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KRISHNAPPA H. K

Dept. of CSE, RVCE., Bangalore, India, hk_krit@yahoo.co.in

N K. SRINATH

Dept. of CSE, RVCE., Bangalore, India, srinath_nk@yahoo.com

S. Manjunath

BNM Institute of Technology, drmanjus@gmail.com

RAMAKANTH KUMAR P

Dept. of ISE, RVCE., Bangalore, India, pramakanth2000@gmail.com

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ALGORITHMS FOR CONSTRUCTING EDGE MAGIC TOTAL LABELING OF COMPLETE BIPARTITE GRAPHS

KRISHNAPPA H K¹, N K SRINATH², S MANJUNATH³ & RAMAKANTH KUMAR P⁴

^{1&2}Dept. of CSE, RVCE., Bangalore, India, ³Dept. of Mathematics, BNMIT., Bangalore, India., ⁴Dept. of ISE, RVCE., Bangalore, India

Email: hk_krit@yahoo.co.in, srinath_nk@yahoo.com, drmanjus@gmail.com, pramakanth2000@gmail.com

Abstract: The study of graph labeling has focused on finding classes of graphs which admits a particular type of labeling. In this paper we consider a particular class of graphs which demonstrates *Edge Magic Total Labeling*. The class we considered here is a complete bipartite graph $K_{m,n}$. There are various graph labeling techniques that generalize the idea of a magic square has been proposed earlier. The definition of a magic labeling on a graph with v vertices and e edges is a one to one map taking the vertices and edges onto the integers $1, 2, 3, \dots, v+e$ with the property that the sum of the label on an edge and the labels of its endpoints is constant independent of the choice of edge. We use $m \times n$ matrix to construct edge magic total labeling of $K_{m,n}$.

Key words: Magic squares, Magic constant, Complete graphs, Complete bipartite graph etc.

1. INTRODUCTION

The graph considered is a finite, simple and undirected. Let us consider such a graph $G = (V, E)$.

The graph G has a vertex set $V = V(G)$ and edge set $E = E(G)$. We usually denote $v = |V|$ and $e = |E|$. A general reference for graph theoretic notations is used in this paper.

The labeling of a graph is a map that takes graph elements V or E or $V \cup E$ to positive numbers. If the domain is only a vertex set then it is called vertex magic. If the domain is only an edge set then the labeling is called edge magic [3]. If the domain consists of both vertices and edges it gives raise to the vertex magic total labeling. The notation of a vertex² magic total labeling was introduced in [4,5,6,7].

There are various ways to label the edges of a graph. A connected graph is called a semi magic if there is a labeling of the edges with integers such that for each vertex u the sum of the labels of all edges incident with u is the same for all u . A semi magic labeling where the edges are labeled with distinct positive integers is called magic labeling. A magic labeling is called super magic if the set of edge labels consists of consecutive positive integers [9, 10].

A magic labeling of a graph $G(V,E)$ is called bijection if from $V \cup E$ to $\{1, 2, \dots, |V \cup E|\}$ such that for all edges $(x,y) \in E$, $f(x)+f(y)+f(x,y)=k$, where k is a constant. This is called as edge magic total labeling [EMTL]. They prove that the following graphs are EMTL. Cycle graph C_n is EMTL for all $n > 2$, Complete graph K_n is EMTL iff $n=1, 2, 3, 5$ or 6 and the complete bipartite graph $K_{m,n}$ is EMTL. The way

by which they showed these graphs admits EMTL is trial and error method[12].

In this paper we proved the result [14] that is, all complete bipartite graphs $K_{m,n}$ admits EMTL by using $m \times n$ matrix. We proposed an algorithms which takes m and n as inputs and produces a $(m+1) \times (n+1)$ matrix which represents the EMTL of $K_{m,n}$. The magic constant obtained for various complete bipartite graphs are by using EMTL algorithms. Our algorithm finds all the magic constants those lies between minimum and maximum magic constant.

Conventional Representation

The algorithms uses $(m+1) \times (n+1)$ matrix to produce EMTL of $K_{m,n}$. The conventional representation of this matrix for minimum magic constant and for the maximum magic constant is described as below.

0	1	2	3		n
n	m	m	m		m
+	n	n+	n+		n
1	+	m	m		+
	m	+n	+n		m
	+	-1	-2		+
	n				1
2	...				
(...				
n					
+					
1					
)					
	.				
	.				
	.				

(3	3n	3n		2
m	n	+1			n
-	+				+
1	2				3
)					
*					
(
n					
+					
1					
)					
m	2	2n	2n		n
(n		-1		+
n	+				2
+	1				
1					
)					

Fig(1): Matrix representation for magic constant.

Let the matrix is M whose order is (m+1) x (n+1). The entry for M[0,0]=0, the remaining entries for the 0th row represents the vertex labels for the second partite. Similarly, the remaining entries for the 0th column of M represents the vertex labels of the first partite. The entries for the remaining cells of the matrix represents the corresponding edge labels of the graph Km,n.

3. Algorithm to construct minimum magic constant.

We know that Km,n has m+n+mn number of components, hence we need to map numbers from 1,2,.....,m+n+mn to each components of the graph. The algorithm below fills the matrix with these numbers. Finally, we can map these numbers to the corresponding graph elements.

Algorithm Minimum_Magic_Constant (m, n)

//M is the global matrix of size at //least (m+1) x (n+1). Here m and //n are integers and represents //number of vertices in first partite //and second partite of the graph //Km,n.

1. M[0,0]←0
2. Repeat for i← 1 to n do
M[0,i]←i
3. Repeat for i←1 to m do
M[i,0]←i*(n+1)
4. At this stage the first row and first column were filled with numbers 1,2,.....n, n+1, 2(n+1), 3(n+1).....m(n+1).

Let the remaining numbers (n+2), (n+3),.....that is the left out numbers from the set

{1,2,3,.....,m+n+mn} are stored in an array A of size m*n, in descending order.

5. Fill the remaining entries of the matrix M with the elements of the array A.
k←1

Repeat for i←1 to m do

Repeat for j←1 to n do

$$M[i,j] \leftarrow A[k]$$

$$k \leftarrow k+1$$

6. Stop.

Working of this algorithm for K_{2,2}, K_{2,3} and K_{3,2} were given below.

Example 1: K_{2,2}

0	1	2
3	8	7
6	5	4

Table (1)

Magic constant computation.

$$f(u_1)+f(v_1)+f(u_1,v_1)=3+1+8=12$$

$$f(u_1)+f(v_2)+f(u_1,v_2)=3+2+7=12$$

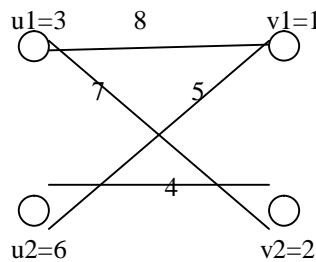
$$f(u_2)+f(v_1)+f(u_2,v_1)=6+1+5=12$$

$$f(u_2)+f(v_2)+f(u_2,v_2)=6+2+4=12$$

Verification by formula: mn+m+2n+2, here m=2, n=2.

$$2*2+2+2*2+2=4+2+4+2=12.$$

Mapping of the above table in to the graph is as shown below.



fig(2)

Example 2: K_{2,3}

0	1	2	3
4	11	10	9
8	7	6	5

Table (2)

Magic constant computation.

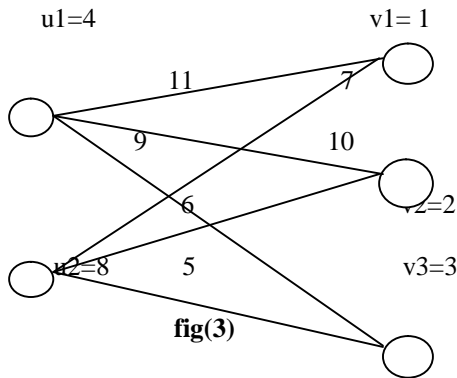
$$\begin{aligned}
 f(u_1)+f(u_1,v_1)+f(v_1) &= 4+11+1=16, \\
 f(u_1)+f(u_1,v_2)+f(v_2) &= 4+10+2=16, \\
 f(u_1)+f(u_1,v_3)+f(v_3) &= 4+9+3=16, \\
 f(u_2)+f(u_2,v_1)+f(v_1) &= 8+7+1=16, \\
 f(u_2)+f(u_2,v_2)+f(v_2) &= 8+6+2=16, \\
 f(u_2)+f(u_2,v_3)+f(v_3) &= 8+5+3=16.
 \end{aligned}$$

Verification by formula.

Here $m=2, n=3$.

$$\begin{aligned}
 \text{Magic constant} &= mn+m+2n+2 \\
 &= 2*3+2+2*3+2=6+2+6+2=16.
 \end{aligned}$$

Mapping of the above table in to the graph is as shown below.



Example 3: $K_{3,2}$

0	1	2
3	11	10
6	8	7
9	5	4

Table(3)

Magic constant computation.

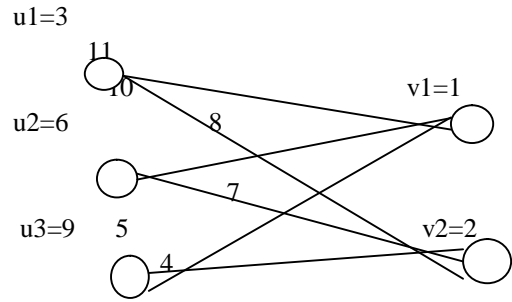
$$\begin{aligned}
 f(u_1)+f(u_1,v_1)+f(v_1) &= 3+11+1=15, \\
 f(u_1)+f(u_1,v_2)+f(v_2) &= 3+10+2=15, \\
 f(u_2)+f(u_2,v_1)+f(v_1) &= 6+8+1=15, \\
 f(u_2)+f(u_2,v_2)+f(v_2) &= 6+7+2=15, \\
 f(u_3)+f(u_3,v_1)+f(v_1) &= 9+5+1=15, \\
 f(u_3)+f(u_3,v_2)+f(v_2) &= 9+4+2=15.
 \end{aligned}$$

Verification by formula.

Here $m=3, n=2$.

$$\begin{aligned}
 \text{Magic constant} &= mn+m+2n+2 \\
 &= 3*2+3+2*2+2 \\
 &= 6+3+4+2=15
 \end{aligned}$$

Mapping of the above table in to the graph is as shown below.



Fig(4).

4. Algorithm to construct maximum magic constant.

To get the maximum magic constant fill the matrix M by using the algorithm given below. Finally use the entries of the first row to label the vertices of the second partite and the entries of the first column to label the vertices of the first partite. The remaining entries are used to label the corresponding edges of the graph.

Algorithm Maximum_Magic_Constant(m, n)
 //M is the global matrix of size at least $(m+1) \times (n+1)$. Here m and n are integers and represents number of vertices in first partite //and second partite of the graph $K_{m,n}$.

- [Initialization]
 $M[0,0] \leftarrow 0$
 $X \leftarrow m+n+m*n$
- [Fill the first row with the highest values]
 Repeat for $i \leftarrow 1$ to n do
 $M[0,i] \leftarrow x$
 $X \leftarrow x-1$
- [Fill the first column as follows]
 $M[1,0] \leftarrow x$
 Repeat for $i \leftarrow 2$ to m do
 $M[i,0] \leftarrow x-(n+1)$
- The remaining numbers from the set $\{1,2,3,\dots,m+n+m*n\}$ which were not used to fill the matrix in step-2, and step-3 store them in an array A in increasing order.
- [Fill the remaining entries of the matrix by using the elements of the array A.]
 $K \leftarrow 1$
 Repeat for $i \leftarrow 1$ to m do
 Repeat for $j \leftarrow 1$ to n do

$$M[i,j] \leftarrow A[k]$$

$$K \leftarrow k+1$$

Verification by formula:
 $(n+1)*(2m+1)=(3+1)*(2*2+1)=4*5=20.$

6. [Stop]
 Working of this algorithm for $K_{2,2}$, $K_{2,3}$ and $K_{3,2}$ were given below.

Example 1: $K_{2,2}$

Here $m=2$ and $n=2$, hence no. of vertices=4 and no. of edges=4.

0	8	7
6	1	2
3	4	5

Table (4).

Magic constant computation.

$$f(u_1)+f(v_1)+f(u_1,v_1)=6+8+1=15$$

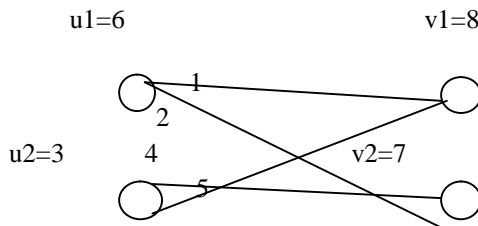
$$f(u_1)+f(v_2)+f(u_1,v_2)=6+7+2=15$$

$$f(u_2)+f(v_1)+f(u_2,v_1)=3+8+4=15$$

$$f(u_2)+f(v_2)+f(u_2,v_2)=3+7+5=15$$

Verification by formula:
 $(n+1)*(2m+1)=(2+1)*(2*2+1)=3*5=15.$

Mapping of the above table in to the graph is as shown below.



fig(5)

Example 2: $K_{2,3}$

Here $m=2$ and $n=3$, hence no. of vertices=5 and no. of edges=6.

0	11	10	9
8	1	2	3
4	5	6	7

Table (5).

Magic constant computation.

$$f(u_1)+f(v_1)+f(u_1,v_1)=8+11+1=20$$

$$f(u_1)+f(v_2)+f(u_1,v_2)=8+10+2=20$$

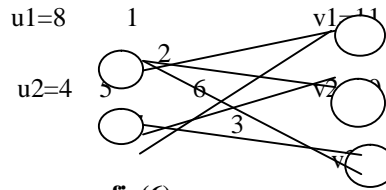
$$f(u_1)+f(v_3)+f(u_1,v_3)=8+9+3=20$$

$$f(u_2)+f(v_1)+f(u_2,v_1)=4+11+5=20$$

$$f(u_2)+f(v_2)+f(u_2,v_2)=4+10+6=20$$

$$f(u_2)+f(v_3)+f(u_2,v_3)=4+9+7=20$$

Mapping of the above table in to the graph is as shown below.



fig(6)

5. CONCLUSION

It has been demonstrated using the two algorithms that the labeling has been successfully done. Construct minimum magic constant algorithm fills the matrix with numbers which could be mapped to the corresponding graph elements. Construction of Maximum magic constant algorithm is used to fill the matrix and the entries that are used to label the corresponding edges of the graph.

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