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Analysis of the Effect of Number of Knots in a Trajectory on Motion Characteristics of a 3R Planar Manipulator

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Abstract - The paper presents a method of trajectory planning and motion characteristics of a robotic manipulator. Main objective is to study the motion characteristics of a manipulator and to explore the scope of minimization of jerk. 8^{th} order polynomial is considered for the trajectory design and the effect of number of intermediate knots between start and final positions of a 3R manipulator within the workspace is studied. Displacements, velocities, accelerations and jerk of end-effectors on a linear path are presented. The simulation for motion of the manipulator is done with the help of AutoLISP on AutoCAD platform.

Key words - *trajectory; spline; knots; jerk; manipulator.*

I. INTRODUCTION

Robotic Manipulators are widely used in almost every leading manufacturing industry - welding shops, assembly sections, machining and many more. In each of these fields one common objective is to move the manipulator according to requirement along a specified trajectory. The problem of trajectory planning is an active area of research in the field of robotics. N. A. Aspragathos [1] worked in the area of generation of Cartesian trajectory under bounded position deviation. Gasparetto and Zanotto [2] developed a new method for smooth trajectory planning. Saramago and Ceccarelli [3] proposed the optimization of trajectory planning taking into account robot actuating energy and grasping forces in manipulator gripper. There are many other similar works in the area of robotic trajectory planning. However, there is a little number of papers published on the effects of a particular trajectory-curve on acceleration and more particularly on jerk of manipulators in motion.

This work is carried out to find the effect of number of knots in polynomial spline used as robotic trajectory on the motion characteristics of manipulator. Many works are there on effects of knots in polynomial spline on kinematics parameters like acceleration and jerk of cam followers [4]. Introduction of knots in polynomial spline and B-spline can play very important role in minimizing acceleration, jerk and ping (time derivative of jerk) of cam followers [5-6]. Taking lead from these works higher order polynomial spline with multiple knots is designed in this work for robotic trajectory.

Considering the jerk at initial and final positions as zero, 8^{th} order polynomial is used to join the knots in the joint space and polynomial spline is constructed. Effects of number of knots on displacement, velocity, acceleration and jerk for the linear path of end-effectors motion are presented. A case study is done with a 3R planar manipulator tracing the newly designed trajectory-curve. The simulation of the motion of the manipulator is done with the help of AutoLISP program on AutoCAD platform.

The problem is approached in the following way: (i) Start point and end point are assumed on a specified manipulator trajectory; (ii) the trajectory is defined in the coordinate space; (iii) some via-points (called here knots) are assumed on the trajectory; (iv) inverse kinematics analysis at these points is done and the points are plotted in the joint coordinate system for each link of the robotic arm; (v) these points are joined by 8^{th} order polynomials to construct the spline; (vi) at each point forward kinematics analysis is done; (vii) simulation of motion is done space using AutoLISP to see whether the manipulator follows the trajectory in the coordinate.

II. THEORETICAL ANALYSIS

Fig. 1 shows a 3R planar manipulator along with its work-space B and target trajectory xy, a straight line with two via-points p and q (called intermediate knots). B is a ring with inner and outer diameters. The number of intermediate knots can be as many as user's choice. Fig. 1(a) shows a spline with two end and intermediate knots. Inverse kinematics analysis at the points a, p, q, b is done and the points are plotted in the joint coordinate systems for each link - a and b being the end knots. In a general way if there is a point P(x, y) in the coordinate space the inverse kinematics analysis requires to find the joint angles $(\theta_1,\theta_2,\theta_3)$ as a function of wrist position and orientation (x,y,φ) as shown in Fig. 2.

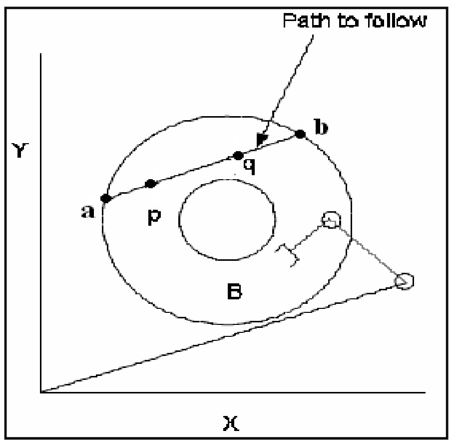

Fig.1 : 3R planar manipulator: work-space and trajectory.

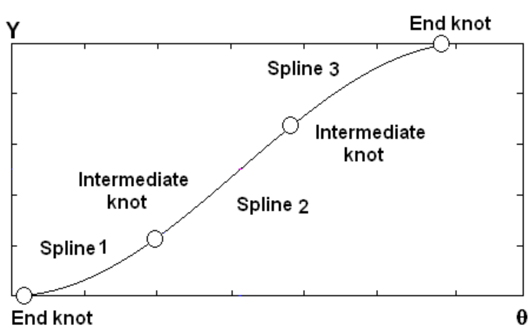

Fig. 1(a) : Spline with knots

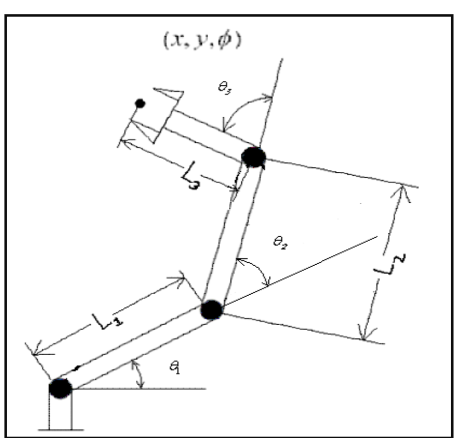

Fig. 2

Solving for θ_1 we rewrite the nonlinear using a change of variables as follows:

$$x=L_1c_1+L_2c_{12}$$
$$y=L_1s_1+L_2s_{12}$$

$$x = k_1c_1 + k_2s_1$$
$$y = k_1s_1 + k_2c_1$$

where $k_1 = L_1 + L_2c_2$ and $k_2=L_2s_2$. Finally we compute θ_2 using the two argument arctangent function

$$\theta_1=\text{atan2}(y,x)-\text{atan2}(k_2,k_1)$$

$$\theta_2=\text{atan2}(s_2,c_2)=\text{atan2}(\pm\sqrt{1-c_2^2},\frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2})$$

$$\theta_3=\varphi-\theta_1-\theta_2$$

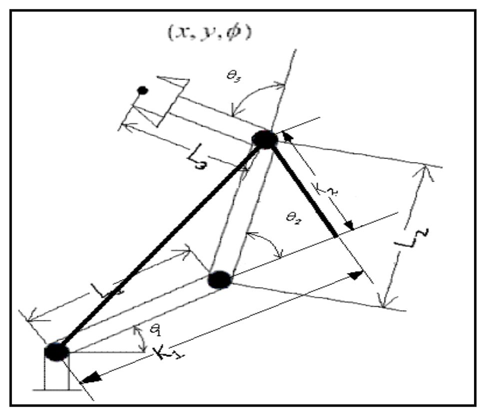

Fig. 3

These points are plotted in the joint co-ordinate system against time. θ_1 and θ_2 for a, p, q, b points are determined.

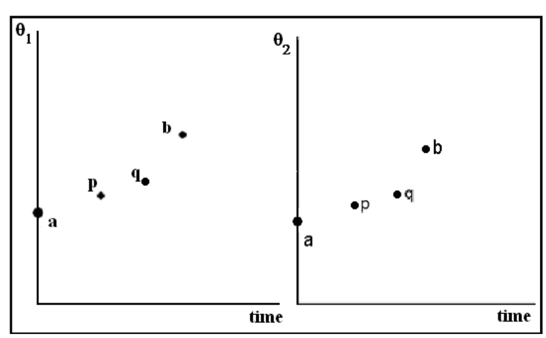

Fig.4 Fig.5

Velocity, acceleration and jerk are considered to be zero at end knots a and b. So we see velocity, acceleration, jerk and position at end knots are known - a total of 8 boundary conditions. Hence these points are joined by 8^{th} order polynomials for trajectory design.

We plan to join the knots (a, p, q, b) by three 8^{th} order polynomials (for k number of knots we use k-1 polynomials). In general if there are k number of knots and m order polynomial then number of unknown quantities: m(k-1), smoothness equations: (k-2)(m-1), interpolation equations: k-2 and boundary conditions: m [4]. Here 8^{th} order polynomials are connected by 4 knots. So the above numbers will be 24, 14, 2 and 8 respectively. We consider here three polynomials to construct the polynomial spline. Eqs. (1-3) are the polynomials, Eqs. (4-17) the smoothness equations, Eqs. (18-19) the interpolation equations and Eqs. (20-27) the boundary condition equations.

Polynomials:

$$Y_1 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 \quad \text{for } 0<t<t_p \tag{1}$$

$$Y_2 = b_0 + b_1(t-t_p) + b_2(t-t_p)^2 + b_3(t-t_p)^3 + b_4(t-t_p)^4 + b_5(t-t_p)^5 + b_6(t-t_p)^6 + b_7(t-t_p)^7 \quad \text{for } t_p<t<t_q \tag{2}$$

$$Y_3 = c_0 + c_1(t-t_q) + c_2(t-t_q)^2 + c_3(t-t_q)^3 + c_4(t-t_q)^4 + c_5(t-t_q)^5 + c_6(t-t_q)^6 + c7(t-t_q)^7 \quad \text{for } t_q<t<t_b \tag{3}$$

Smoothness equations:

$$Y_1 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 \tag{4}$$

$$\dot{Y}_1 = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 + 6a_6 t^5 + 7a_7 t^6 \tag{5}$$

$$\ddot{Y}_1 = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + 30a_6 t^4 + 42a_7 t^5 \tag{6}$$

$$\dddot{Y}_1 = 6a_3 + 24a_4 t + 60a_5 t^2 + 120\, a_6 t^3 + 210a_7 t^4 \tag{7}$$

$$Y_1^{(4)} = 24a_4 + 120a_5 t + 360a_6 t^2 + 840a_7 t^3 \tag{8}$$

$$Y_1^{(5)} = 120a_5 + 720a_6 t + 2520a_7 t^2 \tag{9}$$

$$Y_1^{(6)} = 720a_6 + 5040a_7 t \tag{10}$$

$$Y_2 = b_0 + b_1(t-t_p) + b_2(t-t_p)^2 + b_3(t-t_p)^3 + b_4(t-t_p)^4 + b_5(t-t_p)^5 + b_6(t-t_p)^6 + b_7(t-t_p)^7 \tag{11}$$

$$\dot{Y}_2 = b_1 + 2b_2(t-t_p) + 3b_3(t-t_p)^2 + 4b_4(t-t_p)^3 + 5b_5(t-t_p)^4 + 6b_6(t-t_p)^5 + 7b_7(t-t_p)^6 \tag{12}$$

$$\ddot{Y}_2 = 2b_2 + 6b_3(t-t_p) + 12b_4(t-t_p)^2 + 20b_5(t-t_p)^3 + 30b_6(t-t_p)^4 + 42b_7(t-t_p)^5 \tag{13}$$

$$\dddot{Y}_2 = 6b_3 + 24b_4(t-t_p) + 60b_5(t-t_p)^2 + 120\, b_6(t-t_p)^3 + 210b_7(t-t_p)^4 \tag{14}$$

$$Y_2^{(4)} = 24b_4 + 120b_5(t-t_p) + 360b_6(t-t_p)^2 + 840b_7(t-t_p)^3 \tag{15}$$

$$Y_2^{(5)} = 120b_5 + 720b_6(t-t_p) + 2520b_7(t-t_p)^2 \tag{16}$$

$$Y_2^{(6)} = 720b_6 + 5040b_7(t-t_p) \tag{17}$$

Interpolation equations:

$$b_0 = \theta_1 \quad \text{at } t = t_p \tag{18}$$

$$c_0 = \theta_1 \quad \text{at } t = t_q \tag{19}$$

Boundary conditions:

$$a_0 = Y_1 \quad (\text{At } t=0) \tag{20}$$

$$a_1 = 0 \quad (\text{Since at } t=0,\ \dot{Y}_1 = 0) \tag{21}$$

$$a_2 = 0 \quad (\text{since at } t=0,\ \ddot{Y}_1 = 0) \tag{22}$$

$$a_3 = 0 \quad (\text{since at } t=0,\ \dot{Y}_1 = 0) \tag{23}$$

$$c_0 + c_1(t_b-t_q) + c_2(t_b-t_q)^2 + c_3(t_b-t_q)^3 + c_4(t_b-t_q)^4 + c_5(t_b-t_q)^5 + c_6(t_b-t_q)^6 + c7(t_b-t_q)^7 = Y \quad (\text{at } t=t_b) \tag{24}$$

$$c_1 + 2c_2(t_b-t_q) + 3c_3(t_b-t_q)^2 + 4c_4(t_b-t_q)^3 + 5c_5(t_b-t_q)^4 + 6c_6(t_b-t_q)^5 + 7c_7(t_b-t_q)^6 = 0 \tag{25}$$

(since at $t=t_b$ $\dot{Y}_3 = 0$)

$$2c_2 + 6c_3(t_b-t_q) + 12c_4(t_b-t_q)^2 + 20c_5(t_b-t_q)^3 + 30c_6(t_b-t_q)^4 + 42c_7(t_b-t_q)^5 = 0 \tag{26}$$

(since at $t=t_b$ $\ddot{Y}_3 = 0$)

$$6c_3+24c_4\left(t_b-t_q\right)+60c_5\left(t_b-t_q\right)^2+120\ c_6\left(t_b-t_q\right)^3+210c_7\left(t_b-t_q\right)^4=0 \quad (27)$$

(since at $t=t_b$ $\dddot{Y}_3=0$)

We put $t=t_p$ and $t=t_q$ in Eqs. (4-19) the resulting equations along with Eqs. (20-27) generates a set of 24 simultaneous equations, which can be written in matrix form:

$$M_{24X24}\ x\ N_{24X1}=U_{24X1} \quad (28)$$

where N_{24X1}

$$=\left[a_0\ \ .\ \ .\ \ a_7\ \ b_0\ \ .\ \ .\ \ b_7\ \ c_0\ \ .\ \ .\ \ c_7\right]^T.$$

Since M_{24X24} & U_{24X1} are known we can find N_{24X1}, i.e., the polynomial coefficients a, b, c etc.

We then plot θ_1-t and θ_2-t. From these plots the position of the end-effector can be located at any value of t. For, if we know the joint angles $\theta_1,\theta_2,\theta_3$ (θ_3 is user assigned), using Denavit-Hartenberg algorithm we can find the position of the end-effector in the coordinate space. The necessary equations are:

$$x=L_1\cos\theta_1+L_2\cos\theta_2+L_3\cos\theta_3 \quad (29)$$

$$y=L_1\sin\theta_1+L_2\sin\theta_2+L_3\sin\theta_3 \quad (30)$$

From Eqs. (29-30) the velocity, acceleration and jerk of the end-effector can be calculated using Eq. (31), Eq. (32), Eq. (33) respectively

$$V_R=\sqrt{(\dot{x})^2+(\dot{y})^2} \quad (31)$$

$$a_R=\sqrt{(\ddot{x})^2+(\ddot{y})^2} \quad (32)$$

$$j_R=\sqrt{(\dddot{x})^2+(\dddot{y})^2} \quad (33)$$

III. SIMULATION

Using AutoLISP code generated for the purpose we have simulated the motion of the manipulator arm tracing the newly designed trajectory in the coordinate space. Concepts of loop and file handling of AutoLISP were utilized for this. Drawing an entity and at the same time erasing the previous entity on the graphics screen are done rapidly at a speed selected by the user. This gives an effect of animation in the robotic manipulator on the graphics screen. AutoCAD Figs. (6-7) show simulation of manipulator motion in two positions. In course of simulation number of knots in the trajectory curve is varied and effect of these variations on velocity, acceleration and jerk is computed using MATLAB (R2009b). The results are shown in Figs. (8-10). Figs. (11-13) show the variations of maximum values of velocity, acceleration and jerk with the variation of number of knots. Three cases are shown – for number of knots of 4, 16 and 64.

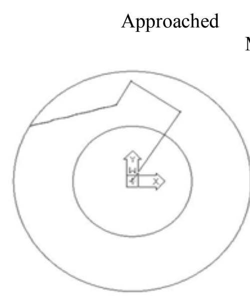

Fig. 6

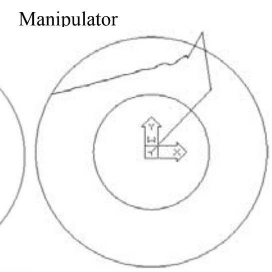

Fig. 7

IV. RESULTS

The curves obtained during simulation are shown below:

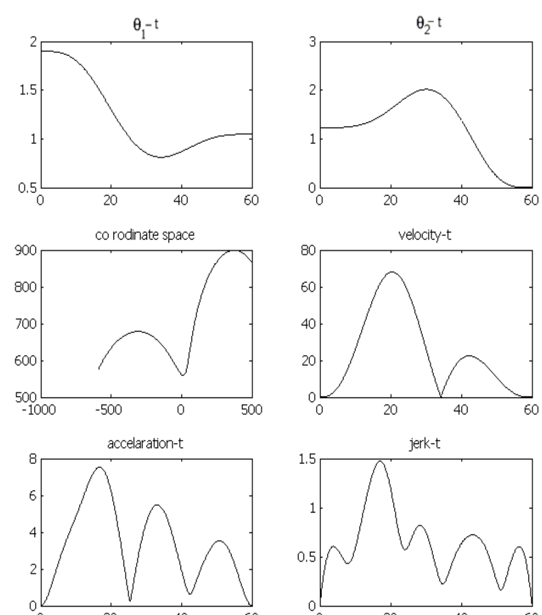

Fig. 8 : Number of knots 4

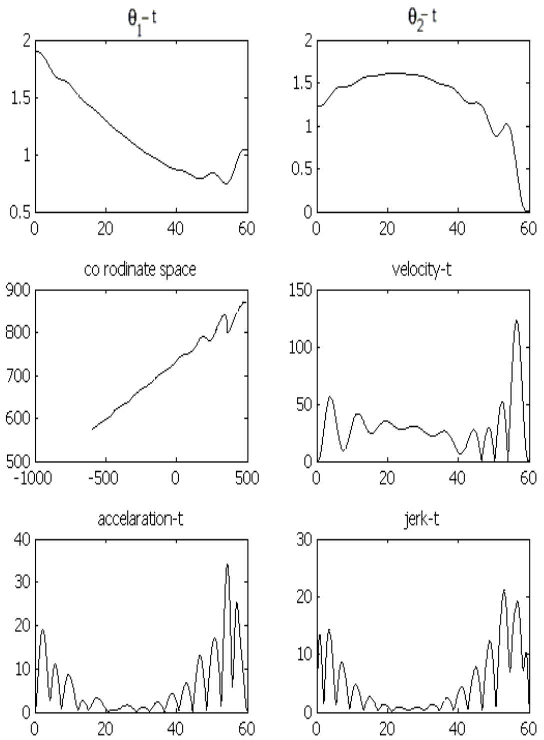

Fig. 9 : Number of knots 16

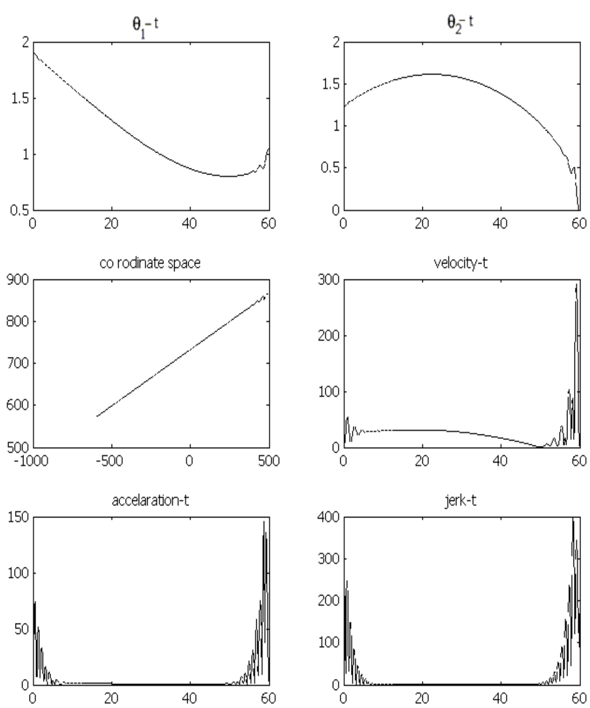

Fig. 10 : Number of knots 64

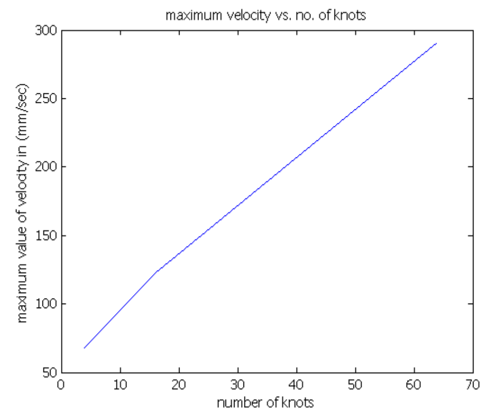

Fig. 11 : Maximum velocity vs. number of knots

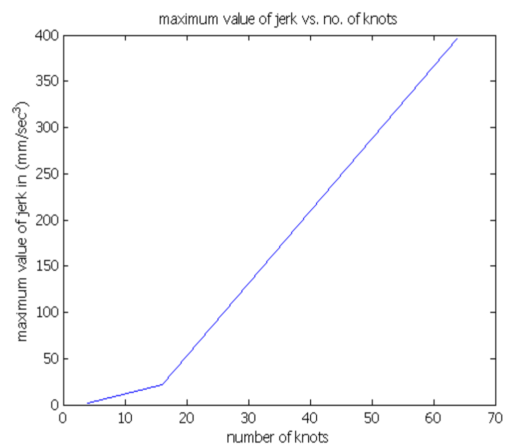

Fig. 13 : Maximum jerk vs. number of knots

From the above analysis the following observations are made:

As we increase the number of knots (i) the trajectory approaches the target curve (ii) the velocity, acceleration and jerk have a very low and smooth value in the middle region, (iii) at the ends of completion of motion high fluctuations with increased number of knots and (iv) maximum value of velocity, acceleration and jerk increase with the rise of number of knots.

V. CONCLUSION

The objective of the work was to study the effect of the number knots on motion characteristics while designing trajectory for a manipulator. In this regard the effect of number of intermediate knots between two positions within the workspace has been studied and an

8th order polynomial is considered for the trajectory design.

From the above analysis it is evident that it has two major advantages. As we increase the number of knots the trajectory approaches the target curve. It is evident from comparing the curves of coordinate space in Figs. (8-10) with that of Fig. 1. It shows how the trajectory approaches the target curve with increasing accuracy. This may be very suitable for industrial machine tool applications like welding where a contour is to be traced by the manipulator with high accuracy. Another advantage we see from Figs. (8-10) is that in spite of the end fluctuations the acceleration and jerk have very low values in the middle region. Regarding velocity the variation is very smooth in the intermediate regions.

However, it is seen that at the ends of motion there are high fluctuations of velocity, acceleration and jerk with the increase of number of knots. The work, therefore, has a scope of further extension by adopting some optimization technique to reduce those fluctuations.

NOMENCLATURES

i. $\theta_1, \theta_2, \theta_3$: Joint angles.

ii. $c_{12....i} = \cos(\theta_1 + \theta_2 + ... + \theta_i)$

iii. $s_{12....i} = \sin(\theta_1 + \theta_2 + ... + \theta_i)$

iv. L_1 = Length of first link of end-effector.

L_2 = Length of second link of end-effector.

L_3 = Length of third link of end-effector.

v. V_R = Resultant velocity of end-effector.

a_R = Resultant acceleration of end-effector.

j_R = Resultant jerk of end-effector.

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