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VIBRATION ANALYSIS OF A ROTATING TAPERED COMPOSITE BEAM WITH TIP MASS

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Abstract- In this study, free vibration responses of a rotating tapered composite beam with tip mass are investigated. The energy expressions for the kinetic and potential energies of a rotating composite beam with tip mass have been formulated. The energy expressions are then applied in Lagrange's equations to develop the equation of motion of a composite beam with tip mass. The stiffness and mass matrices for a standard composite beam element with two end nodes with two degrees of freedom at each node are derived. Various parametric studies are performed to investigate the effect of tip mass and the rotational speed on the variation of natural frequencies of the composite beam. The investigations are also done to study the effect of hub radius on the natural frequencies. It is shown that the addition of tip mass increases the stiffness of the structure and consequently increases the natural frequencies.

Keywords- *Rotating composite beam with tip mass, tapered composite beam*

I. INTRODUCTION

Many engineering systems such as turbo machines, turbine blades, aircraft propellers, helicopter rotors, high speed flexible mechanisms, robot manipulators and spinning space structures cam be modeled by rotating structural beams. The dynamic behavior of composite beams and shafts has been the subject of intensive study for many years. In many structural designs, tip masses are utilized to satisfy the following functions such as: to increase the airflow as in the case of a dynamic inducer for a wind turbine, to modify the vibration frequency of components as in the case of a helicopter rotor or to increase the flexing motion of the blade in flexible blade auto cooling fans.

The free vibration of simply supported symmetric laminated plate were investigated by Noor [1] based on classical laminated theory. The exact solutions for free vibrations of laminated composite beam were investigated by Bertholet [2] and Jones [3]. Miller and Adams [4] studied the dynamic characteristics of orthotropic fixed-free beams based on the classical laminated theory. The free vibration responses of a rotating beam with tip mass is investigated by Hoa [5]. Chen and Yang [6] investigated the static and dynamic responses of a symmetrically laminated beam. Whitney [7] studied the bending behavior of uniform laminated anisotropic plates and beams under transverse load and also analyzed the stability of the structure... Chandrasekhar *et al* [8] presented the exact solutions for the free vibration of symmetrically laminated composite beams with arbitrary boundary conditions by including the first order shear deformation and rotary inertia. Hodges *et al* [9] studied the free vibration of composite beams using exact integration methods and mixed finite element method. They discuss the influences of laminated configuration on the natural frequencies. Krishnaswamy *et al* [10]

derived the governing equations of laminated composite beams and derived the analytical solution using the Hamilton's principle. Singh and Abdelnassar [11] investigated the forced vibration response of composite beams considering a third order shear deformation theory . Nabi and Ganesan [12] obtained a general finite element based on a first order deformation theory to study the free vibration characteristics of laminated composite beams. The natural frequencies of a step wise variable cross-section composite beam were analyzed by Farghaly and Gadelrab [13]. Lin Chen [14] studied the free vibration analysis of various configurations of a tapered composite beam using hierarchical finite element method.

In this study, free vibration responses of a rotating tapered composite beam with tip mass are investigated. The energy expressions for the kinetic and potential energies of a rotating composite beam with tip mass have been formulated. The energy expressions are then applied in Lagrange's equations to develop the equation of motion of a composite beam with tip mass. Various parametric studies are performed to investigate the effect of tip mass and the rotational speed on the variation of natural frequencies of the composite beam. The investigations are also done to study the effect of hub radius on the natural frequencies.

II. MATHEMATICAL MODELING OF A ROTATING TAPERED COMPOSITE BEAM

A rotating tapered composite beam with tip mass, m_t, attached to a hub of radius e, as shown in Figure 1 is considered for the development of the finite element formulation. The beam is considered to be made-off orthotropic layers. The beam is assumed to be a thin, i.e., the thickness H is much smaller than the length L and width b. It is also assumed that the deformation is very small compared to the geometric properties of the structure. The transverse strain and the rotary inertia are neglected in the formulation of equation of motions. The transverse deflection w is considered to be uniform throughout the cross section.

The strain energy of a non-rotating tapered composite beam is given as:

$$U_{sta} = \frac{1}{2}\int_0^L bD_{11}[\frac{\partial^2 w}{\partial x^2}]^2 dx \quad (1)$$

where b is the width of the beam and D_{11} is expressed as:

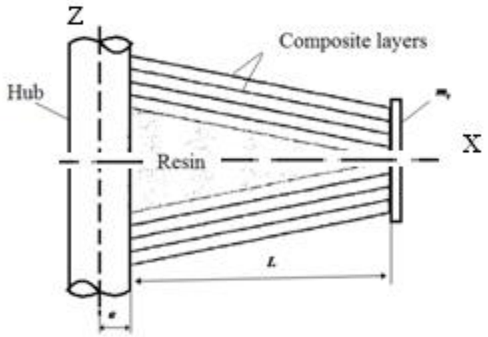

Figure 1. Rotating tapered composite beam with end mass

$$D_{ij} = \int_{-\frac{H}{2}}^{\frac{H}{2}} \overline{Q_{ij}} z^2 dz = \frac{1}{3}\left[\sum_{k=1}^{N-m} \overline{Q}_{ij} \left(z_k^3 - z_{k-1}^3\right) + \sum_{k=N+m+1}^{2N} \overline{Q}_{ij} \left(z_k^3 - z_{k-1}^3\right) + 2\overline{Q}_{rij} z_{N-m}^3\right] \quad (2)$$

Strain energy due to the rotation of the beam is given as:

$$U_{rot} = \frac{1}{2}\int_0^L P(x)[\frac{\partial w}{\partial x}]^2 dx \quad (3)$$

where, P(x) is the axial force against the changes in the horizontal projection and is expressed as:

$$P(x) = \frac{\rho\Omega^2}{2}(L^2 - x^2 + 2e(L-x))(\frac{\partial w}{\partial x})^2 dx \quad (4)$$

where ρ is the total density of the beam and Ω is the rotational speed of the beam .

The increase in axial stress due to addition of the tip mass is given by the expression as:

$$\sigma_x^a = (e+L)m_t\Omega^2 \quad (5)$$

The strain energy due to tip mass is given as,

$$U_{tip} = \frac{1}{2}\int_0^L \sigma_x^a [\frac{\partial w}{\partial x}]^2 dx \quad (6)$$

The kinetic energy for the beam is given by

$$T = \frac{1}{2}\int_0^L \rho b\left(\frac{\partial w}{\partial t}\right)^2 dx \quad (7)$$

where ρ is the density of the beam which is expressed as $\rho = \rho_r H_r + \rho_c H_c$.

III. FINITE ELEMENT FORMULATION

The finite element model is developed by using a standard two-noded-beam element with two degrees of freedom at each node as transverse direction (w) and rotational direction (θ) respectively. The transverse displacement are expressed in terms of nodal displacements and finite element shape functions as,

$$w(x,t) = N_w(x)\ \{d(t)\}$$

where displacement vector, $d(t) = \{w_1, \theta_1, w_2, \theta_2\}$ and $N_W(x)$ are the common linear and cubic polynomial beam shape functions which are expressed as:

$$N_1 = 1 - 3\frac{x^2}{L^2} + 2\frac{x^3}{L^3}$$

$$N_2 = x - 2\frac{x^2}{L} + \frac{x^3}{L^2}$$

$$N_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}$$

$$N_4 = \frac{-x^2}{l} + \frac{x^3}{l^2} \quad (8)$$

The governing equation of motion for the rotating tapered composite beam is formulated using the Lagrange's method. The general Lagrange's equation can be expressed as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \qquad i=1,\ldots n. \quad (9)$$

where n is the total DOF of the system considered for formulation of equation, Q_i is the generalized force corresponding to the i^{th} DOF of the system.

By substituting the total strain energy and total kinetic energy in the Lagrange's equation, an expression for governing equation of motion is obtained in the finite element form as:

$$[m^e]\{\ddot{d}\} + [k^e]\{d\} = \{f^e\} \quad (10)$$

where [m^e] is element mass matrix, [k^e] is element stiffness matrix and $\{f^e\}$ is the element force vector. Assembling all the element mass matrix, stiffness matrix and force vector the global mass matrix, stiffness matrix and force vector can be evaluated, which is expressed as

$$[M]\{\dot{d}\} + [K]\{d\} = \{F\} \quad (11)$$

IV. RESULTS AND DISCUSSIONS

The validity of the developed finite element formulation of the governing differential equation of motion of rotating tapered composite beam with tip mass is investigated by comparing the results in terms of natural frequencies of non-rotating tapered composite beam without tip mass evaluated using FEM with those evaluated using Ritz method. The simulation results are obtained by considering a tapered composite beam made up of NCT301 graphite epoxy. The various geometric and material properties mentioned in Table I have been considered for the simulation. The results of the first five natural frequencies are tabulated in Table II. A good agreement between the results evaluated using FEM and Ritz method can be observed.

TABLE I. THE MATERIAL PROPERTIES OF NCT 301 GRAPHITE EPOXY [14]

Composite	E_1	144 Gpa
	E_2, E_3	12.14 Gpa
	ν_{21}, ν_{31}	0.017
	ν_{23}	0.458
	G_{12}, G_{13}	4.48 Gpa
	G_{23}	3.2 Gpa
	ρ_c	1660.8 kg/m^3
Resin	E	3.93 Gpa
	ν	0.37
	G	1.034 Gpa
	ρ_r	1000 kg/m^3

TABLE II. NATURAL FREQUENCIES OF THE BEAM DERIVED BY DIFFERENT METHODS

Mode	Natural frequencies (rad/s)		%Variation
	FEM	Ritz method	
1	455.46	425.48	6.58
2	2854.30	2716.20	4.84
3	7992.40	7646.40	4.33
4	15663.00	15005.00	4.20
5	25894.00	24821.00	4.14

The properties of a rotating tapered composite beam are strongly influenced by many structure-related parameters, which may include rotational speed, hub radius, taper angle, ply orientation and the number of plies etc. The proposed FE model is used to study the effects of variations in those properties on the natural frequencies of the beam. The simulation results are obtained by considering a tapered composite beam made up of NCT301 graphite epoxy and the various mechanical properties of the beam are shown in Table I.

The various geometric properties of the beam considered are: *L*=0.28 m; Individual ply thickness t = 0.0001524 m. The number of plies considered on the left end and right end are 32 plies and 30 plies, respectively. The configuration of both the ends are $[(0/90)_8]_s$ and $[(0/90)_7/0]_s$ respectively. The height of each ply in z direction is *h*=0.00015240001905 m. It is almost as same as the hickness t=0.0001524, because of tapered angle is only $\alpha = 0.031185^{\circ}$.

A mass of 0.004423 kg is added to the free end and Hub radius of 0.03 m is considered for the simulation. The bending stiffness coefficient is cubic function x of throughout the entire length is expressed as:

$$D_{11} = 817.122 - 490.366x + 95.5097x^2 - 1.01515x^3 \text{ [14]}$$

The effect of the rotational speed on the variation of natural frequencies is investigated by performing the simulation at various rotational speed and the results for the first five modes are shown in Figure 2. It can be seen that the natural frequencies at all the modes considered increase with increase in the rotational speed. It can be related to the increase in the stiffness of the structure with increase in rotational speed.

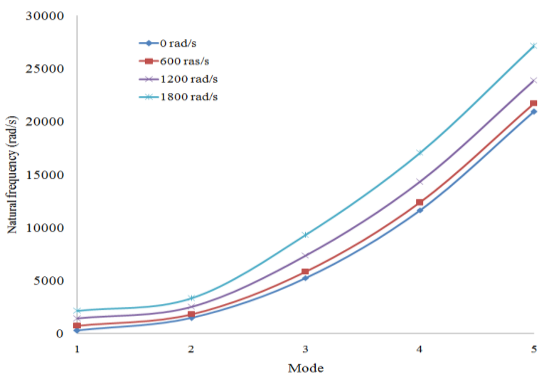

Figure 2 Effect of the rotation speed of the tapered composite beam with tip mass on the natural frequencies.

The effect of variation of tip mass on the natural frequencies of the rotating tapered composite beam is investigated by calculating the natural frequencies at various tip mass at a rotational speed of 250 rad/s and the results are presented in Table III. It can be realized from the results that the natural frequencies decrease with increase in tip mass while it increases with further increase in the tip mass. This is due to the fact that the natural frequencies decrease with increase in mass of the beam by the addition of tip mass by dominating the stiffness of the structure. On the other hand, further increase in the tip mass increases the stiffness of the structure due to the rotational speed of the beam and dominating the increase in the mass of the structure.

TABLE III. EFFECT OF VARIATION TIP MASS ON THE NATURAL FREQUENCIES

Mode	Natural frequency (rad/s)				
	0.002211 kg	**0.004423 kg**	**0.006634 kg**	**0.008846 kg**	**0.011057 kg**
1	451	374	330	300	278

The effect of variation of hub radius on the natural frequencies of the beam is studied by evaluating the natural frequencies at various hub radii. The results evaluated at a rotational speed of

2	1701	1535	1474	1445	1428
3	5440	5340	5312	5302	5299
4	11777	11713	11698	11696	11698
5	20700	20654	20646	20648	20653

250 rad/s and the tip mass of 0.004423 kg are shown in Table IV. The results indicate that the increase in hub radius increases the natural frequencies of the beam. This is due to the fact that the stiffness of the beam is significantly increased with increase in hub radius.

TABLE IV. EFFECT OF VARIATION HUB RADIUS (m) ON THE NATURAL FREQUENCIES OF THE ROTATING TAPERED COMPOSITE BEAM.

Mode	Natural frequency (rad/s)				
	0.01	0.02	0.03	0.04	0.05
1	363	369	374	380	385
2	1528	1531	1535	1538	1542
3	5328	5334	5340	5346	5352
4	11698	11705	11713	11720	11727
5	20639	20646	20654	20662	20669

The effect of variation of taper angle on the natural frequencies of the beam is studied by evaluating the natural frequencies at various taper angle. The results evaluated at a rotational speed of 250 rad/s , hub radius 0.03 m and the tip mass of 0.004423 kg are shown in Figure 3. The results indicate that the increase in taper angle increases the natural frequencies of the beam. Here taper angle is changed by varying the length of the beam and eeping all the other parameters constant.

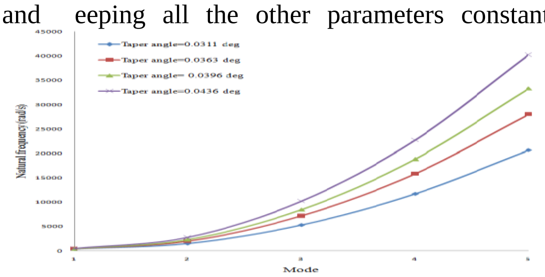

Figure 3 Effect of variation of the natural frequencies with various taper angle of beam

5. CONCLUSION

In this study, free ibration responses of a rotating tapered composite beam with tip mass are investigated. It is shown that the natural frequencies of the rotating tapered composite beam at all the modes considered are increased with increase in the taper angle. It is also shown that the natural frequencies of rotating tapered composite beam at all the modes considered are increased with increase in the rotational speeds. The natural frequencies of rotating tapered composite beam at all the modes considered are decreased with increase in the tip mass except at the higher modes. The natural frequencies of rotating tapered composite beam at all the modes considered are increased with increase in the hub radius.

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