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# MATHEMATICAL MODELLING AND SIMULATION OF GRID CONNECTED SOLAR PHOTOVOLTAIC SYSTEM

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**Abstract** - This paper presents the mathematical modeling of three-phase grid connected inverter fed by Solar Photovoltaic (SPV) system with Maximum Power Point Tracking (MPPT). Analysis has been carried out to choose the proper modulation index for maximum output for three-phase inverter. With this modulation index, the variation of the active and reactive power for different loads has been presented along with major parameters like Transfer ratio and Efficiency. Also the Real and Reactive power output of the SPV has been measured with various solar Radiation levels.

## 1. INDEX TERMS

Solar Photovoltaic cell, Mathematical model, Three Phase Inverter, Grid Modelling, Transfer ratio, Efficiency.

## 2. NOMENCLATURE

$I_{ph}$	: Light current of SPV Module (A)
$I_d$	: Diode reverse saturation current in the equivalent circuit ( $\mu$ A)
$R_{sc}$	: Series Resistance in the equivalent circuit of the module ( $m\Omega$ )
$R_{sh}$	: Shunt Resistance in the equivalent circuit of the module ( $m\Omega$ )
$D$	: Diode used in the equivalent circuit
$I_{sh}$	: Current through the shunt resistance (mA)
$I_{sc}$	: Short circuit current of the module (A)
$V_{oc}$	: Open circuit voltage of the module (V)
$I_N$	: Current flowing through inverter circuit (A)
$I_{AF}, I_{BF}, I_{CF}$	: I. Three phase output of Inverter
$S_a, S_b, S_c$	: Switching pulse for individual phases of Inverter circuit
$M, \phi$	: Modulation index and Phase angle
$I_{pv}$	: Solar PV Module current (A)
$V_{pv}$	: Solar PV Module voltage (V)
$K_i$	: Temperature co-efficient of short circuit current
$I_{pvn}$	: Photovoltaic current at nominal condition
$G_n$	: Nominal insolation level (1000 W/m <sup>2</sup> )
$U_{dc}$	: II. Input DC voltage for Inverter (V)
$I_{scn}$	: Short circuit current at nominal condition
$G$	: Insolation level W/m <sup>2</sup>
$V_{AB}, V_{BC}, V_{CA}$	: Line Voltage – Inverter output (V)
$V_{AN}, V_{BN}, V_{CN}$	: Phase Voltage – Inverter output (V)
$P_1, P_2, P_3$	: Active power for R, Y and B phase respectively
$Q_1, Q_2, Q_3$	: Re-active power for R, Y and B phase respectively

## I. INTRODUCTION

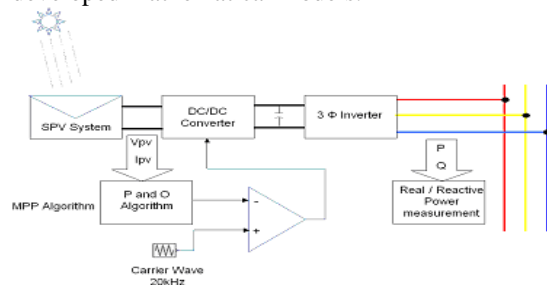
The problem of power injection into the grid must be investigated by modelling the SPV system as well as the grid. The design should be a controlled inverter output suitably interfaced with SPV system and grid, which can be capable of controlling real

and reactive power. The SPV system exhibits non-linear characteristics due to the following reasons.

- Insolation and temperature of SPV cell
- Movement of clouds and other natural factors mask the Sun's ray from the SPV Cells
- The output of the SPV Cells needs to be converted to three phase power and synchronized with the grid
- The load in the grid is also a variable that depends on time, location and season
- A trip command issued by a relay to a major load would often result in power swing and the output of the SPV cells to the grid needs to be stable during this period
- Loss of a generator in the grid would result in power swing and the output of the SPV Cells to the grid needs to be stable during this period

In this paper the mathematical model of Solar Cell, Inverter, load and power measurement is discussed. The active and reactive power measurement with simulation results under different conditions is shown. It helps the user to understand the fundamental parameters of grid connected SPV system. The basic block diagram of the system is shown in Fig.1.

In the following sections, the detailed modeling of each block in Fig.1 is discussed. The required analyses have been carried out with the help of the developed mathematical models.



**Fig. 1. SPV Grid integrated system**

A well-known five-parameter model of a SPV

cell is considered for this work [2]-[3]. The mathematical equations used to simulate the realistic model of SPV cell are given in equations (1) and (2)

There should always be a balance in generation and demand for power system stability. Considering the factors mentioned above, it is evident that solar generation and load variations are dynamic in nature. To manage the two dynamic variables for effective usage of power generated by SPV, a detailed analysis of the system is required.

**II. ONE DIODE MODEL OF SOLAR PHOTOVOLTAIC CELL**

A well-known five-parameter model of a SPV cell is considered for this work [2]-[3]. The

$$I = I_{pv} - I_d - I_{sh} \tag{1}$$

$$I = I_m - I_{sh} \text{ where } I_m = I_{pv} - I_d ;$$

$$I_m = I_{pv} N_{pp} - I_o N_{pp} [\exp A - 1] ;$$

$$A = \frac{V + I(R_{ss} N_{ss} / N_{pp})}{N_{ss} V_{ta}} ;$$

$$I_{pv} = [I_{pvn} + K_i dt] \frac{G}{G_n} \text{ and } I_{pvn} = \frac{[R_s + R_p]}{R_p} I_{SCn} \tag{2}$$

For this work SOLKAR model having the following specifications have been used. The specifications at STC are:  $I_{sc}=2.55A$ ,  $V_{oc}=21.24V$ ,  $I_{mp}=2.25A$ ,  $V_{mp}=16.56V$ ,  $P_{mp}=37.08W$ . The mathematical model of SPV system has been developed using MATLAB/SIMULINK with equations (1) and (2). To get the requisite voltage and current for the grid connected system considered for this work (5A and 230V), number of cells in series and parallel have been properly increased in the model. SPV array consists of 12 panels in series to obtain 230V and 2 panels in parallel to obtain 5A. The simulated characteristic of 12x2 array is shown in Fig. 2.

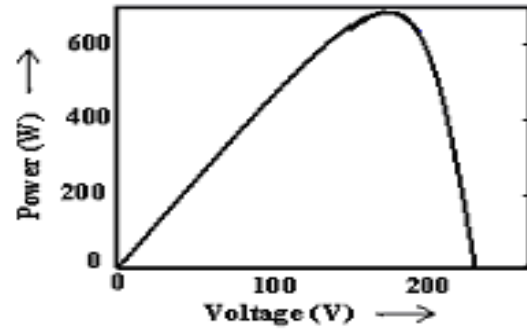
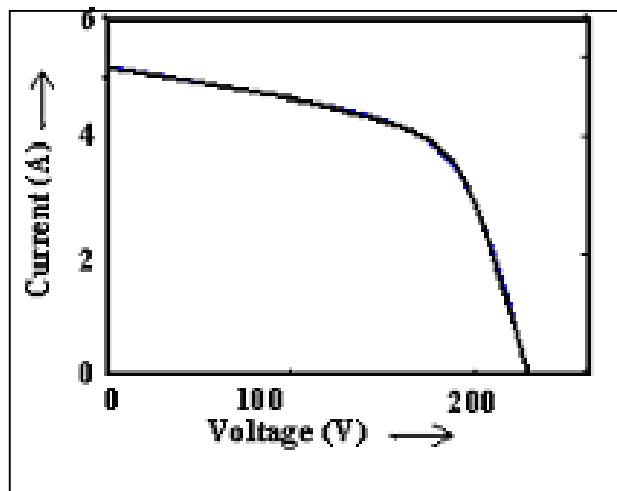


Fig. 2. V-I and V-P Characteristics of 12x2 SPV Array

**I. IMPLEMENTATION OF MAXIMUM POWER POINT TRACKING ALGORITHM**

In this work, Perturb and Observe (P&O) algorithm [4] is used for MPP tracking. It has a simple feedback structure and fewer measured parameters. It operates by periodically perturbing (i.e. incrementing or decreasing) the array terminal voltage and comparing the SPV output power with that of the previous perturbation cycle. In rapidly changing atmospheric conditions, the MPPT takes considerable time to track the MPP. The illumination for 12 hours during a sunny day is obtained (Fig 3). The program runs for the recorded illumination for 12 hours in a sunny day. The simulation of algorithm is also given in Fig.3. From Fig.3, it is inferred that the P and O algorithm tracks MPP for various illumination levels.

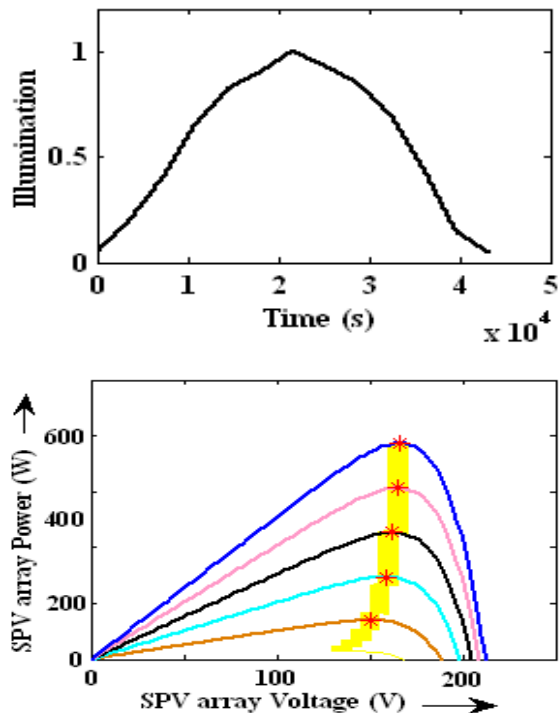


Fig.3 Simulation of illumination for 12 hours and tracking using Perturb and Observe Algorithm

Maximum Power Point Tracker (MPPT) can be realized using different converter topologies. In this paper boost converter is used due to its high efficiency. The boost converter interface is used between SPV array and rest of the system. It acts as MPP tracker. The design equations for boost converter are given from (8) to (10).

$$\frac{V_o}{V_s} = \frac{1}{1-D}$$

Where  $V_o$  = Output voltage,  $V_s$  = Source voltage,  $D$  = Duty cycle. The critical values of the inductance and capacitance can be calculated using the following equations,

$$L_b = \frac{(1-D)^2 DR}{2f}$$

$$C_{min} = \frac{D}{2fR}$$

$f$  = frequency 10 kHz. The inductance and capacitance are calculated as  $L=100\mu H$ ,  $C=220\mu F$ . The value of resistance used is  $100 \Omega$ .

## II. MATHEMATICAL MODEL OF THREE PHASE INVERTER

A Three Phase inverter is used to convert the DC output from the Solar Module to a constant AC three phase 50Hz output. Analytical model is an important tool for prediction of dynamic performance and stability limits using differential control laws and system parameters. Mathematical model of the inverter must be established before the design stage. The typical configuration of a three-phase full-bridge inverter is considered [6]. Semiconductor switching devices ( $Q_1$ – $Q_6$ ) of the inverter are controlled by PWM signals to obtain three-phase near sinusoidal ac voltages of the desired magnitude and frequency at the inverter output [5].

The switching functions can be mathematically represented as follows

$$S_a(\omega t) = \sum_{k=1,3,\dots}^{\infty} A_k \text{Sink}(\omega t + 0);$$

$$S_b(\omega t) = \sum_{k=1,3,\dots}^{\infty} A_k \text{Sink}(\omega t - \frac{2\pi}{3});$$

$$S_c(\omega t) = \sum_{k=1,3,\dots}^{\infty} A_k \text{Sink}(\omega t - \frac{4\pi}{3}) \quad (6)$$

$$\begin{bmatrix} T_{AV} \\ T_{BV} \\ T_{CV} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} \quad (7)$$

The matrix expansion gives the  $T_{AV}$ ,  $T_{BV}$  and  $T_{CV}$  values, which can be used to obtain the Line and Phase voltages as shown.

$$\begin{aligned} T_{AV} &= \frac{1}{2}[S_a - S_b]; T_{BV} = \frac{1}{2}[S_b - S_c]; \\ T_{CV} &= \frac{1}{2}[S_c - S_a] \end{aligned} \quad (8)$$

### Line Voltage Equations

$$\begin{aligned} V_{AB} &= T_{AV} \times T_{DC}; V_{BC} = T_{BV} \times T_{DC}; \\ V_{CA} &= T_{CV} \times T_{DC} \end{aligned} \quad (9)$$

### Phase Voltage Equations

$$\begin{aligned} V_{AN} &= [\frac{1}{3}(T_{AV} \times T_{CV})] \times V_{DC}; \\ V_{BN} &= [\frac{1}{3}(T_{BV} \times T_{AV})] \times V_{DC}; \\ V_{CN} &= [\frac{1}{3}(T_{CV} \times T_{BV})] \times V_{DC}; \end{aligned} \quad (10)$$

## V. VARIATION OF INVERTER PARAMETER WITH SOLAR INSOLATION CHANGES

The Solar radiation levels are modified to study the effect on inverter output. The Real and Reactive power output of the inverter for various solar insolation levels are shown in Fig 4. The maximum insolation for tropical countries like India is around  $1000 \text{ W/m}^2$

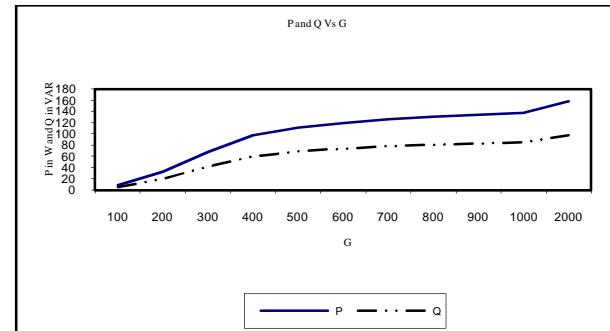


Fig. 4. Active and Reactive Power variation

The transfer ratio of the SPV module is defined as the ration of the AC Power output of the Inverter to the DC Power input. The Transfer ratio for various levels of insolation is depicted in Fig 5. It is observed that, at low radiation levels below  $300 \text{ W/m}^2$  the inverter operates nonlinearly.

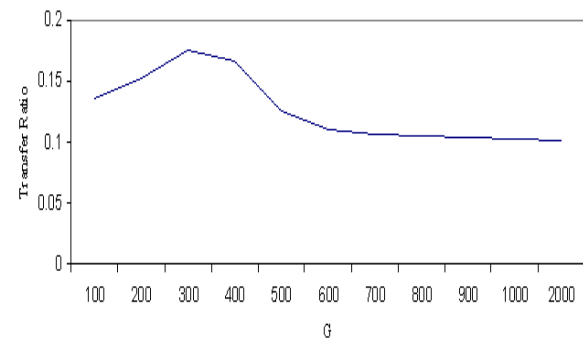


Fig. 5. Transfer ratio variation

The modulation index 'm' and phase angle 'Ø' play a major role in the generation of the PWM pulses for the inverter and hence its voltage. Hence the effect of m and Ø upon Real and Reactive power of the inverter is analysed. The results are given in Fig 6.

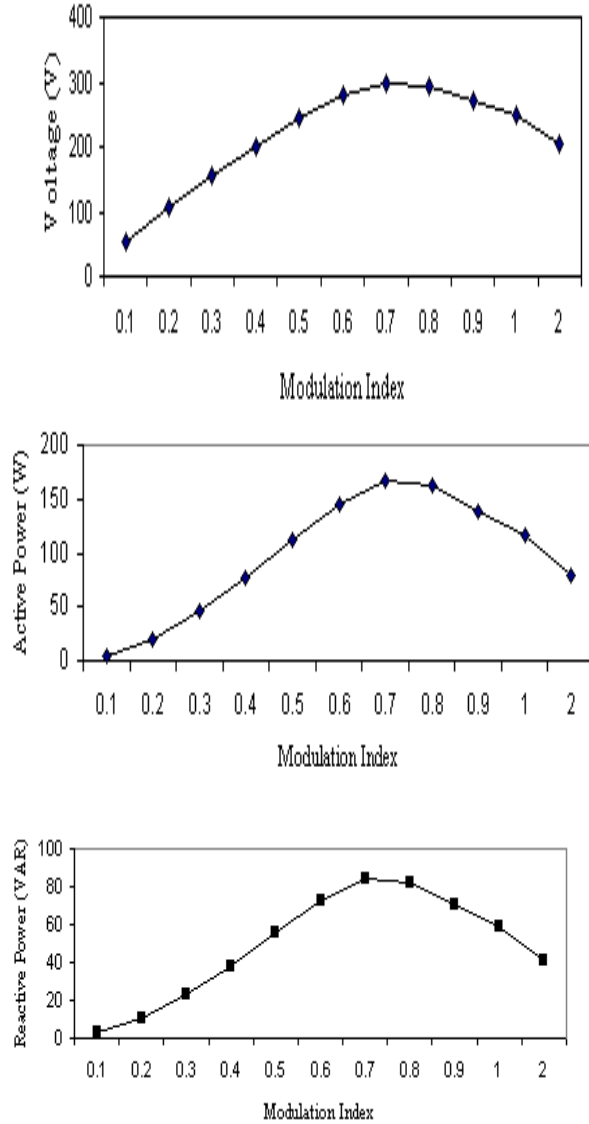


Fig. 6. Voltage, Active Power and Reactive Power for various m values

From the above result the following points can be summarised.

- When the Ø value varies there is no significant change in Active and Reactive power. The above measurement was noted for Ø =0 and Ø =180.
- The Active and Reactive power are at their maximum when m=0.7

Hence for further investigation the m value is taken as 0.7 and real, reactive power and transfer ratio is shown in Fig 7

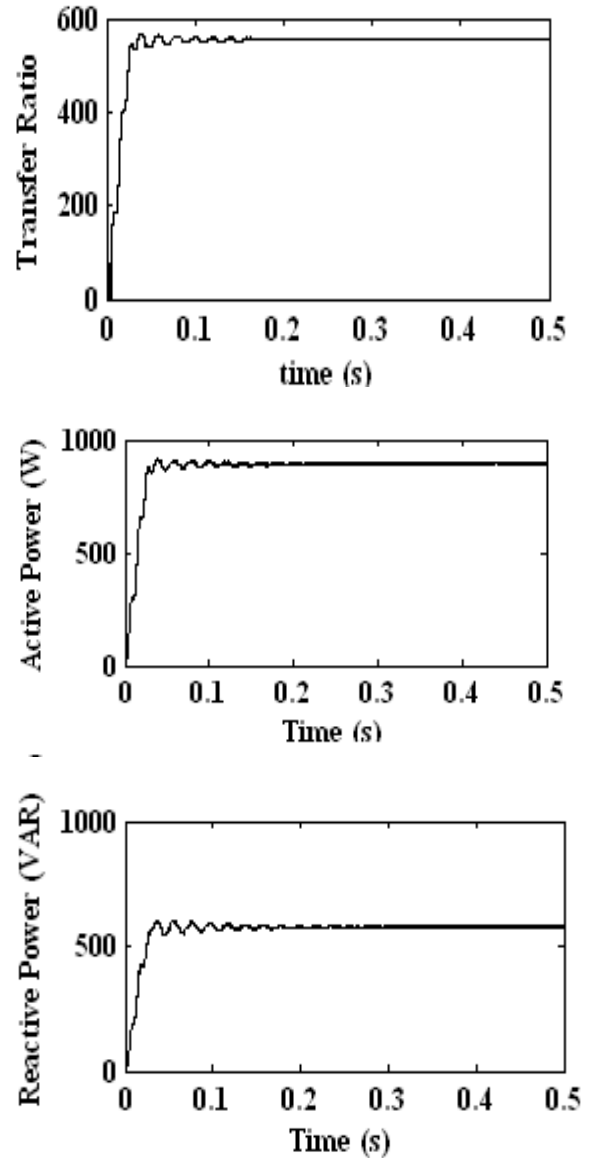


Fig. 7. Transfer Ratio, Active Power and Reactive Power for m=0.7

### III.SPV AND GRID INTEGRATED SYSTEM

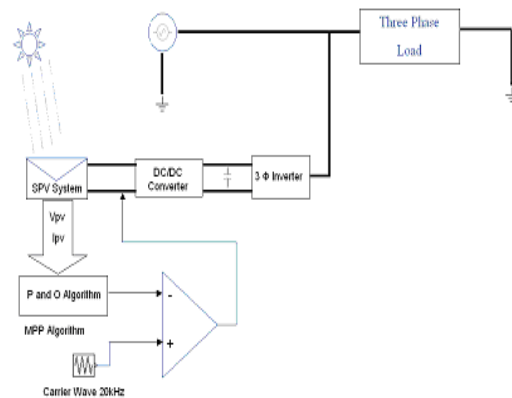
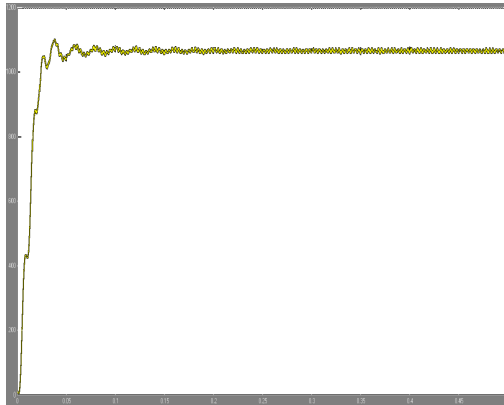
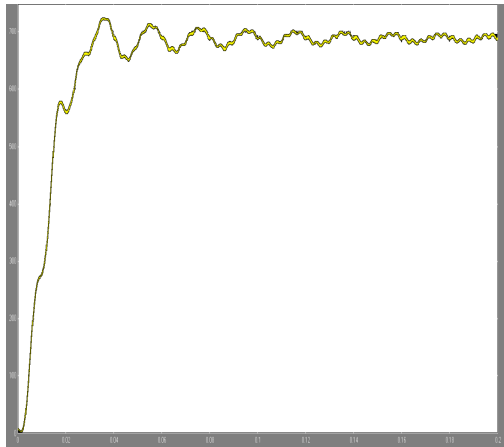


Fig. 8. SPV Grid Connected System

[Fig 8 shows the SPV system connected to the grid. The real and reactive power at the inverter output terminal and at the grid terminal are given in Fig 9.



Time in Sec



Time in Sec

#### IV. CONCLUSION

The mathematical model developed for the Grid connected SPV system has provided a constant Voltage and Power output after 10 cycles. This model can be used as the fundamental building block for further study on Grid connected SPV system

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