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Reduced optimal controller design for the nuclear reactor power control system

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Abstract**—*****The power control system of a nuclear reactor is one of the key systems that concern the safe operation of the plant. Much attention is paid to the power control systems' performance of nuclear reactor in engineering. The goal of this paper is apply balance model reductionto derive reduced order model and then design the reduced optimal controller for nuclear reactor power system.The simulation results with reduced-order model and with optimized controller show that the proposed technique is improved .***

Keywords-Nuclear reactor; control; Model reduction; Optimization

I. INTRODUCTION

The power control system is a key control system for a nuclear reactor, which directly concerns the safe operation of a nuclear reactor. Much attention is paid to improve the power control system performance. Generally, the power control system should operate safely and reliably, should maintain maximum power output of nuclear reactor with less static error and should possess certain stability margin, suitable peak overshoot and transient time [5].Resent control literature shows that an important role is played by the balance realization order reduction technique in model and controller reduction procedure. Reduction of high-order system to low-order models has been an important subject area in the control engineering [9]. In this paper first the optimal control is employed and then reduced optimal controller based on balanced model areimployed to control the nuclear reactor power system.

Model reduction seeks to replace a large-scale system by a system of substantially lower dimension that has nearly the same response characteristics large structures yield large state-space dimensions. In this case, problems related to storage, accuracy and computational speed may arise. Thus, the design of low-order sytems for high-order plants is a challenging problem from a computational point of view. It is then interesting to achieve reduced-order models and controllers while maintaining robustnessproperties.

The paper is organized as follows. In section 2, planet mathematical model are presented.In section 3, balancing approximation technique is reviewed.In section 4 The proposed method is applied to the dynamic of nuclear reactor. Then conclusion is given in section 5.

II. PLANET MATHEMATICAL MODEL

The reactor power is modeled using the point kinetics equations with six groups of delayed neutrons and two thermal feedbacks due to changes in fuel temperature and coolant temperature. The core heat transfer model is composed of one fuel node and two coolant nodes. The point kinetics dynamic linearized equations are given as follows [11]:

$$\frac{dn_r}{dt} = \frac{\rho(t)-\beta}{\Lambda} n_r(t) + \frac{1}{\Lambda}\sum_{i=1}^{6} \beta_i c_{ri} \quad (1)$$

$$\frac{dc_{ri}}{dt} = \lambda_i n_r(t) - \lambda_i c_{ri}(t)\,, \quad i = 1, \ldots, 6 \quad (2)$$

$$\frac{dT_f}{dt} = \frac{f_f p_0}{\mu_f} n_r - \frac{\Omega}{\mu_f} T_f + \frac{\Omega}{2\mu_f} T_l + \frac{\Omega}{2\mu_f} T_e \quad (3)$$

$$\frac{dT_l}{dt} = \frac{(1-f_f)p_0}{\mu_c} n_r + \frac{\Omega}{\mu_c} T_f - \frac{2M+\Omega}{2\mu_c} T_l + \frac{2M-\Omega}{2\mu_c} T_e \quad (4)$$

$$\frac{d\delta\rho r}{dt} = G_r z_r \quad (5)$$

$$\rho = \delta_{\rho_r} + \alpha_f(T_f - T_{f0}) + \alpha_c(T_c - T_{c0}) \quad (6)$$

Linearization of equations (1) through (6) about nominal working point n_r result in the following state-space representation of the reactor model.

$$\dot{X} = AX + B \quad (7)$$
$$Y = CX + DU$$

where

$$x = [\delta_{n_r} \delta_{c_r}\ \delta T_f\ \ \delta T_l\ \ \delta_{\rho_r}]^T \quad (8)$$

$$y = [\delta_{n_r}], u = [z_r] \quad (9)$$

$$A = \begin{bmatrix} \frac{-\beta}{\Lambda} & \frac{\beta}{\Lambda} & \frac{nr_{\alpha_f}}{\Lambda} & \frac{nr_{\alpha_c}}{2\Lambda} & \frac{nr}{\Lambda} \\ \lambda & -\lambda & 0 & 0 & 0 \\ \frac{f_f p_0}{\mu_f} & 0 & \frac{-\Omega}{\mu_f} & \frac{\Omega}{2\mu_f} & 0 \\ \frac{(1-f_f)p_0}{\mu_c} & 0 & \frac{\Omega}{\mu_f} & \frac{-(2M+\Omega)}{2\mu_f} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

$$B = [0\ \ 0\ \ 0\ \ 0\ \ Gr]^T \quad (11)$$

$$C = [1\ \ 0\ \ 0\ \ 0\ \ 0] \quad (12)$$

$$D = [0] \quad (13)$$

Five parameters of the model parameters are depend on relative power level n_r as follows [11], [15].

$$\mu_c(n_r) = \left(\frac{160}{9} n_r + 54.022\right) (MW.s/^{\circ}{}_C) \quad (14)$$

$$\Omega(n_r) = \left(\frac{5}{3} n_r + 4.9333\right) (MW/^{\circ}{}_C) \quad (15)$$

$$M(n_r) = (28\, n_r + 74)(MW/^{\circ}{}_C) \quad (16)$$

$$\alpha_r(n_r) = (n_r - 4.24) * 10^{-5} \left(\frac{\delta K}{K}/^{\circ}{}_C\right) \quad (17)$$

$$\alpha_c(n_r) = (-4n_r - 17.3) * 10^{-5} \left(\frac{\delta K}{K}/^{\circ}{}_C\right) \quad (18)$$

TABLE I. Parameters of a TMI-type PWR reactor

n_r	$= {}^{n}/_{n_0}$nutron density relative to initial equilibrium density,
n	nutron density(n/cm^3),
n_0	Initial equilibrium (steady-state) neutron density,
c_{ri}	${}^{c_i}/_{c_{i0}}$rlativedensityofith group precursor,
c_i	Core averaged ith group precursor density(atom/cm^3),
c_{i0}	Initial equilibrium (steady-state) density of ith group precursor,
ρ	(k-1)/k, reactivity(Δk/k),
K	$= K_{eff}$ effective neutron multiplication factor,
Λ	Effective prompt neutron life time (s),
λ_i	radioactive decay constant of ith group neutron precursor (s^{-1}),
β	Total delayed neutron fraction,
β_i	Ith group delayed neutron fraction,

T_f	average reactor fuel temperature (°K),
T_l	temperature of the water leaving the reactor (°K),
T_e	temperature of the water entering the reactor (°K),
T_c	$(T_f+T_l)/2$, average reactor coolant temperature (°K),
f_f	Fraction of reactor power deposited in the fuel,
p_0	Initial equilibrium power (MW),
μ_f	Total heat capacityof the fuel=weight of fuel times its specific heat (MJ/°K),
μ_c	Total heat capacityof the reactor coolant=weight of coolant times its specific heat (MJ/ °K),
Ω	Heat transfer coefficient between fuel and coolant (MW/°K),
M	Mass flow rate multiplied byheatcapacityof the coolant (MW/°K),
$\delta_{\rho r}$	Reactivity due to the control rod movement,
Z_r	Control input, control rod speed in units of fraction of core length per second
G_r	Total Reactivity worth of control rod,
α_f	Fuel temperature reactivity coefficient (Δk/k/ °K),
α_c	Coolant temperature reactivity coefficient (Δk/k/°K),
T_{f0}	Initial equilibrium (steady-state) fuel temperature (°K),
T_{c0}	Initial equilibrium (steady-state) coolant average temperature (°K).

TABLE I. Value of the Parameters of a TMI-type PWR reactor at the middle of the fuel cycle in full power [15]

parameter	***value***	***parameter***	***value***
β	0.006019	μ_f	26.3(MJ/°$_K$
Δ	0.0001 (s)	α_f	3.24e-5 (k/k/°K)
n_r	1	α_c	21.3e-5 (k/k/°K)
λ	0.15 (s^{-1})	G_r	0.01450
f_f	0.92	M	102.0 (MW/ °K)
p_0	2500 (MW)	μ_c	71.8 (MJ/ °K)
T_e	564.3 (K)	Ω	6.6 (MW/ °K)

III. PROPOSED CONTROLLER

The performance index associated with the system is as follows.

$$j = \int_0^{\infty}[0.01\ \delta T_f^{\,2} + 0.1\ \delta T_l^{\,2} + 3000 z_r^{\,2}]dt \qquad (19)$$

As seen from the above performance index precise control of reactor temperatures are important for us. The value 3000 chosen for penalize the use of fast control rod speed.

with this cost function, when power decreases, deviation of fuel temperature from its equilibrium value is penalized less and instead, coolant exit temperature deviations and control rod speed are penalized more [11].

The optimal control is given by

$$u = -Fx \qquad (20)$$

with the feed back gain

$$F = R^{-1}B^TS \qquad (21)$$

where S is the solution of the algebraic riccati equation.

$$A^TS + SA + Q - SBR^{-1}B^TS = 0 \qquad (22)$$

The optimal cost for full order system is j^*_{op}= 0.1149 and for reduced system is j^*_r= 0.1162.

IV. BALANCING APPROXIMATION

Balancing approximation is a model reduction technique for systems leads to reduced order models.

During the eighties a robust order reduction technique for time invariant linear system, based on the balancing transformation was developed [9],[10]. The technique is briefly reviewed blow.

Given an nth-order, linear time invariant, and asymptotically stable system G(S) with a minimal realization

$$G(S) = C(SI - A)^{-1}B + D \qquad (23)$$

Assummption: The system is asymptotically stable, the pair(A,B)is controllable, and the pair (A,C) is observable.

The system controllability and observabilityGrammian satisfy the algebraic Lyapunov equations [10]

$$PA' + AP + BB' = 0 \qquad (24)$$

$$QA + A'Q + CC' = 0 \qquad (25)$$

For controllable and observable system both controllability and observability Grammians are positive definite, P>0,Q>0.

The balancing transformation is a state transformation that makes the controllability and observability Grammians identical and diagonal , that is

$$P = Q = \Sigma = diag\{\sigma_1, \sigma_1, \dots, \sigma_n\} \qquad (26)$$

Where $\sigma_i \geq \sigma_{i+1}; i = 1,2, \dots, n-1$are Hankel singular values Now partition the balanced system(A,B,C,D) and the Grammian $\sum$ conformably as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \ B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \ C = [C_1 \quad C_2], \qquad (27)$$

$$\Sigma = \begin{bmatrix} \sum 1 & 0 \\ 0 & \sum 2 \end{bmatrix}$$

Where A_{11} and Σ_1 are r×r (r < n) matrices

$$\sum = diag\{\sigma_1, \sigma_1, \dots, \sigma_r\}, \sum = diag\{\sigma_1, \sigma_1, \dots, \sigma_n\}, \quad (28)$$

Assuming that$\sigma_i \geq \sigma_{i+1}$ then the correspondingreduced-order system transfer function is

$$G_r(S) = C_1(SI - A_{11})^{-1}B_1 + D \qquad (29)$$

This reduced-order system is controllable and observable since the corresponding Hankle singular values are all positive. In addition,The reduced-order system is balanced and asymptotically stable .It was shown (k.Glover,1984)that $H_\infty - norm$ of the reduced-order system, obtainedthrough the above defined truncation , satisfies.

$$\|G(S) - G_r(S)\|_\infty \leq 2(\sigma_{r+1} + \sigma_{r+2} + \dots + \sigma_n) \qquad (30)$$

The step response of the reduced-order system and original system is shown in Fig 1.It will be seen that there is a very good match between the responses of the original system, and reduced-order system as expected.

Therefore it is proper to use reduced-order model instead of original system for designing controller.

V. CONCLUSION

the optimal controller for nuclear reactor power control system has been designed. Then the system order reduction for reactor steady state model via balanced realization has been obtained. The simulation result for operation of reactor is shown in figures2,3, and 4.In the simulation the system was operating at power level of 100%. Simulation result shows that the results is very close to the exact optimal solutions. And the reduced order controller design has good performance. It has been shown that reduced-order model match and provides a very good approximation to the step response of the original system .The process of reduced order and designing an optimal controllersuggests that this method is simple and practical for complex and nonlinear nuclear power control system. The overall resultwill be a considerable saving in computation.

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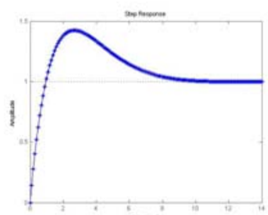

Figure 1. ***Step outputs of reduced order-model(*) and orjinal system (-) at power level 100%.***

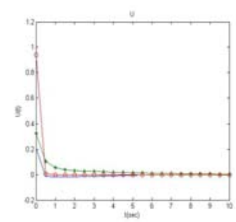

Figure 2. ***Optimal inputs for various value of weighting matrices Q of reduced order-model.***

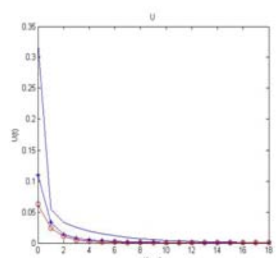

Figure 3. ***Optimal inputs for various value of weighting matrices R (R=1,'-' R=5, '-*' R=10, '-ro') of reduced order-model .***

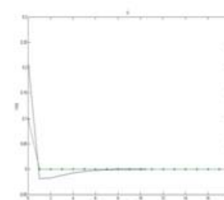

Figure 4. ***Optimal inputs of reduced order-model (*) and orginal system.***