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Performance Analysis of ESPRIT Algorithm for Smart Antenna System

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Abstract- ESPRIT is a high-resolution signal parameter estimation technique based on the translational invariance structure of a sensor array. The ESPRIT algorithm is an attractive solution to many parameter estimation problems due to its low computational cost. The performance of DOA using estimation signal parameter via a rotational invariant technique is investigated in this paper. By exploiting invariance's designed into the sensor array, parameter estimates are obtained directly, without knowledge of the array response and without computation or search of some spectral measure. The exact number of samples and elements used is the most important parameter in the algorithms in order to sustain the accuracy of the direction of arrival of the incident signals. This algorithm is more robust with respect to array imperfections than MUSIC

Keywords: ESPRIT, DOA, MUSIC, invariance's.

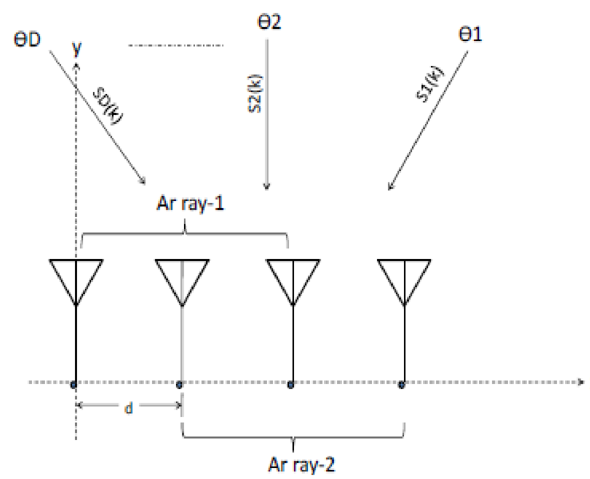
I .Introduction

Most of the algorithms discussed up to now depend on the precise knowledge of the array steering matrix **A**(θ). For every θ*i*, the corresponding array response, **a**(θ*i*), must be known. This is obtained by either direct calibration in the field, or by analytical means using information about the position and the response of each individual element of the array; this is normally an expensive and time-consuming task. Furthermore, errors in the calibration may seriously degrade the estimation accuracy.[1] Also, the spectral-based DOA algorithms involve an exhaustive search through all possible angles or steering vectors to find the locations of the power spectral peaks and to estimate the DOA, which is computationally intensive. ESPRIT (Estimation of Signal Parameters via Rotational Invariance

Techniques) overcomes these problems with a dramatic reduction in computational and storage requirements by exploiting a property called the shift invariance of the array [1] Unlike most DOA estimation methods such as MUSIC, ESPRIT does not require that the array manifold steering vectors be precisely known, so the array calibration requirements are not stringent. The ESPRIT algorithm is applicable to a particular class of sensor arrays whose geometry is shift invariant. Computation complexity and storage requirements are lower than MUSIC as it does not involve extensive search throughout all possible steering vectors [2][5].

II. Mathematical Model for ESPRIT Algorithm

ESPRIT achieves a reduction in computational complexity by imposing a constraint on the structure of an array. The ESPRIT algorithm assumes that an antenna array is composed of two identical subarrays (see Figure). The subarrays may overlap, that is, an array element may be a member of both subarrays.



If there are a total of M elements in an array and m elements in each subarray, the overlap implies that $M \leq 2m$. For subarrays that do not overlap, $M = 2m$. The individual elements of each subarray can have arbitrary polarization, directional gain, and phase response, provided that each has an identical twin in its companion subarray[5][6]. Elements of each pair of identical sensors, or doublet, are assumed to be separated physically by a fixed displacement (translational) vector. The array thus possesses a displacement (translational) invariance (i.e., array elements occur in matched pairs with identical displacement vectors). This property leads to the rotational invariance of signal subspaces spanned by the data vectors associated with the spatially displaced subarrays; the invariance is then utilized by ESPRIT to find DOAs.

Assume *d* signals impinging onto the array. Let $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ represent the signal received by the two subarrays corrupted by additive noise $\mathbf{n}_1(t)$ and $\mathbf{n}_2(t)$. Each of the subarrays has *m* elements., the signals received can then be expressed as

$$X_1(t) = [a(\mu_1),..,a(\mu_d)]s(t)+n_1(t)$$

$$= As(t) + n_1(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$X_2(t) = [a(\mu_1)e^{j\mu}{}_1 ,..,a(\mu_d)\, e^{j\mu}{}_d\,]s(t)+n_2(t)$$

$$A\Phi s(t) + n_2(t)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ are the $m * 1$ vectors representing the data received by the first and second subarrays, respectively. $\mathbf{n}1(t)$ and $\mathbf{n}2(t)$ are the $m * 1$ vectors representing the noises received by the two subarrays, respectively. $\mathbf{A} = [\mathbf{a}(\mu_1), \ldots, \mathbf{a}(\mu_d)]$ is the $m * d$ steering matrix of the subarray. $\mathbf{s}(t)$ is the signals received by the first subarray. [3]

Φ = diag[$e^{j\mu}{}_1$,.., $e^{j\mu}{}_d$] is a $d *d$ diagonal matrix that relates the signals received by the two subarrays and is called the rotation operator. It is caused by the fact that the signals arriving at the second subarray will experience an extra delay. due to the fixed displacement Δ between the two subarrays. Equations (1) and (2) can be combined to form the total array output vector as

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} A \\ A\Phi \end{bmatrix} s(t) + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} = \tilde{A}s(t) + \mathbf{n}(t)$$

Given *N* snapshots, $\mathbf{x}(t_1)$, $\mathbf{x}(t_2)$, …, $\mathbf{x}(t_n)$, the objective of the ESPRIT technique is to estimate the DOAs via an estimation of μ_i by Determining Φ = diag[$e^{j\mu}{}_1$,..., $e^{j\mu}{}_d$] In doing so, two steps are required based on the data received by the array: estimating the signal subspace and then estimating the subspace rotation operator. we take the uniform linear array (ULAs) as an example to illustrate the standard ESPRIT procedure. Consider a ULA consisting of *M* elements.[7] [8]

The two subarrays , array-1 and array-2 are displaced by distance 'd'. The signals induced on each of the arrays are given by

$$x_1(k) = A_1{*}s(k) + n_1(k)$$

and

$$x_2(k) = A_1{*}\Lambda{*}s(k) + n_2(k)$$

where Λ = diag $\{e^{jkd\sin(\theta 1)}\ e^{jkd\sin(\theta 2)}$ ----- $e^{jkd\sin(\theta D)}\}$
= D x D diagonal unitary matrix with phase shifts between doublets for each DOA.

Creating the signal subspace for the two subarrays results in two matrices V1 & V2. Since the arrays are translationally related, the subspaces of eigenvectors are related by a unique nonsingular transformation matrix ϕ such that[4]

$$V1\phi = V2$$

There must also exist a unique nonsingular transformation matrix T such that

$$V1 = AT \text{ and } V2 = A\Lambda T$$

and finally we can derive

$$T\,\Phi T\text{-}1 = \Lambda$$

Thus the eigenvalues of ϕ must be equal to the diagonal elements of Λ such that

$\lambda_1 = e^{jkd\sin(\theta 1)}$, $\lambda_2 = e^{jkd\sin(\theta 2)}$ ----- $\lambda_D = e^{jkd\sin(\theta D)}$

Once the eigenvalues of ϕ, λ_1 ,λ_2, ------ λ_D are calculated, we can estimate the angles of arrivals as

$$\theta_i = \sin^{-1}(\arg(\lambda_i)/kd)$$

ESPRIT eliminates the search procedure & produces the DOA estimation directly in terms of the eigenvalues without much computational and storage requirements. The ESPRIT method estimates signal DOA by finding the roots of two independent equations closest to the unit circle. This method does not require using a scan vector to scan over all possible directions like the MUSIC (Multiple Signal Classification) algorithm.[9]

III. SIMULATION RESULT

Simulation of ESPRIT algorithm for linear uniform array with four elements is carried out by varying different parameters of linear array.

Table no.1 DOA ESTIMATION USING ESPRIT FOR VARYING ANGULAR SEPARATION (M=8, SNR=10 dB, K=100)

Input (?) in deg	ESPRIT (?) in deg
20	20.1081
40	39.9680
60	60.0216
100	100.000

Table no.2 DOA ESTIMATION USING ESPRIT FOR VARYING ANGULAR SEPARATION (M=16 , SNR=10 dB, K=100)

Input (?) in deg	ESPRIT (ϴ) in deg
20	20.0335
40	40.0219
60	60.0026
100	100.0056

After observing table no .1 & 2 illustrates that the percentage error in DOA detection using ESPRIT algorithm decreases as number of array elements increased .

K=10	
θ Input (deg)	ESPRIT (ϴ) in deg
60	59.9676
80	80.0134
100	99.9970
K=1000	
60	60.0008
80	79.9975
100	100.0001

Table 3 . DOA ESTIMATION USING ESPRIT FOR VARYING NUMBER OF SNAPSHOTS K. (M=10, SNR=20 DB)

No. of signals	**θ Input (deg)**	**θ ESPRIT (deg)**
4	10	9.7943
	30	30.0125
	50	49.9803
	80	79.9893
6	20	20.2636
	30	32.1985
	40	46.2640
	60	63.2484
	70	75.8272
	90	106.9828

Table 3 illustrates that when number of samples are increased the error becomes small that is difference between actual & estimated angle goes on reducing.

Table no 4

Table no 4 illustrates how the ESPRIT algorithm can successfully detects 4 incident signals on array of 8 elements and how it completely fails if the number of incident signals increased to 6.Percentage error increases with same no of antenna elements & increased incident signals.

IV. CONCLUSION

This paper shows the performance of ESPRIT Algorithm. The algorithm has successfully estimated the Direction of Arrival of the incidents signals impinging on the antenna array. ESPRIT exploits such a displacement property which translates into an underlying rotational invariance of signal subspaces spanned by two data vectors received by two subarrays. The simulation results show that performance of ESPRIT improves with more elements in the array, with higher number of snapshots of signals and greater angular separation between the signals.

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