. So the ergodic capacity decreases for all values of SNR, as the value of L Increases. This effect can be seen by comparing curves where all parameters are equal but L differs.
- Correlated *fading* is seen to reduce the capacity of the system. However, the *correlation* at the transmitter or at the receiver has the same effect on the capacity of the system.
- Capacity of the *MIMO* system is independent of *fading correlation* (Fig: 1.7, 1.8, 1.9 and 1.10) at very high value of L(104), this is because at L = 10^4, the capacity of the *MIMO* system is due to only deterministic part (LOS).
- When the *fading* is fully correlated, the increase in L factor increases the outage capacity of the system because weighting the deterministic streams emphasizes the rank 1 matrix which is the maximum rank of the fully correlated channels, can be seen from figures 1.3,1.6 and 1.9.

ii) Effect of the number of antennas on the capacity: This section presents the effect of K factor and correlated *fading* parameter (ρ) on the capacity of N × N *MIMO* systems at a given fixed SNR with *correlation* at both ends. Fig. 1 show the ergodic and 1 % outage capacities as functions of the number of antennas (N) for various values of *Rician fading* parameter L, with or without *correlation* (ρ) at 20dB SNR.

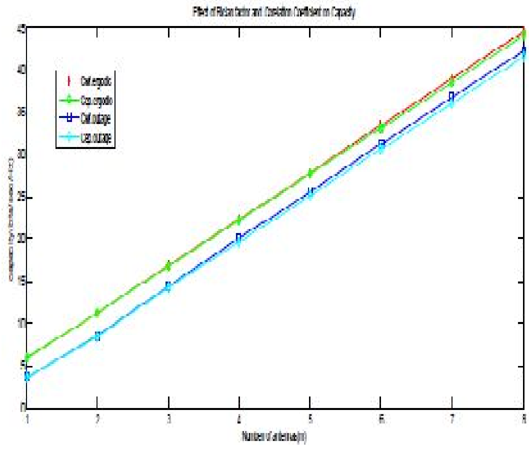

Fig: 2.1: MIMOCapacity vs. N with L=0, ρ=0

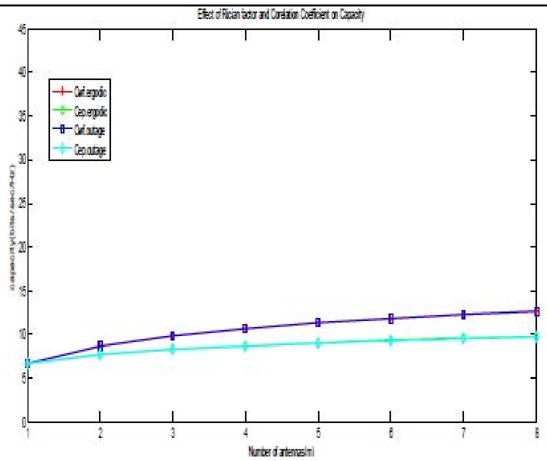

Fig: 2.2: MIMOCapacity vs. N with L=10000, ρ=1

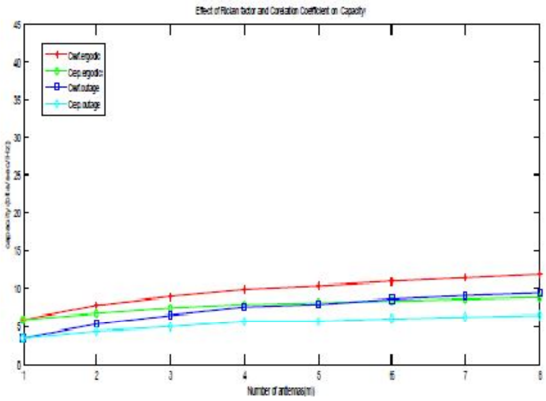

Fig: 2.3: MIMOCapacity vs. N with L=0, ρ=1

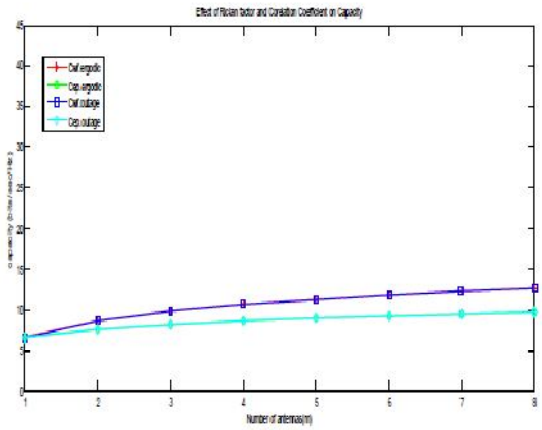

Fig: 2.4: MIMOCapacity vs. N with L=10000, ρ=0

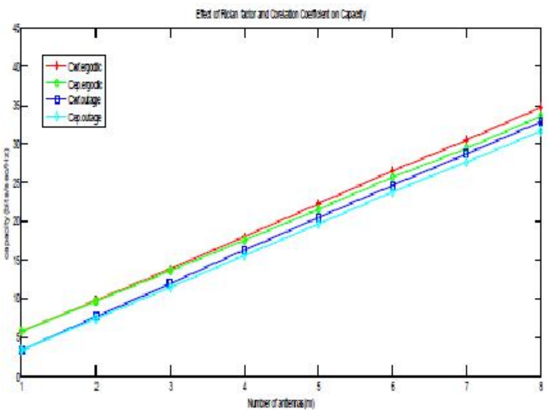

Fig: 2.5: MIMOCapacity vs. N with L=0, ρ=0.7

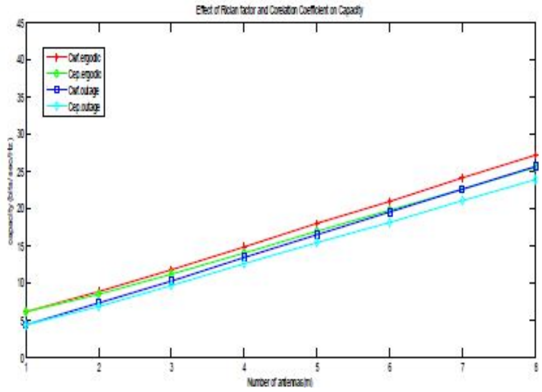

Fig: 2.6: MIMOCapacity vs. N with L=2, ρ=0.7

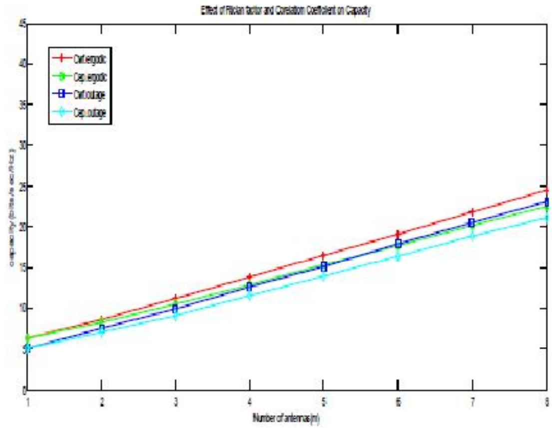

Fig: 2.7: MIMOCapacity vs. N with L=4, ρ=0.7

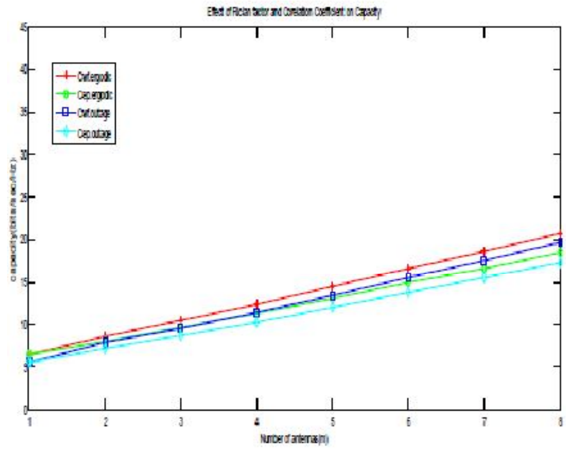

Fig: 2.8: MIMOCapacity vs. N with L=10, ρ=0.7

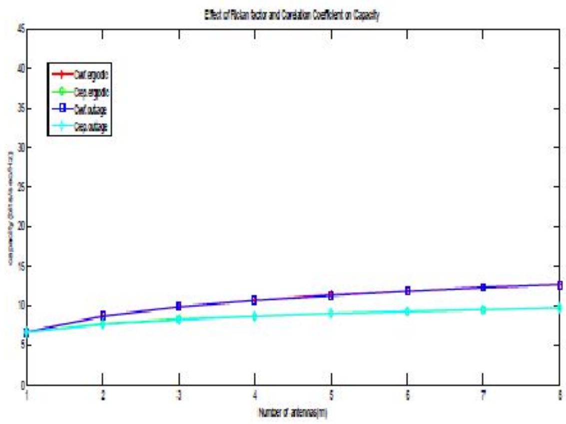

Fig: 2.9: MIMOCapacity vs. N with L=10000, ρ=0.7

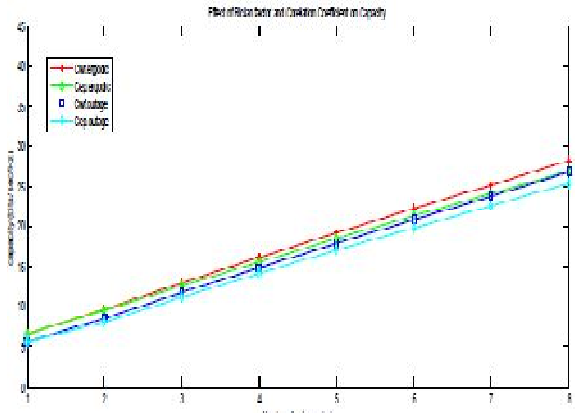

Fig: 2.10: MIMOCapacity vs. N with L=10, ρ=0

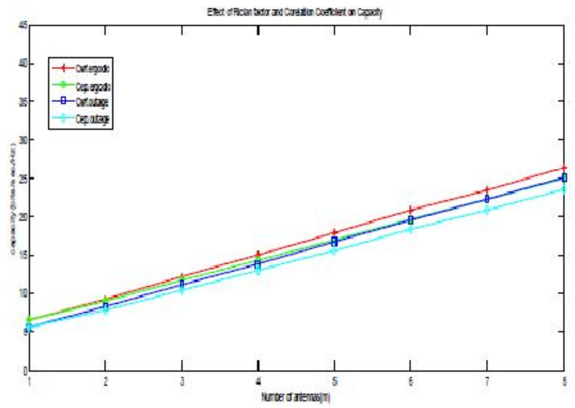

Fig: 2.11: MIMOCapacity vs. N with L=10, ρ=.3

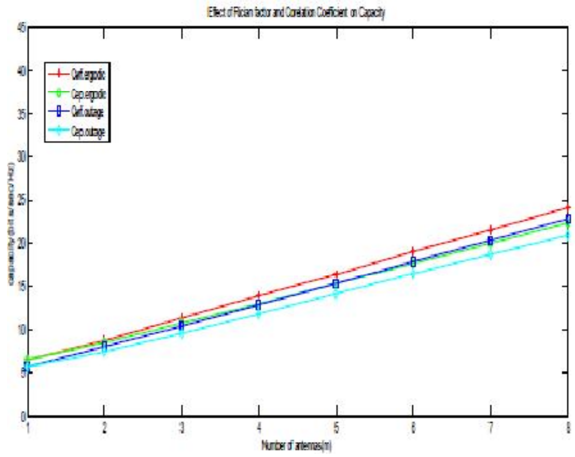

Fig: 2.12: MIMOCapacity vs. N with L=10, ρ=.5

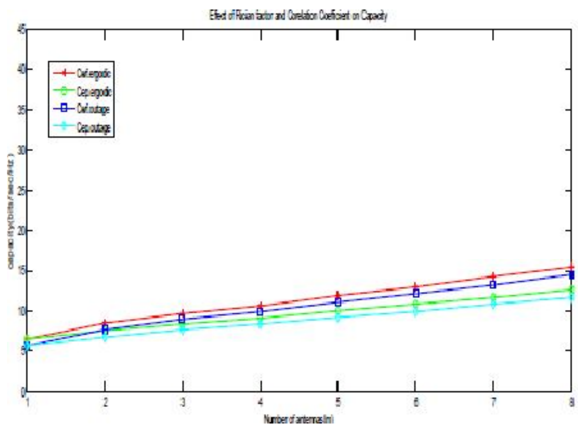

Fig: 2.13: MIMOCapacity vs. N with L=10, ρ=.9

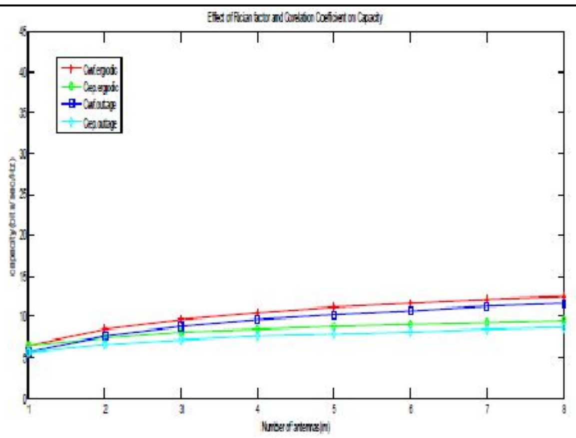

Fig: 2.14: MIMOCapacity vs. N with L=10, ρ=1

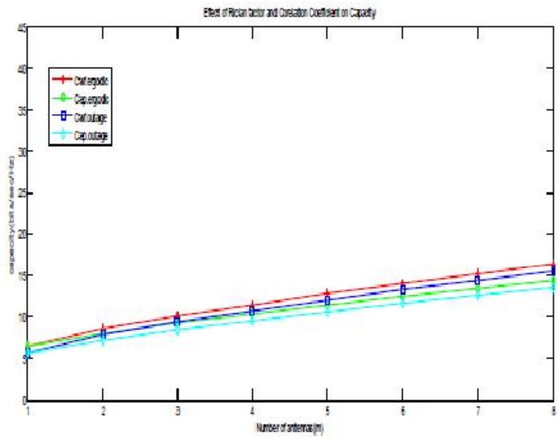

Fig: 2.15: MIMOCapacity vs. N with L=10, ρ=.7 (Uniform co-relation)

- From the figures 2.1 to 2.4,it is clear that:
  - For L=0 and ρ=0, capacity gains highest value and for any other worst values of L=10000 and/or ρ=1 capacity gain going to decrease drastically.
- *From the figures2.5-2.9 , it is clear that:*
  - Increase in L factor emphasizes the deterministic part of the channel, having rank 1 and so the capacity decreases. So for all values of N, as the value of L increases, the ergodic and the 1 % outage capacity decrease. Where, the loss is more at higher values of N.
  - Ergodic and 1 % outage capacities show results in same fashion, however, ergodic capacity is higher. Where, the difference between the ergodic capacity and the 1 % outage capacity reduces at high values of L, because the link stabilizes at higher values of L.
- *From the figures 2.10-2.15 and 2.8, it is clear that:*
  - Ergodic and 1 % outage capacities show results in the same fashion, however, ergodic capacity is higher. For all values of N, as the value of ρ increases, the ergodic and the 1 % outage capacity decrease. However, the loss is more at higher values of N. This is

because the increase in correlated *fading* parameter emphasizes that the fades are less independent and thus reduces the rank of the random channel and so the capacity decreases.

- When the *fading* is fully correlated, the rank of the channel reduces to one, and so the system capacity reduces.
- As the value of L increases, the loss increases even further. This is because both L and ρ contributes to the loss of the overall system capacity

➢ *From the figures1.5,1.10,2.8 and 2.15 , it is clear that:*

- Exponential co-relation model gives better result than uniform co-relation model.

iii) Effect of CSI at the transmitter on the capacity:

In this section, we present the effect of CSI on the capacity of N × N *MIMO* systems under different channel conditions. In all the cases discussed here, we have considered *correlation* at both ends. Above all Figures presents the relative capacity gain, as defined in [17] (in percentage) due to the availability of CSI at the transmitter as function of the number of antennas (N) for various values of SNR, *Rician fading* parameter L, and *correlation* parameter (ρ).

➢ From the above all figures it is clear that:

- CSI or Water-filling gains are reduced at high SNR. This is due to the fact that the relative importance of transmit array gain in boosting average SNR decreases in high SNR region and so the benefit of CSI at the transmitter also reduces.

- It is interesting to note that the relative water-filling gains increases with the increase in the K factor and *correlation* parameter (r). This is due to the fact that K factor and *correlation* parameter (r) reduces the capacity of the *MIMO* system; therefore, the availability of CSI at the transmitter becomes relatively more important at higher values of K factor and *correlation* parameter (r), than when they are small.

VI. CONCLUSIONS

For N × N *MIMO* systems, we found that for all values of N as the value of *Rician* factor L, and correlated *fading* parameter (ρ) increases, the ergodic and the 1 % outage capacity decreases and loss is more at high values of N. Along with that we also found that at higher values of L and r the relative water-filling capacity gain is more compared to equal power distribution method. For low and initial values of SNR, CSI improves capacity for *Rician fading*, where improvement is more for the outage capacity than the ergodic capacity. The effect of CSI is negligible at high SNR's.

However it is noticeable that if there is insufficient space for placing antennas, the decision for choosing the number of antenna should be taken carefully.

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